

If the vector $(2, 6, 1, -3)$ is a linear combination of the other vectors, the following equation should be solvable for constants $c_1, c_2, c_3 \in \mathbb{R}$.

$$\begin{bmatrix} 2 \\ 6 \\ 1 \\ -3 \end{bmatrix} = c_1 \vec{u} + c_2 \vec{v} + c_3 \vec{w}$$

$$\begin{bmatrix} 2 \\ 6 \\ 1 \\ -3 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 3 \\ 3 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ -5 \\ 0 \end{bmatrix}$$

This is equivalent to the following set of equations.

$$1c_1 + 0c_2 + 0c_3 = 2$$

$$3c_1 + 0c_2 + 0c_3 = 6$$

$$3c_1 + 0c_2 + (-5)c_3 = 1$$

$$0c_1 + 1c_2 + 0c_3 = -3$$

We can now solve for c_2

$$c_2 = -3$$

Then for c_1

$$3c_1 = 6$$

$$c_1 = 2$$

Finally, for c_3

$$3c_1 - 5c_3 = 1$$

$$3(2) - 5c_3 = 1$$

$$-5c_3 = -5$$

$$c_3 = 1$$

Then we must verify that the values still work with the first equation.

$$1(2) + 0(1) + 0(-3) = 2$$

$$2 = 2$$

Therefore, $(2, 6, 1, -3)$ is a linear combination of $\vec{v}, \vec{u}, \vec{w}$.