Vector Algebra

$c \in \mathbb{R}$

commutivity → $\vec{u}+\vec{v}=\vec{v}+\vec{u}$

$1 \cdot \vec{u} = \vec{u}$

associativity → $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$

existance of $\vec{0}$ → $\vec{u}+\vec{0}=\vec{u}$

$(0, 0, 0, \dots)$

existance of -ve → $\vec{u}+(-\vec{u})=\vec{0}$

distributive → $c(\vec{u}+\vec{v})$ $= c\vec{u}+c\vec{v}$

$(-u_1, -u_2, -u_3 \dots)$

→ $(c+d)\vec{u}=c\vec{u}+d\vec{u}$

→ $c(d\vec{u})=(cd)\vec{u}$

→ $(u_1, u_2 \dots) + [(v_1, v_2 \dots)+(w_1, w_2 \dots)]$

$= (u_1, u_2 \dots) + (v_1+w_1, v_2+w_2 \dots)$

$= (u_1+v_1+w_1, u_2+v_2+w_2 \dots)$

$[(u_1, u_2 \dots) + (v_1, v_2 \dots)] + (w_1, w_2 \dots)$

$(u_1+v_1, u_2+v_2 \dots) + (w_1, w_2 \dots)$

$(u_1+v_1+w_1, u_2+v_2+w_2 \dots)$

Linear combination $\rightarrow \vec{u} \in \mathbb{R}^n, \vec{u}_1, \vec{u}_2, \vec{u}_3 \ldots \in \mathbb{R}^n$

↳ if there are scalars $c_1, c_2, \ldots \in \mathbb{R}$ where $\vec{u} = c_1 \cdot \vec{u}_1 + c_2 \vec{u}_2 \ldots c_n \vec{u}_n$ — $\vec{u}$ is linear combo of $\vec{u}_1, \vec{u}_2$

$\vec{u} = (2, 2, -1)$

$\vec{u}_1 = (1, 0, 0)$

$\vec{u}_2 = (0, 1, 0)$

$\vec{u}_3 = (0, 0, 1)$

$\vec{u} = 2 \cdot \vec{u}_1 + 2\vec{u}_2 + (-1 \cdot \vec{u}_3)$

$= (2,0,0) + (0,2,0) + (0,0,-1)$

→ connect to matrices for solving

$$\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = x\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} c_2 - c_3 = 2 \\ 2c_1 - 2c_2 - c_3 = -2 \\ c_2 - 2c_3 = -1 \end{cases}$$

$c_3 = 3$

$c_2 - 2(3) = -1$

$c_2 = 5$

$2c_1 - 2(5) - 3 = -2$

$c_1 = \frac{11}{2}$

$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 \ldots$

dot/scalar product

↳ $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

homogenous → $\vec{u} \cdot (c\vec{v}) = c(\vec{u} \cdot \vec{v})$

$$\|\vec{u}\| = \sqrt{\vec{u}\cdot\vec{u}} = \sqrt{u_1^2 + u_2^2 + \dots}$$

length