Describe function:

- domain → all allowable inputs
- the codomain → where the function lives
- Exact rule for taking an element of domain and producing an element in codomain

$$\underset{\text{name}}{F} : \underset{\text{domain}}{A} \to \underset{\text{codomain}}{B}$$

$$f : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto x^2$$
exact rule

for multiple formulas:

$g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = \begin{cases} 1 & \text{if } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$

<u>Image/range</u>: given $f : A \to B$, the range is subset of B, contains all outputs for all possible outputs of A

$$\mathrm{Im}(f) \overset{\text{def}}{=} \{ b \in B \mid f(a) = b \text{ for some } a \in A \}$$

$$\mathrm{Img}(g) = \{0, 1\}$$

$h : \mathbb{R} \to \mathbb{R}$

$h(s) = \begin{cases} \sin(s) & \text{if } s \in [0, 2\pi) \\ \cos(s) & \text{if } s \in [2\pi, 4 \end{cases}$

$$\mathrm{Img}(h) = [-1, 1]$$

Given no codomain or domain
↳ make it the largest subset of $\mathbb{R}$ such that the rule makes sense
↳ codomain is natural → image when applied to natural domain

	domain	Natural codomain
$\sqrt{x}$	$[0,\infty)$	$[0,\infty)$
$\frac{x}{x}$	$\mathbb{R}\setminus\{0\}$	$\{1\}$

all real numbers that aren't 0

two functions are equal if both domains are equal, and codomains are equal. for ever input both outputs are the same

$F: A \to B$ $\quad A = C$

$g: C \to D$ $\quad B = D$

$a \in A \quad F(a) = g(a)$

$F(x) = \frac{x}{x}$

$g(x) = 1$

↳ not the same because domains don't match if we set $g: \mathbb{R}\setminus\{0\} \to \{1\}$

The graph of f, where $f: A \to B$ $A, B \subseteq \mathbb{R}$, is a subset of $\mathbb{R}^2$

↳ $\{(x,y) \in \mathbb{R}^2 \mid x \in A, y \in f(x)\}$

Compositions → $f: A \to B$ and $g: B \to C$

same so we can "chain"

↓

composition

$g \circ f$

↓

g composes $A \xrightarrow{f} B \xrightarrow{g} C$

f: → $(g \circ f)(a) = g(f(a))$

If $f: \mathbb{R} \to \mathbb{R}_{\geq 0}$ and $g: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$

$x \mapsto x^2$ $\quad$ $s \mapsto \sqrt{s}$

order does matter

$g \circ f \neq f \circ g$

$(g \circ f)(x) = g(x^2) = \sqrt{x^2}$

$= |x|$

$g \circ f: \mathbb{R} \to \mathbb{R}_{\geq 0}$