Function → A "rule". Accept input → recieve output for each input (1 only)

↓

my cause or effect

relationship between measurable quantities

many shapes define/are defined by a function

expressed by function

Ways to study functions

1. Derivative (approximate function)
2. Integral (small pieces into bigger)
3. Power Series (represent complex function as inf. polynomial)

think about ideas behind computations

Set Notation

Natural Numbers	$\mathbb{N}$	$\{0, 1, 2, 3, \ldots\}$
Integers	$\mathbb{Z}$	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
Rational Numbers	$\mathbb{Q}$	$\{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\}$
Real #'s	$\mathbb{R}$	$\{\mathbb{Q} \cup \text{irrational \#}\}$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$ $\quad \in \rightarrow$ element of

$\pi + e$ may or may not be rational (we don't know)

lemma $\rightarrow$ helper theorem

↳ Let p be a pos. integer

If p^2 is even $\Rightarrow$ p is even

Proof of lemma

Suppose p is odd $\rightarrow$ ~~x~~ (a

$p = 2q + 1$ (q pos. int)

$p^2 = (2q+1)^2$

$p^2 = 4q^2 + 4q + 1$

$p^2 - 4q^2 - 4q = 1$

~~even~~ even

sum of even numbers = even

$\therefore$ 1 is even $\rightarrow$ ~~x~~ (contradiction!)

$\square$

Proof $\sqrt{2} \notin \mathbb{Q}$

Suppose for contradiction that $\sqrt{2}$ is rational

Then we can write $\sqrt{2} = \frac{m}{n}$

where $m, n \in \mathbb{Z}$, $n \neq 0$, and m and n

have no common factor but 1

- Squaring gives

$2 = \frac{m^2}{n^2}$

so $m^2 = 2n^2$

m^2 is even

Now we apply the lemma to m.

Therefor m is even.

$m = 2m_1$ where m_1 is positive integer

So. substituting gives

$$2n^2 = m^2 = (2m_1)^2$$

$$= 4m_1^2$$

$$n^2 = 2m_1^2$$

so n^2 is even, Apply the lemma once more

therefor n is even
but we assume m and n have no common factor, but since both are even they have a common factor of 2
This ~~xx~~ shows that $\sqrt{2}$ cannot be a rational number

Thinking problem

Prove $\sqrt[3]{3}$ is not rational

$\frac{p}{q} = \sqrt{3}$ $\frac{p^2}{q^2} = 3$