set → collection of things

Ex,

$\underline{X} = \{$ dog, cat, duck $\}$

$S = \{0, 1, 4, 9, \ldots\}$ squares

$\underline{Y} = \{52, 91, 6\}$ no rule

order of elements does not matter, allows repetition $\{51, 2, 3\} = \{2, 3, 51\} = \{2, 2, 3, 2, 51, 3\}$

$A \subseteq B$ → subset, every element of A also element of B

line means A can be equal to B

to prove: for each $a \in A$ it is also true that $a \in B$

$A = B$ → if $A \subseteq B$ AND $B \subseteq A$

$A \cap B$ → set of all elements in both A & B

↳ To prove $x \in A \cap B$: show $x \in A$ and $x \in B$

$A \cup B \rightarrow$ set of elements in either A or B or Both

↳ $x \in A \cup B : x \in A$ or $x \in B$

Empty set → no elements — subset of every set

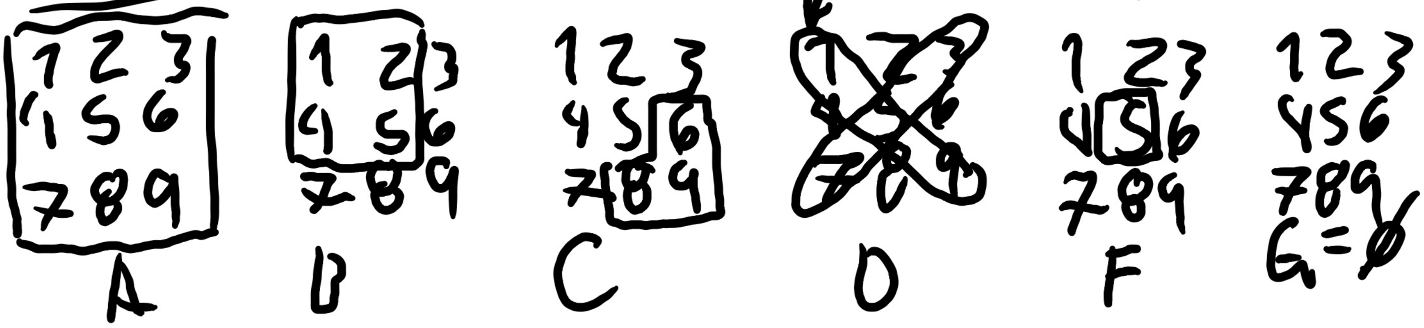
↳ $\emptyset = \{\}$ — like number 0 — for same set A:

$A \cup \emptyset = A$

$A \cap \emptyset = \emptyset$

Example

F



$B \subseteq A \quad D \subseteq A \quad C \not\subseteq D \quad | \quad E \cap D = F, \; A \cap C = C$

$F \subseteq E$

$B \cup C \cup E = A$

$B \cup C \cup D = B \cup C$

$A \cup G = A \quad C \cap G = G$

Set builder notation → describe set by listing test to determine if an element is in a set

$\{x \in \mathbb{N} \mid x \geq 5\} = \{5, 6, 7 \ldots\}$

$\{x^3 \mid x \in \mathbb{R}\} = \mathbb{R}$

$\{a \in \mathbb{N} \mid a \geq 5 \text{ and } a \leq 8\}$

General Form

{variable name | test(s) that elements have to pass to be in set}
(additional restriction)

Intervals → special subsets of $\mathbb{R}$ (often used)

$a, b \in \mathbb{R}$

$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$

$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

$[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$

round bracket: - exclusive
square bracket: inclusive

$(a,b) = (a,\infty) \cap (-\infty, b)$

$[a,b] \subseteq [c,d] \iff c \leq a$ and $b \leq d$

If $a > b$, $(a,b) = \emptyset$ eg. $(3,2) = \emptyset$

Prove $[a,b] \subseteq [c,d] \implies c \leq a$ and $b \leq d$

assume $[a,b] \subseteq [c,d]$

We can assume that $x \in [a,b]$, then $x \in [c,d]$

supose $a \leq b$ and $c \leq d$, so $[a,b], [c,d] \neq \emptyset$

$a \in [a,b] \Leftarrow a \leq a \leq b$

$a \in [a,b] \subseteq [c,d] \implies a \in [c,d]$

so $c \leq a \leq d$, by definition of integral

repeat with b

$b \in [a,b] \subseteq [c,d]$, $b \in [c,d]$

$c \leq b \leq d$, $b \leq d$