

3. (a)  $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$   
 $[c, d] = \{x \in \mathbb{R} \mid c \leq x \leq d\}$
- (b) Since  $x \in [a, b]$ , using the set build form above it can be converted to  $a \leq x \leq b$ . Then, simplify this statement into  $x \leq b$ . Using the premise of  $b \leq d$ , it is known that  $x \leq b \leq d$ . Once again, this can be simplified to  $x \leq d$ , resulting in the desired statement.
- (c) Since  $b \in [a, b]$ , from the premise that  $[a, b] \subseteq [c, d]$  and the fact that any element of a subset must also exist in the superset, it is known that  $b \in [c, d]$ . From the set build form above, this implies that  $c \leq b \leq d$ . This can be simplified to show that  $b \leq d$ , demonstrating the desired statement.



- (d)
- (e) No, for example  $3 \in [3, 5]$ , but it's also the case that  $3 \notin [4, 8]$ . So by definition of a subset, there is an element that exists in  $[3, 5]$  that doesn't exist in  $[4, 8]$ , so  $[3, 5] \not\subseteq [4, 8]$ .
- (f) In this case  $b \leq d$ , but  $a \not\leq c$ .



- (g)
- (h) Yes, as can be seen in the example above every element of  $[4, 7]$  exists in  $[2, 9]$ .
- (i) Since  $[4, 7] \subseteq [2, 9]$ , both statements should be true based on the proof above.