DUE DATE: SEPTEMBER 15, 2023

1. [2 points each]

(a) Let $\mathbf{u} = (1,3,3,0)$, $\mathbf{v} = (0,0,0,1), \mathbf{w} = (0,0,-5,0) \in \mathbb{R}^4$. Is $(2,6,1,-3)$ a linear combination of $\mathbf{u}, \mathbf{v}, \mathbf{w}$?

(b) Let $\mathbf{u} = (0,-1,3), \mathbf{v} = (0,-1,2), \mathbf{w} = (-4,1,1) \in \mathbb{R}^3$. Compute $\mathbf{u} \cdot (\mathbf{v} + 3\mathbf{w})$.

(c) Let $\mathbf{e}_1 = (1,0,0), \mathbf{e}_2 = (0,1,0), \mathbf{e}_3 = (0,0,1) \in \mathbb{R}^3$. Find all real numbers $c \in \mathbb{R}$ such that the angle between the vectors $-\mathbf{e}_1 + 2\mathbf{e}_2 + k\mathbf{e}_3$ and $-\mathbf{e}_1 + k\mathbf{e}_2 + 2\mathbf{e}_3$ is $\pi/2$ (they are orthogonal).

(d) Show that there are no vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ such that $||\mathbf{u}|| = 2, ||\mathbf{v}|| = 2$ and $\mathbf{u}\cdot\mathbf{v} = 5$.

(e) Show that $4\mathbf{u} \cdot \mathbf{v} = ||\mathbf{u}+\mathbf{v}||^2 - ||\mathbf{u}-\mathbf{v}||^2$. Show that $\mathbf{u}$ and $\mathbf{v}$ is orthogonal if and only if $||\mathbf{u}+\mathbf{v}|| = ||\mathbf{u}-\mathbf{v}||$.

2. [5 points each]
Let $\mathbf{u}$, $\mathbf{v} \in \mathbb{R}^n$ be vectors in $\mathbb{R}^n$

(a) State and prove the Cauchy–Schwarz inequality. Give a necessary and sufficient condition on $\mathbf{u}$, $\mathbf{v} \in \mathbb{R}^n$ such that the equality holds.

(b) State and prove the triangle inequality. Show that the equality holds if and only if $\mathbf{u}$ is a scalar multiple of $\mathbf{v}$.

Practice Problems: Section 1.1: 3(a),(d), 4, 6, 12, 16, 18, 28, 41, 43, 55, 57.
Section 1.2: 5, 11, 17(a),(b),(c),(d), 20, 22, 28, 34, 35, 40, 42, 49, 51, 61, 62(a),(b),69.