$$\begin{cases} x+y+z+w=6 \\ \quad\; y+z+w=4 \\ \quad\; y \quad\;\; +w=2 \\ \quad\; y \end{cases}$$

For every $y \in \mathbb{R}$,
you have a different solution

$\boxed{\text{Linear systems}} \longleftrightarrow \boxed{\text{Matrices}}$

$\boxed{\text{Vectors}}$

Linear systems $\longleftrightarrow$ matrices

matrices

vectors

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

coefficients

matrices
↳ generalization of vectors

$$\{(x_1, x_2, \dots, x_n) \mid x_i \in \mathbb{R} \text{ For } i = 1 \dots n\}$$

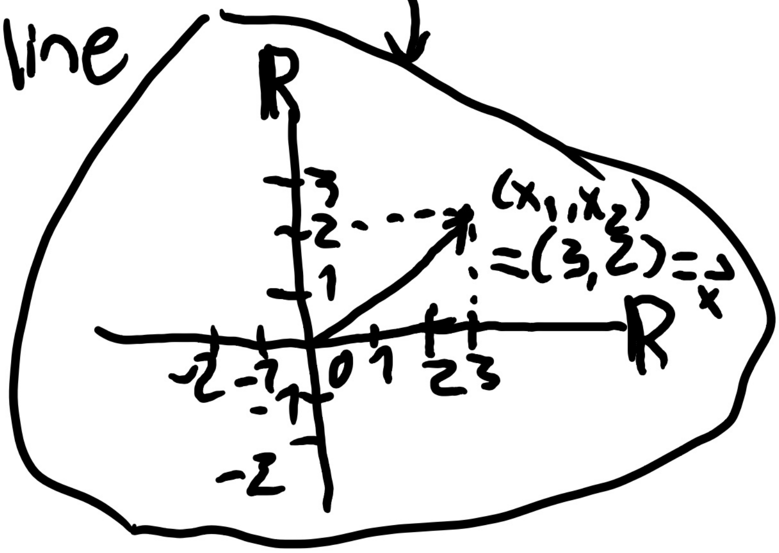
vectors in $\mathbb{R}^n$
↳ $n \in \mathbb{N}$, $\mathbb{R}^n = \underbrace{\mathbb{R} \times \mathbb{R} \times \mathbb{R} \cdots \times \mathbb{R}}_{n \text{ times}}$

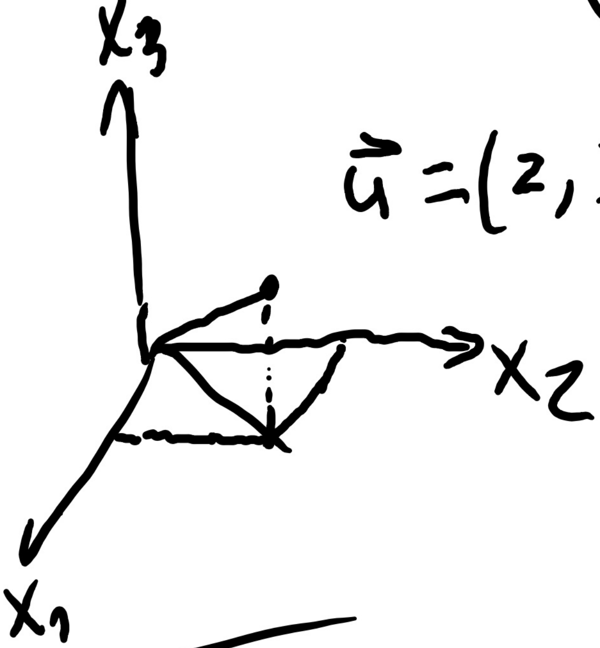
an element of $\mathbb{R}^n$ is "n" real numbers
ie, $\mathbb{R}^6 = (x_1, x_2, x_3, x_4, x_5, x_6)$

$\vec{x}$ → arrow means vector

$\mathbb{R}^2$ → plane

$\mathbb{R}$ → number line





$\vec{u} = (2, 3, 3)$

since when $x, y \in \mathbb{R}$: $x + y \in \mathbb{R}$, it should be when $\vec{u}, \vec{v} \in \mathbb{R}^2$, whatever our addition should be, $\vec{u} + \vec{v} \in \mathbb{R}^2$

<u>addition</u>

$$\vec{u} + \vec{v} = (x_1, x_2, \dots) + (y_1, y_2$$

$$= (\underset{\mathbb{R}}{x_1 + y_1}, \underset{\mathbb{R}}{x_2 + y_2}, \underset{\mathbb{R}}{\dots})$$

$$\mathbb{R}^n \cong \mathbb{R} \times \mathbb{R} \times \mathbb{R} \dots$$

$x, y \in \mathbb{R}$

$x \cdot y \in \mathbb{R}$

scalar multiplication

$\vec{v} \in \mathbb{R}^n$

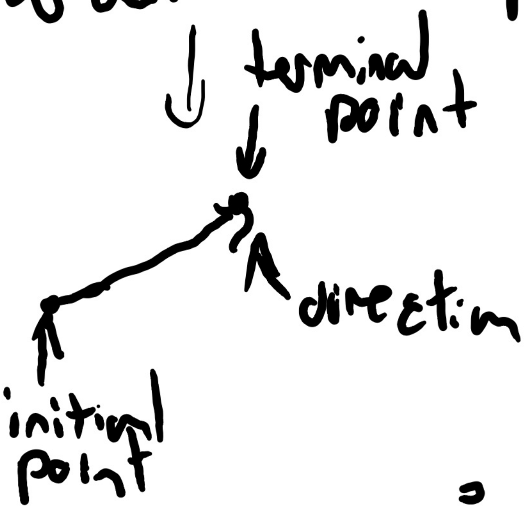
$c \in \mathbb{R}$

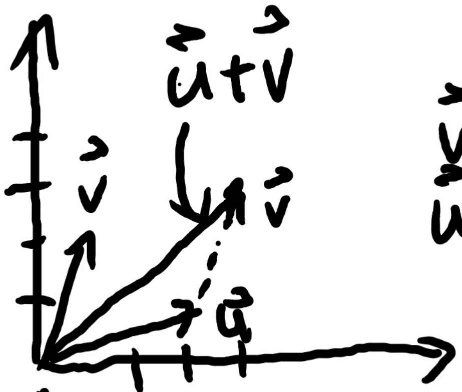
$$c \cdot \vec{v} = (\underset{\mathbb{R}}{c \cdot x_1}, \underset{\mathbb{R}}{c \cdot x_2}, \dots)$$

normally every vector starts at origin

↳ more general definition → 2 points and direction

or initial point, direction, and length





$\vec{v}=(1,2)$

$\vec{u}=(3,1)$

$$\vec{u}-\vec{v}=\vec{u}+(-\vec{v})$$
$$=\vec{u}+(-1\cdot\vec{v})$$
$$=(3,1)+(-1,-2)$$