

Describe function:

- ↳ domain \rightarrow all allowable inputs
- ↳ the codomain \rightarrow where the function lives
- ↳ Exact rule for taking an element of domain producing an element in codomain

$F: A \rightarrow B$
↑ ↑ ↑
name domain codomain

$f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto x^2$
↖ exact rule

for multiple formulas:

$g: \mathbb{R} \rightarrow \mathbb{R}$ given by $g(x) = \begin{cases} 1 & \text{if } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$

Image/range: given $f: A \rightarrow B$, the range is subset of B , contains all outputs for all possible outputs of A

$$\text{Im}(f) \stackrel{\text{def}}{=} \{b \in B \mid f(a) = b \text{ for some } a \in A\}$$

$$h: \mathbb{R} \rightarrow \mathbb{R}$$

$$h(s) = \begin{cases} \sin(s) & \text{if } s \in [0, 2\pi) \\ \cos(s) & \text{if } s \in [2\pi, 4\pi) \end{cases}$$

$$\downarrow \text{Im}(h) = [-1, 1]$$

$$\text{Im}(g) = \{0, 1\}$$

Given no codomain of domain

↳ make it the largest subset of \mathbb{R} such that the rule makes sense

↳ codomain is natural \rightarrow image when applied to natural domain

\sqrt{x} domain Natural codomain
 $[0, \infty)$ $[0, \infty)$

$\frac{x}{x}$ $\mathbb{R} \setminus \{0\}$ $\{1\}$

↑
all real numbers
that aren't 0

two functions are equal if both domains are equal,
and codomains are equal. for every input both outputs
are the same

$$f: A \rightarrow B$$

$$A = C$$

$$g: C \rightarrow D$$

$$B = D$$

$$a \in A \quad f(a) = g(a)$$

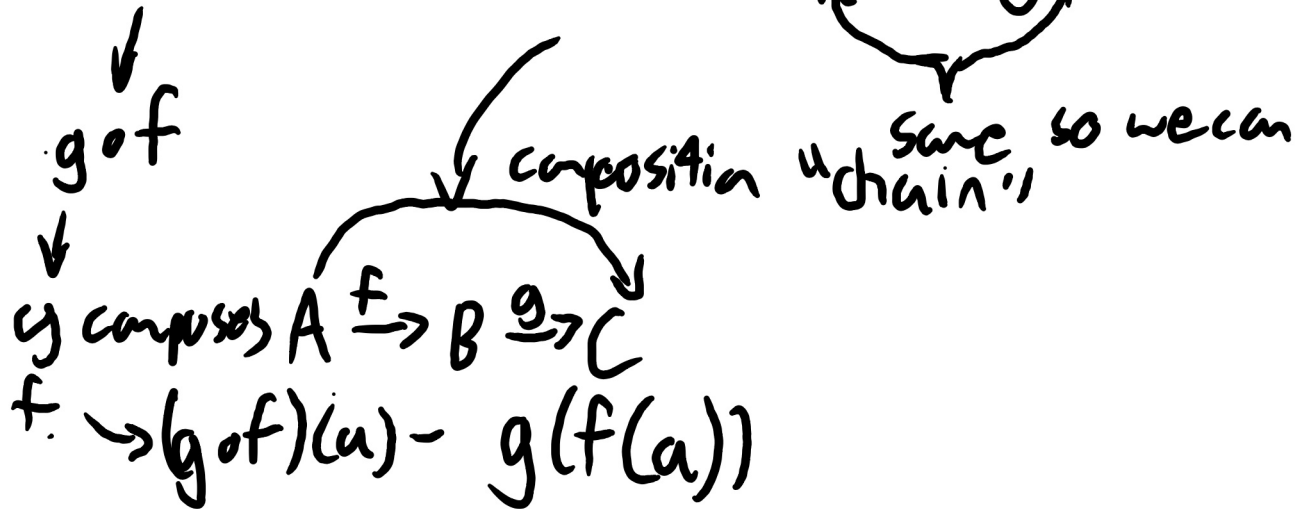
$$\begin{pmatrix} f(x) = \frac{x}{x} \\ g(x) = 1 \end{pmatrix}$$

↳ not the same because domains don't match
if we set $g: \mathbb{R} \setminus \{0\} \rightarrow \{1\}$

The graph of f , where $f: A \rightarrow B$ $A, B \subseteq \mathbb{R}$,
is a subset of \mathbb{R}^2

$$\hookrightarrow \{(x, y) \in \mathbb{R}^2 \mid x \in A, y \in f(x)\}$$

Compositions $\rightarrow f: A \rightarrow B$ and $g: B \rightarrow C$



If $f: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ and $g: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$
 $x \mapsto x^2$ $s \mapsto \sqrt{s}$

order does matter
 $g \circ f \neq f \circ g$

$$(g \circ f)(x) = g(x^2) = \sqrt{x^2} = |x|$$

$$g \circ f: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$$

