

We must first simplify the equations for the two orthogonal vectors.

$$-\vec{e}_1 + 2\vec{e}_2 + k\vec{e}_3 = -\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + k\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ k \end{bmatrix}$$

$$-\vec{e}_1 + k\vec{e}_2 + 2\vec{e}_3 = -\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ k \\ 2 \end{bmatrix}$$

We know that because the vectors are orthogonal, their dot product must be equal to zero. From that we can create the following equation and solve.

$$\begin{bmatrix} -1 \\ 2 \\ k \end{bmatrix} \cdot \begin{bmatrix} -1 \\ k \\ 2 \end{bmatrix} = 0$$

$$(-1) \cdot (-1) + 2 \cdot k + k \cdot 2 = 0$$

$$1 + 4k = 0$$

$$k = -\frac{1}{4}$$

Therefore, the only real number value for k that allows the vectors to be orthogonal is $-\frac{1}{4}$.