We can use the alternate form of $||\vec{v}||$ as $\sqrt{\vec{v}\cdot\vec{v}}$ to rewrite and re-arrange the initial equation.

$$||\vec{u}+\vec{v}||^2 - ||\vec{u}-\vec{v}||^2 = \sqrt{(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})}^2 - \sqrt{(\vec{u}-\vec{v})\cdot(\vec{u}+\vec{v})}^2$$

$$= (\vec{u}+\vec{v})\cdot\vec{u} + (\vec{u}+\vec{v})\cdot\vec{v} - (\vec{u}-\vec{v})\cdot\vec{u} - (\vec{u}-\vec{v})\cdot(-\vec{v})$$

$$= \cancel{\vec{u}\cdot\vec{u}} + \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{v} + \cancel{\vec{v}\cdot\vec{v}} - \cancel{\vec{u}\cdot\vec{u}} + \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{v} - \cancel{\vec{v}\cdot\vec{v}}$$

$$= 4\vec{u}\cdot\vec{v}$$

Therefore the preposition holds. From here we can prove the second part of the question using $\vec{u}\cdot\vec{v}=0$ when $\vec{v}$ and $\vec{u}$ are orthogonal. We start by proving that $\vec{u}\cdot\vec{v}=0 \Rightarrow ||\vec{u}+\vec{v}|| = ||\vec{u}-\vec{v}||$.

$$4\vec{u}\cdot\vec{v} = ||\vec{u}+\vec{v}||^2 - ||\vec{u}-\vec{v}||^2$$

$$4(0) = ||\vec{u}+\vec{v}||^2 - ||\vec{u}-\vec{v}||^2$$

$$0 + ||\vec{u}-\vec{v}||^2 = ||\vec{u}+\vec{v}||^2$$

$$||\vec{u}-\vec{v}|| = ||\vec{u}+\vec{v}||$$

We then prove that $\vec{u}\cdot\vec{v}=0 \Leftarrow ||\vec{u}+\vec{v}|| = ||\vec{u}-\vec{v}||$. To do this we must first rearrange the length equation we initially proved.

$$4\vec{u}\cdot\vec{v} = ||\vec{u}+\vec{v}||^2 - ||\vec{u}-\vec{v}||^2$$

$$||\vec{u}+\vec{v}||^2 = 4\vec{u}\cdot\vec{v} + ||\vec{u}-\vec{v}||^2$$

We can then substitute it into our initial premise after squaring both sides, and rearange.

$$||\vec{u}-\vec{v}||^2 = ||\vec{u}+\vec{v}||^2$$

$$||\vec{u}-\vec{v}||^2 = 4\vec{u}\cdot\vec{v} + ||\vec{u}-\vec{v}||^2$$

$$0 = 4\vec{u}\cdot\vec{v}$$

$$0 = \vec{u}\cdot\vec{v}$$