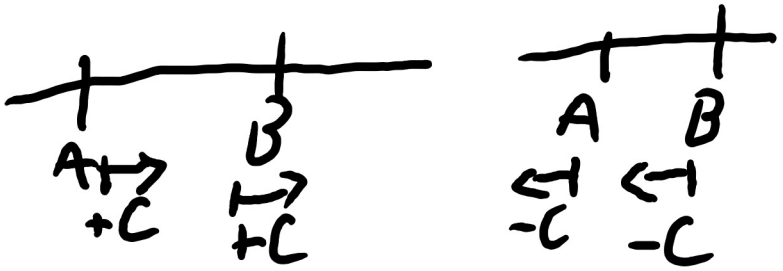
Inequalities

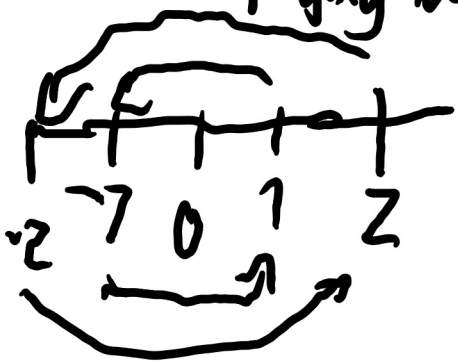
- Any operation you do to two numbers that are equal will result in a still equal number ↪ Here is a way to distinguish them
  ↳ inequalities are different

$A, B \in \mathbb{R}$ $A+C \leq B+C$
$A \leq B$ ↖ inequalities are preserved for addition/subtraction



Inequality is preserved with multi/divide by +ve #. If $C>0$, then $AC \leq CB \wedge \frac{A}{C} \leq \frac{B}{C}$

Multiplying both sides by -1 reverses inequality



Since $A \leq B$, $A-A \leq B-A$, $0 \leq B-A$
$0-B \leq B-A-B$
$-B \leq -A$

× or ÷ by -ve constant, inequality reverses

If $A, B \neq 0$, and $A \leq B$, which is larger? $\frac{1}{A}$ or $\frac{1}{B}$

↳ A and B both +ve, then

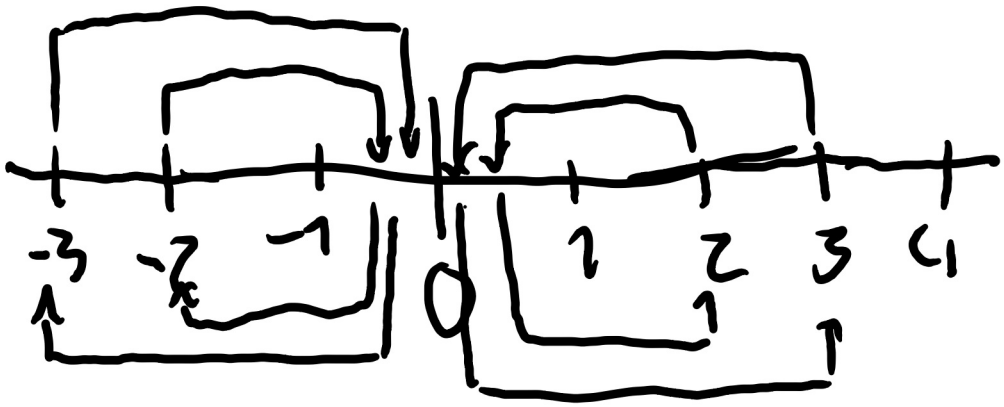
$\frac{A}{A} \leq \frac{B}{A} \Rightarrow 1 \leq \frac{B}{A}$ and $\frac{1}{B} \leq \frac{B}{BA} \Rightarrow \frac{1}{B} \leq \frac{1}{A}$

↳ inequality is flipped

If A and B are -ve, $\frac{A}{A} \geq \frac{B}{A} \Rightarrow \frac{1}{B} \leq \frac{B}{AB} \Rightarrow \frac{1}{B} \leq \frac{1}{A}$

If one is -ve and other is +ve

↳ $\frac{1}{B} \geq \frac{1}{A}$



Common inequalities

for $x \in \mathbb{R}$, $x^2 \geq 0$

for any $x, y \in \mathbb{R}$, then $x^2 + y^2 \geq 2xy$

$(x-y)^2 \geq 0$

$x^2 - 2xy + y^2 \geq 0$

$x^2 + y^2 \geq 2xy$

average $\rightarrow \frac{A+B}{2}$

if $a, b > 0$ or $a, b < 0$ Geometric mean $\hookrightarrow \sqrt{ab}$

AM-GM inequality

$a, b > 0$ then $\sqrt{ab} \leq \frac{a+b}{2}$

$x = \sqrt{a}$ and $y = \sqrt{b}$

$a + b \geq 2\sqrt{a}\sqrt{b}$

$\frac{a+b}{2} \geq \sqrt{ab}$