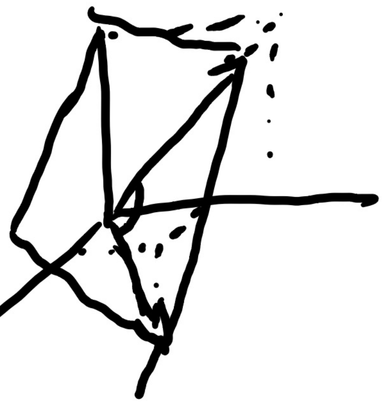
Orthogonal

↳ 2 vectors seperated by angle $\frac{\pi}{2}$

in $\mathbb{R}^n$

$$\boxed{\text{if } \vec{u}\cdot\vec{v}=0}$$

angles in 3d



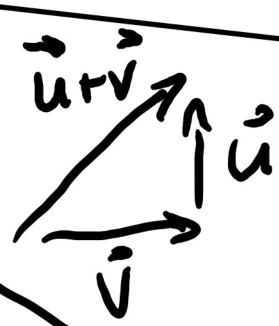
imagine plane containing both vectors, treat that as 2d plane

cosine law

$$\vec{u}\cdot\vec{v}=\|\vec{u}\|\cdot\|\vec{v}\|\cdot\cos\theta$$

allows you to define angles in any vector space $\mathbb{R}^n$

$$\cos\theta=\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}$$



prove $\|\vec{u}+\vec{v}\|^2=\|\vec{u}\|^2+\|\vec{v}\|^2$

$$\sqrt{(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})}^2=$$

because orthogonal

$$(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})=$$

$$\vec{u}\cdot\vec{u}+\cancel{2\vec{u}\cdot\vec{v}}+\vec{v}\cdot\vec{v}=$$

$$\vec{u}\cdot\vec{u}+\vec{v}\cdot\vec{v}=$$

$$\sqrt{\vec{u}\cdot\vec{u}}^2+\sqrt{\vec{v}\cdot\vec{v}}^2=\|\vec{u}\|^2+\|\vec{v}\|^2$$

proof of $\boxed{\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}}$

first proove

$\sin\theta = \dfrac{h}{a}$
$\cos\theta = \dfrac{r}{a}$

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$c^2 = h^2 + (b-r)^2$$

$$c^2 = (a\sin\theta)^2 + (b - a\cos\theta)^2$$

$$c^2 = a^2\sin^2\theta + b^2 - 2ab\cos\theta + a^2\cos^2\theta$$

$$c^2 = a^2\left(\cancelto{1}{\sin^2\theta + \cos^2\theta}\right) + b^2 - 2ab\cos\theta$$

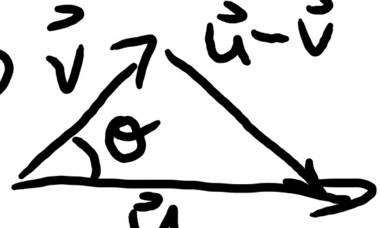
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$\|\vec{u} - \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos\theta$$

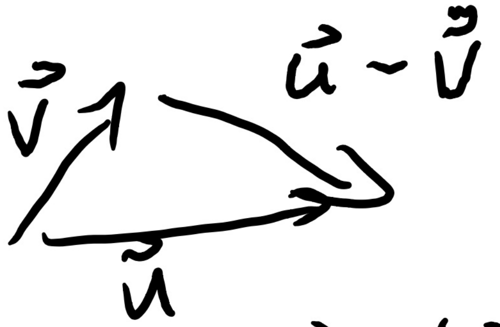
$$(\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} + \vec{v}\cdot\vec{v} - 2\|\vec{u}\|\|\vec{v}\|\cos\theta$$

$$\cancel{\|\vec{u}\|^2} - 2\vec{u}\cdot\vec{v} + \cancel{\|\vec{v}\|^2} = \cancel{\|\vec{u}\|^2} + \cancel{\|\vec{v}\|^2} - 2\|\vec{u}\|\|\vec{v}\| \dots$$

$$-\cancel{2}\,\vec{u}\cdot\vec{v} = -\cancel{2}\,\|\vec{u}\|\|\vec{v}\|\cos\theta$$



$$\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \cos\theta$$



$$\vec{u} = (\vec{u}-\vec{v}) + \vec{v}$$