The Cauchy Shwartz inequality is that for all vectors $\vec{u}, \vec{v} \in \mathbb{R}$, $|\vec{u} \cdot \vec{v}| \leq ||\vec{u}||||\vec{v}||$. To prove this we must start with the fact that $-1 \leq \cos(\theta) \leq 1$. We can take the absolute value of cos, and since all negative values are flipped we know that $0 \leq |\cos(\theta)| \leq 1$. First we remove the zero as it is unnecessary for the proof. We can then multiply $||\vec{v}||||\vec{u}||$ to both sides while maintaining the inequality, since we know that vector lengths must be positive.

$$||\vec{u}||||\vec{v}|| \cdot |\cos(\theta)| \leq ||\vec{u}||||\vec{v}||$$

Since the vector values are both positive, we can expand the absolute value to cover the whole left side without changing the value of that side. Using the identity $\vec{u} \cdot \vec{v} = ||\vec{u}||||\vec{v}|| \cdot \cos(\theta)$ we can substitute it into the left side of out identity to get the Cauchy-Shwartz inequality.

$$|\vec{u} \cdot \vec{v}| \leq ||\vec{u}||||\vec{v}||$$

To show the conditions of equality, we must go back to the equation without the substitution. Setting both sides to equal eachother to state equality, we can rearrange.

$$||\vec{u}||||\vec{v}|| \cdot |\cos(\theta)| = ||\vec{u}||||\vec{v}||$$

$$|\cos(\theta)| = \frac{||\vec{u}||||\vec{v}||}{||\vec{u}||||\vec{v}||}$$

$$|\cos(\theta)| = 1$$

Since the absolute value of $\cos(\theta)$ must be one, we know (θ) must be either 1 or -1. This is only the case when $\theta = 0$ or π, meaning that only when one vector is a scalar multiple of the other does the equality hold.