

CIS 301: Logical Foundations of Programming

Spring 2023

Exam 1 – 100 points

This test is closed-notes and closed-computers.

There are 9 questions worth 8-15 points each.

Name: _____ Solution _____

Score: _____

1. (15 pts) Consider the following questions about logical analysis of conditional statements.

a) (7 pts) Consider the following code fragment:

```
num = 4;
if (x < 10 || (y > 100 && y < 200)) {
    if (x >= 10) {
        num = 18;
    }
}
// ← suppose we are right here
```

Suppose *num* has the value 18 immediately after the above code fragment finishes (i.e., at the point marked with the ← just after the brackets). What can we conclude about *x* and *y*? Explain as concisely as you can.

Answer:

Since *num* was initially 4 and is now 18, we can conclude that we went inside both if-statements. This tells us that $x < 10 \parallel (y > 100 \ \&\& \ y < 200)$ and also that $x \geq 10$. Putting those together, we can conclude that the second side of the \parallel in the first condition must have been true, since *x* can't be both < 10 and also ≥ 10 . This tells us that **$x \geq 10$ and that $100 < y < 200$.**

b) (8 pts) Consider the following code fragment:

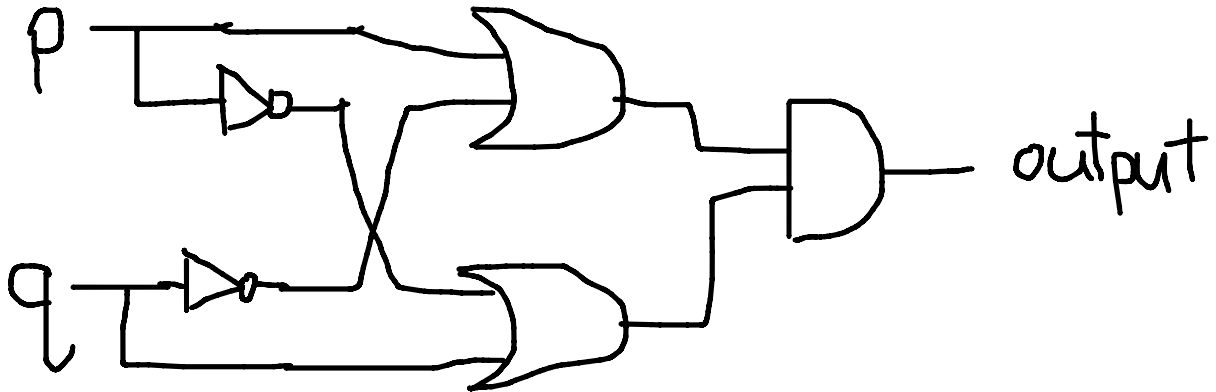
```
num = 4;
if (val1 > 0 || val2 < 10) {
    num = 7;
}
else if (val1 < 0) {
    num = 10;
}
else {
    num = 20;
}
// ← suppose we are right here
```

Suppose *num* has the value 20 immediately after the above code fragment finishes (i.e., at the point marked with the ← just after the brackets). What can we conclude about *val1* and *val2*? Explain as concisely as you can.

Answer:

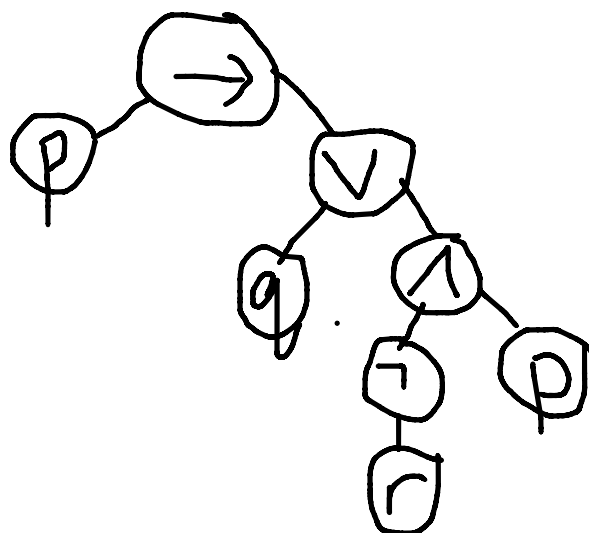
Since *num* was initially 4 and is now 20, we can conclude that we went inside the else statement. This tells us that both the if condition and the else if conditions were false. From the if, we know that *val1* ≤ 0 && *val2* ≥ 10. From the else if, we know that *val1* > 0. Putting them together, we can conclude that ***val1* == 0 and that *val2* ≥ 10.**

2. (8 pts) Draw a circuit for the following logical formula: $(p \text{ OR NOT } q) \text{ AND } (\text{NOT } p \text{ OR } Q)$. Use only a combination of AND, OR, and NOT gates.



3. (8 pts) Draw a parse tree for the following logical formula:

$$p \rightarrow q \vee \neg r \wedge p$$



4. (13 pts) Use two truth tables to demonstrate that the following two statements are logically equivalent:

$$(p \rightarrow q) \wedge (q \rightarrow p)$$

$$(p \vee \neg q) \wedge (q \vee \neg p)$$

Afterwards, give a brief explanation about why your truth tables demonstrate that the statements are equivalent.

*

p	q		$(p \rightarrow q)$	$\wedge (q \rightarrow p)$

T	T		T	T
T	F		F	F
F	T		F	F
F	F		T	T

Contingent

- T : [T T] [F F]
- F : [T F] [F T]

*

p	q		$(p \vee \neg q)$	$\wedge (q \vee \neg p)$

T	T		T F	T F
T	F		T T	F F
F	T		F F	T T
F	F		T T	T T

Contingent

- T : [T T] [F F]
- F : [T F] [F T]

We know the two statements are equivalent because the same set of truth assignments make each statement true.

5. (8 pts) Apply DeMorgan's laws to write an if-statement whose condition is the negation of the condition in the if-statement below. Write your if-statement in such a way that it does not use any ! (not) symbols.

```
if ((total >= 100 && Character.isDigit(ch) == false) || num < 10) {  
    //statements  
}
```

Write your negated if-statement below:

```
if ((total < 100 || Character.isDigit(ch) == true) && num >= 10) {  
    //statements  
}
```

6. (8 pts) Is the statement $(p \vee \neg q) \wedge (\neg p \vee \neg q) \wedge (p \rightarrow q)$ satisfiable? How do we know?

Yes. If we let $p = F$ and $q = F$, then the output of the statement is true. Thus, the statement is satisfiable (there is a truth assignment that makes the output true).

7. (11 pts) Consider the following (invalid) argument:

Premises:

If I order fries, then I get ketchup.

If I get ketchup, then I get a cheeseburger.

I don't get ketchup.

Conclusion:

I don't get a cheeseburger.

a) (4 pts) Translate each premise and conclusion to propositional logic. Start by identifying each propositional atom.

p: I order fries

q: I get ketchup

r: I get a cheeseburger

Premises:

$p \rightarrow q, q \rightarrow r, \neg q$

Conclusion: $\neg r$

b) (7 pts) Provide a truth assignment for your translations in (a) that demonstrates that the argument is NOT valid. Afterward, write 1-2 sentences explaining why your truth assignment shows that the argument is not valid.

Counterexample: $p = F, q = F, r = T$

This assignment makes each premise true and the conclusion false, so the argument is not valid.

8. (15 pts) Complete the following natural deduction proof:

$(c \vee a) \wedge b \vdash (a \wedge b) \vee (b \wedge c)$	
{	
1. $(c \vee a) \wedge b$	premise
2. $c \vee a$	$\wedge e1$ 1
3. b	$\wedge e2$ 1
4. {	
5. c	assume
6. $b \wedge c$	$\wedge i$ 3 5
7. $(a \wedge b) \vee (b \wedge c)$	$\vee i2$ 6
}	
8. {	
9. a	assume
10. $a \wedge b$	$\wedge i$ 9 3
11. $(a \wedge b) \vee (b \wedge c)$	$\vee i1$ 10
}	
12. $(a \wedge b) \vee (b \wedge c)$	$\vee e$ 2 4 8
}	

9. (14 pts) Complete the following natural deduction proof:

$q \rightarrow (p \wedge r), r \rightarrow s \vdash q \rightarrow (s \wedge p)$
{

1. $q \rightarrow (p \wedge r)$ premise

2. $r \rightarrow s$ premise

3. {

4. q assume

5. $p \wedge r$ $\rightarrow e$ 1 4

6. p $\wedge e$ 5

7. r $\wedge e$ 5

8. s $\rightarrow e$ 2 7

9. $s \wedge p$ $\wedge i$ 8 6

}

10. $q \rightarrow (s \wedge p)$ $\rightarrow i$ 3

}