CIS 301: Logical Foundations of Programming

Spring 2023

Exam 1 – 100 points

**This test is closed-notes and closed-computers.**

There are 9 questions worth 8-15 points each.

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Score: \_\_\_\_\_\_\_\_\_\_\_\_

1. (15 pts) Consider the following questions about logical analysis of conditional statements.

a) (7 pts) Consider the following code fragment:

num = 4;

if (x < 10 || (y > 100 && y < 200)) {

if (x >= 10) {

num = 18;

}

}

// 🡨 suppose we are right here

Suppose *num* has the value 18 immediately after the above code fragment finishes (i.e., at the point marked with the 🡨 just after the brackets). What can we conclude about *x* and *y*? Explain as concisely as you can.

x < 10 || (y > 100 && y < 200)

and

x >= 10

Because x >= 10 conflicts with x < 10, we know that the second part of the condition is true.

X >= 10

Y > 100 && y < 200

b) (8 pts) Consider the following code fragment:

num = 4;

if (val1 > 0 || val2 < 10) {

num = 7;

}

else if (val1 < 0) {

num = 10;

}

else {

num = 20;

}

// 🡨 suppose we are right here

Suppose *num* has the value 20 immediately after the above code fragment finishes (i.e., at the point marked with the 🡨 just after the brackets). What can we conclude about *val1* and *val2*? Explain as concisely as you can.

Both if-statements are not valid. This means that val2 >= 10 and val1 == 0.

val1 > 0 || val2 < 10 false means: val1 <=0 && val2 >= 10

val1 <0 being false means: val1 >= 0

So val1 == 0 and val2 >= 10

1. (8 pts) Draw a circuit for the following logical formula: **(p OR NOT q) AND (NOT p OR Q).** Use only a combination of AND, OR, and NOT gates.
2. (8 pts) Draw a parse tree for the following logical formula:

p → q V ¬r ∧ p

1. (13 pts) Use two truth tables to demonstrate that the following two statements are logically equivalent:

(p → q) ∧ (q → p)

(p V ¬q) ∧ (q V ¬p)

Afterwards, give a brief explanation about why your truth tables demonstrate that the statements are equivalent.

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P q | (p → q) ∧ (q → p)

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T T | T T T

T F | F F T

F T | T F F

F F | T T T

\*

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P q | (p V ¬q) ∧ (q V ¬p)

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T T | T F T T F

T F | F

F T | F

F F | T

They are equivalent because both statements have the same output for every truth assignment.

1. (8 pts) Apply DeMorgan's laws to write an if-statement whose condition is the negation of the condition in the if-statement below. Write your if-statement in such a way that it does not use any ! (not) symbols.

if ((total >= 100 && Character.isDigit(ch) == false) || num < 10) {

//statements

}

Write your negated if-statement below:

if ((total < 100 || Character.isDigit(ch) == true) && num => 10) {

//statements

}

1. (8 pts) Is the statement (p V ¬q) ∧ (¬p V ¬q) ∧ (p → q) satisfiable? How do we know?

p = F

q = F

This truth assignment makes the statement true, so it is satisifiable.

1. (11 pts) Consider the following (invalid) argument:

**Premises:**

If I order fries, then I get ketchup.

If I get ketchup, then I get a cheeseburger.

I don’t get ketchup.

**Conclusion**:

I don’t get a cheeseburger.

a) (4 pts) Translate each premise and conclusion to propositional logic. Start by identifying each propositional atom.

P: I order fries

Q: I get ketchup

R: I get a cheeseburger

Premises:

P -> q

Q -> r

!q

Conclusion:

!r

b) (7 pts) Provide a truth assignment for your translations in (a) that demonstrates that the argument is NOT valid. How do you know that truth assignment makes the argument invalid? Explain.

Need a truth assignment that makes all premises true but the conclusion false.

P = F

Q = F

R = T

Conclusion: The above truth statements makes the argument invalid.

1. (15 pts) Complete the following natural deduction proof:

(c ∨ a) ∧ b ⊢ (a ∧ b) ∨ (b ∧ c)

{

1. (c V a) ^ b premise
2. c V a ^e1 1
3. b ^e2 1
4. {
   1. C assume
   2. b ^ c ^i 3 5
   3. (a ^ b) V (b ^ c) Vi 2 6

}

8.{

9. a assume

10. a ^ b unfinished

}

}

1. (14 pts) Complete the following natural deduction proof:

q → (p ∧ r), r → s ⊢ q → (s ∧ p)