Ogni appl. lineare $\tilde{T}: K^n \longrightarrow K^m$ è del tipo $\tilde{T}_B$, per una opportuna matrice B.

$((1,0,\dots,0),(0,1,0,\dots,0),\dots,(0,0,\dots,0,1))$. Siano:

$$\tilde{T}((1,0,\dots,0)) = (b_1^1, b_1^2, \dots, b_1^m) \in K^m$$

$$\tilde{T}((0,1,\dots,0)) = (b_2^1, b_2^2, \dots, b_2^m) \in K^m$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\tilde{T}((0,\dots,0,1)) = (b_n^1, b_n^2, \dots, b_n^m) \in K^m$$

Considero la matrice

$$B = \begin{pmatrix} b_1^1 & b_2^1 & \dots & b_n^1 \\ b_1^2 & b_2^2 & \dots & b_n^2 \\ \vdots & & & \\ b_1^m & b_2^m & \dots & b_n^m \end{pmatrix}$$

e $\tilde{T}_B: \quad K^n \longrightarrow K^n$

$$(x_1, \dots, x_n) \longrightarrow B\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

Vediamo $\tilde{T} = \tilde{T}_B$ possiamo usare il teorema fondamentale delle applicazioni lineari oppure:

th: $\forall (x_1, \dots, x_n) \in K^n$, $\tilde{T}((x_1, \dots, x_n)) = \tilde{T}_B((x_1, \dots, x_n))$

<u>Dim</u> $(x_1, \dots, x_n) = x_1(1, 0 \dots 0) + \dots + x_n(0, \dots, 1)$.

$$\tilde{T}((x_1, \dots, x_n)) = \tilde{T}\Big(x_1(1, \dots, 0) + \dots + x_n(0, \dots, 1)\Big) =$$

$$= x_1 \tilde{T}\big((1, \dots, 0)\big) + \dots + x_n \tilde{T}((0, \dots, 1)) =$$

$$= x_1(b_1^1, \dots, b_1^n) + \dots + x_n(b_n^1, \dots, b_n^n) =$$

$$= (b_1^1 x_1 + \dots + b_n^1 x_n, \dots, b_1^m x_1 + \dots + b_n^m x_n) = {}^t\left(B\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}\right) \underset{\text{def.}}{=} \tilde{T}_B((x_1, \dots, x_n))$$

$\boxtimes$