

$$A = \begin{pmatrix} a_1^1 & \dots & a_h^1 & \dots & a_n^1 \\ \vdots & & \vdots & & \vdots \\ a_1^k & \dots & a_h^k & \dots & a_n^k \\ \vdots & & \vdots & & \vdots \\ a_1^m & \dots & a_h^m & \dots & a_n^m \end{pmatrix} \quad \leftarrow \text{riga } \underline{a}^k$$

\nearrow
 colonna \underline{a}_h

1) preso $k \in \{1, \dots, m\}$ e $\alpha \in K \setminus \{0\}$

$$\underline{a}^k \longrightarrow \alpha \cdot \underline{a}^k$$

$$B = \begin{pmatrix} a_1^1 & \dots & a_h^1 & \dots & a_n^1 \\ \vdots & & \vdots & & \vdots \\ \alpha a_1^k & \dots & \alpha a_h^k & \dots & \alpha a_n^k \\ \vdots & & \vdots & & \vdots \\ a_1^m & \dots & a_h^m & \dots & a_n^m \end{pmatrix}$$

$$b^k = \alpha \cdot \underline{a}^k$$

Esempio

$$A = \begin{pmatrix} 2 & 7 \\ -1 & 3 \\ 0 & -5 \end{pmatrix}$$

$$\underline{a}^2 \longrightarrow 3 \cdot \underline{a}^2$$

$$B = \begin{pmatrix} 2 & 7 \\ -3 & 9 \\ 0 & -5 \end{pmatrix}$$

$$2) \forall k, i \in \{1, \dots, m\} \quad e^k \leftrightarrow e^i$$

$$A = \begin{pmatrix} e_1^1 & \dots & e_m^1 \\ \vdots & & \vdots \\ e_1^k & \dots & e_m^k \\ \vdots & & \vdots \\ e_1^i & \dots & e_m^i \\ \vdots & & \vdots \\ e_1^m & \dots & e_m^m \end{pmatrix} \longrightarrow B = \begin{pmatrix} e_1^1 & \dots & e_m^1 \\ \vdots & & \vdots \\ e_1^i & \dots & e_m^i \\ \vdots & & \vdots \\ e_1^k & \dots & e_m^k \\ \vdots & & \vdots \\ e_1^m & \dots & e_m^m \end{pmatrix}$$

Esempio

$$A = \begin{pmatrix} 2 & 7 \\ -1 & 3 \\ 0 & -5 \end{pmatrix} \quad \underline{e^1} \leftrightarrow \underline{e^3} \quad B = \begin{pmatrix} 0 & -5 \\ -1 & 3 \\ 2 & 7 \end{pmatrix}$$

$$3) \forall k, i \in \{1, \dots, m\} \quad k \neq i, \lambda \in K$$

$$e^k \longrightarrow e^k + \lambda e^i$$

$$A = \begin{pmatrix} e_1^1 & \dots & e_m^1 \\ \vdots & & \vdots \\ e_1^k & \dots & e_m^k \\ \vdots & & \vdots \\ e_1^i & \dots & e_m^i \\ \vdots & & \vdots \\ e_1^m & \dots & e_m^m \end{pmatrix} \dashrightarrow B = \begin{pmatrix} e_1^1 & \dots & e_m^1 \\ \vdots & & \vdots \\ e_1^k + \lambda e_1^i & \dots & e_m^k + \lambda e_m^i \\ \vdots & & \vdots \\ e_1^i & \dots & e_m^i \\ \vdots & & \vdots \\ e_1^m & \dots & e_m^m \end{pmatrix}$$

Esempio:

$$A = \begin{pmatrix} 2 & 7 \\ -1 & 3 \\ 0 & -5 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 7 \\ 0 & \frac{13}{2} \\ 0 & -5 \end{pmatrix}$$

$$K=2, \quad i=1, \quad \alpha=1/2$$

$$\underline{q^2} \rightarrow \underline{q^2} + \frac{1}{2} \underline{q^1} = (-1, 3) + \frac{1}{2} (2, 7) = (0, 1\frac{1}{2})$$