

Esercizio

$$T: \mathbb{R}^2[x] \rightarrow \mathbb{R}^3$$

$$a_0 + a_1x + a_2x^2 \rightarrow (2a_0 + a_1, a_1 + a_2, a_1 - 2a_2)$$

$$\mathcal{B} = (1, x, x^2) \text{ base ordinata di } \mathbb{R}^2[x]$$

$$\mathcal{B}' = ((1, 1, 0), (0, 1, 1), (0, 0, 1)) \text{ base ordinata di } \mathbb{R}^3$$

Rappresentare T nelle basi fissate.

$$\text{Conviene calcolare } \phi_{\mathcal{B}'}^1: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$(x_1, x_2, x_3) = a_1(1, 1, 0) + a_2(0, 1, 1) + a_3(0, 0, 1) = (a_1, a_1 + a_2, a_2 + a_3)$$

$$\begin{cases} a_1 = x_1 \\ a_1 + a_2 = x_2 \Rightarrow a_2 = x_2 - a_1 \rightarrow a_2 = x_2 - x_1 \\ a_2 + a_3 = x_3 \Rightarrow a_3 = x_1 - x_2 + x_3 \end{cases}$$

$$\phi_{\mathcal{B}'}^1: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$(x_1, x_2, x_3) \mapsto (x_1, x_2 - x_1, x_3 - x_2 + x_1)$$

$$\phi_{\mathcal{B}'}^1(T(1)) = \phi_{\mathcal{B}'}^1((2, 0, 0)) = (2, -2, 1)$$

$$\phi_{\mathcal{B}'}^1(T(x)) = \phi_{\mathcal{B}'}^1((3, 1, 1)) = (3, -2, 3)$$

$$\phi_{\mathcal{B}'}^1(T(x^2)) = \phi_{\mathcal{B}'}^1((0, 1, -2)) = (0, 1, -3)$$

$$A = \begin{pmatrix} 2 & 3 & 0 \\ -2 & -2 & 1 \\ 2 & 3 & -3 \end{pmatrix}$$