Siano W_1, W_2 sottospazi vett. di V

$$\underline{0} \in W_1 \cap W_2 \subseteq V \implies W_1 \cap W_2 = \emptyset$$

Vediamo inoltre che $W_1 \cap W_2$ è linearmente chiuso

$+: \ \forall u, v \in W_1 \cap W_2 , \quad u, v \in W_1 \quad e \quad u, v \in W_2$

$$u+v \in W_1 \qquad u+v \in W_2$$

$$\implies u+v \in W_1 \cap W_2$$

$\cdot: \ \forall \alpha \in K \qquad \alpha u \in W_1 \quad e \quad \alpha u \in W_2 \implies \alpha v \in W_1 \cap W_2$

$\forall v \in W_1 \cap W_2 \qquad u \in W_1 \qquad u \in W_2$

Quindi $W_1 \cap W_2$ è un sottospazio vettoriale, detto sottospazio intersezione.

Esempio:

$\mathbb{R}^3$

$$W_1 = \mathcal{L}((1,0,1), (0,0,1))$$
$$W_2 = \mathcal{L}((1,0,2), (0,1,0))$$

Determiniamo $W_1 \cap W_2$

$w_1 \in W_1 \iff \exists\ \alpha, \beta \in \mathbb{R}$ t.c. $w_1 = \alpha(1,0,1) + \beta(0,0,1)$

$$\alpha(1,0,1) + \beta(0,0,1) = (\alpha, 0, \alpha+\beta)$$

$w_2 \in W_2 \iff \exists\ \bar{\alpha}, \bar{\beta} \in \mathbb{R}$ t.c. $w_2 = \bar{\alpha}(1,0,2) + \bar{\beta}(0,1,0)$

$$\bar{\alpha}(1,0,2) + \bar{\beta}(0,1,0) = (\bar{\alpha}, \bar{\beta}, 2\bar{\alpha})$$

$u \in W_1 \cap W_2 \iff \exists\ \gamma_1, \gamma_2 \in \mathbb{R}$ t.c. $u = (\alpha, 0, \alpha+\beta) = (\bar{\alpha}, \bar{\beta}, 2\bar{\alpha})$

Quindi

$$\begin{cases} \alpha = \bar{\alpha} \\ 0 = \bar{\beta} \\ \alpha + \beta = 2\bar{\alpha} \longrightarrow \beta = \bar{\alpha} \end{cases}$$

$$(\alpha, 0, \alpha+\beta) = (\bar{\alpha}, \bar{\beta}, 2\bar{\alpha}) = (\alpha, 0, 2\beta) \Rightarrow \alpha + \beta = 2\beta \Rightarrow \alpha = \beta$$

$$u = (\alpha, 0, 2\alpha) = \alpha(1,0,2)$$

$$W_1 \cap W_2 = \{\alpha(1,0,2) \mid \alpha \in \mathbb{R}\} = \mathcal{L}((1,0,2)).$$