Def

Se $W_1 \cap W_2 = \{\underline{0}\}$, allora $W_1 + W_2$ si dice <u>somma diretta</u> e si indica con:

$$W_1 \oplus W_2$$

Proposizione

La somma $W_1 + W_2$ è diretta $\iff \forall u \in W_1 + W_2 \,, \exists! (w_1 + w_2) \in W_1 \times W_2$ tale che $u = w_1 + w_2$

Dim

"$\Rightarrow$"

$$u = \underbrace{w_1 + w_2}_{\in W_1 \times W_2} = \underbrace{w_1' + w_2'}_{\in W_1 \times W_2} \Rightarrow \underbrace{w_1 - w_1'}_{\in W_1} = \underbrace{w_2' - w_2}_{\in W_2} \in W_1 \cap W_2 \overset{Hyp}{=} \{\underline{0}\}$$

Ma questo implica

$$\begin{cases} w_1 - w_1' = 0 \rightarrow w_1 = w_1' \\ w_2 - w_2' = 0 \rightarrow w_2 = w_2' \end{cases}$$

"$\Leftarrow$" Th: $W_1 \cap W_2 = \{\underline{0}\}$

$$u \in W_1 \cap W_2 \quad {\subseteq W_1 \atop \subseteq W_2}$$

$$u = \overbrace{u}^{\in W} + \overbrace{\underline{0}}^{\in W_2} = \overbrace{\underline{0}}^{\in W_1} + \overbrace{u}^{\in W_2} \overset{Hyp}{\Rightarrow} (u, \underline{0}) = (\underline{0}, u) \Rightarrow$$

$$\Rightarrow u = \underline{0} \quad \blacksquare$$