Esempio

(V, A, π) $\quad \dim A = 2.$ $\quad \mathcal{R} = (O, \mathcal{B})$ $\quad \mathcal{B} = (e_1, e_2)$ base ordin. di V

rappresentare la retta r passante per $P \equiv_{\mathcal{R}} (2,1)$, $P' \equiv_{\mathcal{R}} (3,-1)$

$P(2,1) \neq P'(3,-1)$

$P \longrightarrow P'$ $\qquad 0 \neq \overrightarrow{PP'} \in \vec{r}$

Siccome $\dim \vec{r} = 1$, prendiamo $\vec{r} = \mathcal{L}(\overrightarrow{PP'})$

$$\phi_{\mathcal{B}}(\vec{r}) = \mathcal{L}\left(\phi_{\mathcal{B}}(\overrightarrow{PP'})\right) = \mathcal{L}\left((1,-2)\right) \subseteq \mathbb{R}^2$$

$Q(x_1, x_2) \in r \iff 2x_1 + x_2 - 5 = 0$ rappr di r in $\mathcal{R}$

$$\text{rango} \begin{pmatrix} 1 & x_1 - 3 \\ -2 & x_2 + 1 \end{pmatrix} = 1 \qquad \underline{a}^2 \longrightarrow \underline{a}^2 + 2\underline{a}^1 \begin{pmatrix} 1 & x_1 - 3 \\ 0 & x_2 + 1 + 2x_1 - 6 \end{pmatrix}$$

$\vec{r}: 2x_1 + x_2 = 0$

Esempio 2: $\dim A = 3$, $\mathcal{R} = (O, \mathcal{B})$ $\quad \mathcal{B} = (e_1, e_2, e_3)$

Rappresentare la retta per $P \equiv_{\mathcal{R}} (1, 2, -2)$, $P'(3,3,1)$

$\vec{r} = \mathcal{L}(\overrightarrow{PP'})$ $\qquad \overrightarrow{PP'}(2,1,3)$ $\qquad r = (P, \vec{r})$

$Q\,(x_1, x_2, x_3) \in r \iff \begin{cases} x_2 - 2 - \frac{1}{2}x_1 + \frac{1}{2} = 0 \\ x_3 + 2 - \frac{1}{3}x_1 + \frac{1}{3} = 0 \end{cases}$ $\quad n = 3 \quad h = 1 \quad n - h = 2$

$$\text{rang}\begin{pmatrix} 2 & x_1 - 1 \\ 1 & x_2 - 2 \\ 3 & x_3 + 2 \end{pmatrix} = 1 \qquad \underline{a}^2 \to \underline{a}^2 - \frac{1}{2}\underline{a}^1 \quad \underline{a}^3 \to \underline{a}^3 - \frac{1}{3}\underline{a}^1 \qquad \begin{pmatrix} 2 & x_1 - 1 \\ 0 & x_2 - 2 - \frac{1}{2}x_1 + \frac{1}{2} \\ 0 & x_3 + 2 - \frac{1}{3}x_1 + \frac{1}{3} \end{pmatrix}$$

Esempio 3: $\quad \dim A = 3$

Rappresentare il piano α con giacitura $\vec{\alpha} = \mathcal{L}(\overrightarrow{P_0P_1}, \overrightarrow{P_0P_2})$ e passante per il punto P_0, dove, in un rif. fissato

$R = (O, \mathcal{B})$, $\mathcal{B} = (e_1, e_2, e_3)$, $P_0(0, -2, 1)$, $P_1(1, 1, -1)$, $P_2(2, 1, 0)$

$$\overrightarrow{P_0P_1}(1, 3, -2) \not\parallel \overrightarrow{P_0P_2}(2, 3, -1)$$

$$\alpha = (P_0, \vec{\alpha}) \qquad Q \equiv (x_1, x_2, x_3) \in \alpha \iff \text{rang}\begin{pmatrix} 1 & 2 & x_1 - 0 \\ 3 & 3 & x_2 + 2 \\ -2 & -1 & x_3 - 1 \end{pmatrix} = 2$$

$$\begin{array}{l} a^2 \to a^2 - 3a^1 \\ a^3 \to a^3 + 2a^1 \end{array} \quad \begin{pmatrix} 1 & 2 & x_1 \\ 0 & -3 & x_2 + 2 - 3x_1 \\ 0 & 3 & x_3 - 1 + 2x_1 \end{pmatrix} \quad a^3 \to a^3 + a^2 \quad \begin{pmatrix} 1 & 2 & x_1 \\ 0 & -3 & x_2 - 3x_1 + 2 \\ 0 & 0 & (x_3 - 1 + 2x_1 + x_2 + 2 - 3x_1) \end{pmatrix}$$

$\alpha : -x_1 + x_2 + x_3 + 1 = 0 \qquad \vec{\alpha} : -x_1 + x_2 + x_3 = 0$