ECE 438 Lab, division 1

Lab 07 (week 11): Discrete-Time Random Process (Week 2)

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Section 1.2

1

1. $Z = Y$. $\quad \rho_{XZ} = \dfrac{\mathbb{E}[(X-\mu_X)(Z-\mu_Z)]}{\sigma_X \sigma_Z} = \dfrac{\mathbb{E}[XZ] - \mu_X\mu_Z}{\sigma_X\sigma_Z} = \dfrac{\mathbb{E}[XY] - \mu_X\mu_Y}{\sigma_X\sigma_Y}$

$$= \frac{\mathbb{E}[X]\mathbb{E}[Y] - \mu_X\mu_Y}{\sigma_X\sigma_Y} = 0$$

2. $Z = (X+Y)/2$. $\quad \rho_{XZ} = \dfrac{\mathbb{E}[XZ] - \mu_X\mu_Z}{\sigma_X\sigma_Z}$ $\quad \mu_X = 0,\ \sigma_X^2 = 1$

$\mu_Z = \mathbb{E}[Z] = \mathbb{E}\left[\frac{X+Y}{2}\right] = \frac{1}{2}\mu_X + \frac{1}{2}\mu_Y = 0,$

$\sigma_Z^2 = \mathbb{E}[(Z-\mu_Z)^2] = \mathbb{E}\left[\frac{(X+Y)^2}{4}\right] = \frac{1}{4}\left(\mathbb{E}[X^2 + 2XY + Y^2]\right) = \frac{1}{4}\left(\mathbb{E}[X^2] + 2\mathbb{E}[XY] + \mathbb{E}[Y^2]\right)$

$= \frac{1}{4}\left(\sigma_X^2 + 2\sigma_X\sigma_Y + \sigma_Y^2\right) = \frac{1}{4}(1+1) = \frac{1}{2}.$

$\therefore \rho_{XZ} = \dfrac{\mathbb{E}\left[X \cdot \frac{X+Y}{2}\right] - 0}{1 \cdot \sqrt{\frac{1}{2}}} = \dfrac{\frac{1}{2}\left(\mathbb{E}[X^2] + \mathbb{E}[XY]\right)}{\frac{\sqrt{2}}{2}} = \frac{1}{\sqrt{2}}\left(\sigma_X^2 + \mu_X\mu_Y\right) = \frac{1}{\sqrt{2}} \cdot 1 = \frac{\sqrt{2}}{2}$

3. $Z = \dfrac{(4X+Y)}{5}$.

$\mu_Z = \frac{4}{5}\mu_X + \frac{1}{5}\mu_Y = 0.$

$\sigma_Z^2 = \mathbb{E}\left[\frac{16X^2 + 8XY + Y^2}{25}\right] = \frac{16}{25}\sigma_X^2 + \frac{8}{25}\mu_X\mu_Y + \frac{1}{25}\sigma_Y^2 = \frac{17}{25}$

$\therefore \rho_{XZ} = \dfrac{\mathbb{E}\left[X \cdot \frac{4X+Y}{5}\right] - \mu_X\mu_Z}{\sigma_X\sigma_Z} = \dfrac{\frac{4}{5}\mathbb{E}[X^2] + \frac{1}{5}\cancel{\mathbb{E}[XY]} - 0}{1 \cdot \sqrt{\frac{17}{25}}} = \frac{4}{17}\sqrt{17}$

4. $Z = \dfrac{99X+Y}{100}$.

$\mu_Z = 0$

$\sigma_Z^2 = \mathbb{E}\left[\frac{9801X^2 + 198XY + Y^2}{10000}\right] = \frac{9801}{10000}\sigma_X^2 + \frac{198}{10000}\mu_X\mu_Y + \frac{1}{10000}\sigma_Y^2 = \frac{4901}{5000}$

$\therefore \rho_{XZ} = \dfrac{\mathbb{E}\left[X \cdot \frac{99X+Y}{100}\right] - \mu_X\mu_Z}{\sigma_X\sigma_Z} = \dfrac{\frac{99}{100}\sigma_X^2 + \cancel{\frac{1}{100}\mu_X\mu_Y}}{1 \cdot \sqrt{\frac{4901}{5000}}} = 0.99994\,9.$

numerical estimation:

p1 =

-0.0276

p2 =

    0.6903

p3 =

    0.9686

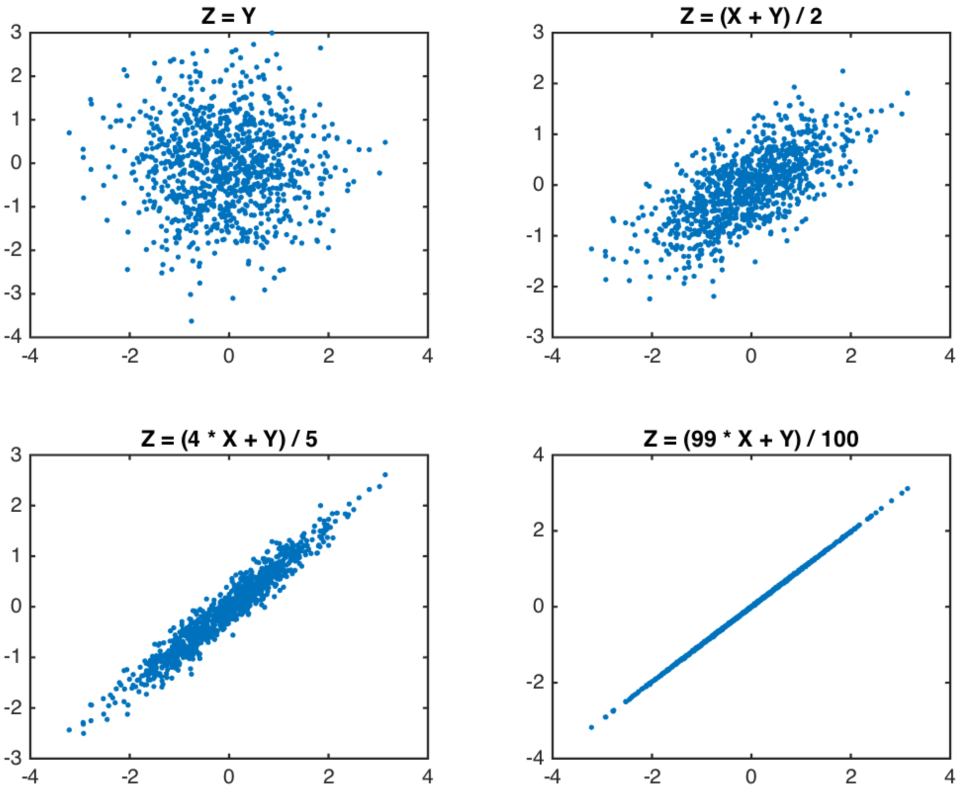
p4 =

    0.9999

2

Because pxz is the theoretical correlation coefficient, while pxz' is just an estimation using the samples. They are not exactly equal, but close enough.

3



4

The greater the pxz is, the plots is more close to the line: Z=X;

The smaller the pxz is, which means X and Z are less correlated, so the plot looks like a bunch of random dots.

Section 2

2.2

1)

2.2/1. $\cancel{r_{YY}(m) = E[Y_n Y_{n+m}] =}$

$$r_{XX}(m) = \sigma_X^2 \delta(m).$$

$$h(m) = \delta(m) - \delta(m-1) + \delta(m-2).$$

$$r_{YY}(m) = h(m) * h(-m) * r_{XX}(m)$$

$$= [\delta(m) - \delta(m-1) + \delta(m-2)] * [\delta(-m) - \delta(-m-1) + \delta(-m-2)] * \sigma_X^2 \delta(m)$$

$$= [\delta(m) - \delta(m-1) + \delta(m-2)] * [\delta(m) - \delta(m+1) + \delta(m+2)] * \sigma_X^2 \delta(m)$$

$$= \Big[\big(\delta(m) - \delta(m+1) + \delta(m+2)\big) - \big(\delta(m-1) - \delta(m) + \delta(m+1)\big) + \big(\delta(m-2) - \delta(m-1) + \delta(m)\big)\Big] * \sigma_X^2 \delta(m)$$

$$= \big(\underline{\delta(m)} - \underline{\delta(m+1)} + \delta(m+2) - \underline{\delta(m-1)} + \underline{\delta(m)} - \underline{\delta(m+1)} + \delta(m-2) - \underline{\delta(m-1)} + \underline{\delta(m)}\big] * \sigma_X^2 \delta(m)$$

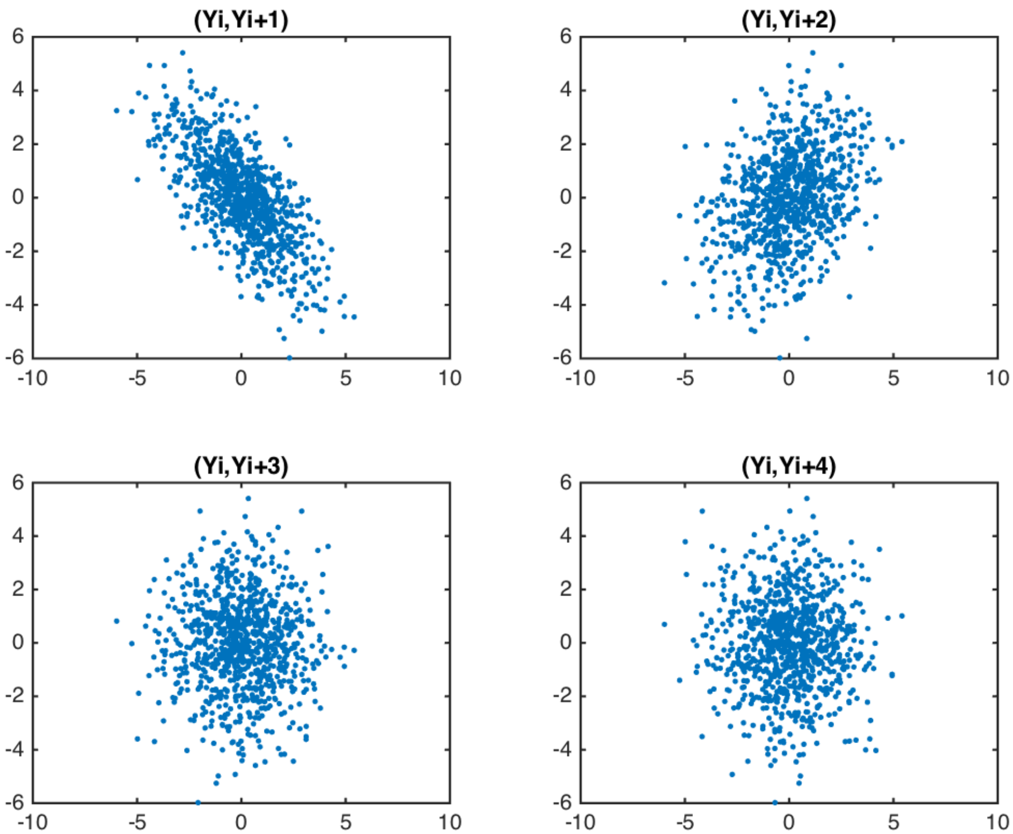
$$\cancel{= (3\delta(m) - 3\delta(m-1) + \delta(m+2) + \delta(m-2) - \delta(m+1)) * \sigma_X^2 \delta(m)}$$

$$= (3\delta(m) - 2\delta(m-1) - 2\delta(m+1) + \delta(m+2) + \delta(m-2))) * \sigma_X^2 \delta(m)$$

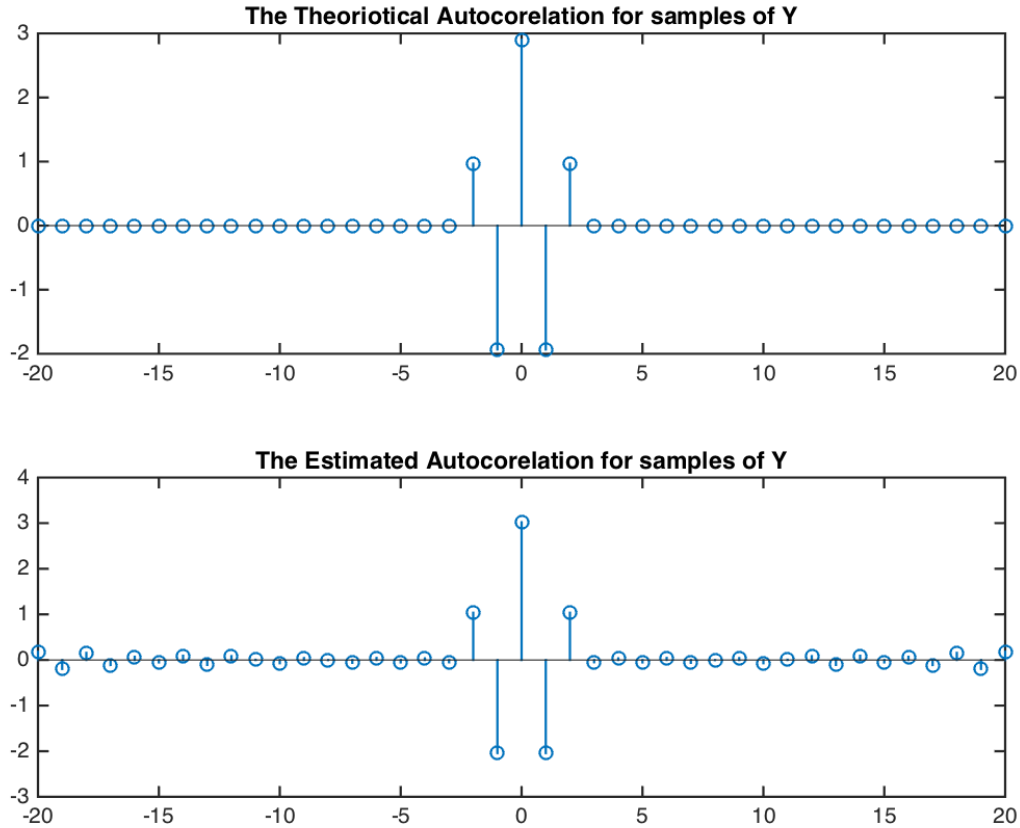
$$= \sigma_X^2 \big(3\delta(m) - 2\delta(m-1) - 2\delta(m+1) + \delta(m+2) + \delta(m-2)\big).$$

$$= 3\delta(m) - 2\delta(m-1) - 2\delta(m+1) + \delta(m+2) + \delta(m-2)$$

2)



3)



The equation (13) does produce a reasonable approximation of true auto correlation.

For both cases, when m = 0, ryy and ryy' both reach their maximum.

4)

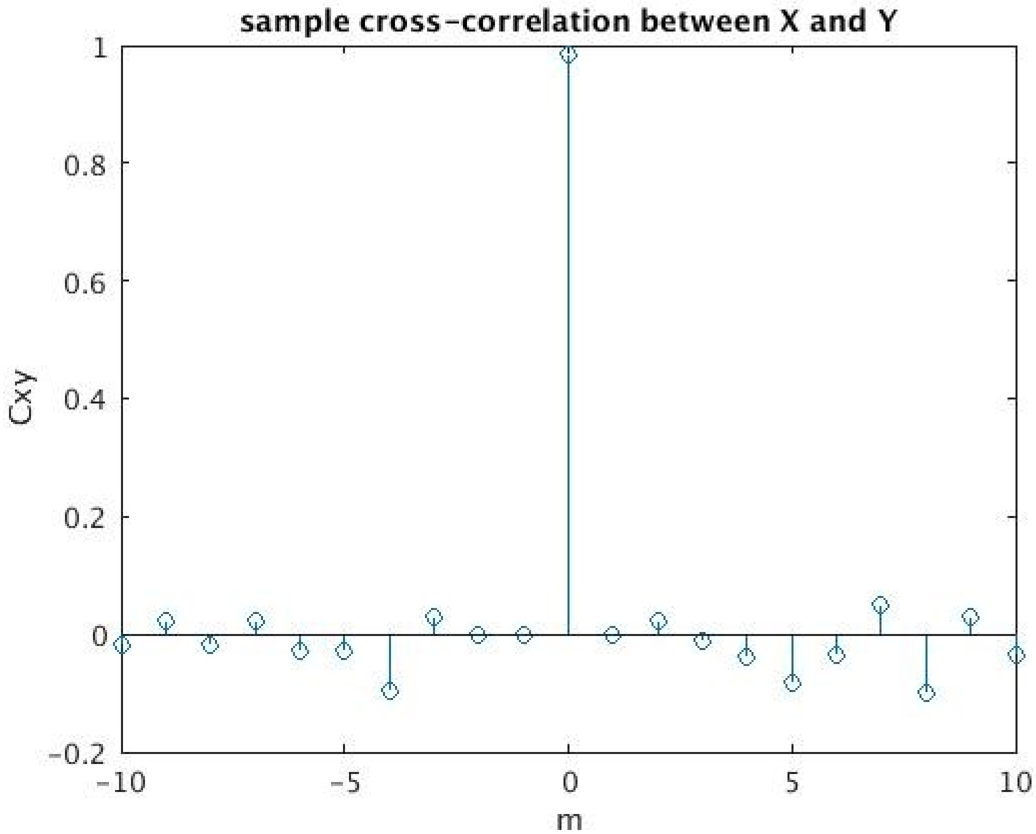
```
%2.2
N = 1000;
X = randn(1,N);
n = 1:1000;
Y = zeros(1,N);
Y(3:1000) = X(3:N) - X(2:N-1) + X(1:N-2);
figure (2)
subplot(2,2,1)
plot(Y(1:900),Y(2:901),'.')
```

```
title('(Yi,Yi+1)')
subplot(2,2,2)
plot(Y(1:900),Y(3:902),'.')
title('(Yi,Yi+2)')
subplot(2,2,3)
plot(Y(1:900),Y(4:903),'.')
title('(Yi,Yi+3)')
subplot(2,2,4)
plot(Y(1:900),Y(5:904),'.')
title('(Yi,Yi+4)')
saveas(gcf,'2.png')
%%
%Estimate sample autocorrelation
N = 1000;
X = randn(1,N);
Y = zeros(1,N);
Y(3:N) = X(3:N) - X(2:N-1) + X(1:N-2);
m = -20:20;
h1 = (m==0) - (m==1) + (m==2);
h2 = (m==0) - (m==-1) + (m==-2);
rxx = var(X).*(m == 0);
ryy_theory = conv(conv(h1,h2),rxx);

ryy_estimate = zeros(1,2*21-1);

part1 = 0;

for i = 1: (2*21 - 1)
    for n = 1 : (N - abs(m(i)))
    part1 = part1 + Y(n) * Y(n+abs(m(i)));
    end
    ryy_estimate(i) = 1 / (N - abs(m(i))) * part1;
    part1 = 0;
end
h1 = (m==0) - (m==1) + (m==2);
h2 = (m==0) - (m==-1) + (m==-2);
rxx = var(X).*(m == 0);
ryy_theory = conv(conv(h1,h2),rxx);
figure (3)
subplot(2,1,1)
stem(m,ryy_theory(41:81))
title('The Theoriotical Autocorelation for samples of Y')
subplot(2,1,2)
stem(m,ryy_estimate)
title('The Estimated Autocorelation for samples of Y')
```

Section 3
3.2

Part1

1)



2)

When m =0, the largest cross-correlation occurs. Because there is no delay between two random variables.

3)

No.

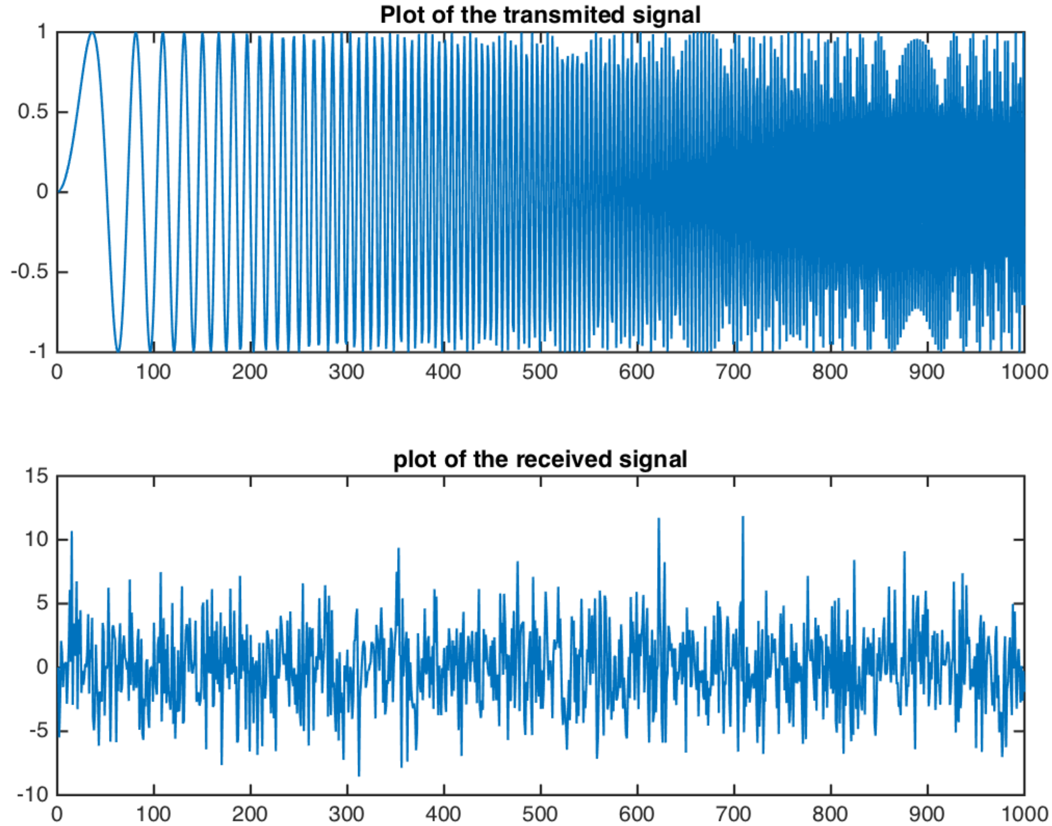
It's not necessary that Cxy(m) = Cxy(-m)

4)

```
function [C] = CorR(X,Y,m)
%CORR Summary of this function goes here

%   Detailed explanation goes here

N = length(X);

if (m >= 0 & m <= N-1)

    C = 1/(N-m)*X(1:N-m)*Y(1+m:N)';

elseif (m >= 1-N & m <0)

    C = 1/(N-abs(m))*X(abs(m)+1:N)*Y(1:N+m)';



end
```

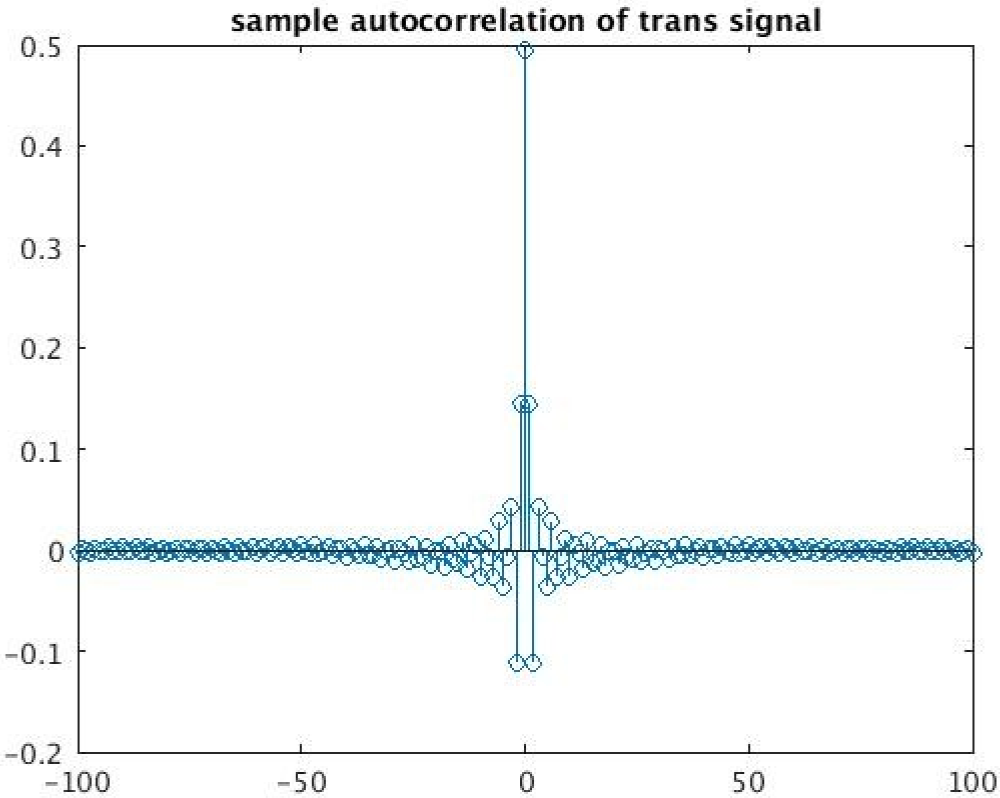
Part 2

1)



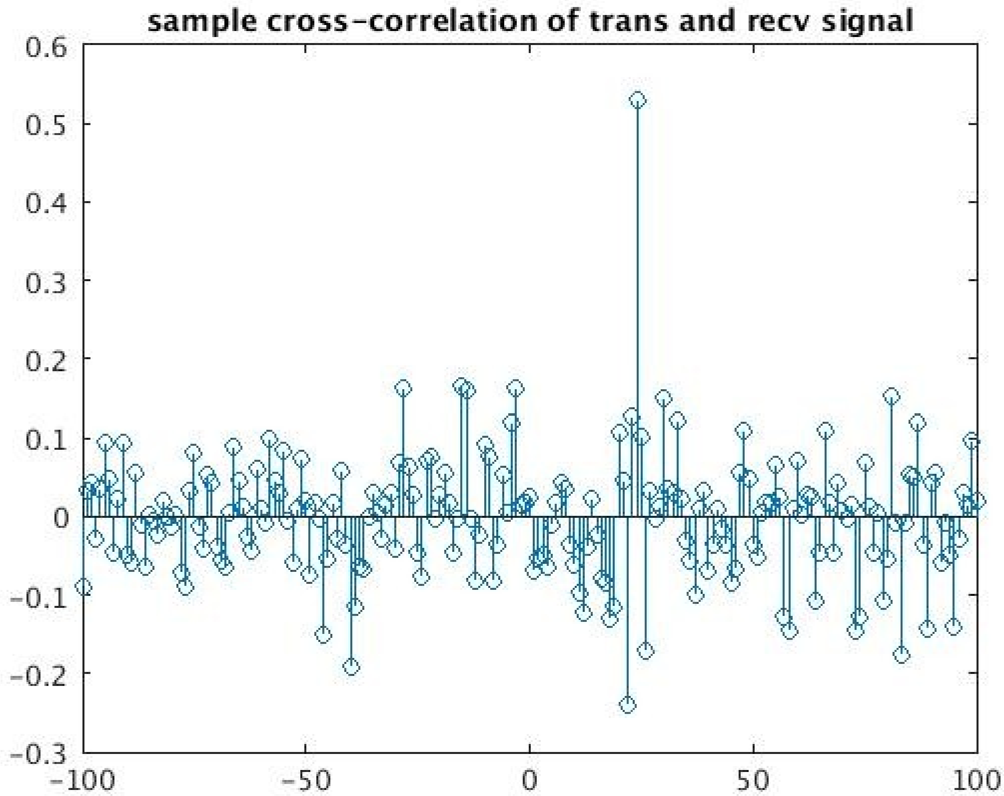
It is hard to estimate the delay by inspection.

2)



sample autocorrelation of trans signal
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-100
-50
0
50
100

3)



4)

D = the value of m when Cxy(m) is max:

```
D = find(cxy==max(cxy)) - 100
D =

   25
```