Math_8452_hw1

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Size 5

```r
#First under the null hypothesis.
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=5, rate = 1)
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated alpha is:", nrej1/nruns))
```

```
## [1] "Estimated alpha is: 0.115"
```

```r
#Now under an alternative.
shift=0.5
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=5, rate = 1) + shift
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated power is:", nrej1/nruns))
```

```
## [1] "Estimated power is: 0.056"
```

Size 10

```r
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=10, rate = 1)
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated alpha is:", nrej1/nruns))
```

```
## [1] "Estimated alpha is: 0.109"
```

```r
#Now under an alternative.
shift=0.5
set.seed(0)
nruns=1000
```

```
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=10, rate = 1) + shift
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated power is:", nrej1/nruns))
```

```
## [1] "Estimated power is: 0.189"
```

Size 20

```
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=20, rate = 1)
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated alpha is:", nrej1/nruns))
```

```
## [1] "Estimated alpha is: 0.089"
```

```
#Now under an alternative.
shift=0.5
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=20, rate = 1) + shift
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated power is:", nrej1/nruns))
```

```
## [1] "Estimated power is: 0.609"
```

size 35

```
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=35, rate = 1)
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated alpha is:", nrej1/nruns))
```

```
## [1] "Estimated alpha is: 0.064"
```

```
#Now under an alternative.
shift=0.5
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=35, rate = 1) + shift
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated power is:", nrej1/nruns))
```

[1] "Estimated power is: 0.924"

Size 50

```
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=50, rate = 1)
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated alpha is:", nrej1/nruns))
```

[1] "Estimated alpha is: 0.07"

```
#Now under an alternative.
shift=0.5
set.seed(0)
nruns=1000
nrej1=0
nrej2=0
for (run in 1:nruns){
  x=rexp(n=50, rate = 1) + shift
  if (t.test(x, mu=1)$p.value<0.05){nrej1=nrej1+1}}
print(paste("Estimated power is:", nrej1/nruns))
```

[1] "Estimated power is: 0.987"

Comment: The t-test is invalid for small samples from non-normal distributions, but it is valid for large samples from non-normal distributions. Non-normal population distributions, especially those that are thick-tailed or heavily skewed, considerably reduce the power of the test. Therefore, the test is robust when the sample size is large enough