

## HW #1

1) a)

C	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	69	77	92	92	92	95	96	96	99	103	115	117	
P(C)	0.0002	0.0003	0.016	0.054	0.121	0.193	0.226	0.193	0.121	0.054	0.016	0.0003	0.0002

$$C \sim \text{Bin}(12, 0.5)$$

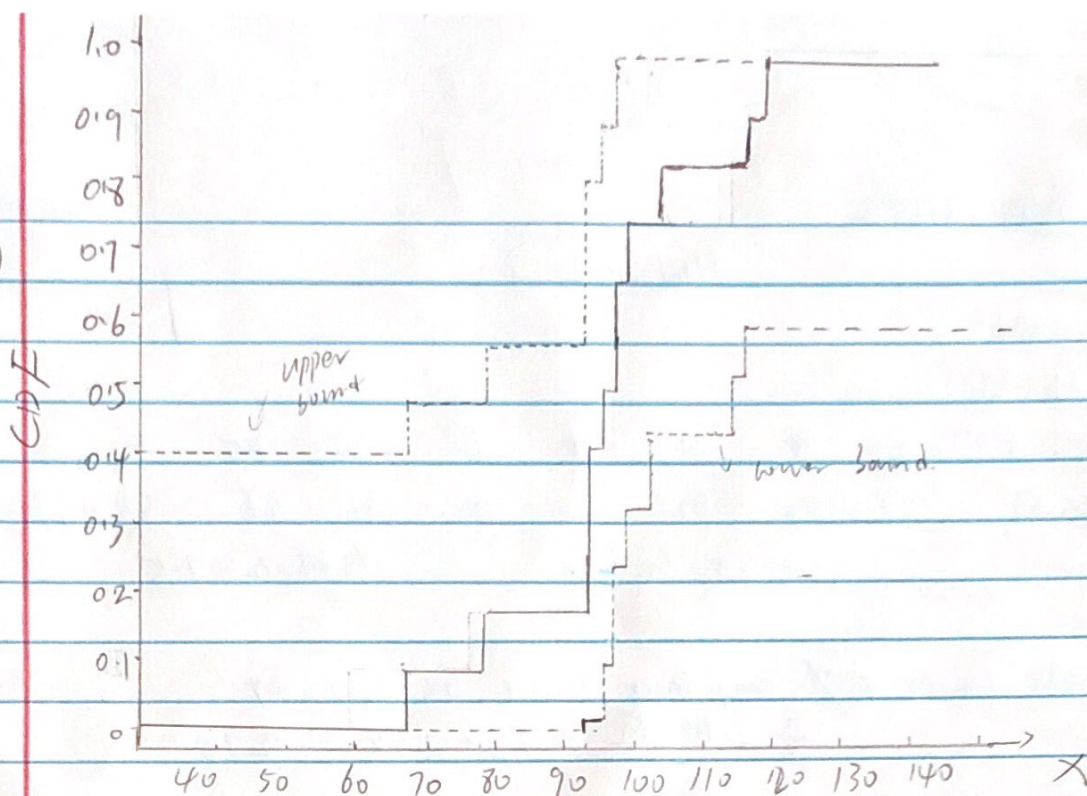
$$1 - [2(0.0002) + 2(0.0003) + 2(0.016) + 2(0.054)] = 0.1464$$

$$r = 4 \quad s = 9 \Rightarrow (X_4, X_9)$$

$$P(4 \leq C \leq 9) = 1 - 0.1464 = 0.8536$$

we are 85.36% confident that the median length of an issue is between 4 and 9 order statistics, which give the value between 92 and 99

b)



c) sample proportion:  $\hat{p} = \frac{3}{12} = \frac{1}{4}$

$$\text{Agresti-Coull CI: } \tilde{p} = \frac{3 + 1.92}{12 + 2(1.92)} \approx 0.311$$

$$\text{and } \tilde{n} = 12 + 2(1.92) = 15.84$$

$$A-C \text{ CI: } 0.311 \pm 1.96 \sqrt{\frac{0.311(1-0.311)}{15.84}}$$

$$\approx (0.082, 0.539)$$

we are 95% confident that the proportion of  
issue length that exceed 100 pages is between  
0.082 and 0.539.



d)  $H_0: \theta = 100$        $H_a: \theta < 100$

Where  $\theta$  is the median length of an issue

Row  $x_i$   $x_i - x_0$

1 69 -31

$B = 3$

2 92 -8

$P = P(B \leq 3 | B \sim \text{Bin}(12, 0.5))$

3 92 -8

4 99 -1

$= \binom{12}{1}(0.5)^{12} + \binom{12}{2}(0.5)^{12} + \binom{12}{3}(0.5)^{12}$

5 96 -4

$= 0.0029 + 0.061 + 0.0537$

6 115 15

$= 0.073$

7 95 -5

Since  $P$  value  $0.073 > 0.05$  Therefore

8 96 -4

We fail to reject  $H_0$  and can't conclude

9 103 3

$H_a$ , that is, we don't have enough evidence

10 92 -8

at 0.05 level to conclude that

11 77 -23

the median length of an issue

12 117 17

is less than 100 pages.



e)  $H_0: \theta_{0.4} = 70$ ,  $H_a: \theta_{0.4} \neq 70$   
 $B \sim \text{Bin}(12, 0.6)$

Let  $B = \# \{X_i > 70\}$  we know they are  
 11 Value exceeding 70

upper-tailed p value:

$$P(11) + P(12) = \binom{12}{11} (0.6)^{11} (0.4)^1 + \binom{12}{12} (0.6)^{12} \\ = 0.0196$$

Two-tail p value:  $2(0.0196) = 0.0392$

Since  $0.0392 < 0.1$  we reject  $H_0$  and  
 conclude  $H_a$ , that is, we have enough evidence  
 at 0.1 level to conclude that 40th percentile  
 is not 70 pages.

2)

C	0	1	2	3	4	5	6	7	8	9
P(C)	0.04	0.156	0.267	0.267	0.172	0.074	0.021	0.004	0	0

$C \sim \text{Bin}(9, 0.3)$

$$0.04 + 0 + 0 + 0.004 + 0.021 = 0.065, \quad 1 - 0.065 = 0.935$$

$$r=1 \quad S=6 \Rightarrow (X_1, X_6)$$

C	0	1	2	3	4	5	6	7	8	9
P(C)	0.002	0.018	0.07	0.164	0.246	0.246	0.164	0.07	0.018	0.002

$C \sim \text{Bin}(9, 0.5)$

$$2(0.002) + 2(0.018) = 0.04$$

$$r=2, \quad S=8 \Rightarrow (X_2, X_8)$$

$$1 - 0.04 = 0.96$$