MATH 8452 Hai Du

HW # 4

1) The # of possibilities to consider is $\frac{6!}{2!2!2!} \cdot \frac{1}{3!} = 15$

1	2	3	KW
1, 2	3, 4	5 6	$\frac{12}{6\times7}\left[2(1.5-3.5)^2+2(3.5-3.5)^2+2(5.5-3.5)^2\right]=4.57$
1 2	3 5	4 6	3.71
1 2	3 6	4 5	3.43
1 3	2 4	5 6	3.71
1 3	2 5	4 6	2.57
1 3	2 6	4 5	2
1 4	2 3	5 6	3.43
1 4	2 5	3 6	1.14
1 4	2 6	3 5	0.86
1 5	2 3	4 6	2
1 5	2 4	3 6	0.86
1 5	2 6	3 4	0.29
1 6	2 3	4 5	1.14
1 6	2 4	3 5	0.29
1 6	2 5	3 4	0

KW	probability
4.57	1/15
3.71	2/15
3.43	2/15
2.57	1/15
2	2/15

KW	probability
1.14	2/15
0.86	2/15
0.29	2/15
0	1/15

5) They are 4 value total, and which sample is which will matter. ⇒ they are $\frac{4!}{2!1!1!} = 12$

possibility	JT Value
1 2 3 4	1+2+2 = 5
1 3 2 4	1+2+1 = 4
1 4 2 3	3
2 1 3 4	4
2 3 1 4	3
2 4 1 3	2
3 1 2 4	3
3 2 1 4	2
3 4 1 2	1
4 1 2 3	2
4 2 1 3	1
4 3 1 2	0

JT Value	Probability
0	1/12
1	2/12
2	2/12
3	3/12
4	2/12
5	1/12

7) H_0: The three population are the same

H_a: The three population are not all the same

$$\frac{5!}{2!2!1!} \cdot \frac{1}{2!} = 15$$

0, 5, 10, 15, 20

permutation	TS
0, 5 10, 15 20	$\frac{0^2}{1} + \frac{15^2}{2} + \frac{35^2}{2} = 725$
0, 5 15, 10 20	650
0, 5 20, 10 15	625
5, 0 10, 15 20,	687.5
5, 0 15, 10 20	587.5
5, 0 20, 10, 15	537.5
10, 0 5, 15 20	725
10, 0 15, 5 20	525
10, 0 20, 5 15	500
15, 0 5, 10 20	687.5
15, 0 10, 5 20	587.5
15, 0 20, 5 10	537.5
20, 0 5, 10 15	725
20, 0 15, 5 10	625
20, 0 10 5 15	650

Test statistic: $10^2 + \frac{35^2}{2} + \frac{5^2}{2} = 725$

p value: $P(TS \geq 725 \mid H_0) = \frac{3}{15} = 0.2$

Since the p value is $0.2 \leq \alpha = 0.2$, we reject H_0 and conclude H_a that is, we have enough evidence at 0.2 level to conclud that three population are all the same