Math 8452 Hai DU

Test

1) The number of possibility to consider is $\binom{3+3}{3} = \binom{6}{3} = 20$

	possibility (1 2 3 4 5 6)	W
1	X X X Y Y Y	4+5+6=15
2	X X Y X Y Y	3+5+6=14
3	X X Y Y X Y	3+4+6=13
4	X X Y Y Y X	3+4+5=12
5	X Y X X Y Y	2+5+6=13
6	X Y X Y X Y	2+4+6=12
7	X Y X Y Y X	2+4+5=11
8	X Y Y X X Y	2+3+6=11
9	X Y Y X Y X	2+3+5=10
10	X Y Y Y X X	2+3+4=9
	~~X Y Y Y X X~~	
11	Y Y Y X X X	1+2+3=6
12	Y Y X Y X X	1+2+4=7
13	Y Y X X Y X	1+2+5=8
14	Y Y X X X Y	1+2+6=9
15	Y X X X Y Y	1+5+6=12
16	Y X X Y X Y	1+4+6=11
17	Y X X Y Y X	1+4+5=10
18	Y X Y X Y X	1+3+5=9
19	Y X Y X X Y	1+3+6=10
20	Y X Y Y X X	1+3+4=8

	W	null probability
0.1	6	1/20
0.1	7	1/20
	8	2/20
	9	3/20
	10	3/20
	11	3/20
	12	3/20
	13	2/20
0.1	14	1/20
0.1	15	1/20

Sinle it is two tail test

$\Delta > 0$ and $\alpha = 0.2$

then

$P(W=6+7+14+15) = \frac{4}{20} = 0.2$

we would reject H_0 when

$W \leq 7$ or $W \geq 14$

2) $\binom{3+2}{3} = \binom{5}{3} = 10$ possibilities

possibility (1 2 3 4 5)	# of value from first sample > second sample
X X X Y Y	0
X Y X X Y	2
X Y X Y X	2
X Y Y X X	2
X X Y X Y	1
X X Y Y X	1
Y Y X X X	3
Y X X Y X	3
Y X X X Y	3
Y X Y X X	3

x: 1 3 4
y: 2 5
y: 1 3 5
y: 2 4
x: 1 4 5
y: 2 3
1 2 4
3 5
1 2 5
3 4
3 4 5
1 2
2 3 5
1 4
2 3 4
1 5
2 4 5
1 3

Value	prob
0	1/10
1	2/10
2	3/10
3	4/10

3) H_0: Listening to statistics talks has no effect on milk yield

a) H_a: Listening to statistics talks has effect on milk yield

Cow	Yield before	Yield after	diff	R_i	ψ_i	$R_i\psi_i$
1	10	12	2	1	1	1
2	15	19	4	3	1	3
3	12	15	3	2	1	2

$T^+ = 6$

Rank 1	Rank 2	Rank 3	T^+
+	+	+	6
+	+	−	3
+	−	+	4
+	−	−	1
−	+	+	5
−	+	−	2
−	−	+	3
−	−	−	0

$T+$	Prob
0	1/8
1	1/8
2	1/8
3	2/8
4	1/8
5	1/8
6	1/8
	1

$\therefore 2P(T^+ \geq 6 | H_0) = 2/8 = 0.25$. we use signed Rank test to obtain $T^+ = 6$ with P value 0.25, since P value $0.25 \leq \alpha = 0.25$ we reject H_0 and Conclud H_a, that is, we have enough evidence at 0.25 level to Conclud that listening to statistics talks has effect on milk yield

3b) H_0: No different in median of milk yield
H_a: True median difference of mild yield is not equal to 0

Cow	Yield befor	Yield after	after-befor	Sign
1	10	12	2	+
2	15	19	4	+
3	12	15	3	+

C	0	1	2	3
P(c)	0.125	0.375	0.375	0.125

$C \sim Bin(3, 0.5)$

upper tail p value: $P(C \geq 3) = 0.125$

two taild p value: $2(0.125) = 0.125$

We can also use Sign test, and [illegible]

A Similar Conclusion can be drawn from part (3a)

4)

X	1	2	3	4	5
Y	-1	-2	-3	-4	-50

$r = -0.743 > -1$

$r_s = -1$

The pearson's corelation r is not at all robust to outlier
The Spearman's rank correlation defined the corelation between the ranks of the X value and ranks of Y Varne. is robust to the outlier, we have outlier -50 in the data set. Thus r_s tend to have stronger association between two variable x and Y

8)

a) $H_0: \theta_{0.5} = 0.5$ vs $H_a: \theta_{0.5} > 0.5$

#	X_i	$X_i - X_0$
1	0.98	0.45
2	0.381	-0.119
3	0.842	0.342
4	0.045	-0.455
5	0.505	0.005
6	0.697	0.197
7	0.773	0.273
8	0.529	0.029

$B = 6 = \#$ values above 0.5

p value $= P(B \geq 6 \mid B \sim Bin(8, \frac{1}{2}))$

$= P(6) + P(7) + P(8)$

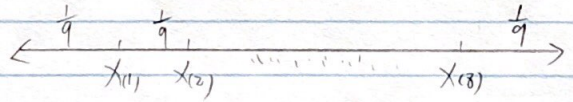
$= 0.11 + 0.031 + 0.004$

$= 0.145$

Since $0.145 > 0.05$, we retain H_0.

We don't have enough evidence to conclude at level 0.05 that the population median exceeds 0.5

b)



The interval $(X_{(1)}, X_{(8)}) = (0.980, 0.529)$ has exact coverage $\frac{7}{9} \approx 77.8\% > 75\%$

Thus, we use this interval as an 75% prediction interval for the next value

See 5) 6) 7) in R markdown