31/1/16 Logarithms

$a^b = c$ ← exponential form.

$\log_a c = b$ ← logarithmic form.

↑ base of the log.

Laws

1) $\log a + \log b = \log (a \times b)$ if bases are same.

↓ base is 10 if nothing is given.

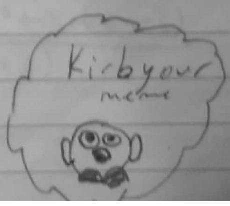
e.g. $\log 3 + \log 2$
$= \log (3 \times 2)$
$= \log 6$

2) $\dfrac{\log a}{\log b} = \log (a - b)$ * $\log \frac{a}{b} = \log (a - b)$

3) $\log_a a = 1$ when base and no. equal

4) $\log 1 = 0$

5) $\log a^m = m \log a$



Log Laws cont.

6) $a^{\log_a x} = x$

7) $\log_b(\sqrt[y]{x}) = \frac{\log_b x}{y}$

8) $x^{\log_b y} = y^{\log_b x}$

9) $\log_b a = \frac{1}{\log_a b}$

10) $\log_{b^n} a = \frac{\log_b a}{n}$

11) $\log_a b \times \log_b a = 1$

12) $\log_{\frac{1}{a}} M = \log_a \frac{1}{M} = -\log_a M$
$= \log_a M^{-1}$

14/2/17 Add and Subtract Algebraic Fractions

Examples

1) $\frac{x+1}{6} + \frac{x+4}{4}$

$= \frac{2(x+1) + 3(x+4)}{12}$

$= \frac{2x+2+3x+12}{12} = \frac{5x+14}{12}$

2) $\frac{2}{3x} - \frac{1}{4x}$

$= \frac{4 \times 2 - 3 \times 1}{12x} = \frac{5}{12x}$

3) $\frac{x+1}{x+3} + \frac{2x-1}{x+2}$

$= \frac{(x+2)(x+1) + (x+3)(2x-1)}{(x+3)(x+2)}$

$= \frac{(x^2+x+2x+2)+(x^2+-x+6x+-3)}{(x+3)(x+2)}$

$= \frac{3x^2+8x-1}{(x+3)(x+2)}$

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14/2/17

Binomial Expansion

Basic rule for expansion -

FOIL

Front Outer Inner Last

$(a+b)\ (c+d)$

$\Rightarrow ac + ad + bc + bd$

Shortcuts -

$(a+b)(a-b)$

$= a^2 - b^2$

$(a+b)(a+b) = (a+b)^2$

$= a^2 + 2ab + b^2$

$(a-b)(a-b) = (a-b)^2$

$= a^2 - 2ab + b^2$

16/2/17

Multiply and divide algebraic frac

Examples.

1) $\dfrac{4}{(x+1)(3x-5)} \div \dfrac{x-7}{x+1}$

$= \dfrac{4}{(x+1)(3x-5)} \times \dfrac{(x+1)}{(x-7)}$

$= \dfrac{4}{(3x-5)(x-7)}$

2) $\dfrac{3xy}{2} \div \dfrac{4x}{9y}$

$= \dfrac{3xy}{2} \times \dfrac{9y}{4x}$

$= \dfrac{27y^2}{8}$

Expanding Brackets

1) $(x+2)^2$

$= 4x + x^2 + 4$

2) $(2x-5)^2$

$= 4x^2 - 20x + 25$

3) $-4(2x+7)^2$

$= -4(4x^2 + 28x + 49)$

$= -16x^2 - 112x - 196$

Factorising

① Common Factor

a) $6x^3 + 3x^2 + 9x^4$

$= 3x^2(2x + 1 + 3x^2)$

b) $3x(a+b) - 6(a+b)$

$= 3(a+b)[x-2]$

② Difference between 2 perfect squares

a) $x^2 - 81y^2$

$= (x+9y)(x-9y)$

$x^2 - y^2 = (x+y)(x-y)$

b) $16x^2 - 9y^2$

$= (4x+3y)(4x-3y)$

c) $16(a+b)^2 - 81$

$= [4(a+b)-9][4(a+b)+9]$

d) $x^4 - y^4$

$= (x^2-y^2)(x^2+y^2)$

$= (x+y)(x-y)(x^2+y^2)$

e) $x^2 - 11$

$= (x+\sqrt{11})(x-\sqrt{11})$

Ctd

Factorising Ctd.

3 Trinomial

Type: $ax^2 + bx + c$, $a = 1$

a) $x^2 + 3x + 2$

$2 = 1 \times 2$

$1 + 2 = 3$

$= (x \quad)(x \quad)$

$= (x+3)(x+2)$ ← Wrong. $= (x+2)(x+1)$

Type: $ax^2 + bx + c$

a) $10x^2 - x - 2$

$5 \times 2 = 10$ $\quad$ $2 \times -1 = -2$

1

$5 \times 2 = 4$

$2 \times -1 = -5$ (cross multiply)

$4 + -5 = -1$

$-x = \boxed{-1}x$

2

$\therefore = (5x+2)(2x-1)$

Completing the Square

1) $x^2 + 4x + 5$

This cannot be factorised with method 3.

$x^2 \quad +4x \quad +5$ $\quad$ $a = 1$

Notice b is 4. $\quad$ $b = 4$ $\quad$ $\frac{1}{2}b = 2$

$c = 5$

$= x^2 + 4x + (\frac{1}{2} \cdot 4)^2 - (\frac{1}{2} \cdot 4)^2 + 5$

We add and subtract $(\frac{1}{2} \text{ of } b)^2$. This does not change the expression.

$= x^2 + 4x + 4 - 4 + 5$

Simplify. Do not evaluate $+4 - 4$.

$= (x+2)^2 - 4 + 5$

Those first 3 terms will always be a perfect square trinomial, so factorise.

$= (x+2)^2 + 1$

Evaluate.

Note - Using Null Factor Law

$x^2 + x = 12$

$x(x+1) = 12$ $\quad$ Wrong. RHS needs to be 0 to use N.F.L

$\therefore x = 12, \; x + 1 = 12$

$\therefore x = 11$

Next page →

2) $x^2 - 7x - 9$

Cannot be factorised using method ⑤.

$= x^2 - 7x + (\frac{1}{2} \cdot 7)^2 - (\frac{1}{2} \cdot 7)^2 - 9$

Same process. $(-7)^2 = (7)^2$ so writing '-' is unecessary.

$= x^2 - 7x + (\frac{7}{2})^2 - (\frac{7}{2})^2 - 9$

Simplify.

$= (x - \frac{7}{2})^2 - (\frac{7}{2})^2 - 9$

$= (x - \frac{7}{2})^2 - \frac{49}{4} - 9$

$= (x - \frac{7}{2})^2 - 21\frac{1}{4}$

3) $2x^2 + 3x - 7$

$= 2(x^2 + \frac{3}{2}x - \frac{7}{2})$

Take 2 out. a needs to be 1 for this to work, then continue as usual.

$= 2[x^2 + \frac{3}{2}x + (\frac{1}{2} \cdot \frac{3}{2})^2 - (\frac{1}{2} \cdot \frac{3}{2})^2 - \frac{7}{2}]$

$= 2[(x + \frac{3}{4})^2 - (\frac{3}{4})^2 - \frac{7}{2}]$

$= 2[(x + \frac{3}{4})^2 - \frac{9}{16} - \frac{7}{2}]$

$= 2[(x + \frac{3}{4})^2 - \frac{65}{16}]$

$= 2(x + \frac{3}{4})^2 - \frac{65}{8}$

Expand and simplify.

Quadratic Functions (Equations)

Solving of quadratic functions.

(purpose is to find x-intercept of a graph)

3 methods:

① Factorise

a) $x^2 - 13x + 42 = \boxed{0}$ since we find x-int, $y = 0$

$= (x-7)(x-6) = 0$

$\therefore x - 7 = 0,\ x - 6 = 0$

$x = 7,\ x = 6$ two x-ints.

Always make sure the trinomial equals 0 on RHS.

② completing the square

a) $x^2 + 2x - 4 = 0$

$0 = x^2 + 2x + (1)^2 - (1)^2 - 4$

$0 = (x+1)^2 - 5$

$\therefore 5 = (x+1)^2$

$x + 1 = \pm\sqrt{5}$

$x = \pm\sqrt{5} - 1$

General form

$(x - p)^2 + q = 0$

The $\pm$ shows that there are two roots, one with $+\sqrt{5}$ and other with $-\sqrt{5}$.

Next page.

③ Quadratic Formula

$$y = ax^2 + bx + c$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) $3x^2 + 4x + 1 = 0$

$a = 3$

$b = 4$ $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$c = 1$

$$\therefore x = \frac{-4 \pm \sqrt{4^2 - [3 \times 4 \times 1]}}{2 \times 3}$$

$$= \frac{-4 \pm \sqrt{16 - 12}}{6}$$

$$= \frac{-4 \pm \sqrt{4}}{6}$$

$x = \frac{-4+2}{6} = \frac{-1}{3}$ and

$x = \frac{-4-2}{6} = -1.$

The Discriminant.

$$\Delta = b^2 - 4ac$$

1. Condition: $\underline{\Delta < 0}$
   - Δ is negative
   - Equn. has no real roots ($\underline{x\text{-ints}}$)
   - The graph does not touch the x-axis.
2. Condition: $\underline{\Delta = 0}$
   - Δ is not negative or +ve.
   - Equn has 1 rational root
   - The graph touches the x-axis once.
3. Condition: $\underline{\Delta > 0}$
   - Δ is positive.

   a) Condition: Δ is a perfect square.
   - Equn. has 2 rational roots.
   - It touches the x-axis twice

   b) Condition: Δ is not a perfect square.
   - Equn, has 2 irrational roots, and has a surd solution
   - It touches the x-axis twice.

Examples on next page.

Discriminant Examples

1. Find the nature of the roots of:

a) $x^2 - 9x - 10 = 0$ $a = 1, b = -9, c = -10$

$\Delta = b^2 - 4ac$ Write the discriminant formula.

$\Delta = (-9)^2 - 4(1)(-10)$ Substitute values into formula.

$\Delta = 81 + 40$ Simplify.

$\Delta = 121$ Evaluate.

$\Delta > 0$, Δ is perfect squ. Give info on Δ (delta).

2 rational roots.

b) $x^2 + 14x = -49$

$\therefore x^2 + 14x + 49 = 0$ $a = 1, b = 14, c = 49$ Expression must $= 0$.

$\Delta = b^2 - 4ac$ Write the discriminant formula.

$\Delta = (14)^2 - 4(1)(49)$ Substitute values into formula.

$\Delta = 196 - 196$ Simplify.

$\Delta = 0$ Evaluate.

$\Delta = 0$, 1 rational root. Describe discriminant.

Simultaneous Equations

a) $y = x^2 + 2x + 2$ ① and $y = 7 - 2x$ ②

Let ① = ②.

$x^2 + 2x + 2 = 7 - 2x$ ① = ②, ① = y, ② = y

$x^2 + 2x + 2x + 2 - 7 = 0$ LHS = 0 to factorise.

$x^2 + 4x - 5 = 0$ Simplify.

$(x+5)(x-1) = 0$ Factorise w/ method 3a.

Null Factor Law: $ab = 0$, $a = 0$ or $b = 0$

$x + 5 = 0$, $x - 1 = 0$ Use null. f. L on both terms

$x = -5$, $x = 1$ Evaluate.

Subst. $x = \underline{-5}$

$y = 7 - 2x$

$= 7 - 2(-5)$

$= 17$ Evaluate.

$P_1(-5, 17)$

Subst. $x = \underline{1}$

$y = 7 - 2x$

$= 7 - 2(1)$

$= 5$ Evaluate.

$P_2(1, 5)$

b) on next page.

b) Show that $y_1 = x^2 + x + 4$ and $y_2 = 2x - 1$ won't intersect.

Assume they will: $y_1 = y_2$ (proof by contradiction)

$x^2 + x + 4 = 2x - 1$

$x^2 + 2x + x + 4 + 1 = 0$ Make RHS $= 0$ to factorise.

$x^2 - x + 5 = 0$ Simplify.

$\Delta = b^2 - 4ac$ Write discriminant formula.

$= (-1)^2 - 4(1)(5)$ Subst. a, b & c from equation.

$= -19$

$\Delta < 0$, no int.

Operations with surds

a) $\sqrt{384}$

$= \sqrt{2\times2\times3\times2\times2\times2\times2\times2}$ Break up into prime fac.

$= \sqrt{4\times4\times4\times3\times2}$ Group together perfect squares

$= 2\times2\times2\times\sqrt{3\times2}$ Simplify perfect squares.

$= 8\sqrt{6}$ simplify.

b) $3\sqrt{405}$

$= 3\sqrt{3\times3\times3\times3\times5}$

$= 3\sqrt{9\times9\times5}$

$= 3\times3\times3\sqrt{5}$

$= 27\sqrt{5}$

d) $5\sqrt{3} + 2\sqrt{12} - 5\sqrt{2} + 3\sqrt{8}$

$= 5\sqrt{3} + 2\sqrt{4\times3} - 5\sqrt{2} + 3\sqrt{4\times2}$ Break up into p.sq.

$= 5\sqrt{3} + 4\sqrt{3} - 5\sqrt{2} + 6\sqrt{2}$ Simplify sq.

$= 9\sqrt{3} + \sqrt{2}$ Add.

Multiply and Divide Surds

Rules:

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
2. $m\sqrt{a} \times n\sqrt{b} = mn\sqrt{ab}$
3. $\sqrt{a} \div \sqrt{b} = \sqrt{a/b}$
4. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
5. $\sqrt{a} \times \sqrt{a} = a$

Examples (surds)

Simplify:

a) $\sqrt{11} \times \sqrt{7}$

$= \sqrt{11 \times 7}$ Rule 1.

$= \sqrt{77}$ Simplify.

b) $6\sqrt{12} \times 2\sqrt{6}$

$= 12\sqrt{12 \times 6}$ Rule 2.

$= 12\sqrt{4 \times 3 \times 2 \times 3}$ Break into primes,

$= 12 \times 2 \times 3\sqrt{2}$ Take out 4 and 9.

$= 72\sqrt{2}$ Simplify.

ctd.

c) $\sqrt{15x^5y^2} \times \sqrt{12x^2y}$

$= \sqrt{5 \times 3 \times 4 \times 3 x^7 y^3}$ Break up, add powers.

$= 3 \times 2 x^3 y \sqrt{5xy}$ Take out 9, 4, x^6 and y^2

$= 6x^3y\sqrt{5xy}$ Simplify

d) $(\sqrt{2} + 1)(\sqrt{2} - 3)$

$= \sqrt{2} \times \sqrt{2} - 3\sqrt{2} + \sqrt{2} - 3$ FOIL method.

$= 2 - 3\sqrt{2} + \sqrt{2} - 3$ Rule 5.

$= -1 - 2\sqrt{2}$ Simplify.

e) $\frac{\sqrt{48}}{\sqrt{3}}$

$= \sqrt{\frac{48}{3}}$ Rule 4.

$= \sqrt{16}$ Evaluate.

$= 4$ Simplify

Rationalising Surds

- Making / changing denominator to a rational number.

Examples:

Rationalise the denominator.

a) $\dfrac{3}{\sqrt{7}}$

$= \dfrac{3}{\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}}$ Times by the denominator over the denominator. This = 1.

$= \dfrac{3\sqrt{7}}{7}$ Simplify.

b) $\dfrac{2\sqrt{12}}{3\sqrt{54}}$

$= \dfrac{2}{3}\sqrt{\dfrac{12}{54}}$ Rule 4.

$= \dfrac{2}{3}\sqrt{\dfrac{2}{9}}$ $\frac{12}{54} = \frac{2}{9}$.

$= \dfrac{2}{3} \times \dfrac{\sqrt{2}}{\sqrt{9}}$ Rule 4.

$= \dfrac{2\sqrt{2}}{9}$ Simplify.

Rationalising Using Conjugate Surds

The 'conjugate' of $a+b$ is $a-b$. Similarly, the conjugate of $a-b$ is $a+b$.

Examples:

Rationalise the denominator.

a) $\dfrac{1}{4-\sqrt{3}}$

$= \dfrac{1}{4-\sqrt{3}} \times \dfrac{4+\sqrt{3}}{4+\sqrt{3}}$ Multiply by the conjugate of the denom.

$= \dfrac{4+\sqrt{3}}{(4+\sqrt{3})(4-\sqrt{3})}$ Simplify.

$= \dfrac{4+\sqrt{3}}{16-3}$ Don't cancel out $4+\sqrt{3}$. Expand denominator.

$= \dfrac{4+\sqrt{3}}{13}$ Simplify.

b) $\dfrac{\sqrt{6}+3\sqrt{2}}{3+\sqrt{3}}$

$= \dfrac{\sqrt{6}+3\sqrt{2}}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}}$ Multiply by the conjugate of the den.

$= \dfrac{(\sqrt{6}+3\sqrt{2})(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$ Share vinculum.

$= \dfrac{3\sqrt{6}-\sqrt{18}+9\sqrt{2}-3\sqrt{6}}{9-3}$ Fully expand.

$= \dfrac{-\sqrt{18}+9\sqrt{2}}{6}$ Simplify.

Rationalising using conjugate surds ctd.

...b) $\dfrac{\sqrt{6}+3\sqrt{2}}{3+\sqrt{3}}$

$= \dfrac{-\sqrt{18}+9\sqrt{2}}{6}$ Prev. pg.

$= \dfrac{-\sqrt{9\times 2}+9\sqrt{2}}{6}$ $18 = 9\times 2$. Break into primes.

$= \dfrac{-3\sqrt{2}+9\sqrt{2}}{6}$ Take 9 out.

$= \dfrac{6\sqrt{2}}{6}$ Simplify.

$= \sqrt{2}$ Cancel out 6.

Logarithms (again)

$y = a^x$ then $\underbrace{\log_a(y) = x}_{\text{Log form}}$

exponential form $a^x = y$

Write in index form.

a) $\log_2 8 = 3$

$2^3 = 8$

b) $\log_{25} 5 = \frac{1}{2}$

$25^{\frac{1}{2}} = 5$

Evaluate logs.

a) $\log_3 81$

$= \log_3 3^4$

$= 4\log_3 3$

$= 4$

b) $\log_4 20 - \log_4 5$

$= \log_4 (20/5)$

$= \log_4 4$

$= 1$

c) $2\log_6 3 + \log_6 4$

$= \log_6 (3^2 \times 4)$

$= \log_6 36$

$= \log_6 6^2$

$= 2\log_6 6$

$= 2$

Laws

1. $\log_a x + \log_a y = \log_a (xy)$
2. $\log_a x - \log_a y = \log_a (x/y)$
3. $\log_a 1 = 0$
4. $\log_a m^n = n\log_a m$
5. $\log_a a = 1$
6. $\log_a (1/x) = -\log_a x$
7. $\log_a a^x = x$

More laws are found at the front.

Change-of-base rule

$\log_a x = \dfrac{\log_b x}{\log_b a}$

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Solving equations w/ Logs

$$\log_a b = c$$

$$\therefore$$

$$a^c = b$$

Examples.

a) $\log_2 x = 3$

$2^3 = x$

$\therefore x = 8$

b) $\log_5 (x-1) = 2$

$5^2 = x-1$

$25 = x-1$

$\therefore x = 26$

c) $\log_2 16 = x$

$2^x = 16$

$2^x = 2^4$ or $\log_2 2^4 = x$

$\therefore x = 4$ $\therefore 4 = x$

d) $\log_x 25 = 2$

$x^2 = 25$

$\therefore x = \pm\sqrt{25} = \pm 5$

$\therefore x = 5$

Solving equations with indices.

$$a^x = a^y$$

$$\therefore$$

$$x = y$$

Examples.

a) $3^x = 81$

$3^x = 3^4$ Express in same base.

$x = 4$ Equate indices.

b) $6^{3x-1} = 36^{2x-3}$

$6^{3x-1} = (6^2)^{2x-3}$ Write 36 w/ base 6

$6^{3x-1} = 6^{4x-6}$ Simplify $2(2x-3)$.

$3x-1 = 4x-6$ Equate indices.

$-1 = x-6$ Solve for x.

$x = 5$ Evaluate.

c) $2^{3n} \times 16^{n+1} = 32$

$2^{3n} \times (2^4)^{n+1} = 2^5$ Write 16 w/ base 2.

$2^{3n} \times 2^{4n+4} = 2^5$ Simplify $4(n+1)$.

$2^{3n+4n+4} = 2^5$ Add indices.

$2^{7n+4} = 2^5$ " "

$7n+4 = 5$ Equate indices.

$7n = 1$ Simplify.

$n = 1/7$ Evaluate.

Solving equations with indices ctd.

a) $5^{2x} - 4(5^x) - 5 = 0$

$(5^x)^2 - 4(5^x) - 5 = 0$ — Rewrite in quadratic form. $5^{2x} =$

Let $y = 5^x$. — Subst. $y \to 5^x$.

$\therefore y^2 - 4y - 5 = 0$ — Rewrite w/ y.

$(y-5)(y+1) = 0$ — Factorise.

$\therefore y = 5, y = -1$ — Use null factor law for y.

$5^x = y$ — Subst. $5^x \to y$.

$5^x = 5^1$, $5^x = -1$* — Solve for y.

$x = 1$, undefined* — Evaluate.

* $5^x = -1$

$\cancel{5^x \neq -5^0}$

$5^x = -1 = -5^0$

No same base, cannot continue.
Undefined.

Quadratic Functions

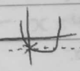
Graph of the parabola:

$y = ax^2 + bx + c$

where a, b and c are numerical values

"5 Point Plan":

1. Shape
   - $a > 0$ ∪ Minimum
   - $a < 0$ ∩ Maximum
2. x-intercepts $(y = 0)$
   - solve x. **
3. y-intercepts $(x = 0)$



c-value $(y = ax^2 + bx + c)$

4. Turning point

   $x = \dfrac{-b}{2a}$ Subst. x-value into original equation to find y-coord. (x, y)

5. Nature of stationary points (or turning point)

   $\Delta = b^2 - 4ac$

   If $\Delta > 0$, 2 roots

   If $\Delta < 0$, no roots

   If $\Delta = 0$, 1 root.

Examples.

(prev. pg)

Examples

e.g. $y = x^2 - 10x + 21$ $a=1$, $b=-10$, $c=21$

this is: a, b, c

1. Shape

$a=1$, $1>0$ ∪ min value

2. x-intercept $(y=0)$

$0 = x^2 - 10x + 21$

$= (x-7)(x-3)$

$\therefore x=7,\ x=3$

3. y-intercept $(x=0)$

$\therefore y = 21$

4. Turning point (Stationary Pts.)

$x = \frac{-b}{2a}$ $a=1$, $b=-10$

$= \frac{-(-10)}{2(1)}$

$= 10/2 = 5$

Subst. $x=5 \rightarrow y = x^2 - 10x + 21$

$y = (5)^2 - 10(5) + 21$

$= 25 - 50 + 21$

$= -4.$ tp$(5, -4)$

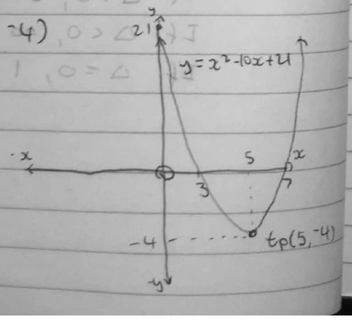
5. Nature

$\Delta = b^2 - 4ac$

$= (-10)^2 - 4(1)(21)$

$= 100 - 84$

$= 16$ 2 roots, ⊕ve



5 Point Plan Expansion

Turning point format:

$$y = a(x-p)^2 + q$$

where tp(p, q)

Examples

1. $y = x^2 - 8x - 20$ Note $a=1$.

$y = x^2 - 8x + (-8 \times \frac{1}{2})^2 - (-8 \times \frac{1}{2})^2 - 20$ Complete the squ.* $b=-8$

$= x^2 - 8x + (-4)^2 - (-4)^2 - 20$ $-8/2 = -4$

$= (x-4)^2 - 16 - 20$ Convert into p. squ. trinomial.

$= (x-4)^2 - 36$ $-16-20 = -36$

tp$(4, -36)$

$x - 4 = 0$

$x = 4$

2. $y = 2x^2 - 4x - 2$ Note $a \neq 1$.

$y = 2(x^2 - 2x - 1)$ Take out 2 so $a=1$.

$= 2[x^2 - 2x + (-2/2)^2 - (-2/2)^2 - 1]$ Complete the squ.

$= 2[x^2 - 2x + (-1)^2 - (-1)^2 - 1]$ $-2/2 = -1$.

$= 2[(x-1)^2 - 1 - 1]$ Convert.

$= 2[(x-1)^2 - 2]$ $-1-1=-2$.

$= 2(x-1)^2 - 4$ Put 2 into the eqn.

tp$(+1, -4)$

$x - 1 = 0$

$x = 1$

$$y = \left(x^2 + bx + (\tfrac{1}{2}b)^2 - (\tfrac{1}{2}b)^2 + c\right)$$

* Completing the square format

Exponential Graphs

Standard form:

$$y = a^x$$

Draw using a table of values.

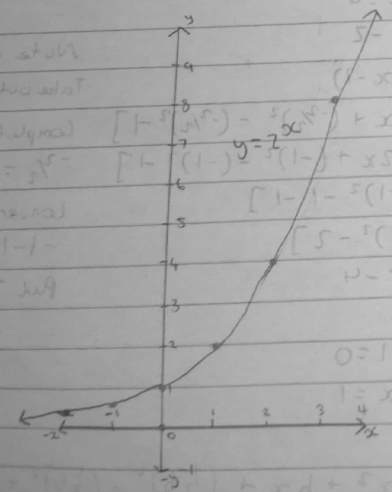
Examples:

1. $y = 2^x$ $(a=2)$

for $\boxed{-2 \leq x \leq 3}$ ← restrictions

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

$y = 2^{-2} = \frac{1}{4}$, $y = 2^3 = 8$ etc.



x-axis is called the asymptote.

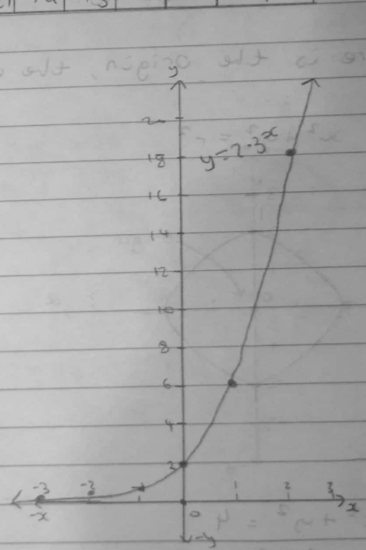
Line will never pass through

Examples

2. $y = 2 \cdot 3^x$ $(a=3)$

for ~~3~~ $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	$\frac{2}{27}$	$\frac{2}{9}$	$\frac{2}{3}$	2	6	18	54



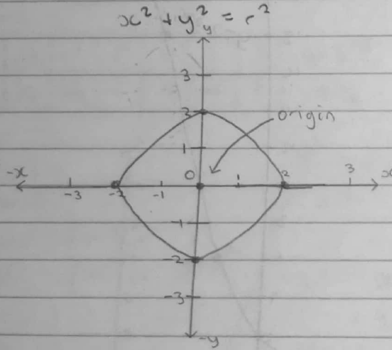
Circles

$$x^2 + y^2 = r^2$$

x and y are points (coordinates) on the graph and r is the radius.

When the centre is the origin, the equation is:

$$x^2 + y^2 = r^2$$



e.g. $x^2 + y^2 = 4$

$\therefore r^2 = 4$

$r = \pm 2$

Circles Ctd

If the centre of the circle moves, then:

$$(x-h)^2 + (y-k)^2 = r^2$$

where

pt (h, k) is centre

r is radius

Examples

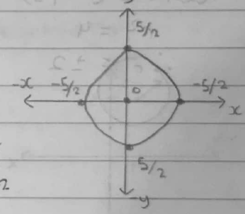
1. $4x^2 + 4y^2 = 25$

$4(x^2 + y^2) = 25$

$x^2 + y^2 = 25/4$

$r^2 = 25/4$

$\therefore r = \pm 5/2$



Graph.

Take 4 out as CF.

Divide by 4.

Write radius.

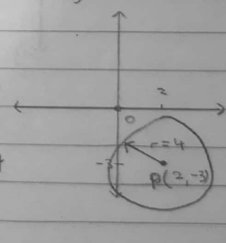
Square root.

2. $(x-2)^2 + (y+3)^2 = 16$

centre $(2, -3)$

$r^2 = 16$

$\therefore r = \pm 4$



Sketch.

Find centre from h & k.

Write radius.

Square root.

Circles Examples Ctd.

Ctd.

3. $x^2 + 2x + y^2 - 6y + 6 = 0$ Graph.

$(x^2 + 2x) + (y^2 - 6y) + 6 = 0$ Group.

$(x^2 + 2x + (\frac{2}{2})^2) + -(\frac{2}{2})^2 + (y^2 - 6y + (\frac{6}{2})^2) - (\frac{6}{2})^2 = -6$ Complete squ

$(x+1)^2 - 1 + (y-3)^2 - 9 = -6$ Simplify.

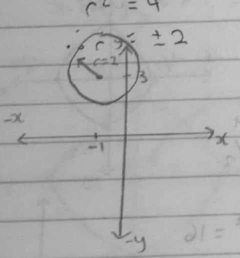
$(x+1)^2 + (y-3)^2 = -6 + 10$ Take $-1-9 \rightarrow 10$

$(x+1)^2 + (y-3)^2 = 4$ $-6 + 10 = 4$

centre $(-1, 3)$

$r^2 = 4$

$\therefore r = \pm 2$



Polynomials

E.g. $P(x) = 6x^3 + 13x^2 - x + 1$ (not a formula)

The 'degree' of a function is its highest power.
The degree of this function is 3.
x is the variable.

6, 13, and -1 are all coefficients of x^3, x^2, and x respectively.

The constant is +1.

Addition, Subtraction and Multiplication of Polynomials

For adding and subtracting:

- Add like terms.
- You cannot add or subtract unlike terms.

For multiplying:

e.g. $x(x+3)(x-1)$

- Expand the brackets first.

$x(x+3)(x-1)$
$= x(x^2+2x-3)$
$= x^3+2x^2-3x$

4 Operations with Polynomials

Adding and Subtracting.

$5x^3+3x^2-2x-1-(x^4+5x^2-4)$
$= 5x^3+3x^2-2x-1-x^4-5x^2+4$ Expand brackets.
$= -x^4+5x^3-2x^2-2x+3$ Add like terms.

Multiplying.

$x(x+2)(x+3)$
$= x(x^2+5x+6)$ use FOIL to exp.
$= x^3+5x^2+6x$ Multiply by x.

Dividing is on the next page to ease reading.

terrible
↓
~~Divy~~

Dividing Polynomials

Let's try $\dfrac{(x^3+3x^2+x+9)}{(x+2)}$

Use Long Division.

$x+2 \;\overline{)\,x^3+3x^2+x+9}$

① Divide x^3/x and write on top. Ans $= x^2$

$$\begin{array}{r} x^2 \\ x+2\;\overline{)\,x^3+3x^2+x+9} \end{array}$$

② Times x^2 by $(x+2)$. Ans $= x^3+2x^2$.
Write Ans under.

$$\begin{array}{rl} & x^2 \\ x+2 & \overline{)\,\boxed{x^3}+3x^2+x+9} \\ \text{should be the same} & \boxed{x^3}+2x^2 \end{array}$$

Dividing Polynomials

ctd.

③ Flip signs on Ans (because you're subtracting)

$$\begin{array}{rl} & x^2 \\ x+2 & \overline{)\,\boxed{x^3}+3x^2+x+9} \\ \text{should cancel out} & \boxed{-x^3}-2x^2 \quad -(x^3+2x^2) = -x^3-2x^2 \end{array}$$

④ Evaluate

$$\begin{array}{rl} & x^2 \\ x+2 & \overline{)\,x^3+3x^2+x+9} \\ & -x^3-2x^2 \\ & \overline{(0)+x^2} \end{array}$$

⑤ Drop down next term (However many terms as the divisor has)

$$\begin{array}{rl} & x^2 \\ x+2 & \overline{)\,x^3+3x^2+x+9} \\ & -x^3-2x^2 \downarrow \\ & \overline{(0)+x^2+x} \end{array}$$

Repeat. The whole process is on the next page.

1. Divide the inside term by the variable. Write ans on top.
2. Multiply the whole divisor by the previous ans. Write new ans under.
3. Flip signs on the new answer because of subtraction.
4. Evaluate. Write the answer under.
5. Drop down as many terms as needed.
6. If no terms are left, your answer is there. Remainder on bottom

Ctd. Dividing Polynomials

Example from prev. pg.

$$
\begin{array}{r l}
 & \{x^2 + x - 1\} \\
x+2 & \overline{)\,x^3 + 3x^2 + x + 9} \\
 & -\;x^3 + 2x^2 \\
 & \overline{(0) + x^2 + x} \\
 & -\;x^2 + 2x \\
 & \overline{(0) - x + 9} \\
 & -\;-x - 2 \\
 & \overline{(0) + \boxed{11}}
\end{array}
$$

$(x^3+3x^2+x+9) \div (x+2) = x^2+x-1 \quad R=11$

1. $3x^2$ does not divide by x. Instead, the new answer is (x^2). Similarly, x doesn't / by x, $-x$ does

Dividing Polynomials

Here is an example of regular long division.

$2581 \div 12$ what answer?

$$
\begin{array}{r l}
 & 215 \leftarrow \text{Ans} \\
12 & \overline{)\,2581} \\
 & -24 \\
 & \overline{\;18} \\
 & -12 \\
 & \overline{\;\;61} \\
 & -60 \\
 & \overline{\;\;\;1} \leftarrow R
\end{array}
$$

215, R = 1

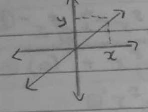
It is more or less the same process.

Functions and Relations

One-to-one relation:

For every one x-value, only 1 corresponding y-value.

E.g.

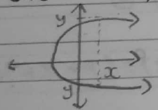


Function

One-to-many relation:

For every one x-value, many y-values.

E.g.

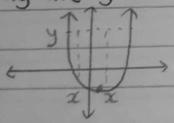


Not Function

Many-to-one relation:

For every one y-value, many x-values.

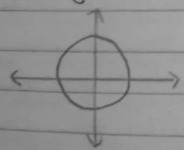
E.g.



Function

Many-to-many relation:

E.g.



Not Function

Functions and Relations, ctd.

Vertical Line Test:

A vertical line is drawn through the graph of a function. If it intercepts more than one point, the line cannot be a function.

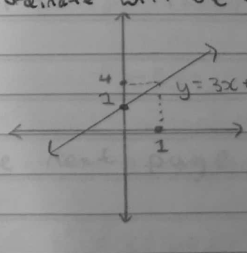
Function Notation

$$y = f(x) = g(x) = h(x)$$

etc.

If $y = 3x+1$ you could say $f(x) = 3x+1$. Use function notation to find the coordinates that go through the graph.

$\therefore$ $f(1) = 4$ means when $x = 1$, the y-coordinate will be 4.



Functions and Relations

Ctd.

Function Notation Examples

1. $g(x) = 3x - 4$, find:

a) $g(-3)$

$g(-3) = 3x - 4$

$= 3(-3) - 4$

$= -13$

b) $g(x) = 5$

$5 = 3x - 4$

$9 = 3x$

$x = 9/3$

$= 3$

Cubic Functions

$$y = ax^3 + bx^2 + cx + d$$

Steps:

1. Shape
   - If $a > 0$, (+)
   - If $a < 0$, (−)
2. x-intercepts
   - Subst. $y = 0$, solve x
   - If function is factorised, use Null Factor Law.
   - If not, use factor theorem, apply long division and find roots.
3. y-intercepts
   - Subst. $x = 0$, solve y.
   - If function is fully expanded, then d is your y-intercept.
4. Turning points

See next page.

Ctd. Cubic Functions

4. Turning points

To find the turning points of a cubic function, find the derivative of the function and solve it.

(Derivatives)

To find the derivative of a term, for example $3x^3$, follow these steps.

1. Make sure the term is in the general form,

$$px^q$$

2. Multiply the power (q) by the coefficient.

$$(pq)x^q$$

3. Subtract 1 from the power.

$$(pq)x^{q-1}$$

(ctd.) (Derivatives)

Example.

Q1. Find the derivative of the expression $3x^3 - 2x^2 + 4x - 1$.

step 1. $3x^3 - 2x^2 + 4x^1 - 1x^0$

Step 2. $9x^3 - 4x^2 + 4x^1 - 0$

step 3. $9x^2 - 4x + 4$

$$\frac{dy}{dx} = 9x^2 - 4x + 4$$

(End Derivatives)

Cubic Functions continue on the next page

4. Turning points, ctd.
After finding the derivative of the equation, solve it. For cubic functions, the derivative should be a quadratic equation. After solving for x, substitute these values back into the function to get the y-coordinate for the turning points.

Example
Graph $f(x) = x^3 - 5x^2 - 2x + 24$

1. shape
$a = 1, a > 0,$ ↗

2. x-intercepts
$0 = x^3 - 5x^2 - 2x + 24$
Try $x+1$ as factor
$= (-1)^3 - 5(-1)^2 - 2(-1) + 24$
$\neq 0$
Try $x+2$ as factor
$= (-2)^3 - 5(-2)^2 - 2(-2) + 24$
$= 0$

Long division.

$$
\begin{array}{r|l}
 & x^2 - 7x + 12 \\
\hline
x+2 & x^3 - 5x^2 - 2x + 24 \\
 & -x^3 \mp 2x^2 \\
\hline
 & -7x^2 - 2x \\
 & +7x^2 \pm 14x \\
\hline
 & 12x + 24 \\
 & -12x - 24 \\
\hline
 & 0
\end{array}
$$

$\therefore f(x) = (x+2)(x^2 - 7x + 12)$
$= (x+2)(x-4)(x-3)$
$\therefore x = -2, x = 4, x = 3$

3. y-intercept
$d = 24$

Continued.

ctd. Cubic Functions

4. Turning points

$f(x) = x^3 - 5x^2 - 2x + 24$

$\frac{dy}{dx} = 3x^2 - 10x - 2 + 0$

$= 3x^2 - 10x - 2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-2)}}{2(3)}$

$= \frac{10 \pm \sqrt{100 + 24}}{6}$

$= \frac{10 \pm 2\sqrt{31}}{6}$

$\therefore x^+ = 3.523, \; x^- = -0.189$

Subst. x^+ into eqn.

$f(x^+) = (3.523)^3 - 5(3.523)^2 - 2(3.523) + 24$

≈ -1.378

$tp_1 (3.523, -1.378)$

ctd. Cubic Functions

Subst. x^- into eqn.

$f(x^-) = (-0.189)^3 - 5(-0.189)^2 - 2(-0.189) + 24$

≈ 24.193

$tp_2 (-0.189, 24.193)$

5. Graph

Summary:

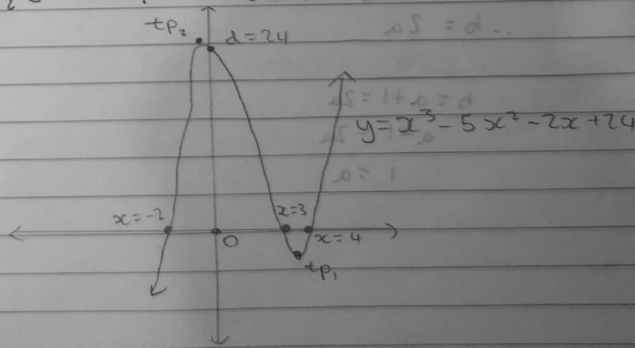
$a > 0$,

roots: $x = -2, 4, 3$

$d = 24$

$tp_1 (3.523, -1.378)$

$tp_2 (-0.189, 24.193)$



Solving Simultaneous Equns w/ Polynomials

Example.

1. $(x-1)$ and $(x-2)$ are two factors of the expression x^3-2x^2-ax+b.

Determine the value of a and b.

$f(1)=0$

$f(2)=0$

$f(1) = (1)^3-2(1)^2-a(1)+b$

$0 = 1-2-a+b$

$= -1-a+b$

$\therefore b = a+1$

$f(2) = (2)^3-2(2)^2-a(2)+b$

$= 8-8-2a+b$

$\therefore b = 2a$

$b = a+1 = 2a$

$a+1 = 2a$

$1 = a$

Subst. $a=1 \rightarrow$ eqn

$b = 2a$

$= 2(1)$

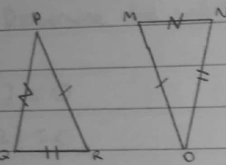
$= 2$

$\therefore b = 2, \quad a = 1$

Congruency

There are 4 cases of congruency.

1. S. S. S (side, side, side)



(Deductive proofing)

In $\triangle PQR$ and $\triangle MNO$

N II I   N II I

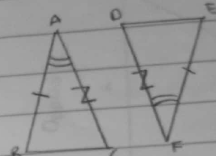
Note the naming order is the same for both $\triangle$'s.

Give 3 reasons.
1. PQ = MN — Given
2. QR = NO — Given
3. RP = OM — Given

state why.

$\therefore \triangle PQR \equiv \triangle MNO$ (S,S,S)

2. S, A, S (side, angle, side) (included)



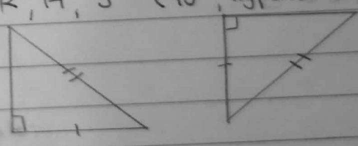
In $\triangle ABC$ and $\triangle FED$

= 1 2   = 1 2

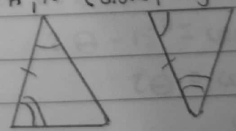
1. AB = FE — Given
2. $\angle A = \angle F$ — Given
3. AC = FD — Given

$\therefore \triangle ABC \equiv \triangle FED$ (S, A, S)

3. R, H, S (90°, hypotenuse, side)

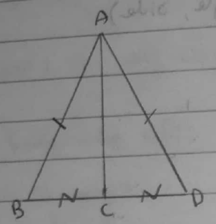


4. S, A, A (side, angle, angle)



Congruency ctd.

Example:



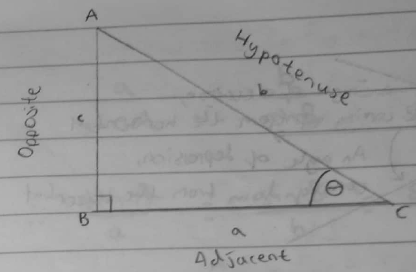
Prove that $\triangle ABC \equiv \triangle ADC$.

1.	$AB = AD$	Given
2.	$AC = AC$	Common
3.	$BC = CD$	Given

$\therefore \triangle ABC \equiv \triangle ADC$ (S,S,S)

spelling

Trigonometry



$\sin(\theta) = {}^{opp}/_{hyp}$; $\operatorname{cosec}(\theta) = \frac{1}{\sin(\theta)} = {}^{hyp}/_{opp}$

$\cos(\theta) = {}^{adj}/_{hyp}$; $\sec(\theta) = \frac{1}{\cos(\theta)} = {}^{hyp}/_{adj}$

$\tan(\theta) = {}^{opp}/_{adj}$; $\cot(\theta) = \frac{1}{\tan(\theta)} = {}^{adj}/_{opp}$

$\sin(\theta) = {}^{opp}/_{hyp}$

$\therefore \theta = \sin^{-1}({}^{opp}/_{hyp})$

etc.

Example:

$3\cos(2\theta - 15^\circ) = 1.987$

$\cos(2\theta - 15^\circ) = {}^{1.987}/_{3}$

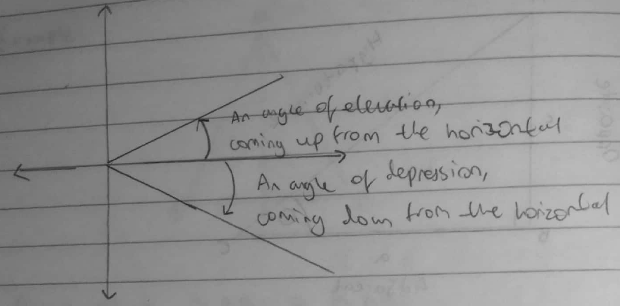
$2\theta - 15^\circ = \cos^{-1}({}^{1.987}/_{3})$

$2\theta = \cos^{-1}({}^{1.987}/_{3}) + 15^\circ$

$\therefore \theta = \dfrac{\cos^{-1}({}^{1.987}/_{3}) + 15^\circ}{2}$

$\simeq 31.761^\circ$

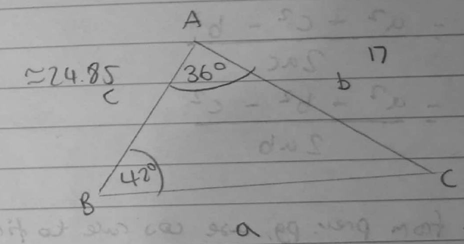
Angles of elevation and depression



Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Example: Find a.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{(a)}{\sin 36} = \frac{(17)}{\sin 42}$$

$\therefore a = {}^{17\sin 36}/_{\sin 42}$

$\simeq 14.93$

Cosine Rule

$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

$\therefore \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Using triangle from prev. pg, use cos rule to find C.

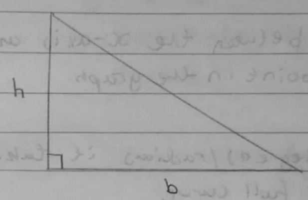
$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\therefore C = \cos^{-1}\left(\dfrac{a^2+b^2-c^2}{2ab}\right)$

$= \cos^{-1}\left(\dfrac{(14.93)^2+(17)^2-(24.95)^2}{2(14.93)(17)}\right)$

$= 102^\circ$

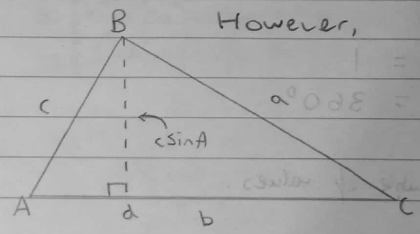
Area Of Triangles



In a right-angled triangle, the area formula is:

$A = \frac{1}{2}\, b\, h$

However,



$(\sin A = {}^{dB}/_{c})$

In a non-right-angled triangle, the area formula is:

$A = \frac{1}{2}\, b\, h$
$\downarrow \quad \downarrow \downarrow \downarrow$
$A = \frac{1}{2}\, b\, c \sin A$

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Basic Trig Functions

Amplitude - the distance between the x-axis and the highest point in the graph.

Period - how many degrees/radians it takes to make 1 full curve

1. $y = \sin x$

Amplitude = 1

Period = 360°

Draw using table of values.

x	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
y	0	0.26	0.5	0.71	0.87	0.97	1	0.97	0.87	0.71	0.5	0.26	0

x	145°	210°	225°	240°	255°	270°	285°	300°	315°	330°
y	-0.26	-0.5	-0.71	-0.87	-0.97	-1	-0.97	-0.87	-0.71	-0.5

x	345°	360°
y	-0.26	0

2. $y = \cos x$

Amplitude = 1

Period = 360°

Draw using table of values.

x	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	130°	165°	180°
y	1	0.97	0.87	0.71	0.5	0.26	0	-0.26	-0.5	-0.71	-0.87	-0.97	-1

x	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°
y	-0.97	-0.87	-0.71	-0.5	-0.26	0	0.26	0.5	0.71	0.87

x	345°	360°
y	0.97	1

Scanned with CamScanner

Radians and Degrees

Radians - another way to measure angles.

Degrees to Radians:

$$\theta^r = \theta^\circ \times \pi/180^\circ$$

Example: 80° to rads

$$80^r = 80 \times \pi/180$$
$$= 4\pi/9^r$$

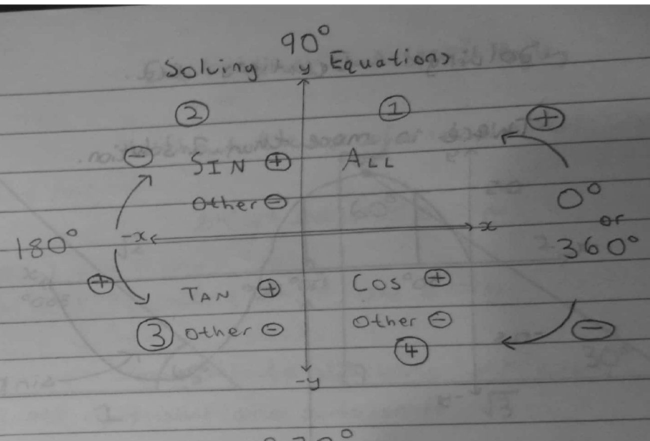
Radians to Degrees:

$$\theta^\circ = \theta^r \times 180^\circ/\pi$$

Example: $3\pi/4$ rad to deg

$$3\pi/4^\circ = 3\pi/4 \times 180^\circ/\pi$$
$$= 135^\circ$$

Solving Equations



Example: $\sin\theta = 0.5$

$\therefore \theta = \sin^{-1}(0.5)$

$= 30^\circ$

$\therefore$ Working in Quadrant 1 and 2.

That is where sin is +.

Continued...

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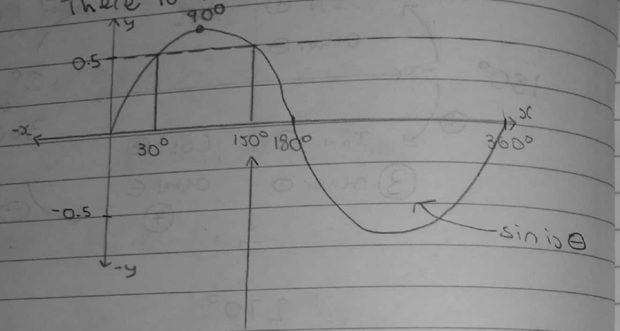
Solving Equations Ctd.

There is more than 1 solution.



In Quad ① $\theta = 30^\circ$ (reference angle)

In Quad ② $\theta = 180^\circ - 30^\circ$

$= 150^\circ$

Example 2. $2\cos\theta = 1.893$ Where $0^\circ \leq \theta \leq 360^\circ$

$\theta = \cos^{-1}(1.893/2)$

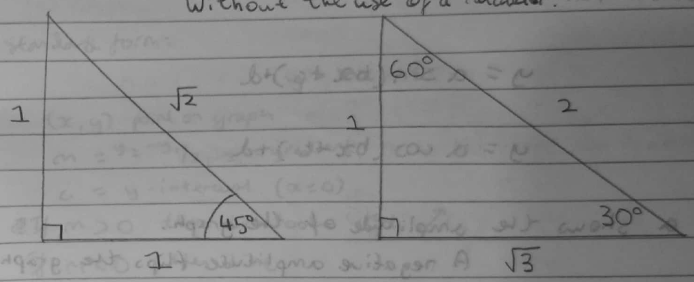
cos θ is ⊕ in 4th → $\approx 18.8^\circ$ (reference angle)

$\theta = 360 - 18.8^\circ$

$= 341.2^\circ$

Exact Values and Special Angles

Without the use of a calculator.

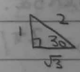


Example 1: $\sin 150^\circ$ +

$= \sin(180^\circ - 30)$

$= \sin(30)$

$= 1/2$

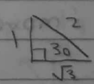


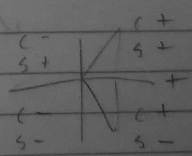
Example 2: $\cos 210^\circ$ + In Q③, cos is ⊖

$= \cos(180 + 30)$

$= -\cos 30$

$= -\sqrt{3}/2$





Features of sin and cos

General Form:

$$y = a\sin(bx + c) + d$$

$$y = a\cos(bx + c) + d$$

a shows the amplitude of the graph.
A negative amplitude flips the graph upside-down.

b shows the period. The period of the graph is $360/b$ or $2\pi/b$.

c shows the phase shift of the graph.
If c is positive, the graph will be moved left c°. Vice versa.

d. shows the vertical translation of the graph.
The graph will be moved up d units.

11B start

Straight Line Graphs
(linear functions)

standard form:

$$y = mx + c$$

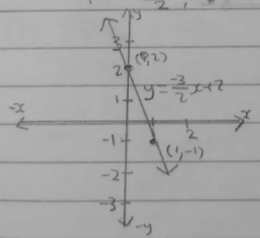
(x, y) point on graph
$m = y_2 - y_1 / x_2 - x_1$ = gradient
c = y-intercept $(x = 0)$
If $m > 0$ then line looks like ↗
If $m < 0$ then line

Graphing

1. Gradient-intercept

$y = \frac{-3}{2}x + 2$

$m = \frac{-3}{2}$, $c = 2$



$\frac{-3}{2}$ go down 3, go across 2

2. Dual-intercept

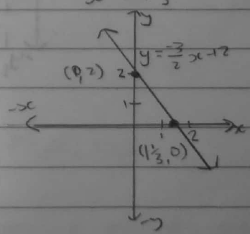
$y = \frac{-3}{2}x + 2$

$c = 2$

$0 = \frac{-3}{2}x + 2$ [x-int, y=0]

⋮

$x = 1\frac{1}{3}$



Graphing Of Quadratic Functions

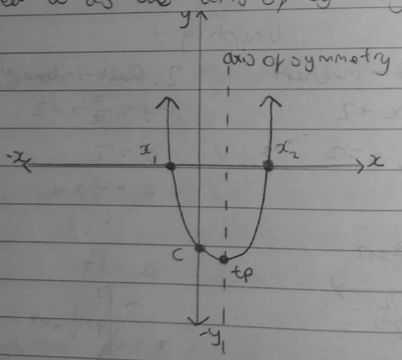
Geeral Form -

$$y = ax^2 + bx + c$$

Almost all info is the same as last year.

$$x = \frac{-b}{2a}$$

is referred to as the "axis of symmetry"



Simultaneous Equations

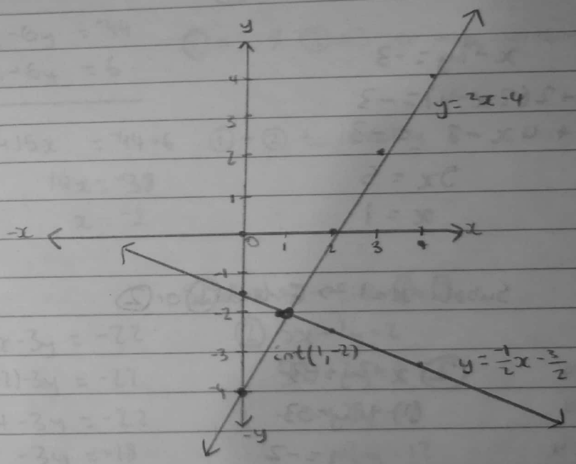
Methods:

1. Graphically

$2x - y = 4$ $\qquad$ $x + 2y = -3$

$\therefore y = 2x - 4$ $\qquad$ $\therefore y = \frac{-1}{2}x - \frac{3}{2}$

$m = 2,\ c = -4$ $\qquad$ $m = \frac{-1}{2},\ c = \frac{-3}{2}$



② Substitution

① $2x - y = 4$ and ② $x + 2y = -3$

$-y = 4 - 2x$

$y = 2x - 4$

Subst. $y = 2x - 4 \rightarrow$ ②

$x + 2y = -3$

$x + 2(2x - 4) = -3$

$x + 4x - 8 = -3$

$5x = 5$

$x = 1$

Subst. $x = 1 \rightarrow$ Either ① or ②

① $2x - y = 4$

$2(1) - y = 4$

$2 - y = 4$

$y = -2$

② $x + 2y = -3$

$(1) + 2y = -3$

$y = -2$

int$(1, -2)$

③ Elimination

$2x - 3y = -22$ and $5x + 2y = 2$

① $2x - 3y = -22$

② $5x + 2y = 2$

$4x - 6y = -44$

$15x + 6y = 6$ ① ×2 & ② ×3 to make y-terms cancel

$4x + 15x = -44 + 6$ ① + ② to cancel y

$19x = -38$

$x = -2$

Subst. $x = -2 \rightarrow$ Either ① or ②

① $2x - 3y = -22$

$2(-2) - 3y = -22$

$-4 - 3y = -22$

$-3y = -18$

$y = 6$

② $5x + 2y = 2$

$5(2) + 2y = 2$

$-10 + 2y = 2$

$2y = 12$

$y = 6$

int$(-2, 6)$

Note - if both y-terms are positive, multiply one equation with a negative number so the terms can cancel.

Discriminant

Same info in last year. See examples

Finding Equations

Straight Line $y = mx + c$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$

1. 2 points given e.g. $pt_1(3,4)$ $pt_2(-2,-6)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad y = 2x + c$, $pt(3,4)$

$= \frac{-6-4}{-2-3}$ $\quad 4 = 2(3) + c$ $\quad y = 2x - 2$

$= \frac{-10}{-5} = 2$ $\quad \therefore c = -2$

Note: if you're given two intercepts, use this method

2. Gradient & point e.g. $m = 3$, $pt(-1,-2)$

$y = 3x + c$ $\quad pt(-1,-2)$

$-2 = 3(-1) + c$ $\quad y = 3x + 1$

$\therefore c = 1$

Parabola $\underset{\text{standard}}{y = ax^2 + bx + c}$ $\quad \underset{\text{turning point form}}{y = a(x-p)^2 + q}$ $\quad \underset{x\text{-intercept form}}{y = a(x - x_1)(x - x_2)}$

1. T.P. and point given e.g. $tp(2,3)$ $pt(-1,-2)$

$y = a(x-p)^2 + q$ $\quad x = -1$, $y = -2$, $p = 2$, $q = 3$

$-2 = a(-1-2)^2 + 3$

$-5 = 9a$ $\quad y = \frac{-5}{9}(x-2)^2 + 3$

$\therefore a = \frac{-5}{9}$

2. 2 roots and point given e.g. $x_1(3,0)$ $x_2(2,0)$ $pt(4,8)$

$y = a(x - x_1)(x - x_2)$ $\quad x = 4$, $y = 8$, $x_1 = 3$, $x_2 = 2$

$8 = a(4-3)(4-2)$

$8 = 2a$ $\quad y = \frac{1}{4}(x-3)(x-2)$

$\therefore a = \frac{1}{4}$

Harder Logarithm Equations

$$1 + 2\log(x+1) = \log(2x+1) + \log(5x+8)$$

$$1 = \log(2x+1)(5x+8) - \log(x+1)^2$$

$$1 = \log\left(\frac{(2x+1)(5x+8)}{(x+1)^2}\right)$$

$$10^1 = \frac{(2x+1)(5x+8)}{(x+1)^2}$$

$$10(x+1)^2 = (2x+1)(5x+8)$$

$$10x^2 + 20x + 10 = 10x^2 + 21x + 8$$

$$10 - 8 = 21x - 20x$$

$$\therefore x = 2$$

Terminologies of Functions

x-value - independent variable

y-value - dependent variable (depends on x-value)

domain - all values of x written D_x

range - all possible values of y written R_y

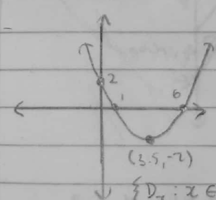
discrete variable - elements of the domain

continuous variable - a variable with infinite possible values

function - a graph which has 1 corresponding y-value for every x-value. use the vertical line test to check whether the graph is a function.

relation - set of ordered pairs, of which more than one x-value can be repeated.

See "Functions and Relations" grade 10 page.
See the same page for information on $f(x)$ not.



$\{D_x : x \in \mathbb{R}\}$

$\{R_y : y \geq -2, y \in \mathbb{R}\}$

Finding the Maximal Domain and Range

Here are some examples.

Type	Example	(what to do) Domain	(what to do) Range
Linear	$y=7-x$	$D_x:\{x \in \mathbb{R}\}$	$R_y:\{y \in \mathbb{R}\}$
Surd	$y=\sqrt{x-2}$	$0 \geq x-2$	Sub. D_x
Hyperbola	$y=\frac{1}{x+2}$	$x+2 \neq 0$	

Type	Example	D_x Sign	D_x Example	R_y Range
Linear	$y=\underline{7-x}$	None	$D_x:\{x \in \mathbb{R}\}$	$R_y:\{y \in \mathbb{R}\}$
Surd	$y=\sqrt{\underline{x-3}}$	$\_ \geq 0$	$x-3 \geq 0, x \geq 3$	$R_y:\{y > 0+c, y \in \mathbb{R}\}$
Hyperbola	$y=\frac{1}{\underline{2x-5}}$	$\_ \neq 0$	$2x-5 \neq 0, x \neq \frac{5}{2}$	$R_y:\{y \neq 0+c, y \in \mathbb{R}\}$
Logarithm	$y=\log_b(\underline{x+4})$	>0	$x+4>0, x>-4$	$R_y:\{y \in \mathbb{R}\}$

For surd, hyperbola & logarithm, a constant in the equation will affect D_x and/or R_y.

e.g. $y=\sqrt{x-3}+2$

$D_x: x-3 \geq 0$

$D_x:\{x \geq 3, x \in \mathbb{R}\}$

$R_y: y \geq 0+c$

$y \geq 0+2$

$R_y:\{y \geq 2, y \in \mathbb{R}\}$

Inverse Functions

To find the inverse of a function, swap the x's and y's.

$f(x) = 3x+2$

$f^{-1}(x) = 3y+2$

$x = 3y+2$

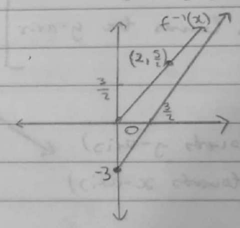
E.g. $f(x)=2x-3,\ x \geq 0$ Find $f^{-1}(x)$

$f^{-1}(x) = 2y-3$

$x = 2y-3$

$x+3 = 2y$

$y = \frac{1}{2}y + \frac{3}{2}$



Example 19 - 24 p.181 - 185

○ (open circle) represents point not included (e.g. $x > 5$)

● (closed circle) represents point included (e.g. $x \geq 5$)

Transformations of Parabolas

$y = a(x-h)^2 + k$

is a translation h units to the right and k units upward.

In General,

1. $y = af(x)$

   The graph $f(x)$ is dilated by factor a.

   e.g. $y = 3x^2 - 6x$

   $= \underline{3}(x^2 - 2x)$ ~~Dilated by factor of 3.~~

2. $y = f(ax)$

   The graph $f(x)$ is compressed towards the y-axis by the factor $\frac{1}{a}$.

If $a > 1$, graph narrower (dilated towards y-axis)

If $a < 1$, graph wider (dilated towards x-axis)

Types of Data

Categorical

- Nominal
  - naming something
  - no order
- Ordinal
  - ranking
  - 1st, ~~2nd~~, 3rd ...
  - yes, maybe, no

Numerical

- Discrete
  - number of
  - integer values
- Continuous
  - amount of
  - real values

Measures of Central Tendency

Mean: $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \dots x_n}{n}$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Sigma Notation

$$\sum_{i=\square}^{\square} \boxed{\ }$$

$\square$ ← last term

Starting value

what to sum

e.g. $\sum_{i=1}^{3} i = 1+2+3=6$

$\sum_{i=1}^{3} 2i = 2(1)+2(2)+2(3)=12$

Median: Score closest to middle

$$\text{median} = \frac{n+1}{2}$$

Data needs to be in order

Mode: Most frequent term

Measures of Spread

Range: $\text{range} = x_{max} - x_{min}$

Interquartile Range (IQR):

1. Data needs to be in order
2. Find median Q_2
3. Find median of data between x_{min} and Q_2, this is Q_1
4. Find median of data between x_{max} and Q_2, this is Q_3
5. $IQR = Q_3 - Q_1$

The IQR measures the spread of data or distribution that describes the range of the middle 50% of data.

Standard Deviation

Population Standard Deviation:

$$\sigma_n = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$$

Sample Standard Deviation:

$$\sigma_{n-1} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$$

Example:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
13	13 - 10 = 3	9
12	12 - 10 = 2	4
14	14 - 10 = 4	16
6	6 - 10 = -4	16
15	15 - 10 = 5	25
12	12 - 10 = 2	4
7	7 - 10 = -3	9
6	6 - 10 = -4	16
7	7 - 10 = -3	9
8	8 - 10 = -2	4

$\Sigma = 100$

$n = 10$

$\bar{x} = 10$

$\Sigma(x_i - \bar{x})^2 = 112$

$\sigma_n = \sqrt{\frac{1}{10}(112)} \approx 3.347$

$\sigma_{n-1} = \sqrt{\frac{1}{9}(112)} \approx 3.528$

Box and whisker Plots

To make a box and whisker plot, the data must always be in order.

5 Number Summary:

first quartile (Q_1), median (Q_2), third quartile (Q_3)

minimum (x_{min}), maximum (x_{max})

~~Even though the median (Q_2) is needed to calculate Q_1 and Q_3, the median is not part of the 5 number summary.~~

Outliers:

An outlier is any number that is more than 1.5 interquartile ranges above or below Q_1 or Q_3.

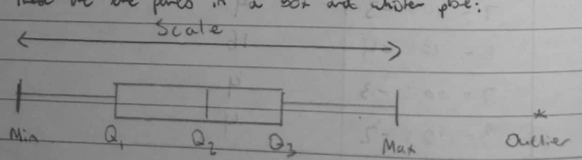
Therefore: Outlier = IQR ×1.5

Lower Limit = $Q_1 - (IQR \times 1.5)$

Upper Limit = $Q_3 + (IQR \times 1.5)$

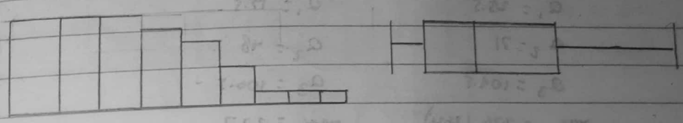
If a number lies outside the lower ~~and upper limits~~, it is an outlier.

These are the parts in a box and whisker plot:

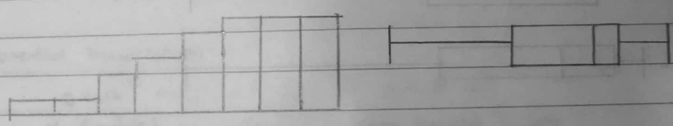


Data skew

If a set of data is positively skewed, there are a large range of values in the upper half of distribution.



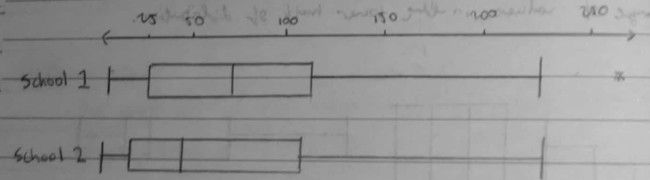
If a set of data is negatively skewed, there are a large range of values in the lower half of distribution.



Comparative Boxplots

Example:

School 1	School 2
min = 2	min = 1
$Q_1 = 25.5$	$Q_1 = 17.5$
$Q_2 = 71$	$Q_2 = 48$
$Q_3 = 104.5$	$Q_3 = 106.5$
max = 226 (264)	max = 227
outlier = 264	



Compare:

- Skew - Centre higher/lower - Spread (width of IQR) - Outliers

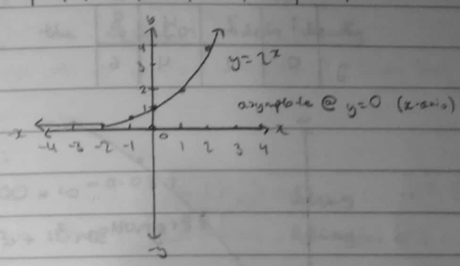
Graphs of Exponential and Logarithmic Functions

Exponential Functions: $y = a^x$; $y = ka^x$; $y = ka^x + c$ etc.

Use a table of values to accurately graph.

Examples: $y = 2^x$

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Exponential Transformations:

$y = a^x + c$

Graph translated c units upwards.

$y = b \cdot a^x$

Graph translated $\frac{b}{a}$ units left, "dilated" by factor b

$y = -a^x$

Graph reflected across x-axis.

Graphs of Exponential and Logarithmic Functions Ctd.

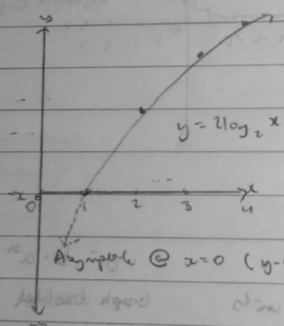
Logarithmic Functions: $a = b^c \Leftrightarrow \log_b a = c$

To change an exponential function to a log function, make x the subject.

Use a table of values to graph.

Example: $y = 2\log_2 x$

x	1	2	4	8
y	0	2	4	6



Exponential Growth & Decay

General form:

$$A = A_0 a^t$$

where A_0 is the initial value

If $|a| > 1$, the equation models growth.

If $|a| < 1$, the equation models decay.

Examples of models:

$A = 300 \times 10^{-0.03T}$ decay

$T = 23 + 10 \times e^{-0.2378T}$ decay

$C_I = P\left(1 + \frac{r}{100n}\right)^{nt}$ growth

Finance Formulae

Effective Interest Rate

$$S\% = 100 \times (e^{0.01r} - 1)\%$$

$S\%$ = effective interest rate, r = nominal rate

Depreciation

$$A = P\left(1 - \frac{r}{100}\right)^n$$

A = final amount, P = principal, r = rate, n = no. of compounds

Inflation

$$F_v = P_v\left(1 + \frac{r}{100}\right)^n$$

F_v = final value, P_v = present value, r = rate, n = no. of compounds

Annuities

$$A_n = \frac{Q(R^N - 1)}{R - 1}$$

A_n = annuity, Q = saved value, R = growth factor, $N = nm$ = no. of years × compound

Growth Rate

$$R = \left(1 + \frac{r}{100m}\right)$$

Annuity w. initial payment followed by periodic payments

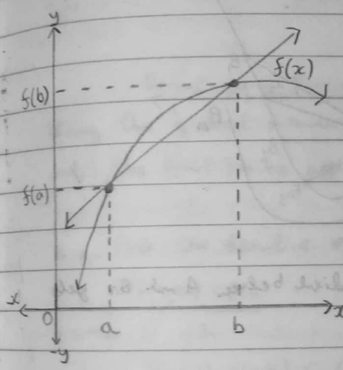
$$A_n = PR^N + \frac{Q(R^N - 1)}{R - 1} \quad (N = nm)$$

Rate of Change

Rate - Ratio between 2 related quantities
kmh^{-1}, ms^{-1} etc.

Change - Final state - initial state

Average rate of change - Average gradient. Occurs in non-linear functions.
Constant rate of change = linear function



Average rate of change $= \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$

Example: $f(x) = x^2 - 2x + 5$ Find av. rate of change between $x = \overset{a}{1}$ to $x = \overset{b}{5}$

$f(1) = (1)^2 - 2(1) + 5 = 4$ so $pt_1 (1, 4)$

$f(5) = (5)^2 - 2(5) + 5 = 20$ so $pt_2 (5, 20)$

Avg. rate of ch. $= \frac{f(b) - f(a)}{b - a}$

$= \frac{20 - 4}{5 - 1}$

$= \frac{16}{4}$

$= 4$

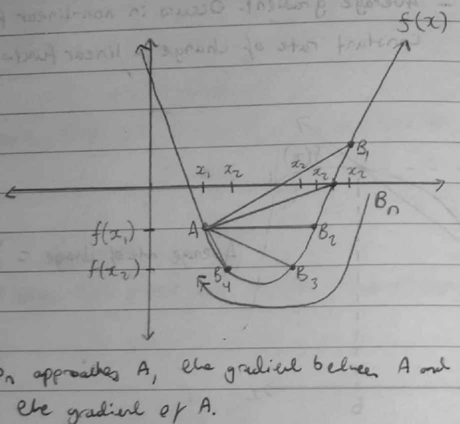
Gradient line eq. $\Rightarrow y = mx + c$

$y = 4x + c$

Example 10 p.498

Instantaneous Rate of Change

In a parabola, a tangent intersects the graph at one point only.
A secant is a line that intersects (cuts) more than one point.



As point B_n approaches A, the gradient between A and B_n gets closer to the gradient of A.

Calculus has to do with the rate of change / average gradient where the difference between x_1 and x_2 becomes so small, it tends to zero.

Useful identities: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

Gradient Function: Gives the value for the gradient at a point of a function.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \frac{\Delta y}{\Delta x}$$

Where h is the horizontal distance between points A and B, i.e. $h = x_2 - x_1$

Derivatives of functions from first principles

Using the gradient formula, find $f(x+h)$. Then, work the limit out until the limit can be applied so $h \to 0$.

e.g. Find the derivative of the function $f(x) = 2x^2+3x+5$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h)^2+3(x+h)+5-(2x^2+3x+5)}{h}$$

$$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+3x+3h+5-2x^2+3x+5}{h}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2+3h}{h}$$

$$= \lim_{h\to 0} \frac{h(4x+2h+3)}{h}$$

$$= \lim_{h\to 0} 4x+2h+3$$

$$= 4x+3$$

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Rules for Differentiation

Operational Notation

If $f(x)$ is a function, then the derived function is

$f'(x)$ or $\frac{d}{dx}$ or $\frac{dy}{dx}$

Newton's Notation

Leibniz's Notation

variable x is being differentiated with respect to.

Rules for Differentiation

Power Rule

1. If $f(x) = kx^n + c$ then $f'(x) = knx^{n-1}$
   c falls away as it is equal to cx^0, as we are finding the derivative w.r.t. x or the variable stated.
2. If $f(x) = kx^{\frac{1}{n}} + c$ then $f'(x) = (k \cdot \frac{1}{n})\, x^{\frac{1}{n}-1}$
   Again, c falls away as it is a constant with no x-term.

Examples

1. If $z = \frac{1}{3}x^3 + x^2$, find $\frac{dz}{dx}$.

$$\frac{dz}{dx} = \frac{1}{3} \cdot 3x^{3-1} + 2x^{2-1}$$
$$= x^2 + 2x$$

2. Find the derivative of $\frac{x^2+3x}{x}$.

$$\frac{x(x+3)}{x} = x+3$$
$$\frac{d}{dx} = 1$$

RB Start

Chain Rule

Definition

If $f(x) = u(v(x))$ then:

$$f'(x) = u'(v(x)) \cdot v'(x)$$

Leibnitz's Notation

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Shortcut

1. Reduce the outside's index by 1, multiply the coefficient by the index... essentially, apply the power rule to the outside function. This gives the $u'(v(x))$ term.
2. Multiply this with the derivative of the inside function. This is the $v'(x)$.

Example

Derive $y = -4(2x^3+5x^2)^3$.

$$\frac{dy}{dx} = -4 \times 3(2x^3+5x^2)^{3-1} \times (6x^2+10x)$$
$$= -12(2x^3+5x^2)^2 \times 2x(3x+5)$$
$$= -24x(2x^3+5x^2)^2(3x+5)$$
$$= -24x \times [x^2(2x+5)]^2(3x+5)$$
$$= -24x^5(2x+5)^2(3x+5)$$

Product Rule

Definition

If $f(x) = u(x)\,v(x)$ then:

$$f'(x) = u'(x)\,v(x) + u(x)\,v'(x)$$

Leibnitz's Notation

$$\frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$

Example

If $r(x) = 6x\sqrt[3]{x+2}$, find $r'(x)$.

$u = 6x$, $\frac{du}{dx} = 6$

$v = \sqrt[3]{x+2}$, $\frac{dv}{dx} = \frac{1}{3\sqrt[3]{(x+2)^2}}$

$$r'(x) = \frac{du}{dx}v + u\frac{dv}{dx}$$

$$= 6\sqrt[3]{x+2} + \frac{6x}{3\sqrt[3]{(x+2)^2}}$$

$$= 6\sqrt[3]{x+2} + \frac{2x}{\sqrt[3]{(x+2)^2}}$$

$$= \frac{\sqrt[3]{(x+2)^2}}{\sqrt[3]{(x+2)^2}} \times 6\sqrt[3]{x+2} + \frac{2x}{\sqrt[3]{(x+2)^2}}$$

$$= \frac{6\sqrt[3]{x+2}\left(\sqrt[3]{x+2}\right)^2 + 2x}{\sqrt[3]{(x+2)^3}}$$

$$= \frac{6(x+2) + 2x}{\sqrt[3]{(x+2)^2}}$$

$$= \frac{8x+12}{\sqrt[3]{(x+2)^2}}$$

Quotient Rule

Definition

If $f(x) = \frac{u(x)}{v(x)}$, $v(x) \neq 0$, then

$$f'(x) = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{[u(x)]^2}$$

Leibnitz's Notation

$$\frac{dy}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

Example

Tangents and Normals

Applications of Calculus

If displacement is $f(x)$ or s,
then velocity is $f'(x)$ or $\frac{ds}{dt}$ (1st derivative of disp. wrt. time)
and acceleration is $f''(x)$ or $\frac{d^2s}{dt^2}$ (2nd derivative of disp. wrt time)

Example 49 p74 - $\frac{dV}{dt}$

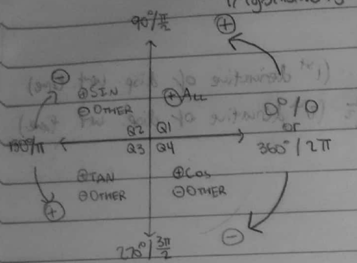
Example 50 p76 - $s, \frac{ds}{dt}, \frac{d^2s}{dt^2}$

Antiderivative / Integral Rules

If $F'(x) = f(x)$ then $\int f(x)\,dx = F(x) + c$.

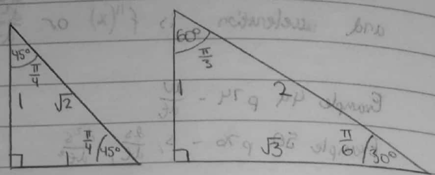
1. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c; \; n \neq -1$
2. $\int f(x) + g(x) \cdot dx = \int f(x) \cdot dx + \int g(x) \cdot dx$
3. $\int k f(x) \cdot dx = k \int f(x) \cdot dx, \; k \in \mathbb{R}$
4. $\int (ax+b)^n \cdot dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$
5. $\int k(ax+b)^n \cdot dx = \frac{k(ax+b)^{n+1}}{a(n+1)} + c$ (3)
6. $\int \frac{k}{ax+b} \cdot dx = \frac{k}{a} \ln|ax+b| + c$
7. $\int \frac{f'(x)}{f(x)} \cdot dx = \ln|f(x)| + c$

Trigonometry Revision content



$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{a}{h}$ $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{o}{a}$

$\theta^\circ = \theta^r \times \frac{180}{\pi}$ $\theta^r = \theta^\circ \times \frac{\pi}{180}$



Features of Trigonometric Graphs (rvsn.)

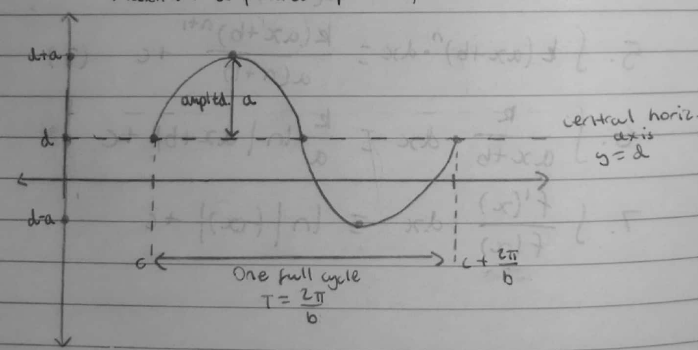
cycle - one full revolution of the curve

period - change in x over one cycle, $T = \frac{2\pi}{b}$

T of $\sin\theta$, $\cos\theta = 2\pi$, T of $\tan\theta = \pi$

frequency - number of periods to complete one unit of θ, $f = \frac{1}{T}$

amplitude - half the distance between local minima and maxima. amplitude of $\sin\theta, \cos\theta = 1$



$y = a\sin(b(x-c)) + d$

General Form

Graphing Cubic Functions

$y = ax^3 + bx^2 + cx + d$

1. shape If $a > 0$, curve looks like ↗

   If $a < 0$, curve looks like ↘

2. y-intercept Solve with $x = 0$ (d-value).
3. x-intercept Solve with $y = 0$.

   Either: - Factorise OR - Factor + Remainder Theorem

   OR - Factor + Remainder Theorem + Long Division

Factor Theorem - If $(x-a)$ is a factor of $f(x)$, then $f(a) = 0$, and vice versa.

Polynomial Long Division - Explained earlier in this book.

4. Stationary Points Solve $f'(x) = 0$, then substitute these values back into the original function to find the y-coordinates.
5. Nature of Stationary Points (Minima & Maxima)

   a) Either look at slope (↗ or ↘) to pick minimum and maximum

   b) Or use second derivative ($f''(x)$ or $\frac{d^2y}{dx^2}$) to investigate concavity at x-coordinate of stationary points (-ve sign is ∩, +ve is ∪)

Probability Revision

Event - a possible outcome

Sample Space - all possible events

Cardinal Number - number of outcomes of an event

Theoretical Probability - $P(E) = \frac{n(E)}{n(S)}$ ← cardinal no. / ← all possible events

Experimental Probability - relative freq. $= \frac{\text{freq.}}{\text{total f.}}$

Lattice Table - table showing all possible events

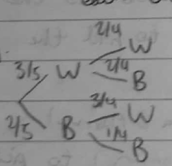
e.g.

2nd coin \ 1st coin	H	T
H	HH	TH
T	HT	TT

Tree Diagram - tree with branches showing every outcome

Probability Tree - tree that showcases probability of each event.

e.g.



WW → $P(WW) = \frac{3}{5} \times \frac{2}{4} = 3/10$

multiply across and add different events

AND events - multiply probabilities

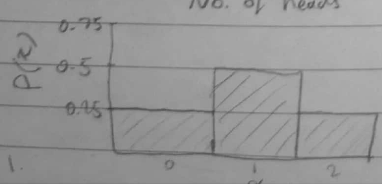
OR events - add probabilities

NOT events - $P(\bar{E}) = 1 - P(E)$

Probability Distribution - Shows probability of each event

x	$P(x)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$

No. of heads



$\Sigma P(x)$ must equal 1.

Differentiation of e^x and $\ln x$

$\frac{d}{dx}(e^x) = e^x$ $\quad \frac{d}{dx}(e^{kx}) = ke^{kx}$ $\quad \frac{d}{dx}(e^u) = \frac{du}{dx} e^u$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

Probability - Expected Values

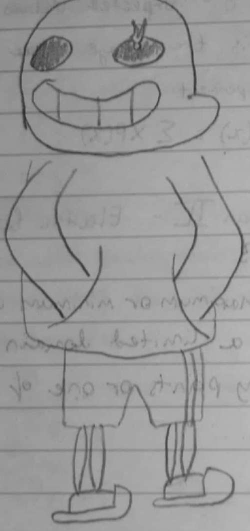
The expected value is the average value expected over a long run of an experiment.

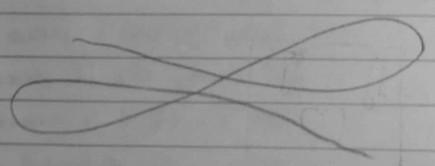
$$E(x) = \Sigma x P(x)$$

Term 3 ## Optimisation II - Electric Boogaloo

Example 3 - p. 393

Note - absolute maximum or minimum value of a function across a limited domain may be one of the boundary points or one of the stationary points.





Antiderivative of e^x

1. $\int ke^{x}\,dx = ke^{x} + c$
2. $\int e^{f(x)}\,dx = \frac{1}{f'(x)}e^{f(x)} + c$

Antiderivatives of Trigonometric Functions

1. $\int k\sin(f(x))\,dx = -\frac{k}{f'(x)}\cos(f(x)) + c$
2. $\int k\cos(f(x))\,dx = \frac{k}{f'(x)}\sin(f(x)) + c$

Antiderivatives of Rational Functions

1. $\int k \times \frac{1}{ax+b}\,dx = \frac{k}{a}\ln|ax+b| + c$
2. $\int \frac{1}{f(x)}\,dx = \frac{1}{f'(x)}\ln|f(x)| + c$

Derivatives and Integrals for Kinematics

$$s \underset{\text{integrate}}{\overset{\text{derive}}{\rightleftarrows}} v \underset{\text{integrate}}{\overset{\text{derive}}{\rightleftarrows}} a$$

More Probability

Bernoulli trial - trial with only two possible outcomes

Independent trial - two trials are independent if the probability of success of the 2nd trial is not affected by the outcome of the first.

Binomial distribution - distribution of the no. of successes in a sequence of independent Bernoulli trials.

$$B(n,p) = {}^nC_r \, p^r q^{n-r}$$

where $B(n,p)$ is the binomial distribution with n trials and probability of success p

n is the number of Bernoulli trials

p is the probability of success for one trial

$(p+q=1)$ q is the probability of failure for one trial

r is the target amount

Probability Density Functions

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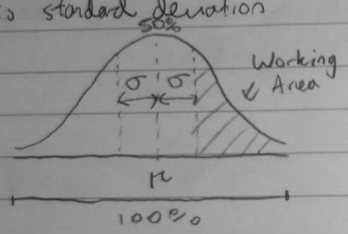
Normal Distribution and Z-Scores

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

where μ is mean of distribution

σ is standard deviation



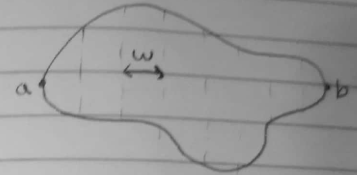
Z-scores calculate or convert μ and σ to a standard score which ~~measures how~~ "usual" how far or "unusual" ~~a set of or data points are.~~ a point is from μ measured in σ

$$z = \frac{x-\mu}{\sigma}$$

where x is the point being calculated

For values $> \mu$, use +ve z. For values $< \mu$, use -ve z.

Trapezoidal Rule



Used to estimate area of enclosed shape.

$$A = \frac{w}{2}(e + 2m), \quad w = \frac{b-a}{n}$$

where w is width of each strip

n is number of strips

e is sum of y-values @ ends

m is sum of y-values in middle

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