Production Optimization

Use Case Inventory

Use Case Name	LP/MILP/NLP	Type of Problem	MC?
Backpack packing	MILP	Knapsack	No
Production Optimization	MILP		No

Production Optimization

Uncle Bob produces chair and tables. Production time for one chair is 1 hour and for one tables is 3 hours. The selling price for one chair is $100 and for one table is $450. Uncle Bob can work up to 40 hours next week.

1. How many chairs and how many tables would you recommend him to build to maximize his revenue?
2. If he could only sell up to 10 tables and 40 chairs, how would your suggested plan change?
3. If he only could sell sets of 1 table and 4 chairs, how would the plan change?
4. If he could work 5 more hours next week (selling only sets), how much extra income would he get?

Manual Solution

1 13 tables
1 chair

2 10 tables
10 chair

3 5 tables
20 chair

4 6 tables
20 chairs

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1. How many chairs and how many tables would you recommend him to build to maximize his revenue?
2. If he could only sell up to 10 tables and 30 chairs, how would your suggested plan change?
3. If he only could sell sets of 1 table and 4 chairs, how would the plan change?
4. If he could work 5 more hours next week (selling only sets), how much extra income would he get?

Layman's Formulation

Decision vars:

* Num. of chairs to prod.
* Num. of tables to prod.

Objective:

Maximize revenue

Constraints:

Only 40 hours of work
Only up to 10 tables
Only up to 30 chairs
Sets of 4 chairs x 1 table

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Expanded Mathematical Formulation

Decision variables

→ Tables: num. tables to produce

→ Chairs: num chairs to produce

Objective function:

max 100·Chairs + 450·Tables

Constraints:

3Tables + Chairs ≤ ~~40~~ 45

Tables ≤ 10   Chairs ≤ 30

Chairs − 4Tables = 0

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totalTime

$limit_i$

Mathematical Formulation

PROD : set of products

Num_i : number of products i to manufacture, $i \in PROD$

$time_i$: number of hours req. to produce i

$price_i$: price of product i

$$\max \sum_{i \in PROD} price_i \, Num_i$$

$$\text{s.t.} \quad \sum_{i \in PROD} time_i \cdot Num_i \leq \overset{40}{totalTime}$$

$$Num_i \leq limit_i \quad \forall i$$

$$Num_{CH} - 4 Num_T = 0$$

Network Optimization

Use Case Inventory

Use Case Name	LP/MILP/NLP	Type of Problem	MC?
Backpack packing	MILP	Knapsack	No
Production Optimization	MILP		No
Network Optimization	LP	Shipment	No

Network Optimization Manual Solution

Yankee Shipping has received a request from VDog company to ship dog food to their customers in Boston, Newark and Toronto. VDog has warehouses in Chicago and Detroit. VDog requests Yankee Shipping to arrange for transportation at the lowest possible cost. The shipping cost per 1,000 lb between origins and destinations is provided as following:

Shipping Cost	Boston	Newark	Toronto
Chicago	$4	$2.5	$1.5
Detroit	$3	$1.5	$1

The dog food demand for Boston, Newark and Toronto is 500, 1,500 and 1,000 lbs respectively. The maximum inventory (supply) for Chicago and Detroit is 2,000 and 1,500 respectively. Find the amount to be transported between each warehouse and customers at a possible minimum cost.

Network Optimization

Layman's Formulation

Yankee Shipping has received a request from VDog company to ship dog food to their customers in Boston, Newark and Toronto. VDog has warehouses in Chicago and Detroit. VDog requests Yankee Shipping to arrange for transportation at the lowest possible cost. The shipping cost per 1,000 lb between origins and destinations is provided as following:

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Python Expanded Syntax

```
m = ConcreteModel()

m.Chi2Bos = Var(domain=NonNegativeReals)
m.Chi2New = Var(domain=NonNegativeReals)
m.Chi2Tor = Var(domain=NonNegativeReals)
m.Det2Bos = Var(domain=NonNegativeReals)
m.Det2New = Var(domain=NonNegativeReals)
m.Det2Tor = Var(domain=NonNegativeReals)

m.Boston_Demand = Constraint(expr = m.Chi2Bos + m.Det2Bos >= 0.5)
m.Newark_Demand = Constraint(expr = m.Chi2New + m.Det2New >= 1.5)
m.Toronto_Demand = Constraint(expr = m.Chi2Tor + m.Det2Tor >= 1)
m.Toronto_Supply = Constraint(expr = m.Chi2Bos + m.Chi2New + m.Chi2Tor <= 2)
m.Detrit_Supply = Constraint(expr = m.Det2Bos + m.Det2New + m.Det2Tor <= 1.5)

m.TotalCost = Objective(expr = 4*m.Chi2Bos + 2.5*m.Chi2New + 1.5*m.Chi2Tor +
                        3*m.Det2Bos + 1.5*m.Det2New + m.Det2Tor,
                        sense=minimize)

opt = SolverFactory('glpk')
sol = opt.solve(m)
print(m.Chi2Bos(), m.Chi2New(), m.Chi2Tor(),
      m.Det2Bos(), m.Det2New(), m.Det2Tor(),  m.TotalCost())
```

```
0.0 0.5 1.0 0.5 1.0 0.0 5.75
```

Network Optimization

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Python Compact Syntax

```
m = ConcreteModel()

m.ORIGINS = Set(initialize=['Chi','Det'])
m.DEST = Set(initialize=['Bos','New','Tor'])
m.Pounds = Var(m.ORIGINS,m.DEST,domain=NonNegativeReals)

m.dem = Param(m.DEST, initialize={'Bos':0.5, 'New':1.5, 'Tor':1})
m.avail = Param(m.ORIGINS, initialize={'Chi':2, 'Det':1.5})
cost = {('Chi', 'Bos'): 4,
        ('Chi', 'New'): 2.5,
        ('Chi', 'Tor'): 1.5,
        ('Det', 'Bos'): 3,
        ('Det', 'New'): 1.5,
        ('Det', 'Tor'): 1}
m.cost = Param(m.ORIGINS, m.DEST, initialize=cost)

def Demand_Rule(m,d):
    return sum(m.Pounds[o,d] for o in m.ORIGINS) >= m.dem[d]
m.Demand = Constraint(m.DEST, rule=Demand_Rule)

def Supply_Rule(m,o):
    return sum(m.Pounds[o,d] for d in m.DEST) <= m.avail[o]
m.Supply = Constraint(m.ORIGINS, rule=Supply_Rule)

def TotalCost_Rule(m):
    return (sum(m.cost[o,d]*m.Pounds[o,d] for o in m.ORIGINS for d in m.DEST))
m.TotalCost = Objective(rule=TotalCost_Rule, sense=minimize)

opt = SolverFactory('glpk')
sol = opt.solve(m)
print([value(m.Pounds[o,d]) for o in m.ORIGINS for d in m.DEST], m.TotalCost())
```

```
[0.0, 0.5, 1.0, 0.5, 1.0, 0.0] 5.75
```

Practice Problem

Z company produces X and Y using two machines (A and B).

Product	Machine A	Machine B
X	50	30
Y	24	33

Currently there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours. This week, it is forecasted that 75 units of X will be sold and 95 units of Y.

Formulate this problem to decide how much of each product to make in the current week to maximize the combined sum of the units of X and the units of Y in stock at the end of the week.