Lecture 1

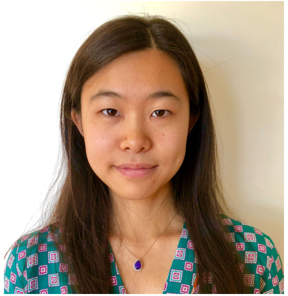
Intro to the Course & Linear Algebra Review

Welcome to Applied Intro to Quantum Computing!

Our Teaching Staff

Edwin Agnew
CS + Philosophy (‘22)
edwin.agnew@duke.edu

Henry Barklam
Math + CS (’23)
henry.barklam@duke.edu



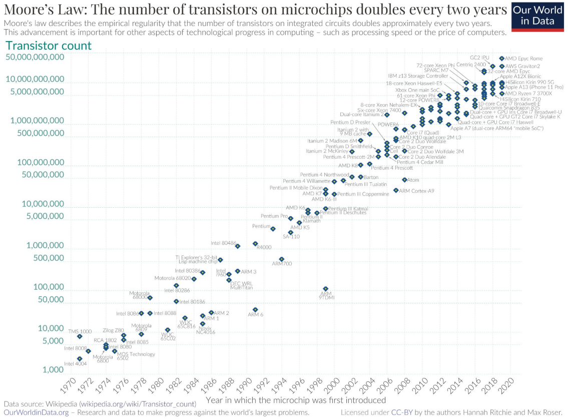
Erin Liu
ECE + Physics (‘23)
erin.liu@duke.edu

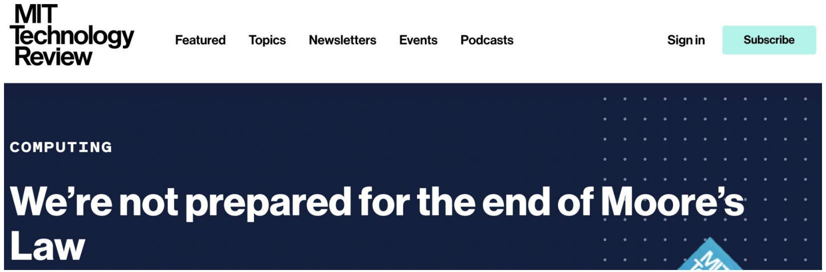
Hunter Kemeny
Math + Physics (‘24)
ahk22@duke.edu

Overview

Context

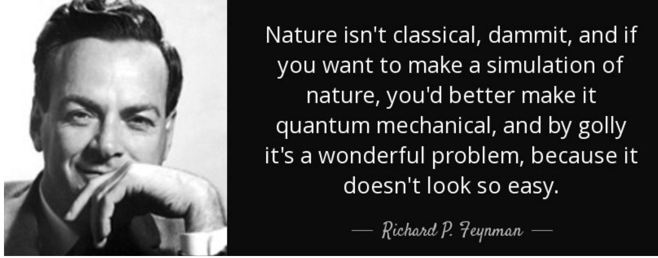
- Since 1970, classical computers have doubled in size every two years (Moore's Law)
- This was largely made possible by shrinking the size of transistors
- We are approaching the physical size limit and experts predict that Moore's Law will end by 2025
- So what happens next?





Most forecasters, including Gordon Moore,[125] expect Moore's law will end by around 2025.[126][123][127] Although Moore's Law will reach a physical limitation, many forecasters are optimistic about the continuation of technological progress in a variety of other areas, including new chip architectures, quantum computing, and AI and machine learning.[128][129]

History



- Quantum Computers were first proposed in the late 1970s by Feynman as a way to more efficiently simulate physical systems
- Applications outside of physics began to be discovered, including:
  - 1984: Quantum Key Distribution
  - 1992: Quantum Teleportation
  - 1994: Shor's factoring algorithm
- Meanwhile, progress was made on actually building quantum computers:
  - 1998: first working 3 qubit NMR quantum computer
  - 2001: 15 is factored using a quantum computer
  - 2019: Google claims to achieve "quantum supremacy" using 53 superconducting qubits

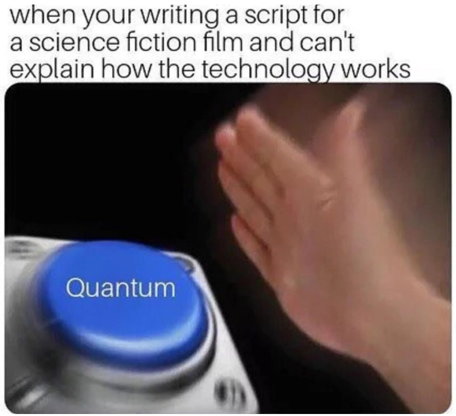
demonstration of this algorithm has proved elusive[8,9,10]. Here we report an implementation of the simplest instance of Shor's algorithm: factorization of $N = 15$ (whose prime factors are 3 and 5). We use seven spin-1/2 nuclei in a molecule as quantum bits[11,12], which can be

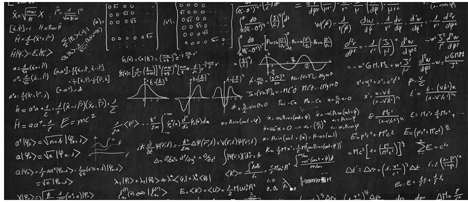
Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

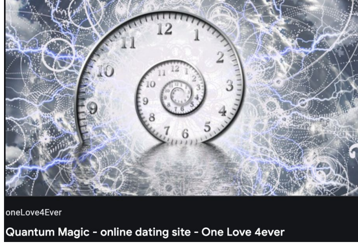
What is Quantum?

- Quantum mechanics is the physics of **very small things**
- It has a number of counter-intuitive properties:
  - Things can be in superposition (“cat problem”)
  - Observation or measurement can change states (“measurement problem”)
- It has a misleading reputation for being “mysterious” or “incomprehensible”
- However, in the language of linear algebra it is surprisingly straightforward
- First, we have to learn to speak the language



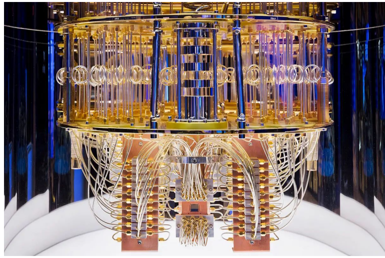


What is Quantum Computer?



- Level 1: A quantum computer is a computer that does quantum stuff
- Level 2: A quantum computer uses parallel universes to compute many things at once

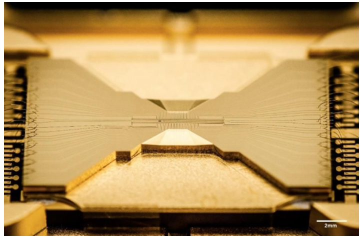
- Level 3: A quantum computer exploits **specific features** of quantum mechanics (superposition, entanglement, interference) to solve *some* problems more efficiently than ordinary computers
- Level 4: ?

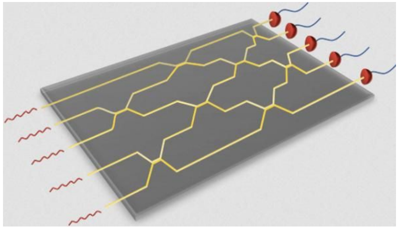


What can it do?

- There are two main fields within the applications of quantum computing
- Quantum Algorithms:
  - Simulation - important for physics/chemistry research; promising for drug development, fertiliser production, etc. **[exponential advantage]**
  - Shor's factoring algorithm (breaks RSA encryption) => structured/ordered problems **[exponential advantage]**
  - Grover's algorithm - useful for search/unstructured problems **[quadratic advantage]**
- Quantum Communication:
  - Quantum Cryptography
  - Quantum Teleportation
  - Quantum Internet
- Much more to be discovered...

How to build?



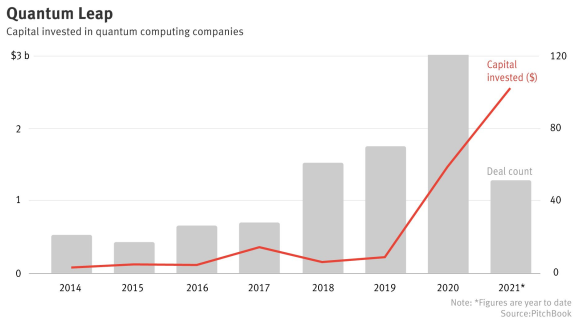


- Similar to how classical computers used vacuum tubes before transistors, there are many different ways to build a quantum computer
- Comparison of 3 currently most popular approaches:

Platform	**Superconducting**	**Trapped Ion**	**Photonic**
Pros	• Easily manufacturable • Fast	• High fidelity • All-to-all connectivity	• Room temp • Fibre optics for communication
Cons	• Limited connectivity • Near 0K temp required	• Slow • Lots of big lasers	• Probabilistic 2-qubit gates
Company (number of qubits)	IBM (127), Google (53)	IonQ (32), Honeywell (10)	Xanadu (12), PsiQuantum (???)

Why should you care?

- Quantum computing is the technology of the future!
- Useful applications; fascinating theory
- Interdisciplinary: physics, maths, computer science, engineering, ...
- Blossoming in both industry and academia:



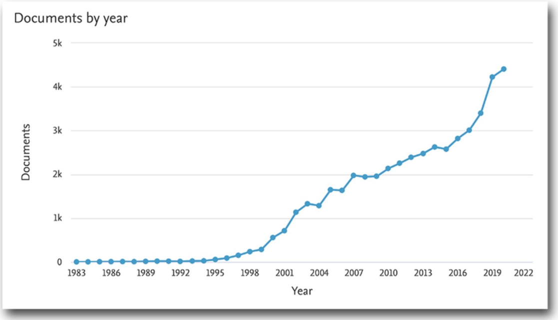


Figure 1. Quantum computing documents by year, 1982–2020

- Just the tip of the iceberg!

Logistics

Course Content Outline

- Part I: Quantum Information Basics
  - Module 1: Linear Algebra, Quantum Mechanics, Quantum Circuits (3 lectures)
  - Module 2: Entanglement, Quantum Communication Protocols (2 lectures)
- Part II: Applications
  - Module 3: Survey of Quantum Algorithms (4 lectures)
  - Module 4: Physical Implementations and Applications (3 lectures)
- Guest Lecture + Final Project Presentation (2 lectures)

Lecture Logistics

Date/time - Every Tuesday 7:30 - 9PM

Location - Zoom, physical location TBD

Format - 1 hr lecture + ½ hr demo in Qiskit

Resources - Lecture slides + demos (Sakai Resources), recording of each session (Sakai Zoom Meetings)

Office Hours - Available upon request, via Zoom

Expectations/Grading

- Attendance and participation are **required for 12/14 lectures** plus the last 15th lecture being **absolutely mandatory**

- There is **no homework**! Instead, there are **optional exercises** with each lecture to help you understand the topics better!

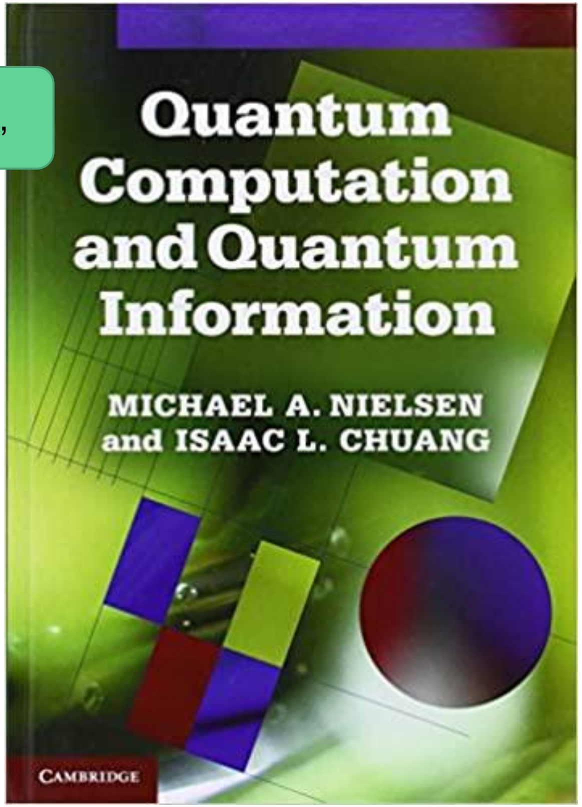
- **Final Project**:
  - Hackathon-style group/individual project of your choice!
  - Guidelines and checklist are posted in the Resources folder in Sakai
  - Present your project during the last mandatory lecture

Useful Resources

Recommended Texts

For guide on these texts, refer to “Useful Resources”

- Qiskit textbook
  https://qiskit.org/textbook/content/ch-ex/
- *Quantum Computation and Quantum Information*, by Nielsen & Chuang
- *An Introduction to Quantum Computing*, by Kaye, Laflamme, & Mosca
- *Quantum Computing: An Applied Approach*, by Jack Hidary



Qiskit and IBM Q Experience

Qiskit

Elements for building a quantum future

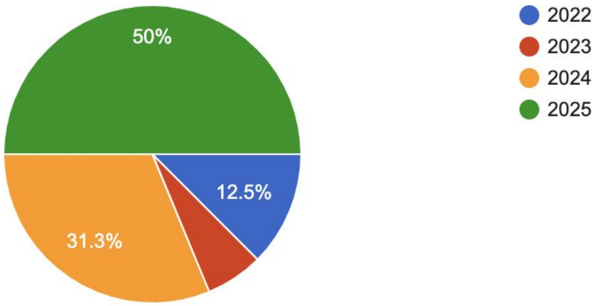
```
pip install qiskit
```

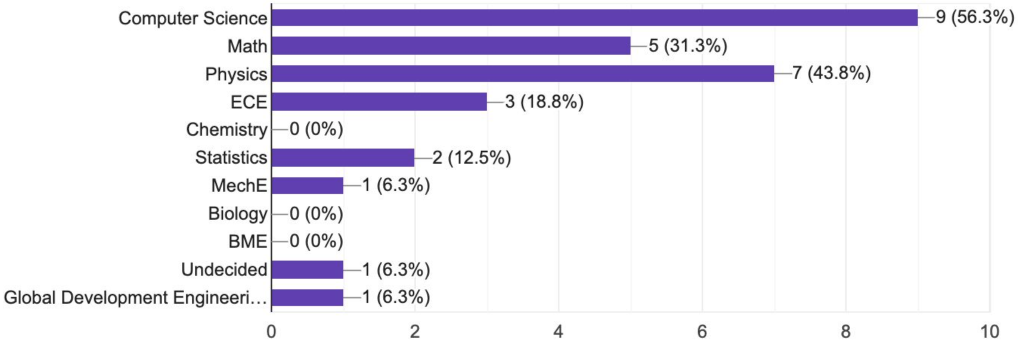
IBM Q Experience: https://quantum-computing.ibm.com

Qiskit Textbook: https://qiskit.org/textbook/preface.html

What to Expect

Demography

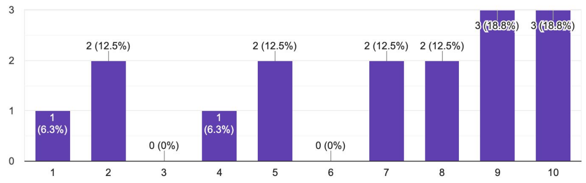

50%
12.5%
31.3%
2022
2023
2024
2025


Computer Science
9 (56.3%)
Math
5 (31.3%)
Physics
7 (43.8%)
ECE
3 (18.8%)
Chemistry
0 (0%)
Statistics
2 (12.5%)
MechE
1 (6.3%)
Biology
0 (0%)
BME
0 (0%)
Undecided
1 (6.3%)
Global Development Engineeri…
1 (6.3%)
0
2
4
6
8
10

Background Experience

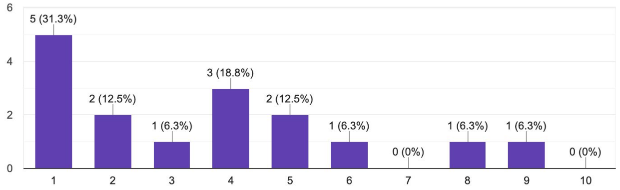
Are you familiar with linear algebra (e.g. vectors, matrices, linear independence, vector spaces, inner product, outer product, determinant, trace, transpose, eigenvalues, eigenvectors, etc.)?

16 responses



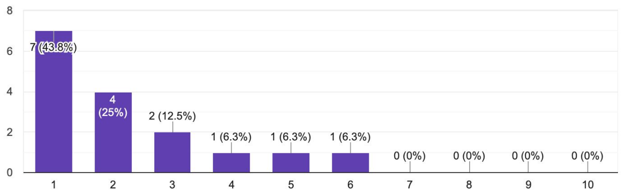
Are you familiar with complex linear algebra (e.g. adjoint , unitary matrices, hermitian matrices, etc.)

16 responses



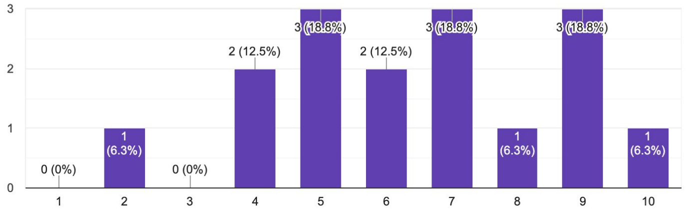
Are you familiar with quantum mechanics (e.g. quantum postulates, superposition, entanglement, etc.)

16 responses



Programing experience (e.g. Python, numpy, Jupyter Notebook, GitHub, etc.)?

16 responses



- We cover all background (except python) in lectures
- **Come to Office Hours!**

DuQIS

DuQIS

Duke Undergraduate
Quantum Information Society

Mailing list: bit.ly/duqis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QUANTUM START-UP PANEL
In the rapidly booming quantum computing industry, start-ups are sprouting up around the world. Join this DuQIS-hosted panel to learn about these up-and-coming players in the industry.
Oct. 8 (Fri.)
4-5p.m.
On Zoom
Yunseong Nam
IonQ (U.S.)
Mark Jackson
CQ (U.K.)
Ilan Tzitrin
Xanadu (Can.)
Peter Johnson
Zapata (U.S.)
Leonardo Castro
Q-CTRL (Aus.)
Naomi Nickerson
PsiQuantum (U.S.)
Register Now:
*Sign up to receive e-mail updates of the event information.
Presented by:
Duke Quantum Center
DuQIS
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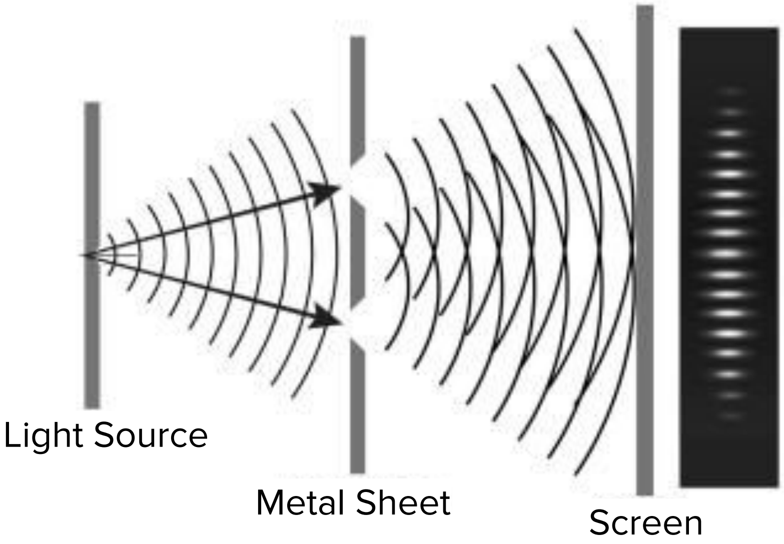
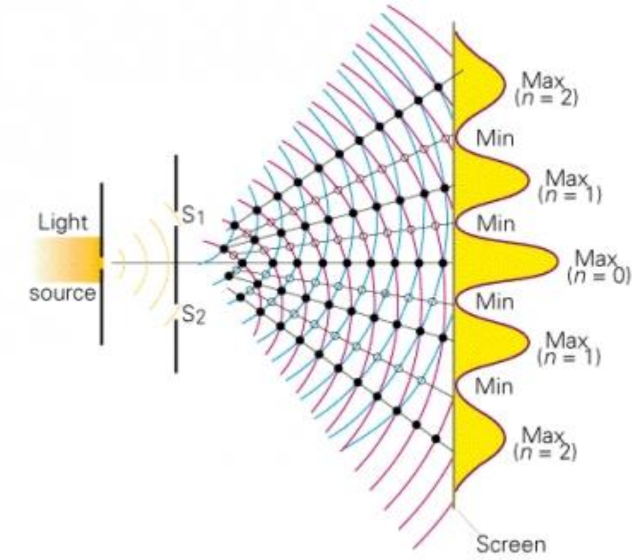
Quantum Weirdness

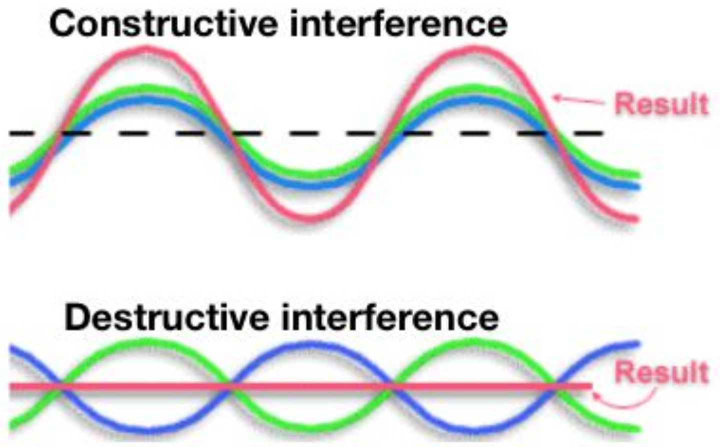
Double-Slit Experiment





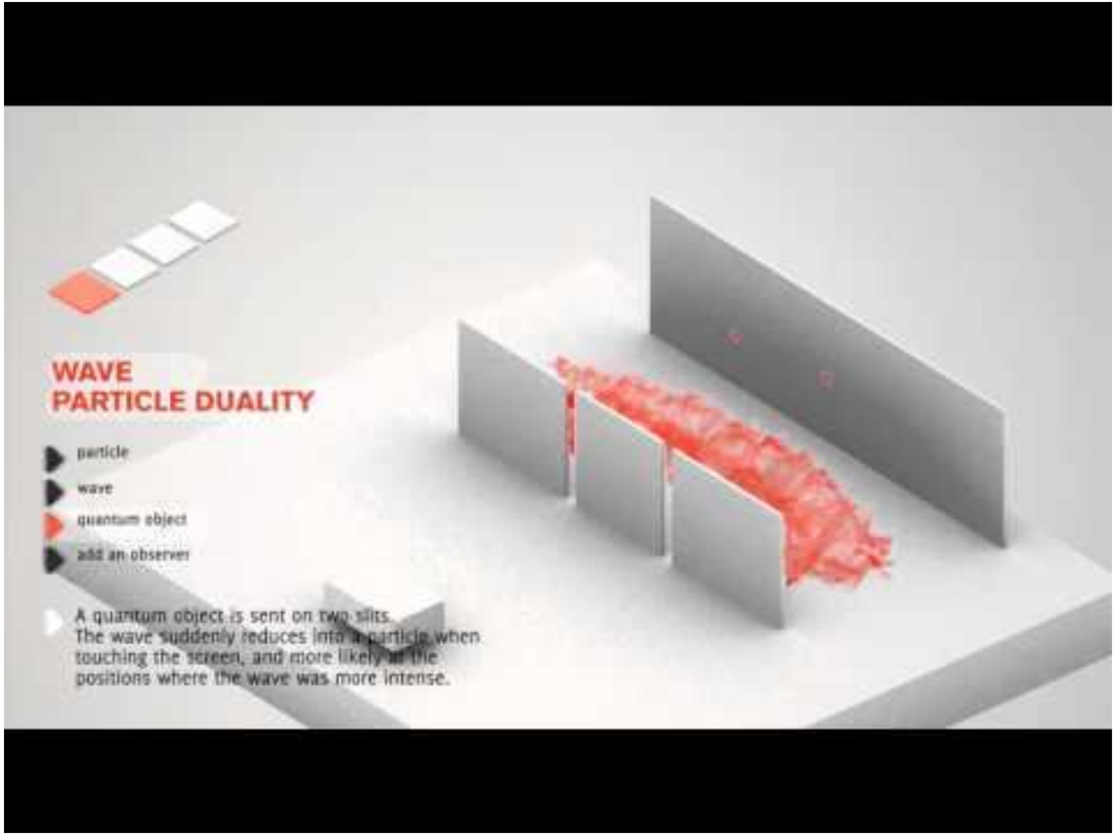
Interference

- Constructive Interference:
    - The two waves combine to create a wave of greater amplitude
- Destructive Interference:
    - The two waves combine to create a wave of lesser amplitude
- If we looked at just one wave, it would look normal, but two waves create interference

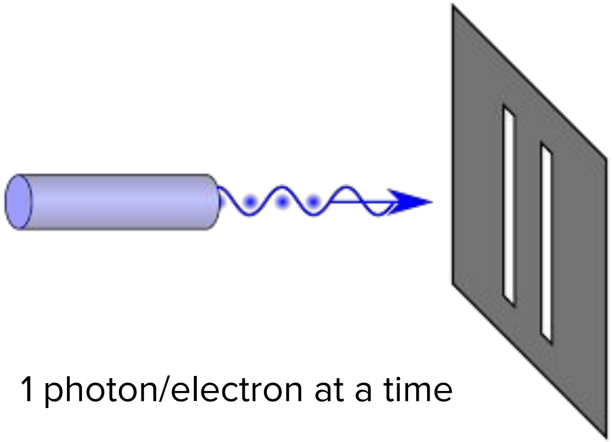


https://www.fizzics.org/interference-of-waves/a

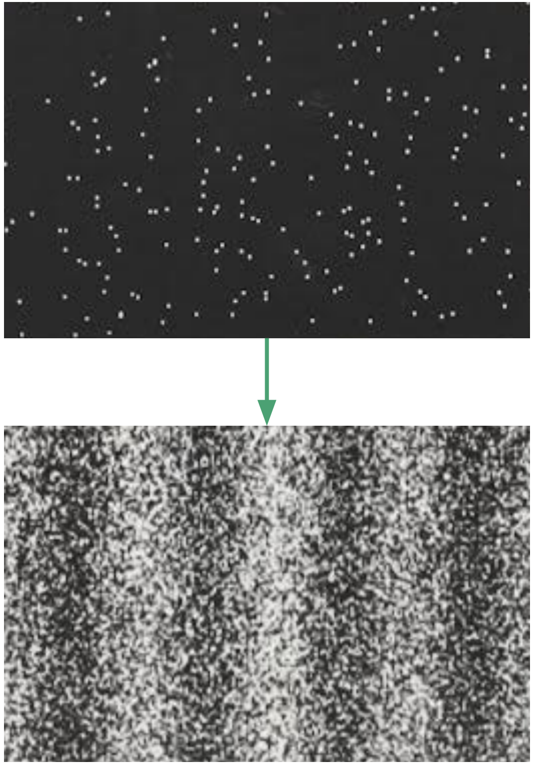
Short Animation on Quantum Objects



Firing Photons One-by-One



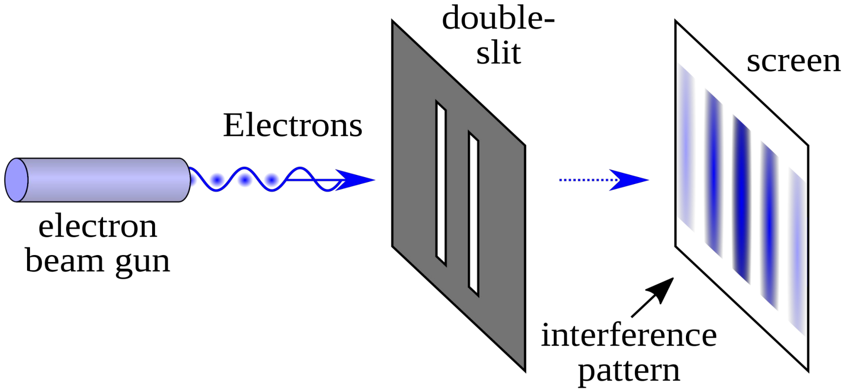
1 photon/electron at a time



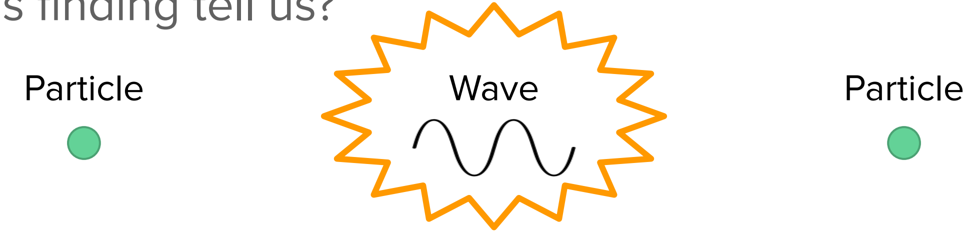
Correspondence Principle!

Double-Slit Experiment

- Even when fired one-by-one, photon produces interference pattern



- What does this finding tell us?



Wave of What? Copenhagen Interpretation

- Niels Bohr and Werner Heisenberg 1925-1927
- Probability wave!
- Wavefunction describes probability distribution of the outcomes of each possible measurement on a system
  - Peaks: more chance particle finds itself there
- Measurement (hit the screen) results in collapse of wave function
  - Return to particle-like certainty
- *Electron travels as a wave, is detected as a particle*

Linear Algebra and Dirac Notation

Why Do We Need Linear Algebra?

- Quantum involves high dimensional spaces
- Very difficult to represent without vectors

$$x, y, z, a, b, c, d, e, f, g, h, i \Rightarrow \begin{pmatrix} x \\ y \\ \dots \\ i \end{pmatrix}$$

- Vectors can be 'operated on' by matrices

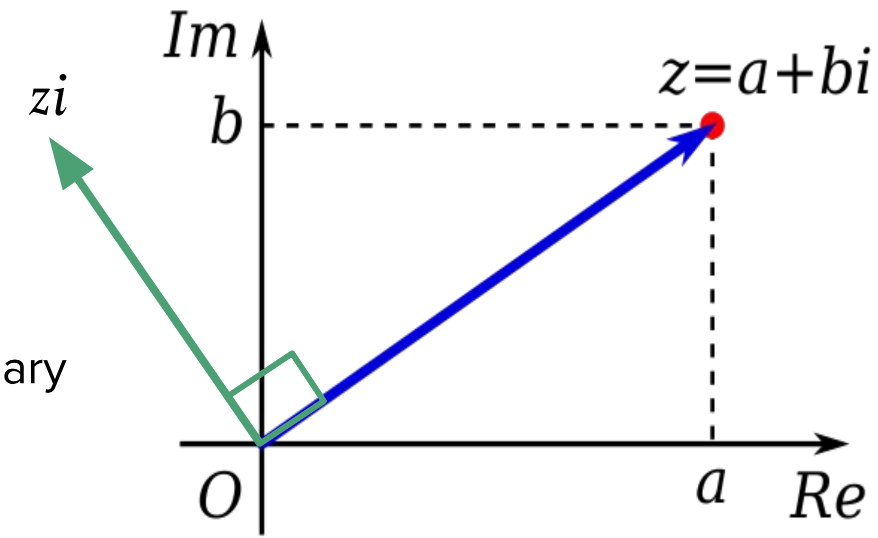
$$\begin{matrix} x = \lambda_1 y + \lambda_2 z + \lambda_3 a + \lambda_4 b + \dots + \lambda_{12} i \\ y = \lambda_{13} x + \lambda_{14} z + \lambda_{15} a + \lambda_{16} b + \dots + \lambda_{24} i \end{matrix} \Rightarrow \begin{pmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_{12} \\ \lambda_{13} & \lambda_{14} & \dots & \lambda_{24} \end{pmatrix}$$

Imaginary Numbers

- $z = a+bi$

Let us investigate what applying an imaginary number does to the vector

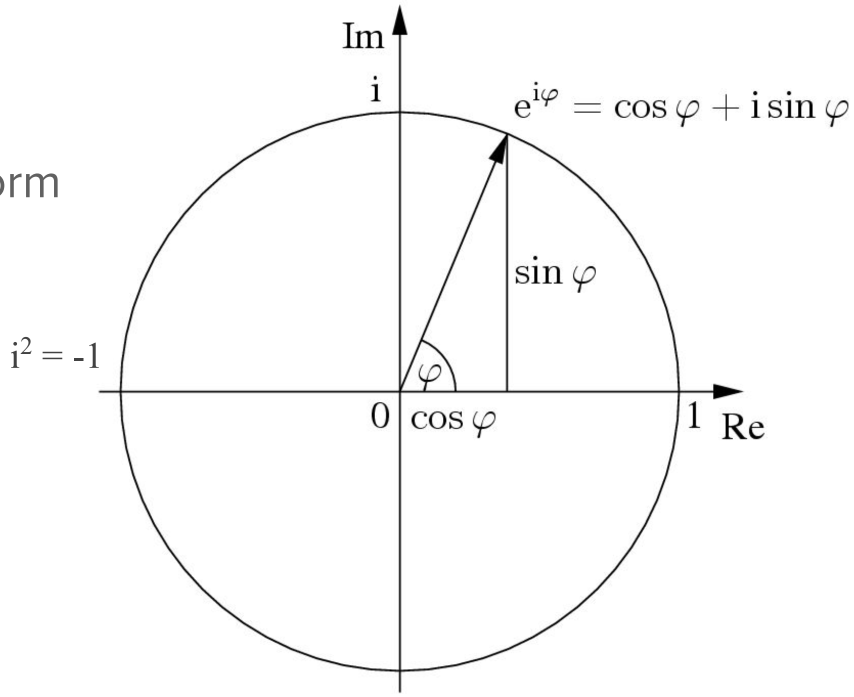
- $zi = (a+bi)i = ai + bii = -b + ai$



- By multiplying any complex number by i, there is a 90° rotation on the complex plane

Complex Numbers

- $i^2 = -1$
- $a+bi$ can be $re^{i\theta}$ written in exponential form
  - Euler's formula: $re^{i\theta} = r\,(\,cos\theta + i\ sin\theta\,)$
- Complex Conjugate (*)
  - If $x = 3 + 5i$, then $x^* = 3 - 5i$
- Complex Modulus (magnitude)
  - $|a + bi| = \sqrt{a^2 + b^2}$
  - $|re^{i\theta}| = |r|$



https://www.expii.com/t/eulers-formula-on-complex-numbers-5283

Dirac Notation (Bra-Ket notation)

- Bras are row vectors, like : $\langle\Psi| = \begin{pmatrix} x & y & z \end{pmatrix}$

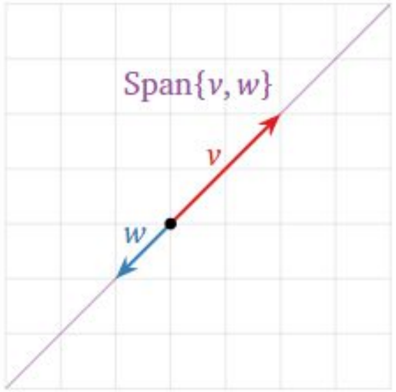
- The dual of this is a ket, or column vector, like this: $|\Psi\rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

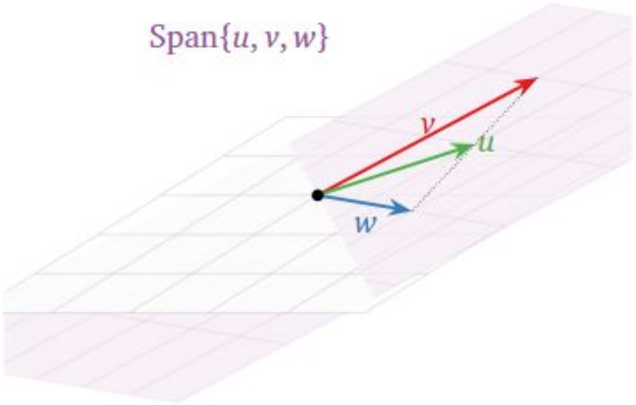
- Notice that $\langle\Psi|$ is not the same as $|\Psi\rangle$
  - x is the complex conjugate of a, etc.

Span

- For some set of vectors $\{|v_1\rangle, |v_2\rangle, \ldots, |v_n\rangle\}$, the span is the subspace consisting of all linear combinations of these vectors

$$c_1 |v_1\rangle + \ldots + c_n |v_n\rangle$$

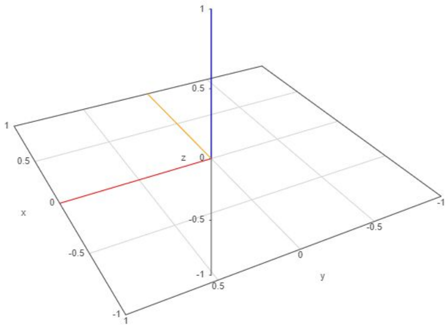




https://textbooks.math.gatech.edu/ila/linear-independence.html

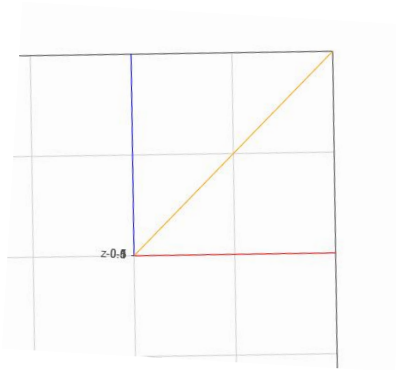
Linear Independence

- A set of vectors is linearly independent if each vector in the set does not fall in the span of the other vectors
- Examples:



$$\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$$

are linearly independent



$$\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$$

are linearly dependent

- If the set has more vectors than the span, there is linear dependence

Basis

- The basis of a space is a set of vectors such that any vector within that space can be written as a linear combination of the basis vectors

- For ℝ3, we might expect something like: $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$

- But this is also valid: $\begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\3\end{pmatrix}$

- You can verify that this spans ℝ3 and is L.I.

Notes on Bases

- Basis vectors are often constrained to unit vectors

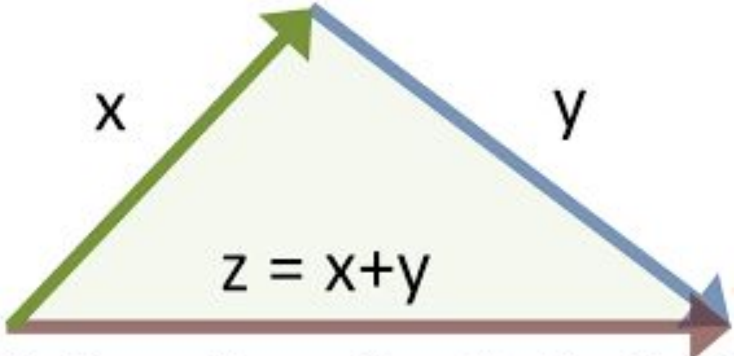
- We typically use 'orthonormal' bases, which means the basis vectors are all orthogonal

Vector Operations

Vector Norm

Take some vector $|x\rangle = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$||x\rangle| = \sqrt{|x_1|^2 + |x_2|^2 + |x_3|^2}$$



https://brilliant.org/wiki/triangle-inequality/

Properties:

- $||x\rangle| \geq 0$
- $||x\rangle| = 0$ iff $|x\rangle = 0$
- $|k|x\rangle| = |k| \times ||x\rangle|$
- $||z\rangle| = ||x\rangle + |y\rangle| \leq ||x\rangle| + ||y\rangle|$ (triangle inequality)

Inner Product

- Inner product of **a** and **b** is traditionally **a** · **b** (dot product)
- More generally it is $\langle A||B\rangle$, or $\langle A|B\rangle$

Dimensions must match!

$$\sum a_n b_n = a_1 b_1 + \ldots + a_n b_n$$

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 = 32$$

- Note: vector norm of $|x\rangle$ can be represented as $\sqrt{\langle x|x\rangle}$

Unit Vectors and Normalization

- Unit vector: vector with a norm of 1

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}, \begin{pmatrix} \sqrt{.5} \\ \sqrt{.4} \\ \sqrt{.1} \end{pmatrix}$$

- Normalization is how we turn any vector into a unit vector $\frac{|x\rangle}{\sqrt{\langle x||x\rangle}}$

- Take vector $|x\rangle = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, which has norm $\sqrt{\langle x|x\rangle} = \sqrt{14}$

- We normalize $|x\rangle$ by dividing by its norm $\frac{|x\rangle}{\sqrt{\langle x||x\rangle}} = \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Outer Product

- Written as $|X\rangle\langle Y|$
- Dimensions don't need to match!

$$|X\rangle\langle Y| = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix} = \begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{pmatrix} x_1y_1 & x_1y_2 & x_1y_3 \\ x_2y_1 & x_2y_2 & x_2y_3 \\ x_3y_1 & x_3y_2 & x_3y_3 \end{pmatrix} \end{array}$$

Matrices

Matrix notation

- Size: the size of a matrix is mxn (m-by-n)
  - m is the number of rows
  - n is the number of columns
- Values of matrix A are written as a_{ij}
  - i is row number
  - j is column number

4×3 matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{pmatrix}$$

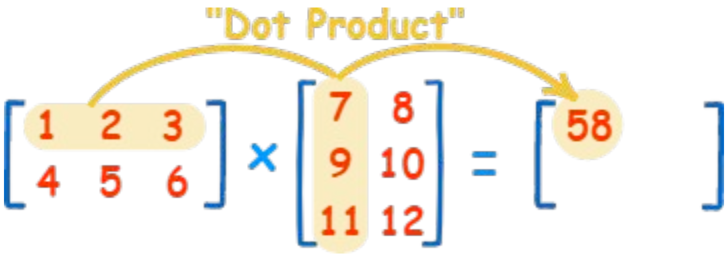
Identity Matrices (I)

- Square matrices with “identity property”
- Any matrix A times the identity matrix I is itself
- $\mathrm{AI} = \mathrm{IA} = \mathrm{A}$

$$\mathrm{I}_1 = [1],\ I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \mathrm{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Matrix Multiplication

- **Order matters!**
- The first matrix must have as many columns as the second one has rows



https://www.mathsisfun.com/algebra/matrix-multiplying.html

2×3 3×2 2×2

- An m×n matrix multiplied by a n×k matrix makes an m×k matrix

Transpose (^{T})

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$$[A^T]_{ij} = [A]_{ji}$$

$[A]_{12} = 2 \longrightarrow [A^T]_{21} = 2$

$[A]_{21} = 4 \longrightarrow [A^T]_{12} = 4$

Conjugate Transpose

- Can be called adjoint, hermitian conjugate, “dagger”, etc
- Represented as A^*, A^H, or $A^\dagger$

$$A = \begin{pmatrix} 3+5i & 4-2i \\ 7 & 3i \\ -7i & -8 \end{pmatrix}$$

$$A^\dagger = \begin{pmatrix} 3-5i & 7 & 7i \\ 4+2i & -3i & -8 \end{pmatrix}$$

$$[A^\dagger]_{ij} = [A]^*_{ji}$$

$[A]_{12} = 4\text{-}2i \;\longrightarrow\; [A^\dagger]_{21} = 4+2i$

$[A]_{31} = \text{-}7i \;\longrightarrow\; [A^\dagger]_{13} = 7i$

$[A]_{32} = \text{-}8 \;\longrightarrow\; [A^\dagger]_{23} = \text{-}8$

Dagger in Dirac Notation

- Start with any bra

$$\langle\Psi| = \begin{pmatrix} 1+3i & -2 & -3i \end{pmatrix}$$

- To get from a bra to a ket, take the dagger

$$\langle\Psi|^\dagger = |\Psi\rangle = \begin{pmatrix} 1-3i \\ -2 \\ 3i \end{pmatrix}$$

- You can go back to a bra by taking the dagger again

$$|\Psi\rangle^\dagger = \langle\Psi| = \begin{pmatrix} 1+3i & -2 & -3i \end{pmatrix}$$

Inner Product with Conjugate Transposes

- Suppose we want to find the unit vector version of $\begin{pmatrix} 1 \\ i \end{pmatrix}$

- Let us first find its normalization constant!

Inner Product with Conjugate Transposes

$$\frac{|x\rangle}{\sqrt{\langle x||x\rangle}}$$

$$\begin{pmatrix} 1 & i \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = 1 + i * i = 1 - 1 = 0$$

$$\begin{pmatrix} 1 & \boxed{-i} \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = 1 + i * (-i) = 1 + 1 = 2 \qquad \boxed{\frac{1}{\sqrt{2}}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

Normalization Constant

- Do not forget to take the dagger for inner product!
- Dagger is also very important in proving if matrices are Hermitian (observables) and/or Unitary (evolutionary operators)

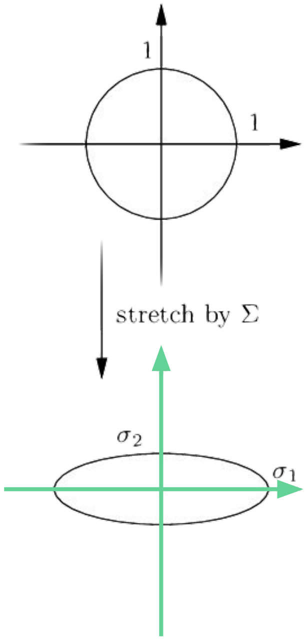
Eigen-Decomposition

- Vector $|\Psi\rangle$ (of length n) is an eigenvector of matrix A ($n \times n$) if

$$A\,|\Psi\rangle = \lambda\,|\Psi\rangle$$

- In this case, $|\Psi\rangle$ is an 'eigenvector' of A

- λ is a scalar 'eigenvalue' of A that corresponds to the eigenvector $|\Psi\rangle$



Hermitian Matrices

- If $A^\dagger = A$ then we call the matrix a "Hermitian Matrix"
- Hermitian matrices are always square matrices
- Have all real eigenvalues
- Observables are represented by Hermitian matrices!

$$\begin{pmatrix} 1 & 4+2i & 0 \\ 4-2i & 3 & 7+4i \\ 0 & 7-4i & 5 \end{pmatrix}$$

Unitary Matrices (U)

- $U^{+}U = UU^{+} = I$, where I is the identity matrix
- $U^{-1} = U^{+}$
- Always square matrices
- Eigenvalues are ±1
- Columns form orthonormal basis
- Operations for evolution are unitary matrices!

Examples

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Next Lecture

- What are the fundamental “rules” of quantum mechanics?

- Programming!

Assignment

- Install Qiskit and set up your environment

https://docs.google.com/document/d/1rT46yQTwawY-GYs-Tzy4c2ZNMsvpOlbGxa0w5RvlWl0/edit?usp=sharing

Exercises (optional)

- Practice converting from $a+bi$ to $re^{i\theta}$ and back
- Prove or demonstrate the vector norm properties
- Prove or demonstrate Hermitian matrices have real eigenvalues
    - Eigen-decompositions will be covered more next week
- Practice matrix multiplication
- Prove or demonstrate that a vector norm can be the root of the inner product with itself
- Practice/review linear independence and dependence