CompSci 116: Lecture 12: Decisions

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Decisions Under Uncertainty

Interpretation by Physicians of Clinical Laboratory Results (1978)

> "We asked 20 house officers, 20 fourth-year medical students and 20 attending physicians, selected in 67 consecutive hallway encounters at four Harvard Medical School teaching hospitals, the following question:
>
> "If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5%, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person's symptoms or signs?"

Decisions Under Uncertainty

Interpretation by Physicians of Clinical Laboratory Results (1978)

> "Eleven of 60 participants, or 18%, gave the correct answer. These participants included four of 20 fourth-year students, three of 20 residents in internal medicine and four of 20 attending physicians. The most common answer, given by 27, was that [the chance that a person found to have a positive result actually has the disease] was 95%.

Probability Review

- P(A): Probability of event A
  - Value in [0,1]
  - How likely is A to happen?
- P(*A*) + P(not *A*) = 1
- P(*A* ∪ *B*) = P(*A*) + P(*B*) – P(*A* ∩ *B*)

  e.g., *x* = roll of a 6-sided die;

  P(*x* is even ∪ *x* > 3)

  = P(*x* is even) + P(*x* > 3) – P(*x* is even ∩ *x* > 3)

  = 0.5 + 0.5 – 1/3 = 2/3

Question #1

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- Assume 500 students enrolled in both Calculus and Physics. Of these students,
  - 82 got an A in calculus,
  - 73 got an A in physics, and
  - 42 got an A in both courses.
- Which of the following probabilities is the smallest?

The probability that a *randomly* chosen student:

a. Got an A in at least one of the two courses
b. Got less than an A in at least one of the two courses
c. Got an A in **both** courses
d. Got an A in calculus but not in **physics**
e. Got an A in physics but not **calculus**

Round One

- Scenario:
  - Class consists of second years (60%) and third years (40%)
  - 50% of the second years have declared their major
  - 80% of the third years have declared their major
  - I pick one student at random.
- Which is more likely: Second year or third year?
  - Second year, because they are 60% of the class

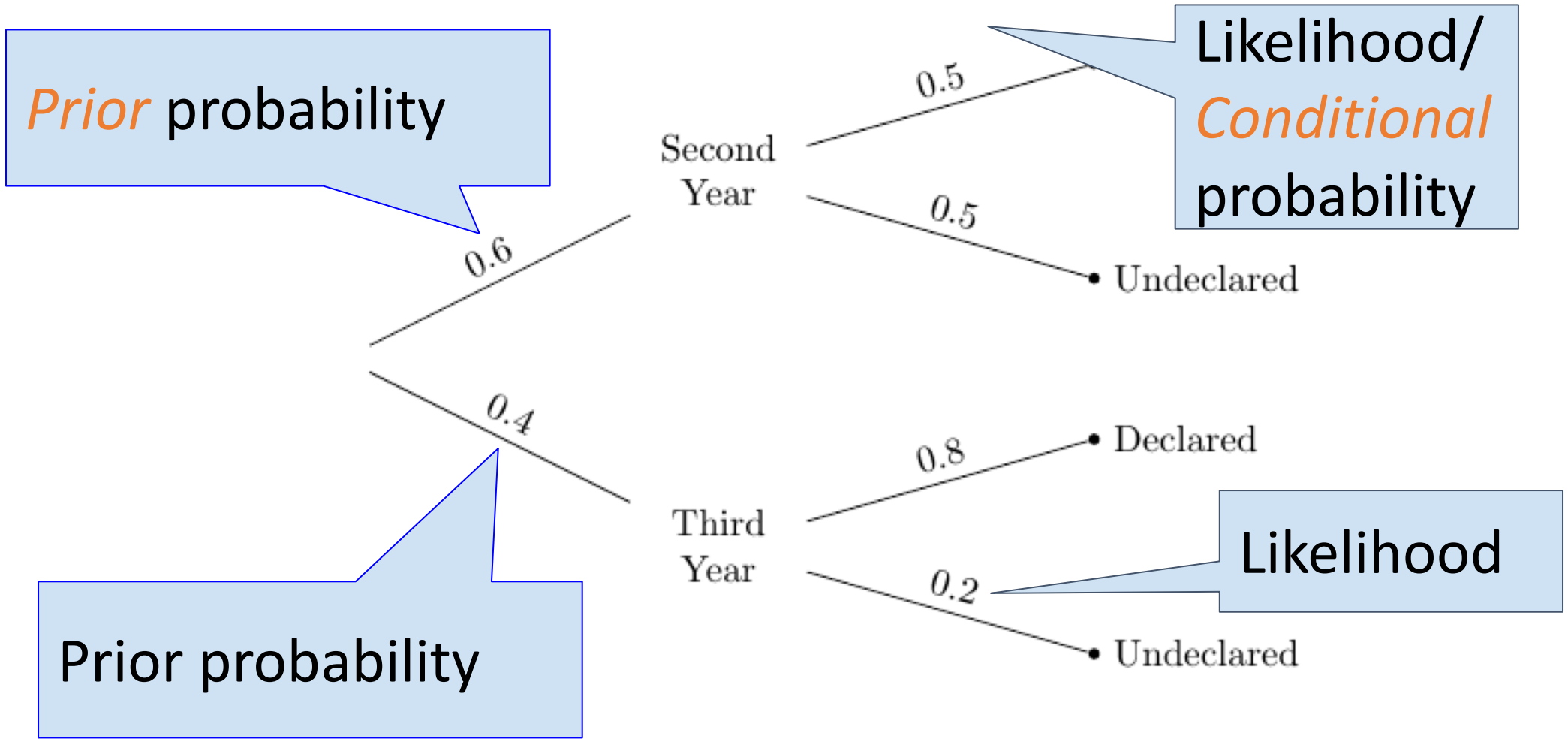
Round Two

- Slightly different scenario:
  - Class consists of second years (60%) and third years (40%)
  - 50% of the second years have declared their major
  - 80% of the third years have declared their major
  - I pick one student at random...
    That student has declared a major!
- Second Year or Third Year?

(Demo)

Terminology

$$P(B \mid A) = P(A \cap B) / P(A)$$



Problem 2: Conditional Probability

http://bit.ly/FoDS-s19-0411-2

- Hat with three cards,
  - Blue on both sides
  - Red on both sides
  - Red/Blue
- A card is drawn at random from the hat and you see that one side is blue

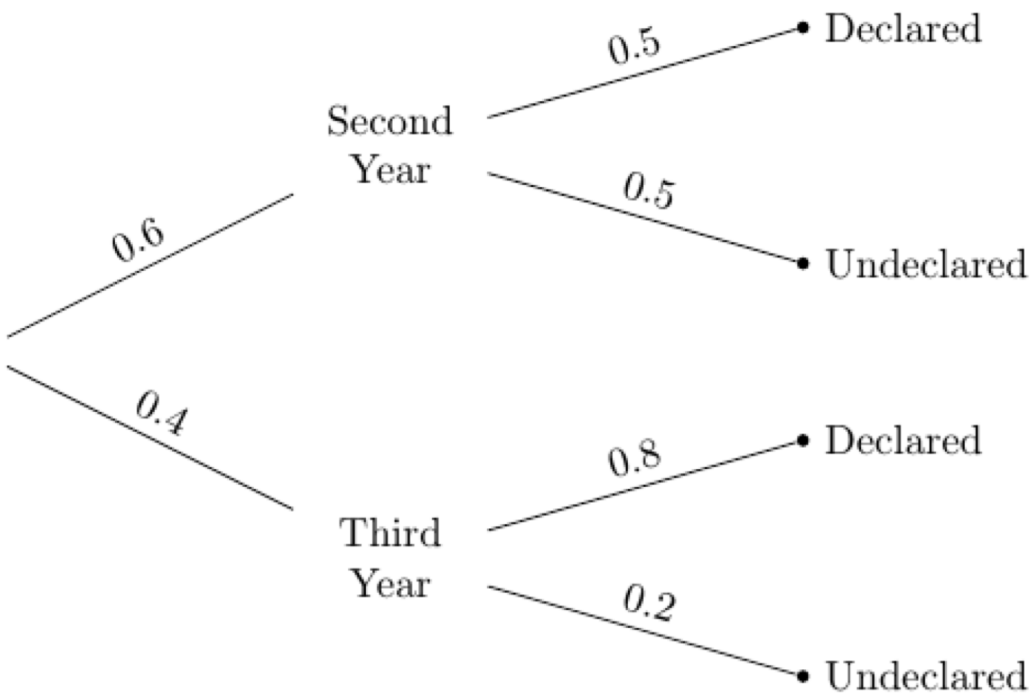
What is the probability the other side is blue?

$a.\ {}^{1}/_{3}$

$b.\ {}^{1}/_{2}$

$c.\ {}^{2}/_{3}$

Bayes' Rule



Pick a student at random.

Posterior probability:

P(Third Year | Declared)

$$= \frac{0.4 \times 0.8}{(0.6 \times 0.5) + (0.4 \times 0.8)}$$

$$= 0.5161\ldots$$

Purpose of Bayes' Rule

- Update your prediction based on new information
- In a multi-stage experiment, find the chance of an event at an earlier stage, given the result of a later stage

(Demo)

Example: Doctors & Clinical Tests

- Out of 1000 patients:

	Positive test result	Negative test result
Has disease	1	0
Doesn't have disease	49.95	949.05

- So only 1 / 50.95 of patients with positive test results have the disease.