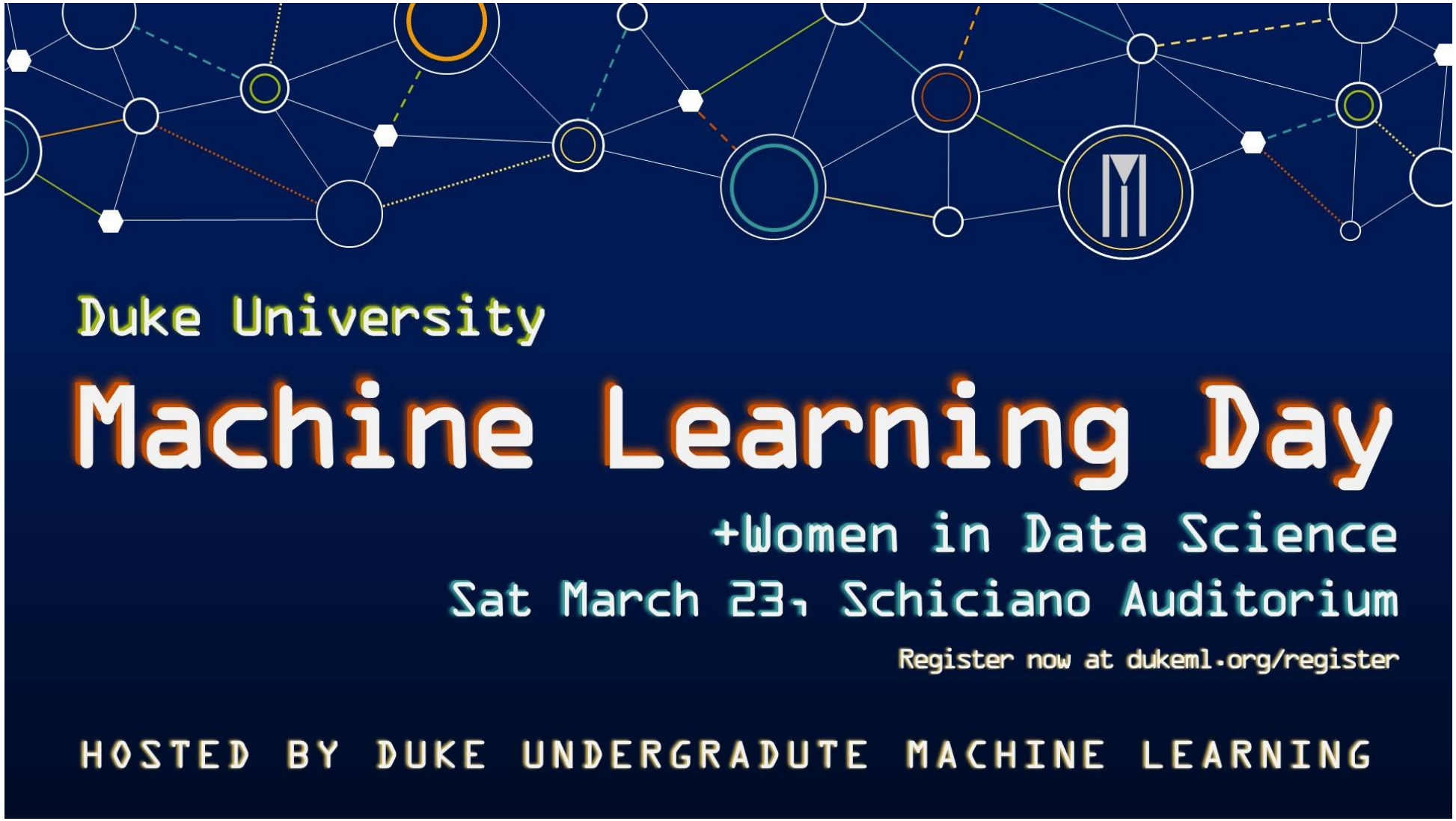
CompSci 116: Lab 6: Resampling & Bootstap

Jeff Forbes

March 7, 2019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Computing Percentiles

The 80th percentile is the value in a set that is at least as large as 80% of the elements in the set

For `s = [1, 7, 3, 9, 5]`, `percentile(80, s)` is 7

The 80th percentile is ordered element 4: `(80/100) * 5`

Percentile

Size of set

For a percentile that does not exactly correspond to an element, take the next greater element instead

The `percentile` Function

- The *p*th percentile is the value in a set that is at least as large as *p*% of the elements in the set
- Function in the `datascience` module:

```
percentile(p, values)
```

- **`p`** is between 0 and 100
- Returns the *p*th percentile of the array

(Readiness Assurance)

How many enemy planes?

Assumptions

- Planes have serial numbers 1, 2, 3, …, N.
- We don't know N.
- We would like to estimate N based on the serial numbers of the planes that we see.

The main assumption

- The serial numbers of the planes that we see are a uniform random sample drawn with replacement from 1, 2, 3, …, N.

Discussion question

If you saw these serial numbers, what would be your estimate of N?

```
170 271 285 290 48
235  24  90 291 19
```

One idea: 291. Just go with the largest one.

The largest number observed

- Is it likely to be close to N?
  - How likely?
  - How close?

Option 1. We could try to calculate the probabilities and draw a probability histogram.

Option 2. We could simulate and draw an empirical histogram.

Verdict on the estimate

- The largest serial number observed is likely to be close to N.
- But it is also likely to underestimate N.

Another idea for an estimate:

Average of the serial numbers observed ~ N/2

New estimate: 2 times the average

(Lab)

Inference: Estimation

- How big is an unknown parameter (e.g., number of planes)?
- If you have a census (that is, the whole population):
  - Just calculate the parameter and you're done
- If you don't have a census:
  - Take a random sample from the population
  - Use a statistic as an estimate of the parameter

Variability of the Estimate

- One sample ➜ One estimate
- But the random sample could have come out differently
- And so the estimate could have been different
- Main question:
  - How different could the estimate have been?
- The variability of the estimate tells us something about how accurate the estimate is:

  estimate = parameter + error

Where to Get Another Sample?

- One sample ➜ One estimate
- To get many values of the estimate, we needed many random samples
- Can’t go back and sample again from the population:
  - No time, no money
- Stuck?

The Bootstrap

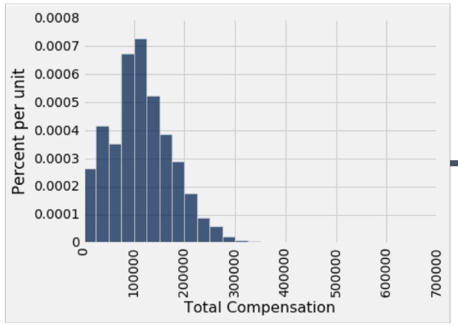
- A technique for simulating repeated random sampling

- All that we have is the original sample
  - … which is large and random
  - Therefore, it probably resembles the population

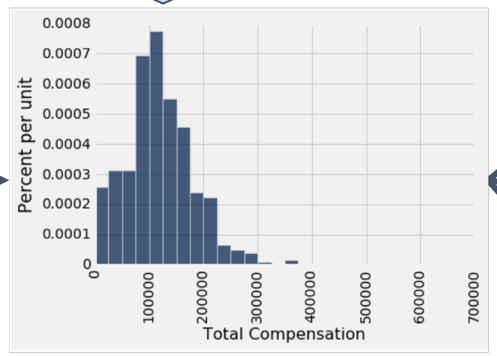
- So we sample at random from the original sample!

Why the Bootstrap Works

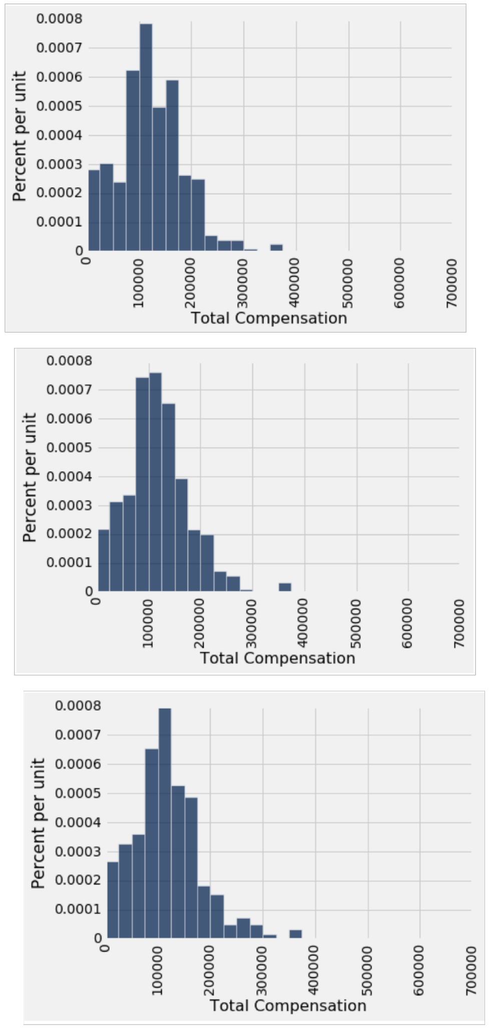
population



sample



resamples



All of these look pretty similar, most likely.

Key to Resampling

- From the original sample,
  - draw at random
  - with replacement
  - as many values as the original sample contained

- The size of the new sample has to be the same as the original one, so that the two estimates are comparable