# Large vocabulary language modeling

Exact sciences Axioms & Deals with theorems Truth is Forever Mathematics Examples C.S. theory F.L. theory

|            | Exact sciences                                   | Empirical<br>sciences              |
|------------|--|------------------------------------|
| Deals with | Axioms & theorems                                | Facts & theories                   |
| Truth is   | Forever  | Temporary                          |
| Examples   | Mathematics<br>C.S. theory<br><b>F.L. theory</b> | Physics Biology <b>Linguistics</b> |

|            | Exact sciences                                   | Empirical<br>sciences                    | Engineering                                  |
|------------|--|--|--|
| Deals with | Axioms & theorems                                | Facts & theories                         | Artifacts                                    |
| Truth is   | Forever  | Temporary                                | It works                                     |
| Examples   | Mathematics<br>C.S. theory<br><b>F.L. theory</b> | Physics<br>Biology<br><b>Linguistics</b> | Many, including applied C.S. e.g. <b>NLP</b> |

Exact sciences Empirical sciences Engineering

morphological properties of words (facts)

Exact sciences

Empirical sciences

Engineering

morphological properties of words (facts)

optimality theory

Exact sciences

Empirical sciences

Engineering

optimality theory is finite-state morphological properties of words (facts)

optimality theory

Exact sciences

Empirical sciences

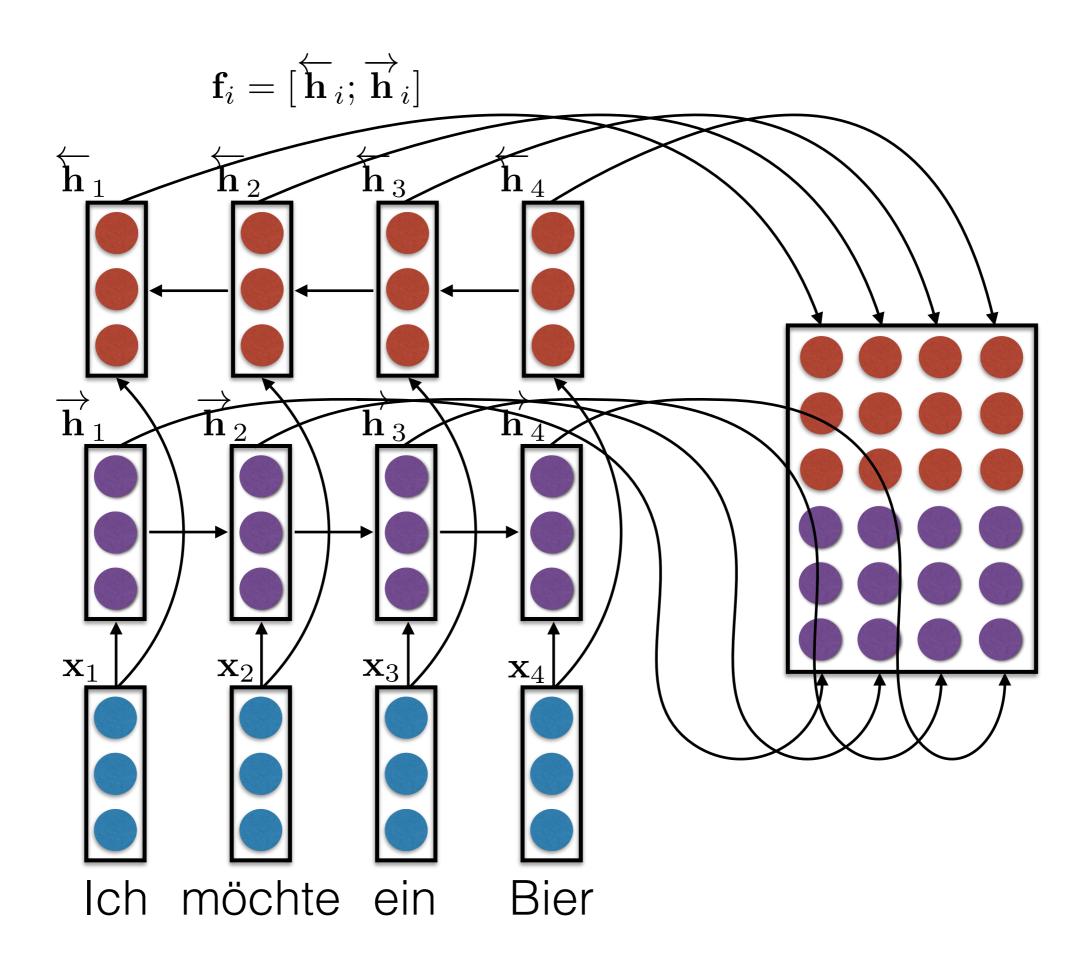
Engineering

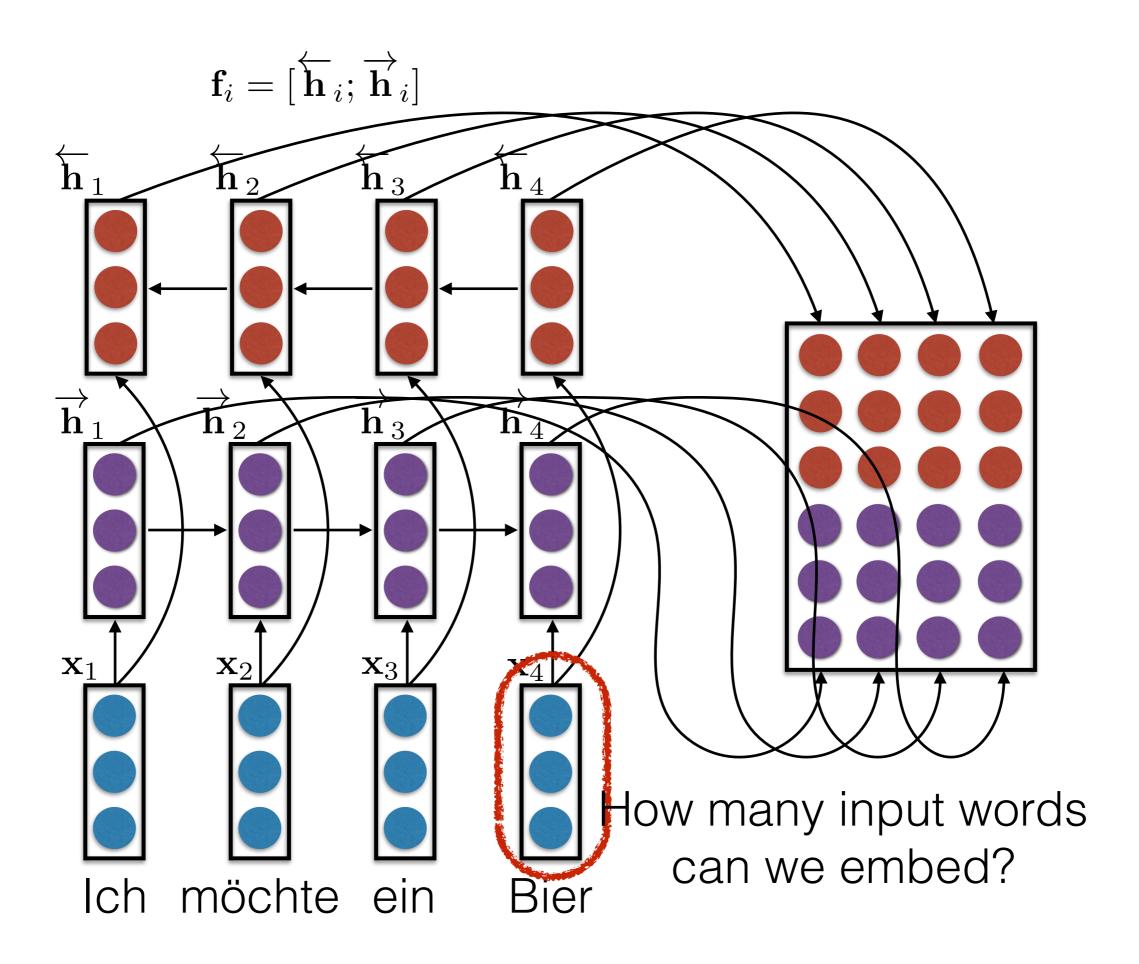
optimality theory is finite-state morphological properties of words (facts)

optimality theory

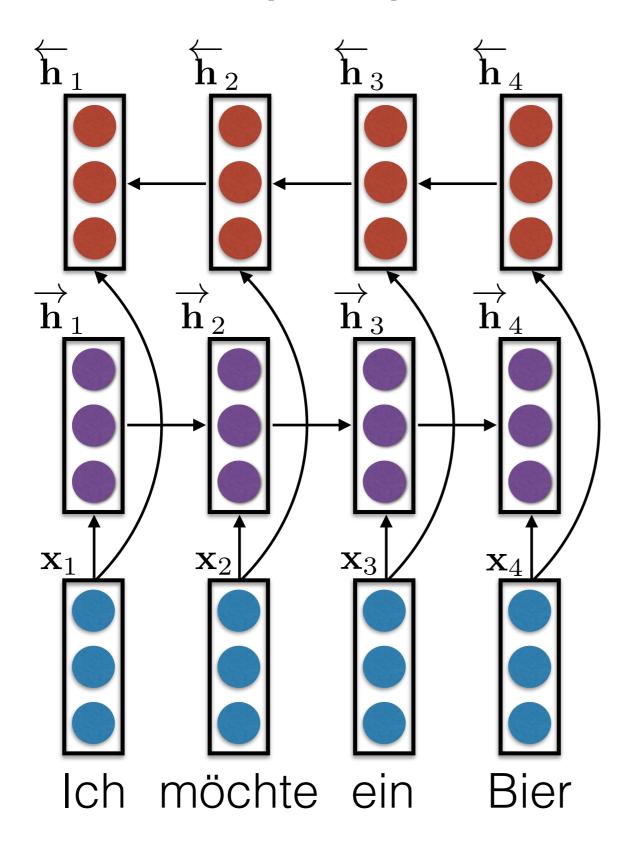
represent
words using
finite-state
machines
(or continuousstate
machines, i.e.

NNs)

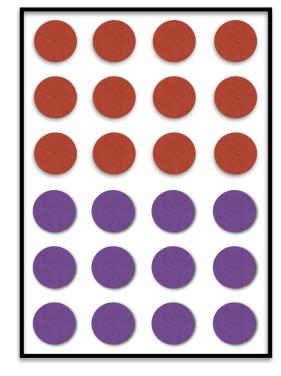




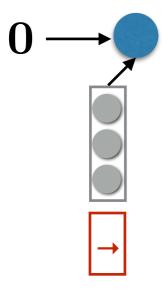
$$\mathbf{f}_i = [\overleftarrow{\mathbf{h}}_i; \overrightarrow{\mathbf{h}}_i]$$

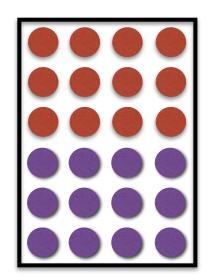


 $\mathbf{F} \in \mathbb{R}^{2n imes |oldsymbol{f}|}$ 

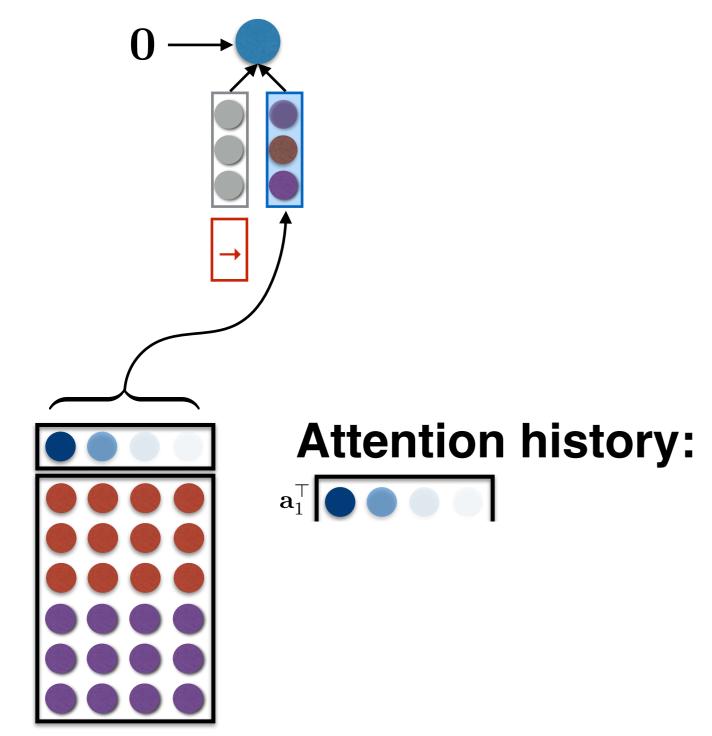


Ich möchte ein Bier

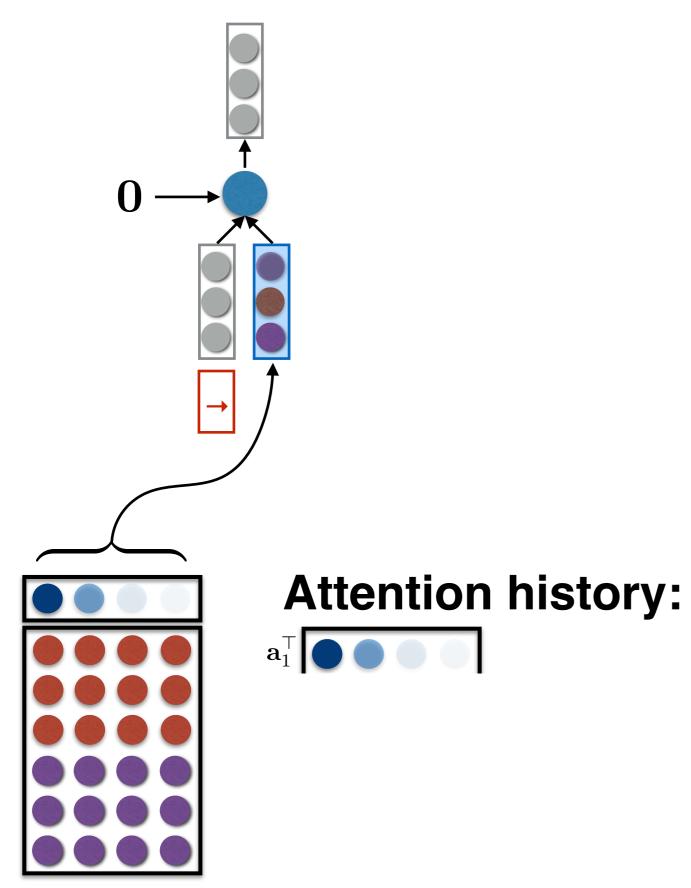




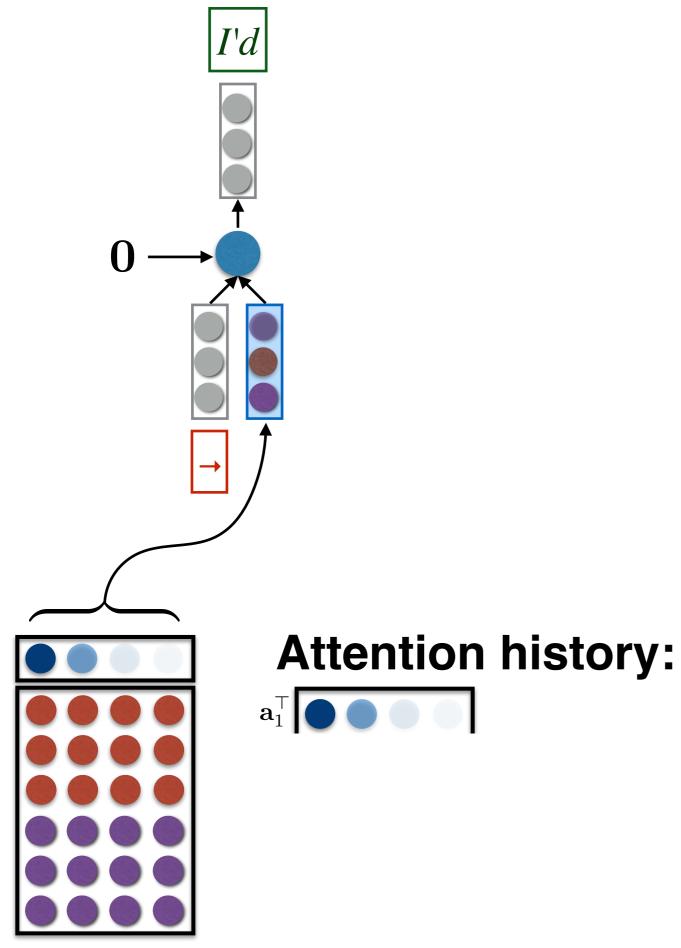
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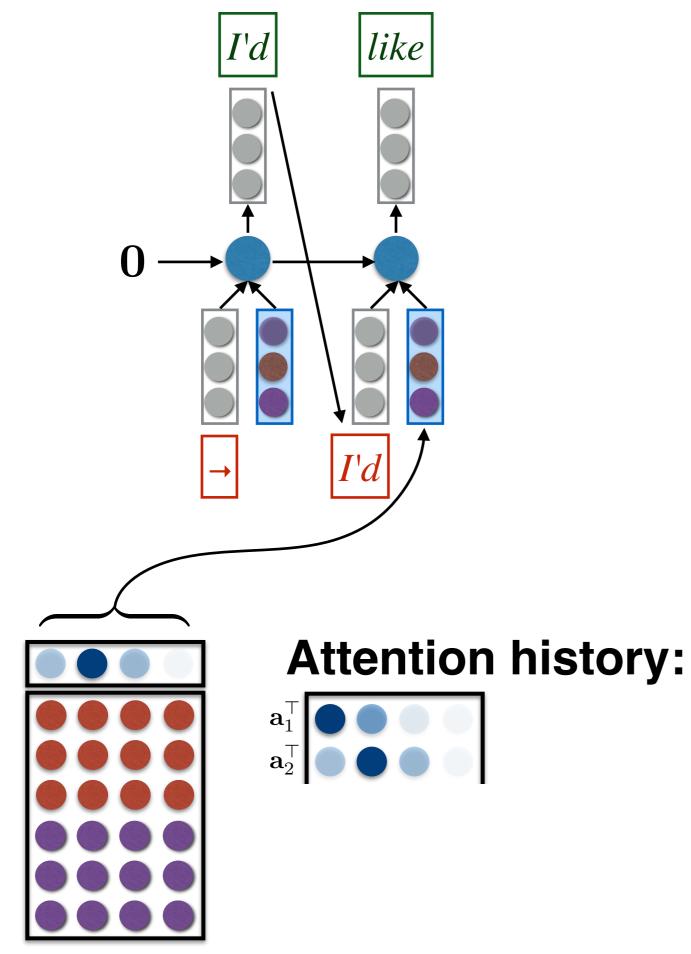
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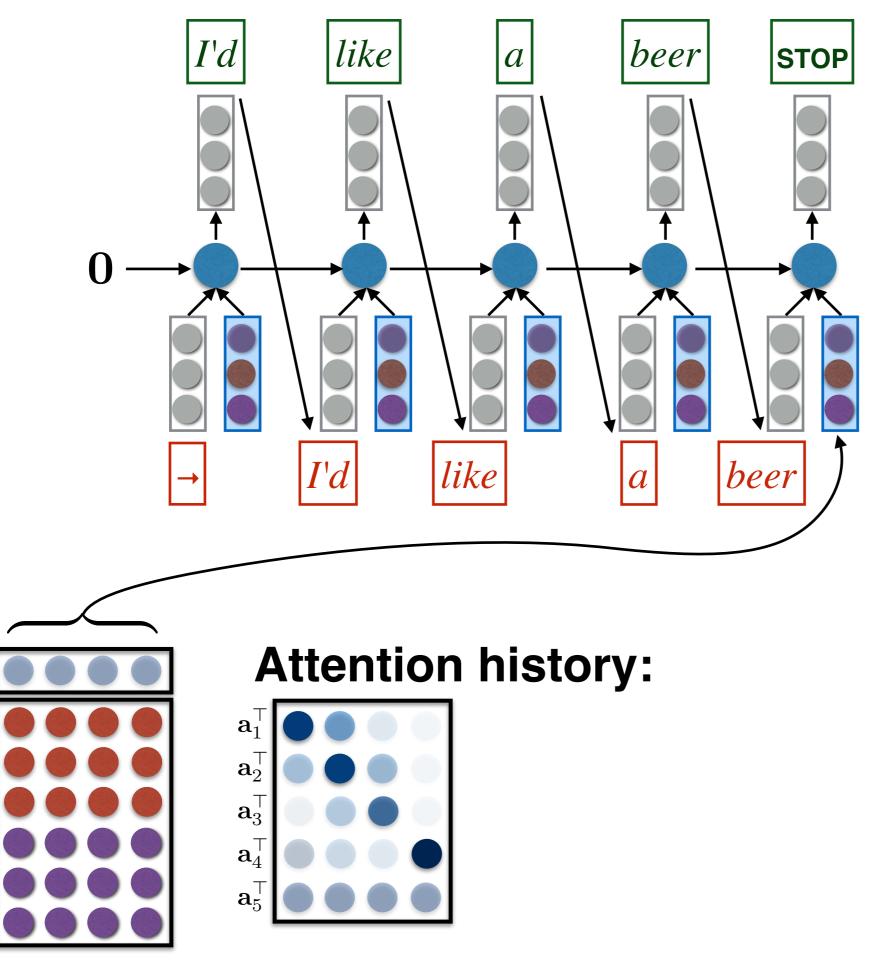
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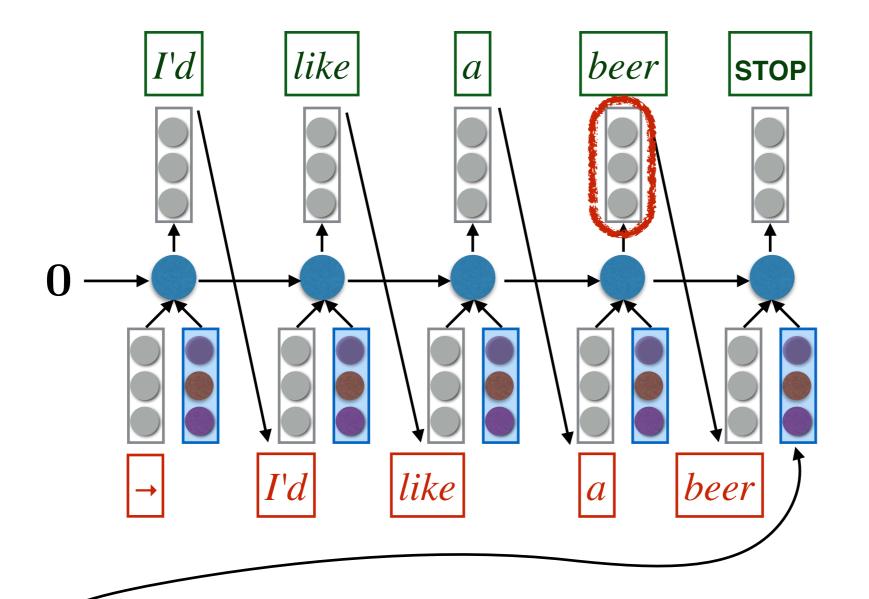
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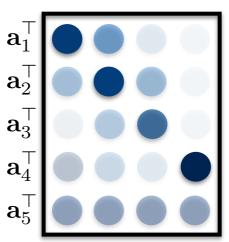
Ich möchte ein Bier



Ich möchte ein Bier







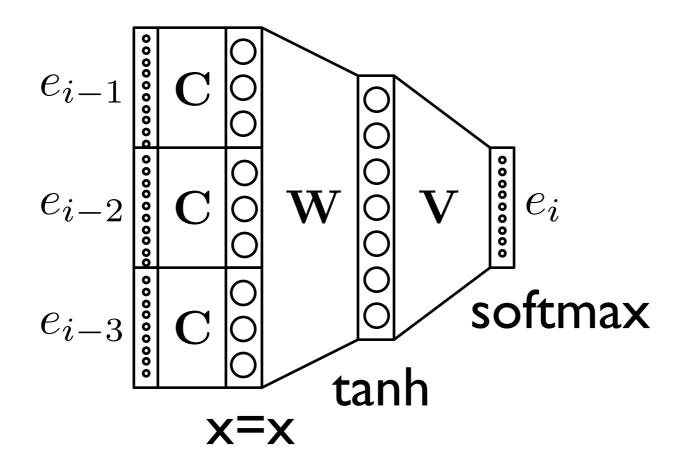
Ich möchte ein Bier

How many output words can we predict?

# Bengio et al. (2003)

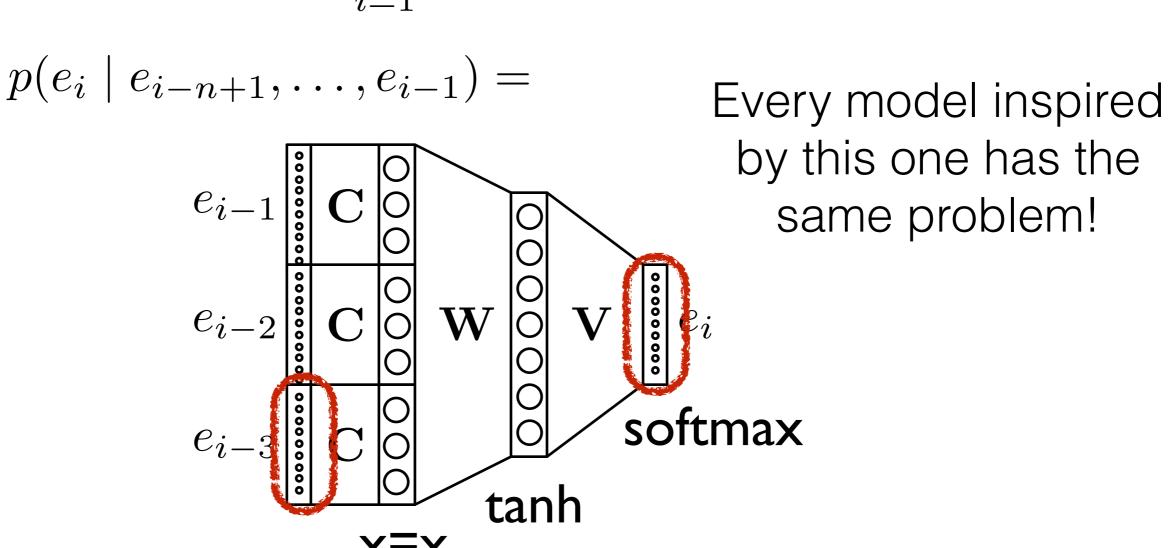
$$p(\mathbf{e}) = \prod_{i=1}^{|\mathbf{e}|} p(e_i \mid e_{i-n+1}, \dots, e_{i-1})$$

$$p(e_i \mid e_{i-n+1}, \dots, e_{i-1}) =$$



# Bengio et al. (2003)

$$p(\mathbf{e}) = \prod_{i=1}^{|\mathbf{e}|} p(e_i \mid e_{i-n+1}, \dots, e_{i-1})$$



## Actually, this problem is even older...

# Language modeling

For language modeling, we seek

$$p(\mathbf{e}) > 0 \quad \forall \mathbf{e} \in \Sigma^*$$

We will assume that  $\Sigma$  is known and finite.

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# Language modeling

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$$p(\mathbf{e}) > 0 \quad \forall \mathbf{e} \in \Sigma^*$$

We will assume that  $\Sigma$  is known and finite.

When does this assumption make sense for language modeling?

#### Known and finite

 Practical problem: softmax computation is linear in vocabulary size.

#### Problems with this?

- Bengio et al.: "Rare words with frequency ≤ 3 were merged into a single symbol, reducing the vocabulary size to |V| = 16,383."
- Bahdanau et al.: "we use a shortlist of 30,000 most frequent words in each language to train our models. Any word not included in the shortlist is mapped to a special token ([UNK])."

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Src | 日本の主要作物は米である。 Ref | the main crop of japan is rice. Hyp | the \_UNK is popular of \_UNK . \_EOS

\_\_\_\_\_

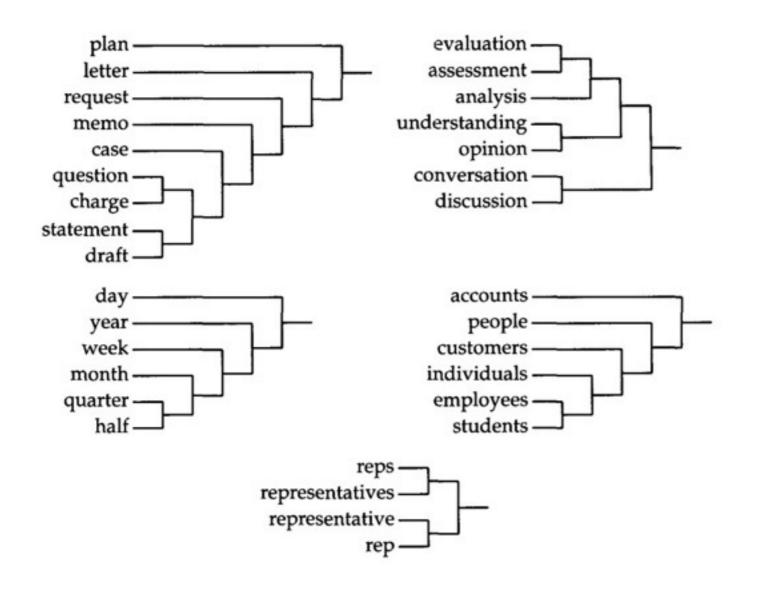
## Approaches

Partition the vocabulary into smaller pieces.

$$p(w_i|h_i) = p(c_i|h_i)p(w_i|c_i,h_i)$$
  
Class-based LM

## Approaches

 Partition the vocabulary into smaller pieces hierarchically (hierarchical softmax).



Brown clustering: hard clustering based on mutual information

## Approaches

 Differentiated softmax: assign more parameters to more frequent words, fewer to less frequent words.

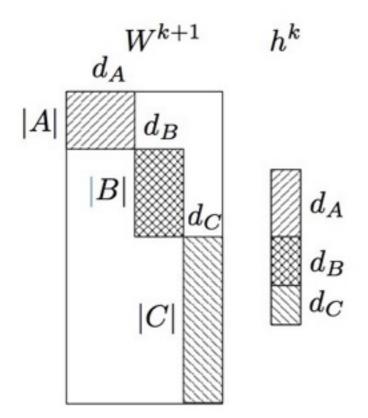
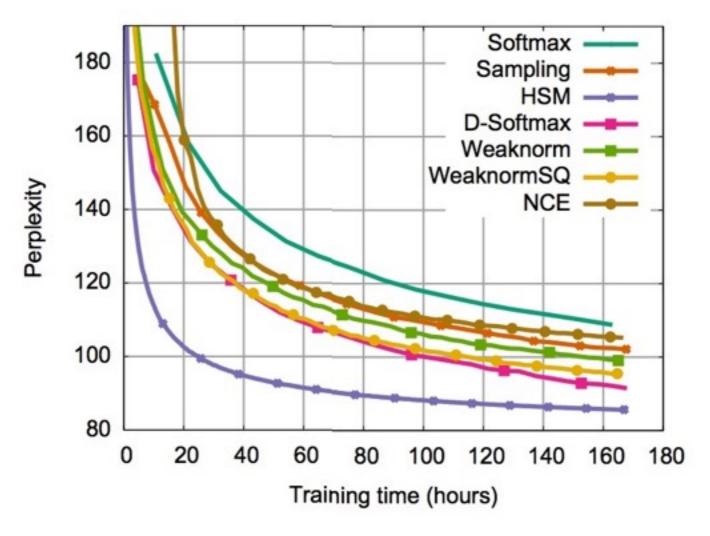
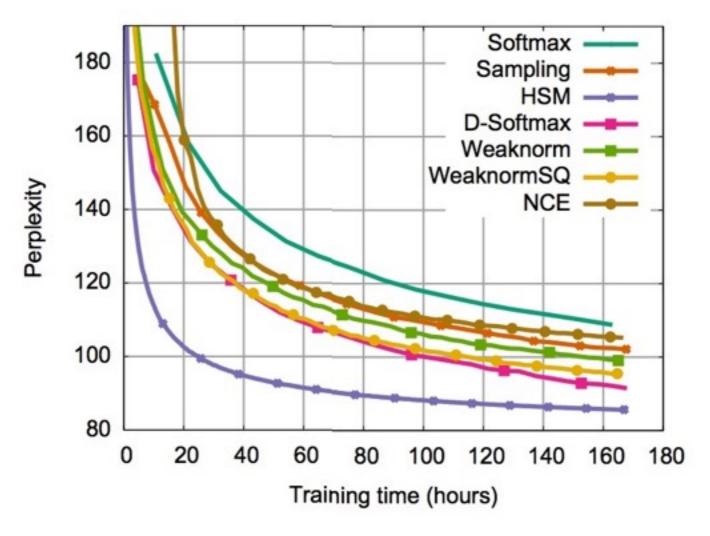


Figure 1: Final weight matrix  $W^{k+1}$  and hidden layer  $h^k$  for differentiated softmax for partitions A, B, C of the output vocabulary with embedding dimensions  $d_A, d_B, d_C$ ; non-shaded areas are zero.



| Dataset  | Train  | Test  | Vocab | OOV  |
|----------|--------|-------|-------|------|
| PTB      | 1M     | 0.08M | 10k   | 5.8% |
| gigaword | 4,631M | 279M  | 100k  | 5.6% |
| billionW | 799M   | 8.1M  | 793k  |      |

Table 1: Dataset statistics. Number of tokens for train and test set, vocabulary size and ratio of out-of-vocabulary words in the test set.



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|---------------|-------|----------|----------|
| KN            | 141.2 | 57.1     | 70.2     |
| Softmax       | 123.8 | 56.5     | 108.3    |
| D-Softmax     | 121.1 | 52.0     | 91.2     |
| Sampling      | 124.2 | 57.6     | 101.0    |
| HSM           | 138.2 | 57.1     | 85.2     |
| NCE           | 143.1 | 78.4     | 104.7    |
| Weaknorm      | 124.4 | 56.9     | 98.7     |
| WeaknormSQ    | 122.1 | 56.1     | 94.9     |
| KN+Softmax    | 108.5 | 43.6     | 59.4     |
| KN+D-Softmax  | 107.0 | 42.0     | 56.3     |
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Table 2: Test perplexity of individual models and interpolation with Kneser-Ney.

Noise contrastive estimation

|               | PIB   | gigaword | billionW |
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Skip normalization step altogether

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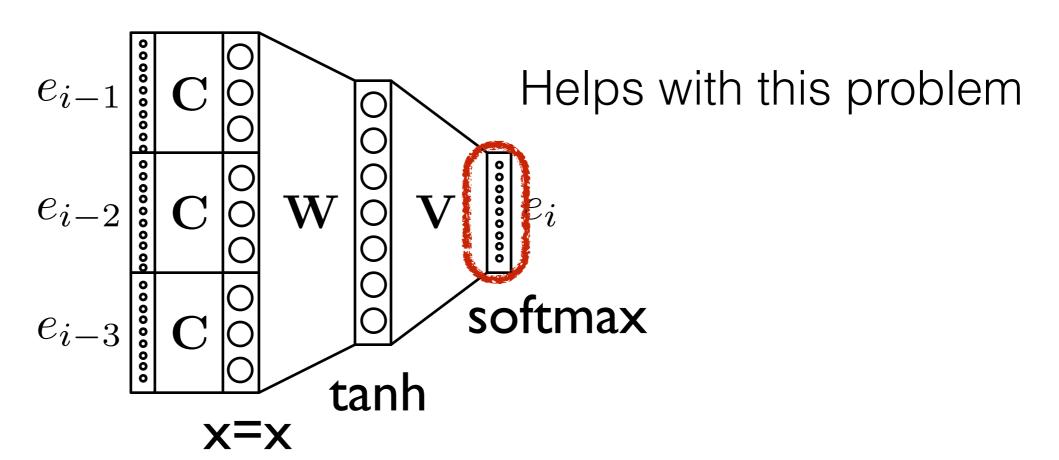
Room for improvement

Table 2: Test perplexity of individual models and interpolation with Kneser-Ney.

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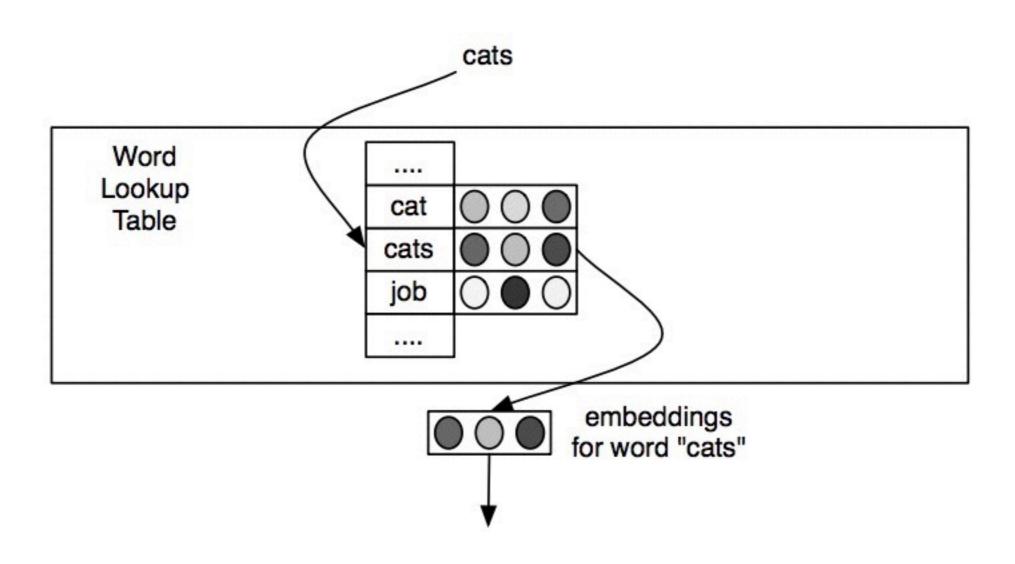
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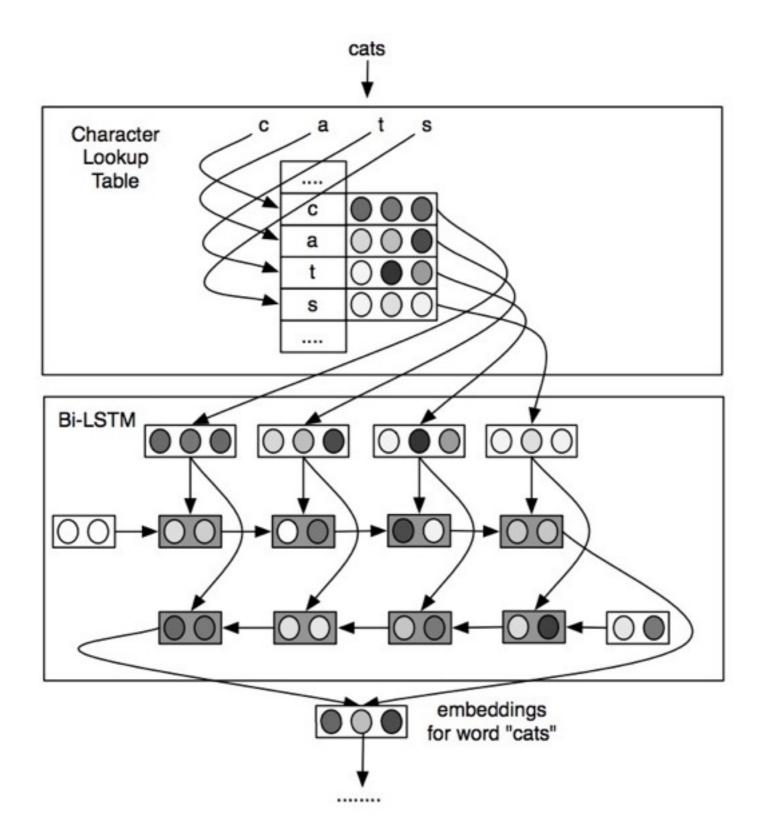
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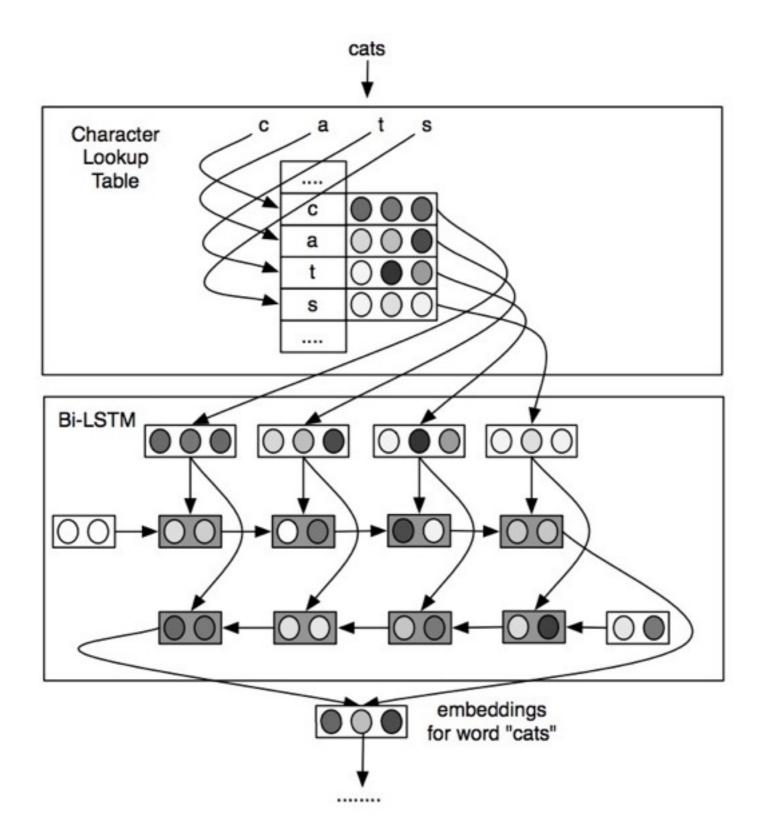


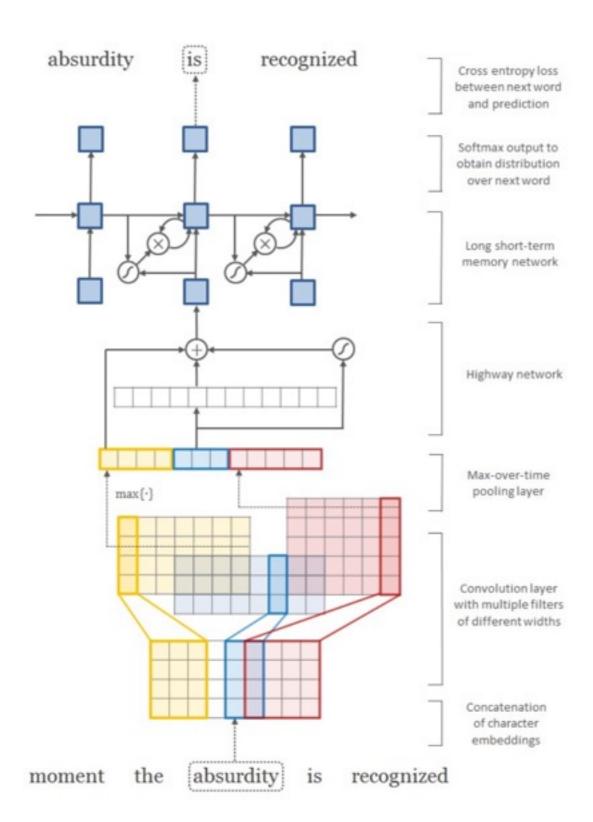
#### Known and finite

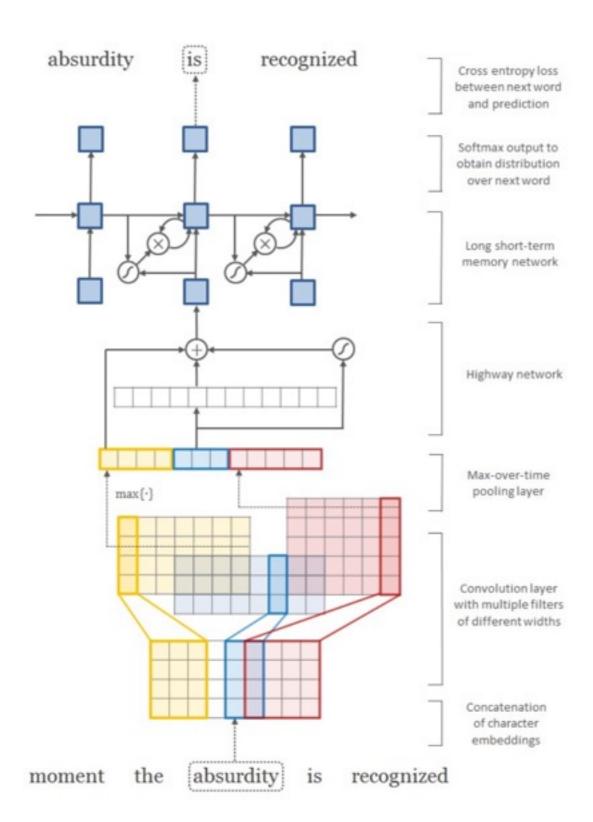
- Practical problem: softmax computation is linear in vocabulary size.
- Theorem. The vocabulary of word types is infinite.
   Proof 1. productive morphology and loanwords.
  - **Proof 2.** 1, 2, 3, 4, ...







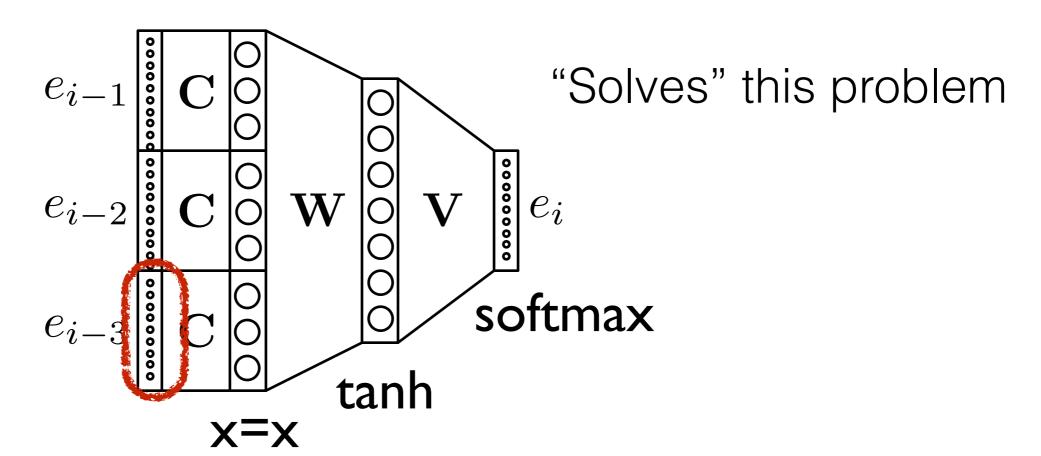




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