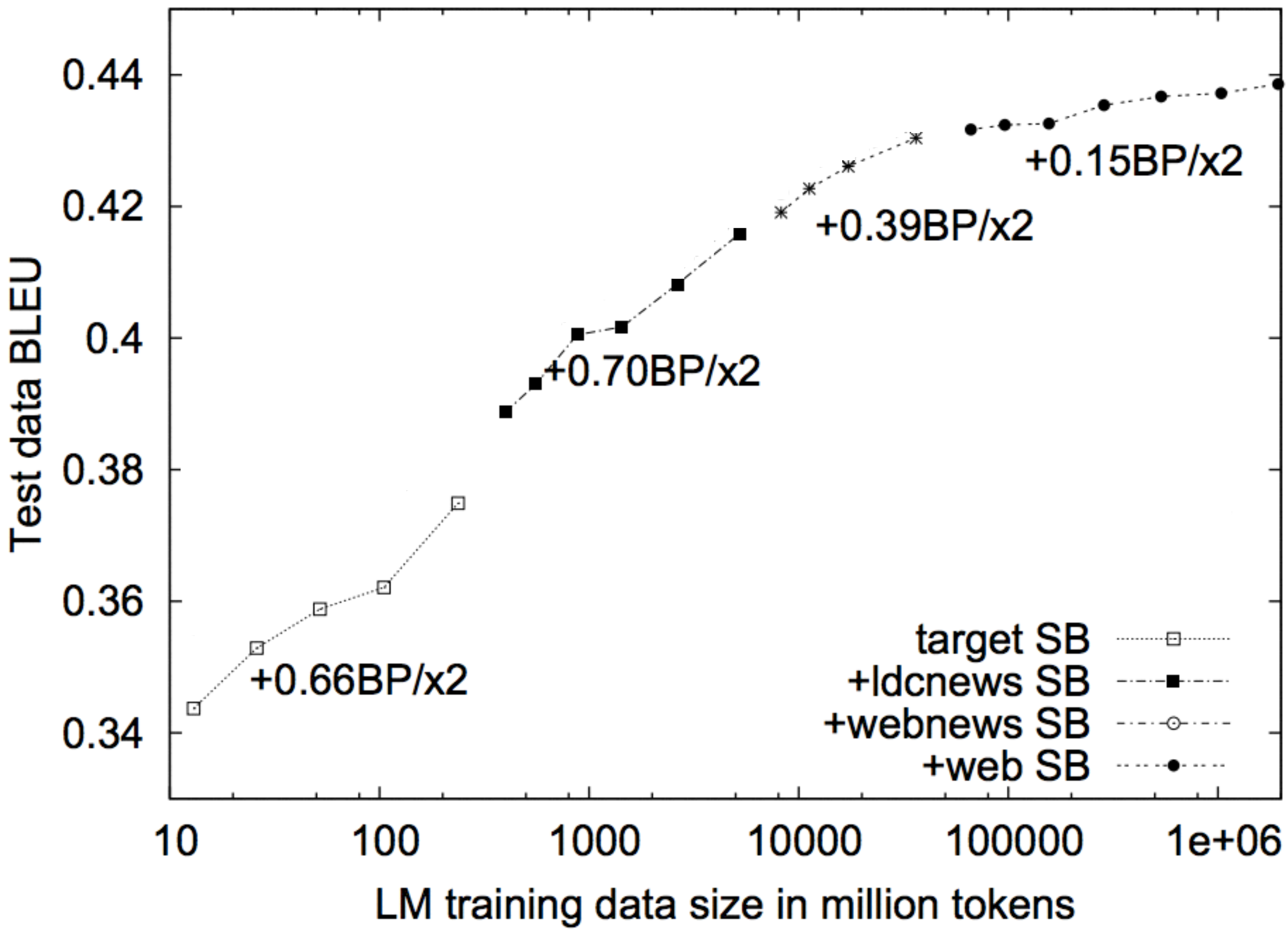
“Neural” language models



Test data BLEU
0.44
0.42
0.4
0.38
0.36
0.34
10
100
1000
10000
100000
1e+06
LM training data size in million tokens
+0.66BP/x2
+0.70BP/x2
+0.39BP/x2
+0.15BP/x2
target SB
+ldcnews SB
+webnews SB
+web SB

LMs in MT

$$p(\mathbf{e}, \mathbf{f}) = p_{LM}(\mathbf{e}) \times p_{TM}(\mathbf{f} \mid \mathbf{e})$$

What is the probability of a sentence?

- Requirements
  - Assign a probability to *every sentence* (i.e., string of words)

$$\sum_{\mathbf{e} \in \Sigma^*} p_{\mathrm{LM}}(\mathbf{e}) = 1$$

$$p_{\mathrm{LM}}(\mathbf{e}) \geq 0 \quad \forall \mathbf{e} \in \Sigma^*$$

n-gram LMs

$$p_{\mathrm{LM}}(\mathbf{e})$$

Vector-valued random variable

Whence parameters?

Whence parameters?

Estimation.

$$
\begin{aligned}
p(x \mid y) &= \frac{p(x, y)}{p(y)} \\
\hat{p}_{\mathrm{MLE}}(x) &= \frac{\mathrm{count}(x)}{N} \\
\hat{p}_{\mathrm{MLE}}(x, y) &= \frac{\mathrm{count}(x, y)}{N} \\
\hat{p}_{\mathrm{MLE}}(x \mid y) &= \frac{\mathrm{count}(x, y)}{N} \times \frac{N}{\mathrm{count}(y)} \\
&= \frac{\mathrm{count}(x, y)}{\mathrm{count}(y)}
\end{aligned}
$$

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$$\boxed{= \frac{\mathrm{count}(x, y)}{\mathrm{count}(y)}}$$

$$
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&= \frac{\mathrm{count}(x, y)}{\mathrm{count}(y)}
\end{aligned}
$$

$$
\hat{p}_{\mathrm{MLE}}(\texttt{call} \mid \texttt{friends}) = \frac{\mathrm{count}(\texttt{friends call})}{\mathrm{count}(\texttt{friends})}
$$

LM Evaluation

- Extrinsic evaluation: build a new language model, use it for some task (MT, ASR, etc.)
- Intrinsic: measure how good we are at modeling language

We will use **perplexity** to evaluate models

Given: $\mathbf{w}, p_{\mathrm{LM}}$

$$\mathrm{PPL} = 2^{\frac{1}{|\mathbf{w}|} \log_2 p_{\mathrm{LM}}(\mathbf{w})}$$

$$0 \leq \mathrm{PPL} \leq \infty$$

Perplexity

- Generally fairly good correlations with BLEU for *n*-gram models
- Perplexity is a generalization of the notion of branching factor
  - How many choices do I have at each position?
- State-of-the-art English LMs have PPL of ~100 word choices per position
- A uniform LM has a perplexity of $|\Sigma|$
- Humans do much better
- ... and bad models can do even *worse* than uniform!

MLE & Perplexity

- What is the **lowest (best) perplexity possible** for your model class?
- Compute the MLE!
- Well, that's easy...

START my friends call me Alex STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{call} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{call}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

START my friends dub me Alex STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{dub} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{dub}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

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MLE

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MLE -3.65172

START my friends dub me Alex STOP

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MLE	-3.65172	-2.07101

START my friends dub me Alex STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{dub} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{dub}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

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MLE -3.65172 -2.07101 -3.32231

START my friends dub me Alex STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{dub} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{dub}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

MLE -3.65172 -2.07101 $-\infty$

START my friends call me Alex STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{call} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{call}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

MLE -3.65172 -2.07101 -3.32231 -0.271271

START my friends dub me Alex STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{dub} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{dub}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

MLE -3.65172 -2.07101 $-\infty$ -2.54562

START	my	friends	call	me	Alex	STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{call} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{call}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

MLE	-3.65172	-2.07101	-3.32231	-0.271271	-4.961

START	my	friends	dub	me	Alex	STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{dub} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{dub}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

MLE	-3.65172	-2.07101	$-\infty$	-2.54562	-4.961

START	my	friends	call	me	Alex	STOP
	$p(\texttt{my} \mid \texttt{START})\times$	$p(\texttt{friends} \mid \texttt{my})\times$	$p(\texttt{call} \mid \texttt{friends})\times$	$p(\texttt{me} \mid \texttt{call})\times$	$p(\texttt{Alex} \mid \texttt{me})\times$	$p(\texttt{STOP} \mid \texttt{Alex})$
MLE	-3.65172	-2.07101	-3.32231	-0.271271	-4.961	-1.96773

START	my	friends	dub	me	Alex	STOP
	$p(\texttt{my} \mid \texttt{START})\times$	$p(\texttt{friends} \mid \texttt{my})\times$	$p(\texttt{dub} \mid \texttt{friends})\times$	$p(\texttt{me} \mid \texttt{dub})\times$	$p(\texttt{Alex} \mid \texttt{me})\times$	$p(\texttt{STOP} \mid \texttt{Alex})$
MLE	-3.65172	-2.07101	$-\infty$	-2.54562	-4.961	-1.96773

START	my	friends	call	me	Alex	STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{call} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{call}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

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MLE assigns probability zero to unseen events

Zeros

- Two kinds of zero probs:
  - **Sampling zeros**: zeros in the MLE due to impoverished observations
  - **Structural zeros**: zeros that should be there. *Do these really exist?*
- Just because you haven't seen something, doesn't mean it doesn't exist.
- In practice, we don't like probability zero, even if there is an argument that it is a structural zero.

Smoothing

Smoothing an refers to a family of *estimation techniques* that seek to model important general patterns in data while avoiding modeling noise or sampling artifacts. In particular, for language modeling, we seek

$$p(\mathbf{e}) > 0 \quad \forall \mathbf{e} \in \Sigma^*$$

We will assume that Σ is known and finite.

Add-α Smoothing

$$\mathbf{p} \sim \mathrm{Dirichlet}(\boldsymbol{\alpha})$$
$$x_i \sim \mathrm{Categorical}(\mathbf{p}) \quad \forall 1 \leq i \leq |\mathbf{x}|$$

Assuming this model, what is the most probable value of **p**, having observed training data **x**?

(bunch of calculus - read about it on Wikipedia)

$$p_x^* = \frac{\mathrm{count}(x) + \alpha_x - 1}{N + \sum_{x'} (\alpha_{x'} - 1)} \quad \forall \alpha_x > 1$$

Add-α Smoothing

- Simplest possible smoother
- Surprisingly effective in many models
- Does not work well for language models
- There are procedures for dealing with 0 < alpha < 1
- When might these be useful?

Interpolation

- "Mixture of MLEs"

$$\begin{aligned}\hat{p}(\texttt{dub} \mid \texttt{my friends}) =& \lambda_3 \hat{p}_{\mathrm{MLE}}(\texttt{dub} \mid \texttt{my friends}) \\ &+ \lambda_2 \hat{p}_{\mathrm{MLE}}(\texttt{dub} \mid \texttt{friends}) \\ &+ \lambda_1 \hat{p}_{\mathrm{MLE}}(\texttt{dub}) \\ &+ \lambda_0 \frac{1}{|\Sigma|}\end{aligned}$$

Where do the lambdas come from?

Discounting

Discounting adjusts the frequencies of observed events downward to reserve probability for the things that have not been observed.

Note $f(w_3 \mid w_1, w_2) > 0$ only when $\mathrm{count}(w_1, w_2, w_3) > 0$

We introduce a **discounted frequency**:

$$0 \leq f^*(w_3 \mid w_1, w_2) \leq f(w_3 \mid w_1, w_2)$$

The total discount is the zero-frequency probability:

$$\lambda(w_1, w_2) = 1 - \sum_{w'} f^*(w' \mid w_1, w_2)$$

Back-off

Recursive formulation of probability:

$$\hat{p}_{\mathrm{BO}}(w_3 \mid w_1, w_2) = \begin{cases} f^*(w_3 \mid w_1, w_2) & \text{if } f^*(w_3 \mid w_1, w_2) > 0 \\ \alpha_{w_1,w_2} \times \lambda(w_1, w_2) \times \hat{p}_{\mathrm{BO}}(w_3 \mid \cancel{w_1}, w_2) & \text{otherwise} \end{cases}$$

"Back-off weight"

Question 1: how do we discount?

Kneser-Ney Discounting

- State-of-the-art in language modeling for 15 years
- Two major intuitions
  - Some contexts have lots of new words
  - Some words appear in lots of contexts
- Procedure
  - Only register a lower-order count the *first time* it is seen in a backoff context
  - Example: bigram model
    - “San Francisco” is a common bigram
    - But, we only count the unigram “Francisco” the first time we see the bigram “San Francisco” - **we change its unigram probability**

Back-off

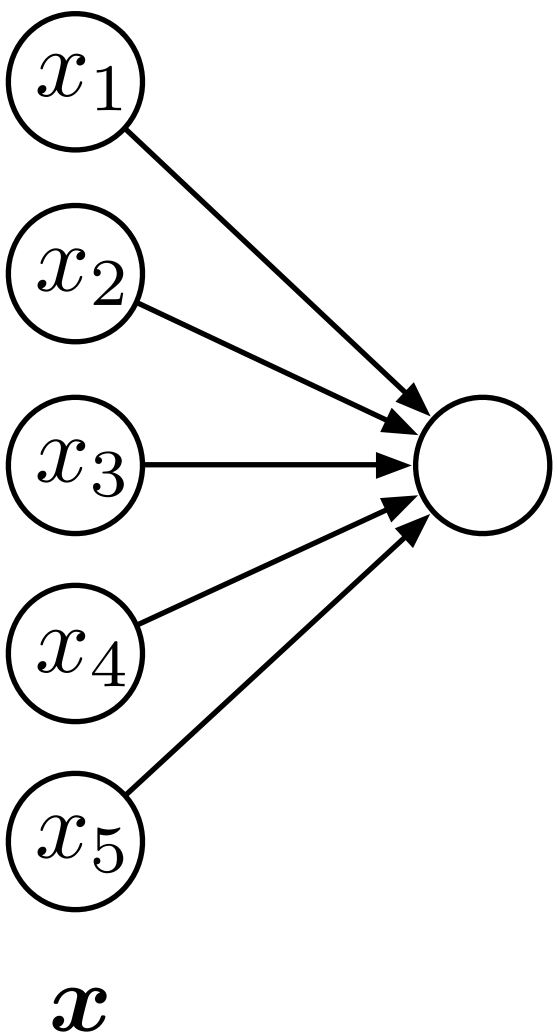
Recursive formulation of probability:

$$\hat{p}_{\mathrm{BO}}(w_3 \mid w_1, w_2) = \begin{cases} f^*(w_3 \mid w_1, w_2) & \text{if } f^*(w_3 \mid w_1, w_2) > 0 \\ \underbrace{\alpha_{w_1,w_2} \times \lambda(w_1, w_2)}_{\text{“Back-off weight”}} \times \hat{p}_{\mathrm{BO}}(w_3 \mid \cancel{w_1}, w_2) & \text{otherwise} \end{cases}$$

Question 1: how do we discount?

Question 2: how many parameters?

“Neurons”



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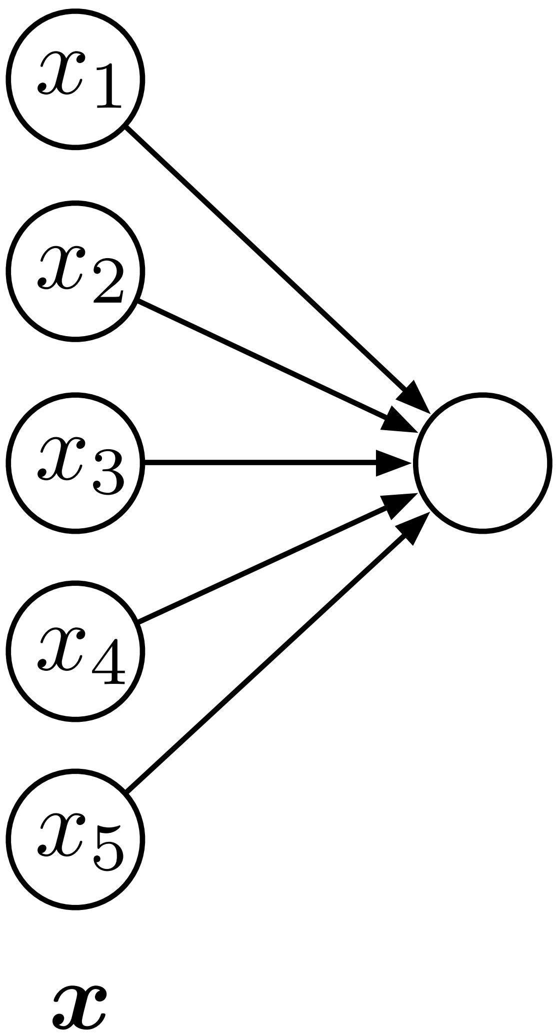

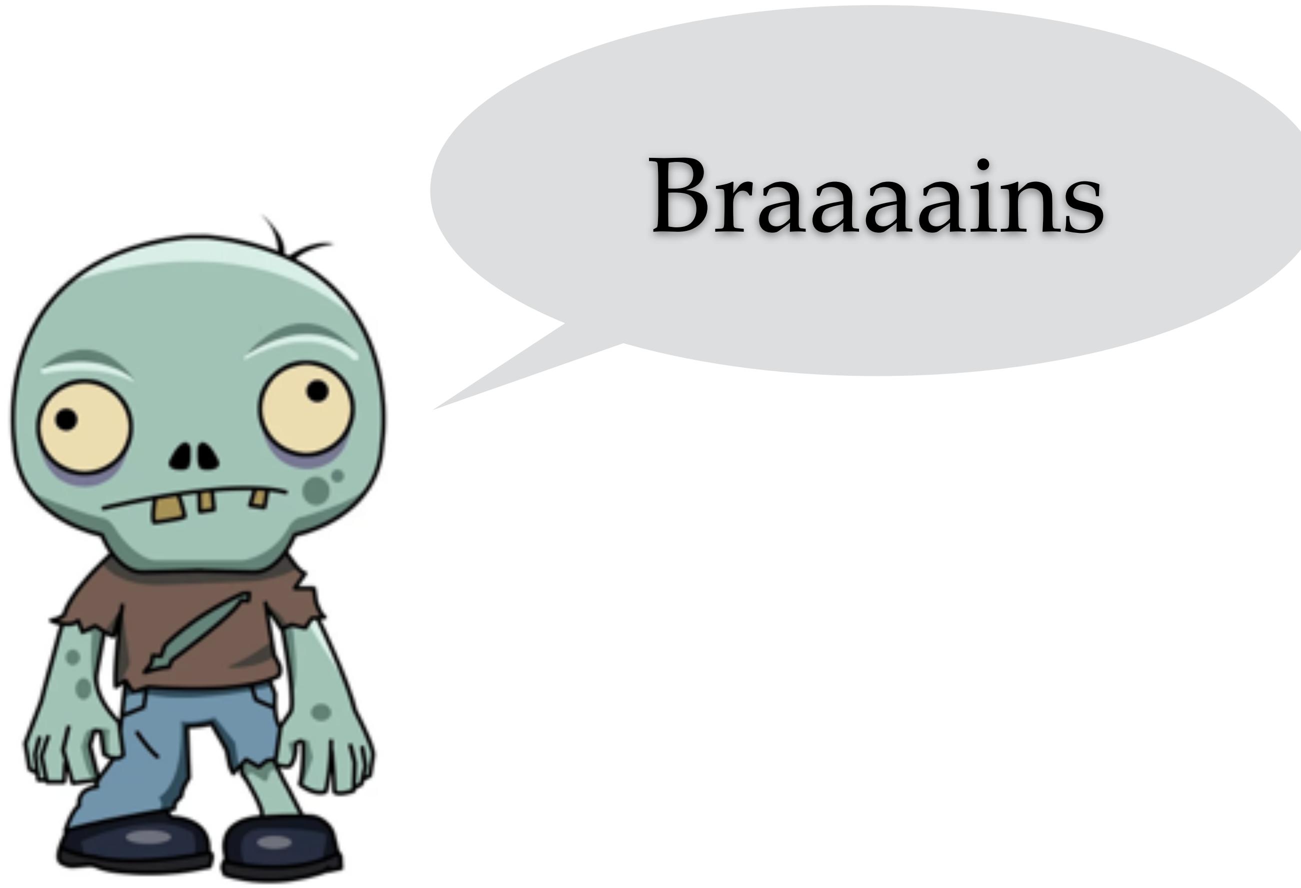

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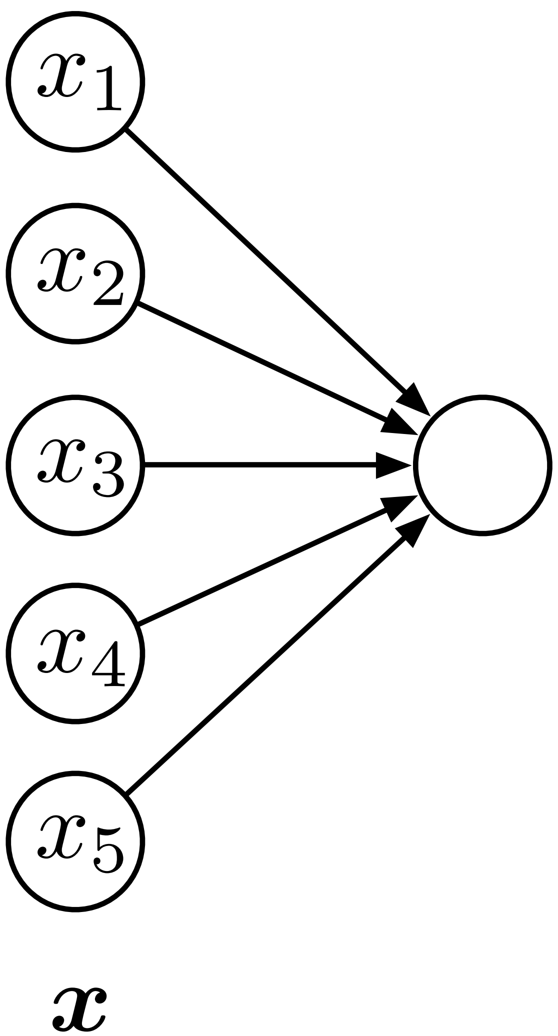
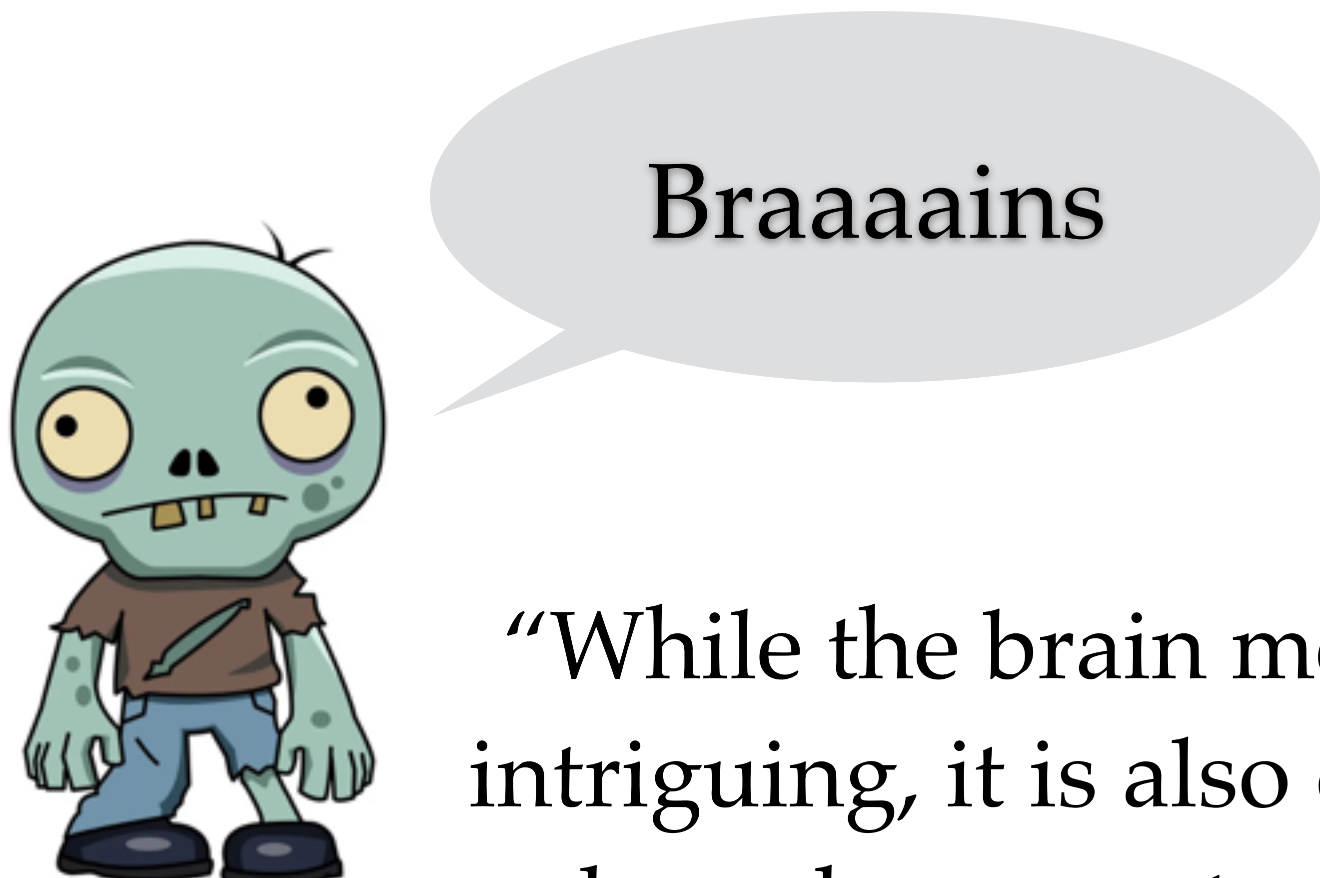
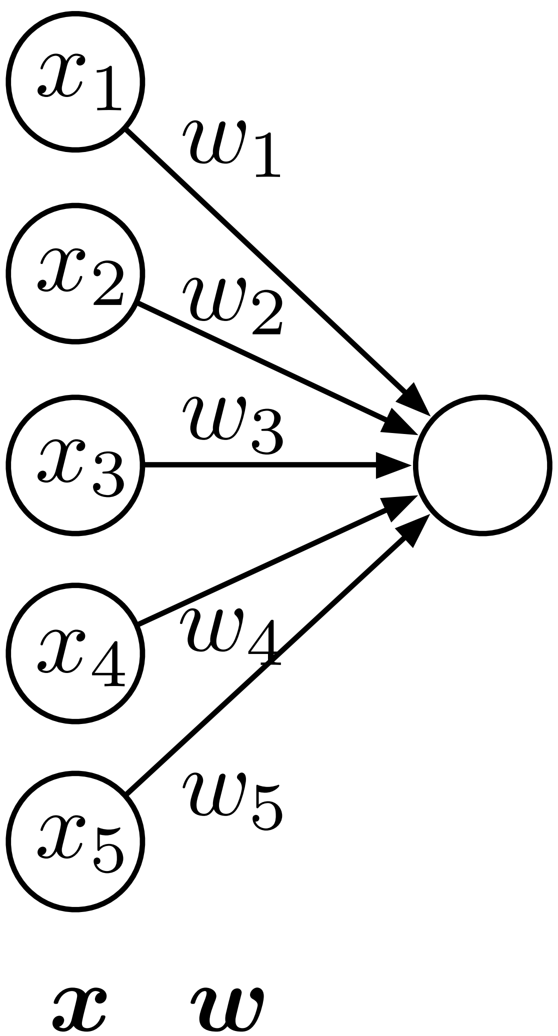




“While the brain metaphor is intriguing, it is also distracting and cumbersome to manipulate mathematically. We therefore switch to using more concise mathematical notation.” (Goldberg, 2015)

“Neurons”



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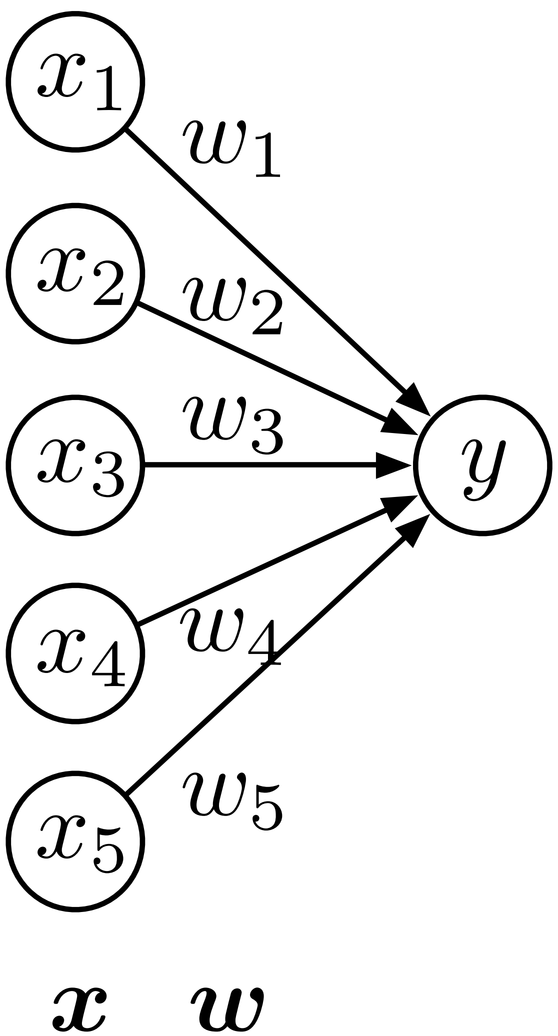
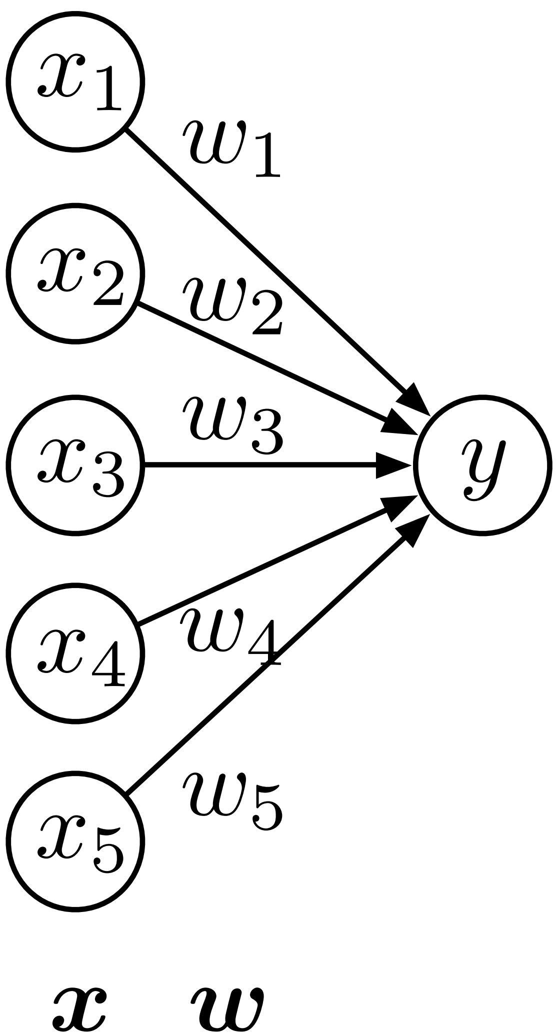


$$y = \boldsymbol{x} \cdot \boldsymbol{w}$$

"Neurons"



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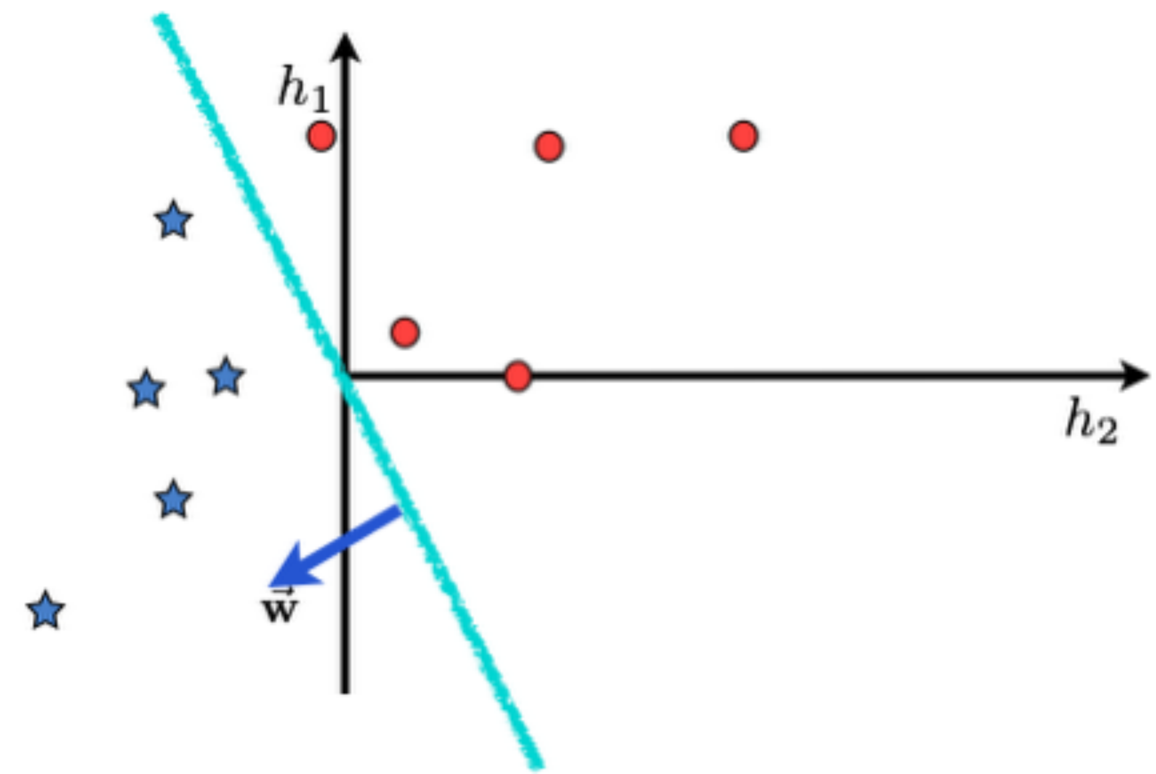

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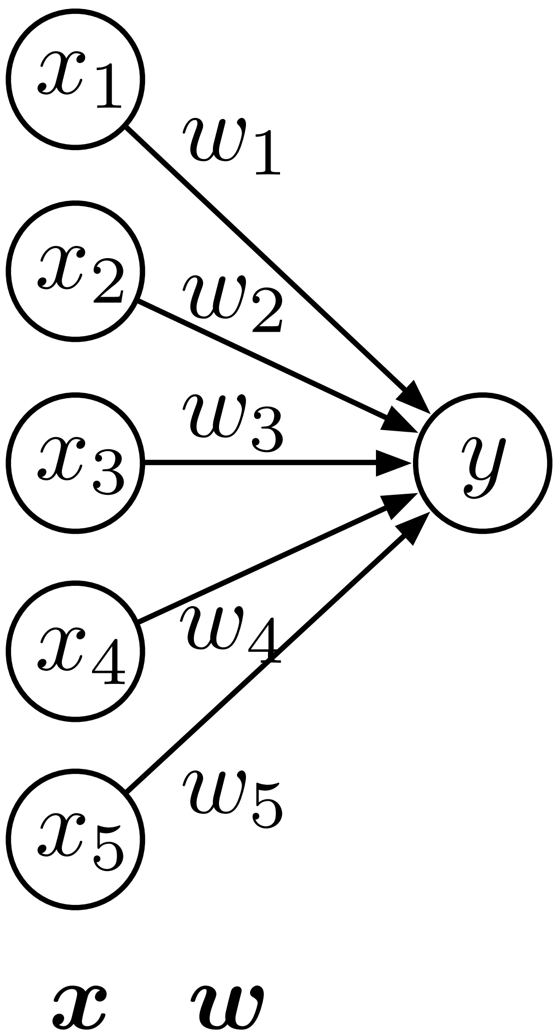
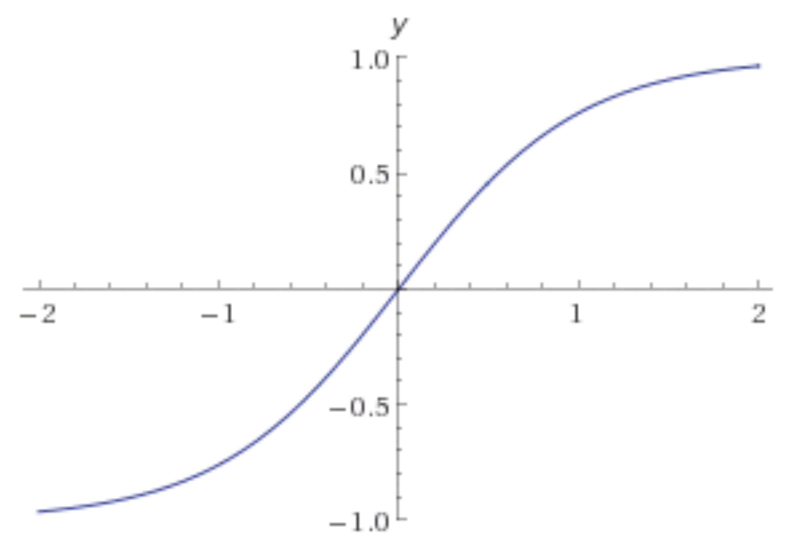
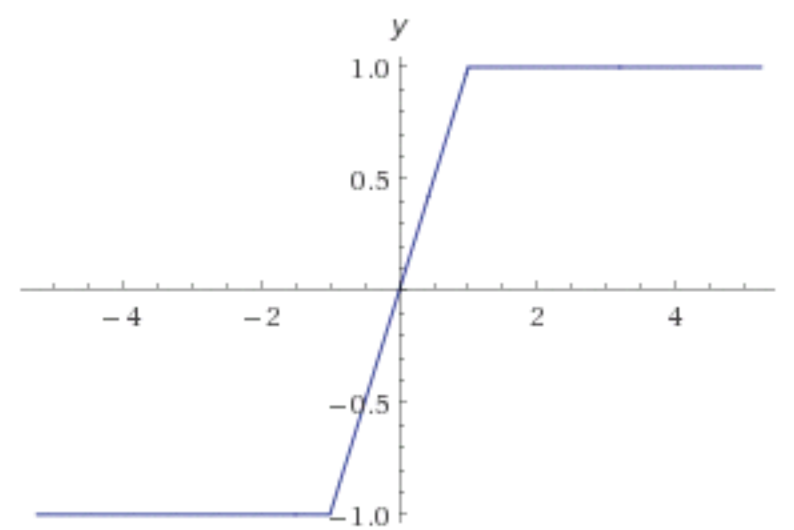


$$\cancel{y = \boldsymbol{x} \cdot \boldsymbol{w}}$$

$$y = g(\boldsymbol{x} \cdot \boldsymbol{w})$$





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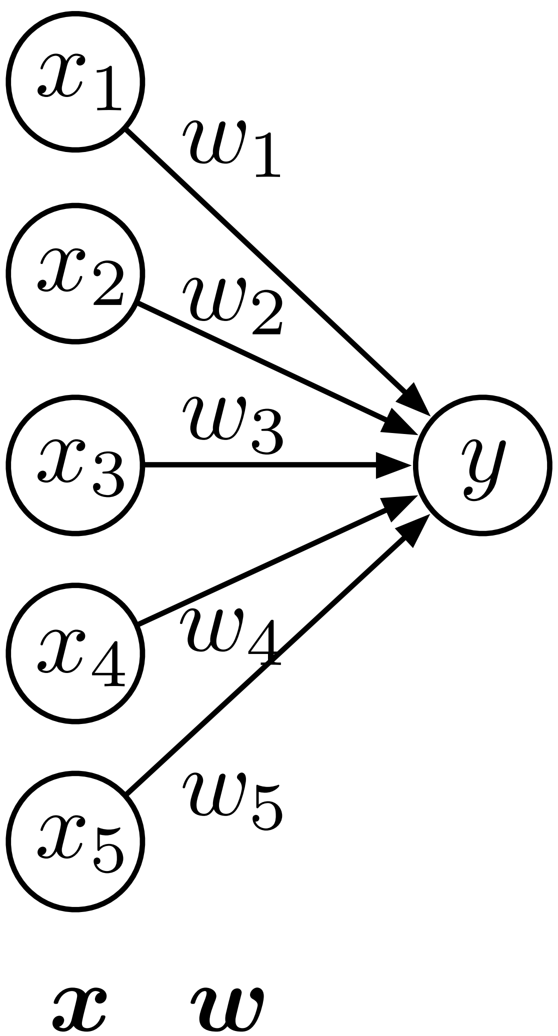
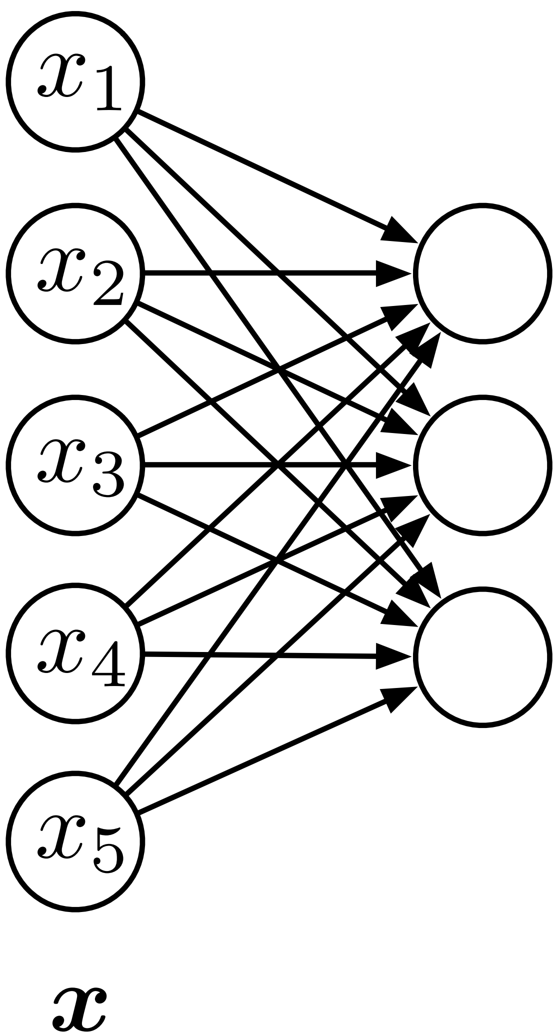


“Neural” Networks



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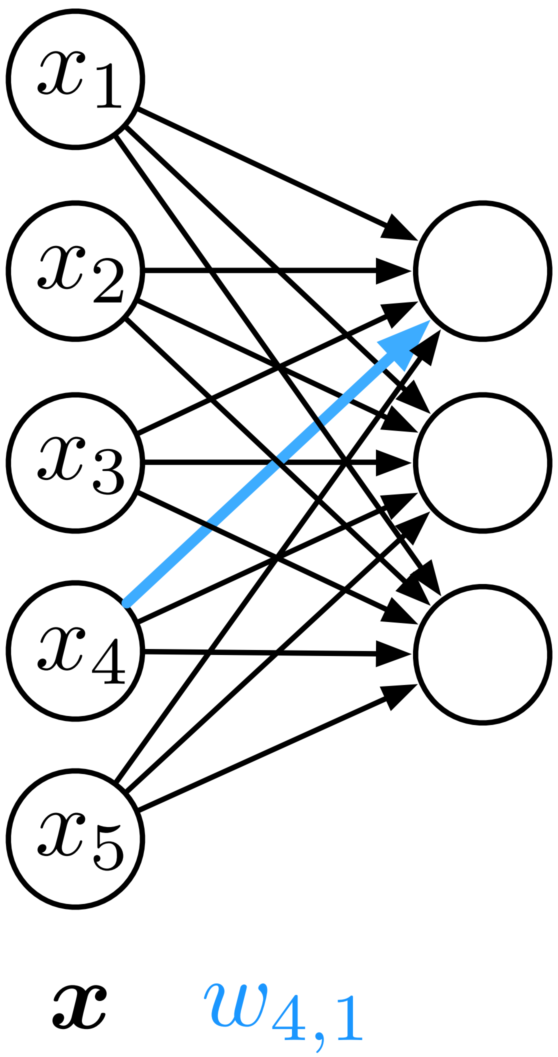

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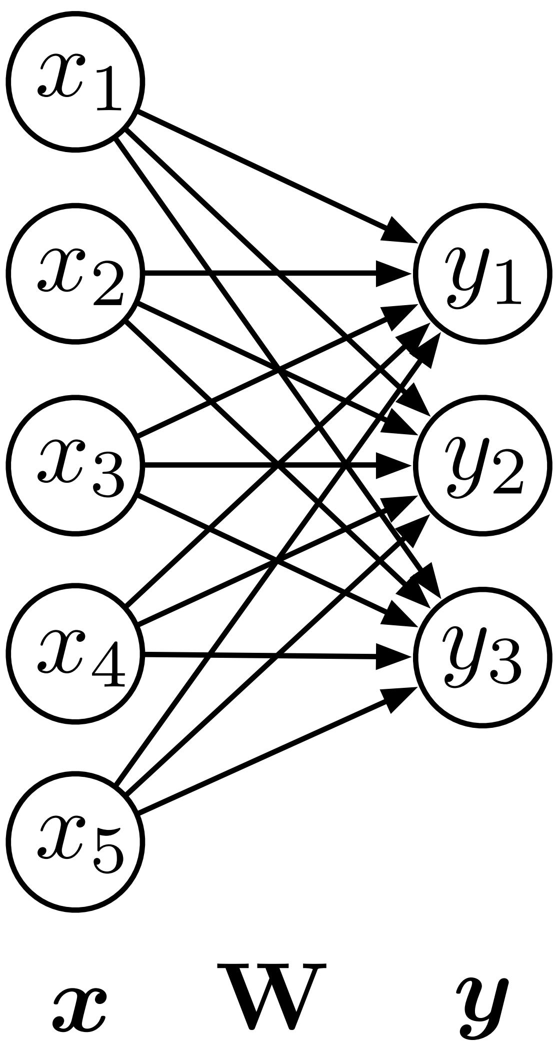
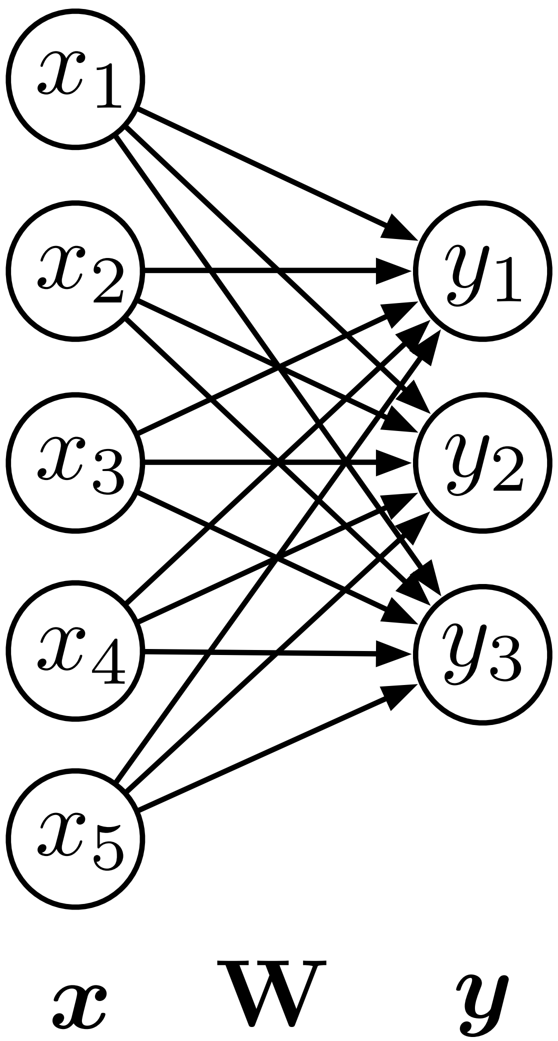


$$\boldsymbol{y} = \boldsymbol{x}^\top \mathbf{W}$$

“Neural” Networks



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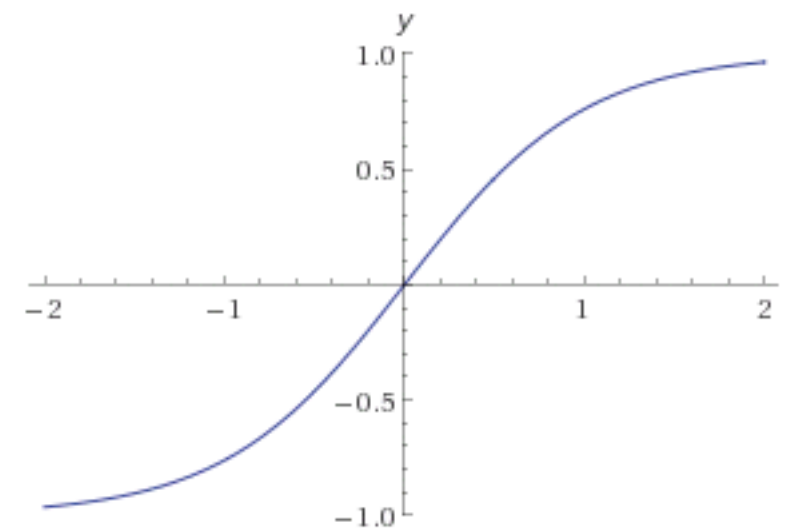

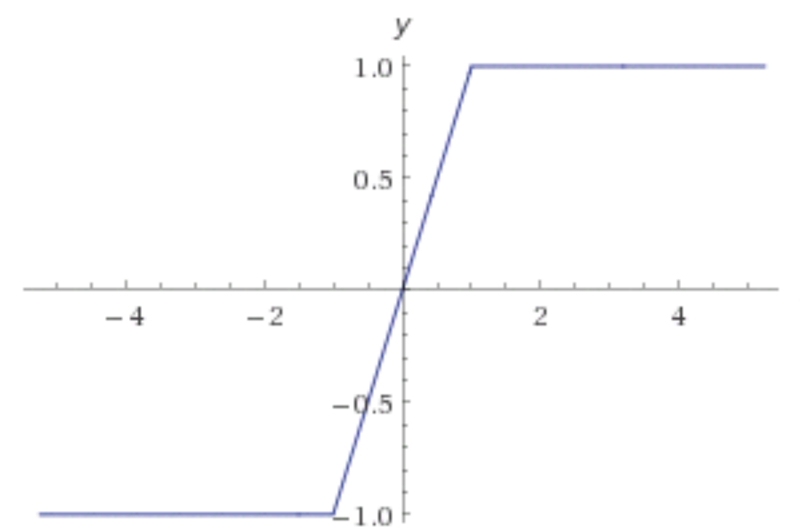

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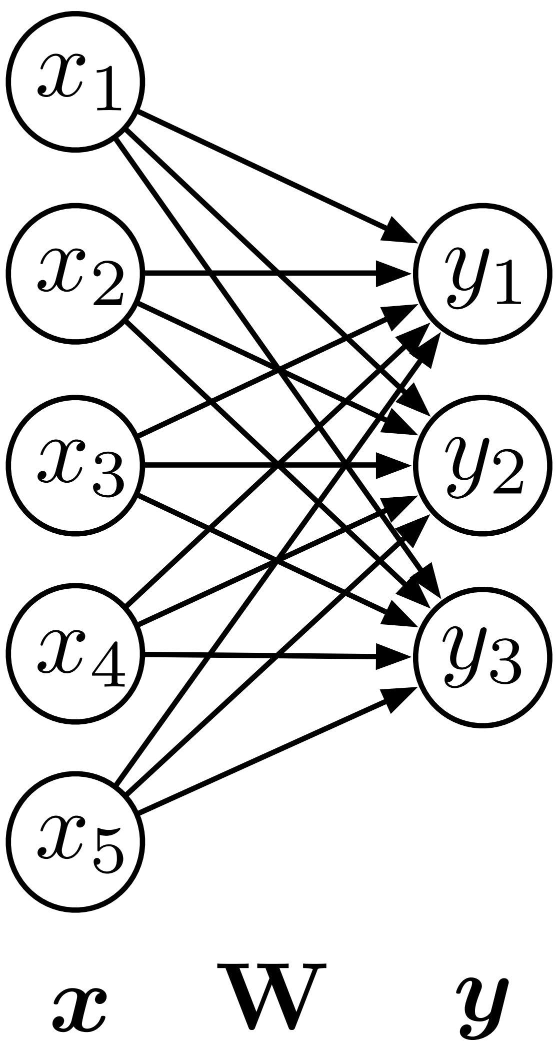

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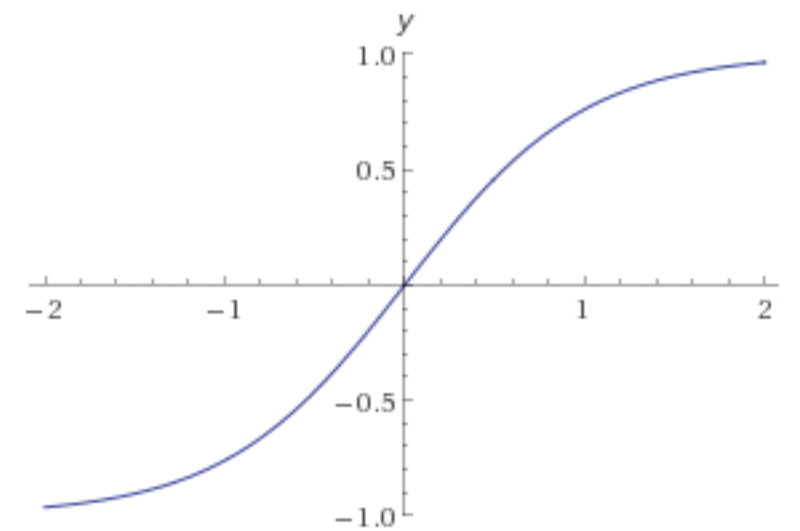
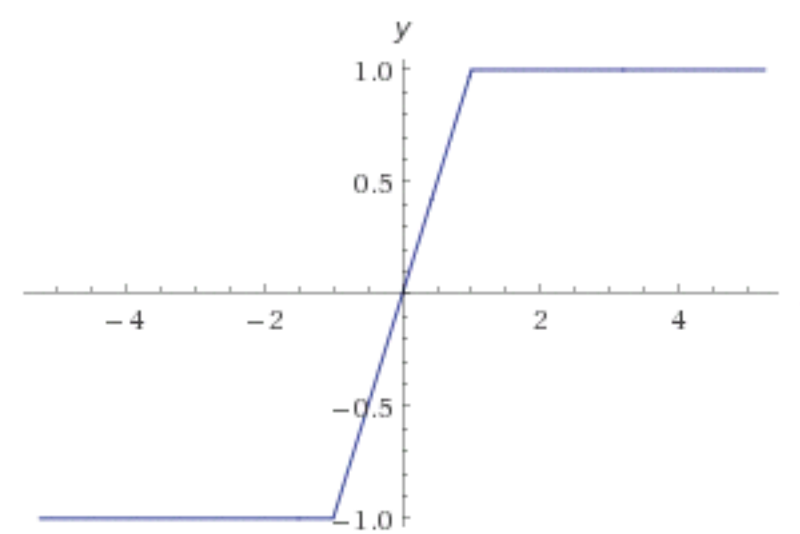
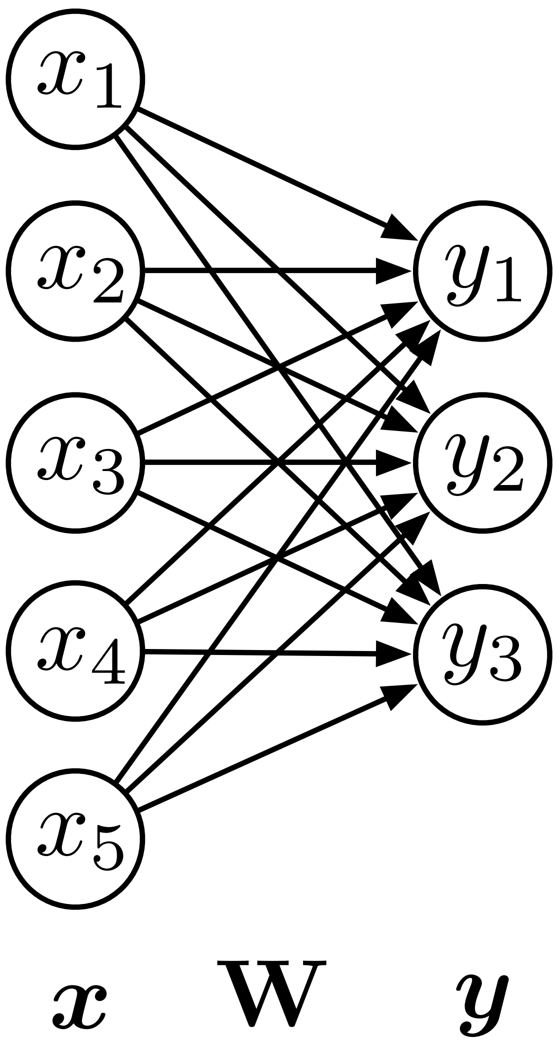




What about probabilities?

Let each y_i be an outcome.

“Neural” Networks



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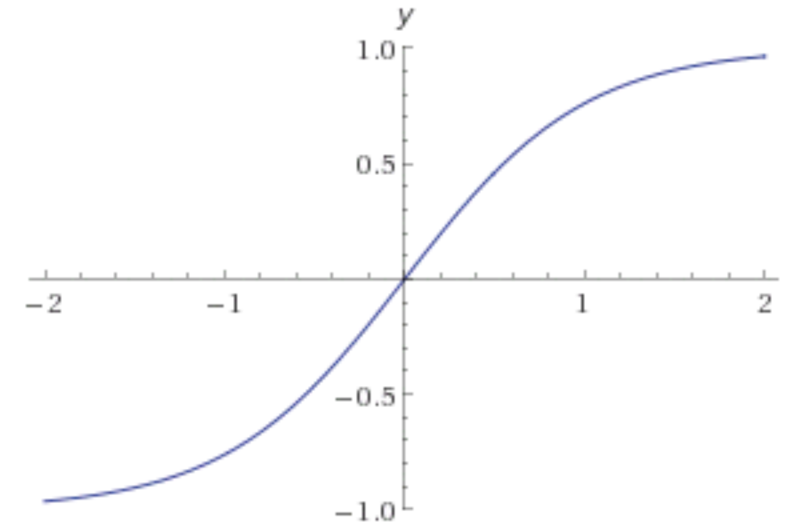
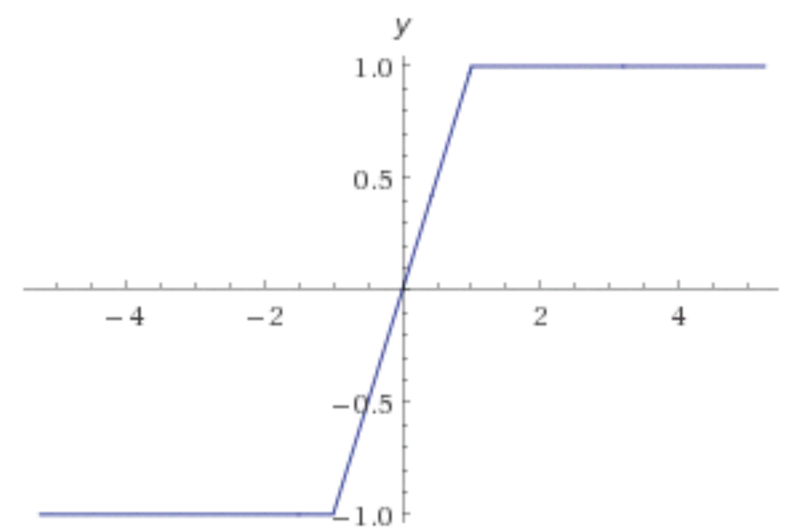


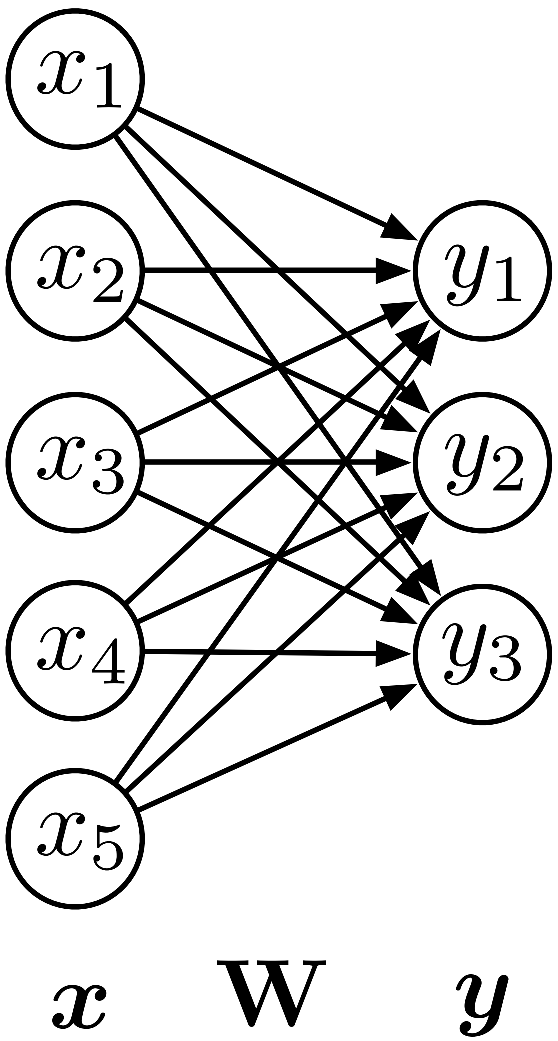


What about probabilities?
Let each y_i be an outcome.

“Soft max”

$$g(\boldsymbol{u})_i = \frac{\exp u_i}{\sum_{i'} \exp u_{i'}}$$

“Neural” Networks



$$\cancel{\boldsymbol{y} = \boldsymbol{x}^\top \mathbf{W}}$$

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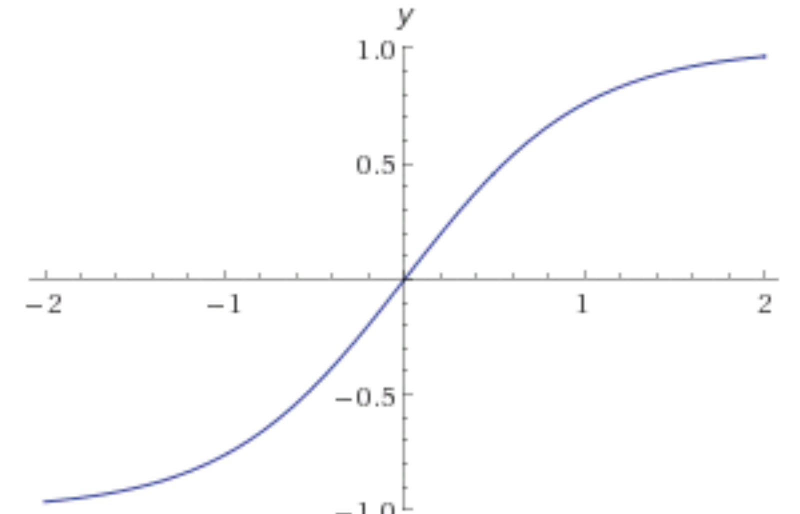


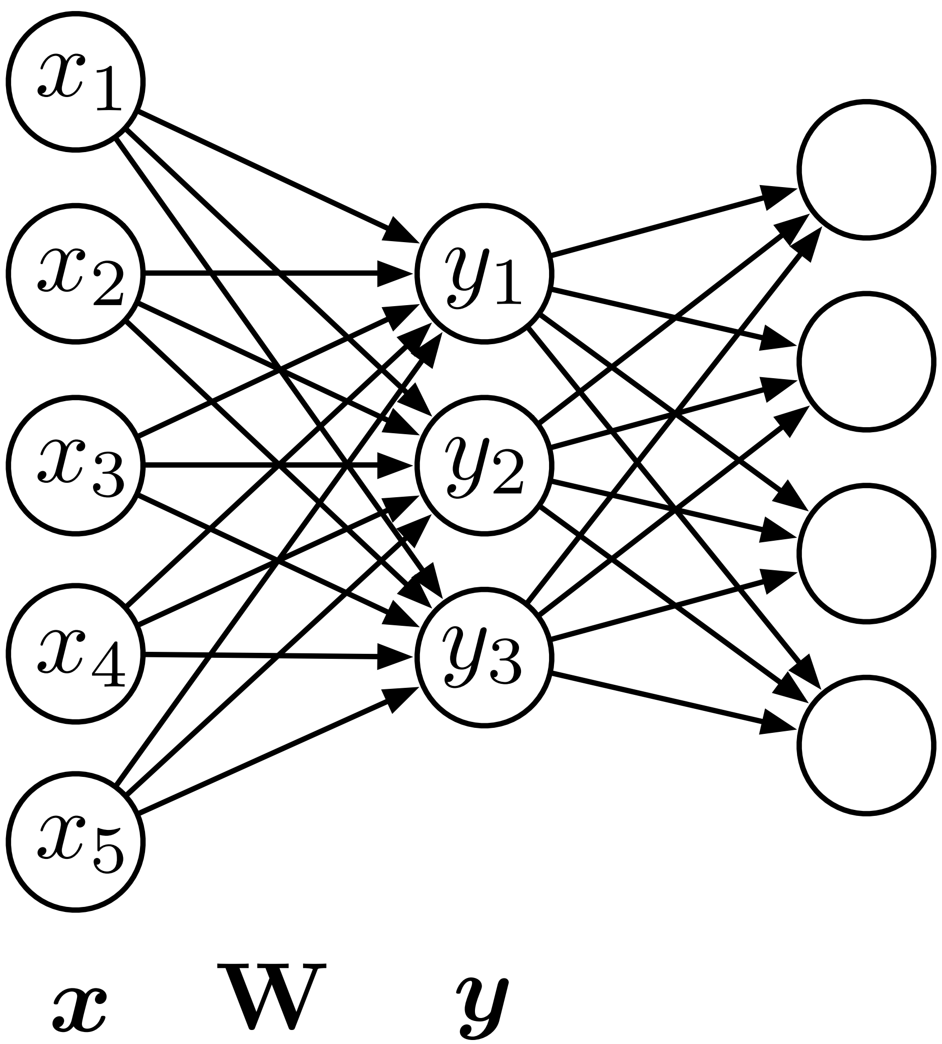
Look familiar?

What about probabilities?
Let each y_i be an outcome.

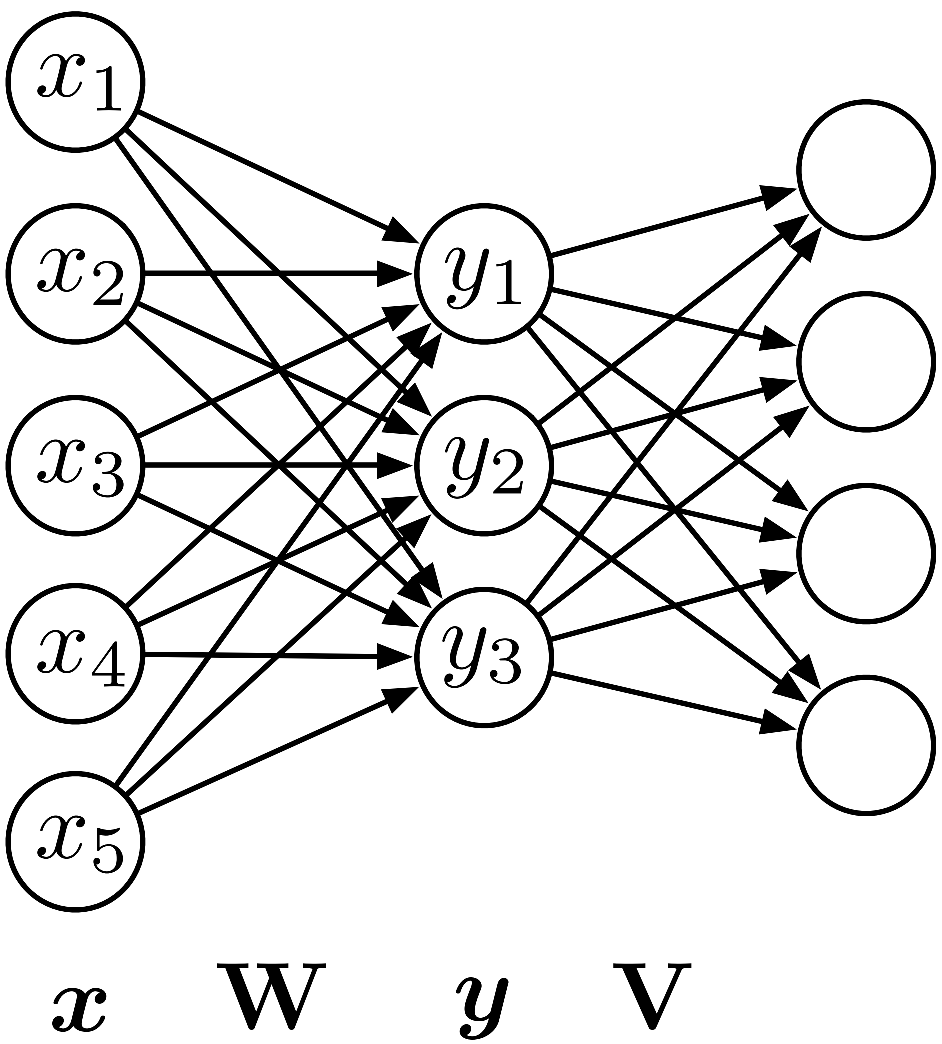
“Soft max”

$$g(\boldsymbol{u})_i = \frac{\exp u_i}{\sum_{i'} \exp u_{i'}}$$

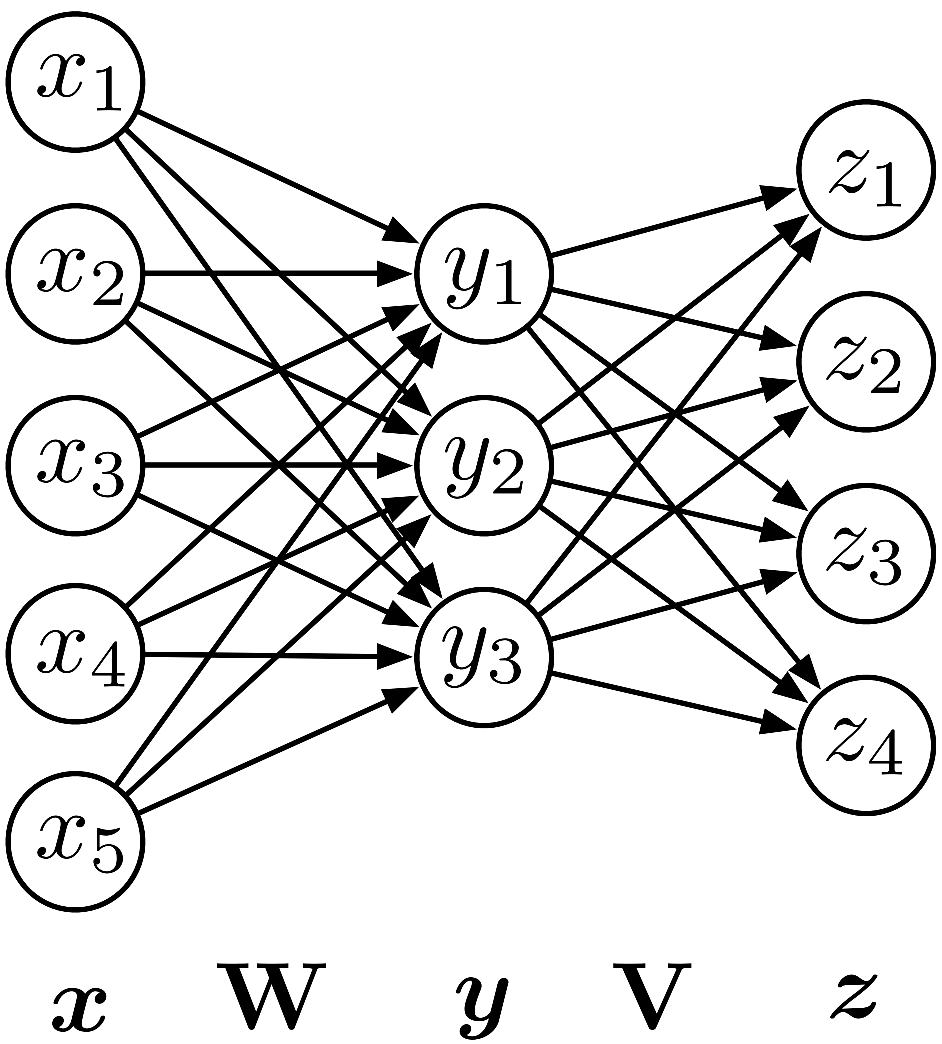
“Deep”



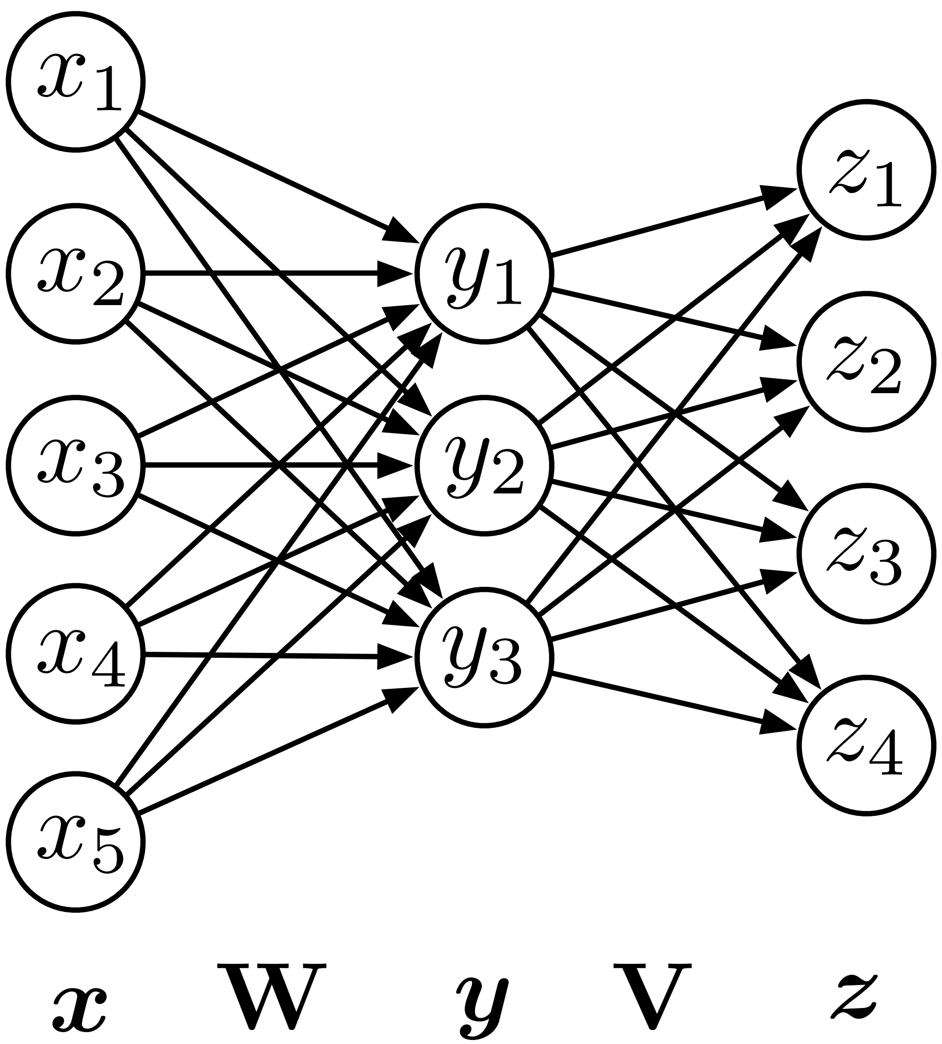
“Deep”


x_1
x_2
x_3
x_4
x_5
y_1
y_2
y_3
$\boldsymbol{x}$ $\mathbf{W}$ $\boldsymbol{y}$ $\mathbf{V}$

“Deep”

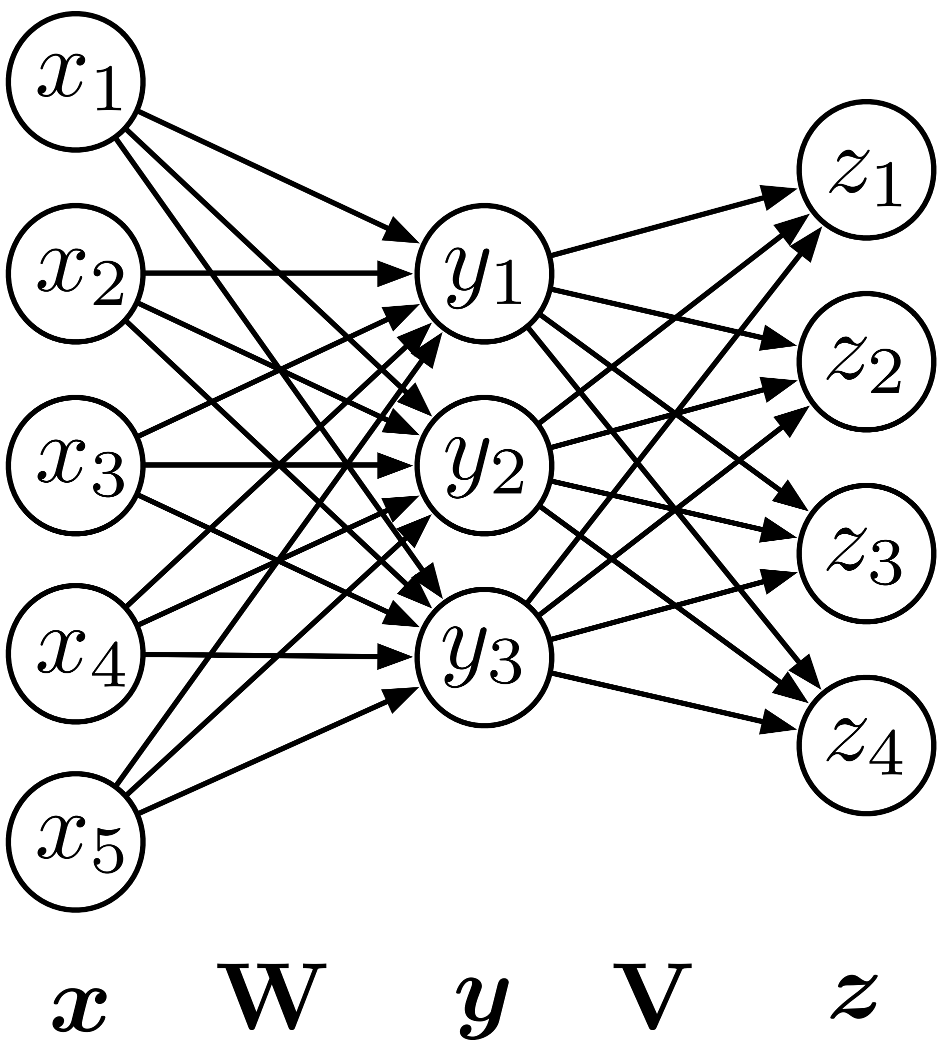

x_1
x_2
x_3
x_4
x_5
y_1
y_2
y_3
z_1
z_2
z_3
z_4
$\boldsymbol{x}$ $\mathbf{W}$ $\boldsymbol{y}$ $\mathbf{V}$ $\boldsymbol{z}$

"Deep"



$$\boldsymbol{z} = g(\boldsymbol{y}^\top \mathbf{V})$$

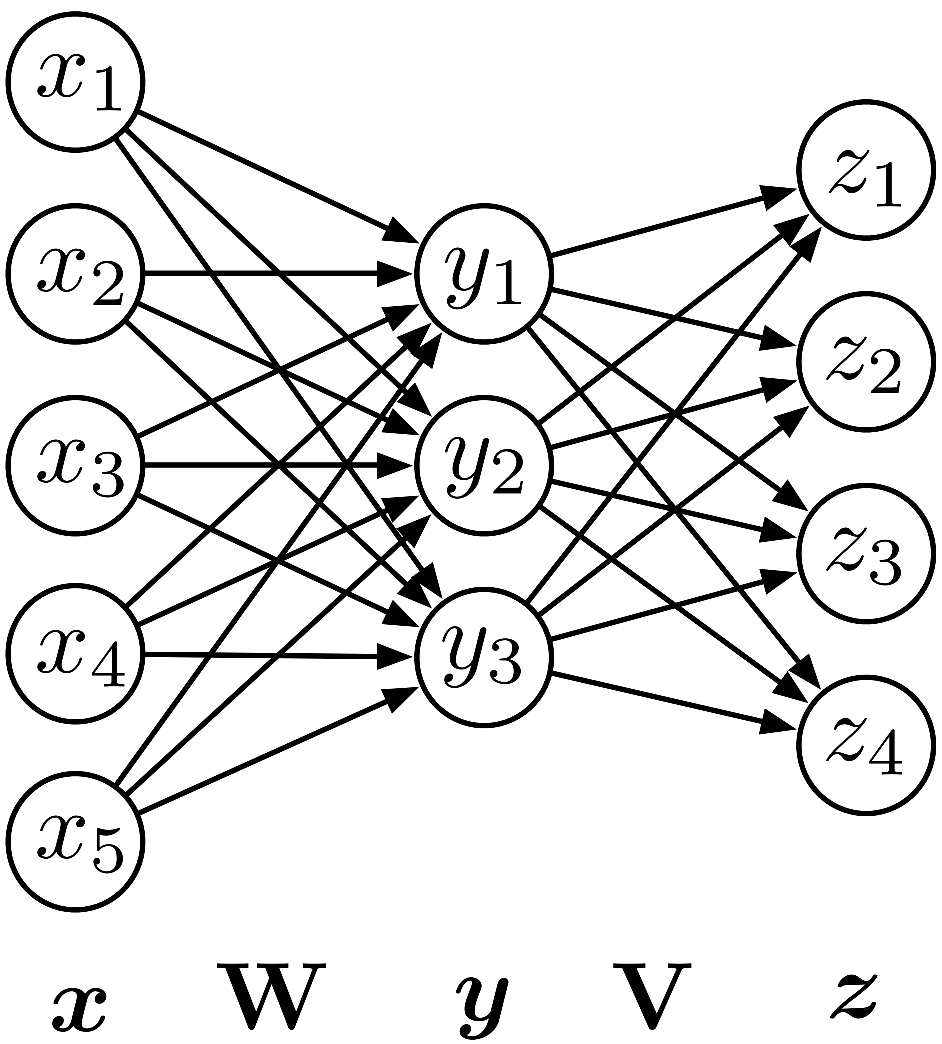
"Deep"



$$\boldsymbol{z} = g(\boldsymbol{y}^\top \mathbf{V})$$

$$\boldsymbol{z} = g(h(\boldsymbol{x}^\top \mathbf{W})^\top \mathbf{V})$$

“Deep”

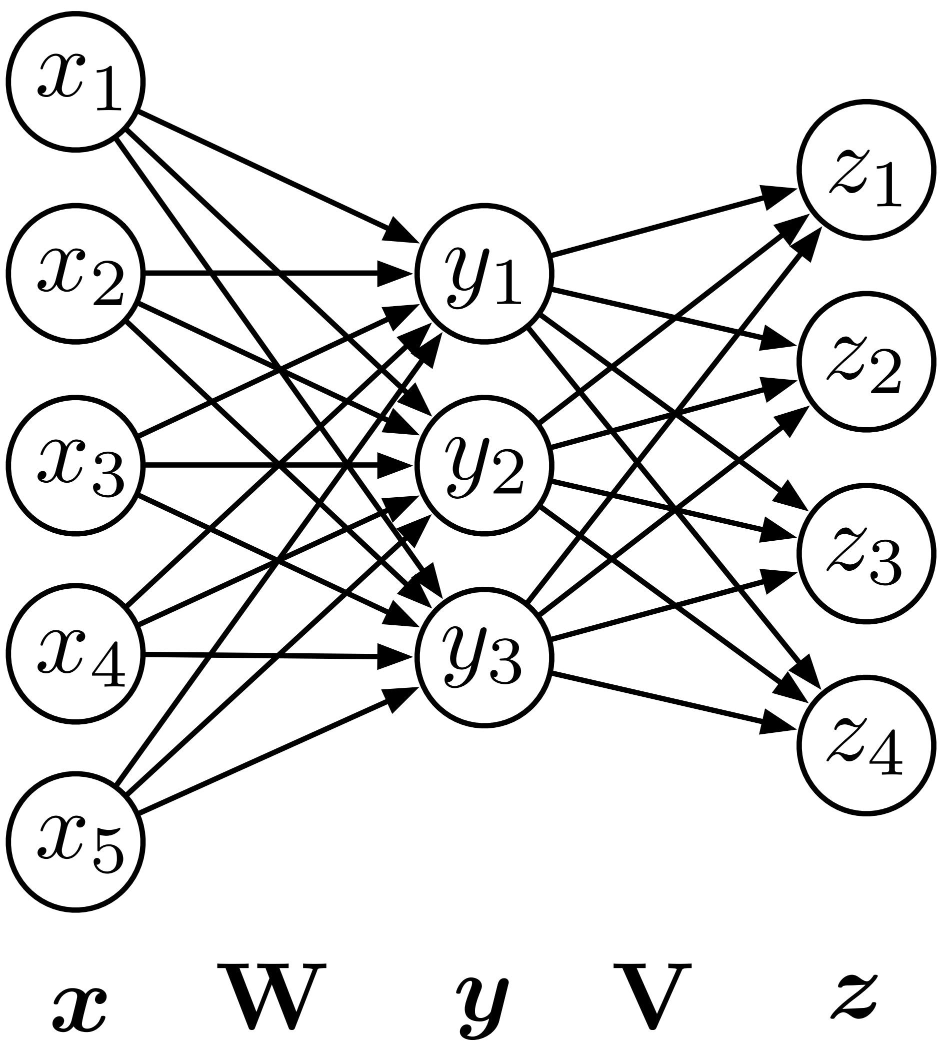


$$\boldsymbol{z} = g(\boldsymbol{y}^\top \mathbf{V})$$

$$\boldsymbol{z} = g(h(\boldsymbol{x}^\top \mathbf{W})^\top \mathbf{V})$$

$$\boldsymbol{z} = g(\mathbf{V} h(\mathbf{W} \boldsymbol{x}))$$

"Deep"



$$\boldsymbol{z} = g(\boldsymbol{y}^\top \mathbf{V})$$

$$\boldsymbol{z} = g(h(\boldsymbol{x}^\top \mathbf{W})^\top \mathbf{V})$$

$$\boldsymbol{z} = g(\mathbf{V} h(\mathbf{W}\boldsymbol{x}))$$

Note:

if $g(\boldsymbol{x}) = h(\boldsymbol{x}) = \boldsymbol{x}$

$$\boldsymbol{z} = \underbrace{(\mathbf{V}\mathbf{W})}_{\mathbf{U}} \boldsymbol{x}$$

Design Decisions

- How to represent inputs and outputs?
- Neural architecture?
  - How many layers? (Requires non-linearities to improve capacity!)
  - How many neurons?
  - Recurrent or not?
- What kind of non-linearities?

Representing Language

- “One-hot” vectors
  - Each position in a vector corresponds to a word type
  - Sequence of words, sequence of vectors
  - Bag of words: multiple vectors
- Distributed representations
  - Vectors encode “features” of input words (character n-grams, morphological features, etc.)

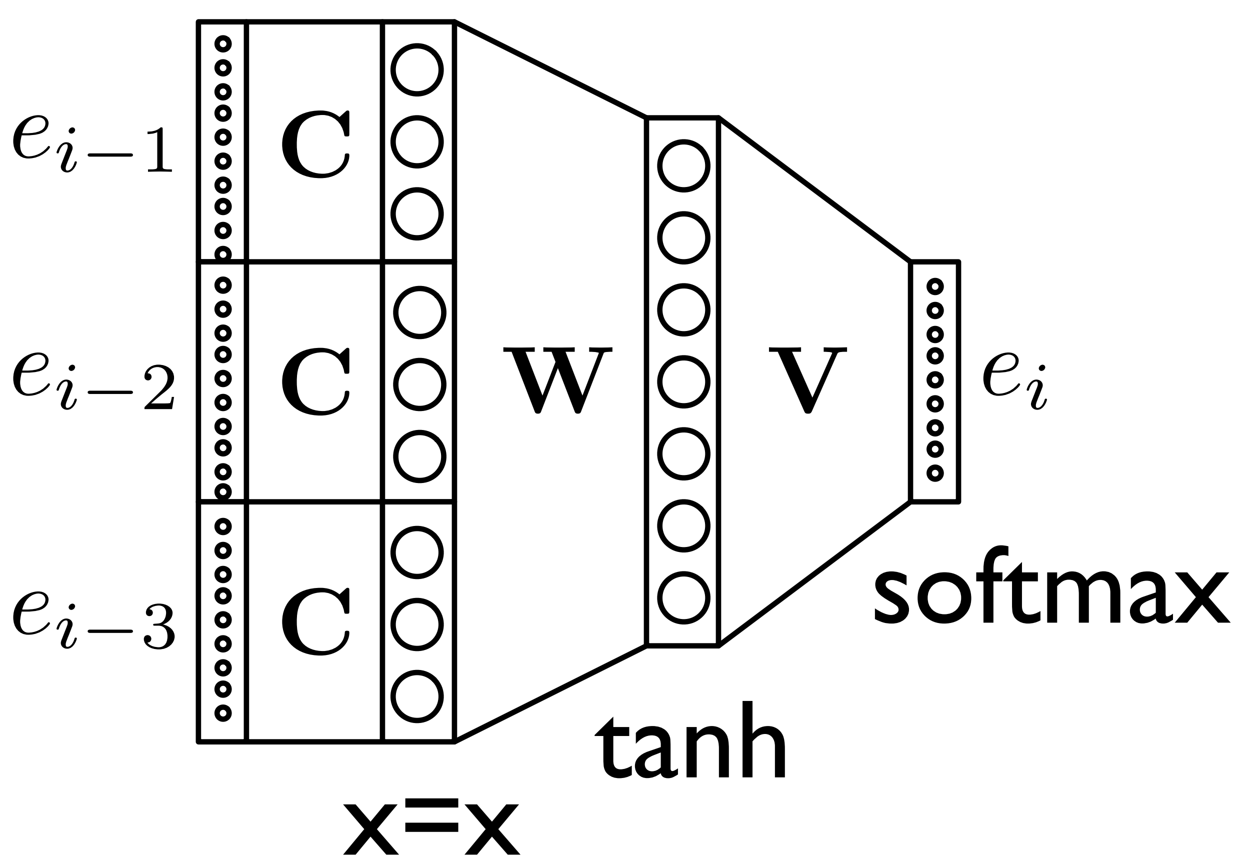
Training Neural Networks

- Neural networks are supervised models - you need a set of inputs paired with outputs
- Algorithm
  - Run for a while
    - Give input to the network, see what it predicts
    - Compute loss(y,y*) and (sub)gradient with respect to parameters. Use the chain rule, aka “back propagation”
    - Update parameters (SGD, AdaGrad, LBFGS, etc.)
- Algorithm is automated, just need to specify model structure

Bengio et al. (2003)

$$p(\mathbf{e}) = \prod_{i=1}^{|\mathbf{e}|} p(e_i \mid e_{i-n+1}, \ldots, e_{i-1})$$

$$p(e_i \mid e_{i-n+1}, \ldots, e_{i-1}) =$$



Bengio et al. (2003)

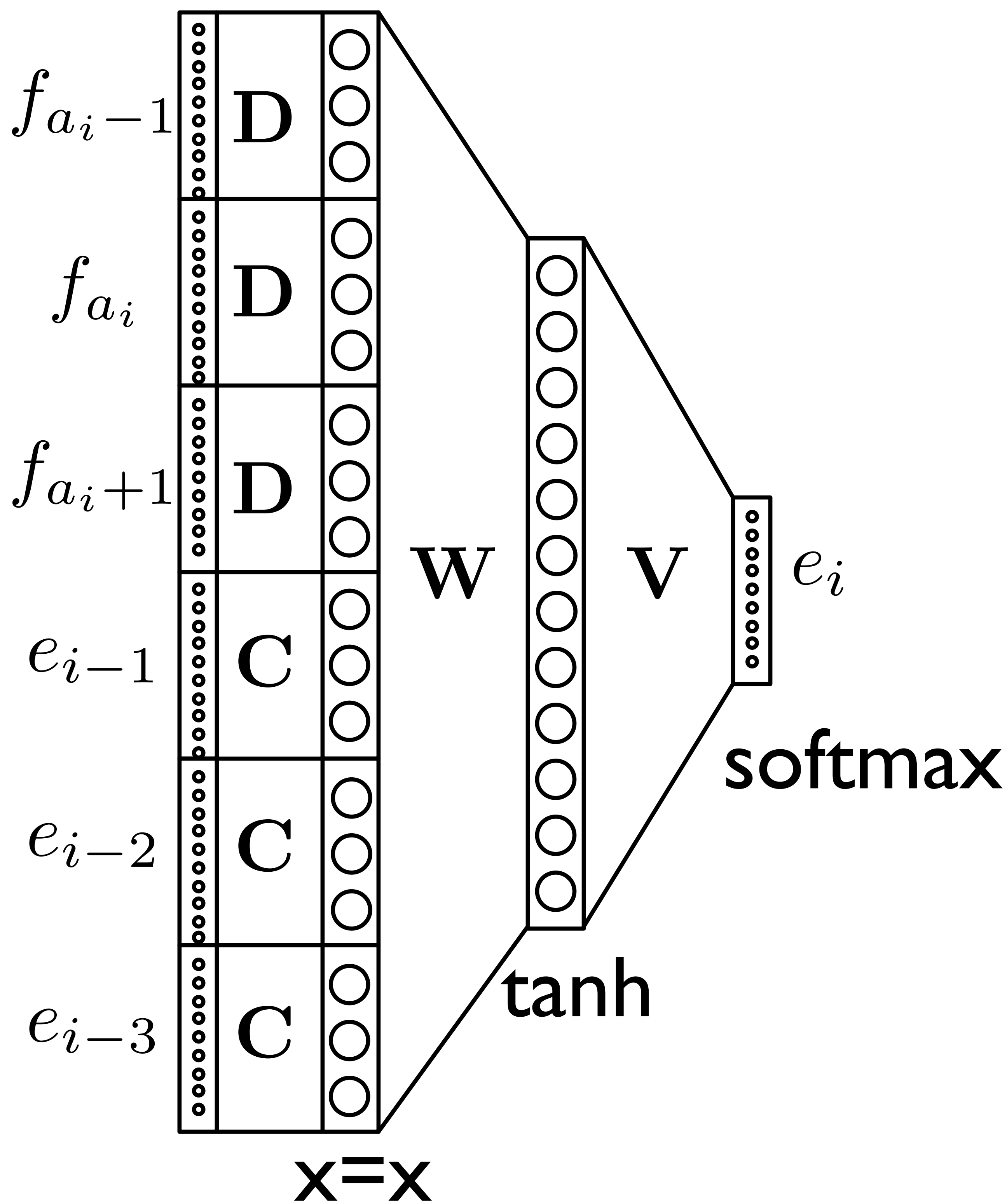
	n	c	h	m	direct	mix	train.	valid.	test.
MLP1	5		50	60	yes	no	182	284	268
MLP2	5		50	60	yes	yes		275	257
MLP3	5		0	60	yes	no	201	327	310
MLP4	5		0	60	yes	yes		286	272
MLP5	5		50	30	yes	no	209	296	279
MLP6	5		50	30	yes	yes		273	259
MLP7	3		50	30	yes	no	210	309	293
MLP8	3		50	30	yes	yes		284	270
MLP9	5		100	30	no	no	175	280	276
MLP10	5		100	30	no	yes		265	**252**
Del. Int.	3						31	352	336
Kneser-Ney back-off	3							334	323
Kneser-Ney back-off	4							332	321
Kneser-Ney back-off	5							332	321

Devlin et al. (2014)

- Turn Bengio et al. (2003) into a translation model
- Conditional model; generate the next English word conditioned on
  - The previous *n* English words you generated
  - The aligned source word, and its *m* neighbors

$$p(\mathbf{e} \mid \mathbf{f}, \mathbf{a}) = \prod_{i=1}^{|\mathbf{e}|} p(e_i \mid e_{i-2}, e_{i-1}, f_{a_i-1}, f_{a_i}, f_{a_i+1})$$

$$p(e_i \mid e_{i-2}, e_{i-1}, f_{a_i-1}, f_{a_i}, f_{a_i+1}) =$$



Devlin et al. (2014)

BOLT Test		
	Ar-En	
	BLEU	**% Gain**
"Simple Hier." Baseline	33.8	-
S2T/L2R NNJM (Dec)	38.4	100%
Source Window=7	38.3	98%
Source Window=5	38.2	96%
Source Window=3	37.8	87%
Source Window=0	35.3	33%
Layers=384x768x768	38.5	102%
Layers=192x512	38.1	93%
Layers=128x128	37.1	72%
Vocab=64,000	38.5	102%
Vocab=16,000	38.1	93%
Vocab=8,000	37.3	83%
Activation=Rectified Lin.	38.5	102%
Activation=Linear	37.3	76%

Summary of neural LMs

- Two problems in standard statistical models
  - We don't condition on enough stuff
  - We don't know what features to use when we condition on lots of structure
- Neural networks let us condition on a lot of stuff without an exponential growth in parameters
- But: they are just reparameterized probability distributions. **Probability is central.**

LMs in MT

$$p(\mathbf{e}, \mathbf{f}) = p_{LM}(\mathbf{e}) \times p_{TM}(\mathbf{f} \mid \mathbf{e})$$

LMs in MT

$$
\begin{aligned}
p(\mathbf{e}, \mathbf{f}) &= p_{LM}(\mathbf{e}) \times p_{TM}(\mathbf{f} \mid \mathbf{e}) \\
&= \prod_{i=1}^{|\mathbf{e}|} p(e_i \mid e_{i-1}, \ldots, e_1) \times \prod_{j=1}^{|\mathbf{f}|} p(f_j \mid f_{j-1}, \ldots, f_1, \mathbf{e})
\end{aligned}
$$

LMs in MT

$$
\begin{aligned}
p(\mathbf{e}, \mathbf{f}) &= p_{LM}(\mathbf{e}) \times p_{TM}(\mathbf{f} \mid \mathbf{e}) \\
&= \prod_{i=1}^{|\mathbf{e}|} p(e_i \mid e_{i-1}, \ldots, e_1) \times \prod_{j=1}^{|\mathbf{f}|} p(f_j \mid f_{j-1}, \ldots, f_1, \mathbf{e})
\end{aligned}
$$

Conditional language model

LMs in MT

$$
\begin{aligned}
p(\mathbf{e}, \mathbf{f}) &= p_{LM}(\mathbf{e}) \times p_{TM}(\mathbf{f} \mid \mathbf{e}) \\
&= \prod_{i=1}^{|\mathbf{e}|} p(e_i \mid \underbrace{e_{i-1}, \ldots, e_1}) \times \prod_{j=1}^{|\mathbf{f}|} p(f_j \mid \underbrace{f_{j-1}, \ldots, f_1}, \mathbf{e})
\end{aligned}
$$

Conditioned on entire sentence!
With direct multinomial parameterizations (n-gram models, IMB Model 1), we had to make independence assumptions for this to work.

LMs in MT

$$
\begin{aligned}
p(\mathbf{e}, \mathbf{f}) &= p_{LM}(\mathbf{e}) \times p_{TM}(\mathbf{f} \mid \mathbf{e}) \\
&= \prod_{i=1}^{|\mathbf{e}|} p(e_i \mid e_{i-1}, \ldots, e_1) \times \prod_{j=1}^{|\mathbf{f}|} p(f_j \mid f_{j-1}, \ldots, f_1, \mathbf{e})
\end{aligned}
$$