

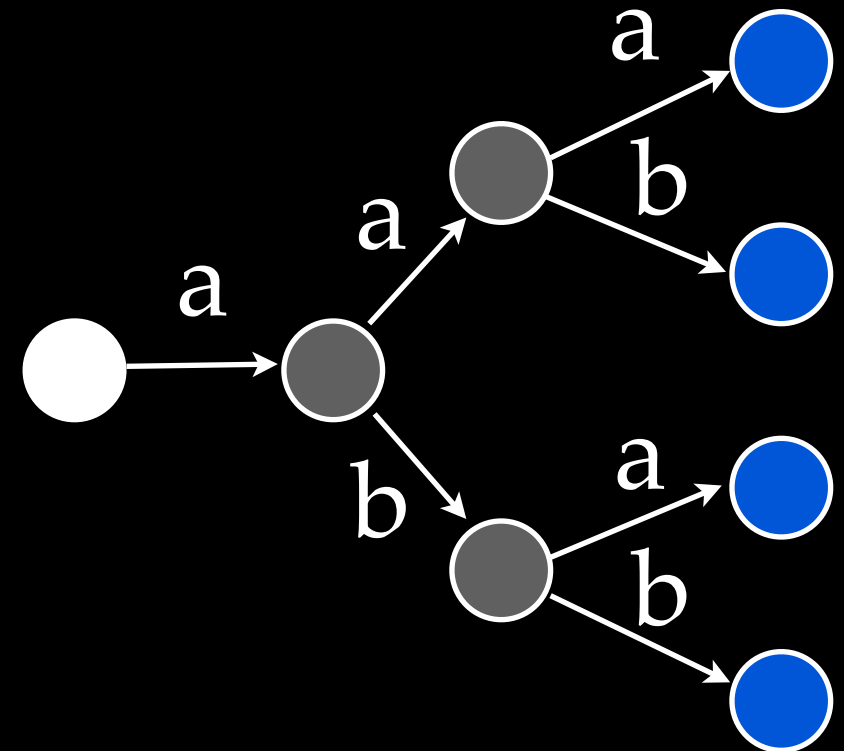
# Syntax-Based Translation with Weighted Automata

# Review

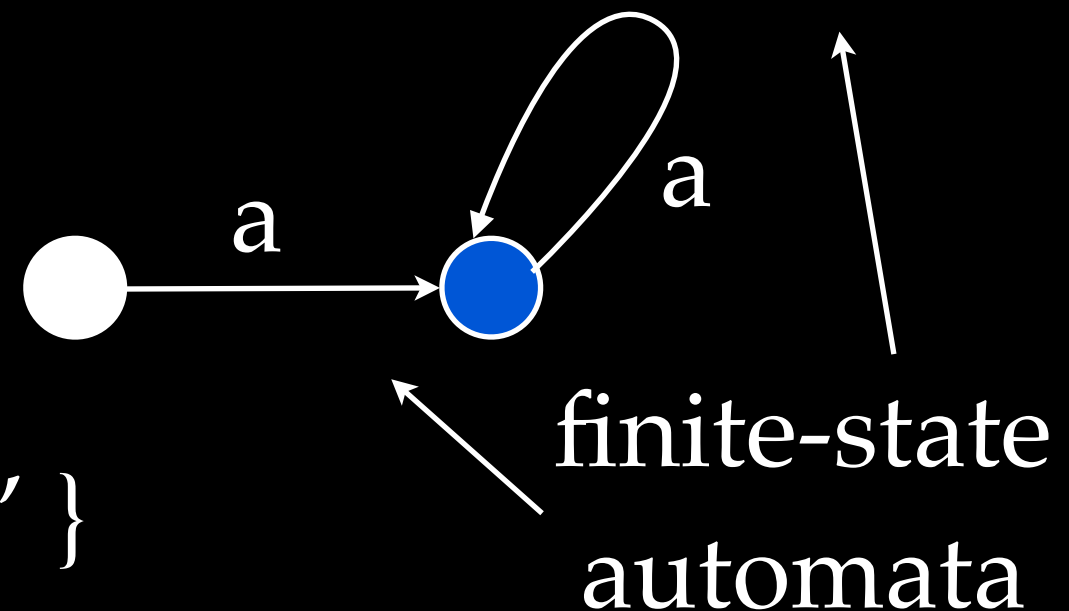
- We need efficient algorithms and data structures to:
  - Encode all of the strings in the language.
  - Assign probabilities to all of those strings.
    - Via products such as  $p(e)p(f|e)$ .
  - Find the string with the highest probability.
  - Compute expectations over substrings.
  - Compute mappings between strings.

# Regular Languages

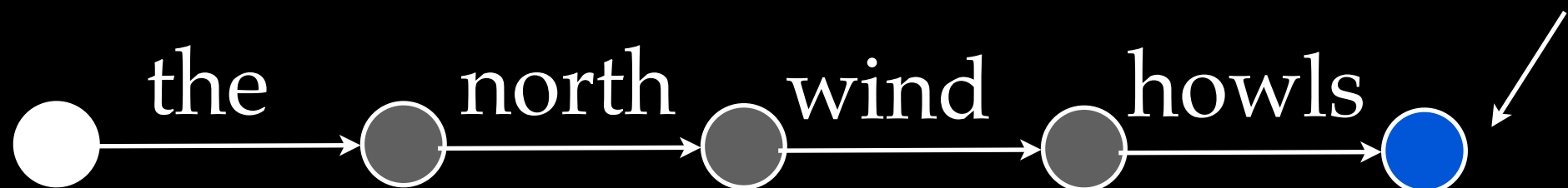
$$\mathcal{L}_1 = \left\{ \begin{array}{c} a a a \\ a b a \\ a a b \\ a b b \end{array} \right\}$$



$$\mathcal{L}_2 = a^* = \{a, aa, aaa, \dots\}$$

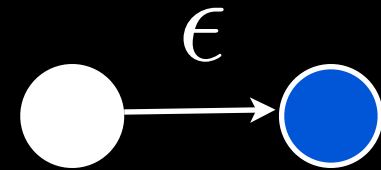


$$\mathcal{L}_3 = \{ \text{"the north wind howls"} \}$$

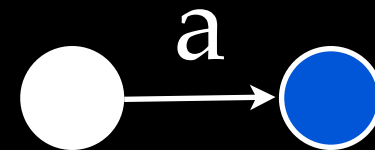


# Regular Languages

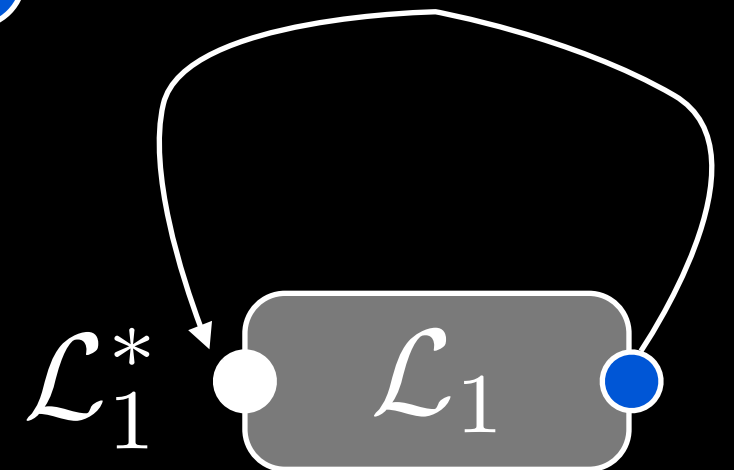
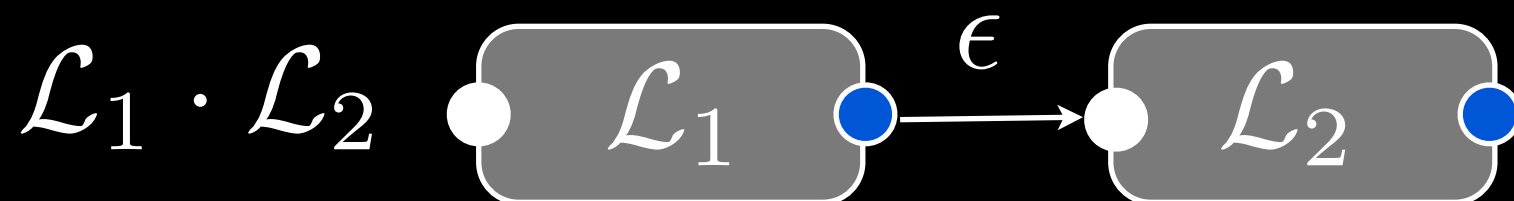
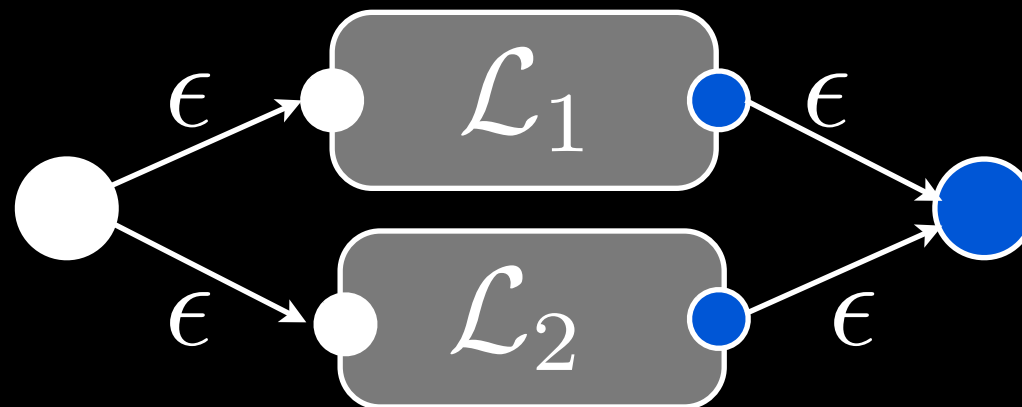
$\{\epsilon\}$  is regular



$\{a\}$  is regular



$\mathcal{L}_1 \cup \mathcal{L}_2$  is regular if  $\mathcal{L}_1$  and  $\mathcal{L}_2$  are regular



# Regular Languages

Not all languages are regular!

$$\mathcal{L}_4 = \{ab, aabb, aaabbb, \dots\} = \forall_{n \in [1, \infty)} a^n b^n$$

Over the last two weeks we saw *context-free* languages.

# Context-Free Grammar

# Context-Free Grammar

$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$

# Context-Free Grammar

S

$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$



# Context-Free Grammar

S

$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$

# Context-Free Grammar

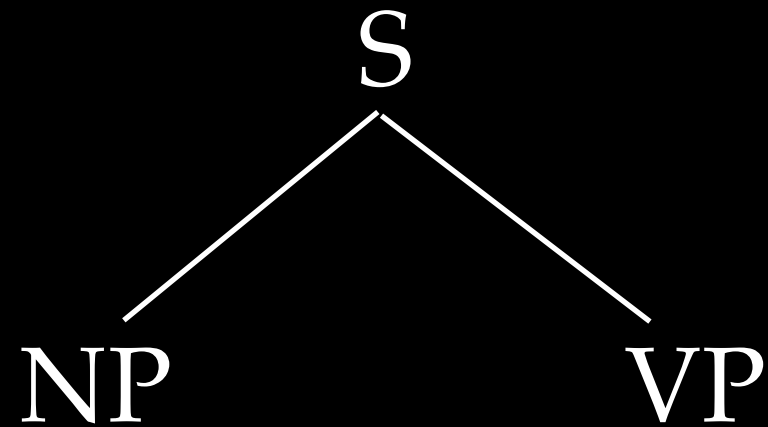
$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$



# Context-Free Grammar

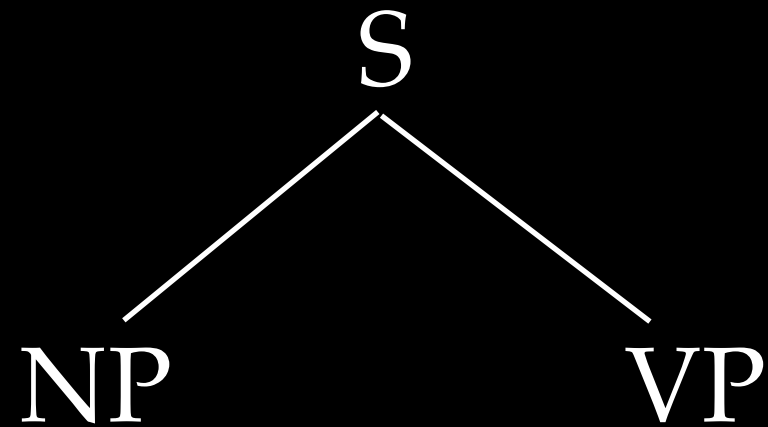
$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$



# Context-Free Grammar

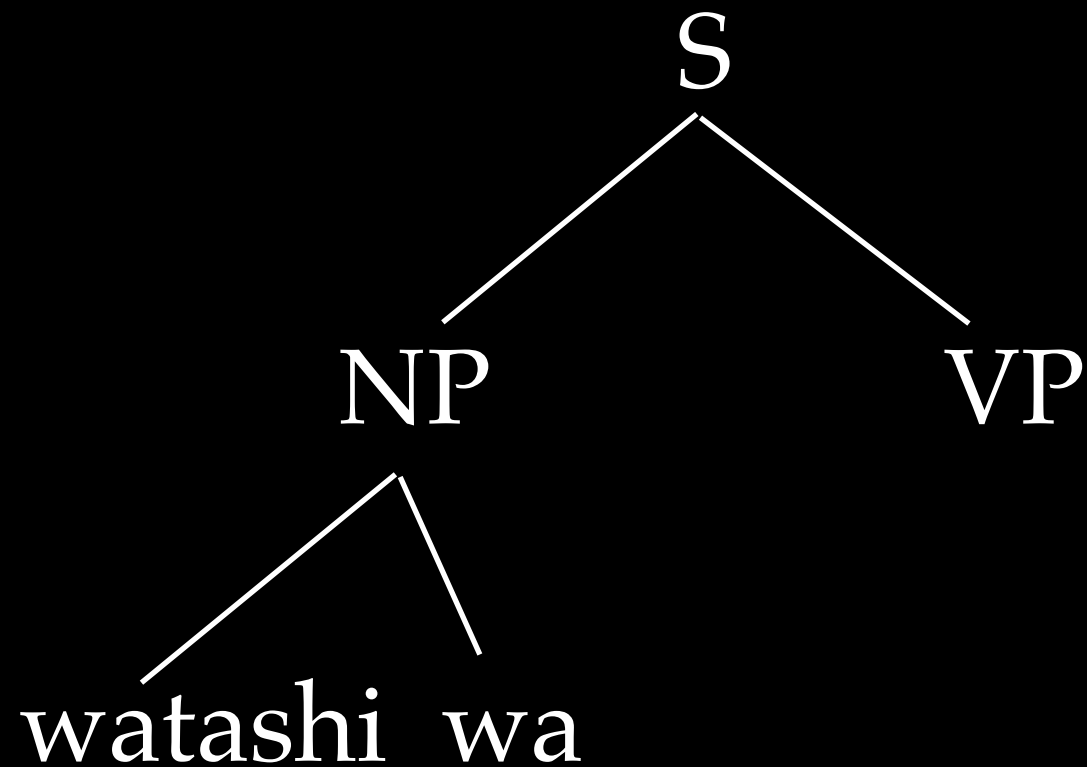
$S \rightarrow NP VP$

$NP \rightarrow \text{watashi wa}$

$NP \rightarrow \text{hako wo}$

$VP \rightarrow NP V$

$V \rightarrow \text{akemasu}$



# Context-Free Grammar

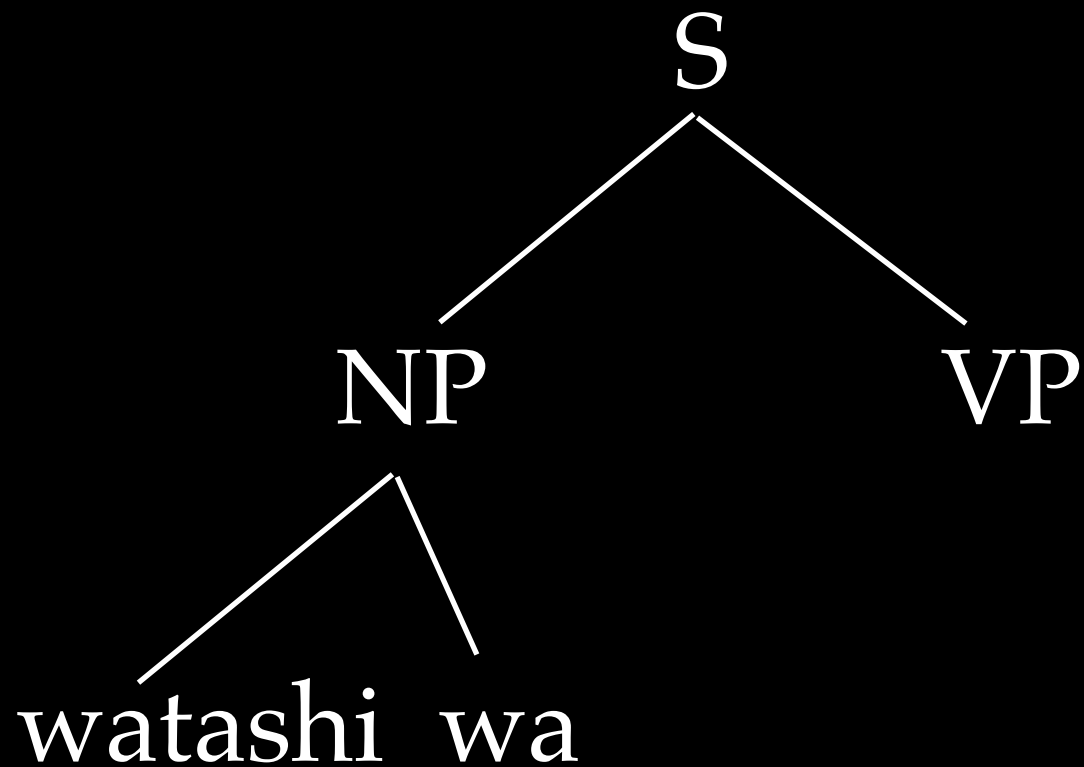
$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$



# Context-Free Grammar

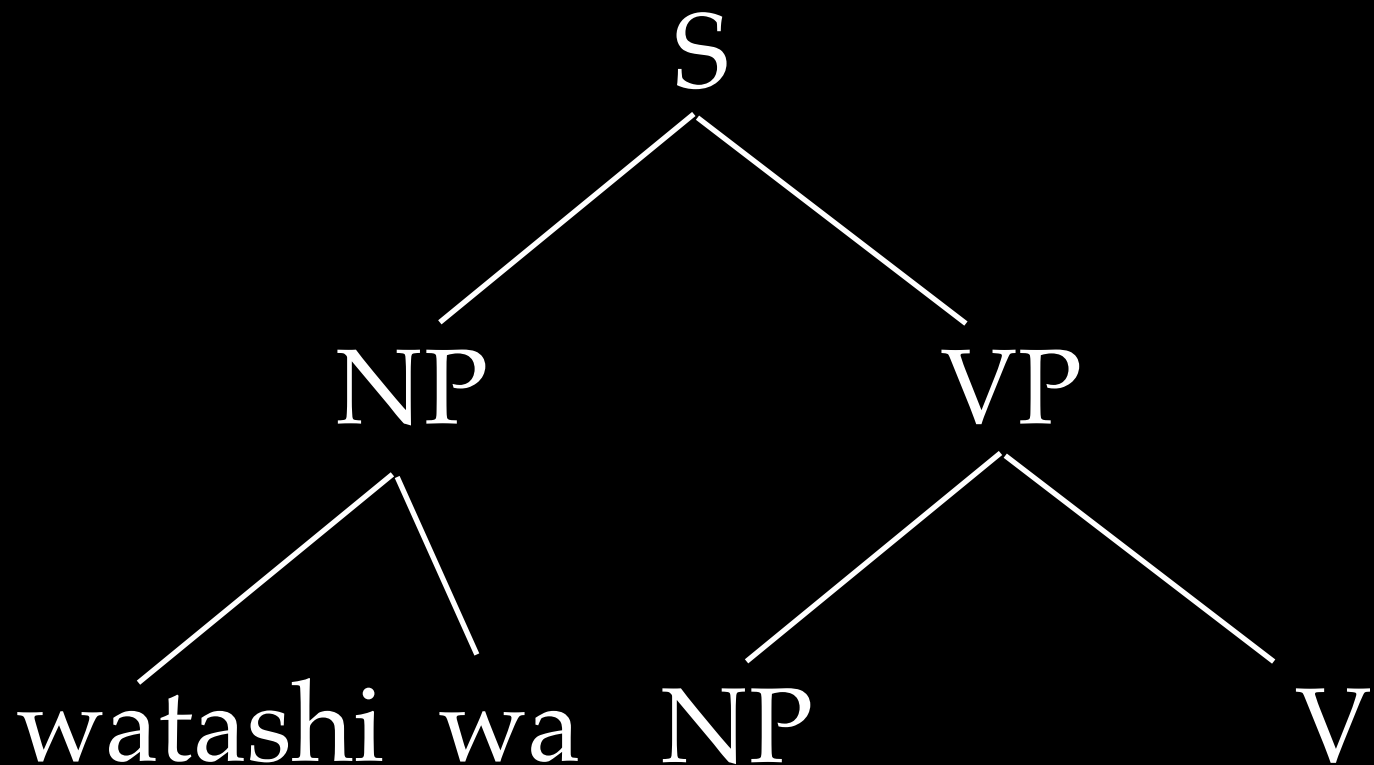
$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$



# Context-Free Grammar

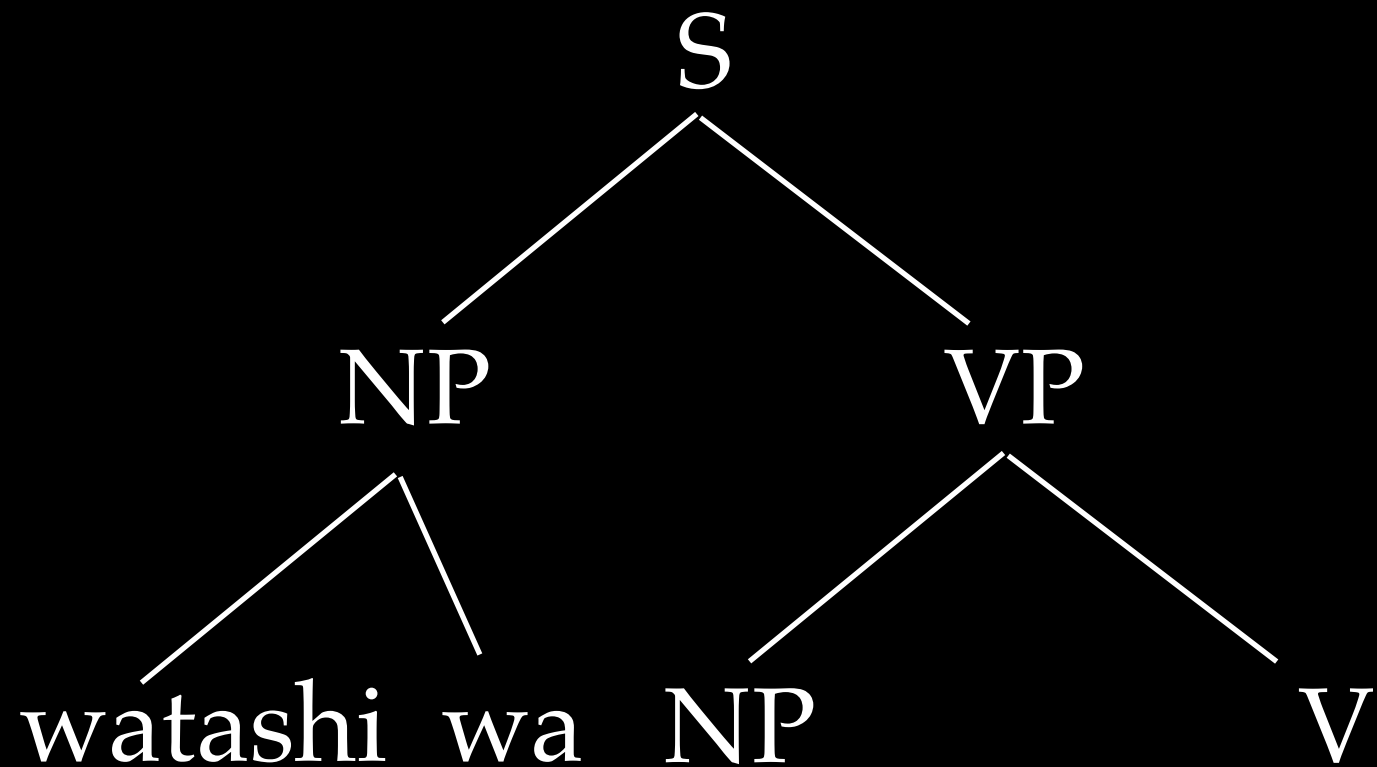
$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$



# Context-Free Grammar

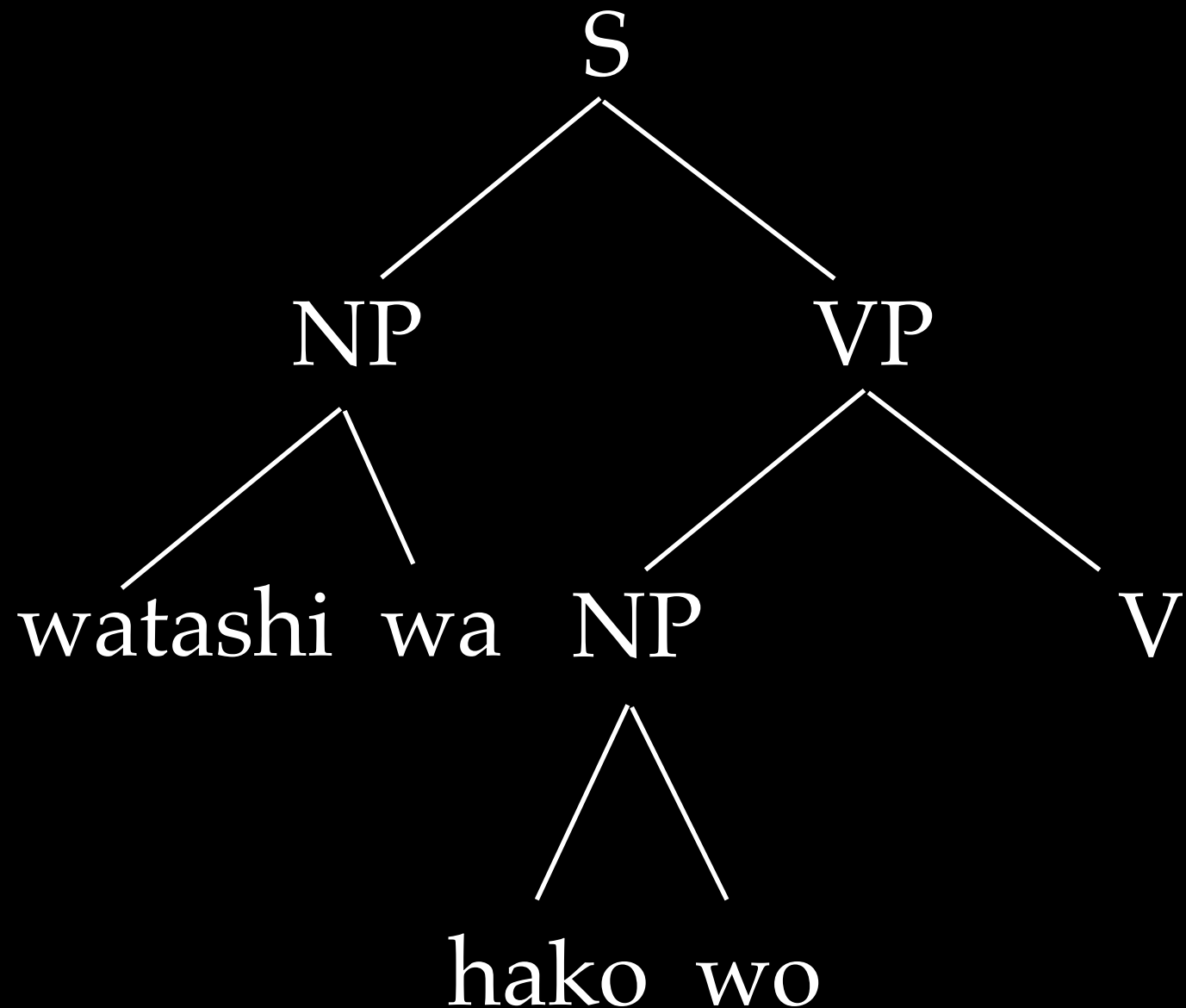
$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$





# Context-Free Grammar

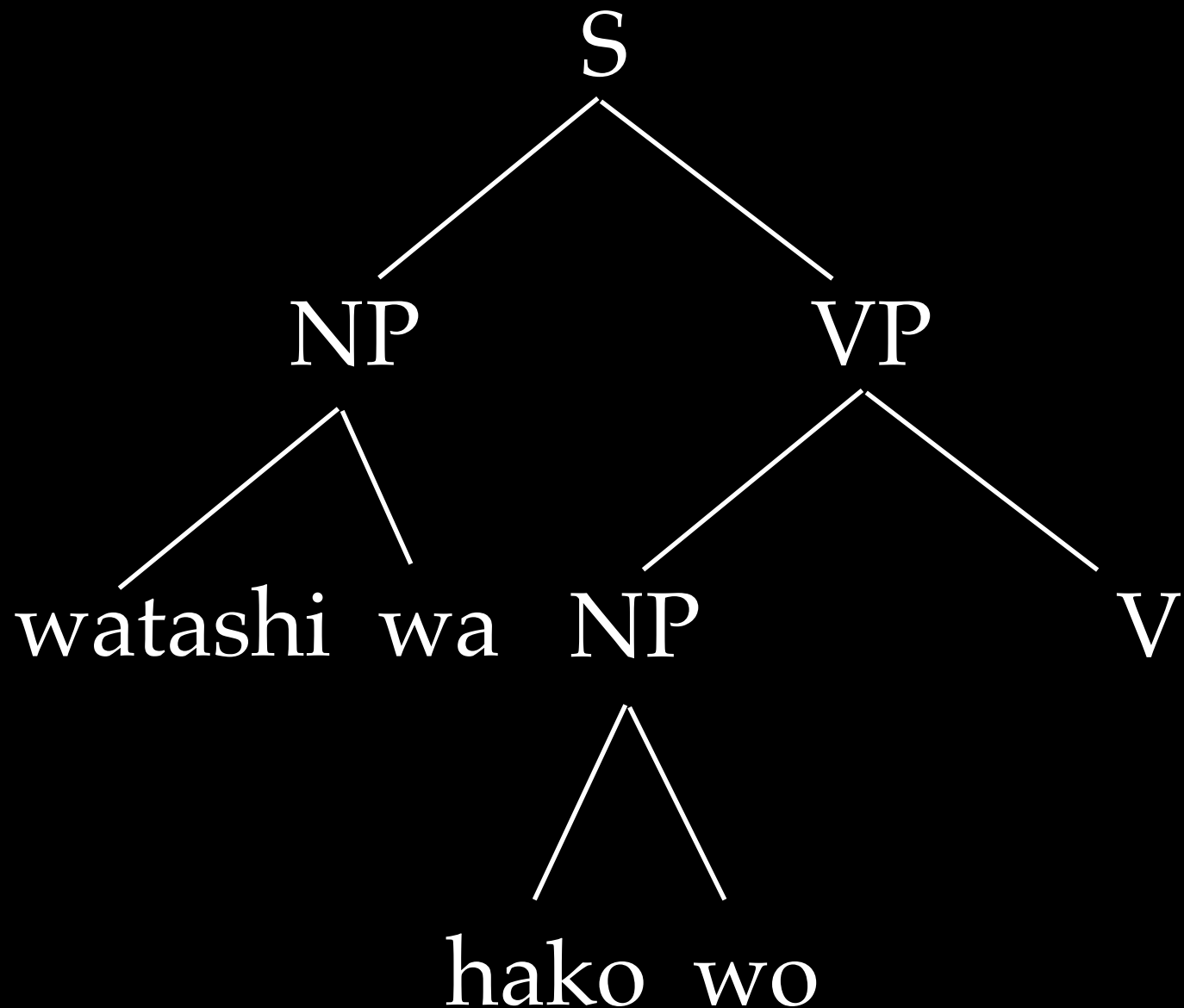
$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$



# Context-Free Grammar

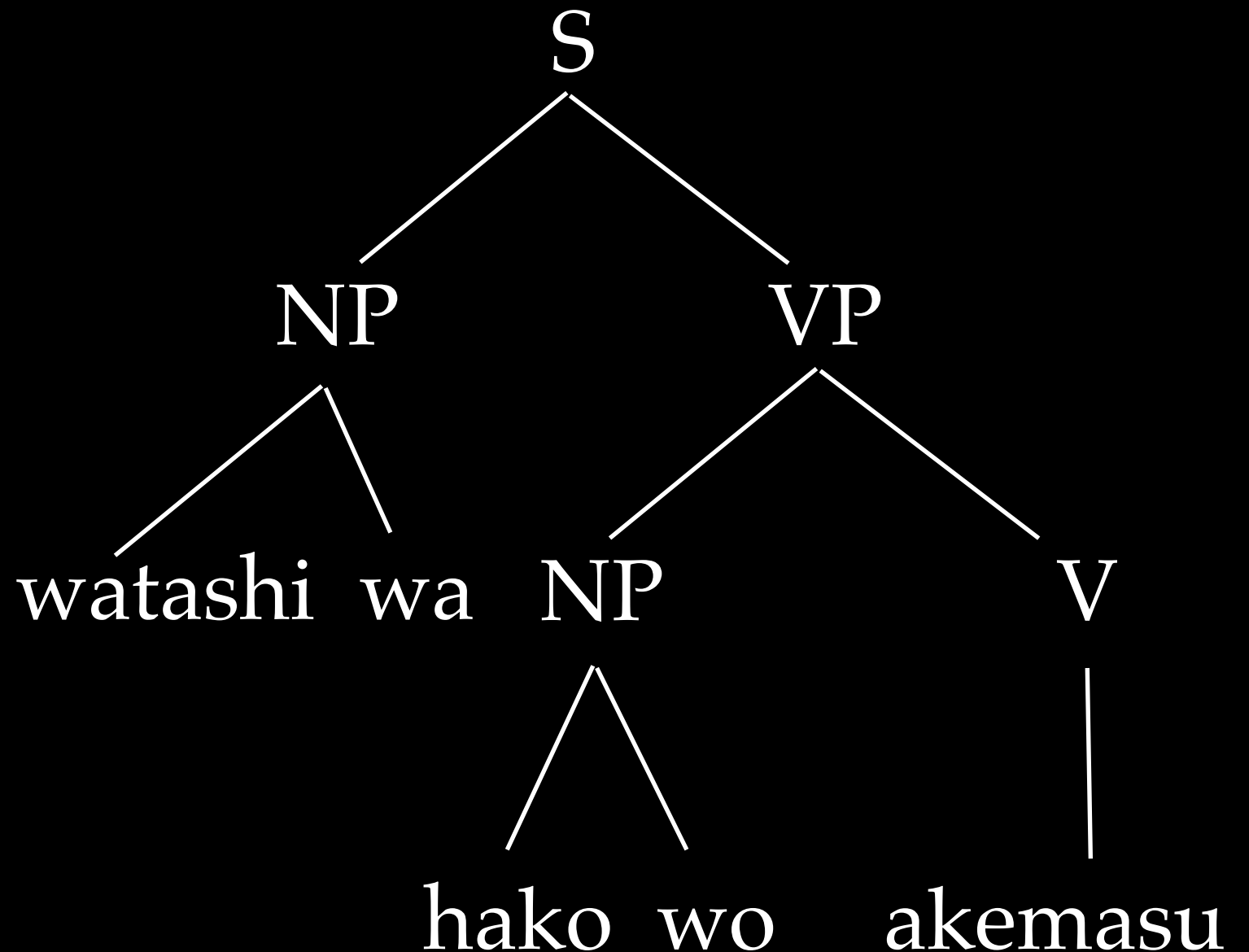
$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

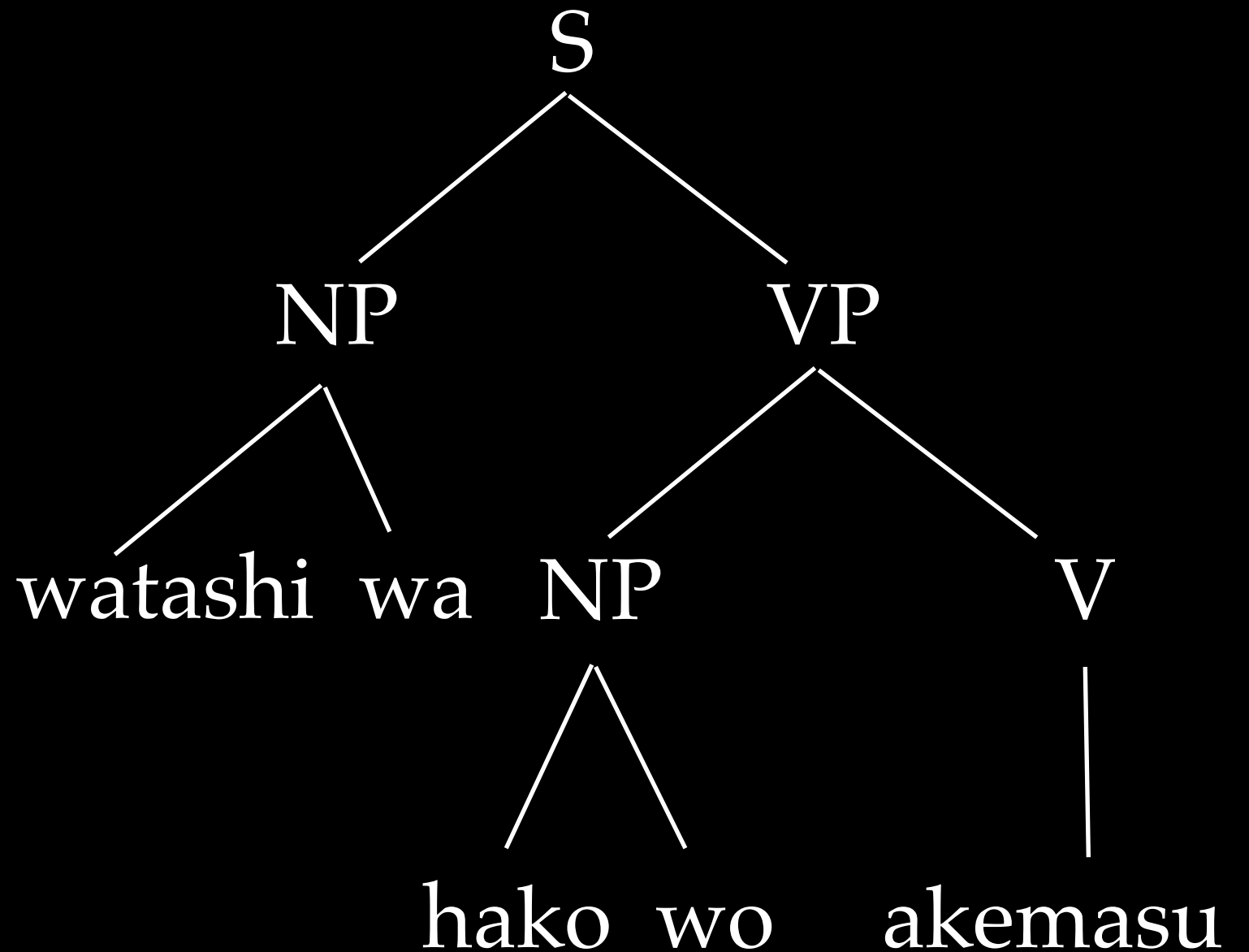
$VP \rightarrow NP V$

$V \rightarrow akemasu$



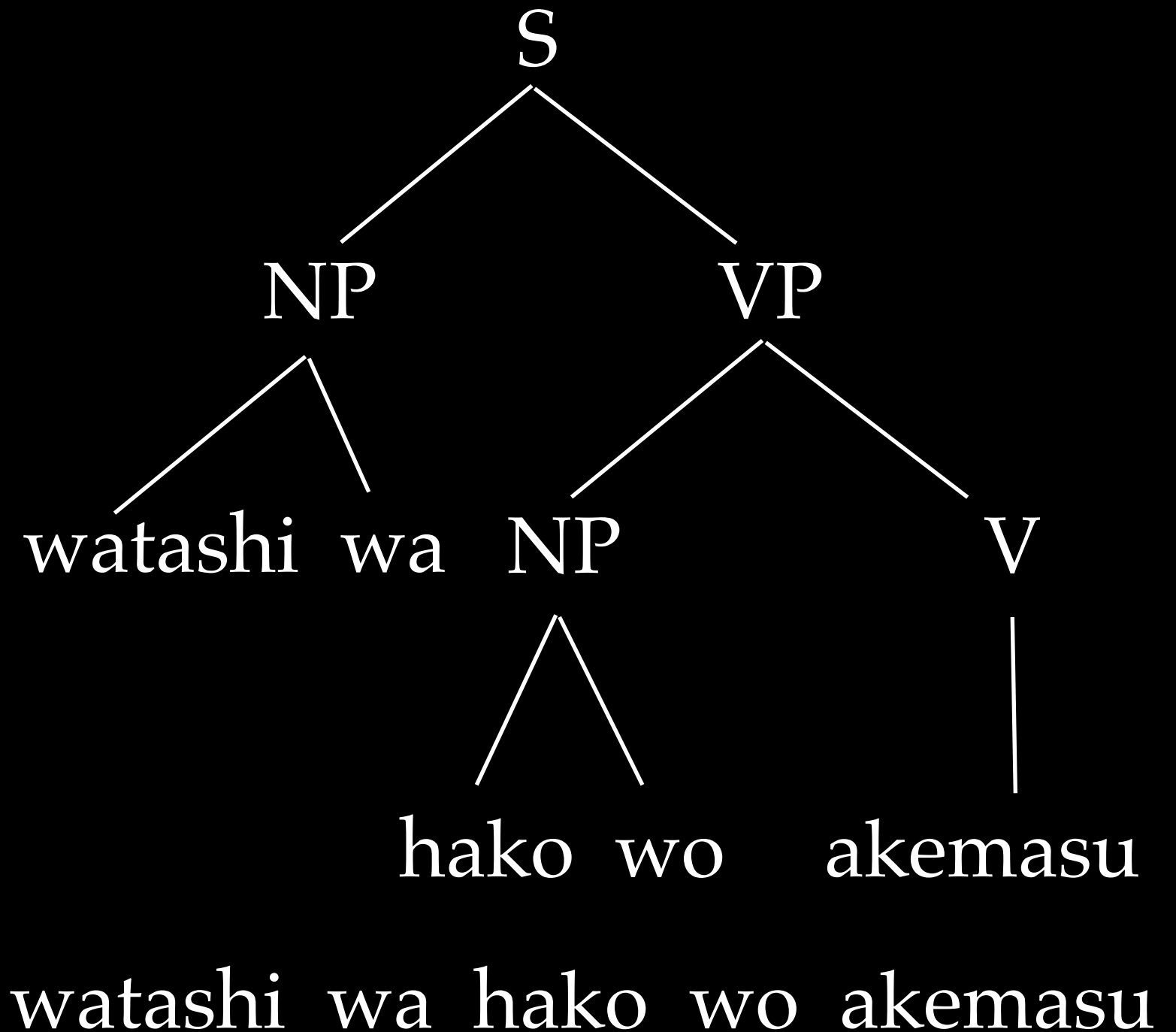
# Context-Free Grammar

$S \rightarrow NP VP$   
 $NP \rightarrow watashi wa$   
 $NP \rightarrow hako wo$   
 $VP \rightarrow NP V$   
 $V \rightarrow akemasu$



# Context-Free Grammar

$S \rightarrow NP VP$   
 $NP \rightarrow watashi wa$   
 $NP \rightarrow hako wo$   
 $VP \rightarrow NP V$   
 $V \rightarrow akemasu$



# Context-Free Grammar

$S \rightarrow NP VP$

$NP \rightarrow \text{watashi wa}$

$NP \rightarrow \text{hako wo}$

$VP \rightarrow NP V$

$V \rightarrow \text{akemasu}$

Note: this particular grammar  
is finite, hence regular.

$\left\{ \begin{array}{l} \text{watashi wa watashi wa akemasu} \\ \text{watashi wa hako wo akemasu} \\ \text{hako wo hako wo akemasu} \\ \text{hako wo watashi wa akemasu} \end{array} \right\}$

# Context-Free Grammar

$S \rightarrow A B$

$S \rightarrow A S B$

$A \rightarrow a$

$B \rightarrow b$

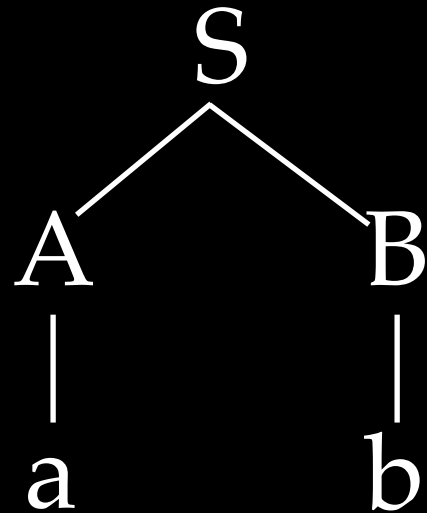
# Context-Free Grammar

$S \rightarrow A B$

$S \rightarrow A S B$

$A \rightarrow a$

$B \rightarrow b$



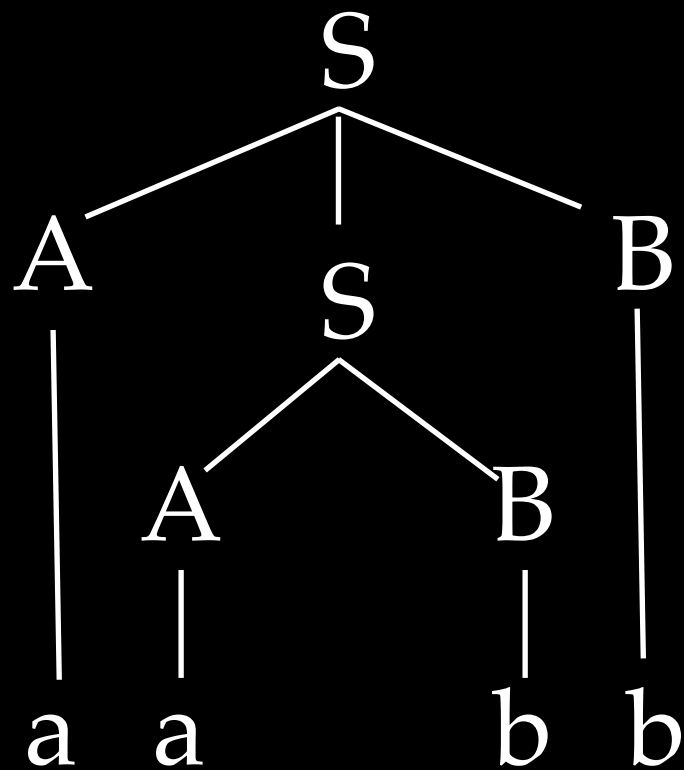
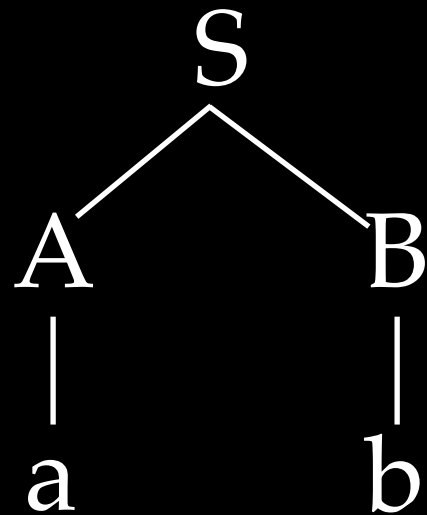
# Context-Free Grammar

$S \rightarrow AB$

$S \rightarrow ASB$

$A \rightarrow a$

$B \rightarrow b$





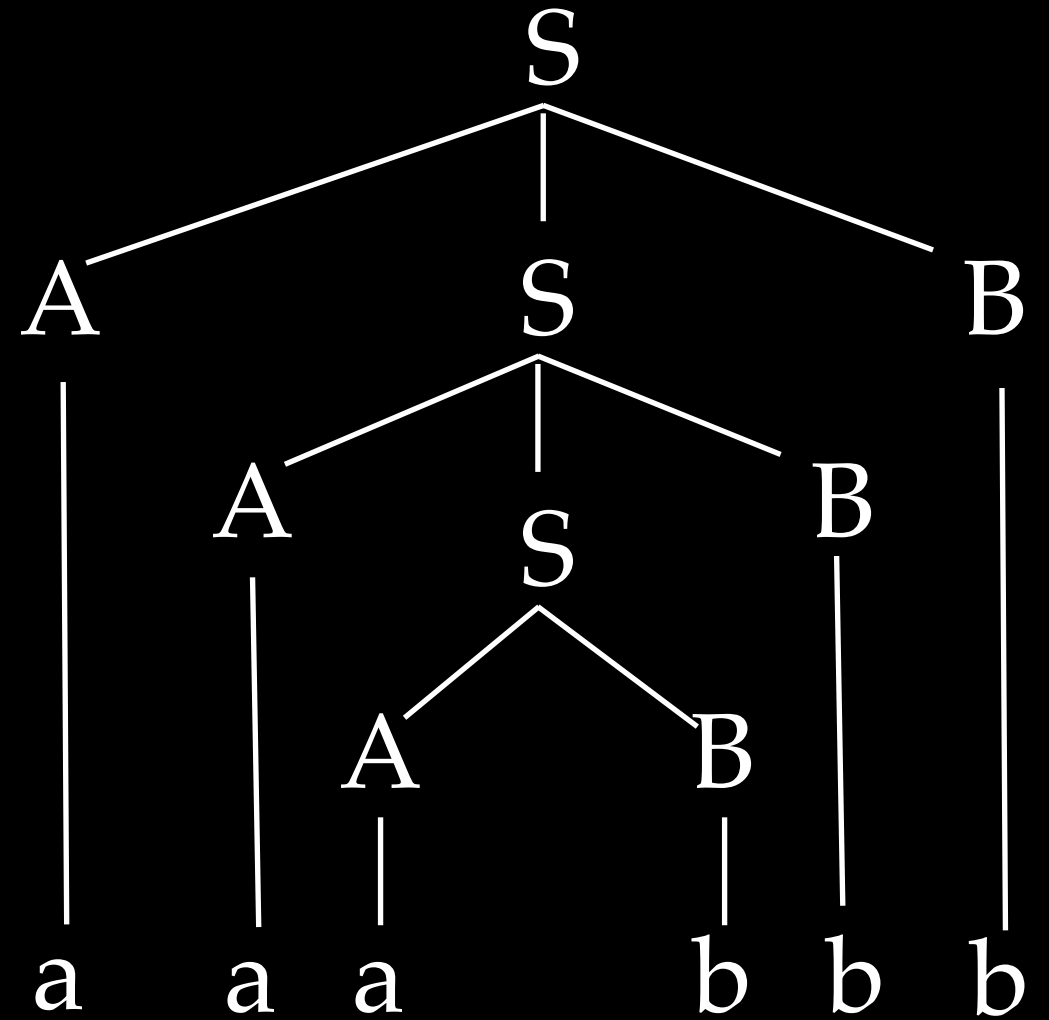
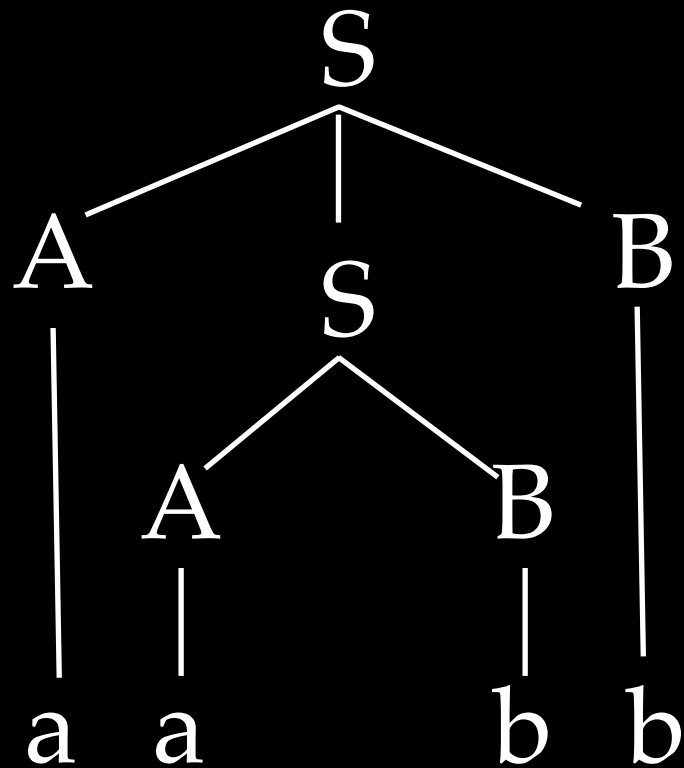
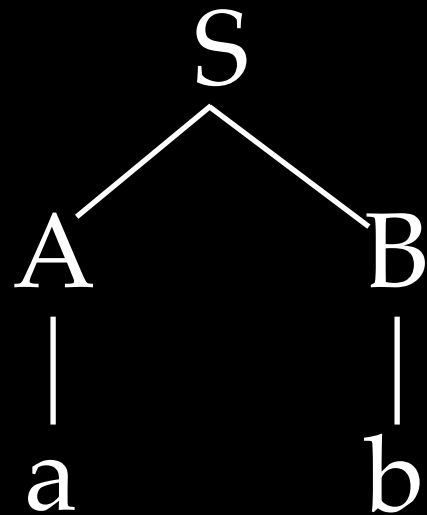
# Context-Free Grammar

$S \rightarrow AB$

$S \rightarrow ASB$

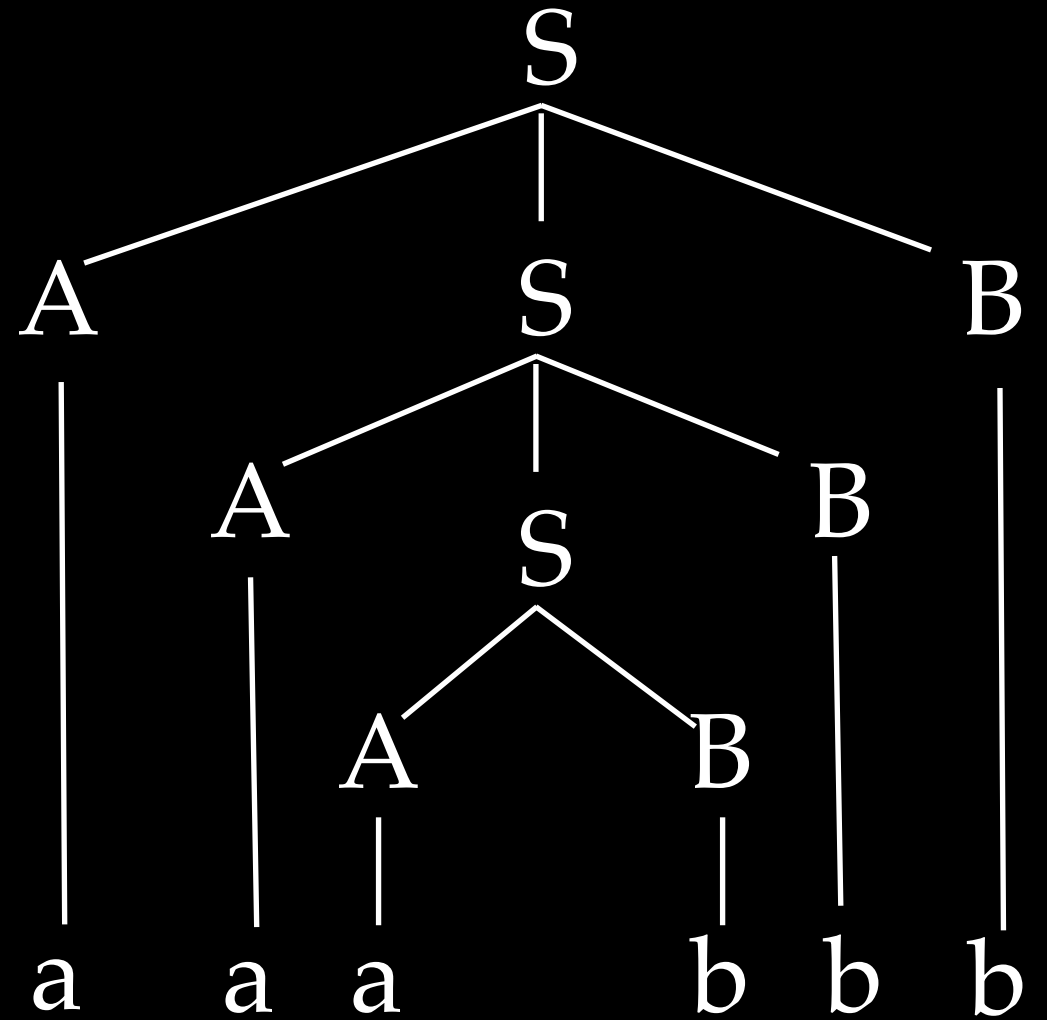
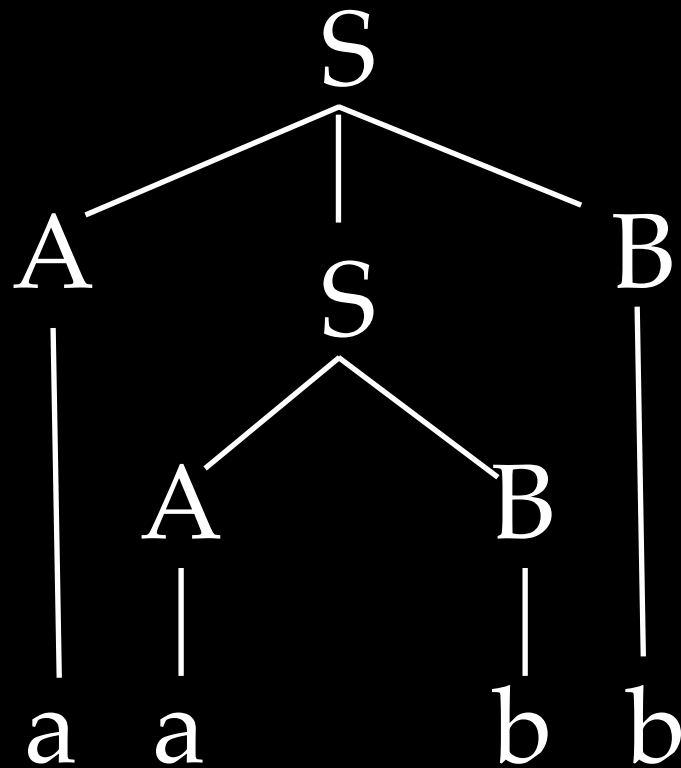
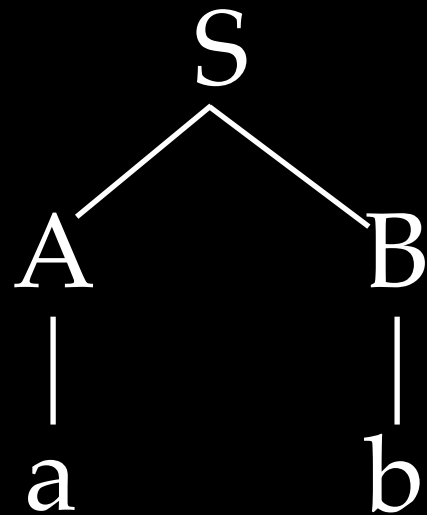
$A \rightarrow a$

$B \rightarrow b$



# Context-Free Grammar

$S \rightarrow AB$   
 $S \rightarrow ASB$   
 $A \rightarrow a$   
 $B \rightarrow b$



$$\mathcal{L}_4 = \{ab, aabb, aaabbb, \dots\} = \forall_{n \in [1, \text{inf})} a^n b^n$$

# Context-Free vs. Regular

# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages

# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages
- Composition of languages:

# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages
- Composition of languages:
  - Regular  $\cap$  Regular = Regular

# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages
- Composition of languages:
  - Regular  $\cap$  Regular = Regular
  - Regular  $\cap$  Context-free = Context-free

# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages
- Composition of languages:
  - Regular  $\cap$  Regular = Regular
  - Regular  $\cap$  Context-free = Context-free

$$A \rightarrow BC \in \mathcal{G}_{CFL}$$



# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages
- Composition of languages:
  - Regular  $\cap$  Regular = Regular
  - Regular  $\cap$  Context-free = Context-free

$$A \rightarrow BC \in \mathcal{G}_{CFL} \qquad s, r, t \in \text{states}(\mathcal{G}_{RL})$$

# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages
- Composition of languages:
  - Regular  $\cap$  Regular = Regular
  - Regular  $\cap$  Context-free = Context-free

$$A \rightarrow BC \in \mathcal{G}_{CFL} \qquad s, r, t \in \text{states}(\mathcal{G}_{RL})$$

$${}_sA_t \rightarrow {}_sB_r{}_rC_t \in \mathcal{G}_{CFL} \cap \mathcal{G}_{RL}$$

# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages
- Composition of languages:
  - Regular  $\cap$  Regular = Regular
  - Regular  $\cap$  Context-free = Context-free

$$A \rightarrow BC \in \mathcal{G}_{CFL} \qquad s, r, t \in \text{states}(\mathcal{G}_{RL})$$

$${}_s A_t \rightarrow {}_s B_r r C_t \in \mathcal{G}_{CFL} \cap \mathcal{G}_{RL}$$

Bar-Hillel 1964

# Context-Free vs. Regular

- Regular languages  $\subset$  Context-free languages
- Composition of languages:
  - Regular  $\cap$  Regular = Regular
  - Regular  $\cap$  Context-free = Context-free
  - Context-free  $\cap$  Context-free = Undecidable

$$A \rightarrow BC \in \mathcal{G}_{CFL} \qquad s, r, t \in \text{states}(\mathcal{G}_{RL})$$

$${}_sA_t \rightarrow {}_sB_r r C_t \in \mathcal{G}_{CFL} \cap \mathcal{G}_{RL}$$

Bar-Hillel 1964

# *Synchronous* Context-Free Grammar

$S \rightarrow NP VP$

$NP \rightarrow \text{watashi wa}$

$NP \rightarrow \text{hako wo}$

$VP \rightarrow NP V$

$V \rightarrow \text{akemasu}$

# *Synchronous* Context-Free Grammar

$S \rightarrow NP VP$

$NP \rightarrow watashi wa$

$NP \rightarrow hako wo$

$VP \rightarrow NP V$

$V \rightarrow akemasu$

$S \rightarrow NP VP$

$NP \rightarrow I$

$NP \rightarrow the\ box$

$VP \rightarrow V NP$

$V \rightarrow open$

# *Synchronous* Context-Free Grammar

$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

$NP \rightarrow watashi wa / I$

$NP \rightarrow hako wo / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow akemasu / \text{open}$

# *Synchronous* Context-Free Grammar

$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

$NP \rightarrow watashi wa / I$

$NP \rightarrow hako wo / the box$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow akemasu / open$



# *Synchronous* Context-Free Grammar

S

S

$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

$NP \rightarrow watashi wa / I$

$NP \rightarrow hako wo / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow akemasu / \text{open}$

# *Synchronous* Context-Free Grammar

S ..... S

$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

$NP \rightarrow watashi wa / I$

$NP \rightarrow hako wo / the box$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow akemasu / open$

# *Synchronous* Context-Free Grammar

$S \dots\dots\dots S$

$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

$NP \rightarrow watashi wa / I$

$NP \rightarrow hako wo / the box$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow akemasu / open$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

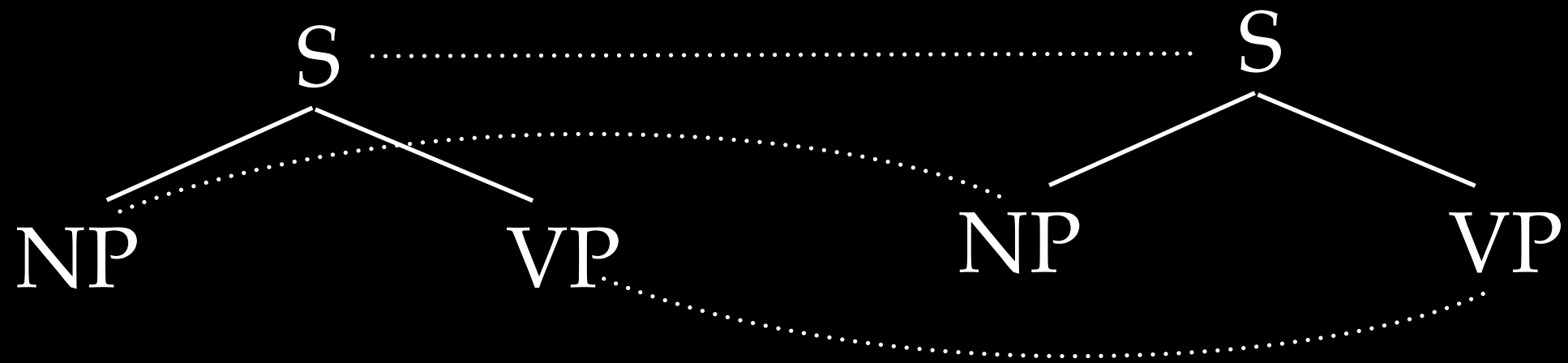
$NP \rightarrow \text{watashi wa} / \text{I}$

$NP \rightarrow \text{hako wo} / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

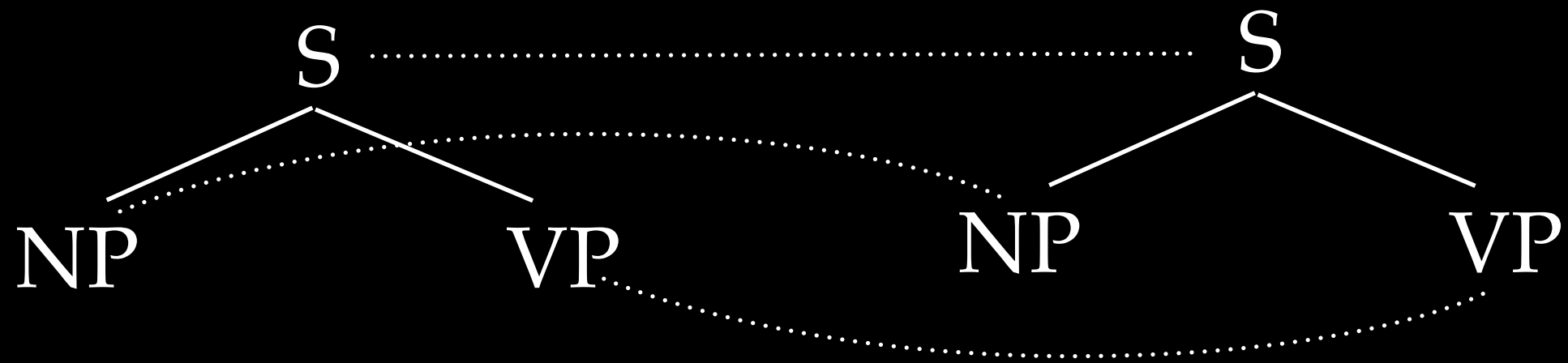
$NP \rightarrow watashi wa / I$

$NP \rightarrow hako wo / the box$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow akemasu / open$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

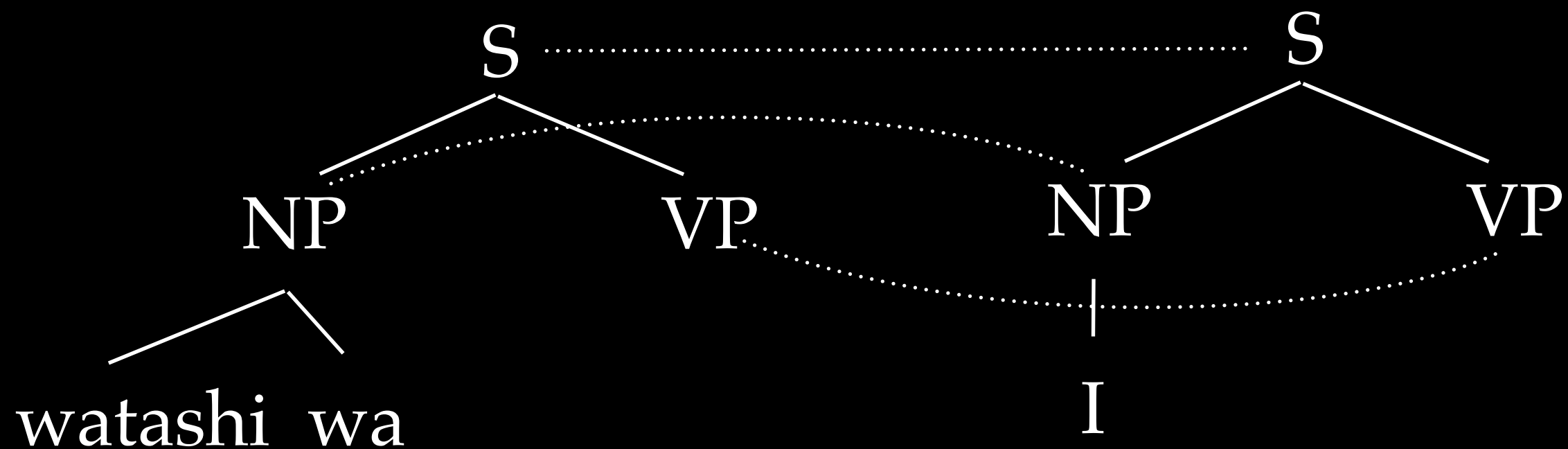
$NP \rightarrow \text{watashi wa} / \text{I}$

$NP \rightarrow \text{hako wo} / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

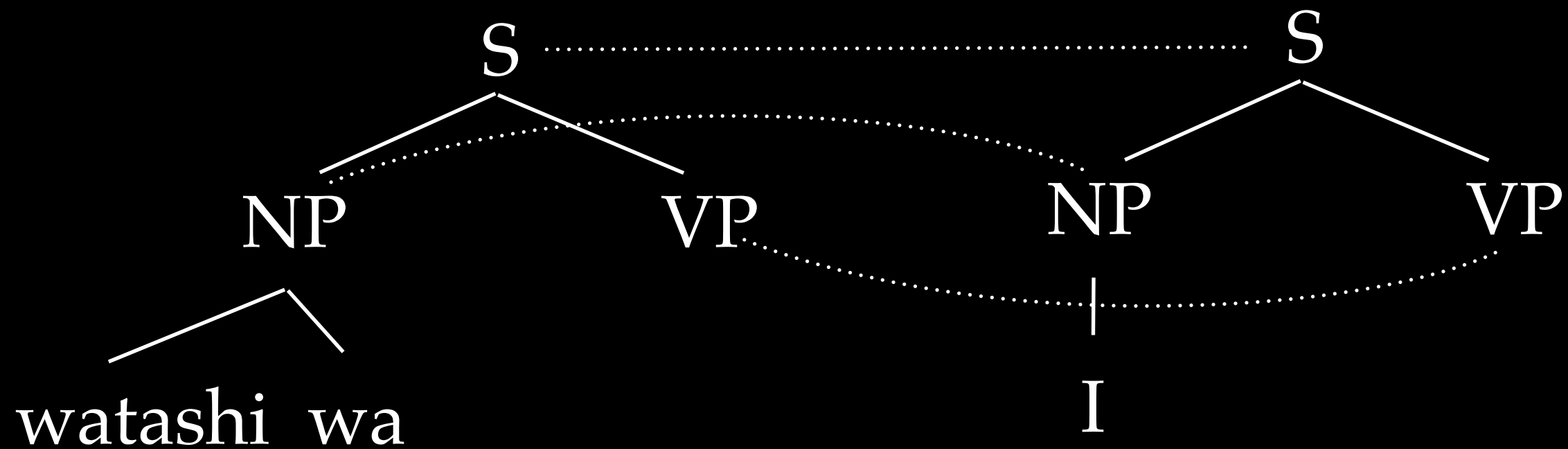
$NP \rightarrow \text{watashi wa} / I$

$NP \rightarrow \text{hako wo} / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

$NP \rightarrow \text{watashi wa} / I$

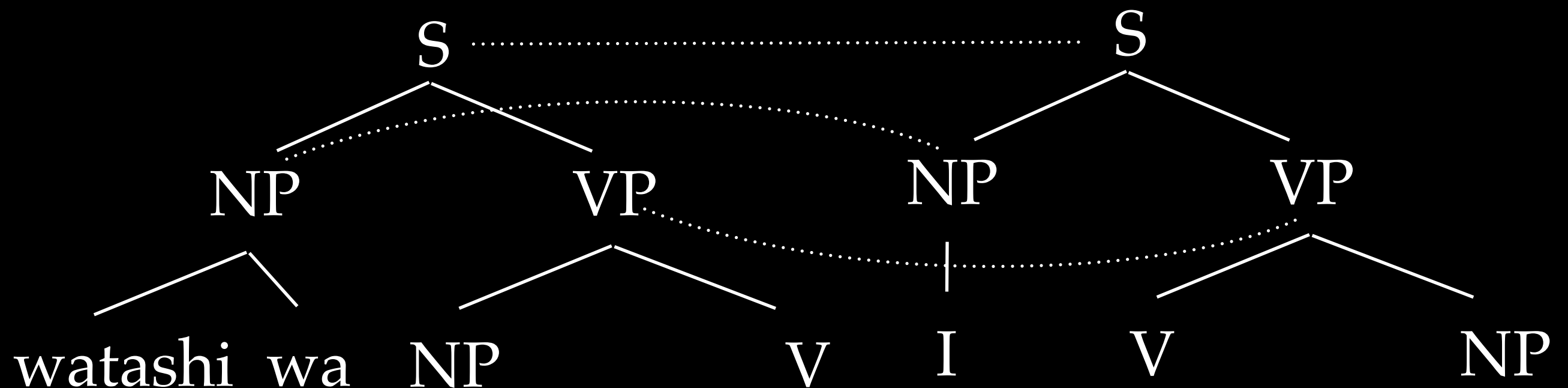
$NP \rightarrow \text{hako wo} / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$



# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

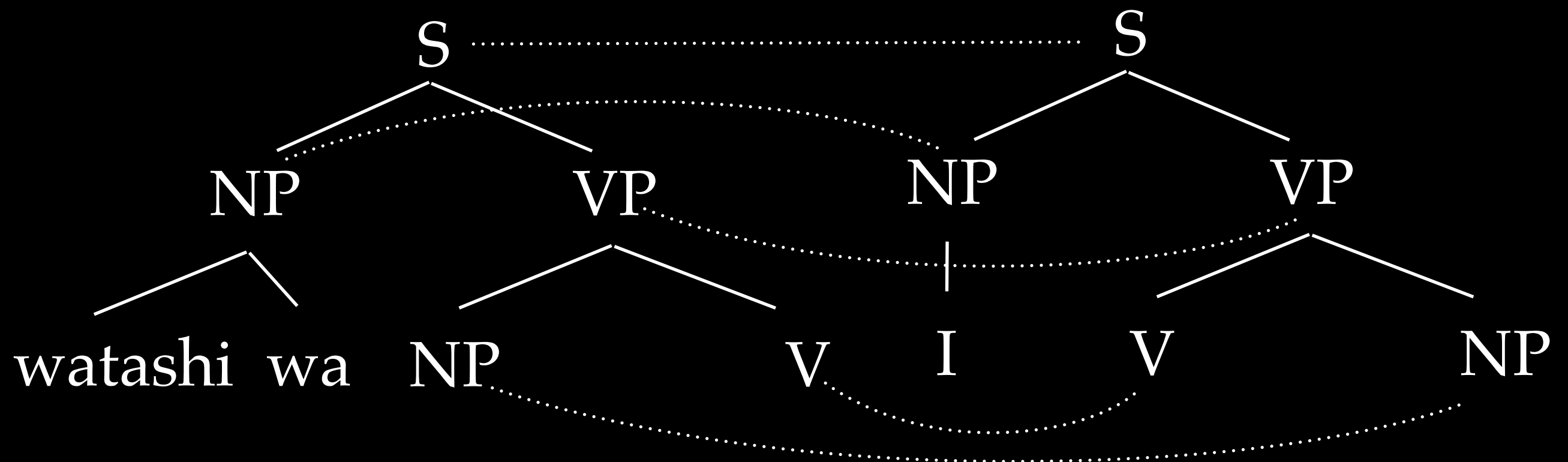
$NP \rightarrow \text{watashi wa} / I$

$NP \rightarrow \text{hako wo} / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

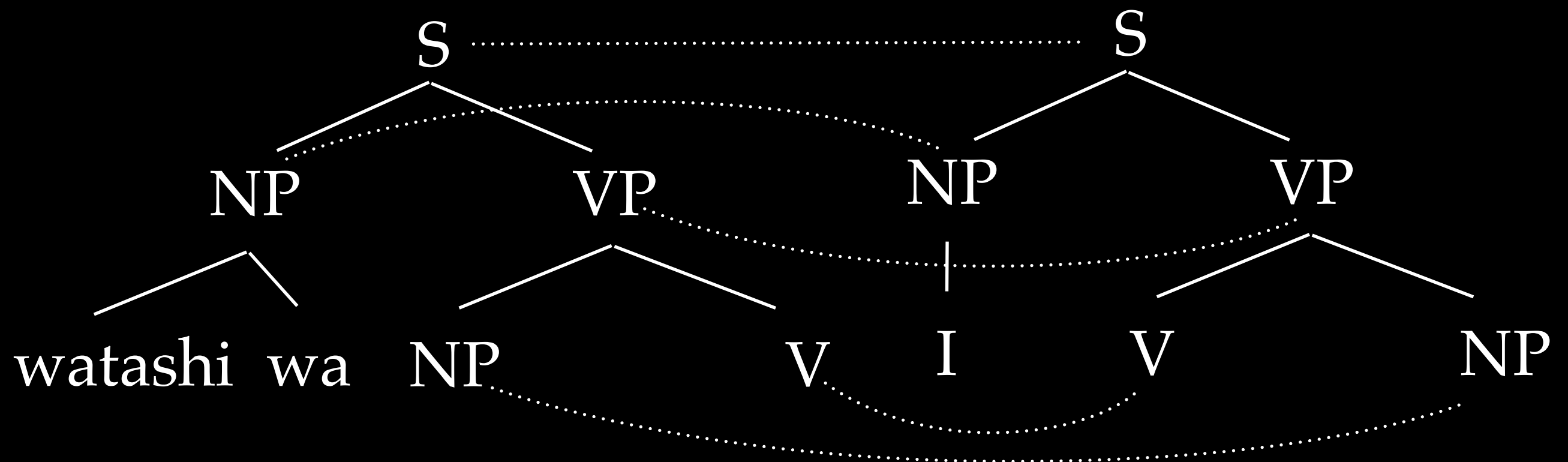
$NP \rightarrow \text{watashi wa} / I$

$NP \rightarrow \text{hako wo} / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

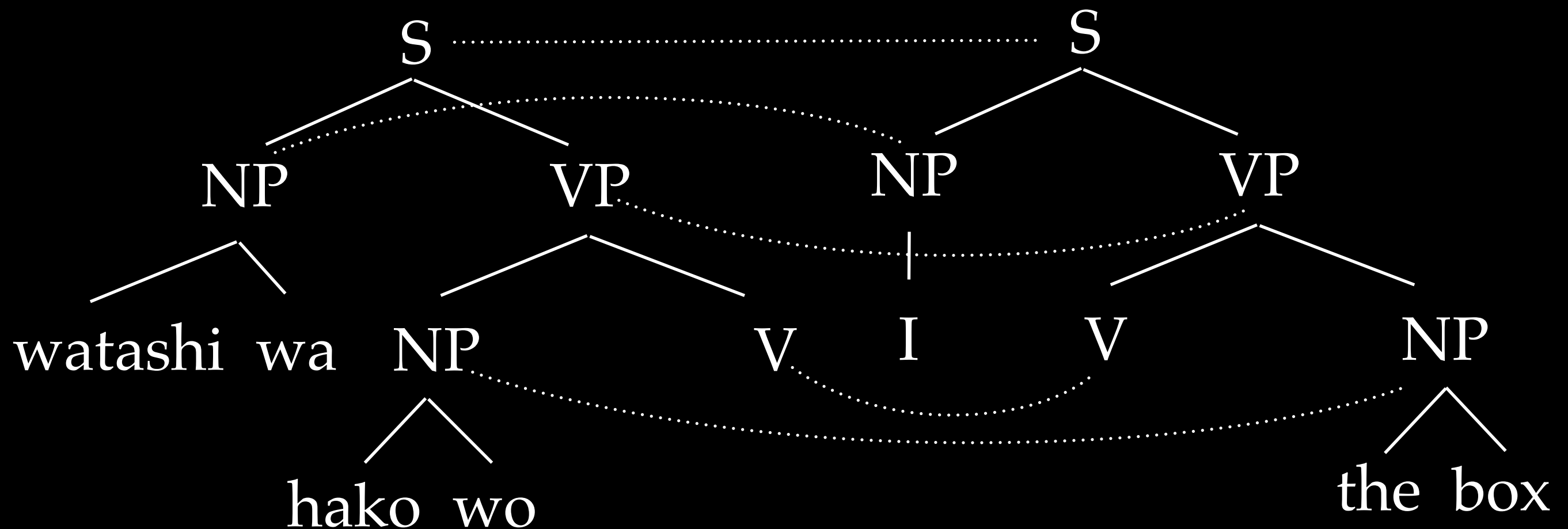
$NP \rightarrow \text{watashi wa} / I$

$NP \rightarrow \text{hako wo} / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

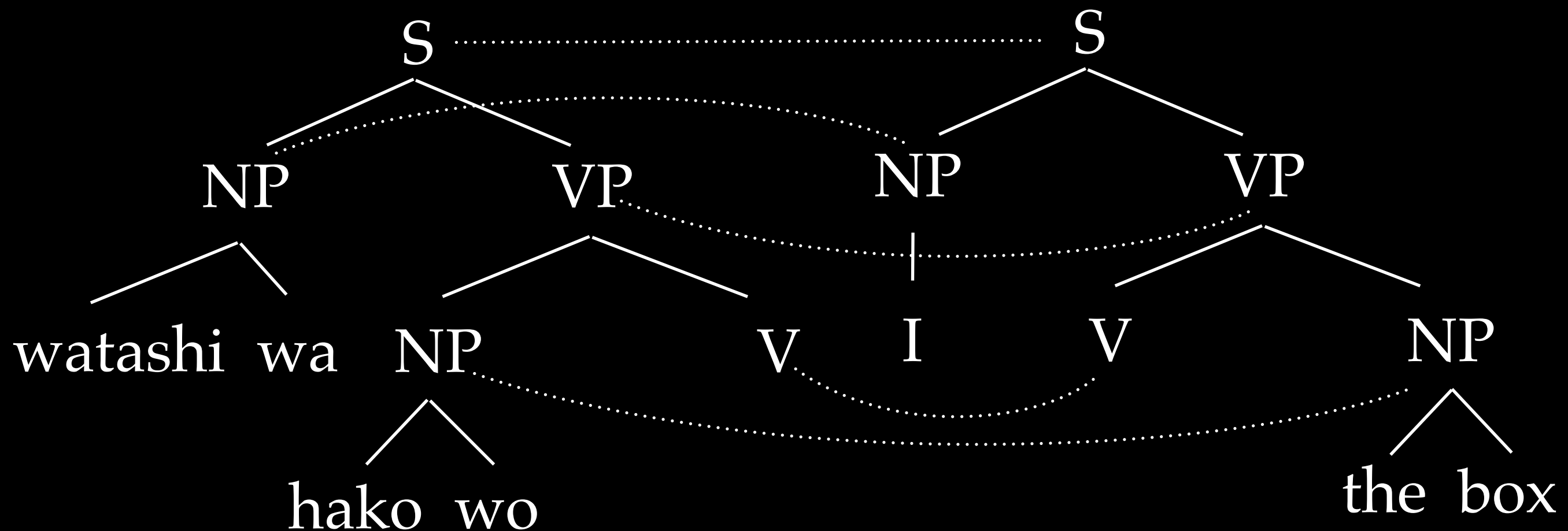
$NP \rightarrow watashi wa / I$

$NP \rightarrow hako wo / the box$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow akemasu / open$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

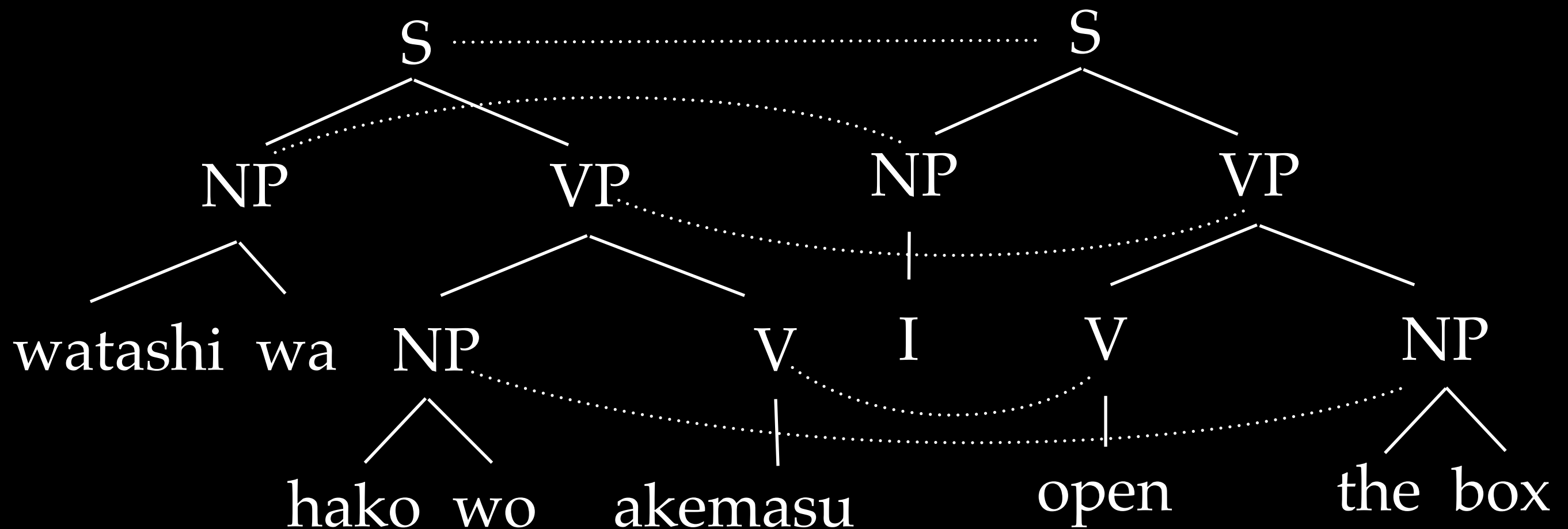
$NP \rightarrow \text{watashi wa} / I$

$NP \rightarrow \text{hako wo} / \text{the box}$

$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$

# *Synchronous* Context-Free Grammar



$S \rightarrow NP_1 VP_2 / NP_1 VP_2$

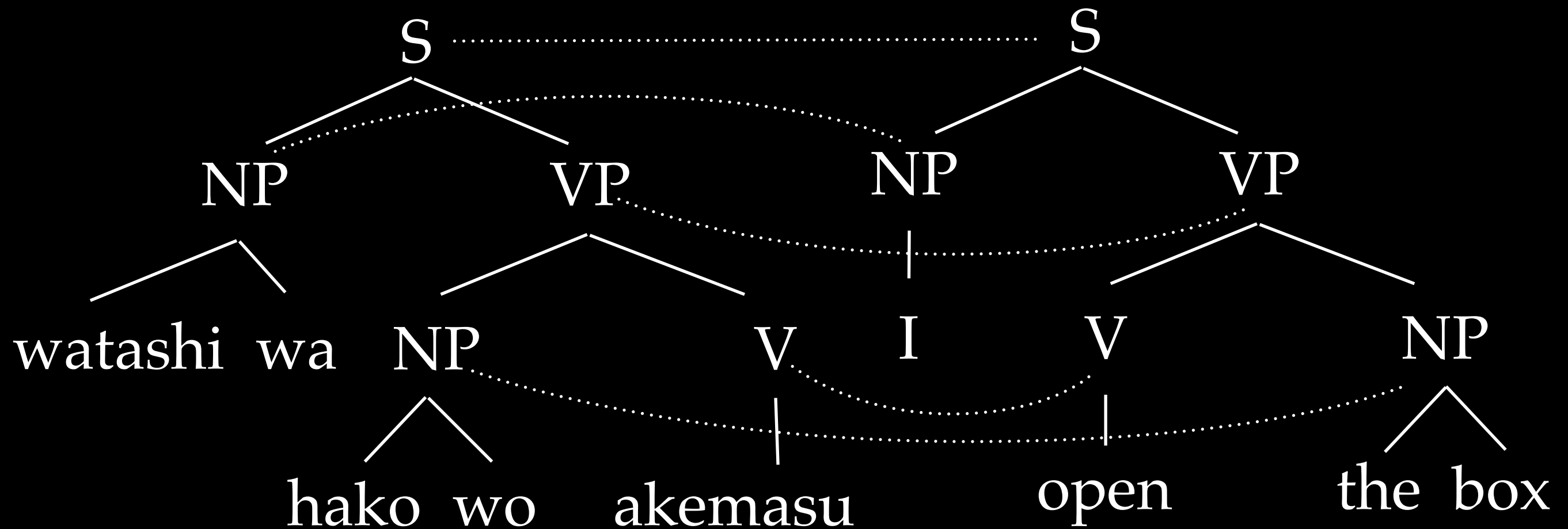
$NP \rightarrow \text{watashi wa} / I$

$NP \rightarrow \text{hako wo} / \text{the box}$

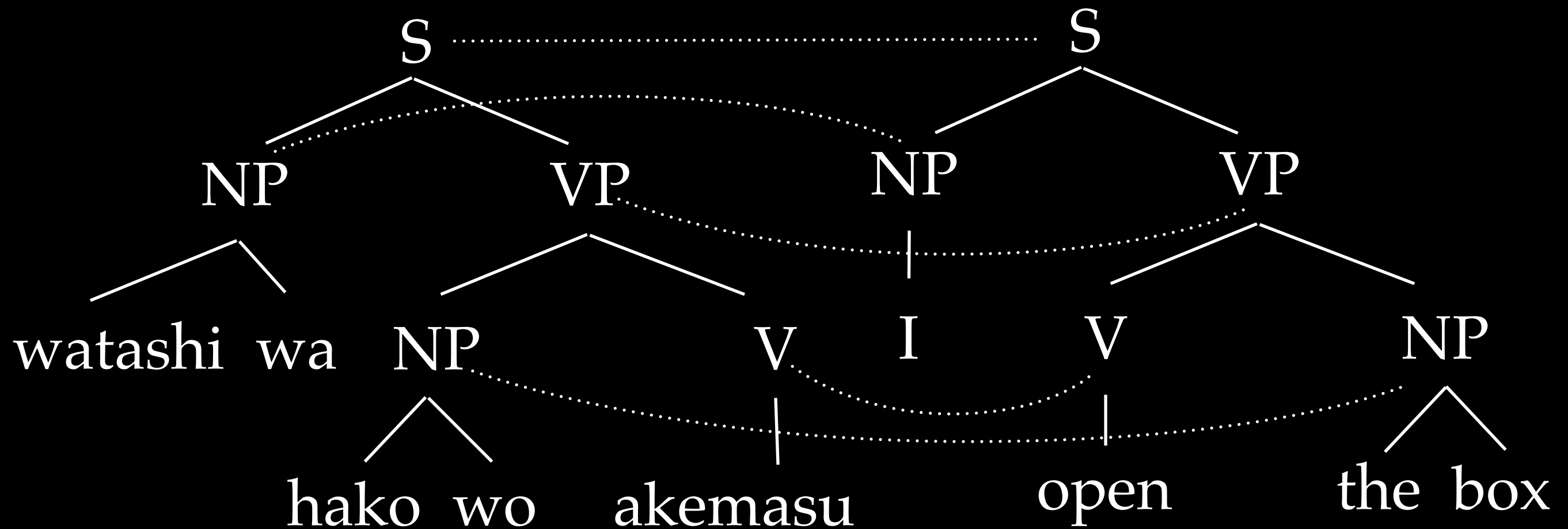
$VP \rightarrow NP_1 V_2 / V_2 NP_1$

$V \rightarrow \text{akemasu} / \text{open}$

# *Synchronous* Context-Free Grammar



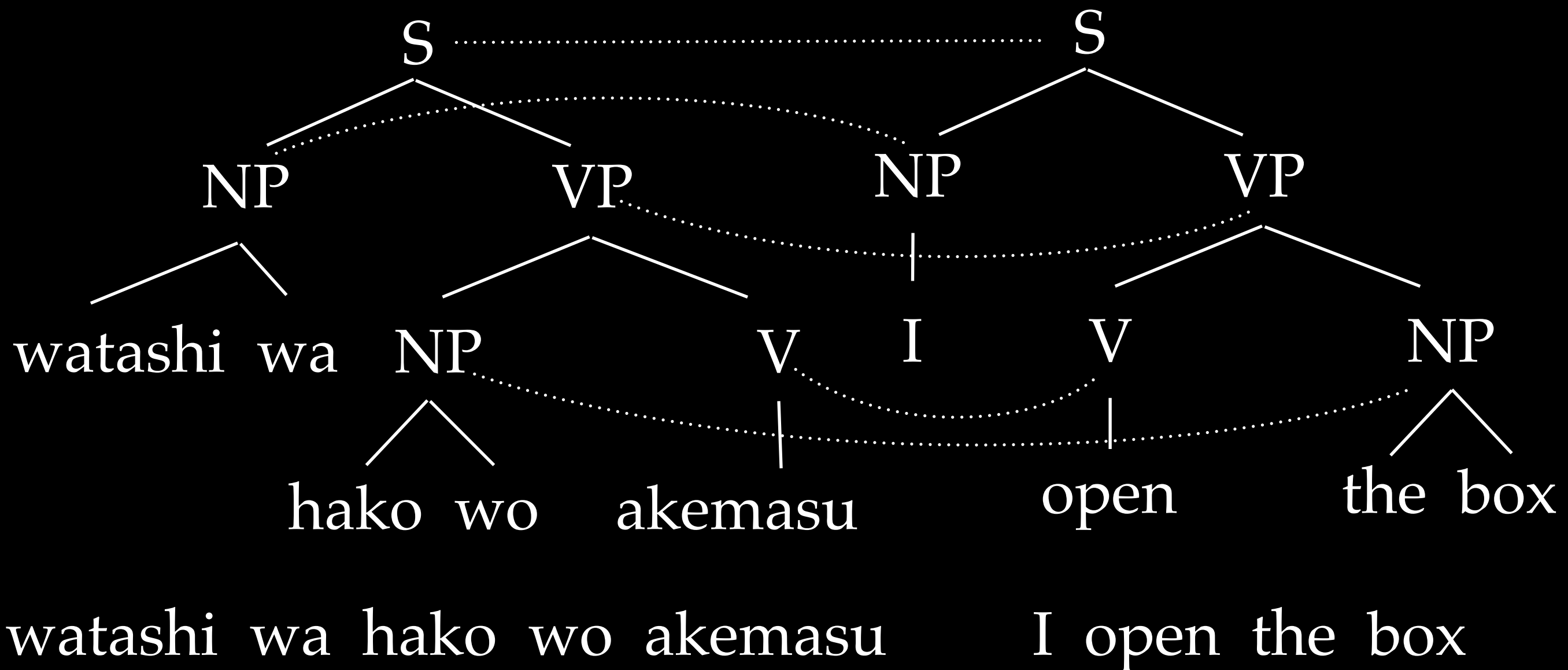
# *Synchronous* Context-Free Grammar



watashi wa hako wo akemasu



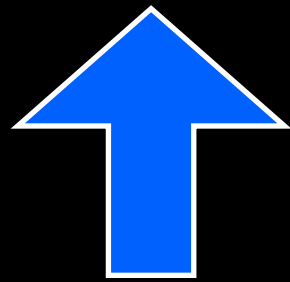
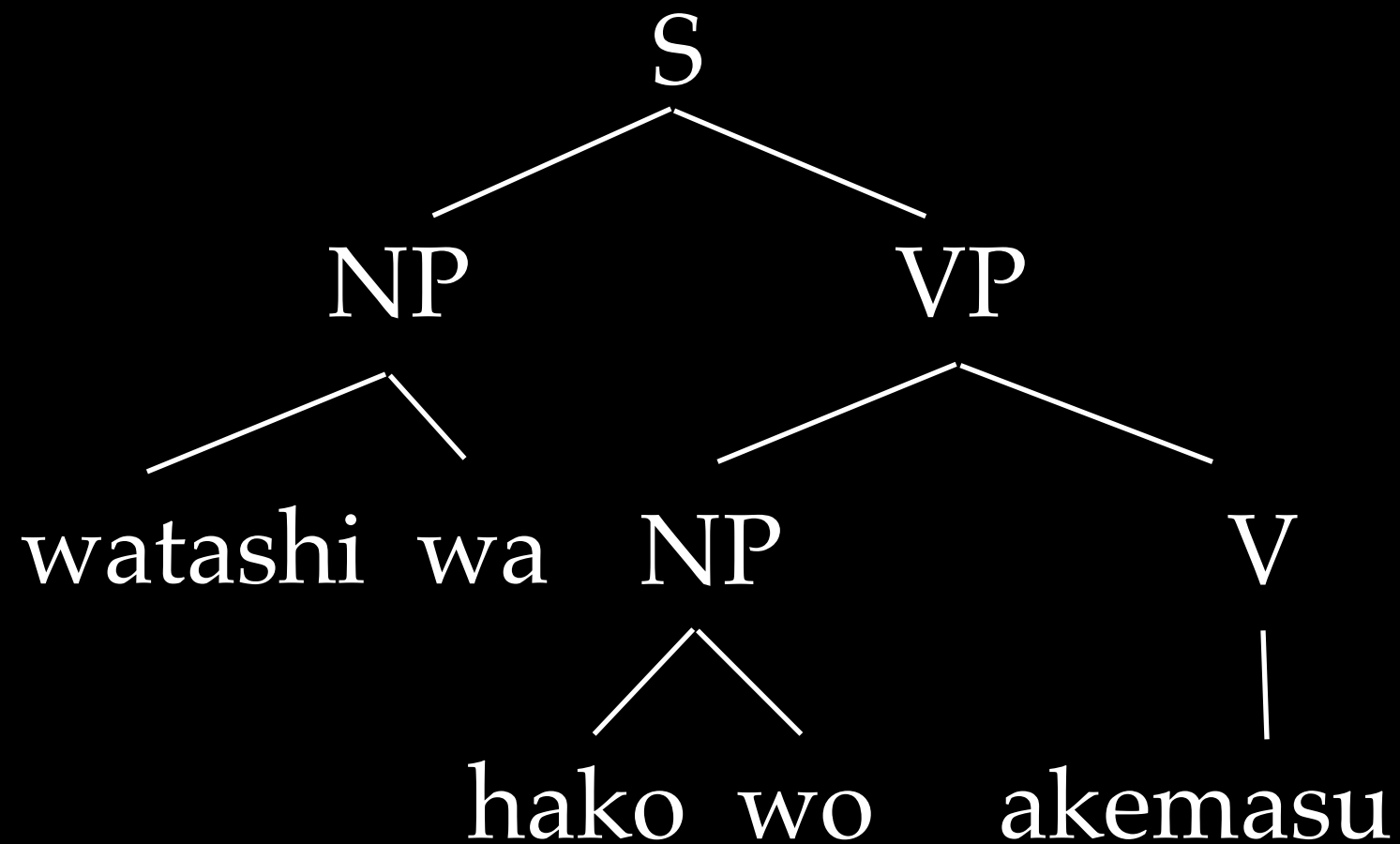
# *Synchronous* Context-Free Grammar



# Translation is Parsing

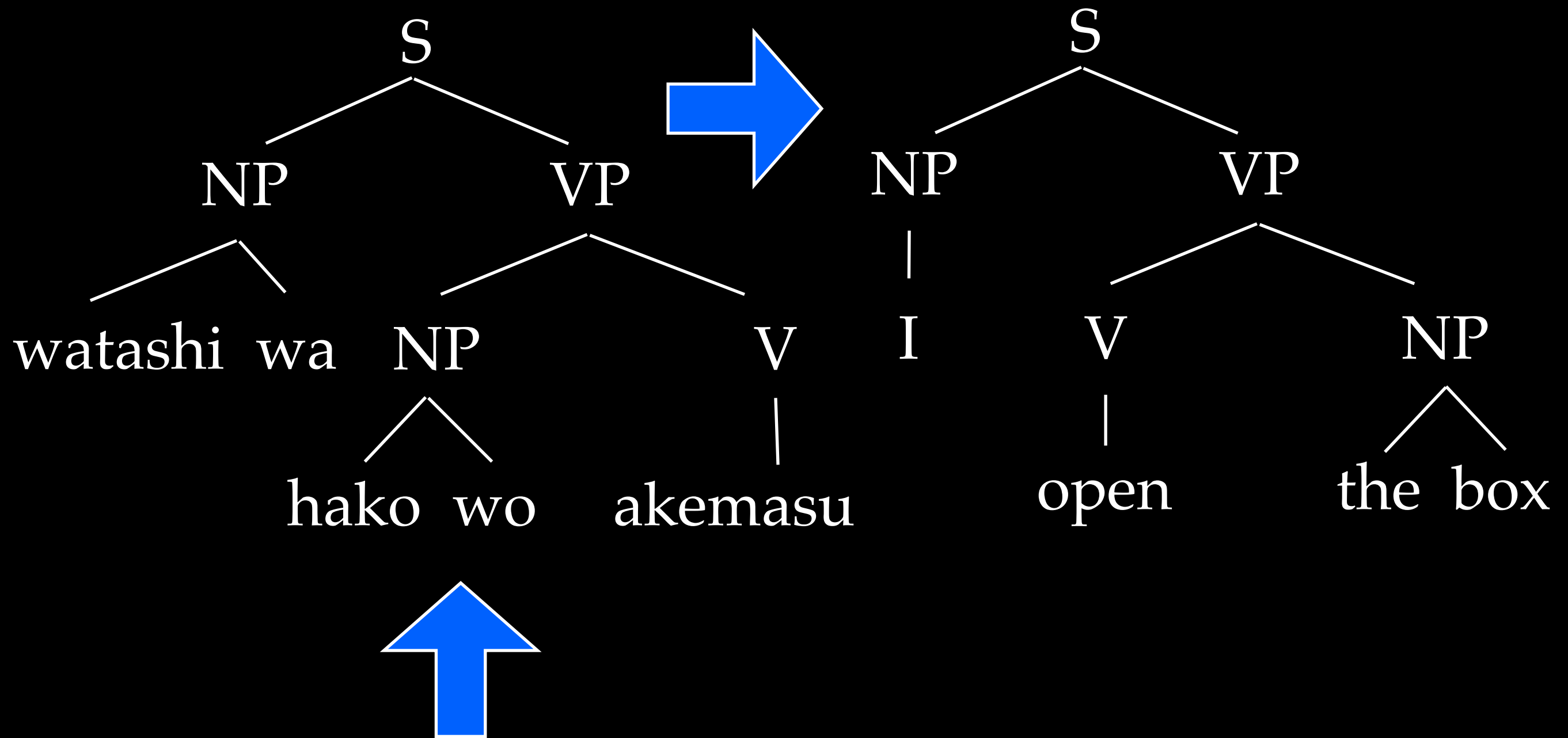
watashi wa hako wo akemasu

# Translation is Parsing



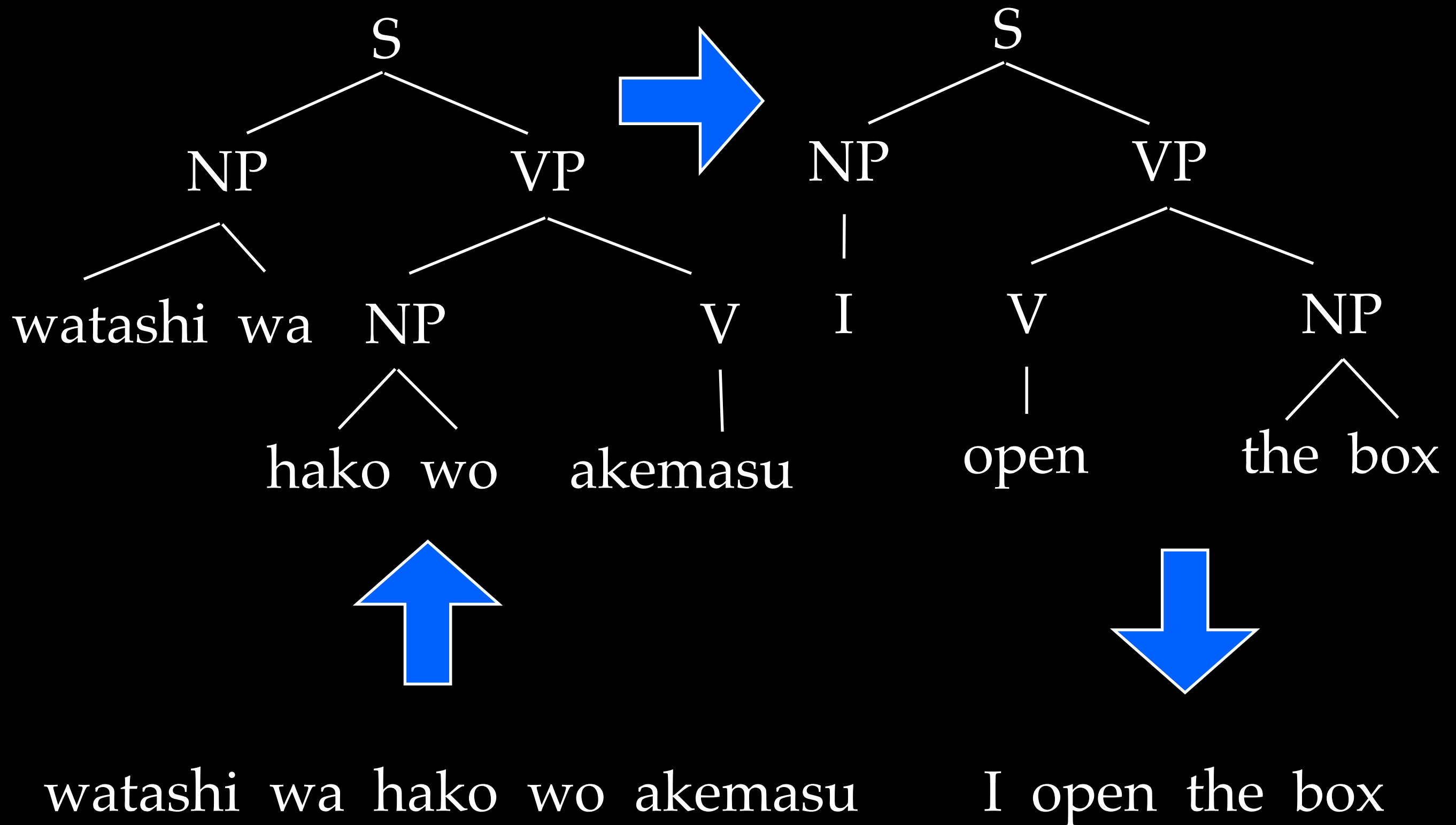
watashi wa hako wo akemasu

# Translation is Parsing



watashi wa hako wo akemasu

# Translation is Parsing



# Translation is Parsing

# Translation is Parsing

- How many parses of a sentence are there?

# Translation is Parsing

- How many parses of a sentence are there?
  - For binary grammar: Catalan number.



# Translation is Parsing

- How many parses of a sentence are there?
  - For binary grammar: Catalan number.  $O(\frac{(2n)!}{(n+1)!n!})$

# Translation is Parsing

- How many parses of a sentence are there?
  - For binary grammar: Catalan number.  $O(\frac{(2n)!}{(n+1)!n!})$
- Dynamic programming to the rescue!

# Parsing

# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

I<sub>1</sub> saw<sub>2</sub> her<sub>3</sub> duck<sub>4</sub>

# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

I<sub>1</sub>   saw<sub>2</sub>   her<sub>3</sub>   duck<sub>4</sub>

# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

I<sub>1</sub> saw<sub>2</sub> her<sub>3</sub> duck<sub>4</sub>

# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

I<sub>1</sub>

saw<sub>2</sub> her<sub>3</sub> duck<sub>4</sub>



# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$PRP_{0,1} \leftarrow (w_1 = I) \wedge (PRP \rightarrow I)$$

I<sub>1</sub>

saw<sub>2</sub> her<sub>3</sub> duck<sub>4</sub>

# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$PRP_{0,1} \leftarrow (w_1 = I) \wedge (PRP \rightarrow I)$$

$PRP_{0,1}$

I<sub>1</sub>

saw<sub>2</sub> her<sub>3</sub> duck<sub>4</sub>

# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$PRP_{0,1}$



$I_1$

saw<sub>2</sub>

her<sub>3</sub>

duck<sub>4</sub>

# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

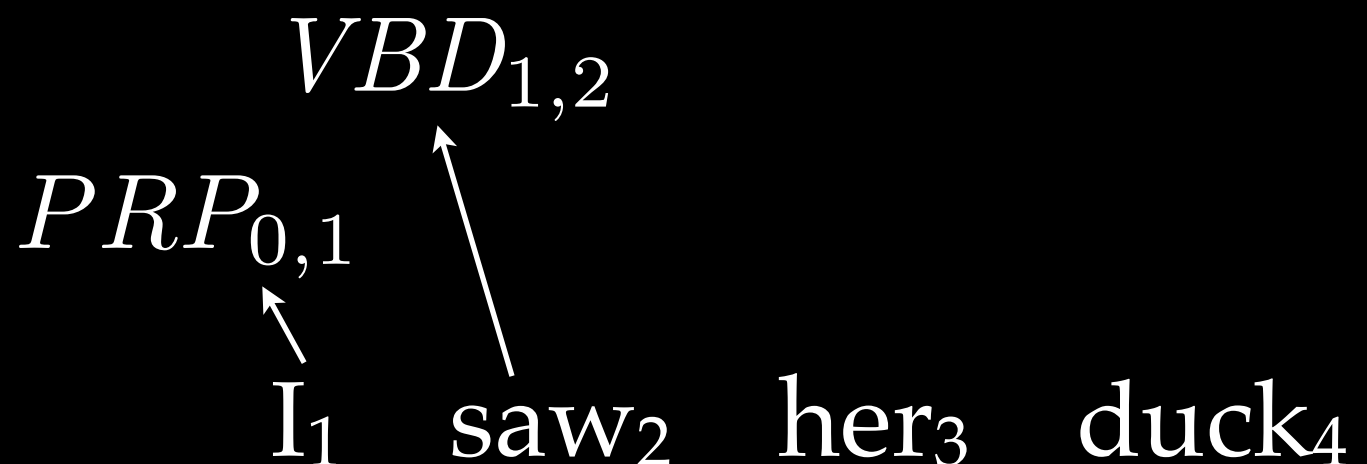
VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$



# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

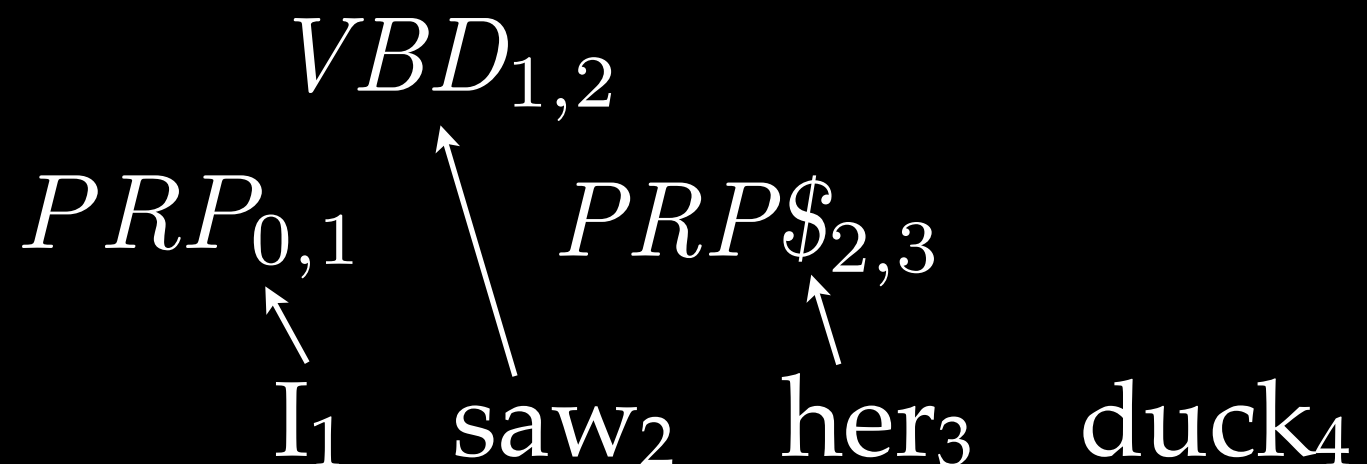
VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$



# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

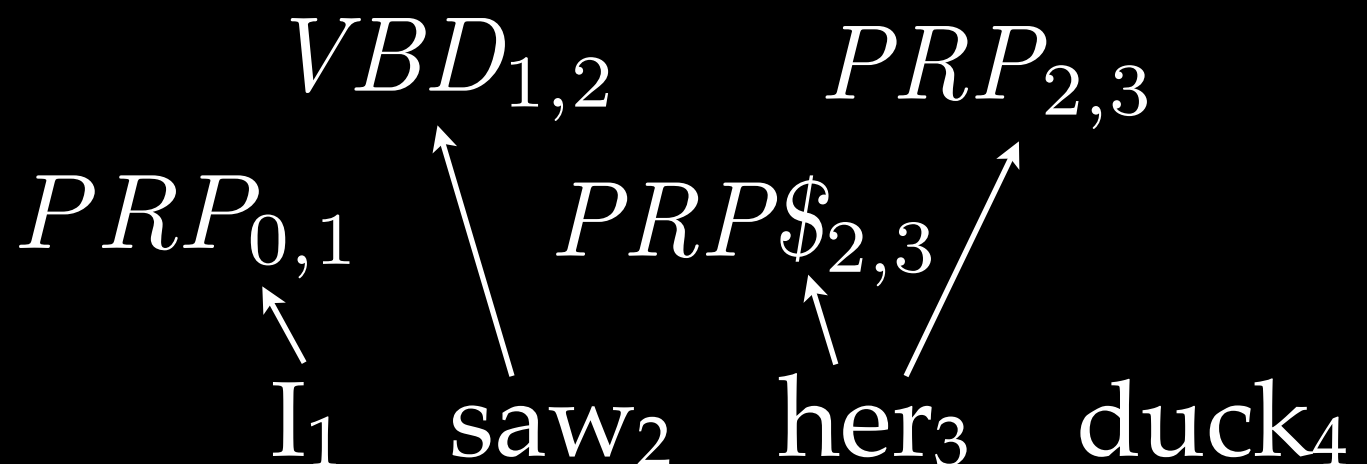
VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$



# Parsing

NN → duck

$$\text{NP} \rightarrow \text{PRP\$ NN}$$
PRP  $\rightarrow$  her
$$\text{PRP} \rightarrow \text{I}$$

PRP\$ → her

$$S \rightarrow \text{PRP VP}$$

SBAR → PRP VB

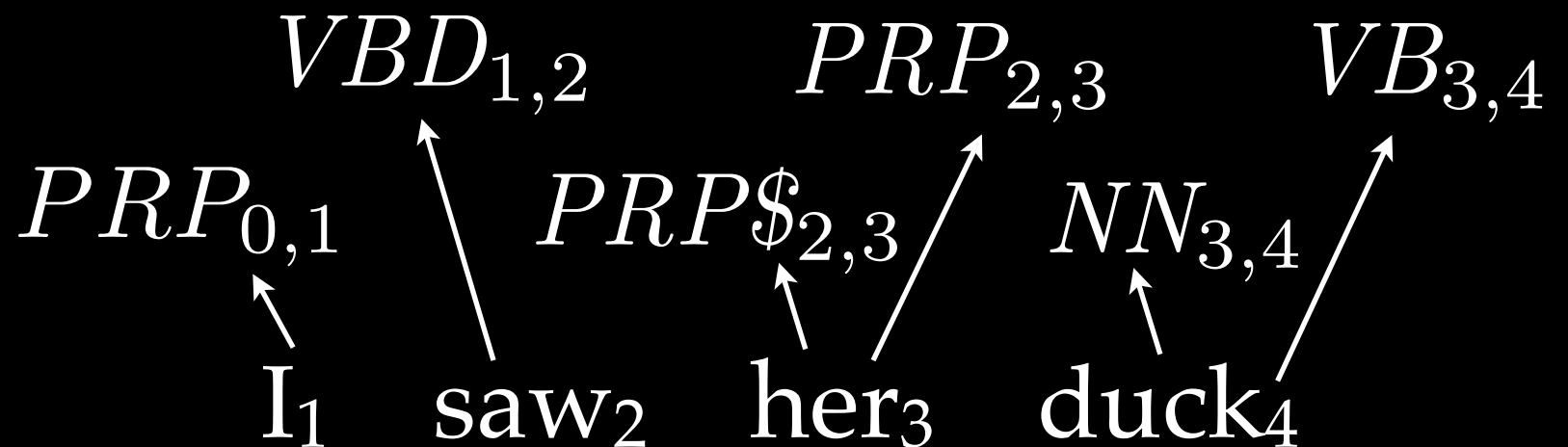
VB  $\rightarrow$  duck

VP → VBD NP

# VP $\rightarrow$ VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$



# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

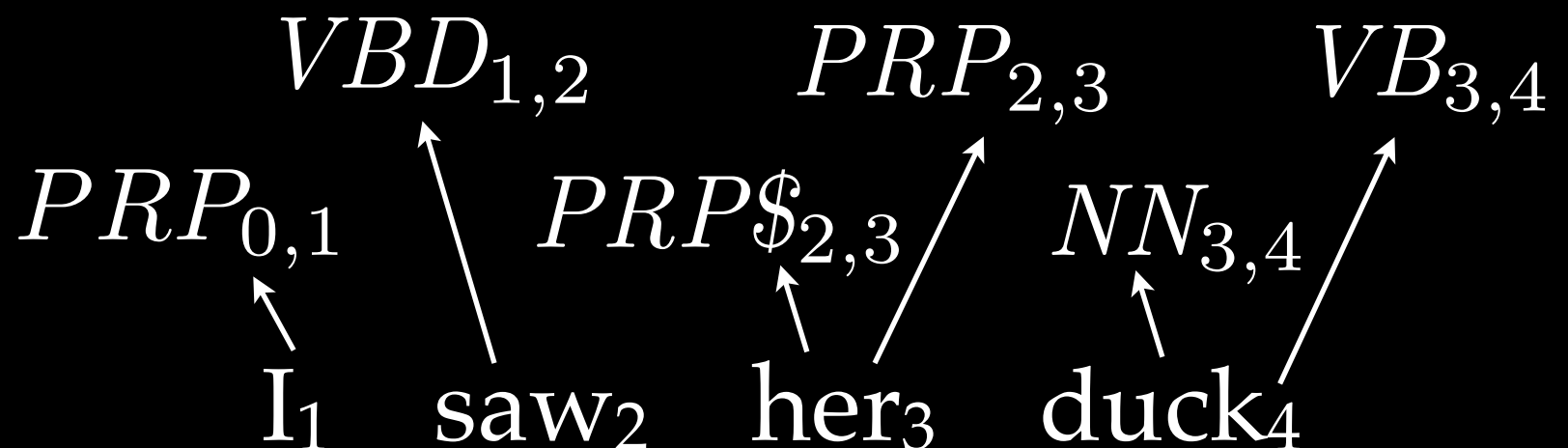
VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$







# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

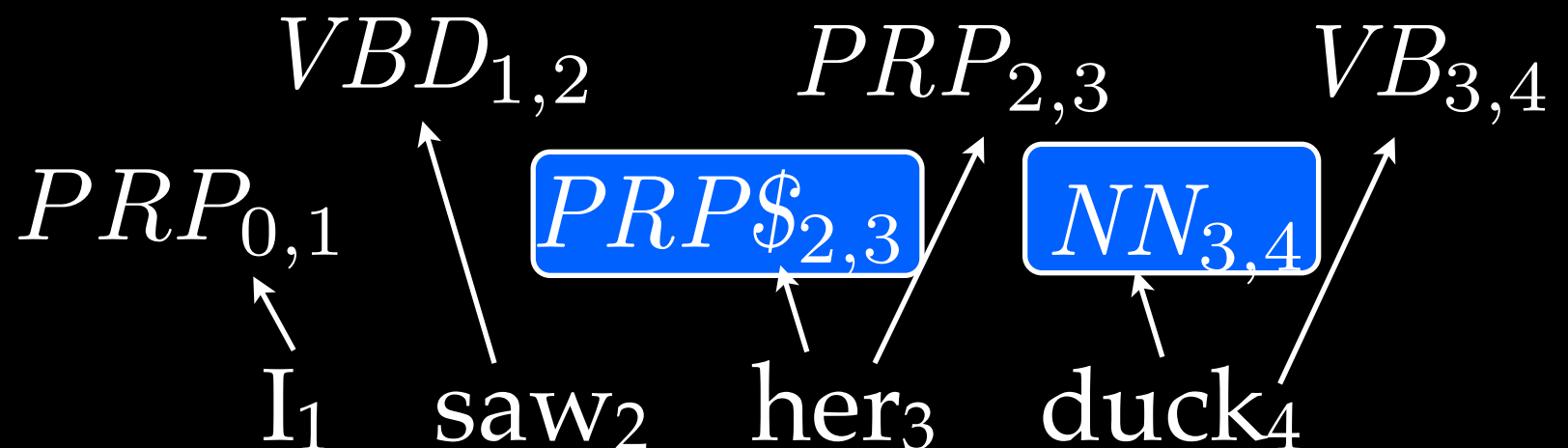
VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$



# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

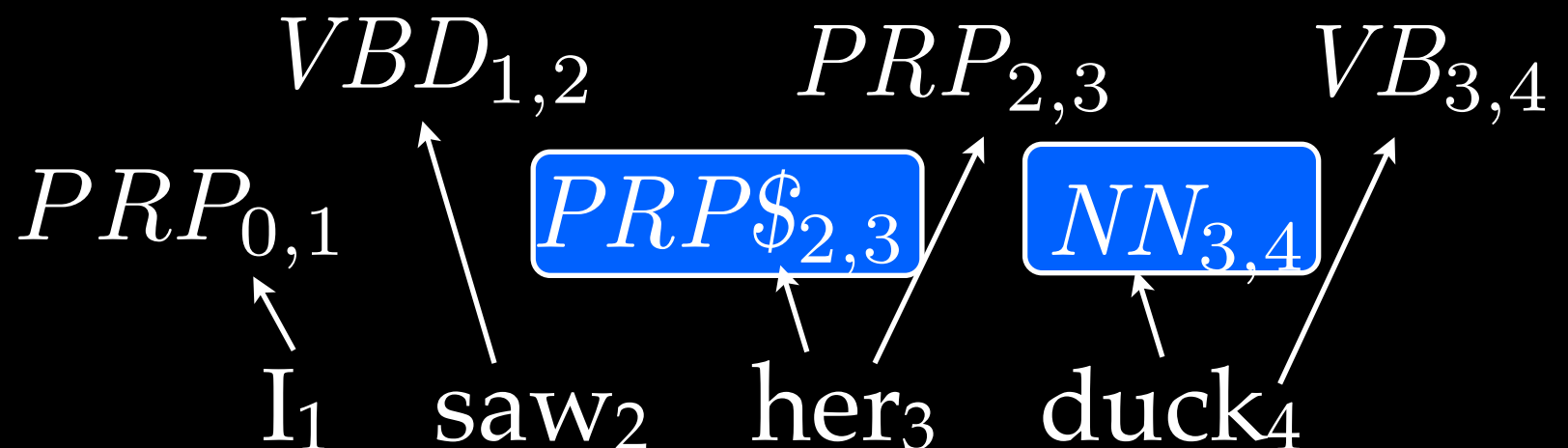
VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$



# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

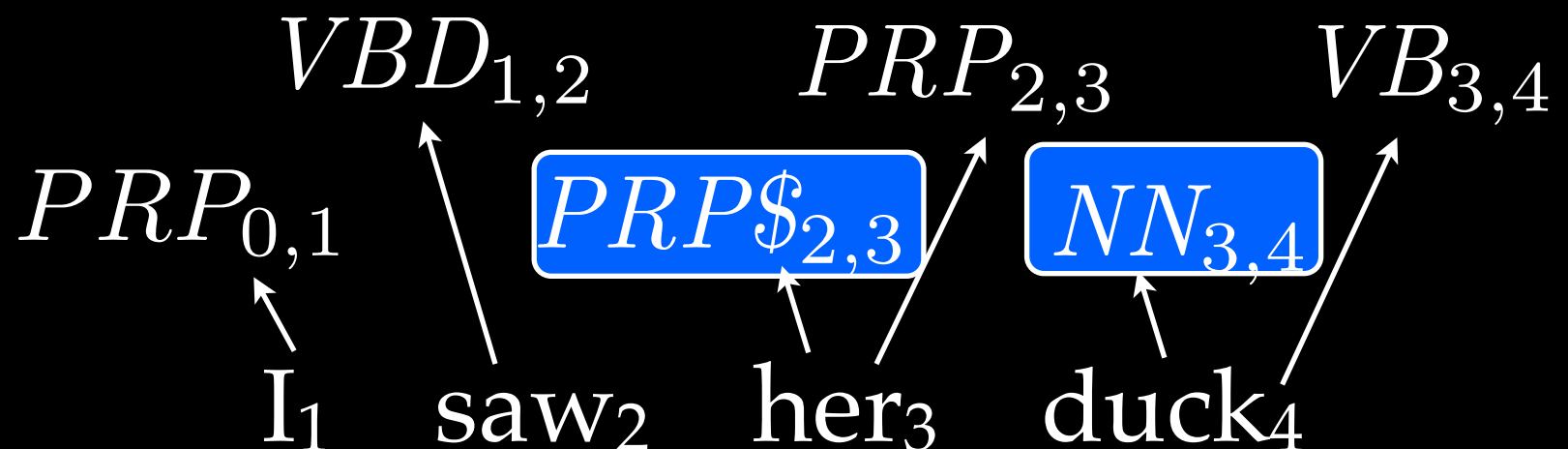
VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$

$$NP_{2,4} \leftarrow PRP\$_{2,3} \wedge NN_{3,4} \wedge (NP \rightarrow PRP\$ NN)$$



# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

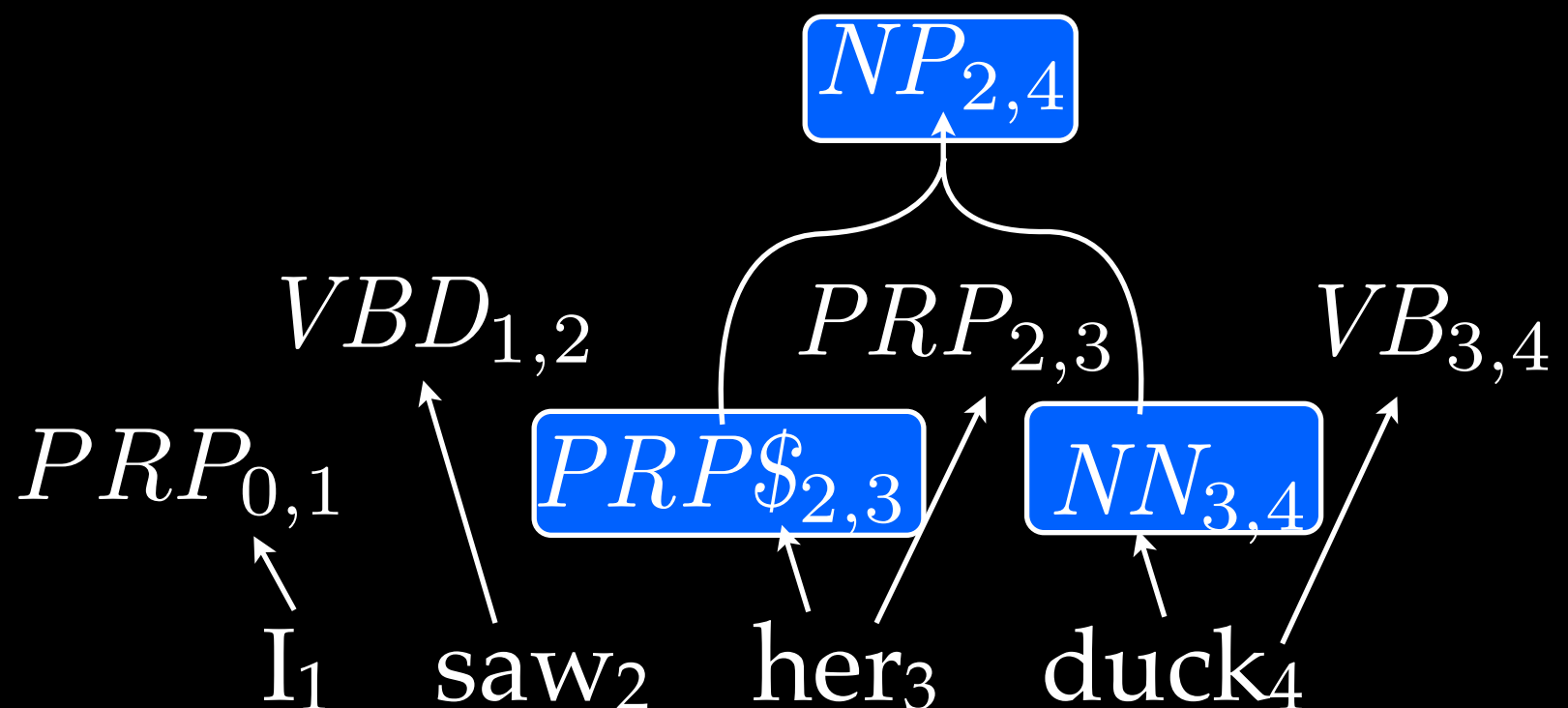
VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$

$$NP_{2,4} \leftarrow PRP\$_{2,3} \wedge NN_{3,4} \wedge (NP \rightarrow PRP\$ NN)$$



# Parsing

$NN \rightarrow \text{duck}$

$NP \rightarrow PRP\$ NN$

$PRP \rightarrow \text{her}$

$PRP \rightarrow I$

$PRP\$ \rightarrow \text{her}$

$S \rightarrow PRP VP$

$SBAR \rightarrow PRP VB$

$VB \rightarrow \text{duck}$

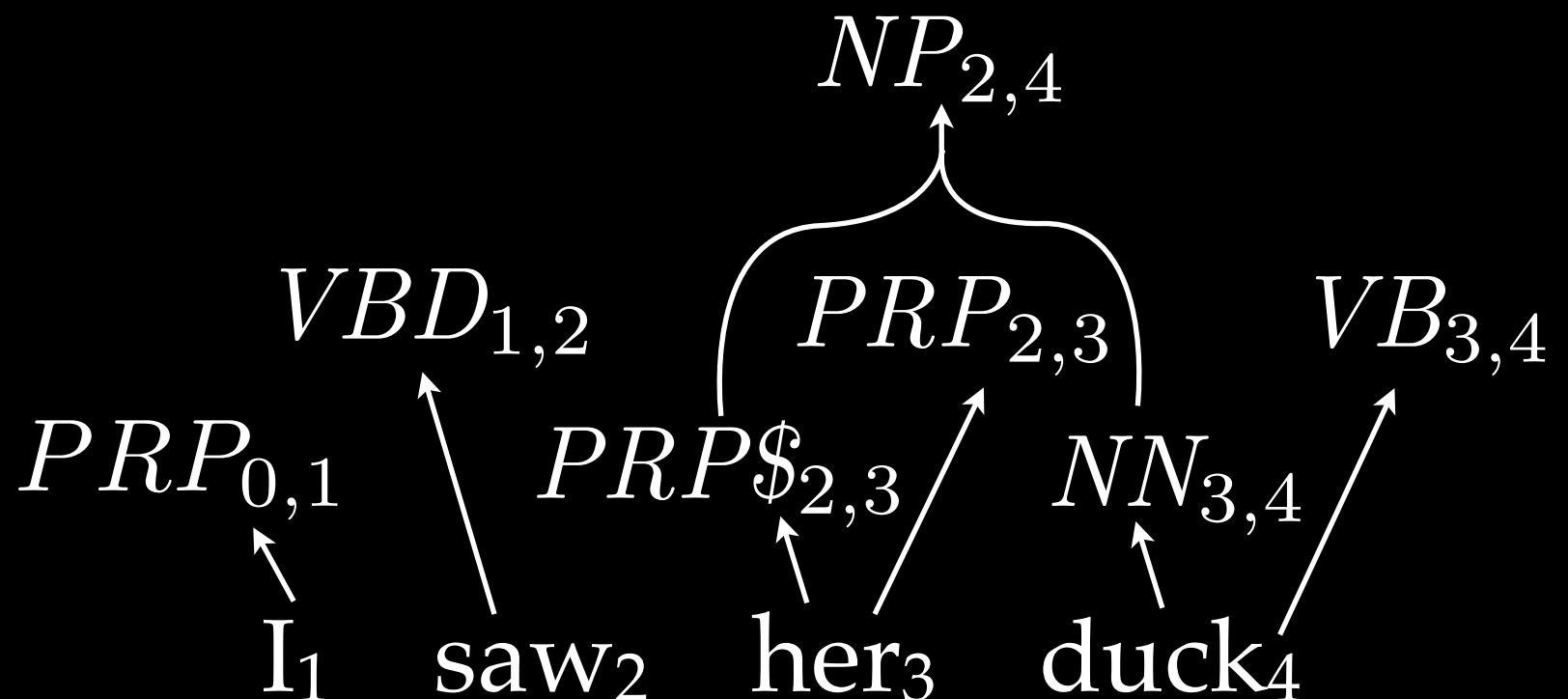
$VP \rightarrow VBD NP$

$VP \rightarrow VBD SBAR$

$VBD \rightarrow \text{saw}$

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$



# Parsing

$NN \rightarrow \text{duck}$

$NP \rightarrow PRP\$ NN$

$PRP \rightarrow \text{her}$

$PRP \rightarrow I$

$PRP\$ \rightarrow \text{her}$

$S \rightarrow PRP VP$

$SBAR \rightarrow PRP VB$

$VB \rightarrow \text{duck}$

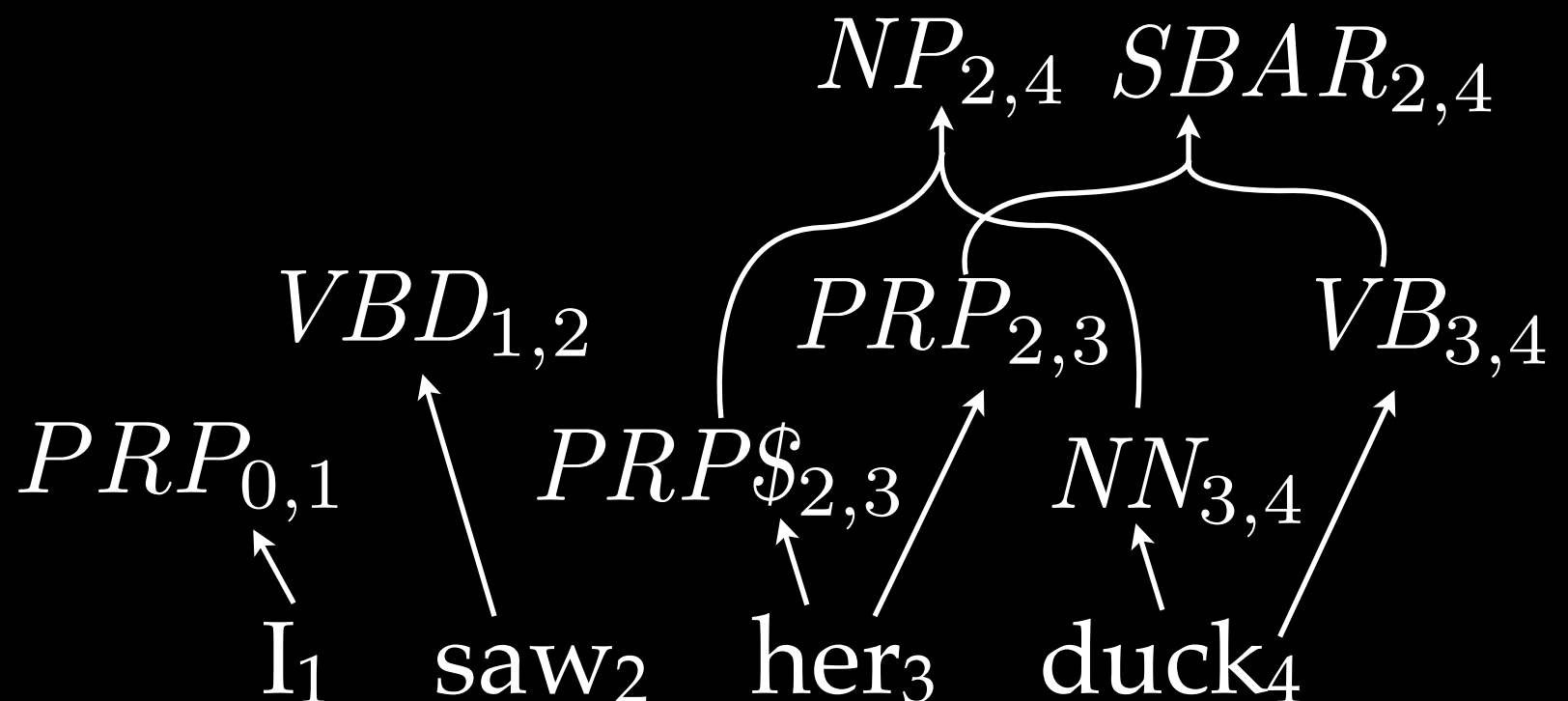
$VP \rightarrow VBD NP$

$VP \rightarrow VBD SBAR$

$VBD \rightarrow \text{saw}$

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$



# Parsing

$NN \rightarrow \text{duck}$

$NP \rightarrow PRP\$ NN$

$PRP \rightarrow \text{her}$

$PRP \rightarrow I$

$PRP\$ \rightarrow \text{her}$

$S \rightarrow PRP VP$

$SBAR \rightarrow PRP VB$

$VB \rightarrow \text{duck}$

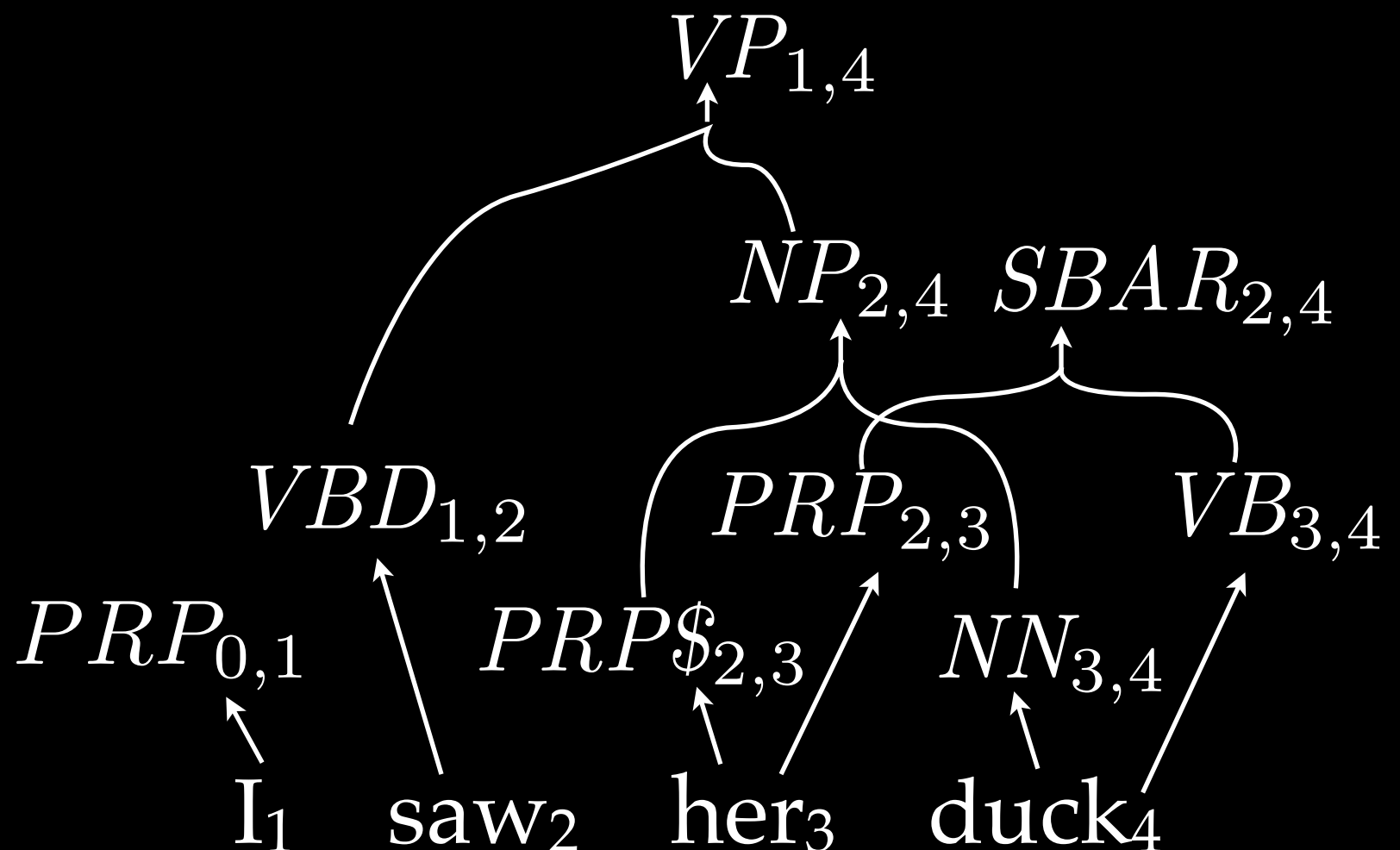
$VP \rightarrow VBD NP$

$VP \rightarrow VBD SBAR$

$VBD \rightarrow \text{saw}$

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$





# Parsing

$NN \rightarrow \text{duck}$

$NP \rightarrow PRP\$ NN$

$PRP \rightarrow \text{her}$

$PRP \rightarrow I$

$PRP\$ \rightarrow \text{her}$

$S \rightarrow PRP VP$

$SBAR \rightarrow PRP VB$

$VB \rightarrow \text{duck}$

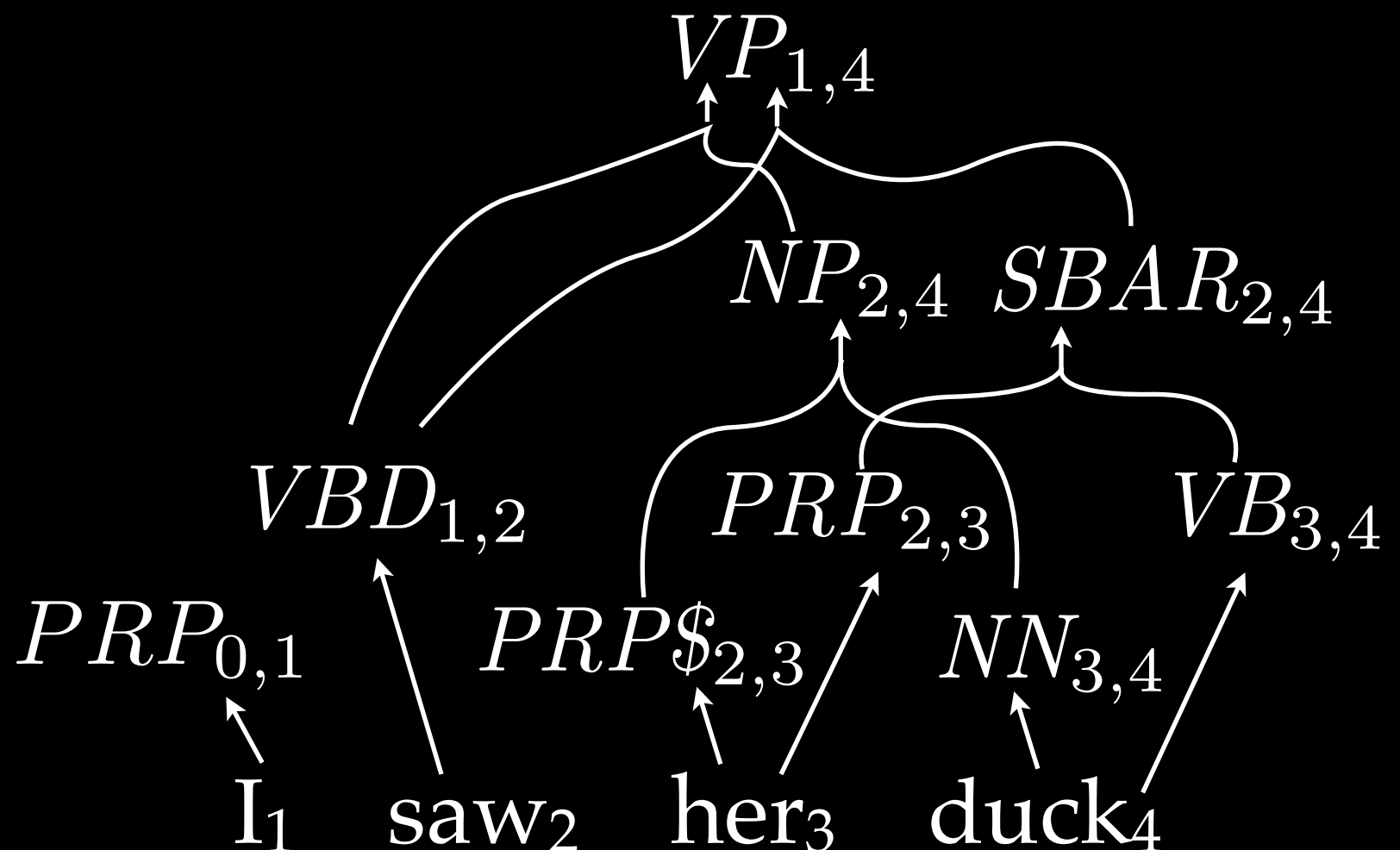
$VP \rightarrow VBD NP$

$VP \rightarrow VBD SBAR$

$VBD \rightarrow \text{saw}$

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$



# Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

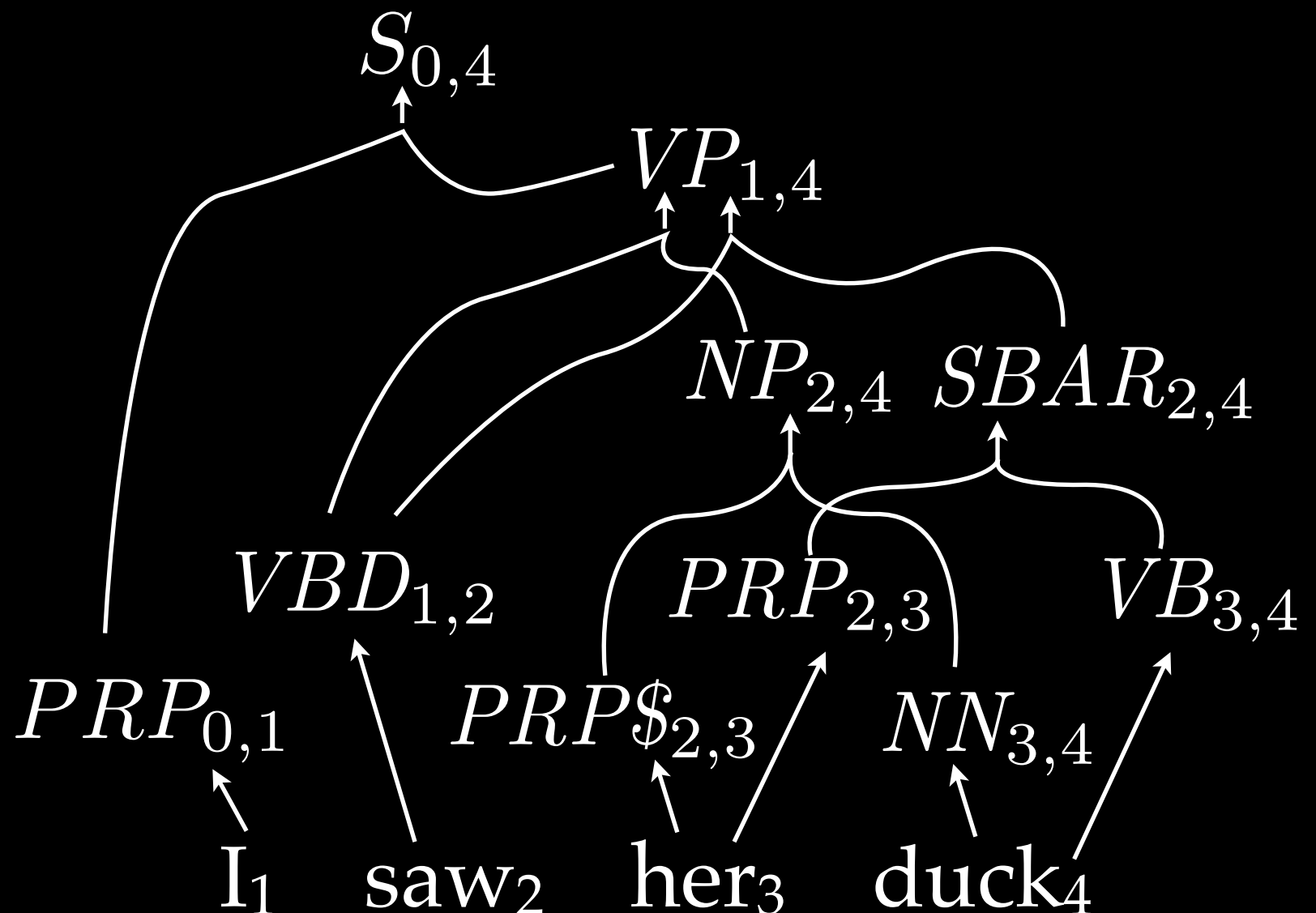
VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

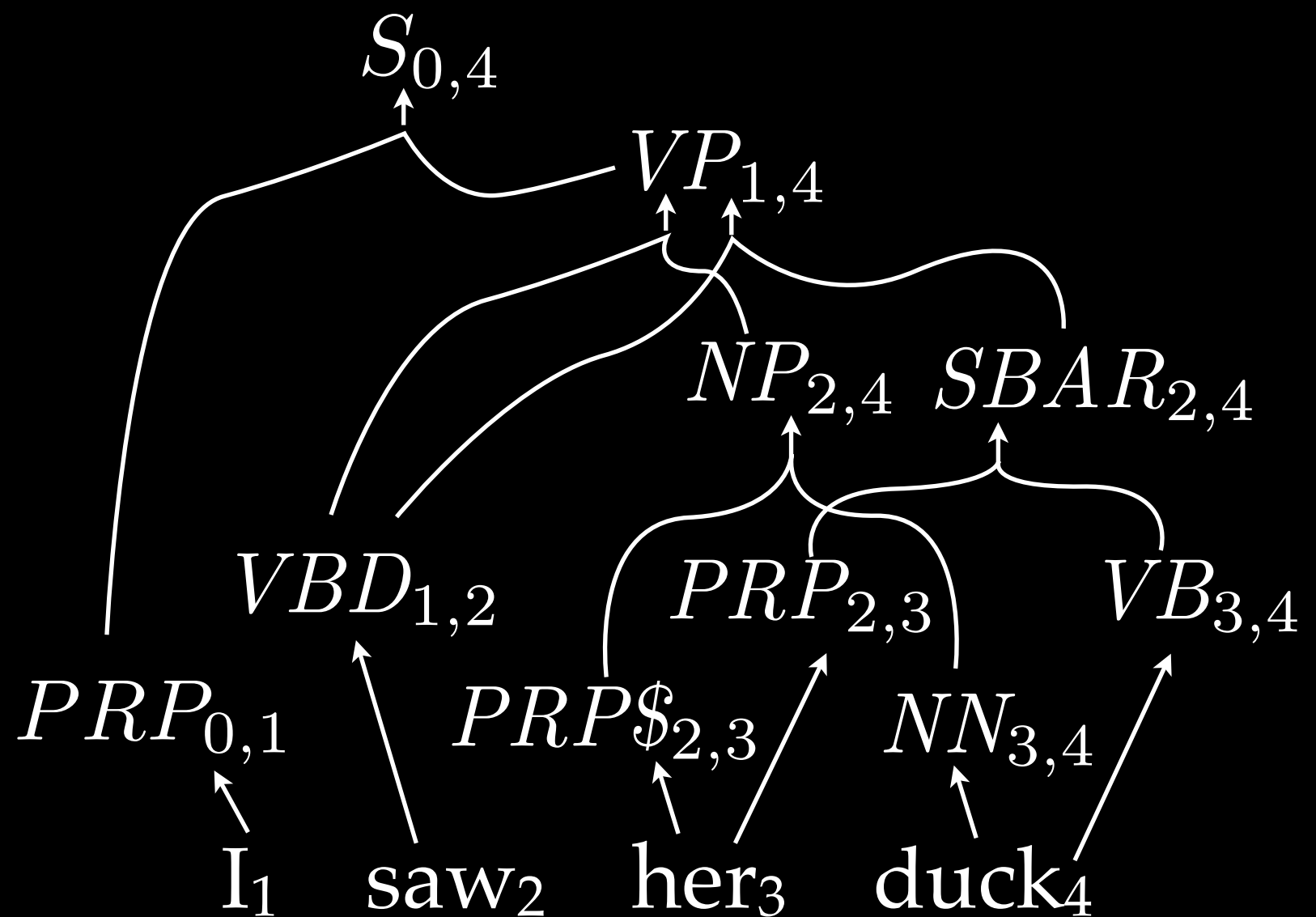
VBD  $\rightarrow$  saw

$$X_{i,i+1} \leftarrow (w_{i+1} = w) \wedge (X \rightarrow w)$$

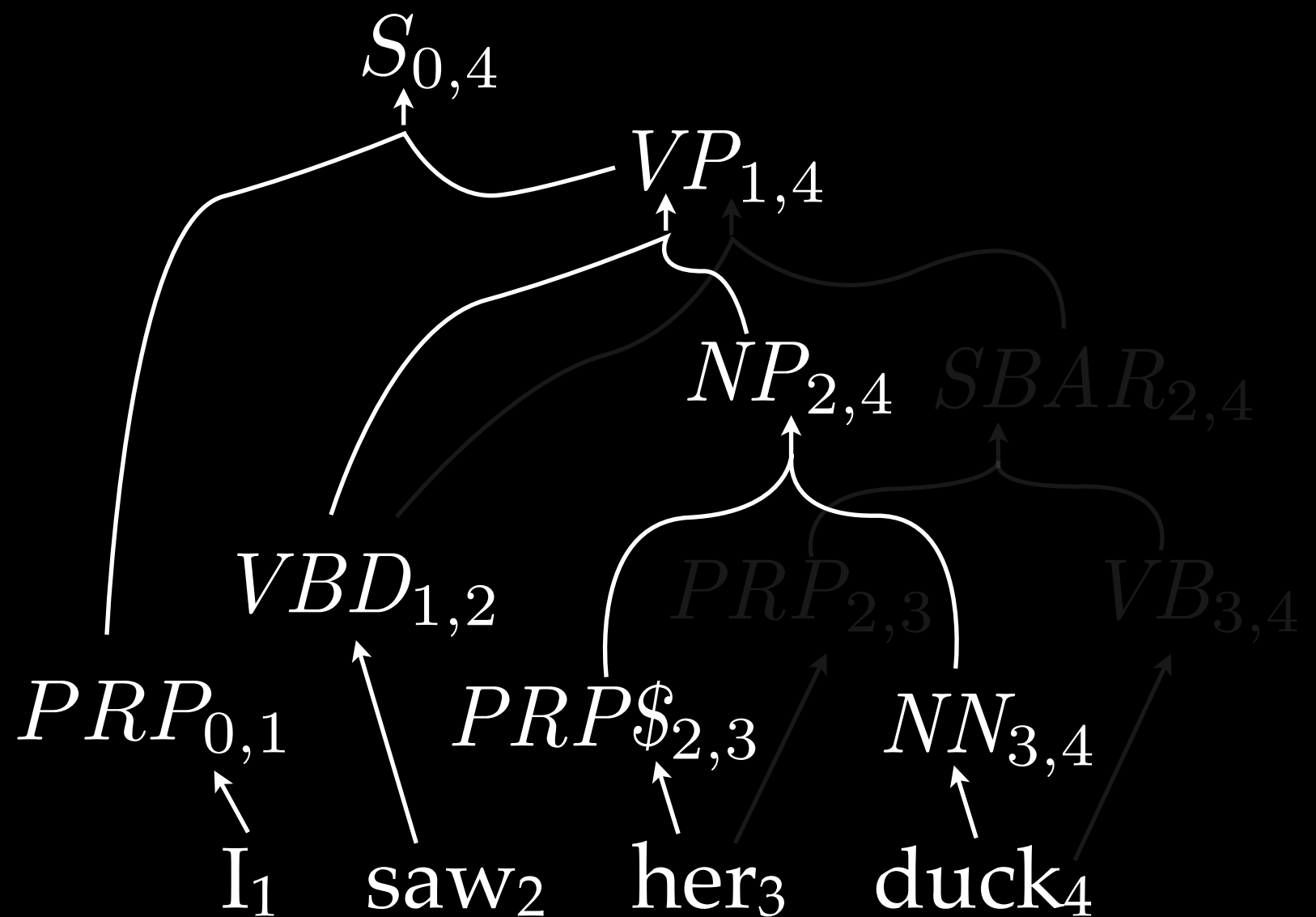
$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$



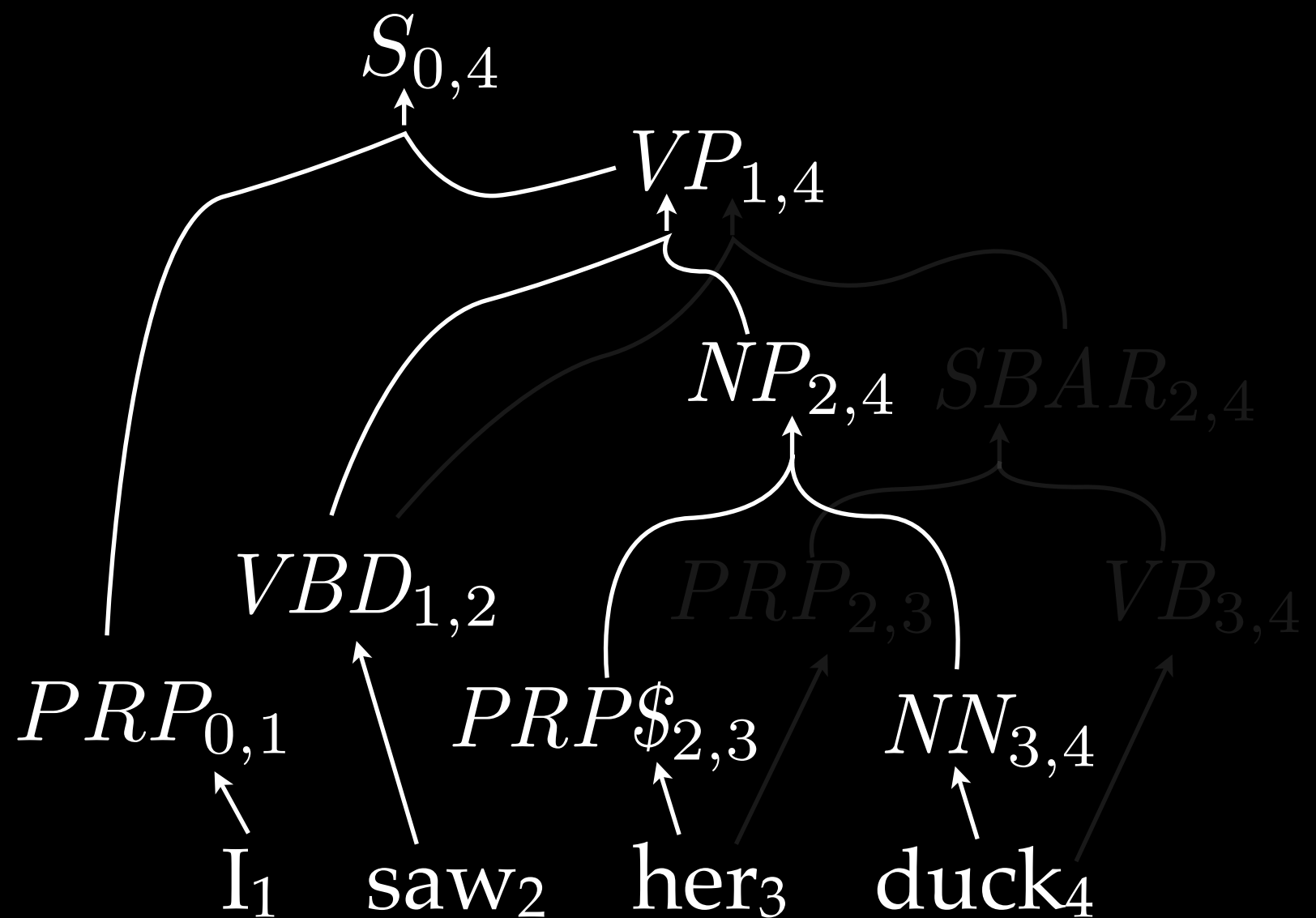
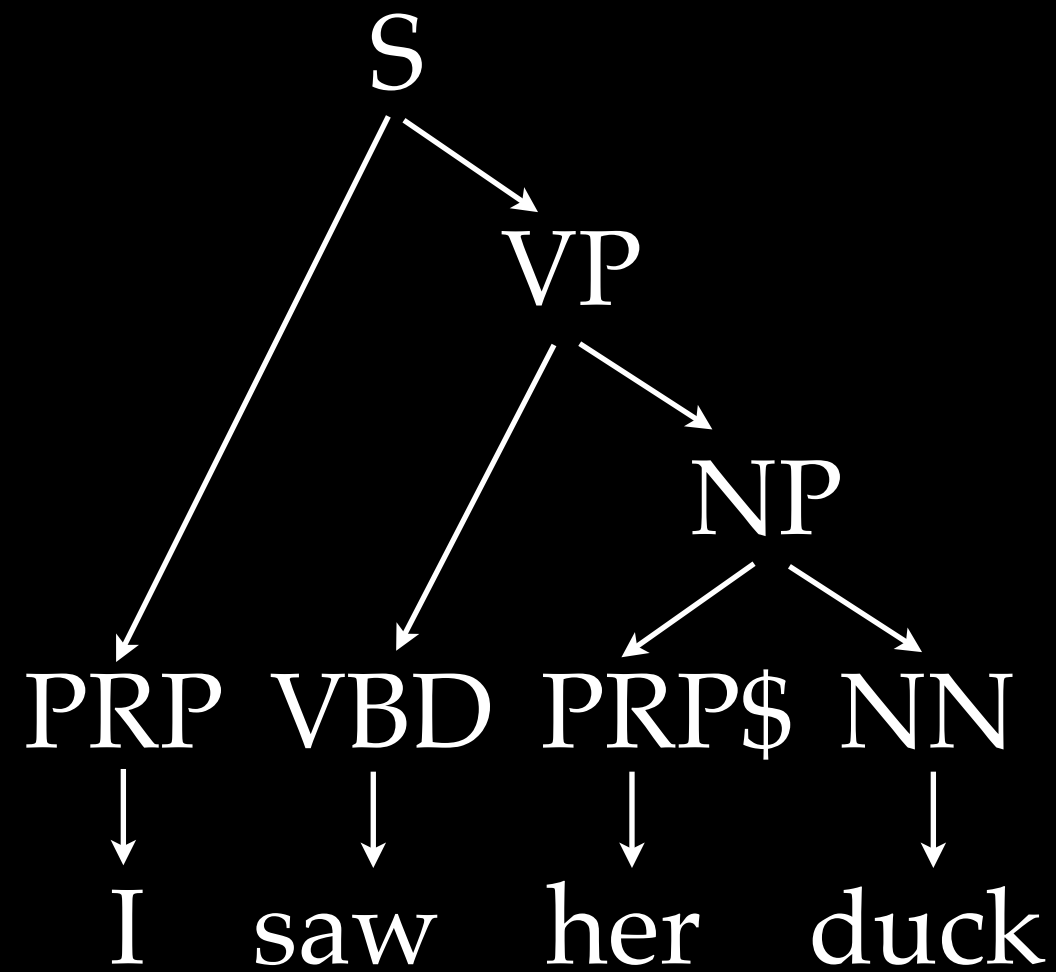
# Parsing



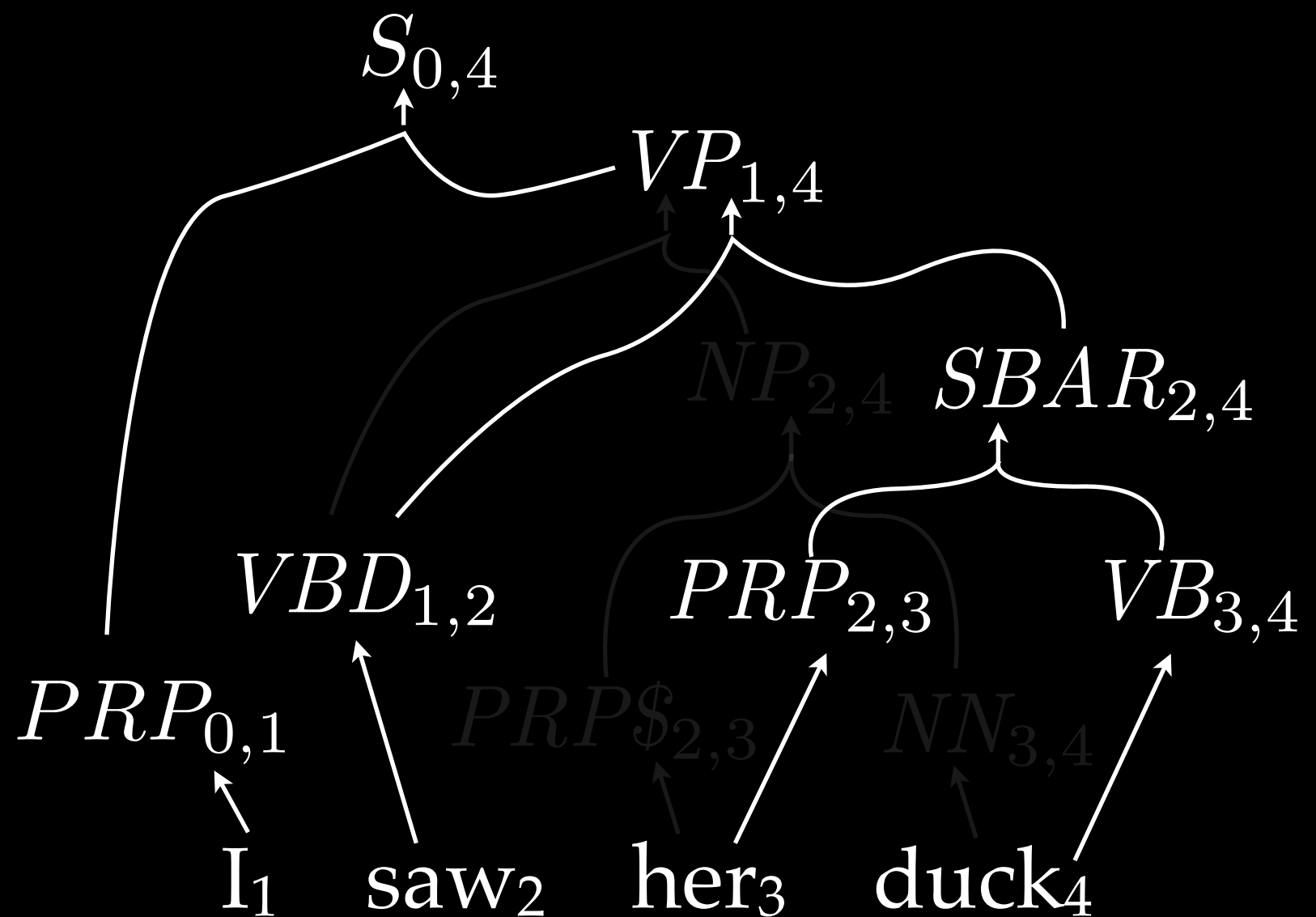
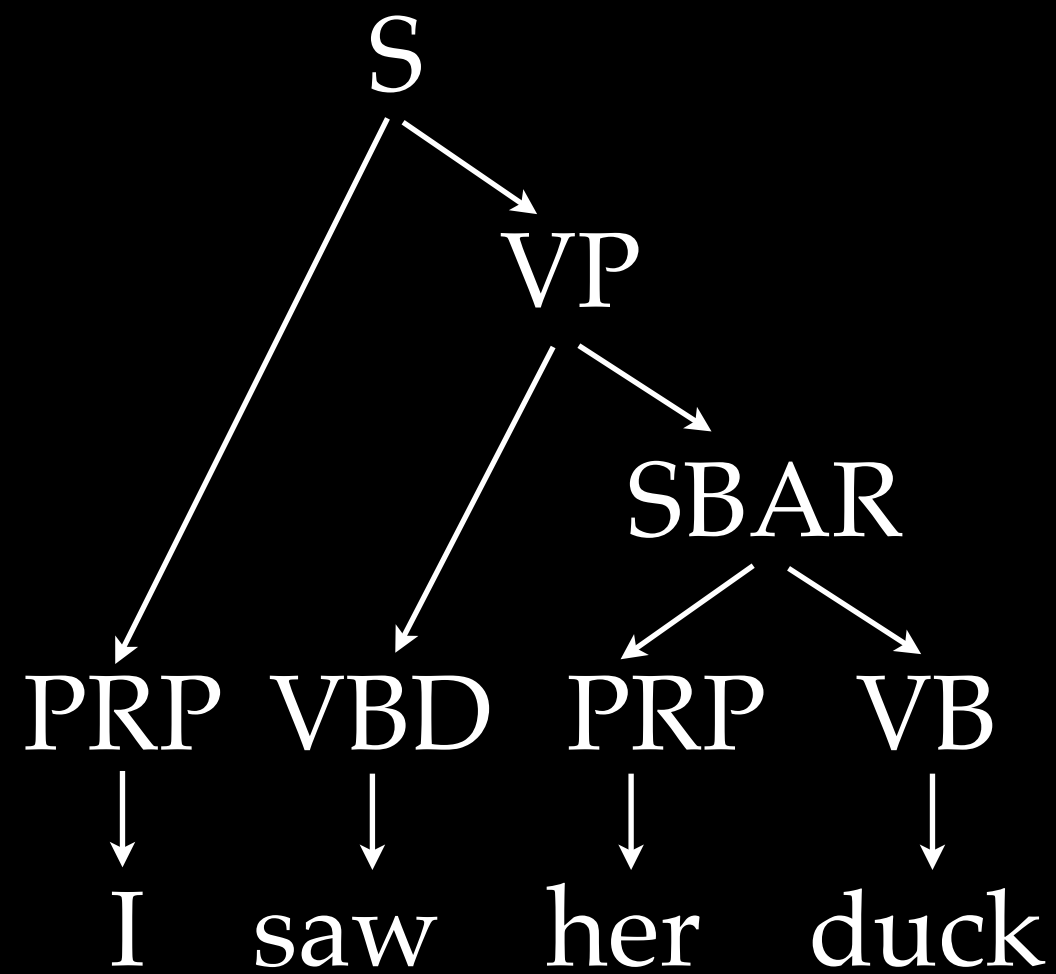
# Parsing



# Parsing

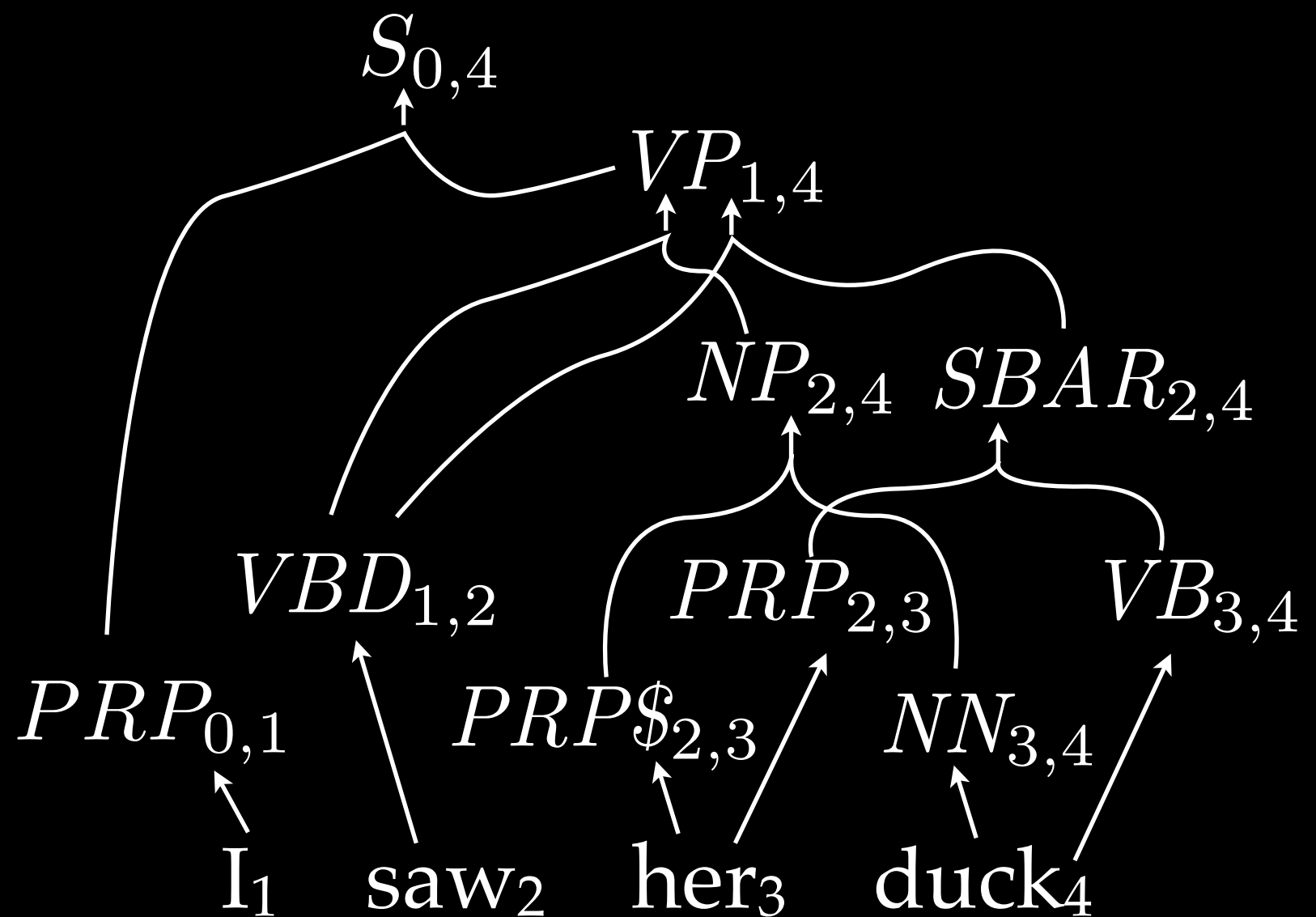


# Parsing



# Parsing

## Analysis

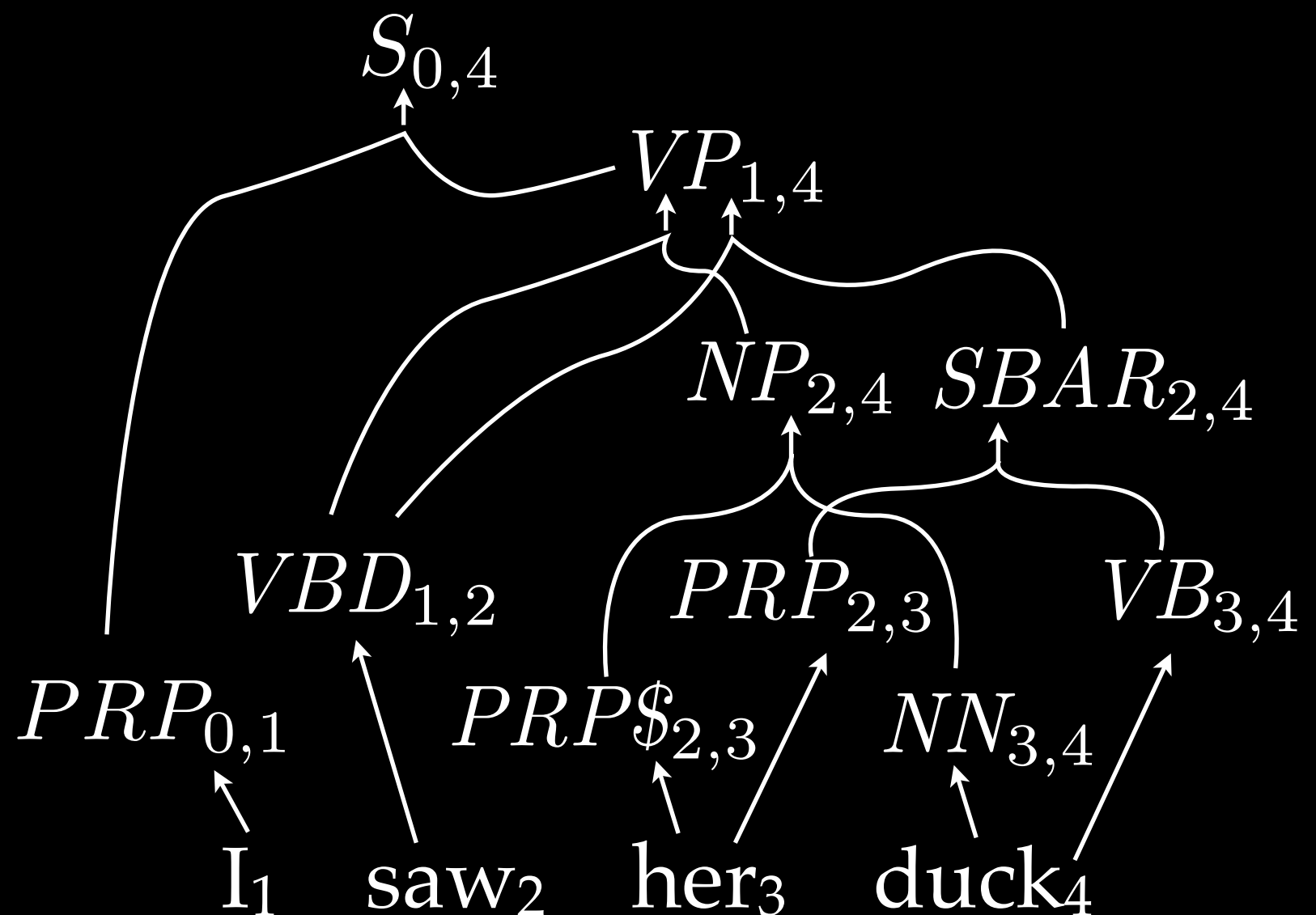


# Parsing

## Analysis

$O(Nn^2)$  nodes

$O(Gn^3)$  edges





# Probabilistic Parsing

NN  $\rightarrow$  duck

NP  $\rightarrow$  PRP\$ NN

PRP  $\rightarrow$  her

PRP  $\rightarrow$  I

PRP\$  $\rightarrow$  her

S  $\rightarrow$  PRP VP

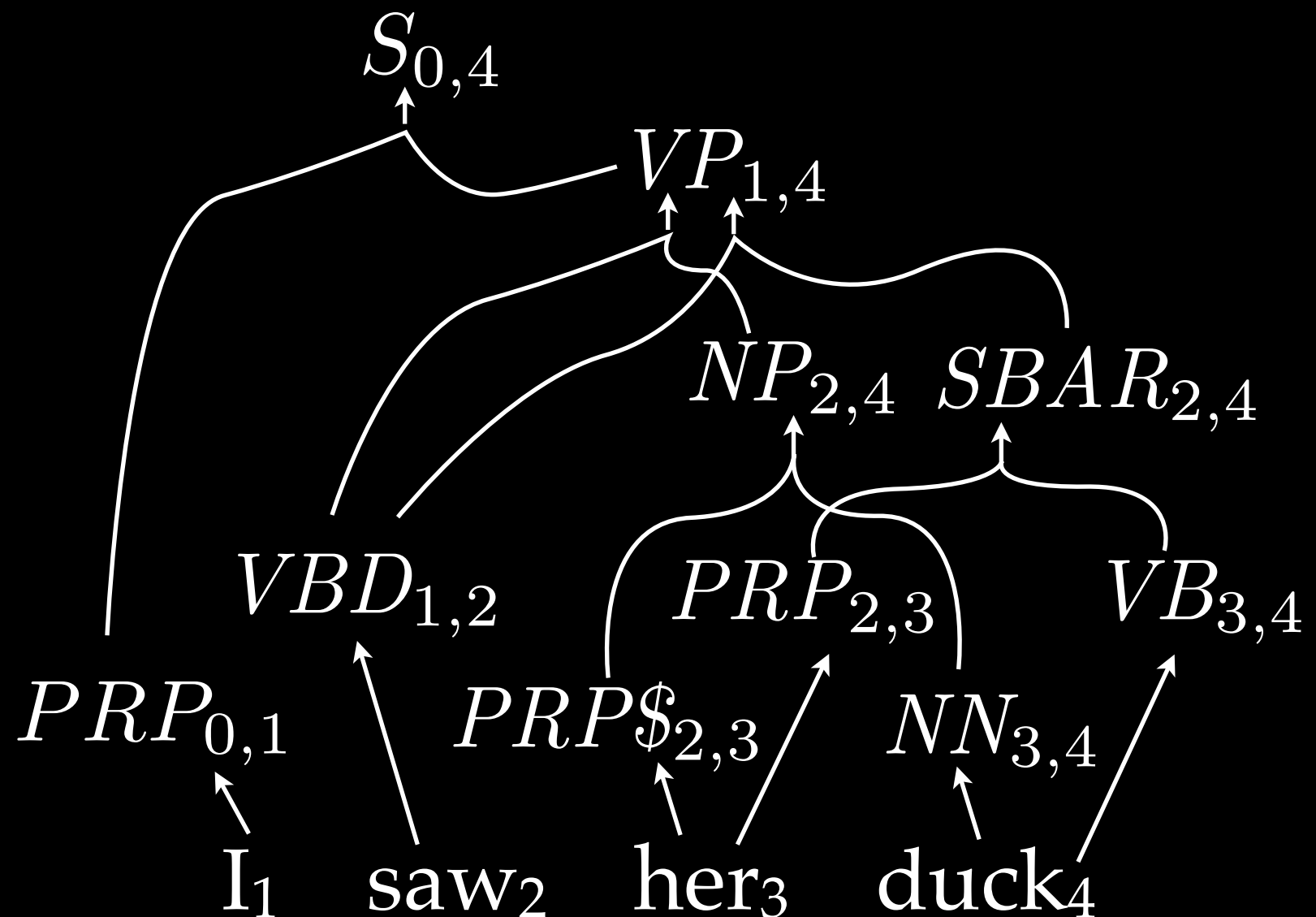
SBAR  $\rightarrow$  PRP VB

VB  $\rightarrow$  duck

VP  $\rightarrow$  VBD NP

VP  $\rightarrow$  VBD SBAR

VBD  $\rightarrow$  saw



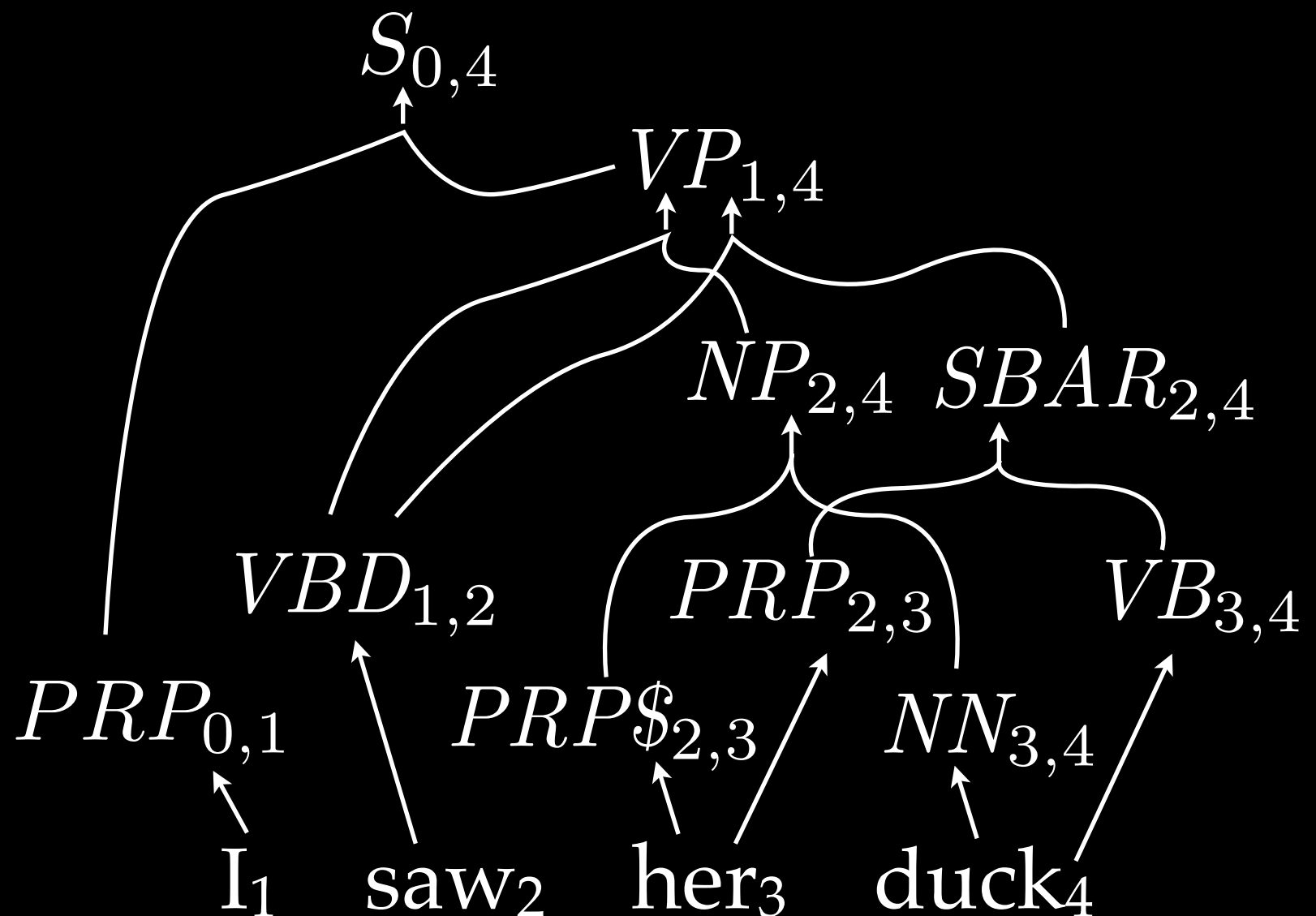
# Probabilistic Parsing

NN  $\rightarrow$  duck (1.0)

$$\text{NP} \rightarrow \text{PRP\$ NN} \quad (1.0)$$
$$\text{PRP} \rightarrow \text{her} \quad (0.3)$$
$$\text{PRP} \rightarrow \text{I} \quad (0.7)$$
$$\text{PRP\$} \rightarrow \text{her} \quad (1.0)$$
$$S \rightarrow \text{PRP VP} \quad (1.0)$$

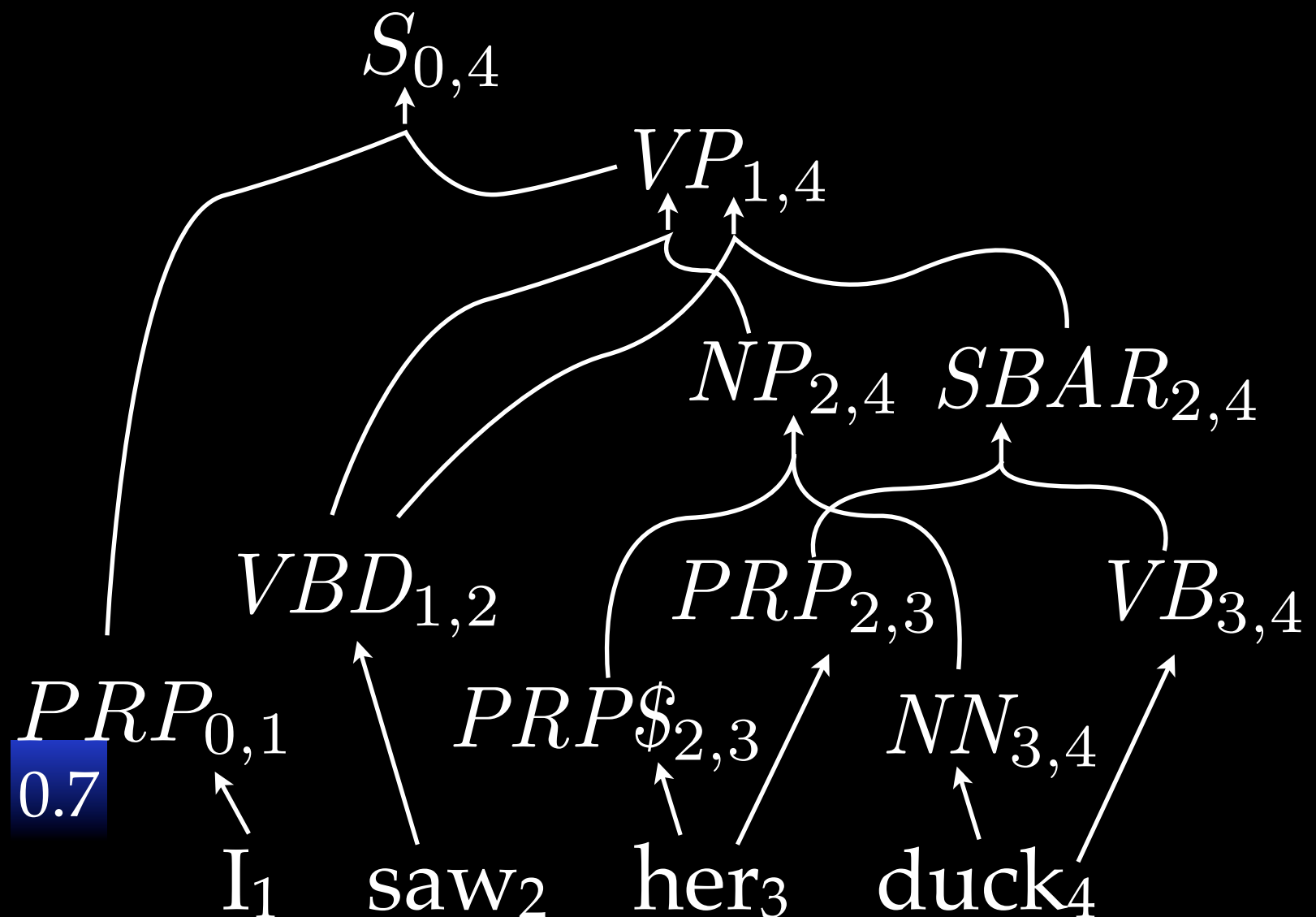
SBAR  $\rightarrow$  PRP VB (1.0)

VB → duck (1.0)

$$\text{VP} \rightarrow \text{VBD NP} \quad (0.8)$$
$$\text{VP} \rightarrow \text{VBD SBAR} \quad (0.2)$$
$$\text{VBD} \rightarrow \text{saw} \quad (1.0)$$


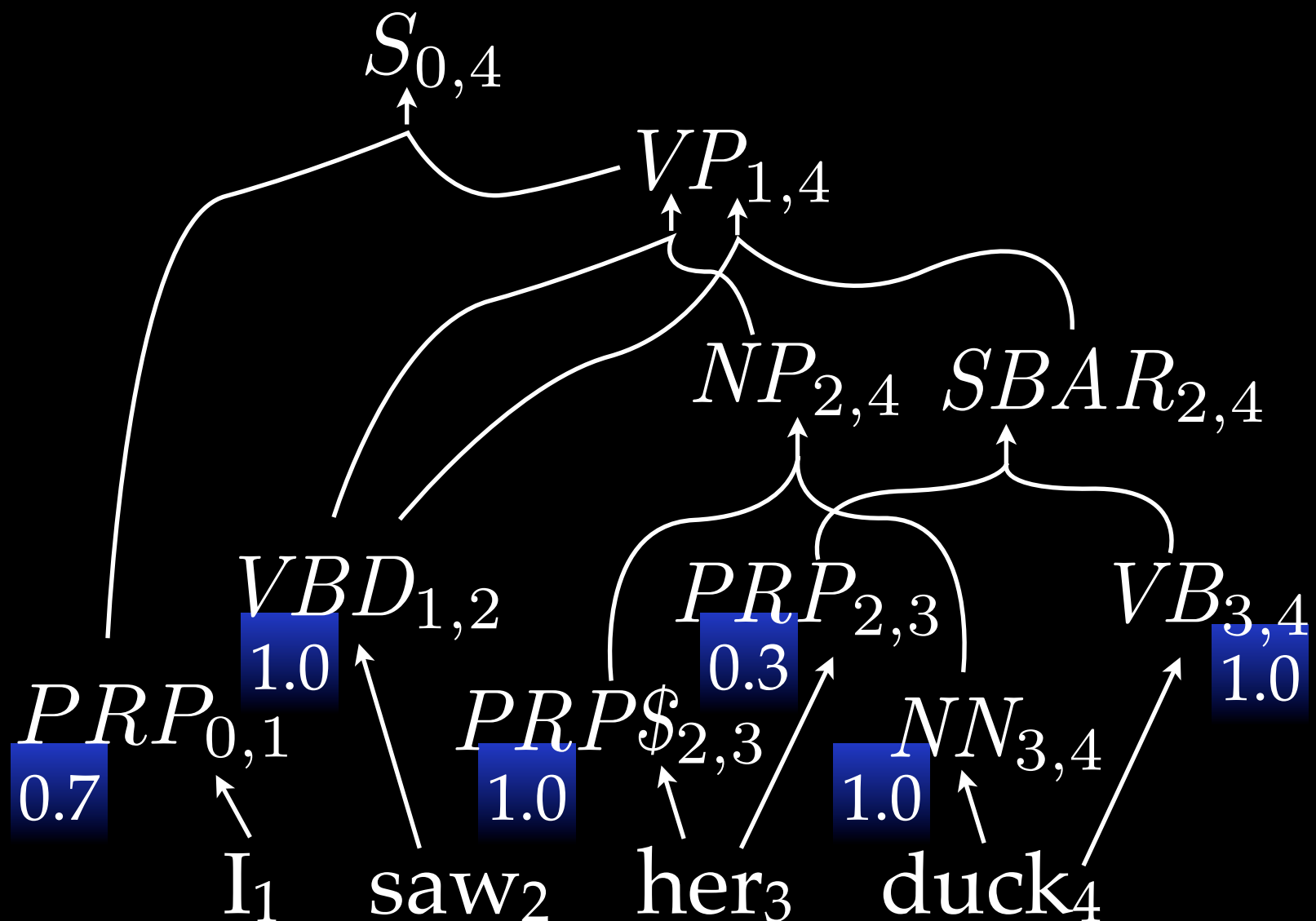
# Probabilistic Parsing

$NN \rightarrow \text{duck}$	(1.0)
$NP \rightarrow PRP\$ NN$	(1.0)
$PRP \rightarrow \text{her}$	(0.3)
$PRP \rightarrow I$	(0.7)
$PRP\$ \rightarrow \text{her}$	(1.0)
$S \rightarrow PRP VP$	(1.0)
$SBAR \rightarrow PRP VB$	(1.0)
$VB \rightarrow \text{duck}$	(1.0)
$VP \rightarrow VBD NP$	(0.8)
$VP \rightarrow VBD SBAR$	(0.2)
$VBD \rightarrow \text{saw}$	(1.0)



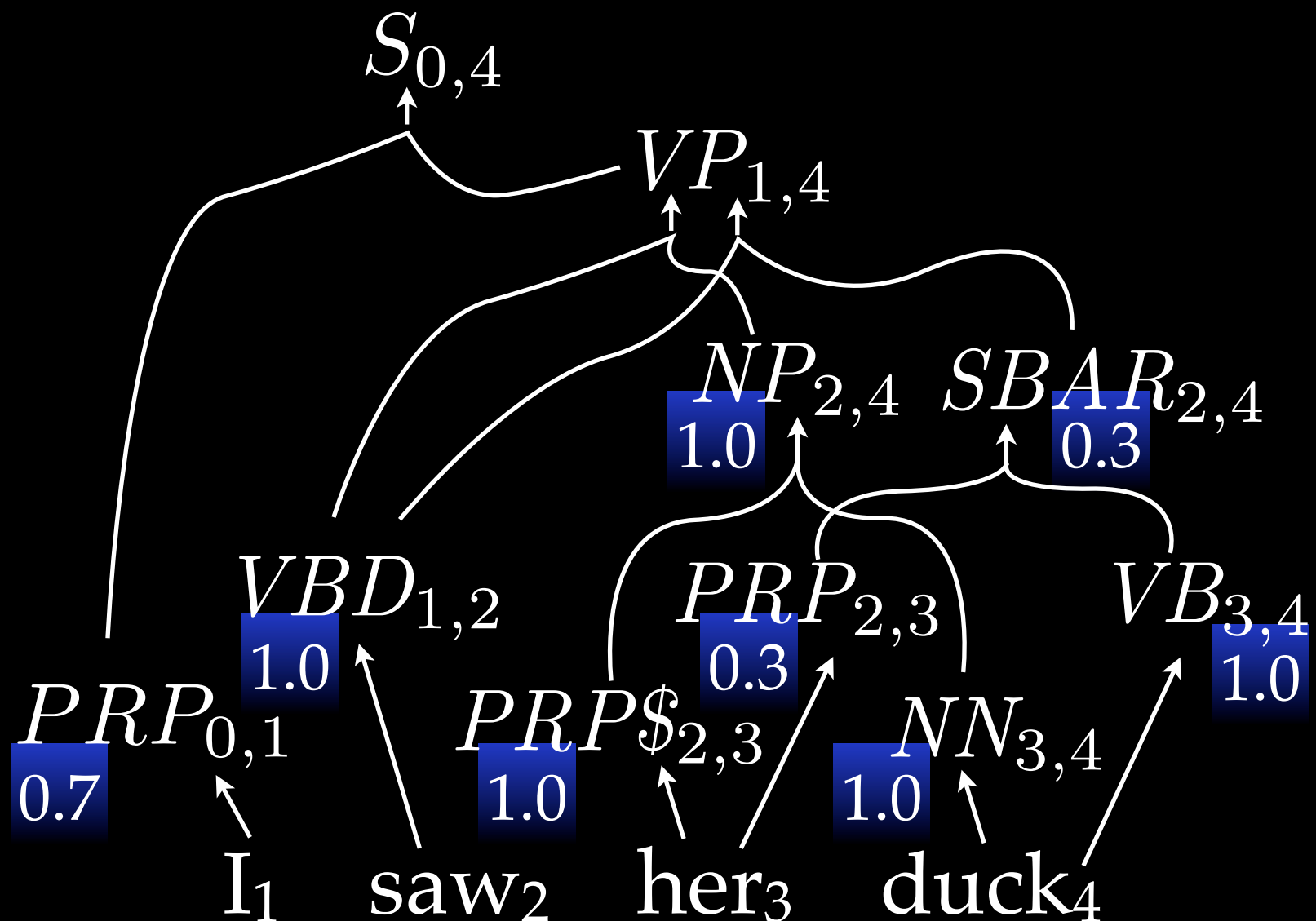
# Probabilistic Parsing

$NN \rightarrow \text{duck}$	(1.0)
$NP \rightarrow PRP\$ NN$	(1.0)
$PRP \rightarrow \text{her}$	(0.3)
$PRP \rightarrow I$	(0.7)
$PRP\$ \rightarrow \text{her}$	(1.0)
$S \rightarrow PRP VP$	(1.0)
$SBAR \rightarrow PRP VB$	(1.0)
$VB \rightarrow \text{duck}$	(1.0)
$VP \rightarrow VBD NP$	(0.8)
$VP \rightarrow VBD SBAR$	(0.2)
$VBD \rightarrow \text{saw}$	(1.0)



# Probabilistic Parsing

$NN \rightarrow \text{duck}$	(1.0)
$NP \rightarrow PRP\$ NN$	(1.0)
$PRP \rightarrow \text{her}$	(0.3)
$PRP \rightarrow I$	(0.7)
$PRP\$ \rightarrow \text{her}$	(1.0)
$S \rightarrow PRP VP$	(1.0)
$SBAR \rightarrow PRP VB$	(1.0)
$VB \rightarrow \text{duck}$	(1.0)
$VP \rightarrow VBD NP$	(0.8)
$VP \rightarrow VBD SBAR$	(0.2)
$VBD \rightarrow \text{saw}$	(1.0)



# Probabilistic Parsing

NN → duck (1.0)

NP → PRP\$ NN (1.0)

PRP → her (0.3)

PRP → I (0.7)

PRP\$ → her (1.0)

S → PRP VP (1.0)

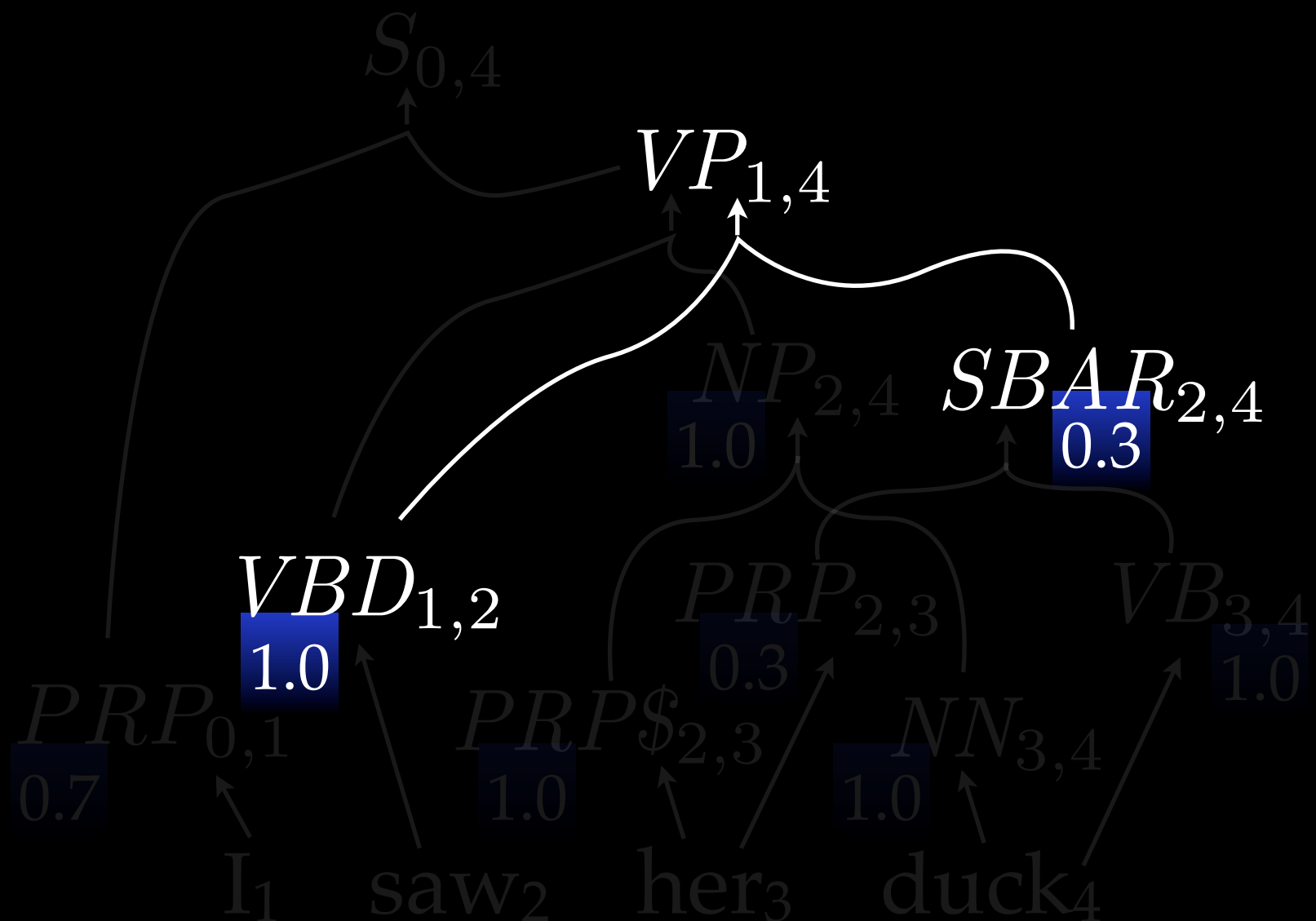
SBAR → PRP VB (1.0)

VB → duck (1.0)

VP → VBD NP (0.8)

**VP → VBD SBAR (0.2)**

VBD → saw (1.0)



# Probabilistic Parsing

NN → duck (1.0)

NP → PRP\$ NN (1.0)

PRP → her (0.3)

PRP → I (0.7)

PRP\$ → her (1.0)

S → PRP VP (1.0)

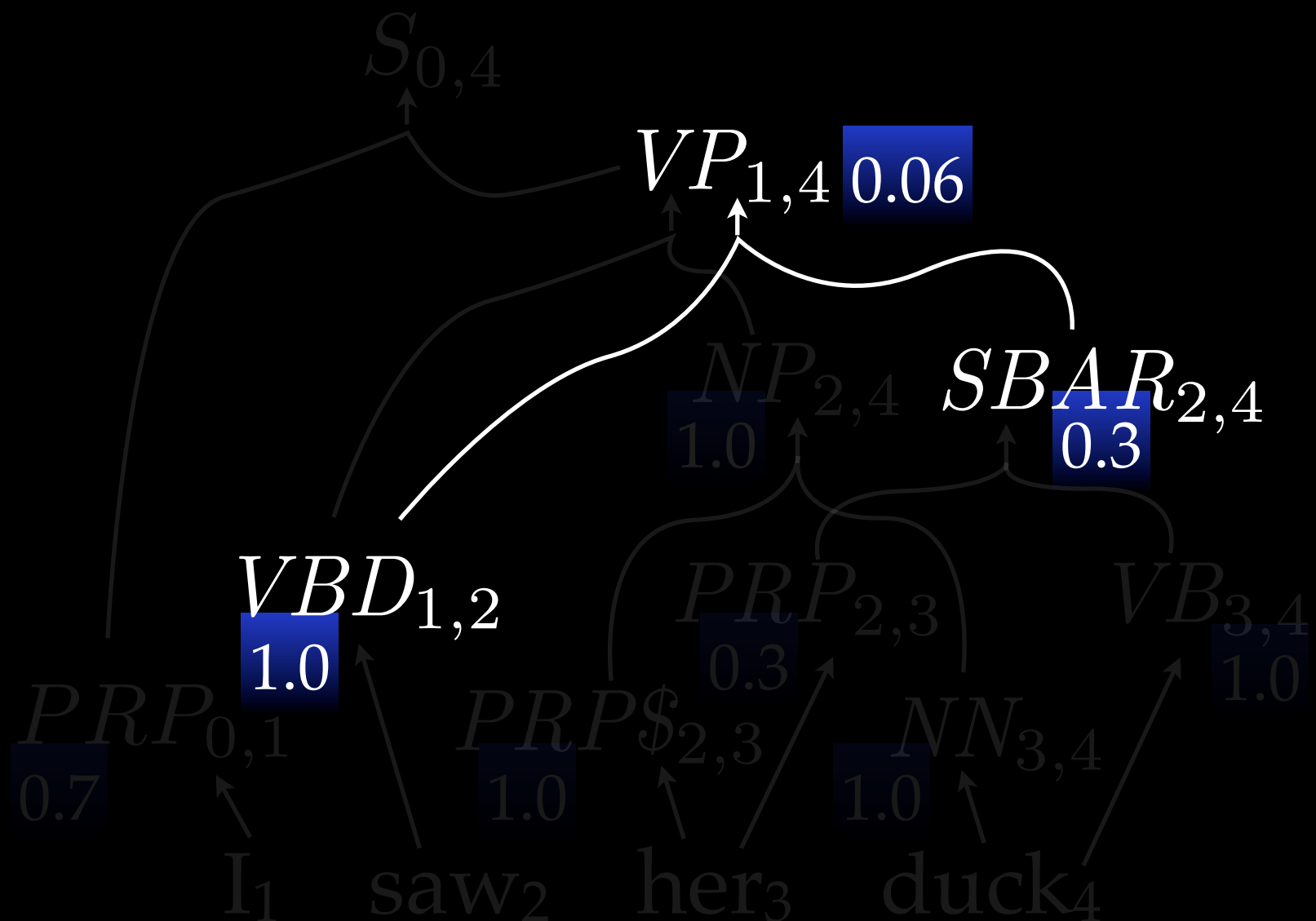
SBAR → PRP VB (1.0)

VB → duck (1.0)

VP → VBD NP (0.8)

**VP → VBD SBAR (0.2)**

VBD → saw (1.0)



# Probabilistic Parsing

NN  $\rightarrow$  duck (1.0)

NP  $\rightarrow$  PRP\$ NN (1.0)

PRP  $\rightarrow$  her (0.3)

PRP  $\rightarrow$  I (0.7)

PRP\$  $\rightarrow$  her (1.0)

S  $\rightarrow$  PRP VP (1.0)

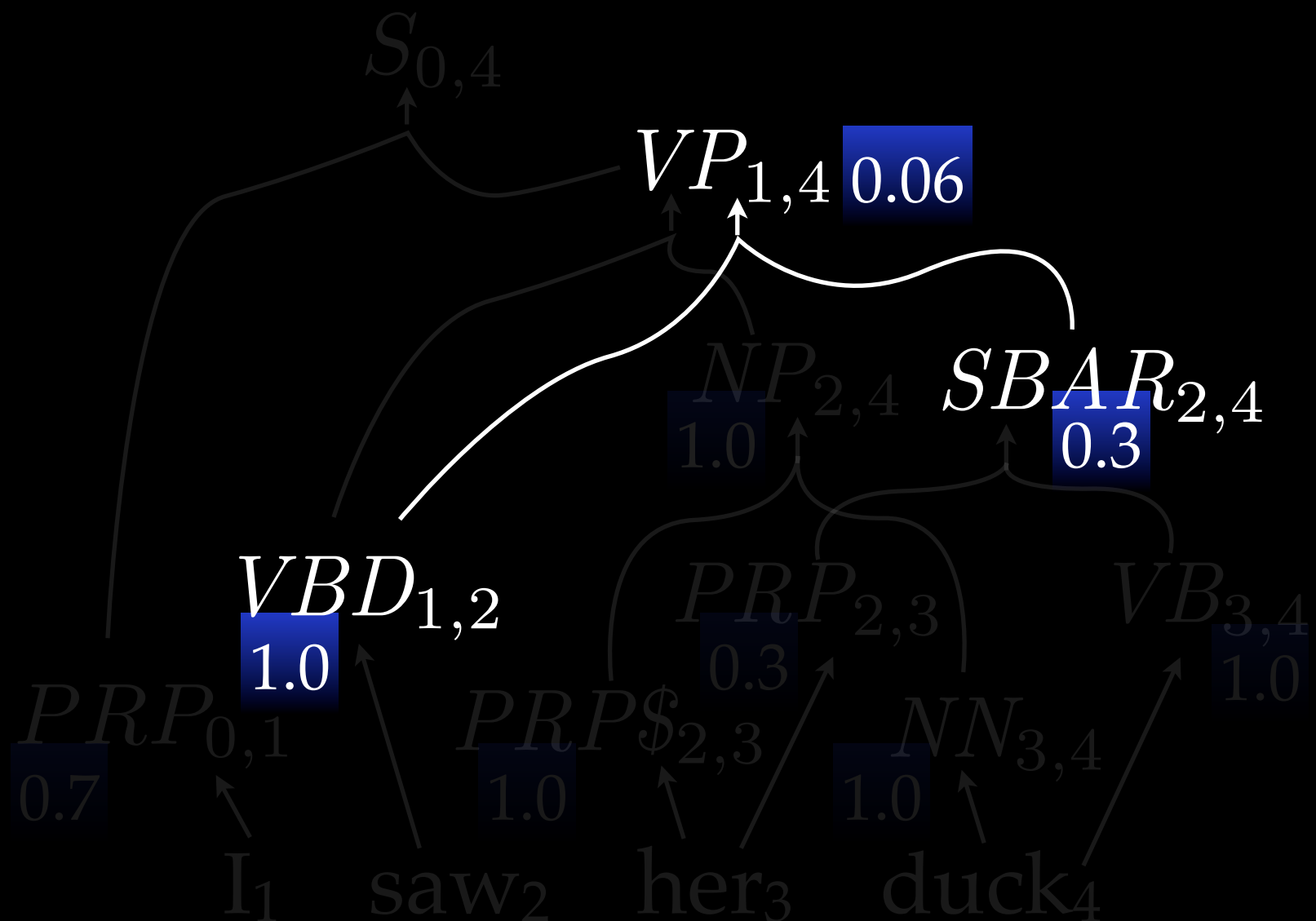
SBAR  $\rightarrow$  PRP VB (1.0)

VB  $\rightarrow$  duck (1.0)

VP  $\rightarrow$  VBD NP (0.8)

VP  $\rightarrow$  VBD SBAR (0.2)

VBD  $\rightarrow$  saw (1.0)





# Probabilistic Parsing

NN → duck (1.0)

NP → PRP\$ NN (1.0)

PRP → her (0.3)

PRP → I (0.7)

PRP\$ → her (1.0)

S → PRP VP (1.0)

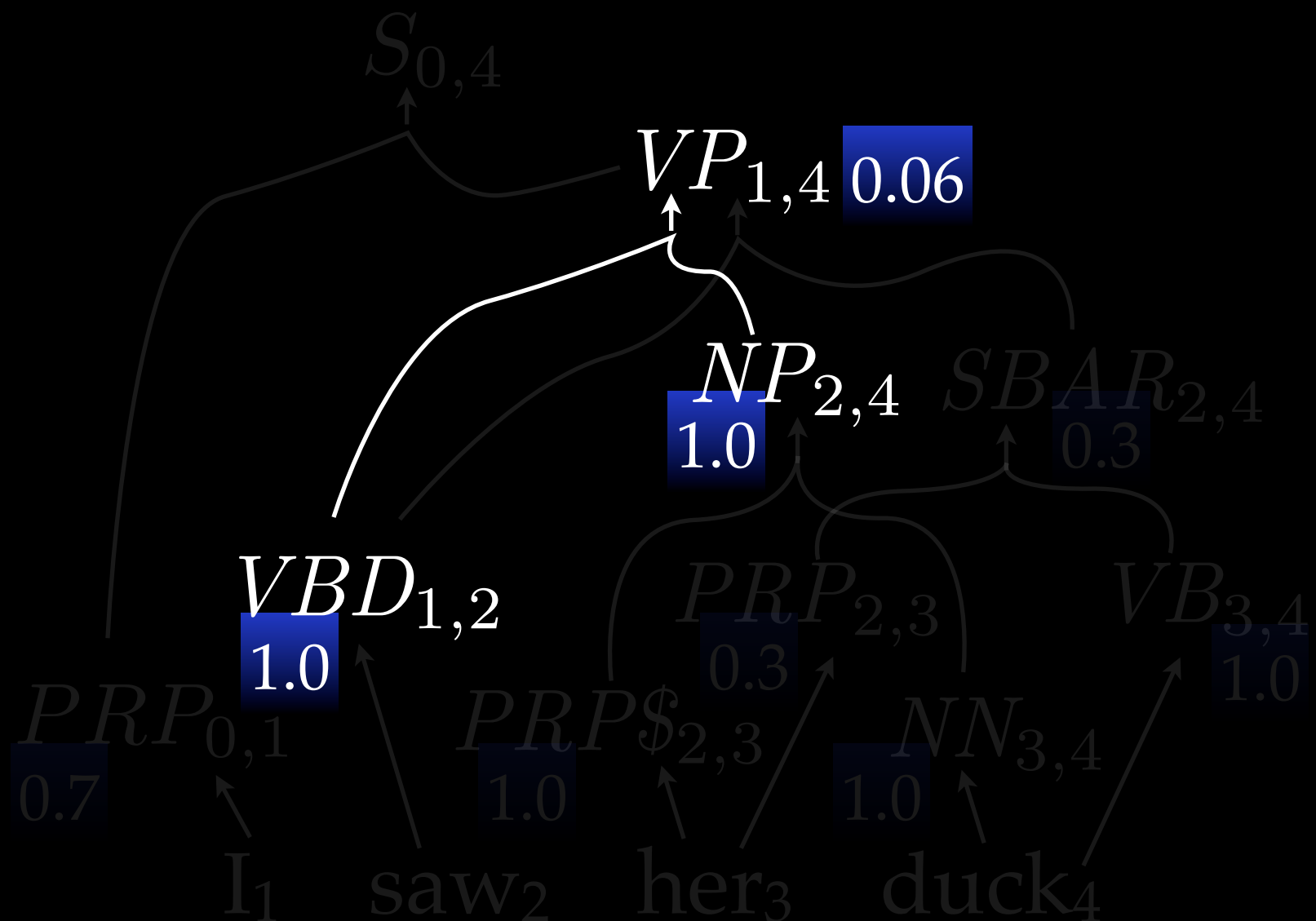
SBAR → PRP VB (1.0)

VB → duck (1.0)

VP → VBD NP (0.8)

VP → VBD SBAR (0.2)

VBD → saw (1.0)



# Probabilistic Parsing

NN → duck (1.0)

NP → PRP\$ NN (1.0)

PRP → her (0.3)

PRP → I (0.7)

PRP\$ → her (1.0)

S → PRP VP (1.0)

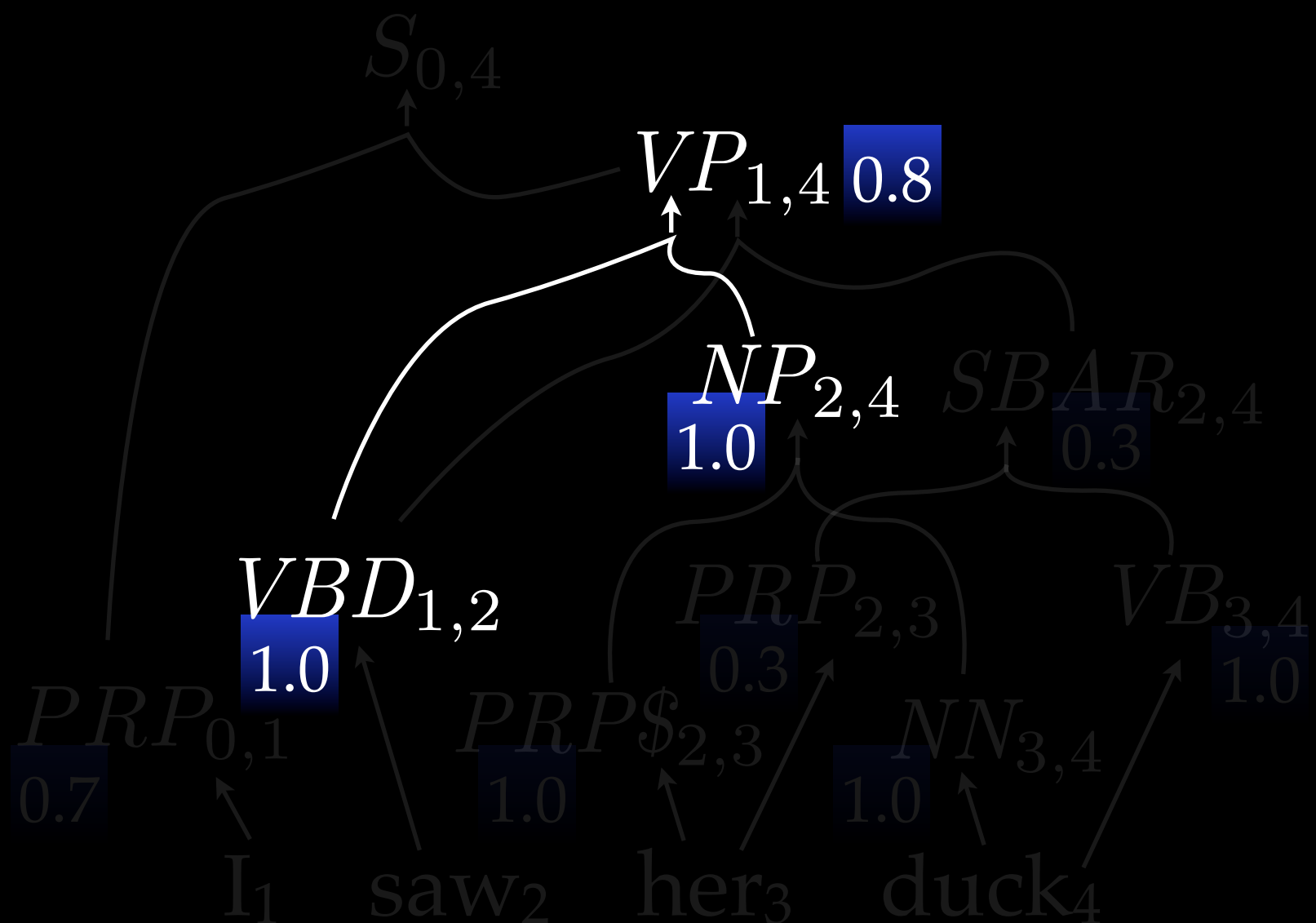
SBAR → PRP VB (1.0)

VB → duck (1.0)

**VP → VBD NP (0.8)**

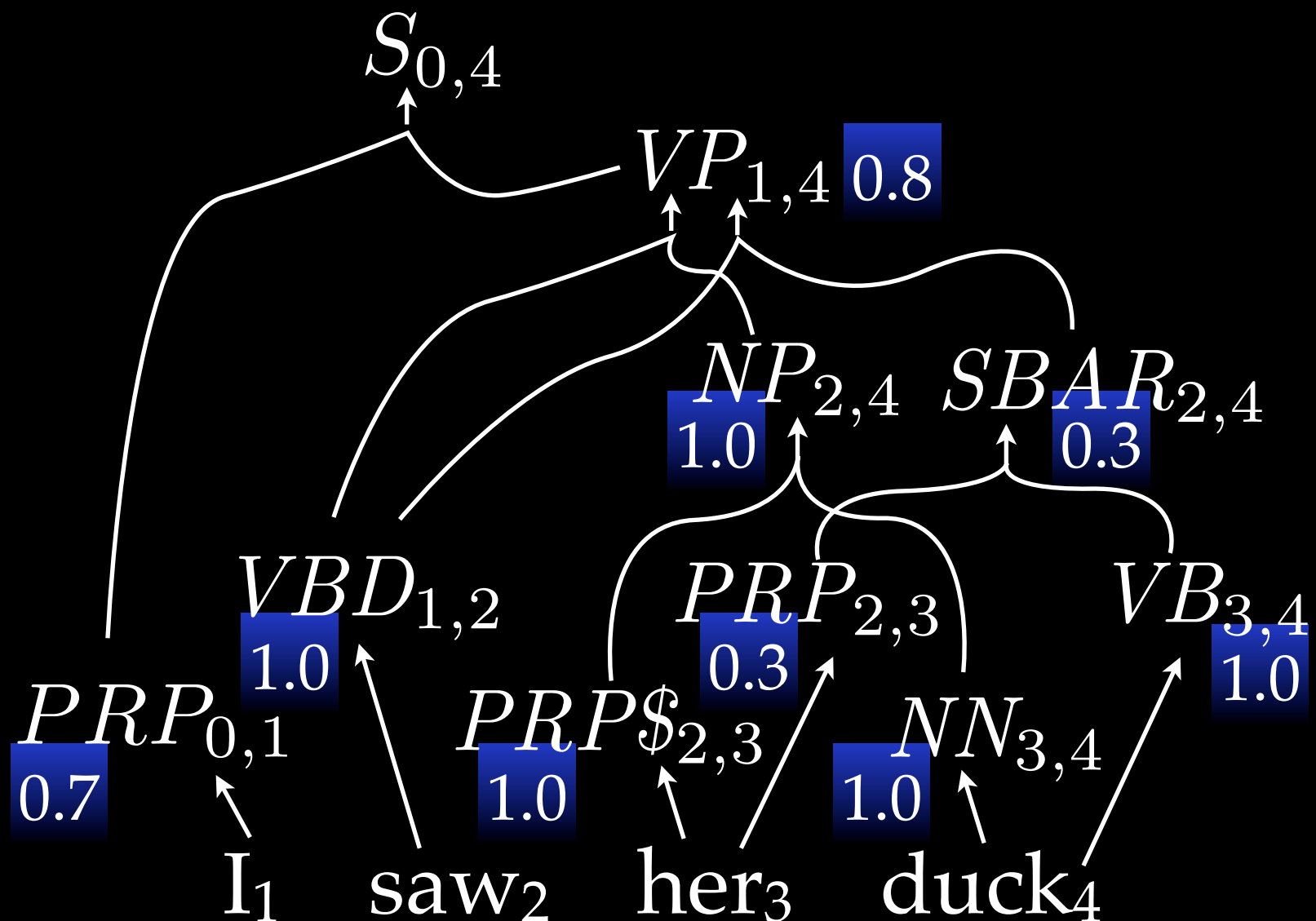
VP → VBD SBAR (0.2)

VBD → saw (1.0)



# Probabilistic Parsing

$NN \rightarrow \text{duck}$	(1.0)
$NP \rightarrow PRP\$ NN$	(1.0)
$PRP \rightarrow \text{her}$	(0.3)
$PRP \rightarrow I$	(0.7)
$PRP\$ \rightarrow \text{her}$	(1.0)
$S \rightarrow PRP VP$	(1.0)
$SBAR \rightarrow PRP VB$	(1.0)
$VB \rightarrow \text{duck}$	(1.0)
$VP \rightarrow VBD NP$	(0.8)
$VP \rightarrow VBD SBAR$	(0.2)
$VBD \rightarrow \text{saw}$	(1.0)



# Probabilistic Parsing

NN → duck (1.0)

NP → PRP\$ NN (1.0)

PRP → her (0.3)

PRP → I (0.7)

PRP\$ → her (1.0)

S → PRP VP (1.0)

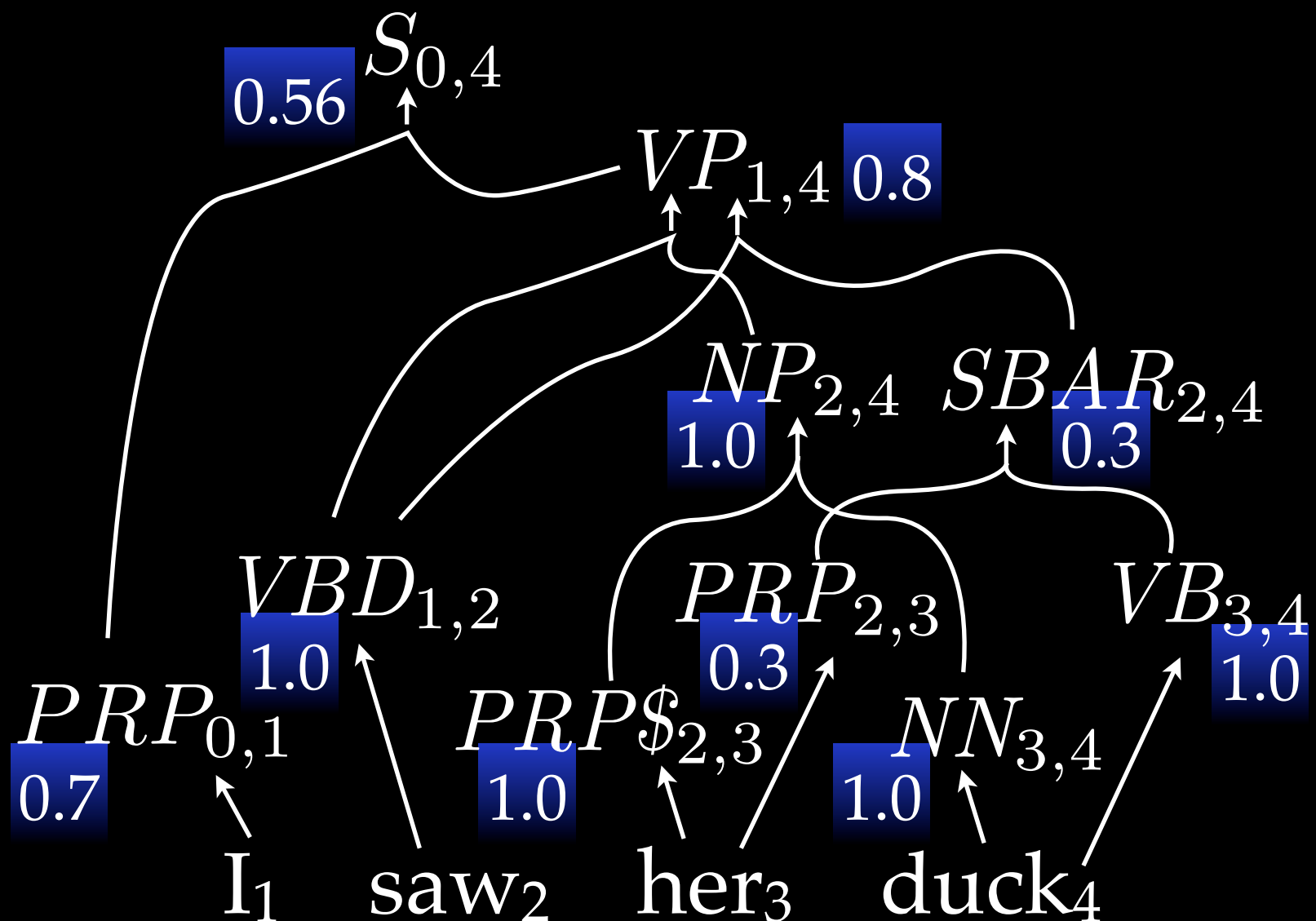
SBAR → PRP VB (1.0)

VB → duck (1.0)

VP → VBD NP (0.8)

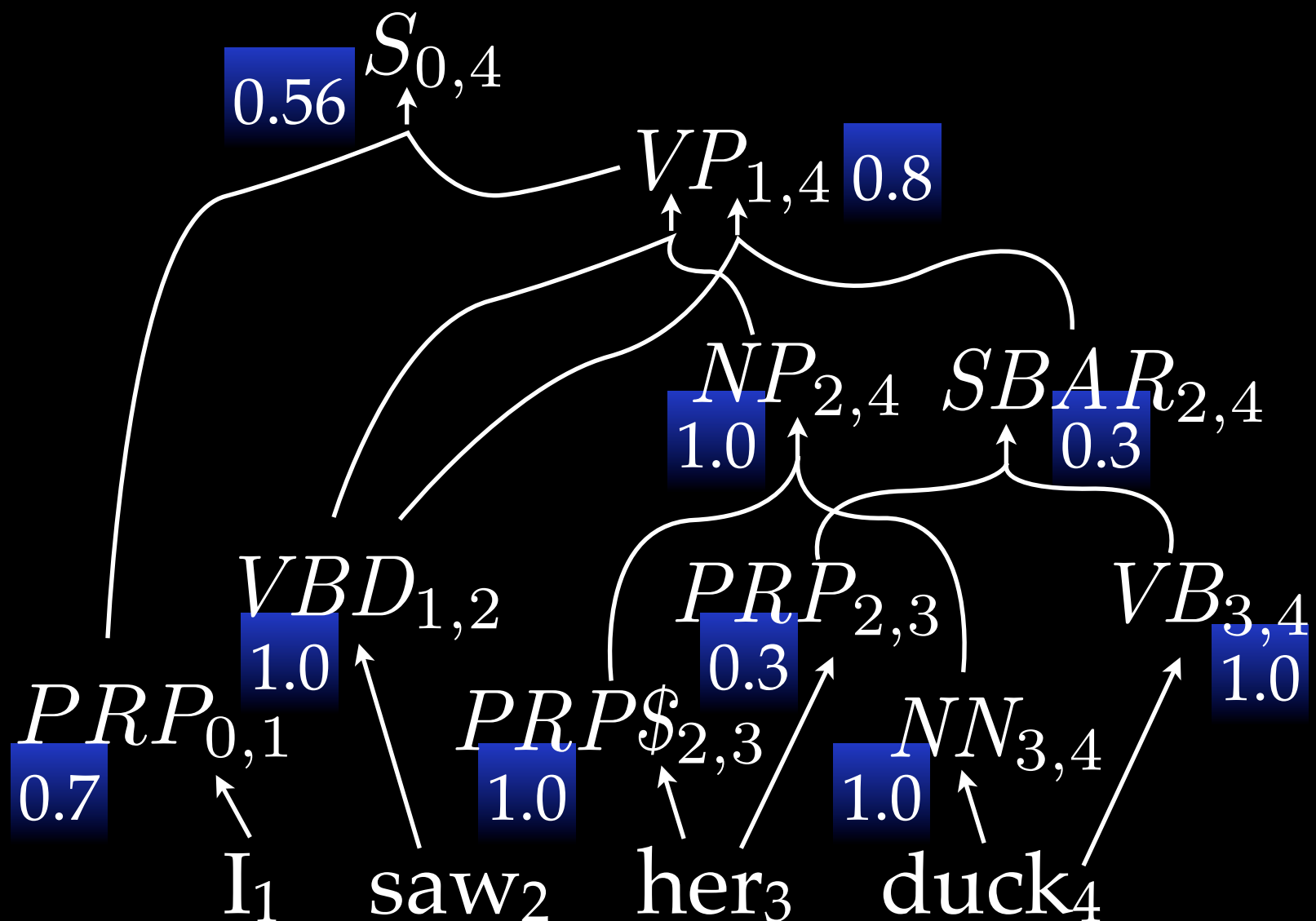
VP → VBD SBAR (0.2)

VBD → saw (1.0)



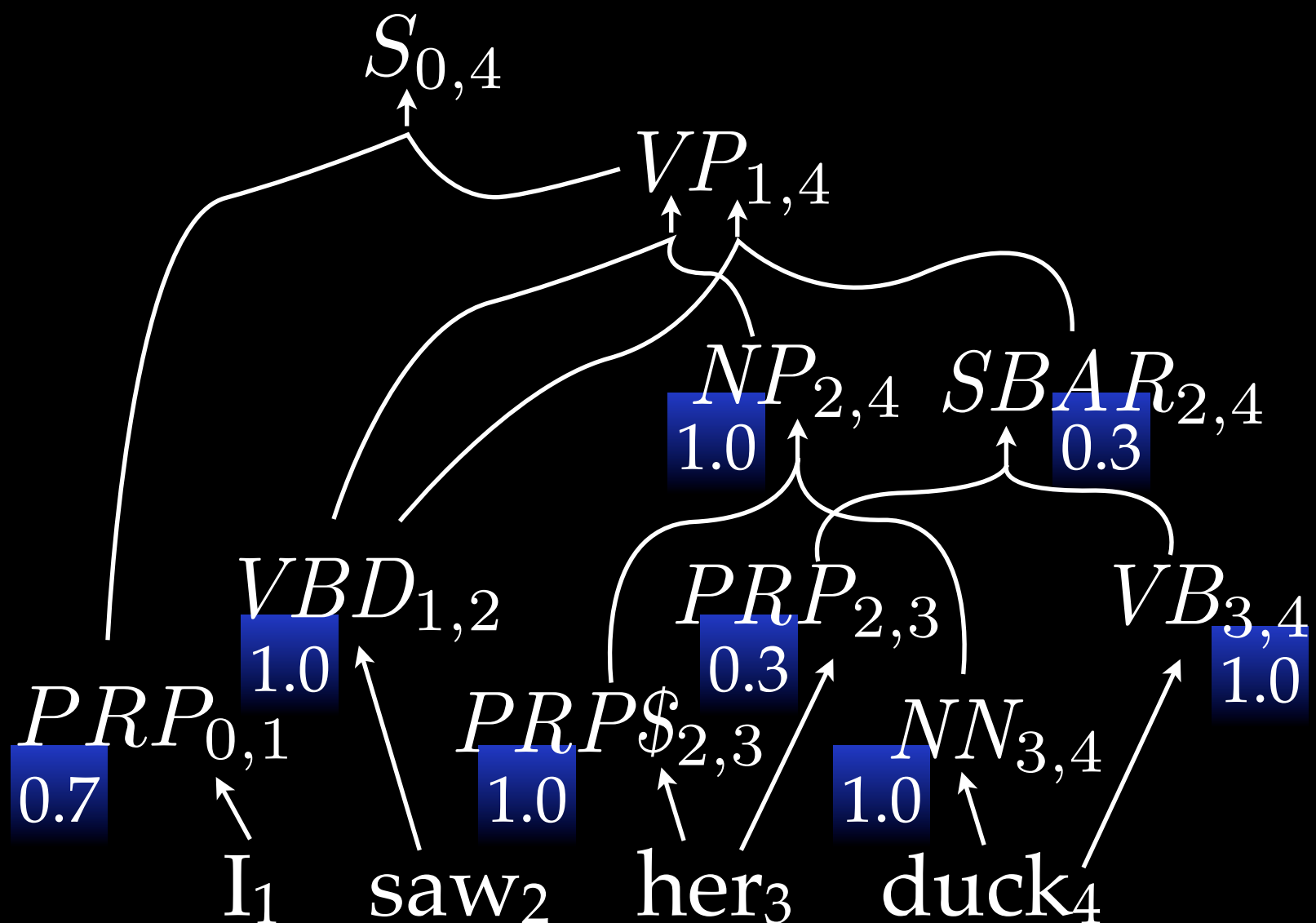
# Probabilistic Parsing

$$X_{i,j} = \max(X_{i,j}, Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$



# Computing Expectations

NN  $\rightarrow$  duck (1.0)

$$\text{NP} \rightarrow \text{PRP\$ NN} \quad (1.0)$$
$$\text{PRP} \rightarrow \text{her} \quad (0.3)$$
$$\text{PRP} \rightarrow \text{I} \quad (0.7)$$
$$\text{PRP\$} \rightarrow \text{her} \quad (1.0)$$
$$S \rightarrow \text{PRP VP} \quad (1.0)$$
$$\text{SBAR} \rightarrow \text{PRP VB} \quad (1.0)$$
$$\text{VB} \rightarrow \text{duck} \quad (1.0)$$
$$\text{VP} \rightarrow \text{VBD NP} \quad (0.8)$$
$$\text{VP} \rightarrow \text{VBD SBAR} \quad (0.2)$$
$$\text{VBD} \rightarrow \text{saw} \quad (1.0)$$


# Computing Expectations

NN → duck (1.0)

NP → PRP\$ NN (1.0)

PRP → her (0.3)

PRP → I (0.7)

PRP\$ → her (1.0)

S → PRP VP (1.0)

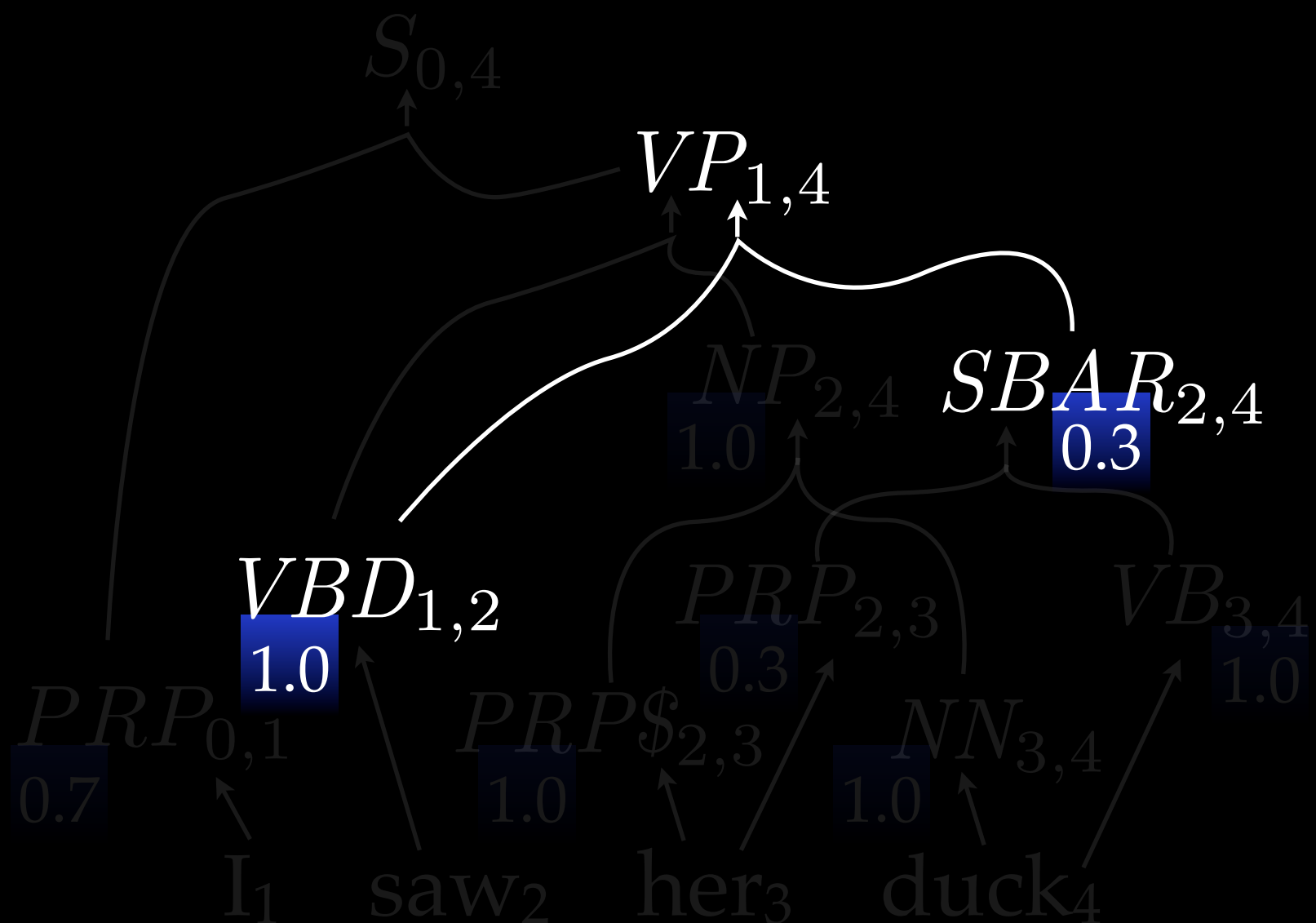
SBAR → PRP VB (1.0)

VB → duck (1.0)

VP → VBD NP (0.8)

VP → VBD SBAR (0.2)

VBD → saw (1.0)



# Computing Expectations

NN → duck (1.0)

NP → PRP\$ NN (1.0)

PRP → her (0.3)

PRP → I (0.7)

PRP\$ → her (1.0)

S → PRP VP (1.0)

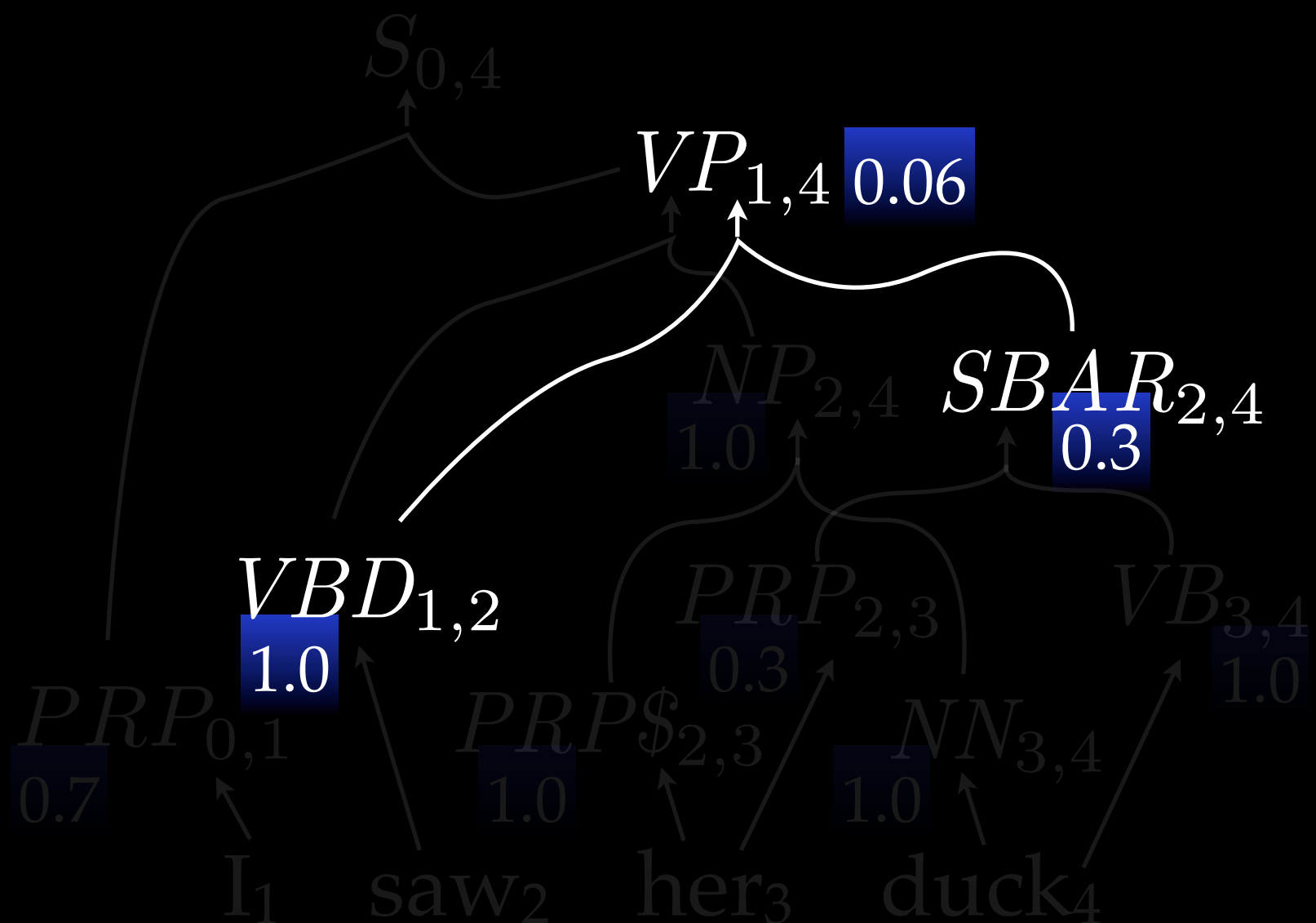
SBAR → PRP VB (1.0)

VB → duck (1.0)

VP → VBD NP (0.8)

VP → VBD SBAR (0.2)

VBD → saw (1.0)





# Computing Expectations

NN  $\rightarrow$  duck (1.0)

NP  $\rightarrow$  PRP\$ NN (1.0)

PRP  $\rightarrow$  her (0.3)

PRP  $\rightarrow$  I (0.7)

PRP\$  $\rightarrow$  her (1.0)

S  $\rightarrow$  PRP VP (1.0)

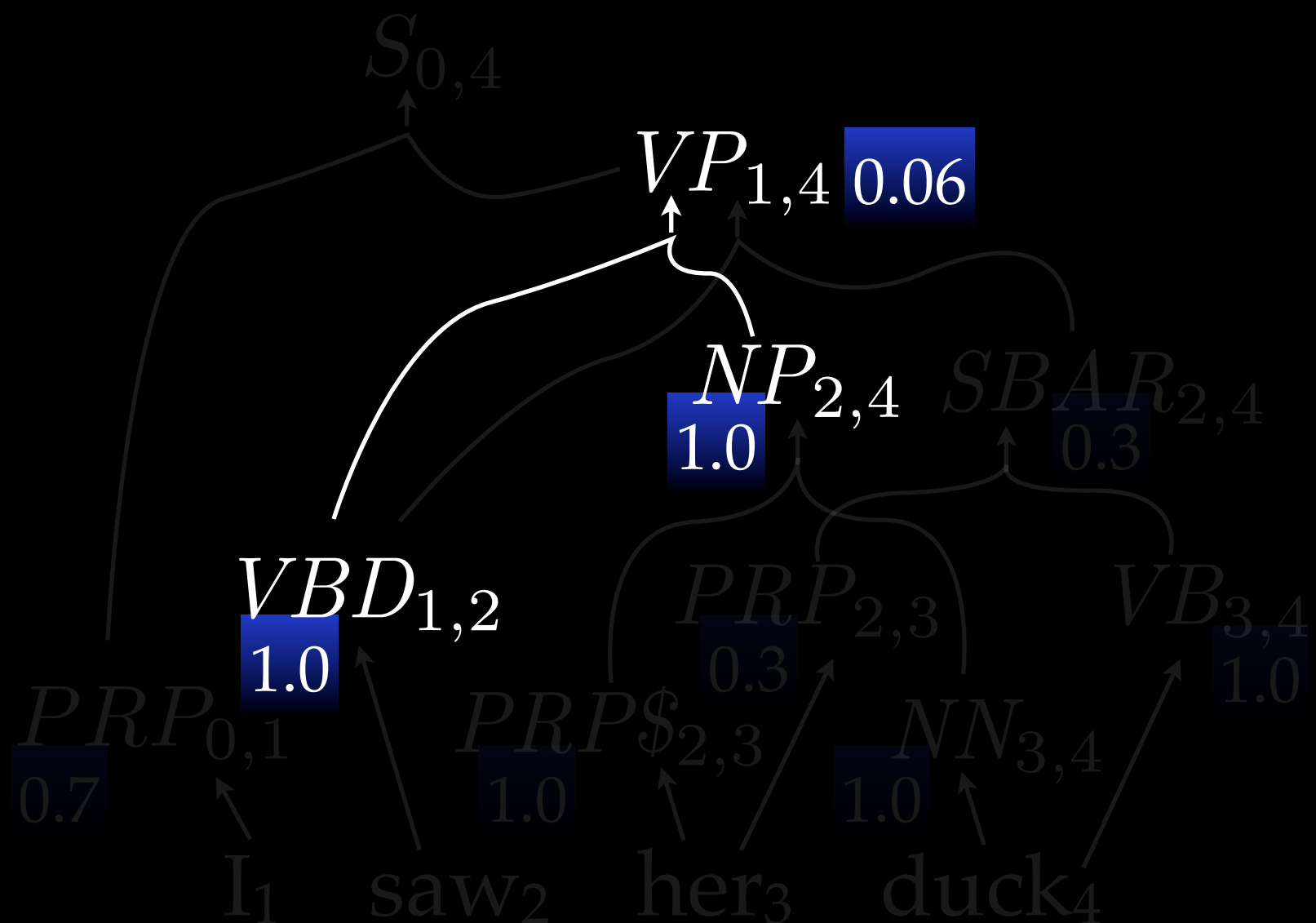
SBAR  $\rightarrow$  PRP VB (1.0)

VB  $\rightarrow$  duck (1.0)

VP  $\rightarrow$  VBD NP (0.8)

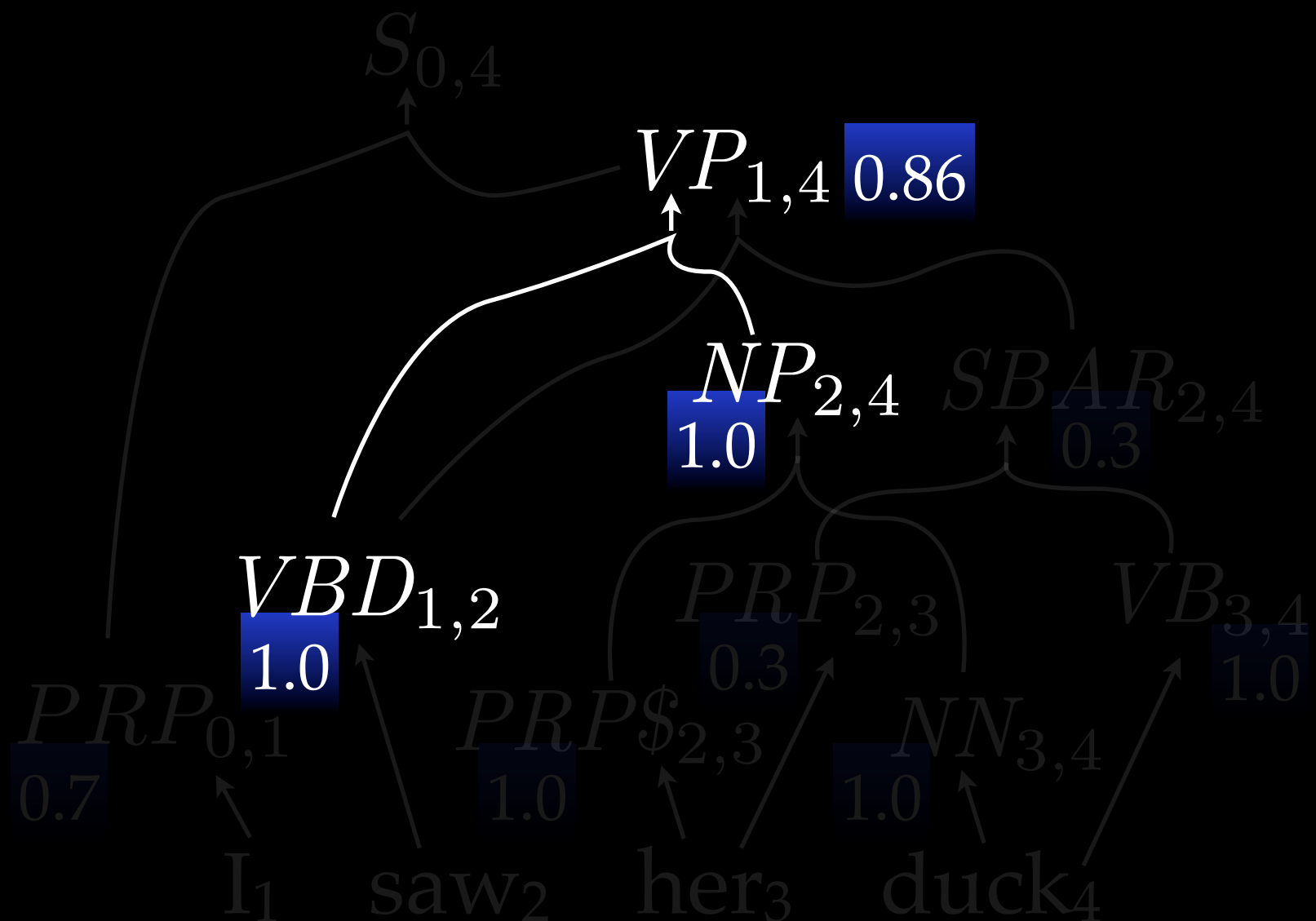
VP  $\rightarrow$  VBD SBAR (0.2)

VBD  $\rightarrow$  saw (1.0)



# Computing Expectations

$NN \rightarrow \text{duck}$	(1.0)
$NP \rightarrow \text{PRP\$ } NN$	(1.0)
$\text{PRP} \rightarrow \text{her}$	(0.3)
$\text{PRP} \rightarrow \text{I}$	(0.7)
$\text{PRP\$} \rightarrow \text{her}$	(1.0)
$S \rightarrow \text{PRP } VP$	(1.0)
$\text{SBAR} \rightarrow \text{PRP } VB$	(1.0)
$VB \rightarrow \text{duck}$	(1.0)
$VP \rightarrow \text{VBD } NP$	(0.8)
$VP \rightarrow \text{VBD } \text{SBAR}$	(0.2)
$\text{VBD} \rightarrow \text{saw}$	(1.0)



# Computing Expectations

NN → duck (1.0)

NP → PRP\$ NN (1.0)

PRP → her (0.3)

PRP → I (0.7)

PRP\$ → her (1.0)

S → PRP VP (1.0)

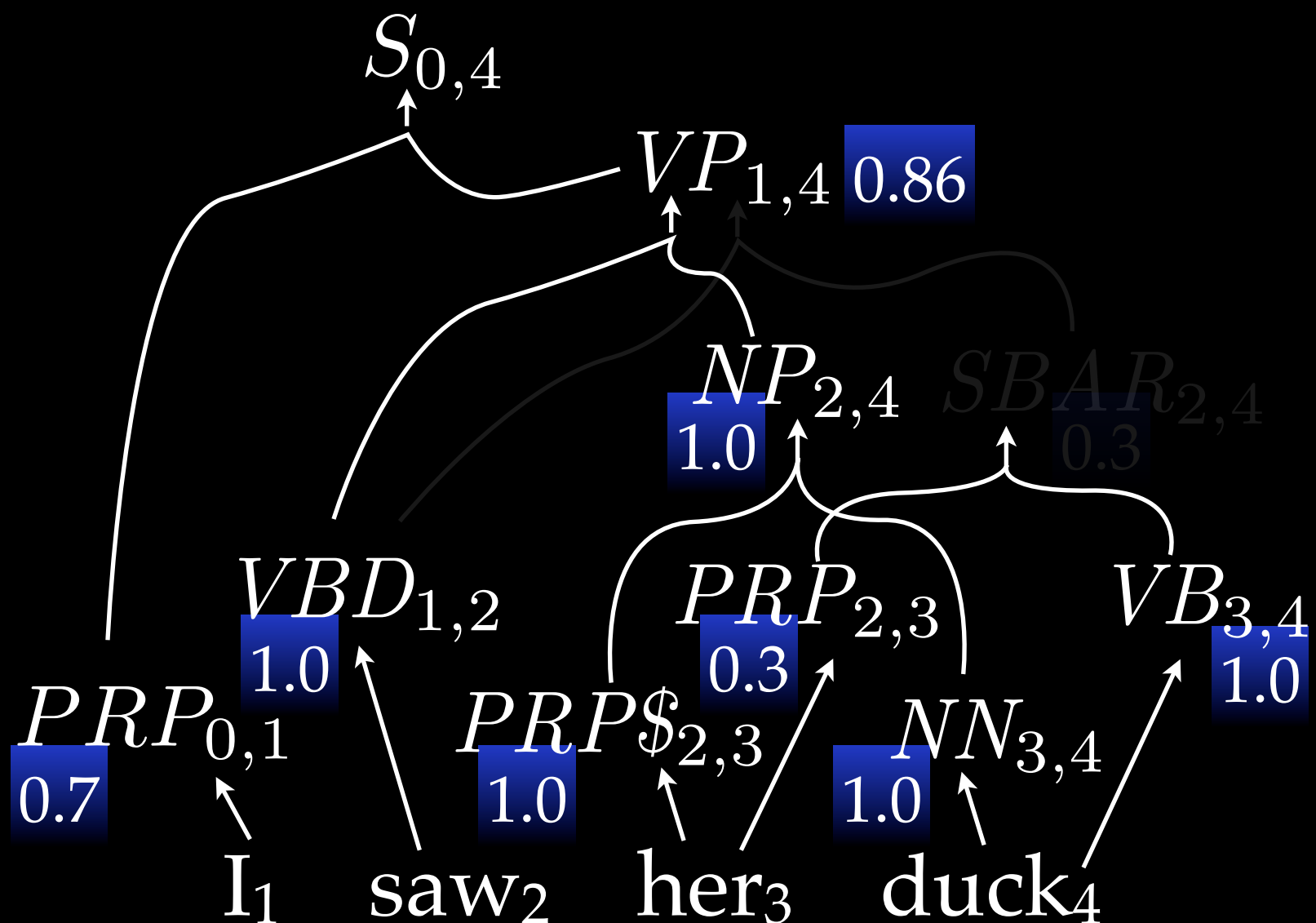
SBAR → PRP VB (1.0)

VB → duck (1.0)

VP → VBD NP (0.8)

VP → VBD SBAR (0.2)

VBD → saw (1.0)



# Computing Expectations

NN  $\rightarrow$  duck (1.0)

NP  $\rightarrow$  PRP\$ NN (1.0)

PRP  $\rightarrow$  her (0.3)

PRP  $\rightarrow$  I (0.7)

PRP\$  $\rightarrow$  her (1.0)

S  $\rightarrow$  PRP VP (1.0)

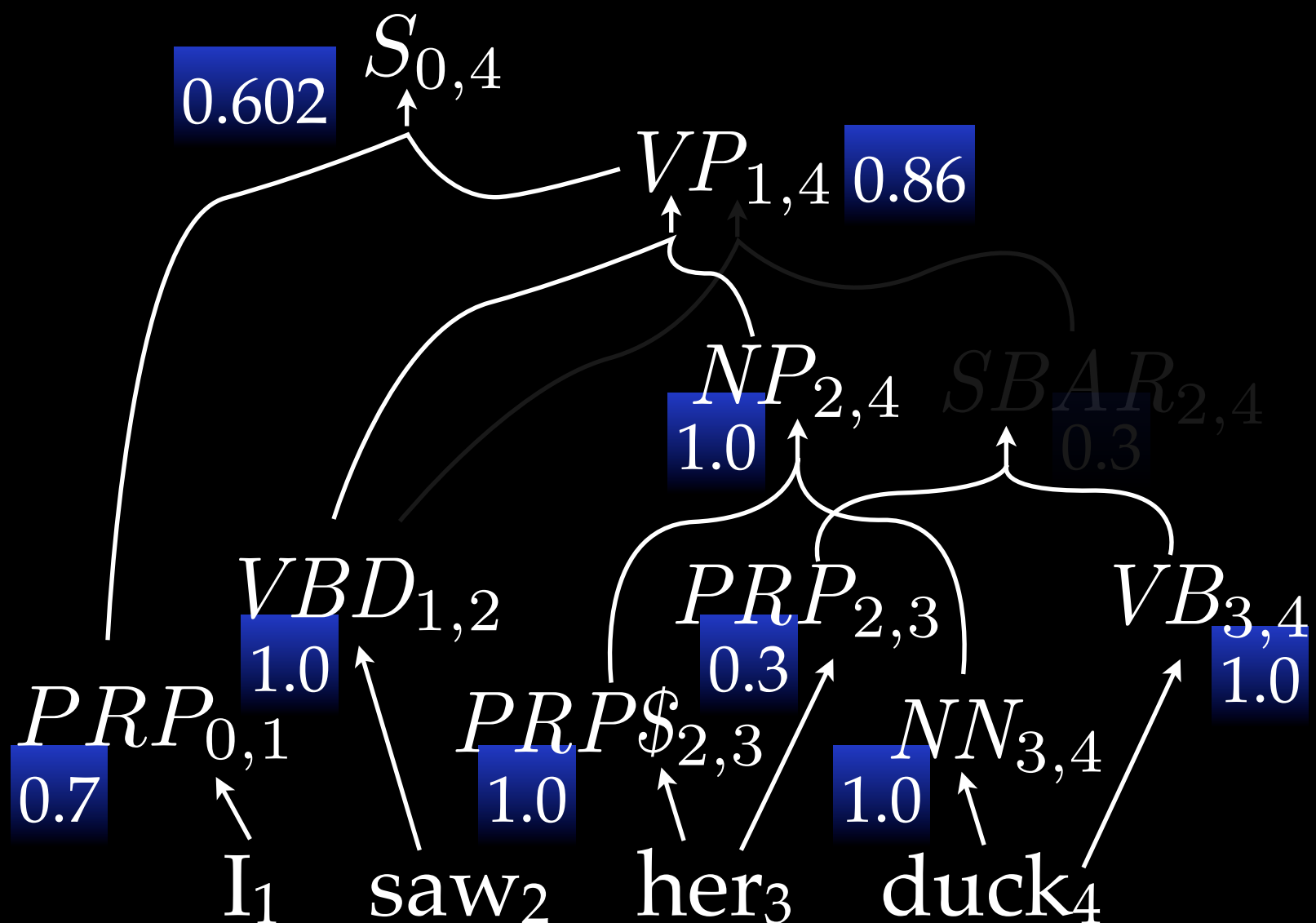
SBAR  $\rightarrow$  PRP VB (1.0)

VB  $\rightarrow$  duck (1.0)

VP  $\rightarrow$  VBD NP (0.8)

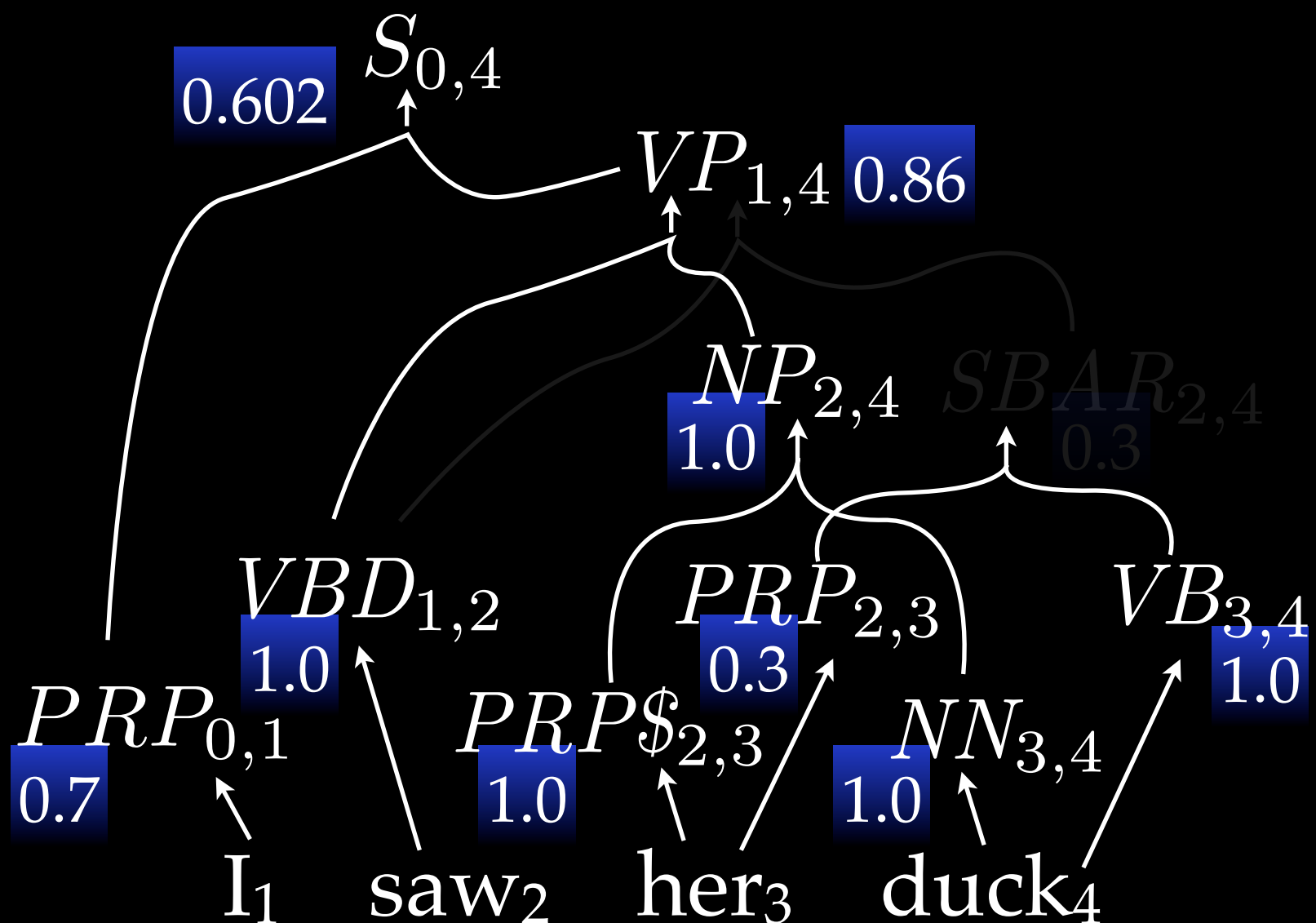
VP  $\rightarrow$  VBD SBAR (0.2)

VBD  $\rightarrow$  saw (1.0)



# Computing Expectations

$$X_{i,j} = X_{i,j} + (Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$



# Semiring Parsing

$$X_{i,j} \leftarrow Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ)$$

$$X_{i,j} = \max(X_{i,j}, Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$X_{i,j} = X_{i,j} + (Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

# Semiring Parsing

$$X_{i,j} = X_{i,j} \vee (Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ))$$

$$X_{i,j} = \max(X_{i,j}, Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$X_{i,j} = X_{i,j} + (Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

# Semiring Parsing

$$X_{i,j} = X_{i,j} \vee (Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ))$$

$$\langle \{T, F\}, \vee, \wedge \rangle$$

$$X_{i,j} = \max(X_{i,j}, Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$X_{i,j} = X_{i,j} + (Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$



# Semiring Parsing

$$X_{i,j} = X_{i,j} \vee (Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ))$$

$$\langle \{T, F\}, \vee, \wedge \rangle$$

$$X_{i,j} = \max(X_{i,j}, Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$\langle \mathbb{R}, \max, \times \rangle$$

$$X_{i,j} = X_{i,j} + (Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

# Semiring Parsing

$$X_{i,j} = X_{i,j} \vee (Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ))$$

$$\langle \{T, F\}, \vee, \wedge \rangle$$

$$X_{i,j} = \max(X_{i,j}, Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$\langle \mathbb{R}, \max, \times \rangle$$

$$X_{i,j} = X_{i,j} + (Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$\langle \mathbb{R}, +, \times \rangle$$

# Semiring Parsing

$$X_{i,j} = X_{i,j} \vee (Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ))$$

$$\langle \{T, F\}, \vee, \wedge \rangle$$

$$X_{i,j} = \max(X_{i,j}, Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$\langle \mathbb{R}, \max, \times \rangle$$

$$X_{i,j} = X_{i,j} + (Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$\langle \mathbb{R}, +, \times \rangle$$

$$X_{i,j} = X_{i,j} \oplus (Y_{i,k} \otimes Z_{k,j} \otimes R(X \rightarrow YZ))$$

# Semiring Parsing

$$X_{i,j} = X_{i,j} \vee (Y_{i,k} \wedge Z_{k,j} \wedge (X \rightarrow YZ))$$

$$\text{boolean} \quad \langle \{T, F\}, \vee, \wedge \rangle$$

$$X_{i,j} = \max(X_{i,j}, Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$\text{Viterbi} \quad \langle \mathbb{R}, \max, \times \rangle$$

$$X_{i,j} = X_{i,j} + (Y_{i,k} \times Z_{k,j} \times p(X \rightarrow YZ))$$

$$\text{inside} \quad \langle \mathbb{R}, +, \times \rangle$$

$$X_{i,j} = X_{i,j} \oplus (Y_{i,k} \otimes Z_{k,j} \otimes R(X \rightarrow YZ))$$

# Parsing

## Is Intersection!

$NN_{3,4} \rightarrow \text{duck}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{her}$

$PRP_{0,1} \rightarrow \text{I}$

$PRP\$_{2,3} \rightarrow \text{her}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

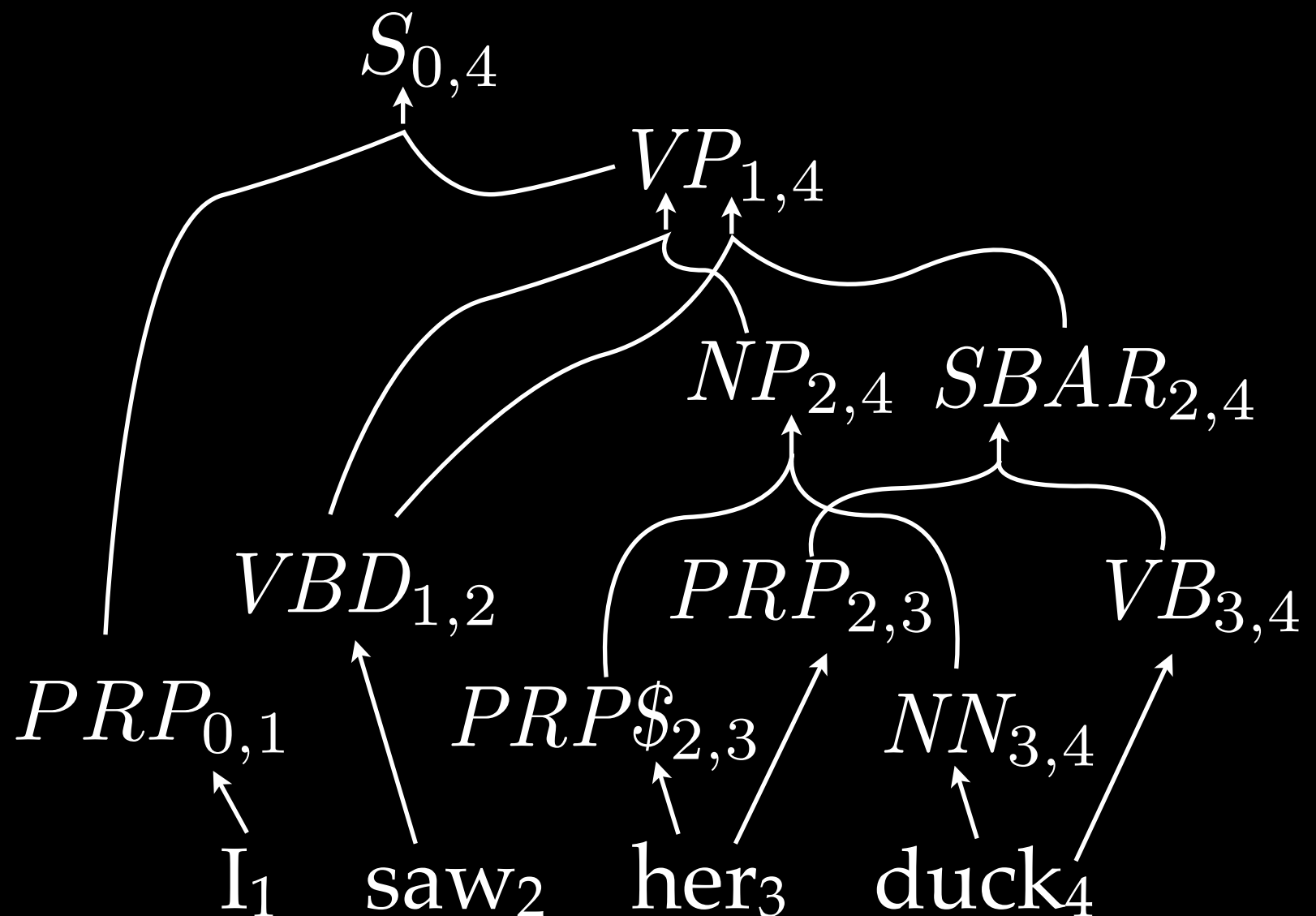
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{duck}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

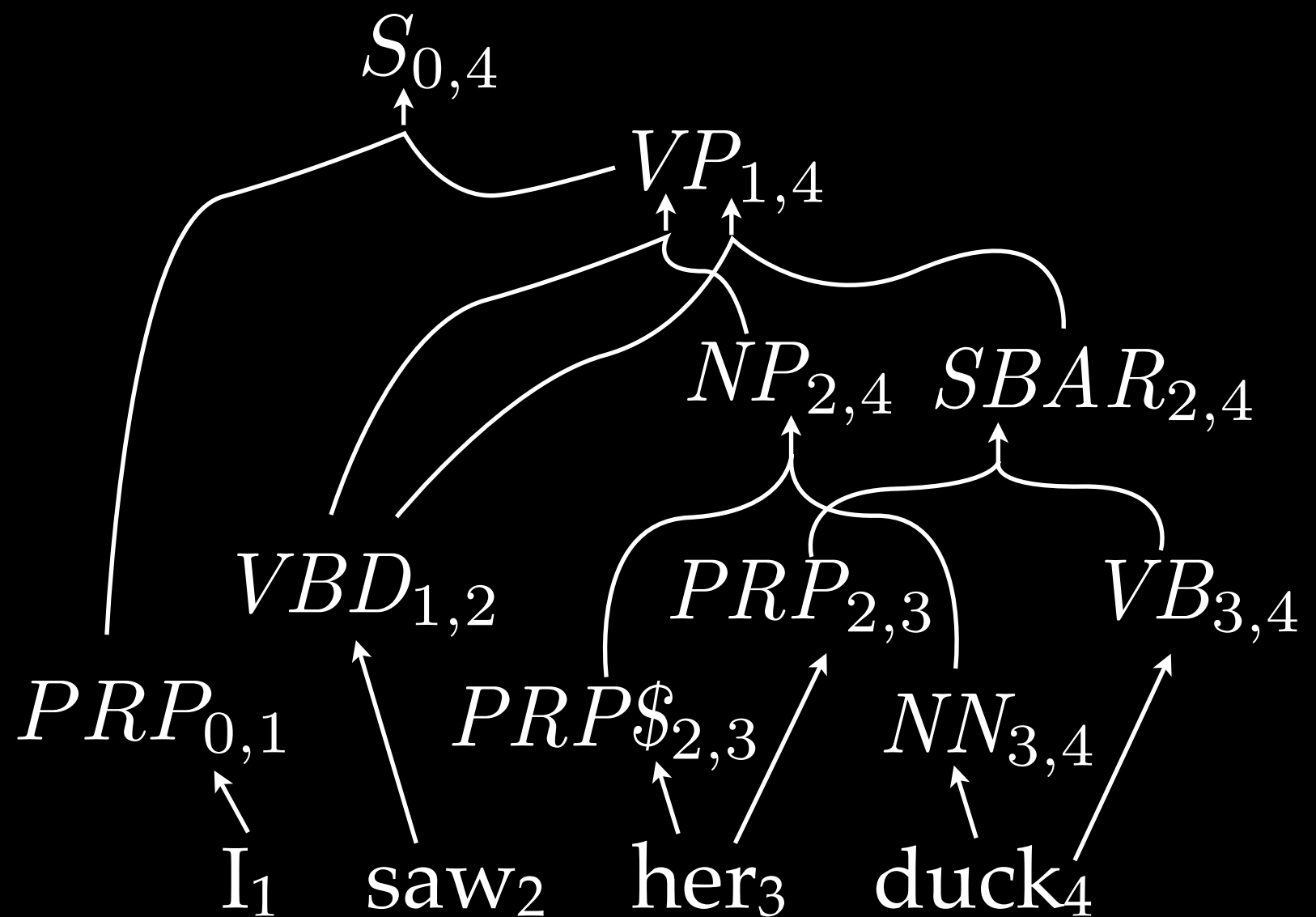
$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{saw}$



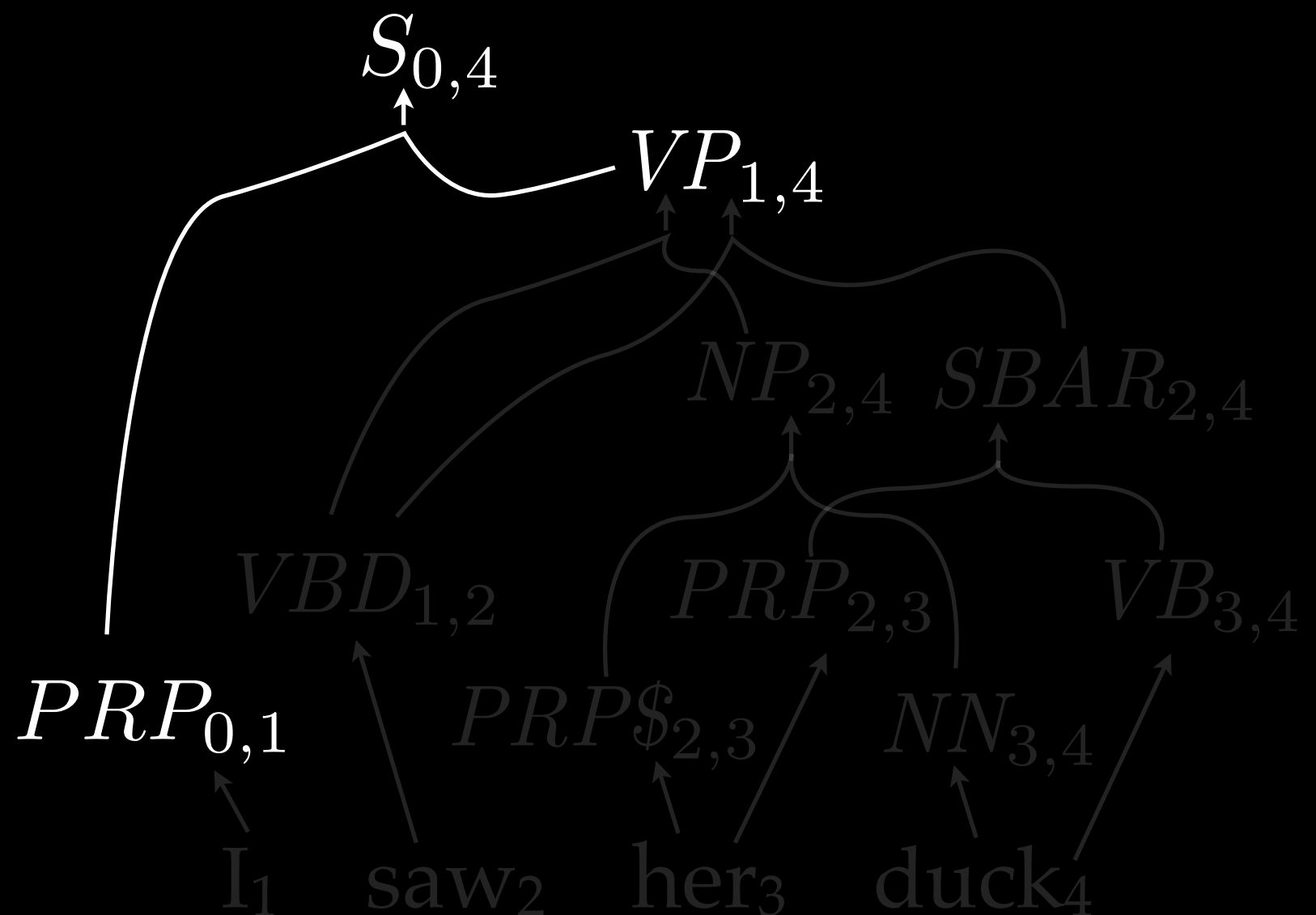
# Parsing

Is Intersection!



# Parsing

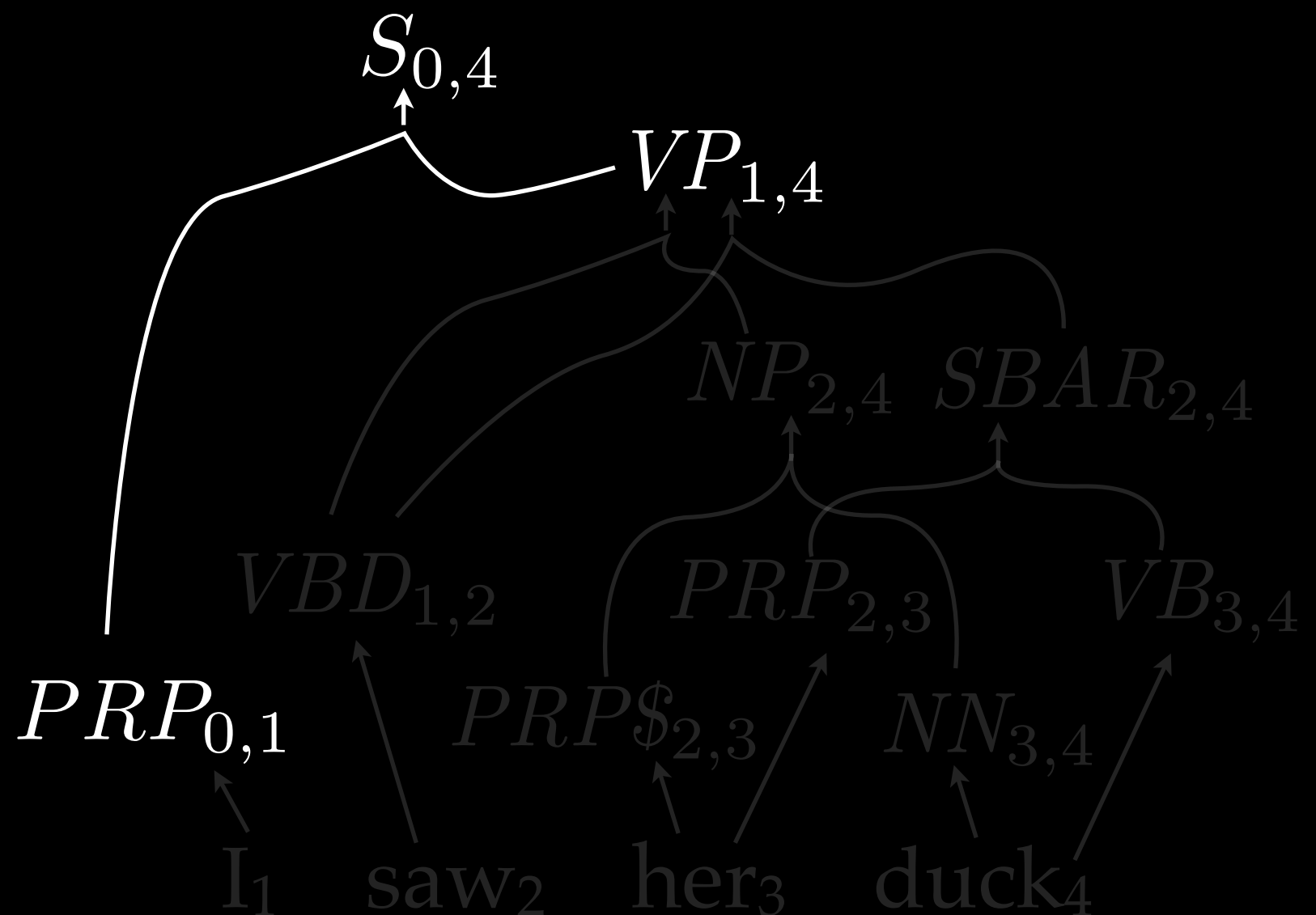
Is Intersection!



# Parsing

## Is Intersection!

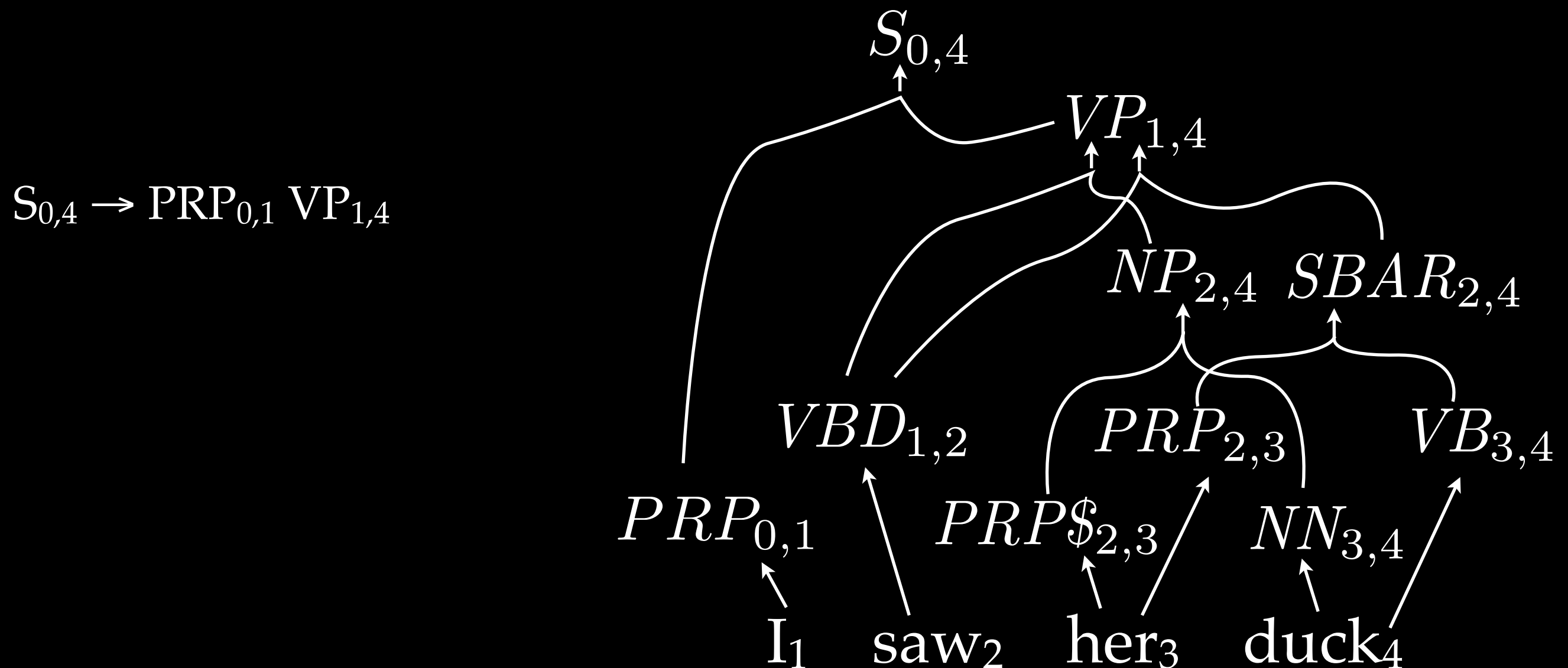
$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$





# Parsing

Is Intersection!



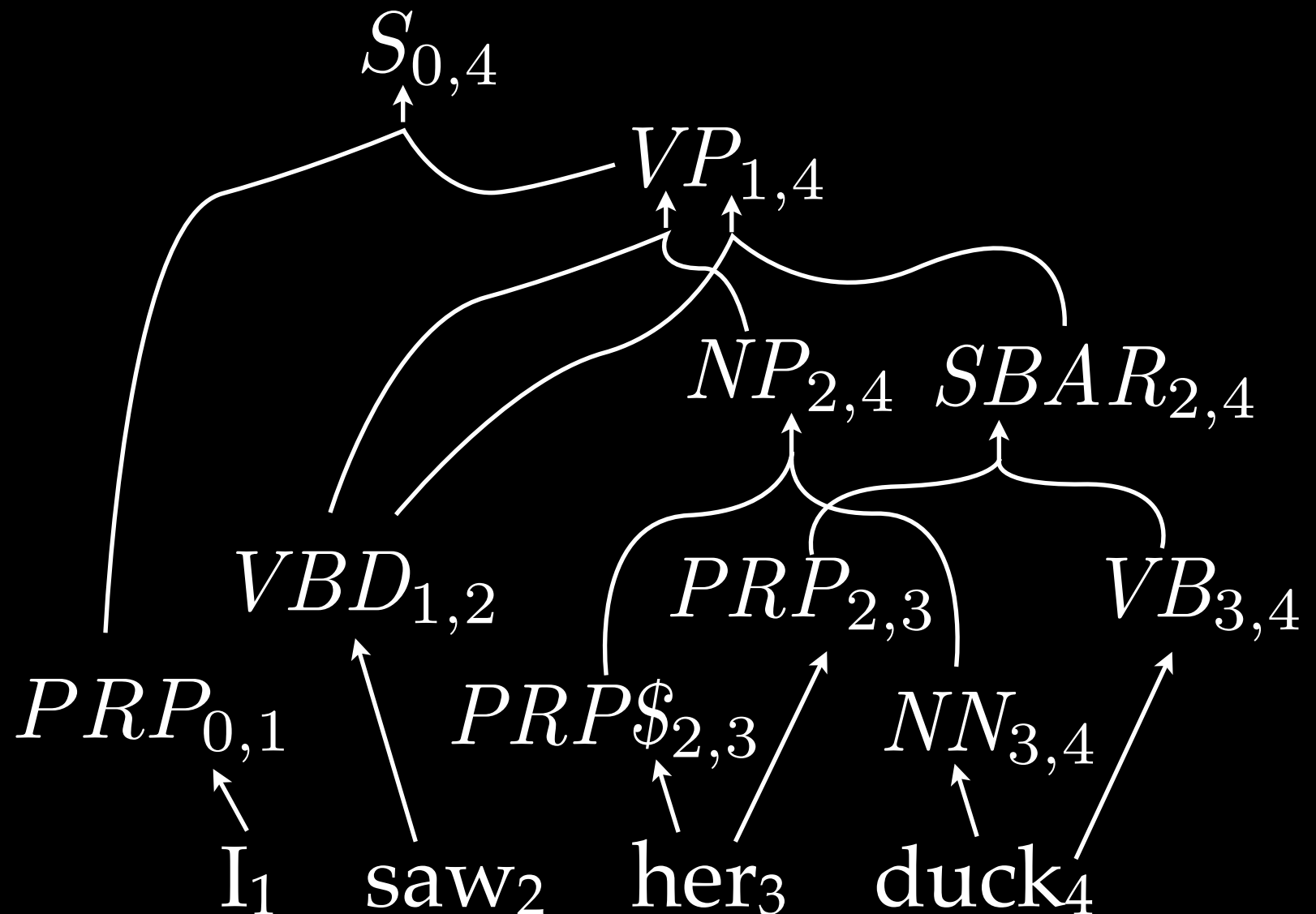
# Parsing Is Intersection!

# NN<sub>3,4</sub> → duck

$$\text{NP}_{2,4} \rightarrow \text{PRP\$}_{2,3} \text{NN}_{3,4}$$
$$\text{PRP}_{2,3} \rightarrow \text{her}$$
$$\text{PRP}_{0,1} \rightarrow \text{I}$$
PRP\$<sub>2,3</sub> → her
$$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$$

SBAR<sub>2,4</sub> → PRP<sub>2,3</sub> VB<sub>3,4</sub>

# VB<sub>3,4</sub> → duck

$$\text{VP}_{1,4} \rightarrow \text{VBD}_{1,2} \text{NP}_{2,4}$$
$$\text{VP}_{1,4} \rightarrow \text{VBD}_{1,2} \text{ SBAR}_{2,4}$$
$$\text{VBD}_{1,2} \rightarrow \text{saw}$$


# Parsing

## Is Intersection!

$NN_{3,4} \rightarrow \text{duck}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{her}$

$PRP_{0,1} \rightarrow \text{I}$

$PRP\$_{2,3} \rightarrow \text{her}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{duck}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{saw}$

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$

# Parsing

## Is Intersection!

$NN_{3,4} \rightarrow \text{duck}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{her}$

$PRP_{0,1} \rightarrow \text{I}$

$PRP\$_{2,3} \rightarrow \text{her}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{duck}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{saw}$

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$

yo vi ella agacharse

yo vi su pato

# Synchronous Parsing as Intersection

# Synchronous Parsing as Intersection

- Parse the English sentence (intersection).

# Synchronous Parsing as Intersection

- Parse the English sentence (intersection).
- Project grammar into French.

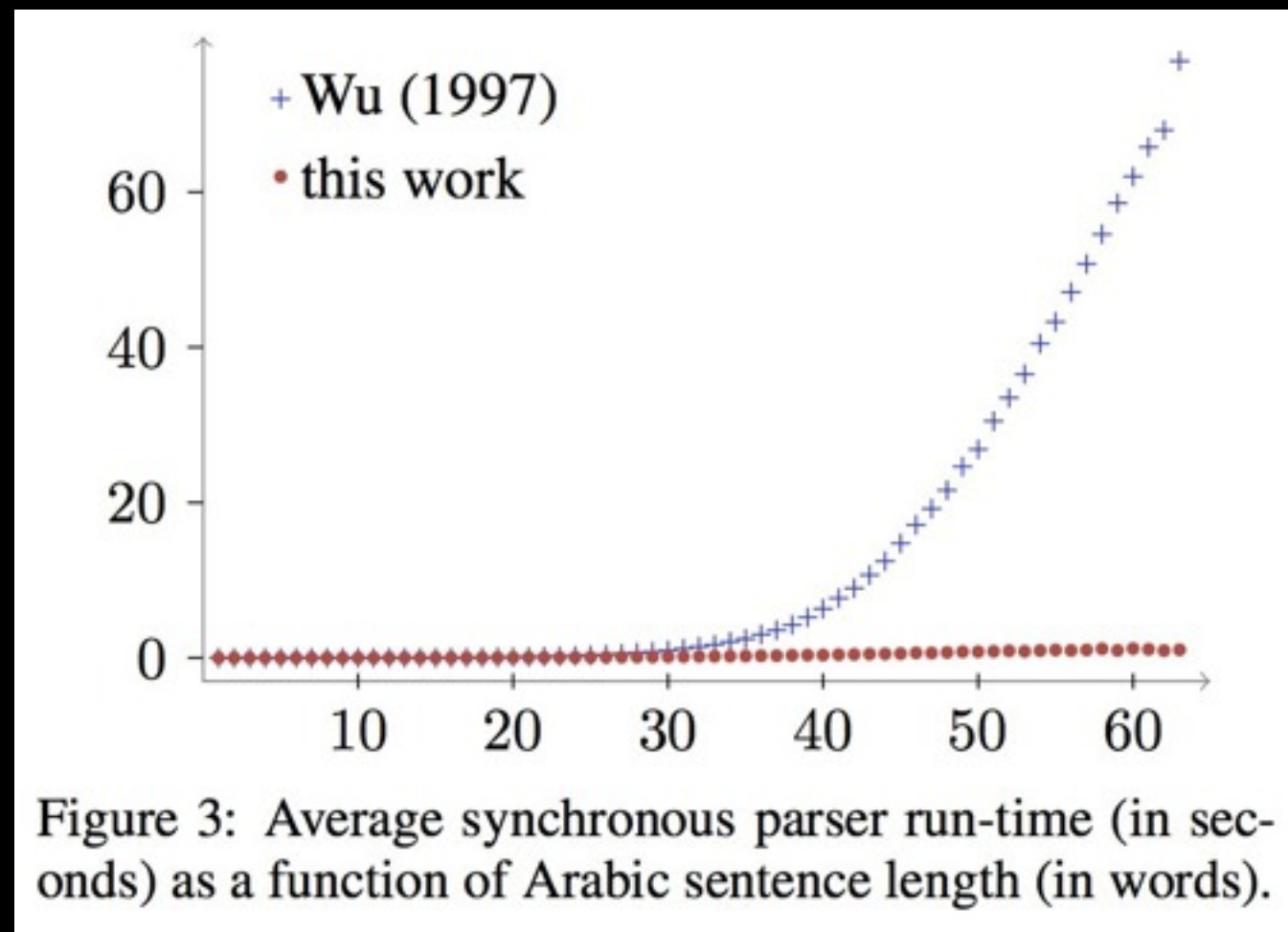
# Synchronous Parsing as Intersection

- Parse the English sentence (intersection).
- Project grammar into French.
- Parse the French sentence (intersection).



# Synchronous Parsing as Intersection

- Parse the English sentence (intersection).
- Project grammar into French.
- Parse the French sentence (intersection).



Dyer, NAACL 2010

# Translation as Intersection?

$NN_{3,4} \rightarrow \text{duck}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{her}$

$PRP_{0,1} \rightarrow \text{I}$

$PRP\$_{2,3} \rightarrow \text{her}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{duck}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{saw}$

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$

yo vi ella agacharse

yo vi su pato

# Translation as Intersection?

Observation: target grammar generates a *finite language*

$NN_{3,4} \rightarrow \text{duck}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{her}$

$PRP_{0,1} \rightarrow \text{I}$

$PRP\$_{2,3} \rightarrow \text{her}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{duck}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{saw}$

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$

yo vi ella agacharse

yo vi su pato

# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$

# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$

# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

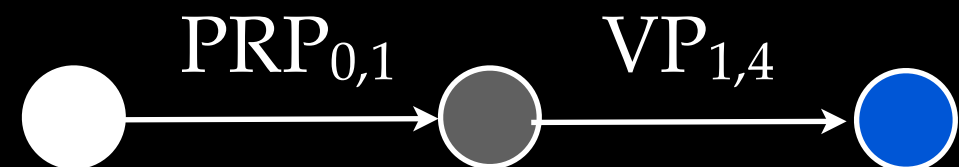
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

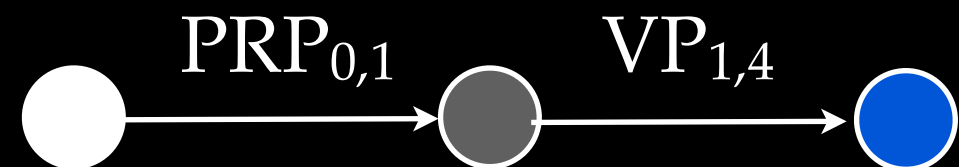
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

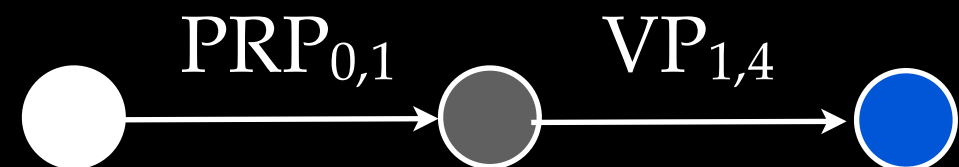
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$





# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

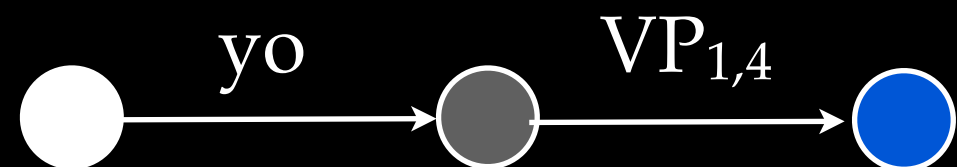
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

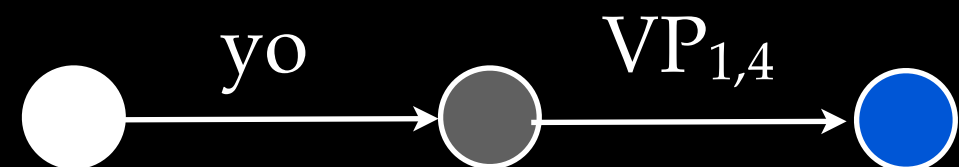
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

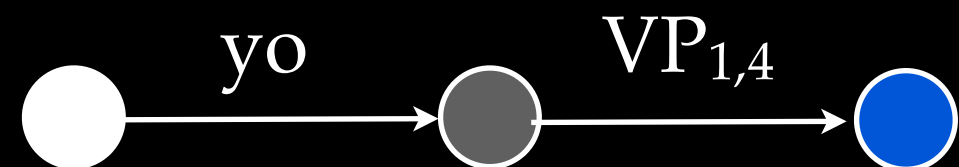
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

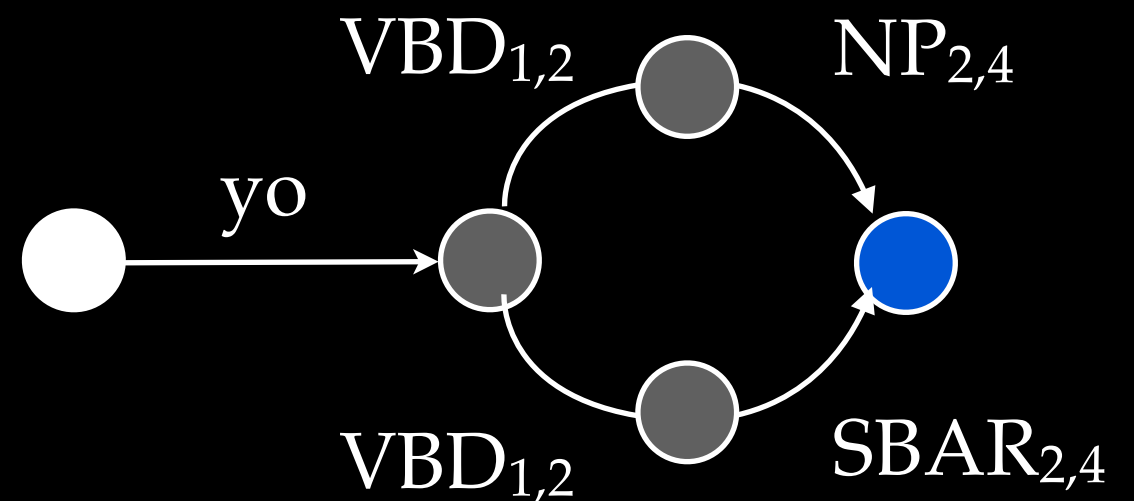
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

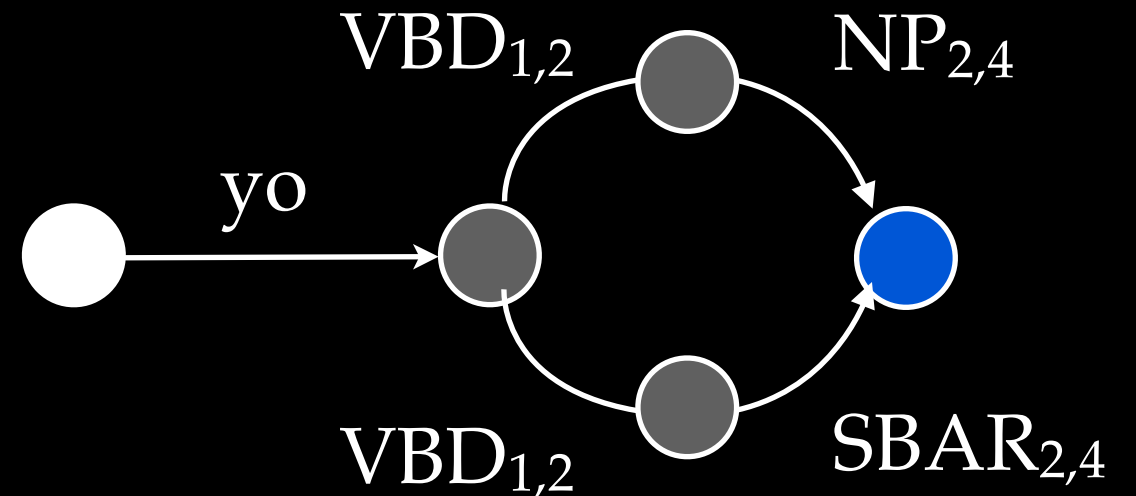
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

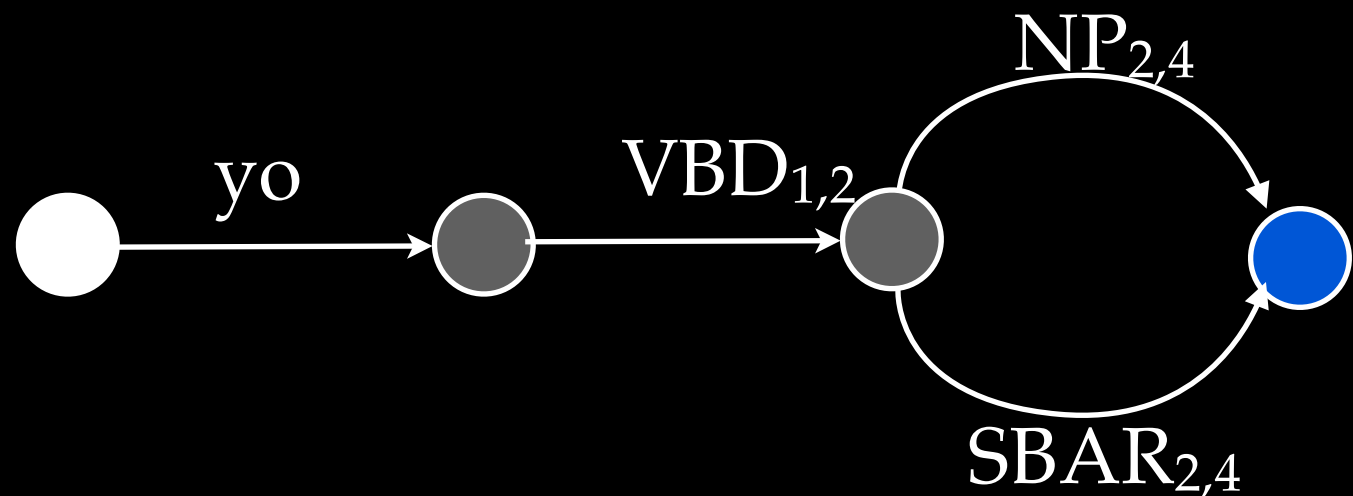
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

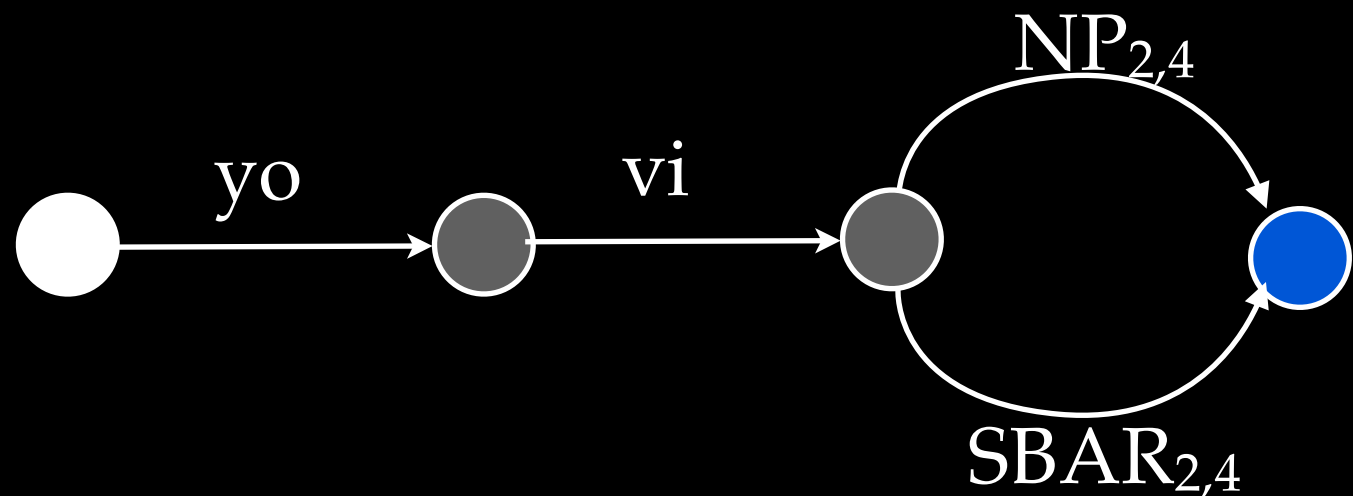
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

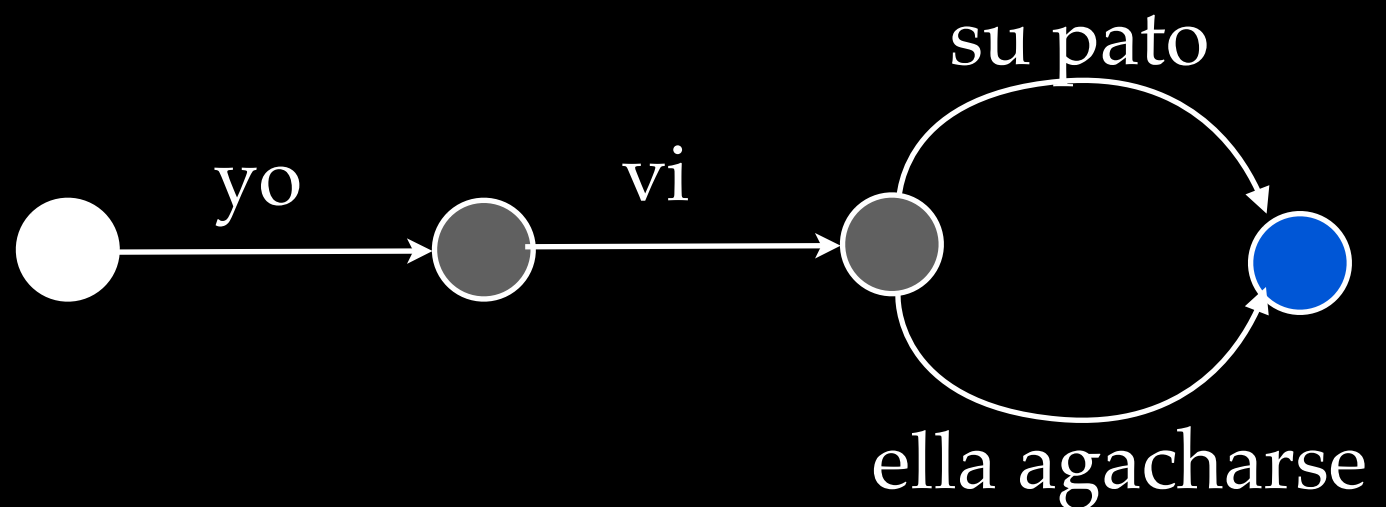
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$





# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

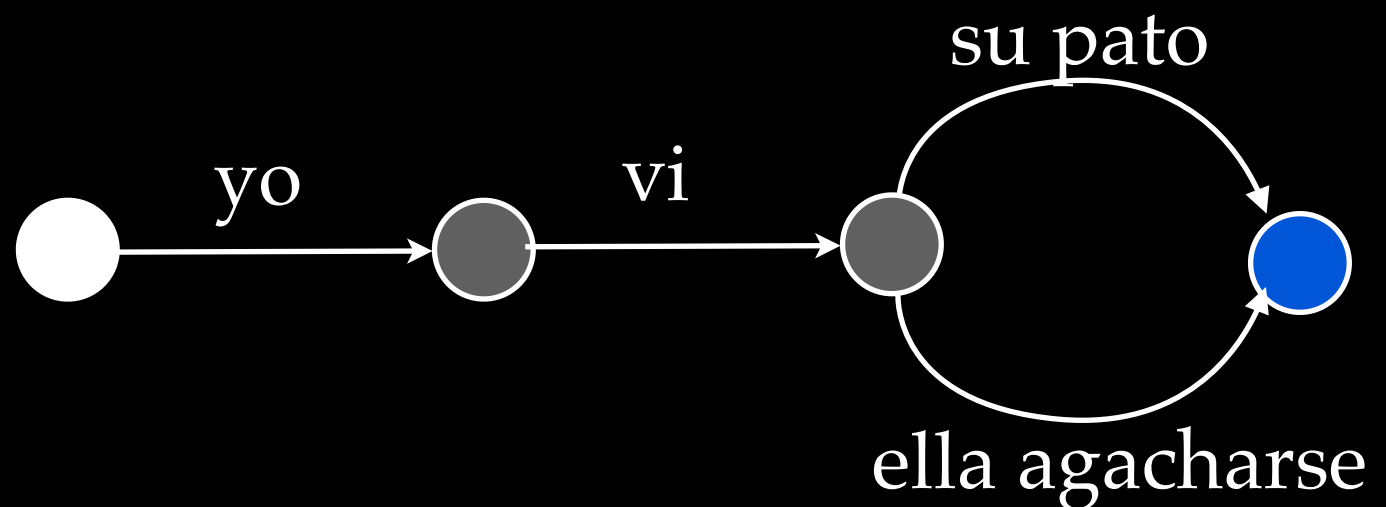
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



Better: lazy algorithm

# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

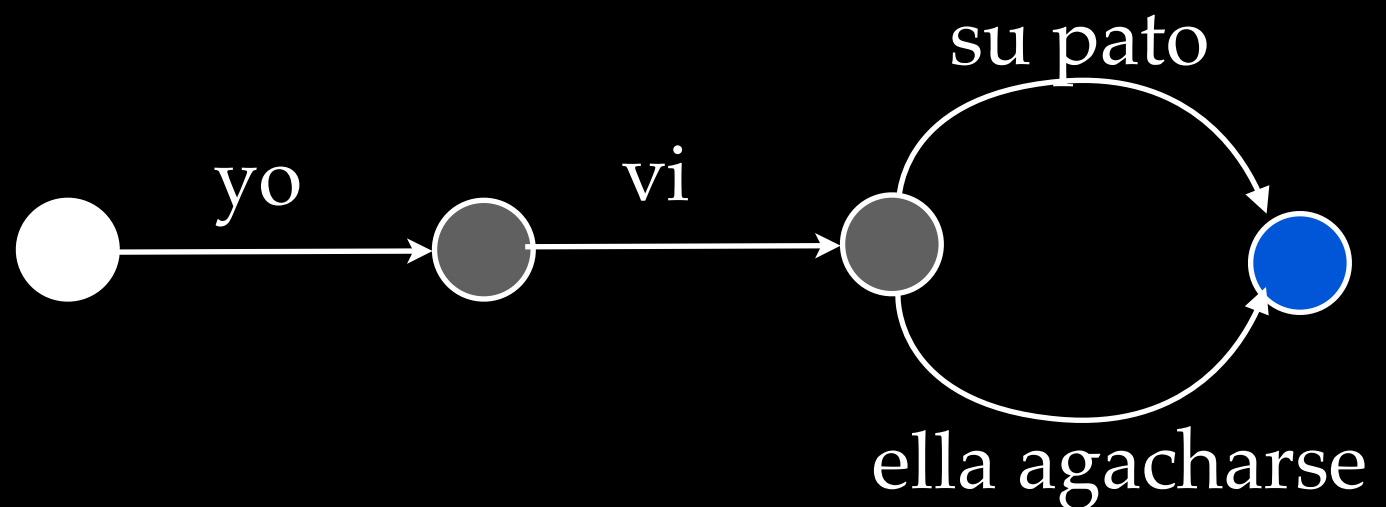
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$



Better: lazy algorithm

Even better: convert to PDA

# Translation as Intersection?

$NN_{3,4} \rightarrow \text{pato}$

$NP_{2,4} \rightarrow PRP\$_{2,3} NN_{3,4}$

$PRP_{2,3} \rightarrow \text{su}$

$PRP_{0,1} \rightarrow \text{yo}$

$PRP\$_{2,3} \rightarrow \text{ella}$

$S_{0,4} \rightarrow PRP_{0,1} VP_{1,4}$

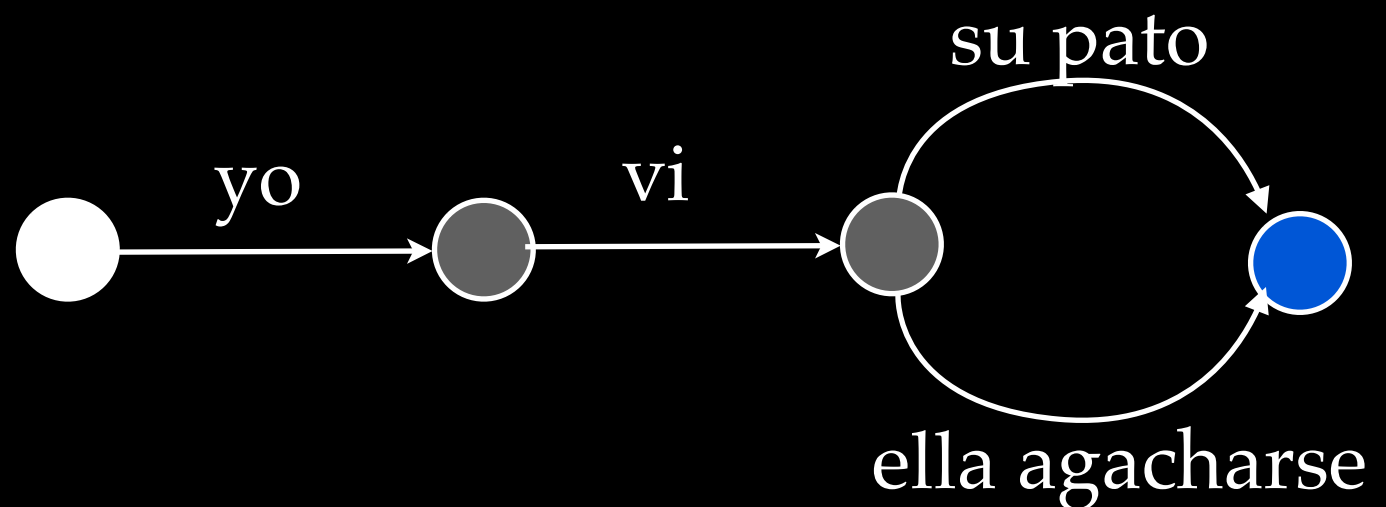
$SBAR_{2,4} \rightarrow PRP_{2,3} VB_{3,4}$

$VB_{3,4} \rightarrow \text{agacharse}$

$VP_{1,4} \rightarrow VBD_{1,2} NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2} SBAR_{2,4}$

$VBD_{1,2} \rightarrow \text{vi}$

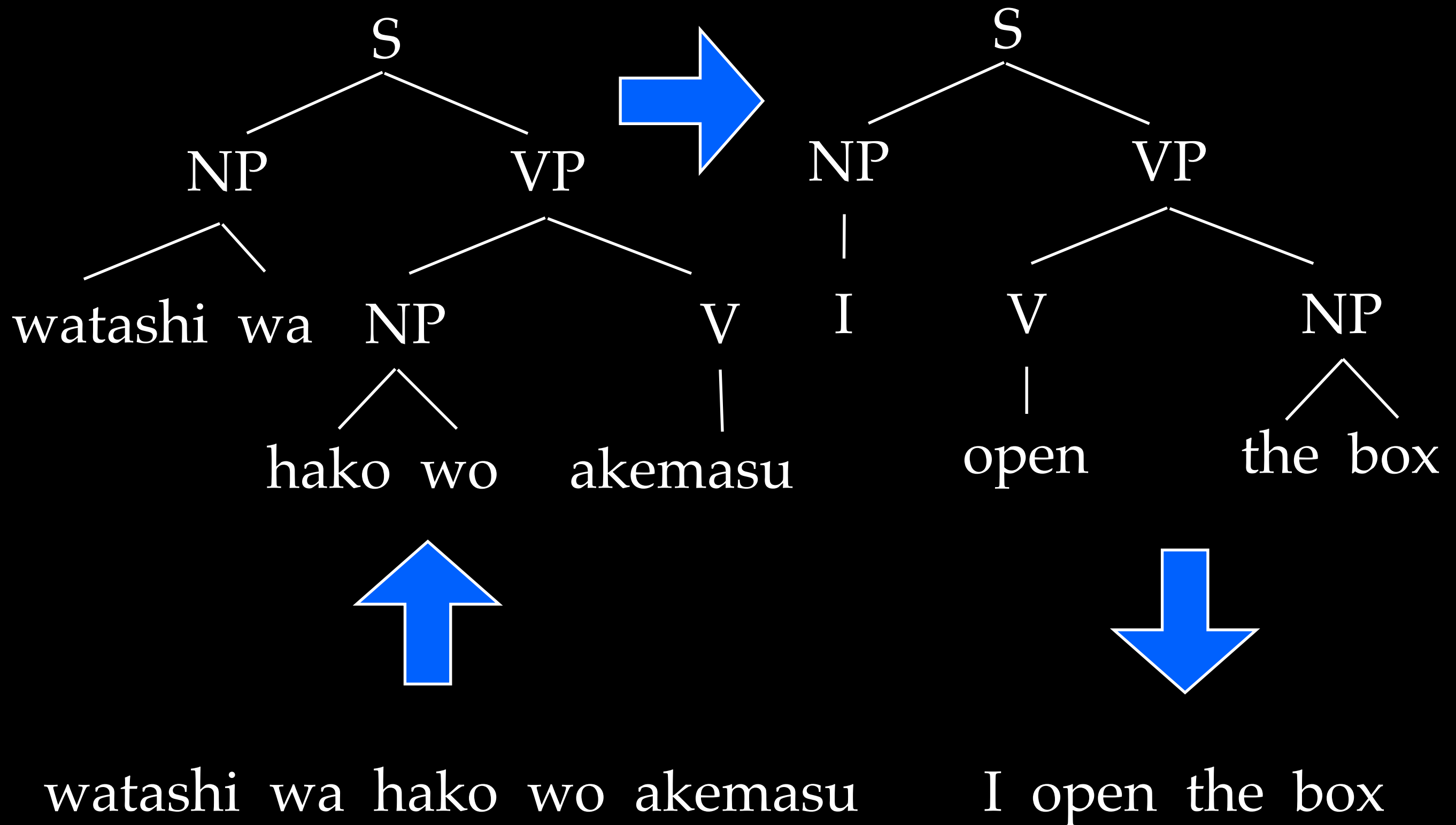


Better: lazy algorithm

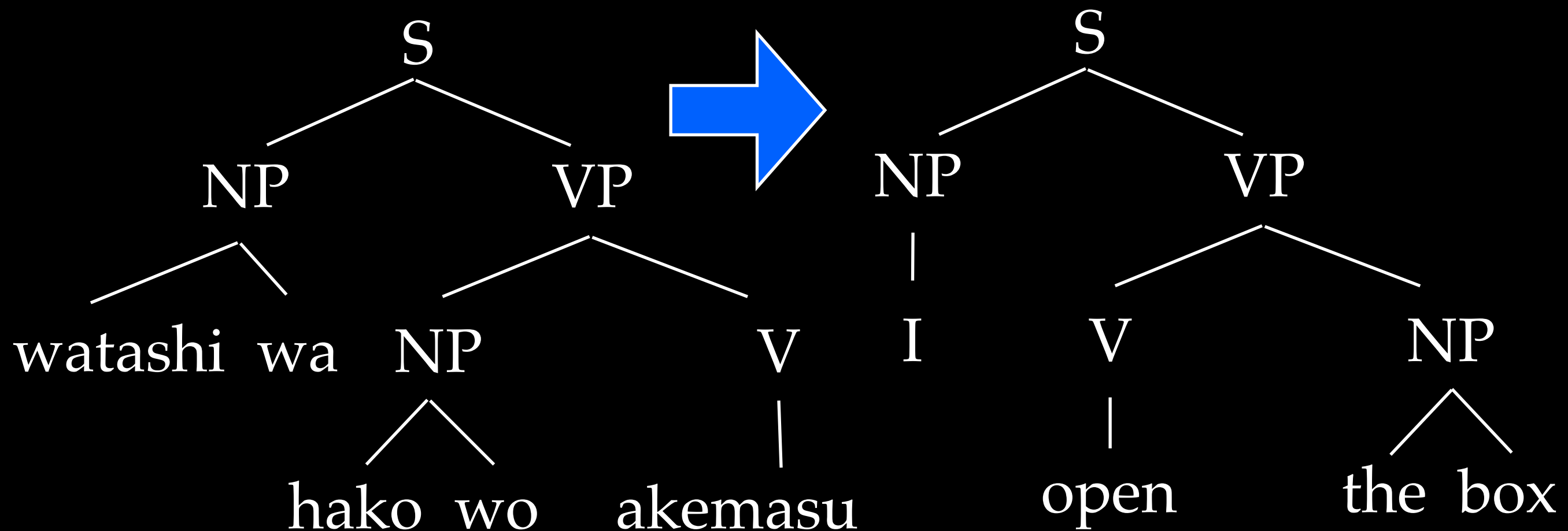
Even better: convert to PDA

Cambridge: best NIST 2009 Arabic system

# Parting Thoughts



# Parting Thoughts

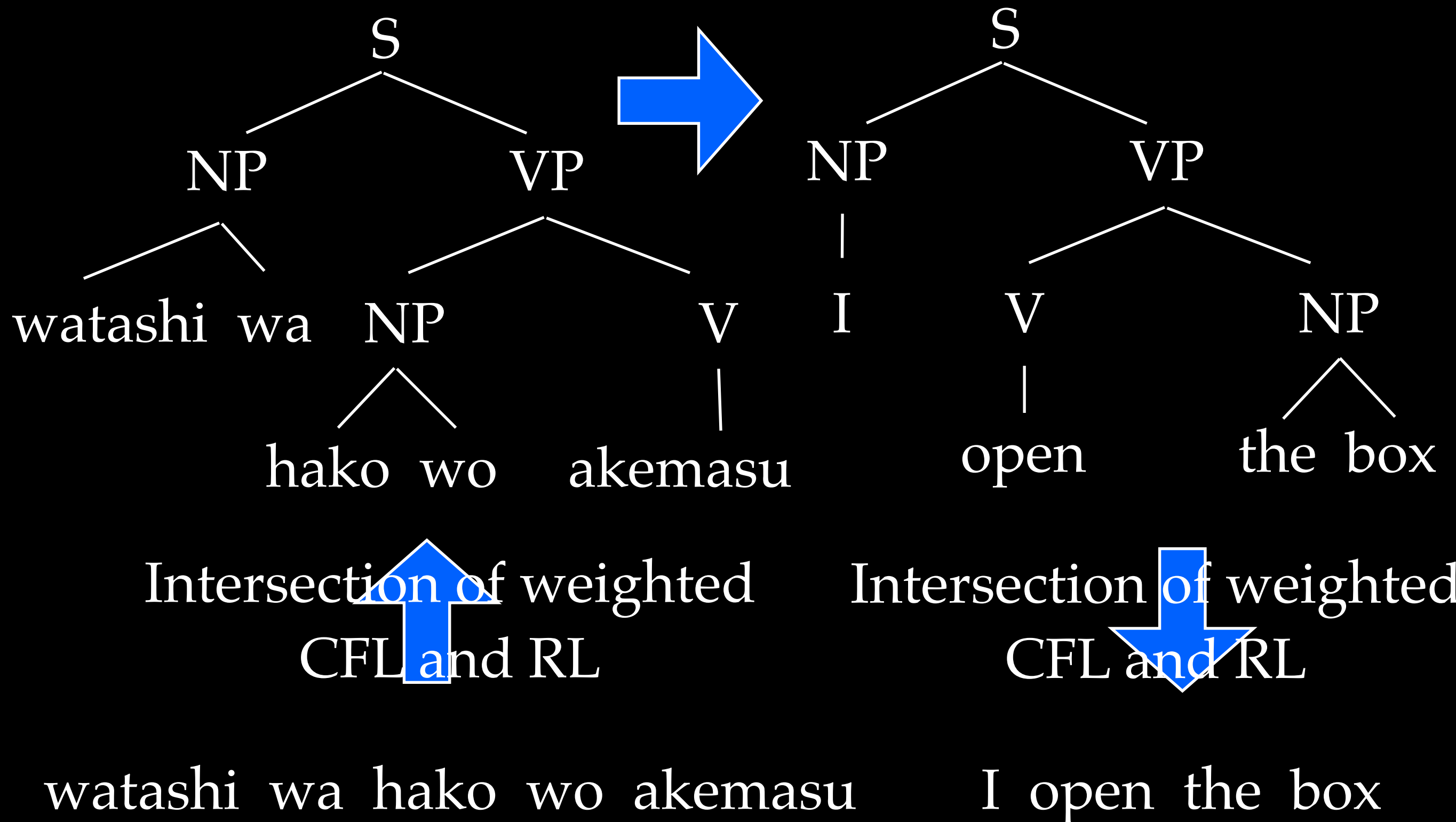


Intersection of weighted  
CFL and RL

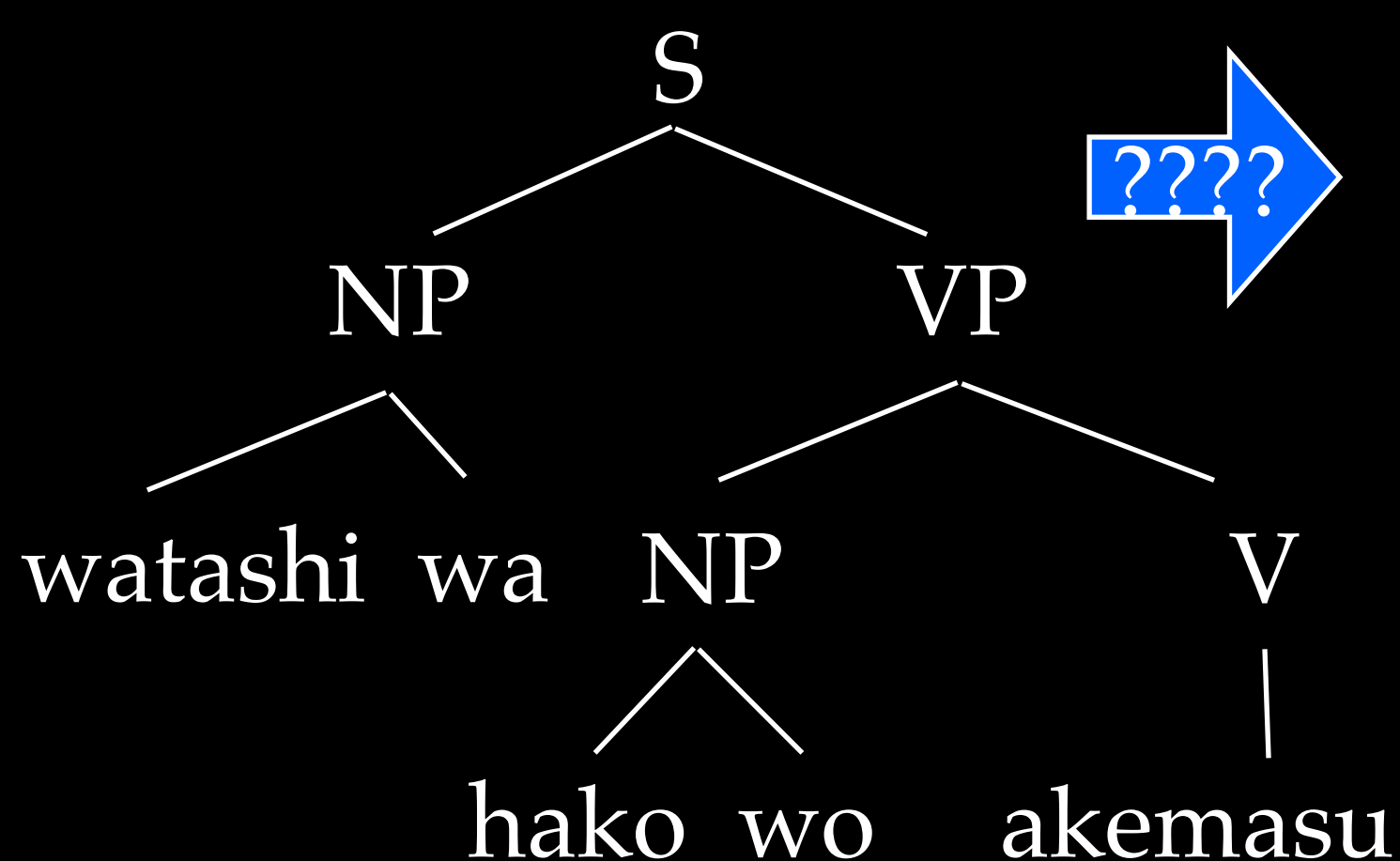
*watashi wa hako wo akemasu*

*I open the box*

# Parting Thoughts

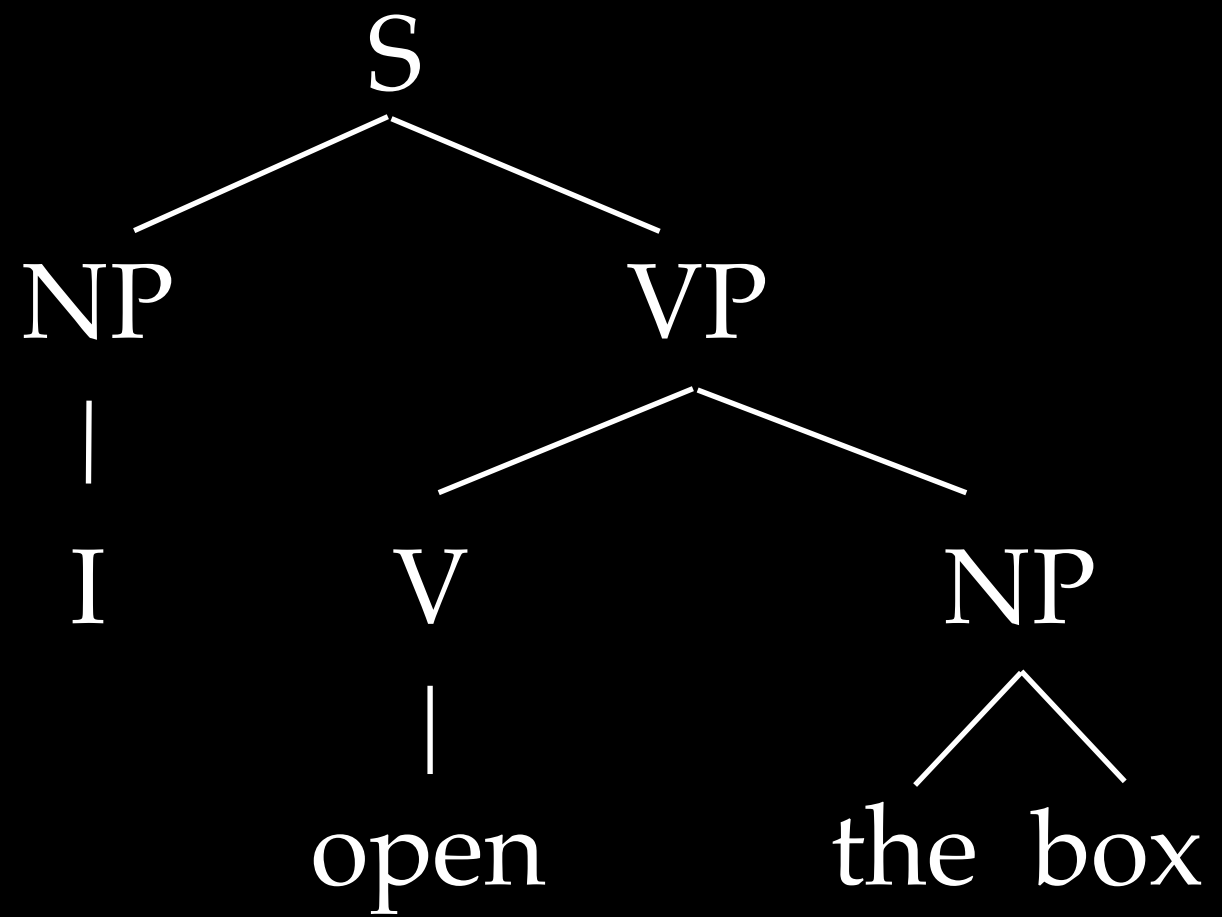


# Parting Thoughts



Intersection of weighted  
CFL and RL

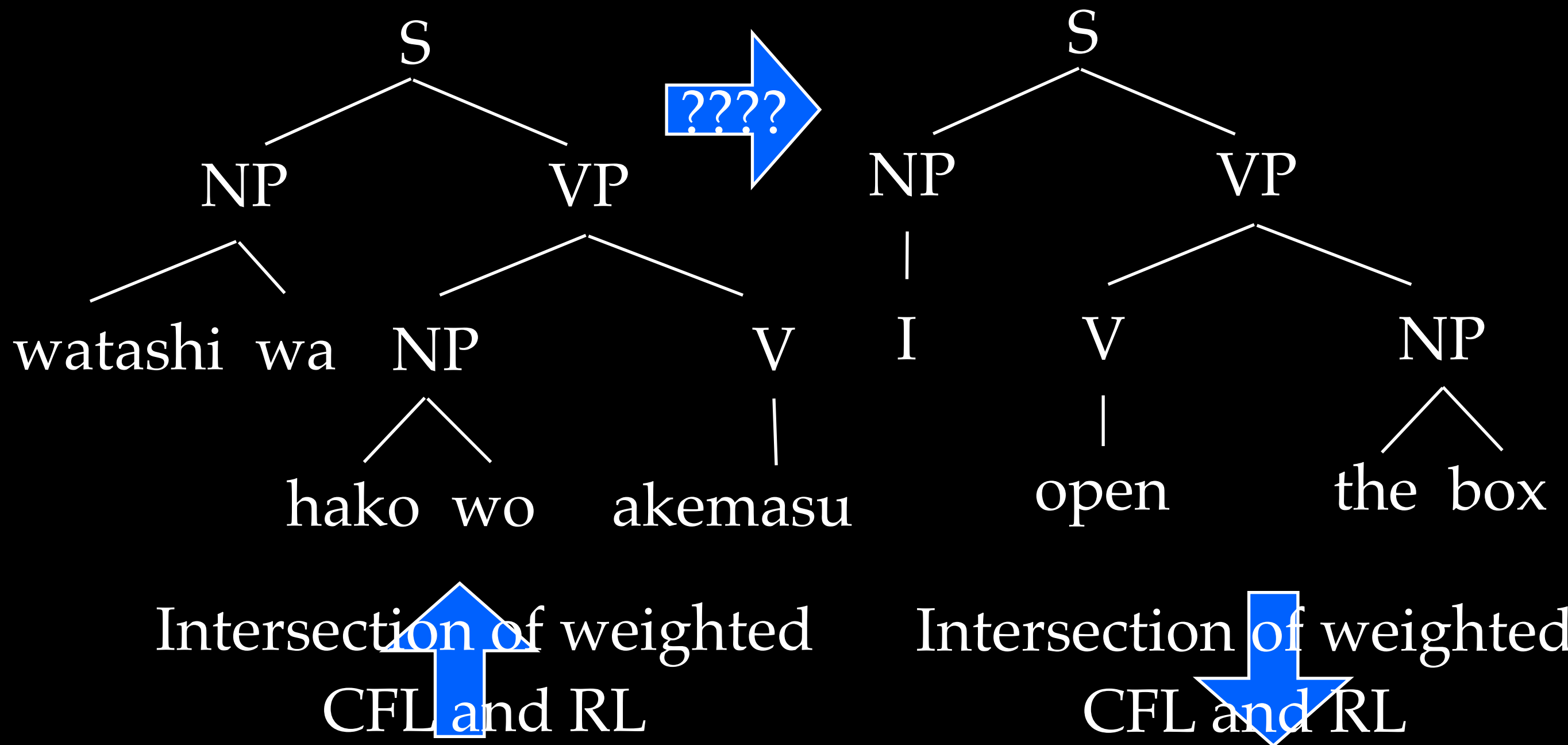
watashi wa hako wo akemasu



Intersection of weighted  
CFL and RL

I open the box

# Parting Thoughts



*watashi wa hako wo akemasu*      *I open the box*

Weighted *tree* languages, automata, and transducers.



# Parting Thoughts

# Parting Thoughts

Not all languages are context-free!

# Parting Thoughts

Not all languages are context-free!

$$\mathcal{L} = \{abc, aabbcc, aaabbbccc, \dots\} = \forall_n \in [1, \infty) a^n b^n c^n$$

# Parting Thoughts

Not all languages are context-free!

$$\mathcal{L} = \{abc, aabbcc, aaabbbccc, \dots\} = \forall_n \in [1, \infty) a^n b^n c^n$$

CS Theory: *context-sensitive* languages (Turing equivalent)

# Parting Thoughts

Not all languages are context-free!

$$\mathcal{L} = \{abc, aabbcc, aaabbbccc, \dots\} = \forall_n \in [1, \infty) a^n b^n c^n$$

CS Theory: *context-sensitive* languages (Turing equivalent)

FL Theory: *mildly context-sensitive* languages.

# Parting Thoughts

Not all languages are context-free!

$$\mathcal{L} = \{abc, aabbcc, aaabbbccc, \dots\} = \forall_n \in [1, \infty) a^n b^n c^n$$

CS Theory: *context-sensitive* languages (Turing equivalent)

FL Theory: *mildly context-sensitive* languages.  
superset of CFL, subset of CSL, polynomial-time

# Parting Thoughts

Not all languages are context-free!

$$\mathcal{L} = \{abc, aabbcc, aaabbbccc, \dots\} = \forall_n \in [1, \infty) a^n b^n c^n$$

CS Theory: *context-sensitive* languages (Turing equivalent)

FL Theory: *mildly context-sensitive* languages.  
superset of CFL, subset of CSL, polynomial-time  
Tree-adjoining grammar, Combinatory categorial  
grammar, many others.