Probability and features

Bayes' Rule

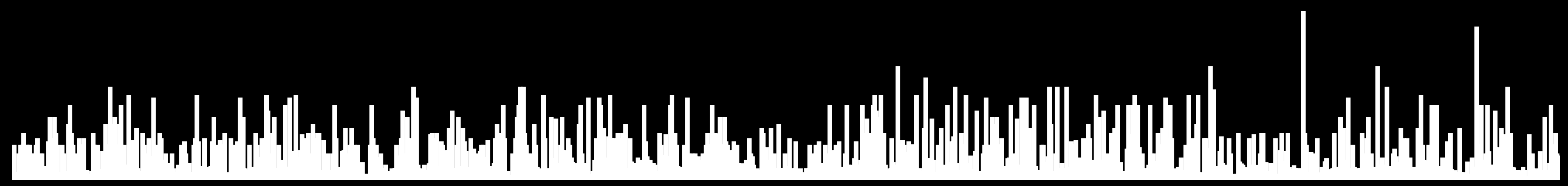
$$p(English|Chinese) \sim$$

$$p(English) \times p(Chinese|English)$$

language model

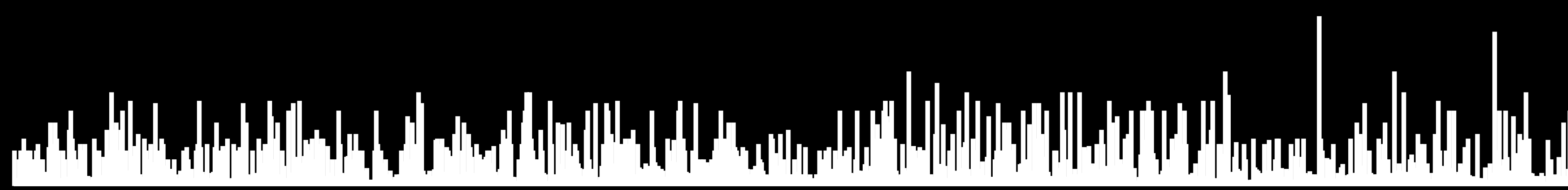
translation model

$p(Chinese|English)$

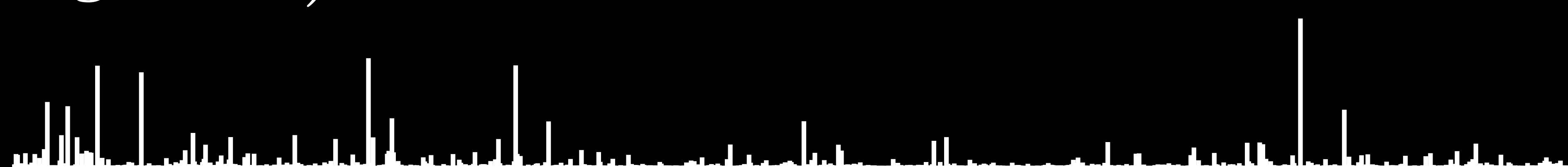


$English$

$$p(Chinese|English)$$

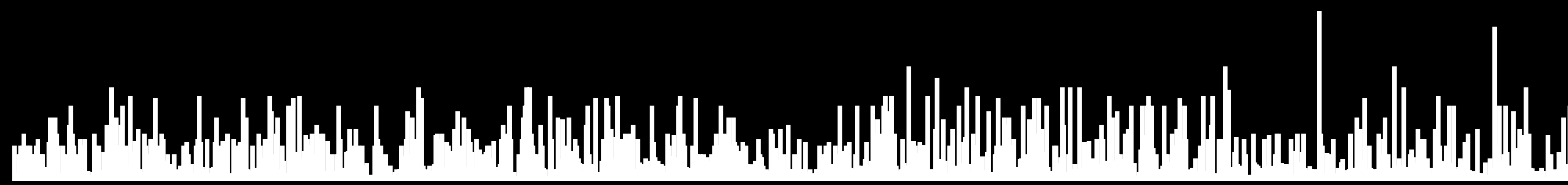


$$\times\ p(English)$$

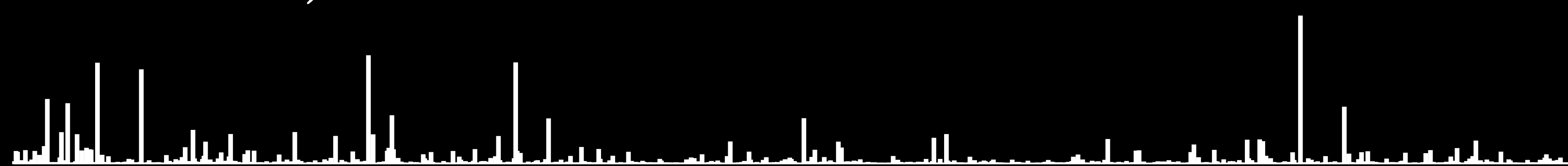


$English$

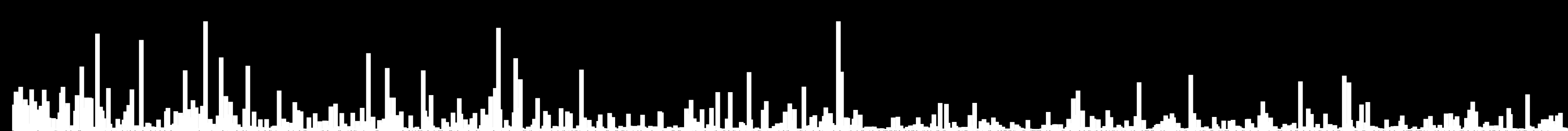
$p(Chinese|English)$



$\times\ p(English)$

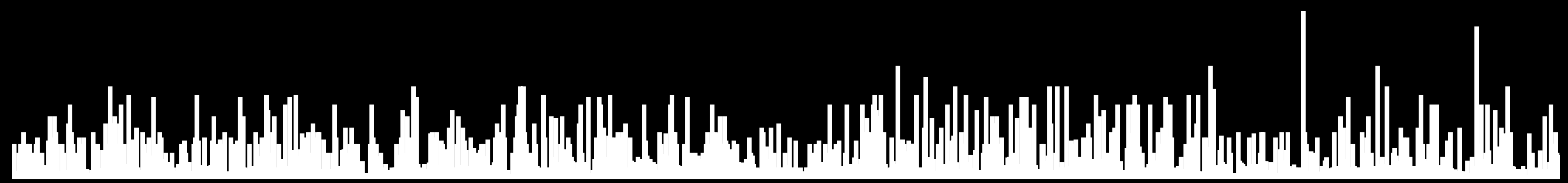


$\sim p(English|Chinese)$

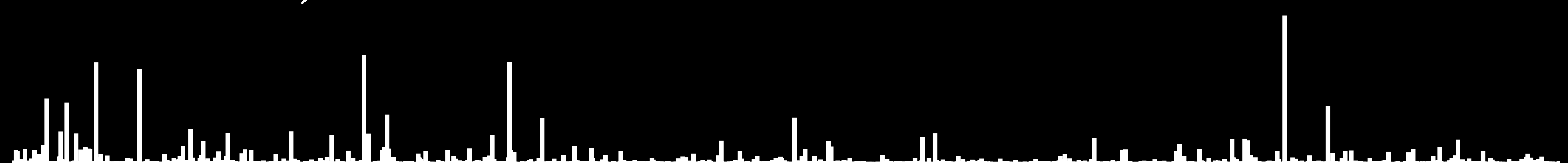


$English$

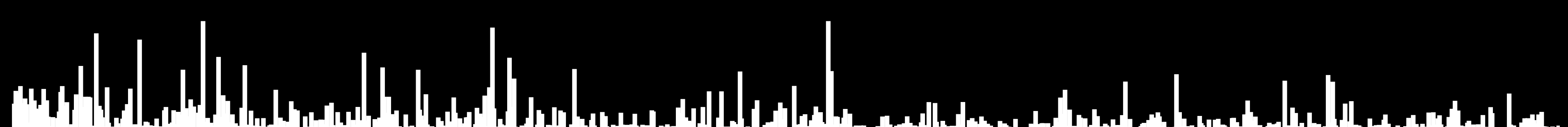
$$p(Chinese|English)^1$$



$$\times\ p(English)^1$$

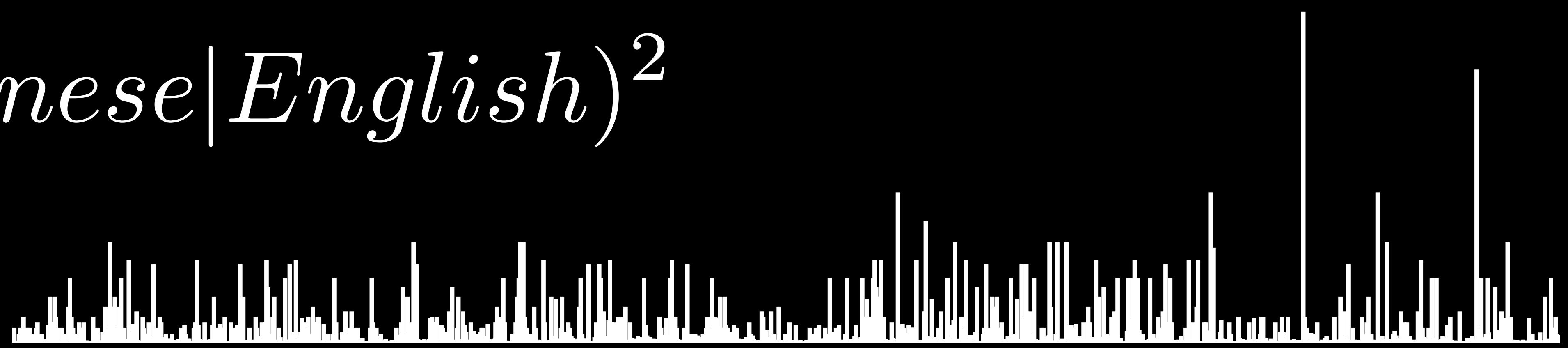


$$\sim p(English|Chinese)$$

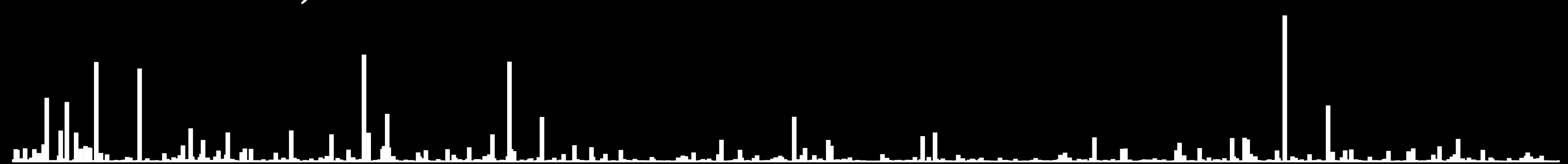


$English$

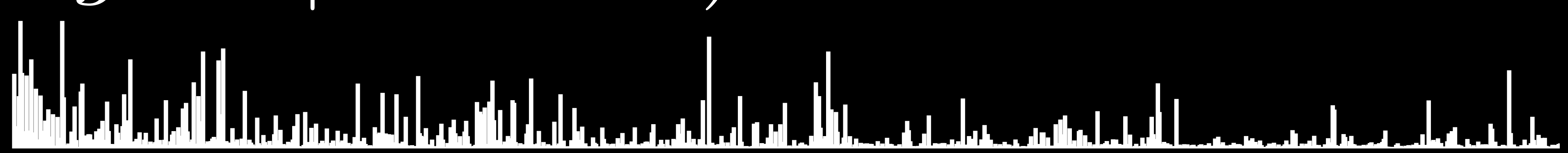
$$p(Chinese|English)^2$$



$$\times\ p(English)^1$$

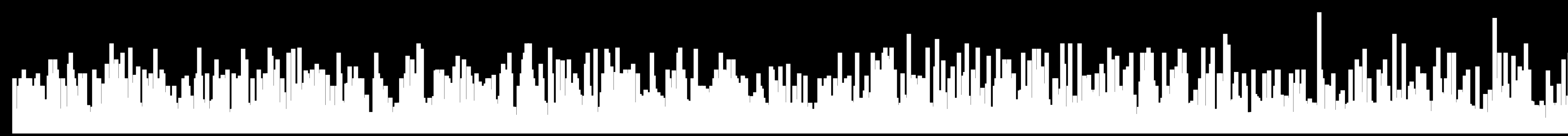


$$\sim p(English|Chinese)$$

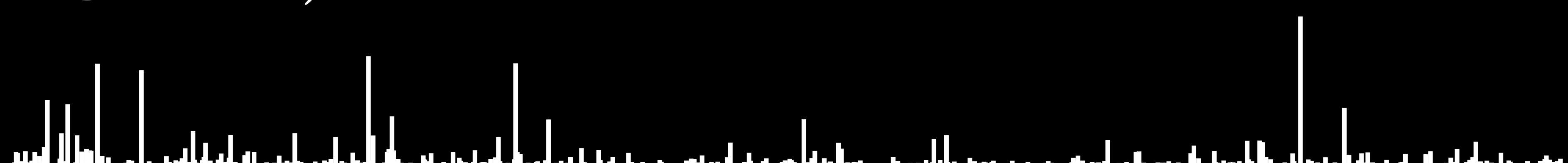


English

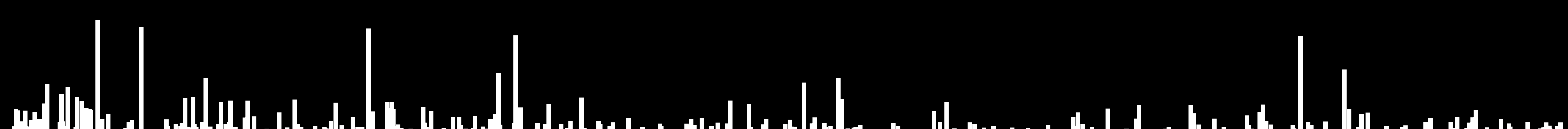
$$p(Chinese|English)^{1/2}$$



$$\times\ p(English)^1$$



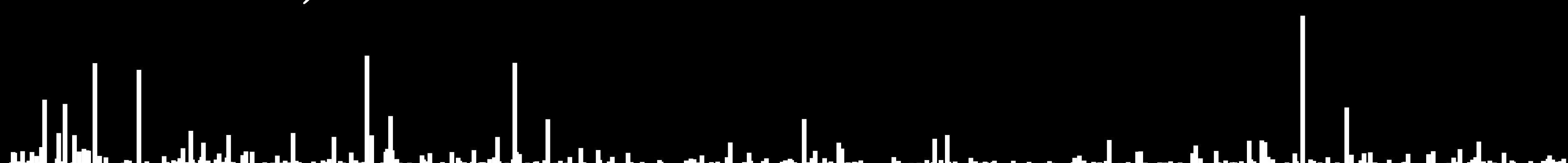
$$\sim p(English|Chinese)$$



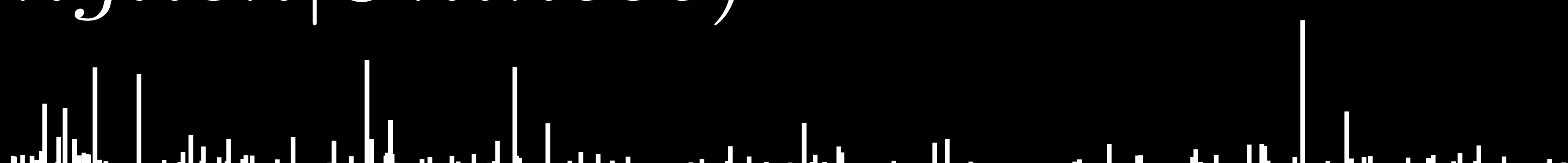
$English$

$$p(Chinese|English)^0$$

$$\times\ p(English)^1$$



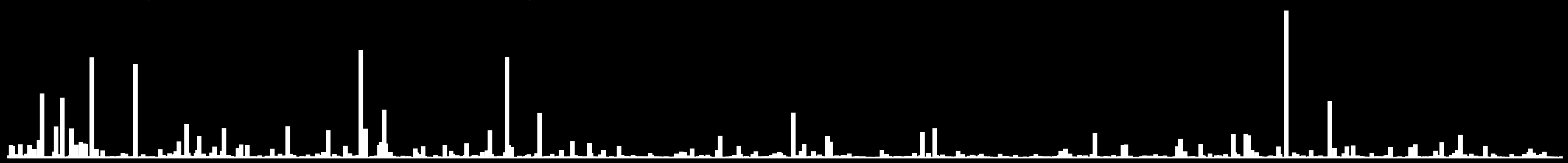
$$\sim p(English|Chinese)$$



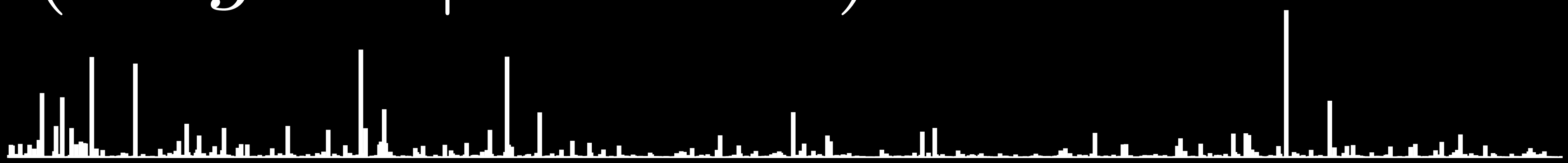
$English$

$$0 \cdot \log p(Chinese|English)$$

$$+1 \cdot \log p(English)$$



$$\sim \log p(English|Chinese)$$



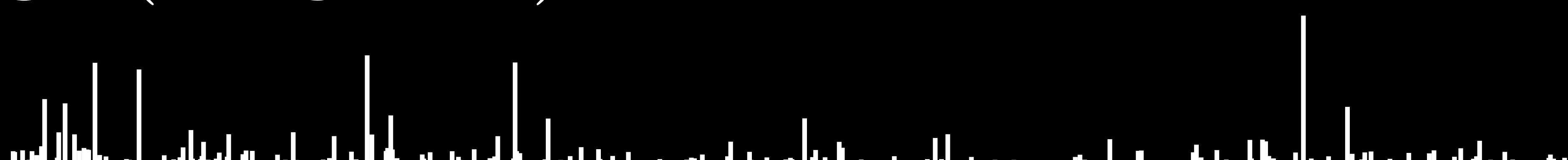
$English$

log(x) is monotonic for positive *x*:

log(x) > *log(y)* iff *x*>*y*

$$0 \cdot \log p(Chinese|English)$$

$$+1 \cdot \log p(English)$$



$$\sim \log p(English|Chinese)$$

$$English$$

log(x) is monotonic for positive *x*:

log(x) > *log(y)* iff *x*>*y*

$$0 \cdot \log p(Chinese|English)$$

$$+1 \cdot \log p(English)$$

$$\sim \log p(English|Chinese)$$

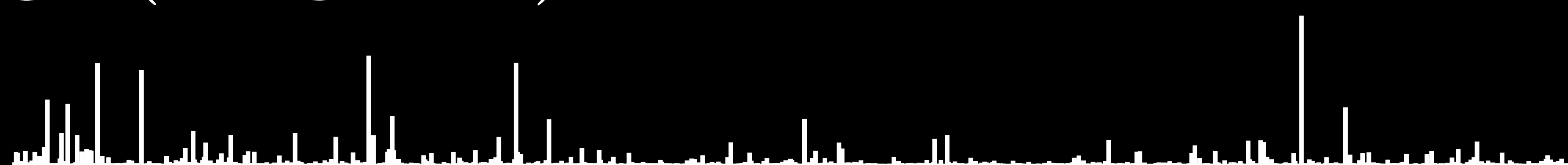
Local historic footnote: logarithms were invented in Edinburgh by John Napier, whose ancient family home in Merchiston is now part of Edinburgh Napier University.

$log(x)$ is monotonic for positive x:

$log(x) > log(y)$ iff $x>y$

$$0 \cdot \log p(Chinese|English)$$

$$+1 \cdot \log p(English)$$

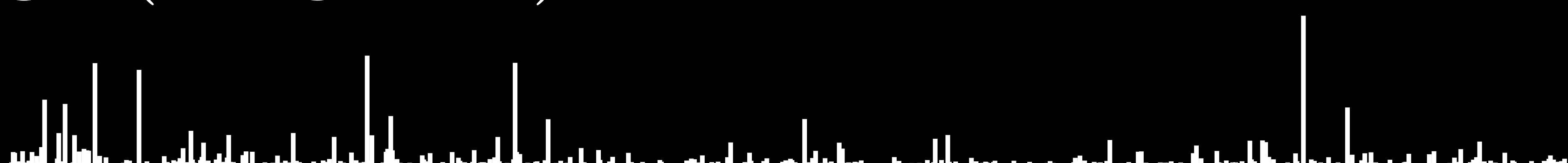


$$\sim \log p(English|Chinese)$$

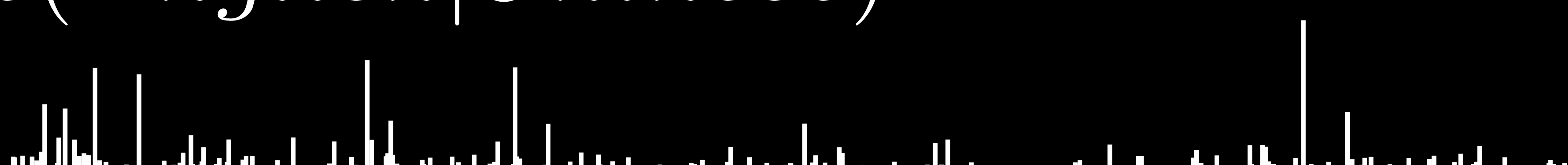
$$English$$

$$0 \cdot \log p(Chinese|English)$$

$$+1 \cdot \log p(English)$$



$$= score(English|Chinese)$$



English

$$score(English|Chinese) =$$

$$\lambda_1 \log p(Chinese|English) + \lambda_2 \log p(English)$$

$$score(English|Chinese) =$$

$$\exp(\lambda_1 \log p(Chinese|English) + \lambda_2 \log p(English))$$

$$p(English|Chinese) =$$

$$\frac{\exp(\lambda_1 \log p(Chinese|English) + \lambda_2 \log p(English))}{\sum_{nglish} \exp(\lambda_1 \log p(Chinese|English) + \lambda_2 \log p(English}$$

$$p(English|Chinese) =$$

$$\frac{\exp(\lambda_1 \log p(Chinese|English) + \lambda_2 \log p(English))}{\sum_{nglish} \exp(\lambda_1 \log p(Chinese|English) + \lambda_2 \log p(English)}$$

log-linear model

maximum entropy model

conditional random field

undirected model

$$p(English|Chinese) =$$

$$p(English) \times p(Chinese|English)$$

Note: Original model is a special case of this model!

log-linear model
maximum entropy model
conditional random field
undirected model

$$p(English|Chinese) =$$

$$\frac{\exp(\lambda_1 \log p(Chinese|English) + \lambda_2 \log p(English))}{\sum_{nglish} \exp(\lambda_1 \log p(Chinese|English) + \lambda_2 \log p(English)}$$

log-linear model

maximum entropy model

conditional random field

undirected model

$$p(English|Chinese) = \frac{\exp\left\{\sum_k \lambda_k h_k(English, Chinese)\right\}}{\sum_{English'} \exp\left\{\sum_k \lambda_k h_k(English', Chinese)\right\}}$$

log-linear model

maximum entropy model

conditional random field

undirected model

$$p(English|Chinese) = \frac{1}{Z}\exp\left\{\sum_k \lambda_k h_k(English, Chinese)\right\}$$

log-linear model
maximum entropy model
conditional random field
undirected model

$$p(English|Chinese) =$$
$$\frac{1}{Z}\exp\left\{\sum_k \lambda_k h_k(English, Chinese)\right\}$$

Z is the normalization term or *partition function*

log-linear model
maximum entropy model
conditional random field
undirected model

$$p(English|Chinese) =$$

$$\frac{1}{Z} \exp \left\{ \sum_k \lambda_k h_k(English, Chinese) \right\}$$

Z is the normalization term or *partition function*

The functions h_k are *features* or *feature functions*
They are deterministic (fixed) functions of the input/output pair.

The parameters of the model are the λ_k terms.

What’s a Feature?

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A feature can be *any* function in the form:

$$h_k : English \times Chinese \rightarrow \mathbb{R}_+$$

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- Language model: *p(English)*
- Translation model: *p(Chinese | English)*
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- The number of words in the English sentence.

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A feature can be *any* function in the form:

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- Language model: *p(English)*
- Translation model: *p(Chinese | English)*
- Reverse translation model: *p(English | Chinese)*
- The number of words in the English sentence.
- The number of verbs in the English sentence.

What's a Feature?

A feature can be *any* function in the form:

$$h_k : English \times Chinese \rightarrow \mathbb{R}_+$$

- Language model: *p(English)*
- Translation model: *p(Chinese | English)*
- Reverse translation model: *p(English | Chinese)*
- The number of words in the English sentence.
- The number of verbs in the English sentence.
- 1 if the English sentence has a verb, 0 otherwise.

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- A word-based translation model: *p(Chinese | English)*

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A feature can be *any* function in the form:

$$h_k : English \times Chinese \rightarrow \mathbb{R}_+$$

- A word-based translation model: *p(Chinese | English)*
- Agreement features in the English sentence.

What's a Feature?

A feature can be *any* function in the form:

$$h_k : English \times Chinese \rightarrow \mathbb{R}_+$$

- A word-based translation model: *p(Chinese | English)*
- Agreement features in the English sentence.
- Features over part-of-speech sequences in the English sentence.

What's a Feature?

A feature can be *any* function in the form:

$$h_k : English \times Chinese \rightarrow \mathbb{R}_+$$

- A word-based translation model: *p(Chinese | English)*
- Agreement features in the English sentence.
- Features over part-of-speech sequences in the English sentence.
- How many times the sentence pair includes the English word *north* and Chinese word 北.

What's a Feature?

A feature can be *any* function in the form:

$$h_k : English \times Chinese \rightarrow \mathbb{R}_+$$

- A word-based translation model: *p(Chinese | English)*
- Agreement features in the English sentence.
- Features over part-of-speech sequences in the English sentence.
- How many times the sentence pair includes the English word *north* and Chinese word 北.
- Do words *north* and 北 appear in a dictionary?

Probability again

$$p(English|Chinese) =$$

$$\frac{1}{Z}\exp\left\{\sum_k \lambda_k h_k(English, Chinese)\right\}$$

Probability again

$$p(y|x) = \frac{1}{Z} \exp \left\{ \sum_k \lambda_k h_k(x, y) \right\}$$

Probability again

x and y can be anything here.

This is a ***maximum entropy language model***

Suppose: $y = w_i$

$x = w_{i-1} \ldots w_{i-n+1}$

$$p(y|x) = \frac{1}{Z} \exp(\lambda^\top \cdot h_k(x, y))$$

Now design h so that it is sensitive to morphology