Bayesian Analysis of Player Performance over Time

A Thesis
Presented to
Department of Statistical Science
Duke University

Nathaniel Brown

April 10, 2018

Approved for the
Bachelor of Science in Statistical Science

Mike West

Merlise Clyde

Cliburn Chan

Mine Cetinkaya-Rundel, DUS

Acknowledgements

Thank you to Mike West, for advising me throughout this project, to Merlise Clyde, for introducing me to the ones trick in JAGS, and to Kevin Cullen, for providing the data for this research.

Table of Contents

List of Tables

List of Figures

Abstract

This study is an investigation of Bayesian statistical models and analyses for problems arising in shooting a basketball. The data consists of player-tracking data from the Duke Men's Basketball team, which is recorded on the SportVU cameras from STATS, LLC. The goals are to explore, develop, and apply Bayesian models to existing and new data on shooting outcomes. In addition, we want to understand and evaluate questions of inherent random variation, changes over time in shooting performance, and issues related to the "hot hand" concept in sports.

The models we use to investigate this data are a Bayesian logistic generalized linear model, a hierarchical model with mixed effects on the shooter identity, and a discounted likelihood model that reduces the influence of shots as their time difference from the current shot increases.

The results of our analysis models show some weak support for time-dependency in shooting outcomes in this dataset. The discounted models, where predictors of shot success are allowed to shift based on recent outcomes, perform better in likelihood, while the models with fixed parameters over time perform marginally better in out-of-sample prediction.

Introduction

In the sport of basketball, points are awarded by the binary event of shooting the ball into the goal. Some factors that may affect the success rate include the location of the shooter, the individual skill of the shooter, whether the shooter is shooting on his team's home court or on an away court, and his shooting success in recent games. There have been previous studies investigating the effect of recent shooting success on current shooting success, and the results vary. For example, Gilovich, Vallone, & Tversky (1985) use Walf-Wolfowitz runs tests, autocorrelation tests on consecutive shot attempts, goodness-of-fit tests for the distribution of successes, and paired t-tests comparing the mean of makes following a make to that of makes following a miss. These statistical tests did not detect significant evidence supporting streakiness in basketball shooting data. In addition, Wetzels et al. (2016) found evidence that a Hidden Markov Model with two states (representing a high shot success rate and a low rate) better fits Shaquille O'Neal's free throw shooting data than a Binomial model with one constant state. Bar-Eli, Avugos, & Raab (2006) completed a review of previous statistical papers whose authors research the concept of streaky success rates in data with a binomial response; the applications include basketball shooting, baseball hitting, baseball pitching, horeshoes, cognitive science, and economics. They summarize 11 papers that support evidence of streakiness in binomial data, and 13 that do not.

The purpose of this paper is to investigate Bayesian modelling techniques on basketball shooting data, and to learn more about time-dependency in the data.

Literature Review

Albert (1993)

In this paper, Albert uses a Markov switching model to analyze streakiness in baseball pitching data. He concludes that a few players exhibit streakiness, but not enough to reject the null hypothesis. An exploratory technique that we take from this paper is to examine the peaks and valleys in a moving average plot to observe streakiness. A strength of this paper is that Albert controls for situational variables such as home field advantage, the handedness of the pitcher, and the runners on the bases.

Albert (2013)

In this paper, Albert analyzes streakiness in baseball hitting data. His analysis techniques include using Bayes Factors to compare models of the form $f(y_j|p_j) = p_j(1-p_j)^{y_j}, y_j = 0, 1, 2, ...$; a consistent model with a constant p_j, and a streaky model with a varying p_j from a beta distribution. A useful insight that we apply to this paper is the concept that the existence of streakiness depends on the definition of "success" in binary outcome data. He found substantially more evidence for streakiness for when a success was coded as "not a strikeout" instead of a "hit". From this paper, we learn a technique for comparing Bayesian models, and that the organization of the data can affect the outcomes.

Albert & Williamson (1999)

In this paper, Jim Albert attempts to improve upon the low-powered frequentist tests of Gilovich, Vallone, and Tverky's 1985 paper on the "hot hand". Albert formally defines "streakiness" as the presence of nonstationarity (nonconstant probability between trials) or autocorrelation (sequential dependency). He uses Gibbs sampling to approximate posterior densities and to simulate data, then fits two types of models on binary data from baseball and basketball to try to characterize streakiness. He fits an overdispersion model to detect nonstationarity, and a Markov switching model to detect sequential dependencies. While he did not uncover strong evidence for the hot hand, one of his takeaways was that overdispersion decreases as time goes on in basketball free throw shooting data. A weakness of this paper is that Albert does not show the results of both the Markov model and the overdispersion model on the same data. We use Albert's formal definitions of streakiness as well as his motivation for fitting Bayesian models over running frequentist tests.

Ameen & Harrison (1984)

This journal article contains information about discounted likelihood regression using exponential weights, which generalizes to discount weighted estimation (DWE). There is also a section that elaborates on how these models apply to time series. Although the material in this writing focuses on data with a Normal response instead of a Binomial, we use the concepts in this article to help explain our discounted likelihood models in Section 2.1.3.

Bar-Eli et al. (2006)

This paper is a review of previous hot hand research. It reviews several papers investigating the concept of the hot hand in sports such as basketball, baseball, volleyball, and horseshoe, and other fields such as cognitive science and economics. Bar-Eli, Avugos, and Raab evaluate the datasets, the tests and statistics used, and the conclusions of each study. Overall, the authors summarize 13 papers that oppose the hot hand phenomenon, and 11 that support it; they also acknowledge that the scientific evidence for the hot hand is weaker than the evidence against it, and it is typically more controversial. Instead of just looking to answer whether the hot hand exists, Bar-Eli, Avugos, and Raab also examine how people define a "hot hand", and the psychological factors behind the belief in it, such as the gambling and game strategy. The strengths of this paper are that it evaluates the strengths and weaknesses of many competing claims, and concisely summarizes the information into a table. A weakness is that they do not make any claim of their own. This paper is useful in this thesis because it describes several data analysis techniques to detect streaks in a binary sequence.

Gilovich et al. (1985)

In this research paper from *Cognitive Psychology*, Thomas Gilovich, Robert Vallone, and Amos Tversky investigate peoples' belief in the hot hand in basketball. The hot hand is the concept that the probability of a success increases for trials that follow a success in a binary sequence; in basketball, these binary events are shot attempts. The methods in this paper include an analysis of in-game shot attempts from the Philadelphia 76ers of the National Basketball Association (NBA) in the 1981 season, analysis of free-throw attempts from the Boston Celtics in the 1981 and 1982 seasons, and a controlled shooting drill using male and female varsity basketball players at Cornell University. Statistical techniques they used to attempt to detect streakiness in the data included Walf-Wolfowitz run tests, autocorrelation tests on consecutive shot attempts, goodness-of-fit tests for the distribution of successes, and paired t-tests comparing the mean of makes following a make to that of makes following a miss. In addition to this analysis of shooting, this research also contained a survey of basketball fans, that gauged how much people believed success probabilities changed following a success or a failure. The statistical tests did not detect significant evidence supporting the Hot Hand in basketball. The lack of statistical power in Gilovich, Vallone, and Tversky's frequentist tests motivates the use of Bayesian models in this thesis. Strengths of this paper include the fact that it was one of the first research papers to analyze streakiness in basketball data, and many future papers build off of it. Some weaknesses in this paper are the assumptions it makes in its analysis, such as all shots being independent of each other, and not accounting for shot location.

Joseph (n.d.)

This web page provided a sample of code describing how to implement the “ones trick” in JAGS. The source is from the Lawrence Joseph, a professor in the Department of Epidemiology and Biostatistics of McGill University in Montreal, Quebec. The information from this source was used in the code where we build the model, and in the descriptions of the discounted likelihood hierarchical model in Section 3.1.3.

Prado & West (2010)

This textbook provides theory, applications, and examples of time series models such Dynamic Generalized Linear Models (DGLMs). More specifically, Section 14.4 in this book provides an example of a DGLM for a binomial response variable. The concept of a time-varying parameter in a binomial model is applied in this paper, through our use of discounted likelihood models.

Wetzels et al. (2016)

In this research paper, Wetzels conducts a simulation study to investigate the Hot Hand Phenomenon. His analysis consists of calculating Bayes Factors to compare evidence between a Hidden Markov Model with two states and a binomial model with one state. He applies this method to data from basketball foul shots and from visual discernment tests. In the basketball data, he found that Shaquille O’Neal’s free-throws show evidence for a two-state Markov model, while Kobe Bryant’s show more evidence for a one-state binomial model. In the data from the visual discernment tests, he found no strong evidence supporting one model over the other. A strength of this paper is Wetzel’s formal comparison of a Bayesian Markov model to a binomial model. A weakness is that the Bayes Factors only compare evidence between the two models; it does not mean that either model is “good”.

Chapter 1

Data

1.1 Description of Dataset

The data for this analysis comes from SportVU, a player-tracking system from STATS, LLC. that provides precise coordinates for all ten players and the ball at a rate of 25 times per second. The Duke University Men's Basketball team permitted us to use their SportVU data from the 2014 to 2017 basketball seasons for this project. Since the ability to record this data depends on specialized tracking cameras, Duke does not have this data for every game they play—only home games, and a few road games in arenas that had the technology installed. Therefore, there is a substantial amount of missing data between games. More specifically, between the 2014 and 2017 seasons, the Duke Men's Basketball team played 147 games; this dataset contains 94 games, with 82 at Duke and 12 at other arenas.

For our analysis, we use the following files for each game:

- Final Sequence Play-by-Play Optical:

This dataset comes in an a semi-structured Extensible Markup Language (XML) file, where there is a unique element for each "event" (an event is a basketball action such as a dribble, pass, shot, foul, etc.). Each event element has attributes describing the type of event, the time of the event, and the player who completed the action. We use these files to uncover when a shot is attempted in a game, who attempted the shot, and the result of the shot attempt.

- Final Sequence Optical:

These XML files contain the locations of all ten players and the ball during precise time intervals within the game (25 times per second). Each time unit has a unique element, and these elements have attributes describing the locations. We merge this with the Final Sequence Play-by-Play Optical data on the time attribute to obtain the shooter's location at the moment of a shot attempt.

1.2 Data Cleaning

Steps taken to clean the merged shooter IDs with shot locations include translating the locations to a half-court setting (the teams switch sides of the court halfway through every game, which means that we have to flip the coordinates across the middle of the court for about half of the shots in every game), converting the x-y coordinates to polar coordinates (in the units of feet and radians), and adding an indicator for home games. We only use the shots that Duke players attempt, because there is an inadequate amount of data for players on other teams—no opposing players appear in more than 5 games. The final dataset contains 5,467 observations from 31 shooters over 94 games. A summary of the cleaned dataset is in Table 1.1:

Table 1.1: Summary of Dataset

Name	Type	Values	Extra Details
season	categorical	$\{2014, \ldots, 2017\}$	
gameid	categorical	NA	94 unique values
time	continuous	NA	13-digit timestamp in milliseconds
globalplayerid	categorical	NA	31 unique values
r	continuous	$[0, \infty)$	Distance of shot from basket (feet)
theta	continuous	$[-\pi, \pi]$	Angle of shot (radians)
home	categorical	$\{0,1\}$	1 if shot occured in a home game
result	categorical	$\{0,1\}$	1 if shot was made (response)

A small subset of the cleaned data is displayed below in Table 1.2:

Table 1.2: Sample of Dataset

season	gameid	time	globalplayerid	r	theta	home	result
2014	201401070173	1389141733839	603106	4.2076	1.0746	1	1
2014	201401070173	1389141844712	601140	16.6537	1.2973	1	0
2014	201401070173	1389143172185	696289	18.7901	-0.0581	1	1
2014	201401070173	1389143196303	601140	23.4629	0.9539	1	1
2014	201401070173	1389143220261	756880	6.5365	0.0696	1	0

Figure 1.1 shows the locations of all the shots in the dataset, translating the locations to one half of the court, and excluding heaves from beyond half court. The variable θ has a range of 2π radians, but this plot shows that most of the attempts occur within the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$. This figure also shows the bimodal distribution of shot distance over all players.

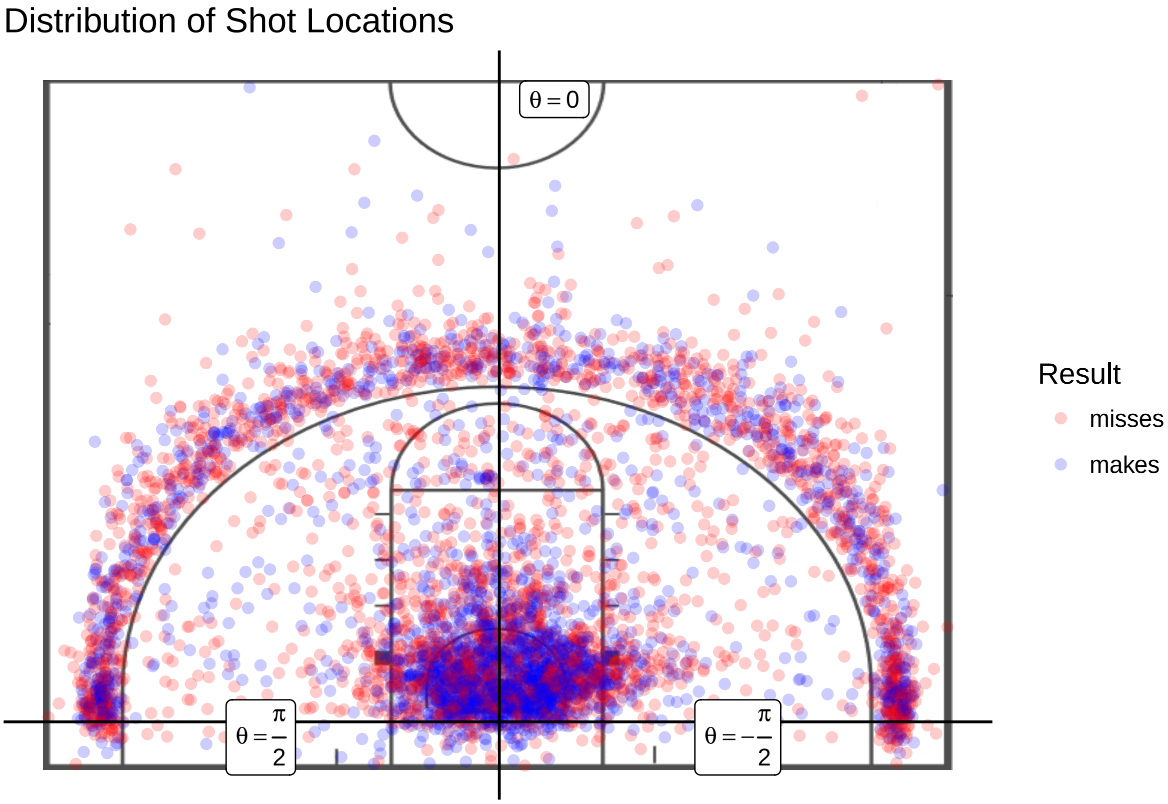


Figure 1.1: Locations and Results of All Shots

1.3 Exploratory Data Analysis

The exploratory data analysis plots in Figure 1.2 examine how consistent the probability of a made shot is, using a loess smooth curve on the binary outcomes. We present these smoothed plots for four high-usage basketball players at Duke University between the 2014 and 2017 seasons. Each plot represents a single player's ordered shooting outcomes for a single season. These plots do not account for the amount of time in between shots, but simply shot order and outcome.

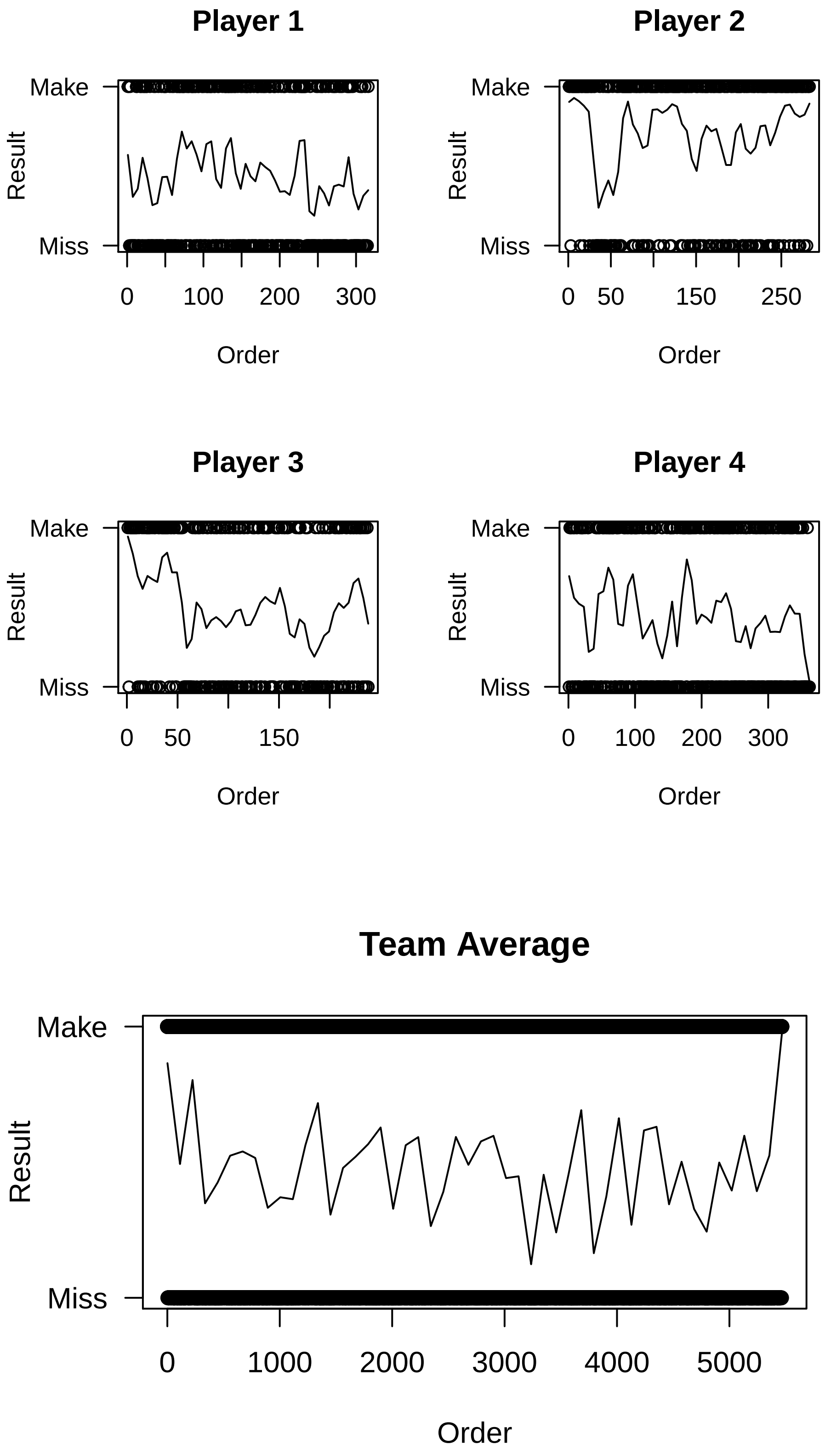


Figure 1.2: Moving Average of Shot Success Rate

We can see that the players vary in the consistency of their made shots, since they all contain spikes and trends. For example, Player 3 initially has a very high success rate, which quickly falls to the middle after about 30 shot attempts, and the Player 2 has a noticeable upward trend in shot success beginning around shot number 150.

We investigate the shooting outcomes using Bayesian models, and present the results in the next chapter.

Chapter 2

Models & Analysis

2.1 Description of Models

For our models, we consider the shot location, a home court indicator, the shooter's identity, and his shooting outcomes in nearby games as factors that can affect a shot's outcome. We use the Just Another Gibbs Sampler library in R (`R2jags`) to build these models. Each one is based on a logistic regression model that provides the posterior distribution of the shot location parameters (distance and angle) and an additional intercept to capture the influence of home-court advantage. The models do not account for covariance between these predictors. We expand upon this model by adding mixed effects and discounted likelihood models to control for shooter identity and between-game variability, respectively. In our Gibbs Samplers, we estimate the posterior distributions using 10,000 simulations and a burn-in of 500. Our prior distributions are constructed from the corresponding maximum likelihood estimates for the first four games in the dataset, and we initialize our Monte Carlo Markov Chains using values of 0 for all means, and 1 for all variances. The `R2jags` code used to build these models can be found in Appendix A.

2.1.1 Generalized Linear Model

We start with a logistic regression model for each shot attempt i:

$$\text{logit}(p_i) = \beta_{\text{int}} + x_{\text{r,i}}\beta_{\text{r}} + x_{\theta,\text{i}}\beta_{\theta} + x_{\text{H,i}}\beta_{\text{H}}.$$

In this model, the x refers to the data, and the βs are the parameters from the model. The subscripts *int*, r, θ, and H respectively refer to the intercept, the log-distance of the shot, the angle of the shot, and whether shot was taken on Duke's home court or another gym. This fourth β accounts for the possibility of "home-court advantage", which can affect shot outcomes.

2.1.2 Hierarchical Generalized Linear Model

Our second model is a hierarchical model, with random effects on the j players in the dataset. These random effects occur for each of the four parameters of interest—the intercept, the distance effect, the angle effect, and the home effect. Each individual player's parameter values are sampled from a Normal distribution centered at the population values. A benefit of this type of model is that the parameters for players with few shot attempts are shrunk towards the population means. The model has the form below for each player j and shot attempt i:

$$\text{logit}(p_{\text{ji}}) = \beta_{\text{int, j}} + x_{\text{r,ji}}\beta_{\text{r, j}} + x_{\theta,\text{ji}}\beta_{\theta,\text{j}} + x_{\text{H, ji}}\beta_{\text{H, j}},$$

$$\beta_{\text{int, j}} \sim N(\beta_{\text{int}}, \tau^2_{\text{int}}),$$

$$\beta_{\text{r, j}} \sim N(\beta_{\text{r}}, \tau^2_{\text{r}}),$$

$$\beta_{\theta,j} \sim N(\beta_{\theta}, \tau^2_{\theta}),$$

$$\beta_{\text{H, j}} \sim N(\beta_{\text{H}}, \tau^2_{\text{H}}).$$

2.1.3 Discounted Likelihood Hierarchical Model

In the discounted likelihood models, the likelihood function for all model parameters at a given time is more heavily influenced by the observations close to that specific time point than the observations far away from it. We measure the "distance" between observations by the number of games between them; a shot attempt that occurs in the next or the preceding game will influence the likelihood function for the parameters "anchored" at that game more than a shot that occurs two games away. We chose to analyze time-dependency in the data using a discounted likelihood model instead of a full dynamic model because a discounted likelihood model would have fewer complications in the context of a hierarchical model. The methodology behind our discounted likelihood model relates closely to Bayesian dynamic modeling that uses power discounting to reduce the effect of data that occurs further in the past. We begin defining this model with the concept of forward filtering. This is implied by a power-discount Bayesian time series model (Smith, 1979) that uses a "discounted" Bayes Theorem in which the posterior disitribution of the model parameters at any chosen time t is proportional to the product of the prior, $p(\Theta)$, and the discount likelihood function, which has the form

$$p_t(X_{1:t}|\Theta) = p_t(X_{1:t-1}|\Theta)p(X_t|\Theta),$$

where

$$p_t(X_{1:t-1}|\Theta) \propto p(X_1|\Theta)^{\delta^{t-1}}...p(X_{t-2}|\Theta)^{\delta^2}p(X_{t-1}|\Theta)^{\delta}$$

for some discount factor $0 < \delta < 1$, typically closer to 1 to allow for the discounting of past data without omitting its information content.

This equation shows that as distance increases backward in time, the effect on the likelihood function—and hence on the resulting posterior for Θ at our current time t—decreases. The corresponding discount likelihood for Θ at time t given both past and future data up to a time $T > t$ has a similar form, but with two-sided discounting. This relates to dynamic models with time-varying parameters, which apply backwards updating to update their posteriors. The form of the two-sided likelihood function of Θ at a chosen time t is:

$$p_t(X_{1:T}|\Theta) = p_t(X_{1:t-1}|\Theta)p(X_t|\Theta)p_t(X_{t+1:T}|\Theta),$$

where the past data component $p_t(X_{1:t-1}|\Theta)$ is the same as above, and the future data component is:

$$p_t(X_{t+1:T}|\Theta) \propto p(X_{t+1}|\Theta)^{\delta}p(X_{t+2}|\Theta)^{\delta^2}...p(X_T|\Theta)^{\delta^{T-t}}.$$

Specifically for our model, given an observed shot i in game g, attempted by player j, we apply the above reasoning to discount the likelihood of a made shot in the hierarchical model. First we begin with a hierarchical logistic regression model:

$$\begin{aligned}
\text{logit}(p_{\text{gji}}) &= \beta_{\text{int, gj}} + x_{\text{r,gji}}\beta_{\text{r, gj}} + x_{\theta,\text{gji}}\beta_{\theta,\text{gj}} + x_{\text{H, gji}}\beta_{\text{H, gj}}, \\
\beta_{\text{int, j}} &\sim N(\beta_{\text{int}}, \tau^2_{\text{int}}), \\
\beta_{\text{r, j}} &\sim N(\beta_{\text{r}}, \tau^2_{\text{r}}), \\
\beta_{\theta,j} &\sim N(\beta_{\theta}, \tau^2_{\theta}), \\
\beta_{\text{H, j}} &\sim N(\beta_{\text{H}}, \tau^2_{\text{H}}).
\end{aligned}$$

Without discounting, the likelihood term controbuted from a shot's outcome, y_{gji}, that is attempted by player j in game g, is:

$$L_{\text{gj}}(\Theta) = \prod_{\text{i=1}}^{\text{n}_{\text{gj}}} p(\text{y}_{\text{gji}}|\Theta) \propto \prod_{\text{i=1}}^{\text{n}_{\text{gj}}} p_{\text{gji}}^{\text{y}_{\text{gji}}}(1 - p_{\text{gji}})^{1-\text{y}_{\text{gji}}}.$$

When we apply the exponential discounting to the outcomes like so:

$$\pi_{\mathrm{gji}} = \left(p_{\mathrm{gji}}^{\mathrm{y_{gji}}}(1 - p_{\mathrm{gji}})^{1-\mathrm{y_{gji}}}\right)^{\delta^{|g-g_0|}},$$

our likelihood becomes:

$$\Lambda_{\mathrm{gj}}(\Theta) = \prod_{\mathrm{i=1}}^{\mathrm{n_{gj}}} \pi_{\mathrm{gji}},$$

where g is the game index of the current shot i, and g_0 is a fixed game index, which we refer to as the "anchor game".

In this model, p represents the binomial probability, and π is the discounted probability. Both of these quantities are probabilities that are bounded in the interval [0,1]. Similarly, L is the likelihood and Λ is the discounted likelihood. The contribution of shot outcomes from games g that are more distant from the anchor game g_0 are more heavily discounted, and the discounting increases for smaller values of δ. In a model with $\delta = 0$, only shots taken in the same game as shot i can contribute to the likelihood, while $\delta = 1$ is equivalent to a model with no discounting. If models with larger values of δ best fit the data, this suggests that shooting success is consistent throughout a career. If smaller values of δ are more likely in the data, however, then we can assume there is a substantial amount of time variation, or "streakiness", in the data on the game level. Figure 2.1 illustrates how the exponential weight on the likelihood depends on the selected value of δ and the distance from the anchor game $(g - g_0)$.

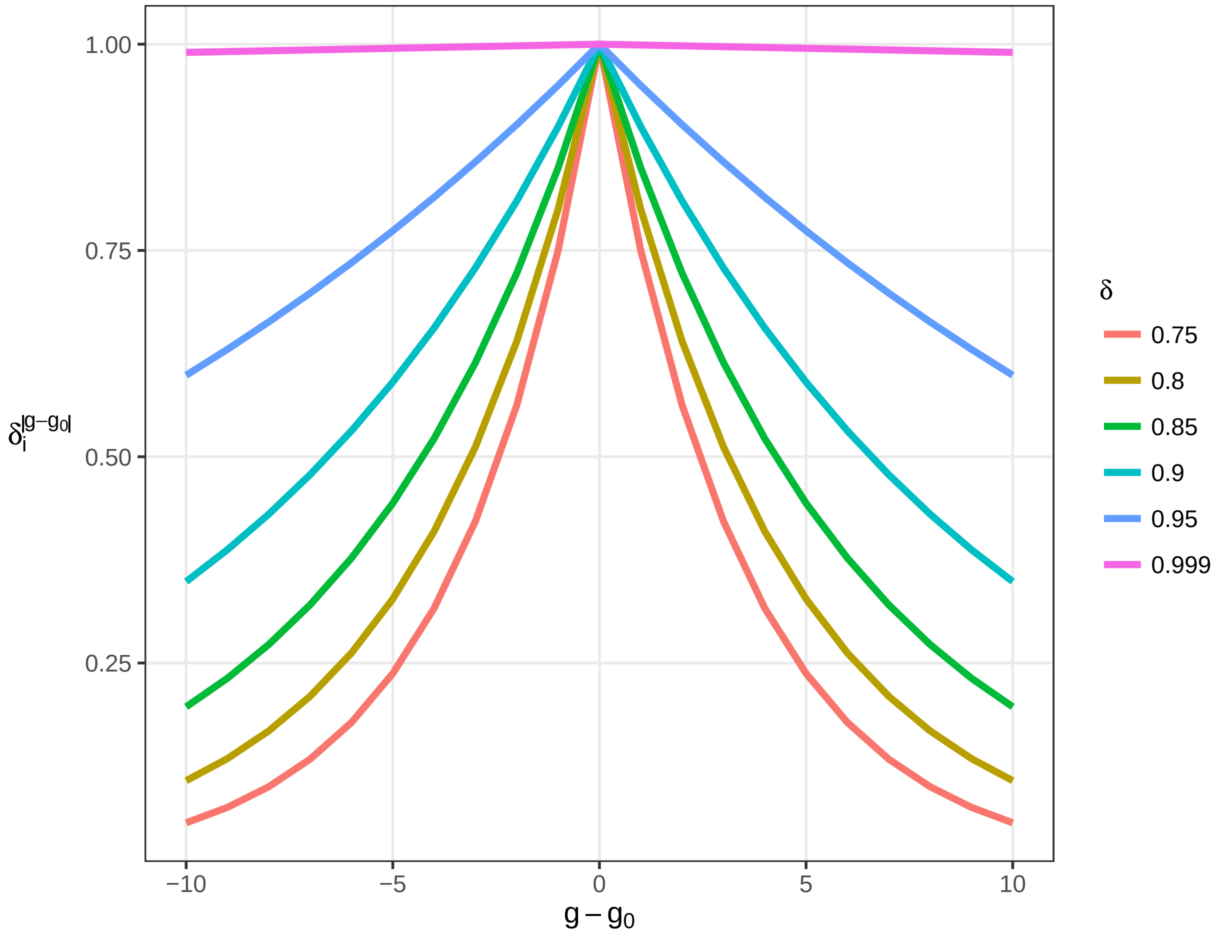


Figure 2.1: Illustration of Discounted Weighting

This model specification is specific to the chosen anchor game g_0, so that the resulting posterior distribution for all model parameters is indexed by g_0 and represents inferences "local" to that game. Repeating the analysis across all games as anchors results in a sequence of posteriors, where their differences reflect time variation. As a result, we refit the model using MCMC for each combination of game index and δ, which involves substantial computation.

To build these discounted likelihood models in the `R2jags` library, we apply the "ones trick". This technique allows us to use a sampling distribution that does not exist in the library by modifying a common distribution—in this case, the Binomial. The probability p is estimated in the same way as the Bayesian hierarchical model. We discount this probability to estimate π, and then we specify that it comes from Binomial data that consists only of ones; this is equivalent to sampling from a distribution with discounted outcomes. In the code excerpt below, `result` and `prob` are the binomial outcomes and probabilities, while `y` represents the "trick" outcomes (a vector of 1s), and `pi` is the discounted probability.

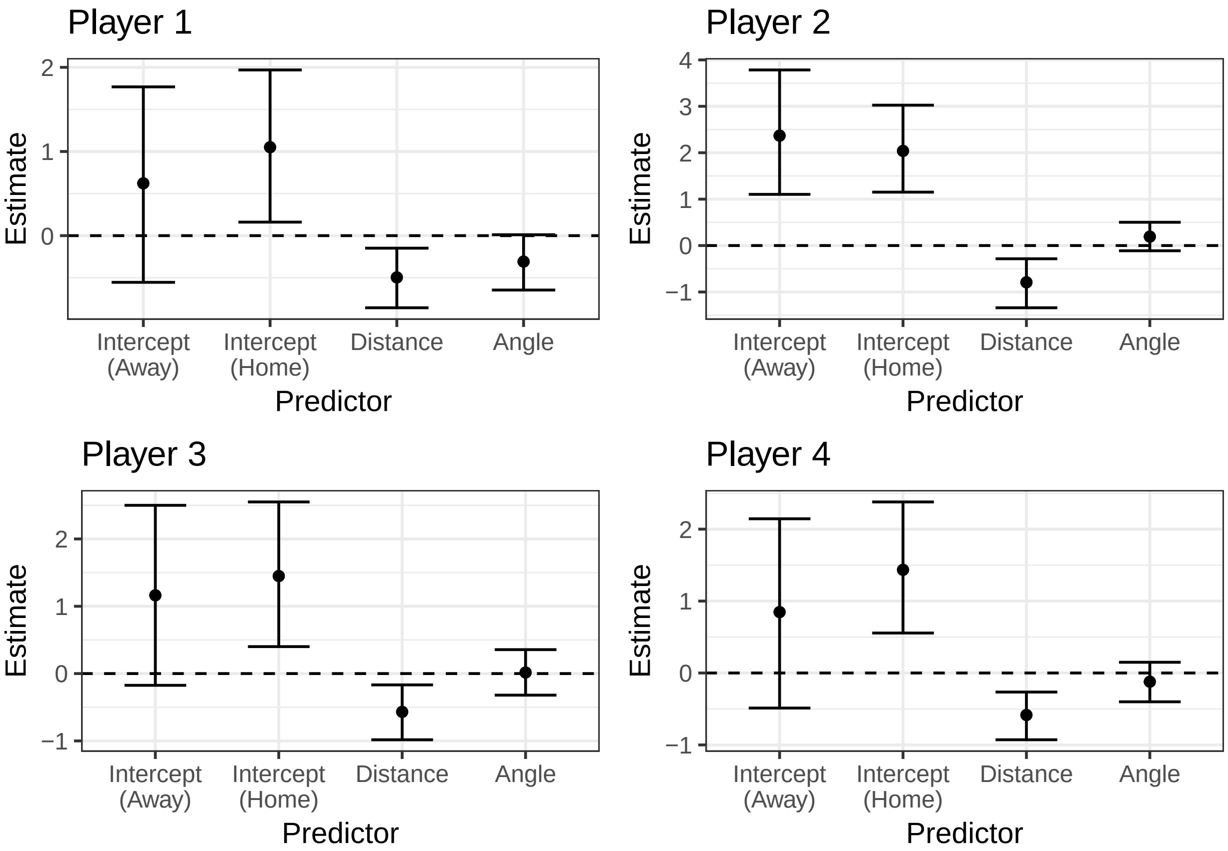
```
for(i in 1:N){

  # delta = discount rate for game g relative to anchor game g0
  wt[i] <- delta^abs(games[i]-g0)

  # player-level random effects
  logit(prob[i]) <- beta_int[player[i]]*int[i] +
                    beta_home[player[i]]*home[i] +
                    beta_r[player[i]]*logr[i] +
                    beta_theta[player[i]]*theta[i]

  # likelihood function
  p1[i] <- prob[i]^result[i]
  p2[i] <- (1-prob[i])^(1-result[i])

  # discounted likelihood function
  pi[i] <- (p1[i] * p2[i])^wt[i]

  # defines correct discounted likelihood function
  y[i] ~ dbern(pi[i])

}

# Priors
for(j in 1:M){
  beta_int[j] ~ dnorm(beta_int0,tau_int)
  beta_home[j] ~ dnorm(beta_home0, tau_int)
  beta_r[j] ~ dnorm(beta_r0,tau_r)
  beta_theta[j] ~ dnorm(beta_theta0,tau_theta)
}
```

See Appendix A.3 for the full `R` code.

2.2 Analysis

2.2.1 Generalized Linear Model

In our generalized linear model, we only look at shot location and the home court indicator as predictors of shot outcome. This is a logistic regression model where the intercepts correspond to the log-odds of making a shot when the angle is zero (the middle of the court) and the log-distance is zero (one foot away from the rim). To illustrate these effects for individual players, we simply subset the dataset to include only shots attempted by that player before running the Gibbs Sampler. In Figure 2.2, The 95% credible intervals of the posterior parameters are reported for the same four players that were introduced in Figure 1.2.



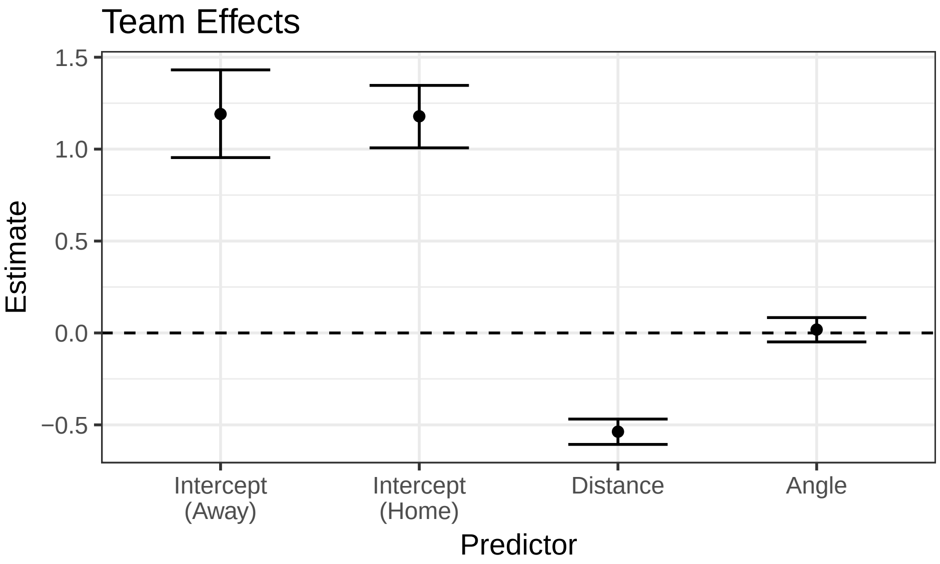


Figure 2.2: GLM Posterior Distributions for Four Players

From these plots, we see that the team-wide 95% credible interval of the angle effect is centered at zero, and it is therefore probably not predictive of a made shot. The average distance effect shows us that the log-odds of a made shot decrease by $\beta_r = 0.5372$ as the log-distance increases by one unit and the other predictors remain constant. This effect is translated to the probability scale using the expression

$$\frac{e^{\beta_r}}{1 + e^{\beta_r}},$$

which equals 0.3689.

We also see that the 95% credible interval on the effect of distance is completely negative, which follows the intuitive idea that the probability of a made shot significantly decreases as distance from the basket increases. The intercepts show us that there is not a substantial difference in baseline shooting performance between home games and away games. The interval for away games is wider because there are fewer of them in the dataset.

2.2.2 Hierarchical Generalized Linear Model

To make the hierarchical model, we add random effects that allow the parameters to vary for each player in the dataset. For every linear covariate in the model, we model player-specific effects as sampled from a Normal distribution roughly centered at the covariate's population mean. We present the results in Figure 2.3 by comparing the effects of the four high-usage players of interest to the population density of each covariate.

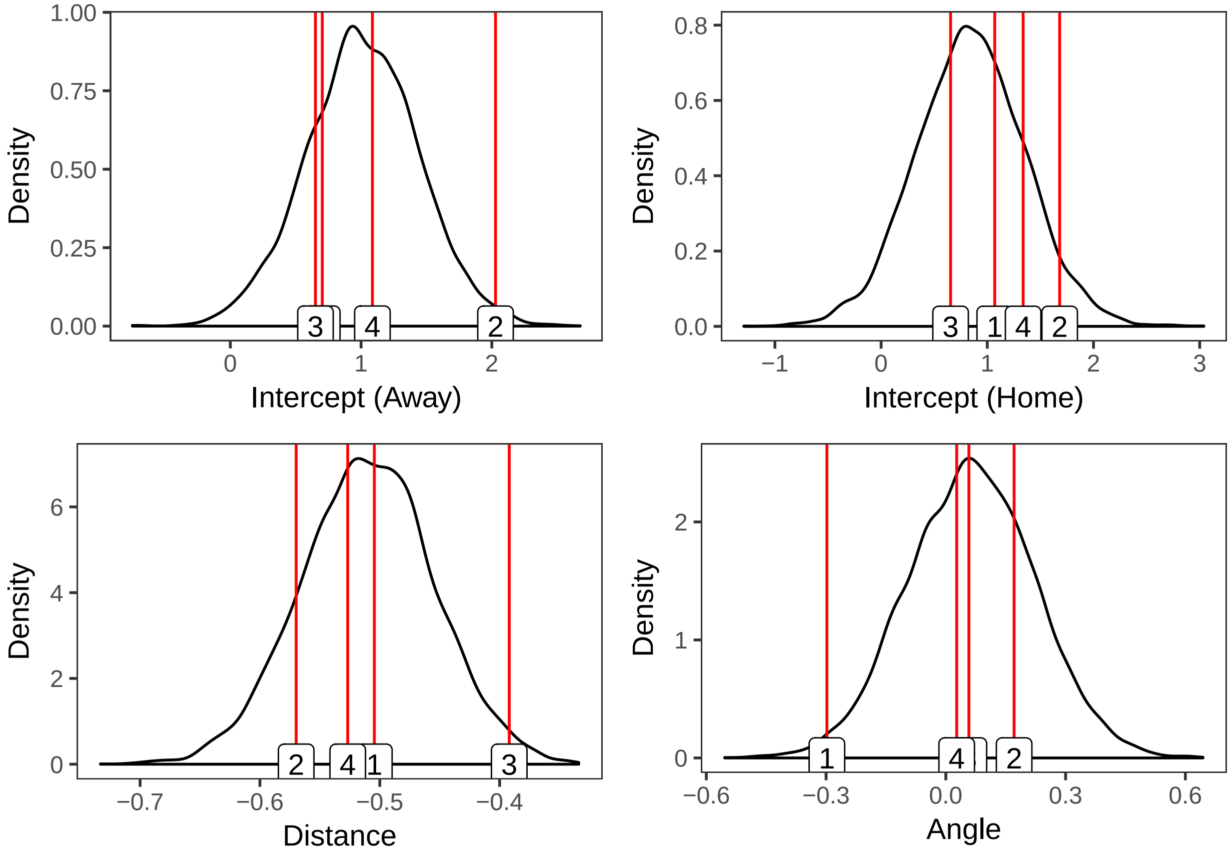


Figure 2.3: Population Distribution with Four Player Effects

The plots in Figure 2.3 show us that Player 2 excels at scoring under baseline conditions (close to the basket), but he has a steeper-than-average drop in his odds of scoring as his distance from the basket increases. We can also see that Player 1 strongly increases his odds of scoring when his angle is negative, which corresponds to the left side of the basket, while the other three players' effects are all close to zero.

In Figure 2.4, we present contour plots showing the players' expected field goal percentages at all locations on the half of the court where they are shooting. We compute these from the fitted probabilities of shot success from the hierarchical model. This plot confirms that Player 1 is more effective on the left side of the basket than the others. Other takeaways that were not noticeable in Figure 2.3 are that Player 2 has the darkest overall contour plot, which suggests that he has the highest overall probability of scoring, and that Player 4 has lightest plot, suggesting he is the least reliable scorer among these four players.

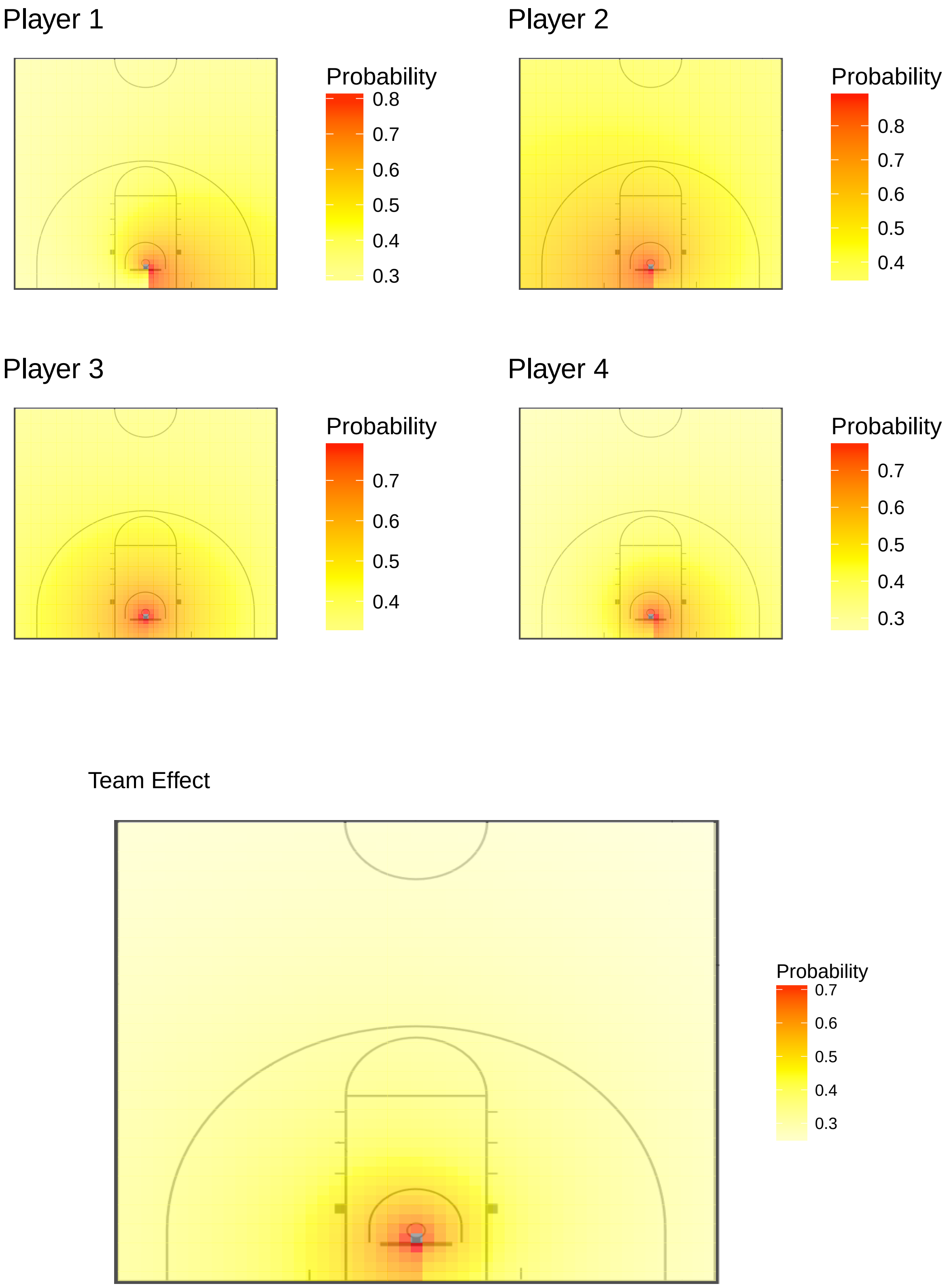


Figure 2.4: Contour Plots for Four Players and Population of Players

2.2.3 Discounted Likelihood Hierarchical Model

The values of δ that we use to fit the discounted likelihood models are 0.750, 0.800, 0.850, 0.900, 0.950, and 0.999. For each of these values, we refit the model to generate a full MCMC sample from the posterior, using every game as the anchor game g_0. We calculate predictions and fitted values for a particular shot in game g using the posterior median of the MCMC chain where g is the anchor game g_0. The plots in Figure 2.5 and 2.6 show how the posterior parameter distributions (95% credible intervals) change over the course of one season on the team level and for two players (the other two players are not shown here because they did not play during this season). Figure 2.5 illustrates the results for our smallest value of δ, 0.750, and 2.6 shows them for our largest value of δ, 0.999.

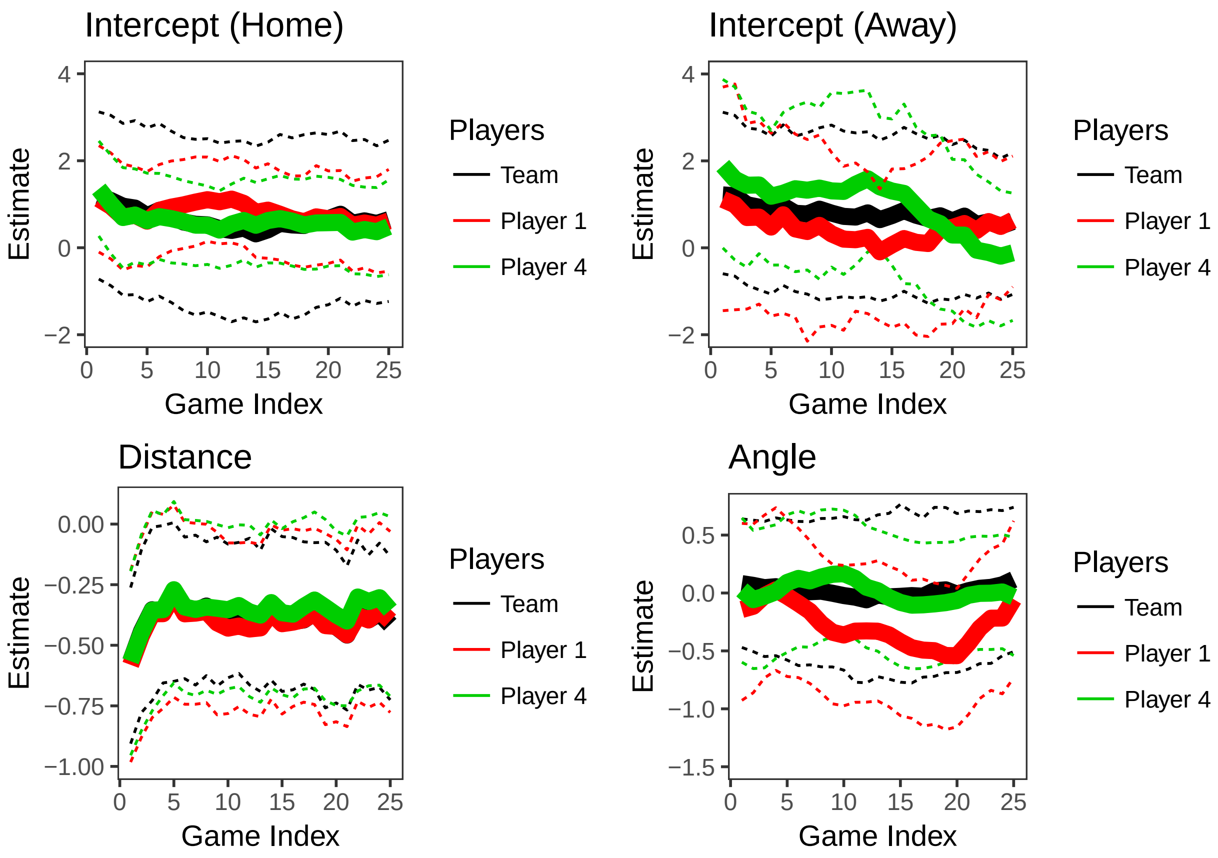


Figure 2.5: Parameters for Two Players and Population over Time, $\delta = 0.750$

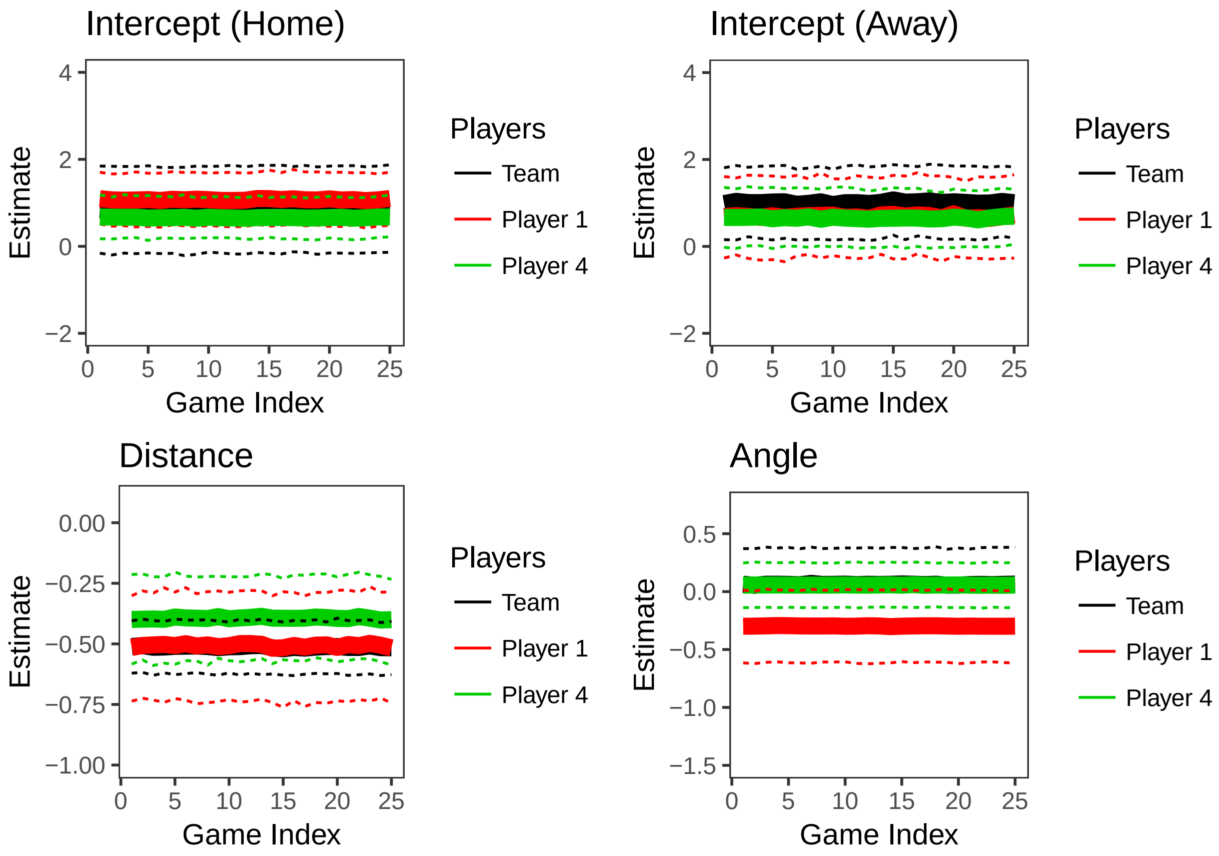


Figure 2.6: Parameters for Two Players and Population over Time, $\delta = 0.999$

These plots illustrate how smaller values of δ allow for more variation in the posteriors over time. Another observation from this plot is that the credible intervals are wider when δ is smaller. Between the two plots of the home game intercepts, we can see that the 95% credible in the first game when $\delta = 0.750$ ranges from about -1 to 3, while the corresponding interval in the plot where $\delta = 0.999$ ranges from about 0 to 2. The uncertainty in the posteriors increases as the discounting parameter δ decreases because a smaller δ corresponds to fewer observations contributing to the likelihood.

Chapter 3

Discussion

3.1 Evaluation of Models

To evaluate these models, we use 5-fold cross-validation. In each train-test split, we evaluate the models' out-of-sample classification rates (using a cutoff probability of 0.5), Brier scores (mean squared error), and log-likelihoods. The predictions and fitted values are obtained using MCMC averages; to calculate the probability for an individual shot, we calculate a response for each of the 9,500 posterior simulations, then take the average of those responses. This process used up to 20 simultaneous RStudio Pro servers provided by the Duke University Statistical Science Department. The results are plotted below in Figure 3.1:

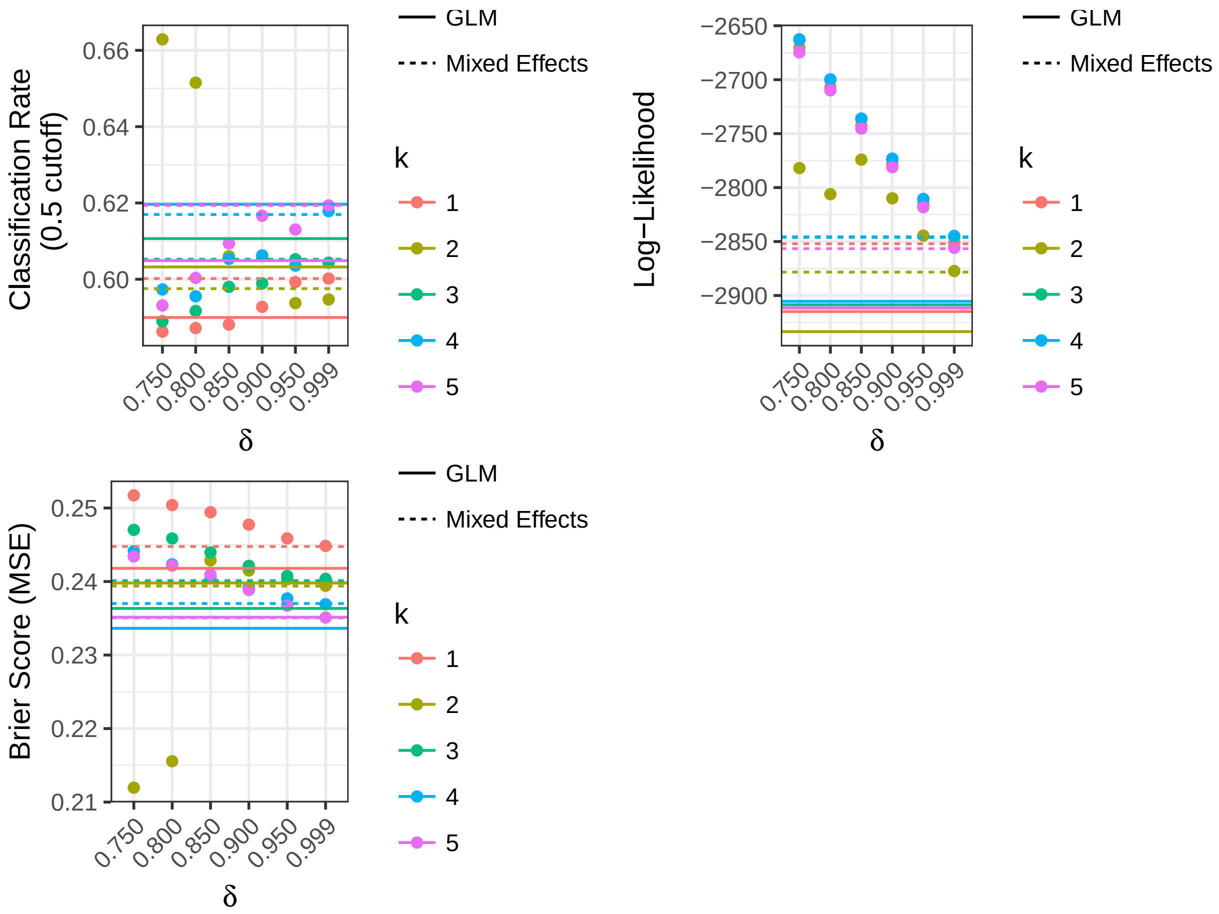


Figure 3.1: Model Evaluation

From Figure 3.1, we can observe that all of the models have different strengths. The discounted likelihood model with the smallest value of δ consistently has the highest likelihood. However, it does not test as well as the other models in areas of out-of-sample classification rate and Brier score. This suggests that models with smaller values of δ, where the likelihood of an observed shot is more heavily influenced by shots closer to it, may overfit the model to the training data. The generalized linear models perform best in Brier score, but worst in log-likelihood. The hierarchical models are about the same as the GLMs, but they have a better log-likelihood performance. A model that balances the trade-off between predictive accuracy and likelihood is a discounted likelihood model with $\delta = 0.850$.

In addition, we can see that the overall variation in model performance is small. For example, most of the out-of-sample classification rates fall between 0.58 and 0.62. This is within the 95% confidence interval for a random binomial proportion of 0.6 using a sample size of 40 (because there are 8 different models and 5 train-test splits for each model), which is (0.5225, 0.6775). Therefore, the evidence that the models without discounting predict better than the ones with discounting is not particularly strong.

For the discounted likelihood model with $\delta = 0.850$, we build calibration plots to assess how well the estimated probabilities fit the actual proportions. To make these plots, we divide the predicted probabilities into 20 equally-sized bins, then plot these bins on the x-axis with the proportions of the actual outcomes within the bins on the y-axis. The horizontal bars represent the bin width, and the vertical bars represent a 95% confidence interval of the proportions. The red line of slope 1 represents equality between the bin medians and the empirical probabilities within the bins. In Figure 3.2, we present these plots for a full training set and a testing set.

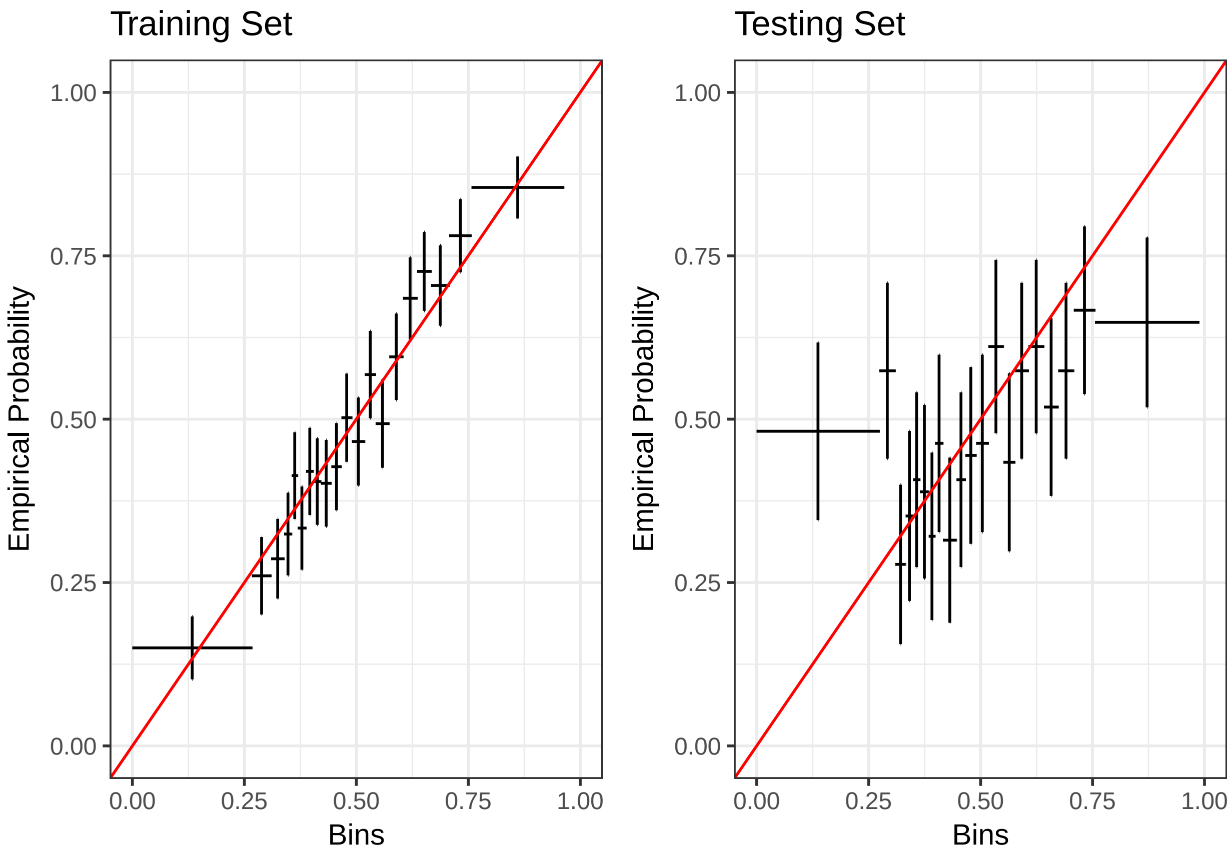


Figure 3.2: Calibration Plots for Discounted Likelihood Model, $\delta = 0.850$

We can see that the confidence intervals on the training set all cross the line of slope 1, which shows that the model output reliably fits the probabilities. In the testing set, however, the predictions only cross this line between about 0.3 and 0.75. In addition, the widths of the bins on the edges show that the model is not likely to predict values close to 0 or 1.

3.2 Results from Model with $\delta = 0.850$

To illustrate results from the discounted likelihood model with $\delta = 0.850$, we replicate the plots in Figures 2.5 and 2.6. The results are shown in Figure 3.3.

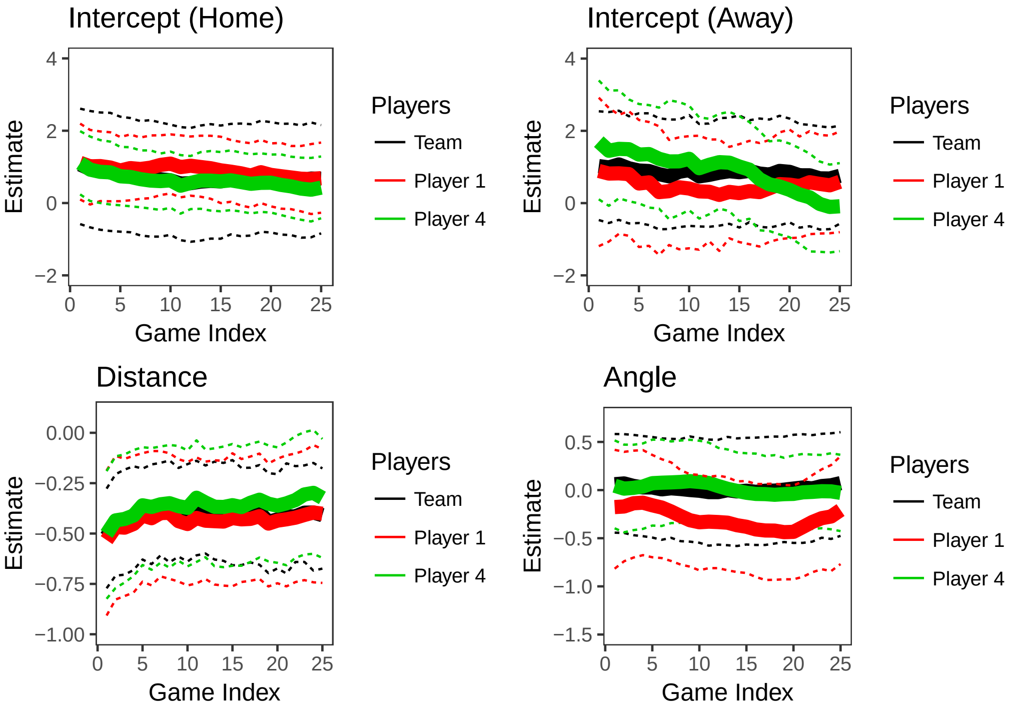


Figure 3.3: Parameters for Two Players and Population over Time, $\delta = 0.850$

Figure 3.3 has smoother changes over time than in Figure 2.5, where $\delta = 0.750$, which could indicate that there is less overfitting. One surprising result from this model is how the distance parameters slightly increase over time, and the intercepts slightly decrease. This could be a result of team shot selection evolving throughout the season, or just a coincidental signal.

3.3 Conclusion

The evaluations of the models show that there is some weak evidence for time-dependency in shooting success rate in this dataset of player-tracking data from the Duke Men's Basketball team. Allowing predictors of shot success to shift based on recent success does not significantly improve the predictive accuracy of a model. However, we do see a systematic improvement in likelihood for smaller discount factors (i.e., more emphasis on recent shots, and therefore support of "streakiness"). Weaknesses of the discounting model include a smaller sample size and poorer out-of-sample prediction. Takeaways that we observed in other models include the fact that the angle of the shot only matters for certain players, and it is not a significant predictor of shot success between all players. Also, the effects of home-court advantage are not strong in this dataset, possibly due to the fact that most of the games away from home are missing.

To account for possible unexplained variation between seasons, and for variation introduced from having such a small population of road games, I repeated this analysis on a subset of the data that only consisted of shots from available games in one season (25 games), and shots from all home games (82 games). The results were similar, except for increased uncertainty due to smaller sample sizes. The model evaluation plots show similar patterns to the ones in Figure 3.1, and they are presented in Appendix B.

3.4 Future Goals

Future goals for this research are to build a better-fitting model to predict basketball shots using more advanced factors that can be approximated from the dataset. Possibilities for this include reparametrizing the location of a shot using categories (e.g., corner three-point shots, heaves from half-court) using the distance of the nearest defender as a proxy for defense quality, or using the amount of time a player has played without a substitution or timeout to approximate fatigue. In addition, we make the unrealistic assumption that every player exhibits the same amount of shooting streakiness with our discounted likelihood models. We could improve upon this assumption in the discounted likelihood models by adding a random effect on δ, to allow it to vary by player.

Appendix A

Code for Models

A.1 Generalized Linear Model

```
Xtrainsub <-
  Xtrain %>%
  filter(as.integer(as.factor(gameid)) < 5)

priormod <- glm(formula = result ~ log(r) + theta,
                data = Xtrainsub,
                family = "binomial")

mu0r <- summary(priormod)[["coefficients"]]["log(r)","Estimate"]
mu0theta <- summary(priormod)[["coefficients"]]["theta","Estimate"]

fit_glm <- function(dat, S = 10000, B = 500){

  model.glm <- function(){

    # Liklihood function (for N observations)
    for(i in 1:N){

      logit(prob[i]) <- beta_int*int[i] +
        beta_home*home[i] +
        beta_r*logr[i] +
        beta_theta*theta[i]

      result[i] ~ dbern(prob[i])
    }

    # Priors
  # we expect less variation in the distance parameter,
  # because shot success rate should get worse
  # as distance increases under baseline circumstances.
  beta_int   ~ dnorm(0, 0.1)
  beta_home  ~ dnorm(0, 0.1)
  beta_r     ~ dnorm(mu0r, 0.01)
  beta_theta ~ dnorm(mu0theta, 0.1)
}

datlist.glm <-
  list(
    int = rep(1, nrow(dat)),
    logr = log(dat$r),
    theta = dat$theta,
    result = dat$result,
    home = dat$home,
    N = nrow(dat),
    mu0r = mu0r,
    mu0theta = mu0theta
  )

params.glm <- c("beta_int",
                "beta_home",
                "beta_r",
                "beta_theta")

initslist <- list(list("beta_int"=0,
                       "beta_r"=0,
                       "beta_theta"=0,
                       "beta_home"=0))

sim <-
  jags(data = datlist.glm,
       n.chains = 1, n.iter = S, n.burnin = B, n.thin = 1,
       inits=initslist,
       parameters.to.save = params.glm,
       model.file=model.glm
  )

sim.mcmc <- as.data.frame(as.mcmc(sim)[[1]])

# Changing from a baseline mean + a shift amount
# to two different means based on the type of game.
sim.mcmc <-
```

```
    sim.mcmc %>%
    mutate(beta_intA = beta_int,
           beta_intH = beta_int + beta_home) %>%
    select(beta_intA, beta_intH, beta_r, beta_theta)

  return(sim.mcmc)
}
```

A.2 Hierarchical Generalized Linear Model

```
Xtrainsub <-
  Xtrain %>%
  filter(as.integer(as.factor(gameid)) < 5)

priormod <- glm(formula = result ~ log(r) + theta,
                data = Xtrainsub,
                family = "binomial")

mu0r <- summary(priormod)[["coefficients"]]["log(r)","Estimate"]
mu0theta <- summary(priormod)[["coefficients"]]["theta","Estimate"]

fit_players <- function(dat = NA, S = 10000, B = 500){

  model.player <- function(){

    # Likelihood function for N observations
    for(i in 1:N){

      # the parameters now vary by player id.
      logit(prob[i]) <- beta_int[player[i]]*int[i] +
        beta_home[player[i]]*home[i] +
        beta_r[player[i]]*logr[i] +
        beta_theta[player[i]]*theta[i]

      result[i] ~ dbern(prob[i])
    }

    # Priors
    for(j in 1:M){
        beta_int[j] ~ dnorm(beta_int0,tau_int)
        beta_home[j]  ~ dnorm(beta_home0, tau_int)
        beta_r[j] ~ dnorm(beta_r0,tau_r)
      beta_theta[j] ~ dnorm(beta_theta0,tau_theta)
  }

  # Hyperpriors
  beta_int0   ~ dnorm(0, 0.1)
  beta_home0  ~ dnorm(0, 0.1)
  beta_r0     ~ dnorm(mu0r, 0.01)
  beta_theta0 ~ dnorm(mu0theta, 0.1)
  tau_int ~ dgamma(10, 100)
  tau_r ~ dgamma(10, 0.2)
  tau_theta ~ dgamma(10, 10)
}

datlist.player <-
  list(
    logr = log(dat$r),
    theta = dat$theta,
    home = dat$home,
    result = dat$result,
    player = as.integer(as.factor(dat$globalplayerid)),
    N = nrow(dat),
    int = rep(1, nrow(dat)),
    M = n_distinct(dat$globalplayerid),
    mu0r = mu0r,
    mu0theta = mu0theta
  )

# we want posteriors for the overall effects
# and for the individual player effects
params <- c("beta_int",
            "beta_home",
            "beta_r",
            "beta_theta",
            "beta_int0",
            "beta_home0",
            "beta_r0",
            "beta_theta0")

M <- datlist.player$M

initslist <- list(
  list("beta_int"=rep(0,M),
       "beta_home"=rep(0,M),
       "beta_r"=rep(0,M),
       "beta_theta"=rep(0,M),
       "beta_int0"=0,
       "beta_home0"=0,
       "beta_r0"=0,
       "beta_theta0"=0,
       "tau_int"=1,
       "tau_r"=1,
       "tau_theta"=1
))

sim.player <-
  jags(data = datlist.player,
       n.iter = S, n.chains = 1, n.burnin = B, n.thin = 1,
       inits=initslist,
       parameters.to.save = params,
       model.file=model.player
)

sim.mcmc.player <- as.data.frame(as.mcmc(sim.player)[[1]])


# Changing from a baseline mean + a shift amount
# to two different means based on the type of game.
hometext <- paste0("`beta_intH[",1:M,"]` =
                   `beta_int[",1:M,"]` +
                   `beta_home[",1:M,"]`",
                   collapse=",\n")

awaytext <- paste0("`beta_intA[",1:M,"]` =
                   `beta_int[",1:M,"]`",
                   collapse=",\n")

sim.mcmc.player <- eval(parse(text=
  paste0("sim.mcmc.player %>%
            mutate(",hometext,",
                   beta_intH0 = beta_int0 + beta_home0)","%>%
            rename(",awaytext,",
                   beta_intA0 = beta_int0)"
         ))) %>%
  select(grep("(beta_int)|(beta_theta)|(beta_r)",names(.)))

colorder <- order(colnames(sim.mcmc.player))
sim.mcmc.player <- sim.mcmc.player[ , colorder]
```

```
  # Renaming mixed effects columns from default factor levels
  # (integers) to the corresponding player ids

  factorids <-
    str_extract_all(names(sim.mcmc.player), "[[:digit:]]+") %>%
    as.numeric()

  fids <- data.frame(factorid = factorids,
                     order = 1:length(factorids))

  datmap <- dat %>%
    mutate(factorid = as.integer(as.factor(globalplayerid))) %>%
    select(globalplayerid, factorid)

  gameids <- merge(datmap, fids, all.x=FALSE,all.y=TRUE) %>%
    unique() %>%
    mutate(globalplayerid = ifelse(is.na(globalplayerid),
                                   0,
                                   globalplayerid)) %>%
    arrange(order)

  names(sim.mcmc.player) <-
    str_replace_all(names(sim.mcmc.player), "[[:digit:]]+",
                    as.character(gameids$globalplayerid))

  return(sim.mcmc.player)

}
```

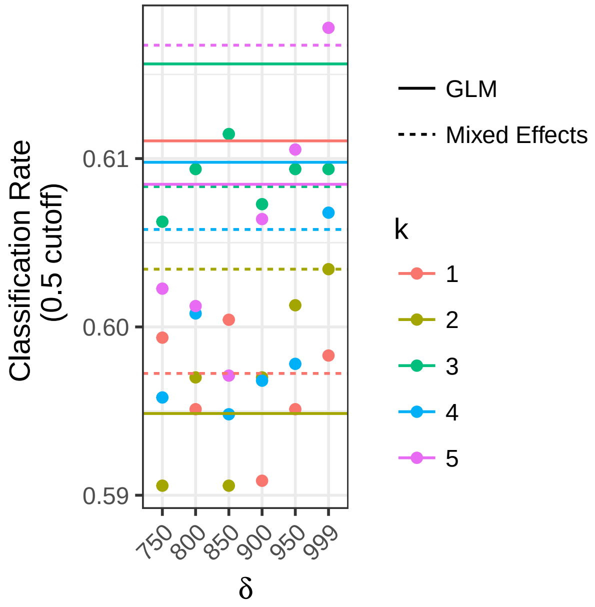
A.3 Discounted Likelihood Hierarchical Model

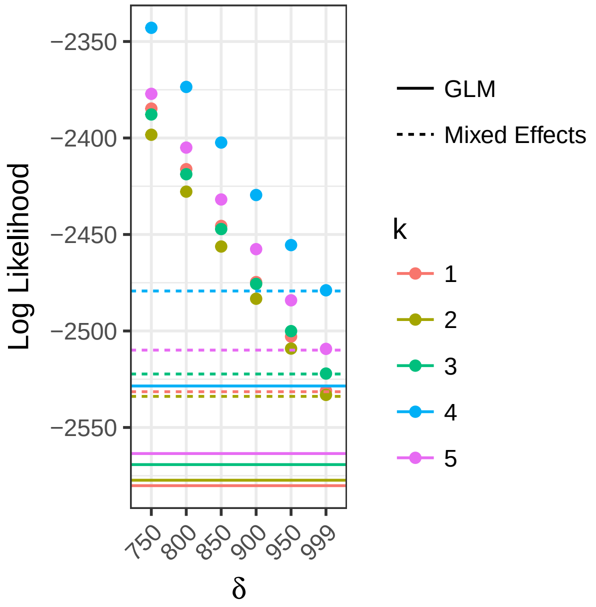
```
fit_game <- function(dat = NA, g0 = NA, delta = NA,
                     S = 10000, B = 500){

  model.game <- function(){

    for(i in 1:N){

      # delta = discount rate for game g relative to anchor game g0
      wt[i] <- delta^abs(games[i]-g0)
    # player-level random effects
    logit(prob[i]) <- beta_int[player[i]]*int[i] +
                      beta_home[player[i]]*home[i] +
                      beta_r[player[i]]*logr[i] +
                      beta_theta[player[i]]*theta[i]

    # likelihood function
    p1[i] <- prob[i]^result[i]
    p2[i] <- (1-prob[i])^(1-result[i])

    # discounted likelihood function
    pi[i] <- (p1[i] * p2[i])^wt[i]

    # defines correct discounted likelihood function
    y[i] ~ dbern(pi[i])

  }

  # Priors
  for(j in 1:M){
    beta_int[j] ~ dnorm(beta_int0,tau_int)
    beta_home[j] ~ dnorm(beta_home0, tau_int)
    beta_r[j] ~ dnorm(beta_r0,tau_r)
    beta_theta[j] ~ dnorm(beta_theta0,tau_theta)
  }

  # Hyperoriors
  beta_int0   ~ dnorm(0, 0.1)
  beta_home0  ~ dnorm(0, 0.1)
  beta_r0     ~ dnorm(mu0r, 0.01)
  beta_theta0 ~ dnorm(mu0theta, 0.1)
  tau_int ~ dgamma(10, 100)
  tau_r ~ dgamma(10, 0.2)
  tau_theta ~ dgamma(10, 10)
}

datlist.game <-
  list(
    int = rep(1, nrow(dat)),
    logr = log(dat$r),
    theta = dat$theta,
    result = dat$result,
    home = dat$home,
    player = as.integer(as.factor(dat$globalplayerid)),
    N = nrow(dat),
    M = n_distinct(dat$globalplayerid),
    mu0r = mu0r,
    mu0theta = mu0theta,
    delta = delta,
    games = as.integer(as.factor(dat$gameid)),
    g0 = g0,
    y = rep(1, nrow(dat))
  )

params <- c("beta_int",
            "beta_r",
            "beta_home",
            "beta_theta",
            "beta_int0",
            "beta_home0",
            "beta_r0",
            "beta_theta0")

M <- n_distinct(dat$globalplayerid)

initslist <- list(list("beta_int"=rep(0,M),
                       "beta_r"=rep(0,M),
                       "beta_theta"=rep(0,M),
                       "beta_int0"=0,
                       "beta_r0"=0,
                       "beta_theta0"=0,
                       "tau_int"=1,
                       "tau_r"=1,
                       "tau_theta"=1
                       ))

sim.game <- jags(data = datlist.game,
            n.iter = S, n.chains = 1, n.burnin = B, n.thin = 1,
            inits = initslist,
            parameters.to.save = params,
            model.file=model.game
)
sim.mcmc.game <- as.data.frame(as.mcmc(sim.game)[[1]])

# Changing from a baseline mean + a shift amount
# to two different means based on the type of game.

hometext <- paste0("`beta_intH[",1:M,"]` =
                   `beta_int[",1:M,"]` +
                   `beta_home[",1:M,"]`",
                   collapse=",\n")

awaytext <- paste0("`beta_intA[",1:M,"]` =
                   `beta_int[",1:M,"]`",
                   collapse=",\n")

sim.mcmc.game <- eval(parse(text=
  paste0("sim.mcmc.game %>%
            mutate(",hometext,",
                   beta_intH0 = beta_int0 + beta_home0)"," %>%
            rename(",awaytext,",
                   beta_intA0 = beta_int0)"
         ))) %>%
  select(grep("(beta_int)|(beta_theta)|(beta_r)",names(.)))

colorder <- order(colnames(sim.mcmc.game))
sim.mcmc.game <- sim.mcmc.game[ , colorder]

# Renaming mixed effects columns from default factor levels
# (integers) to the corresponding player ids


factorids <-
  str_extract_all(names(sim.mcmc.game), "[[:digit:]]+") %>%
  as.numeric()
fids <- data.frame(factorid = factorids,
                   order = 1:length(factorids))

datmap <- dat %>%
  mutate(factorid = as.integer(as.factor(globalplayerid))) %>%
  select(globalplayerid, factorid)

gameids <- merge(datmap, fids, all.x=FALSE,all.y=TRUE) %>%
  unique() %>%
  mutate(globalplayerid = ifelse(is.na(globalplayerid),
                                 0,
                                 globalplayerid)) %>%
  arrange(order)

names(sim.mcmc.game) <-
  str_replace_all(names(sim.mcmc.game),
                  "[[:digit:]]+",
                    as.character(gameids$globalplayerid))

  return(sim.mcmc.game)

}
```

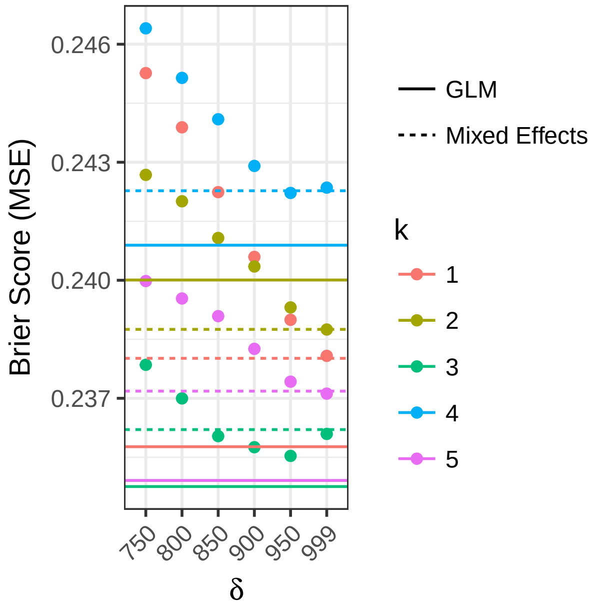
Appendix B

Reproducing Evaluation Plots

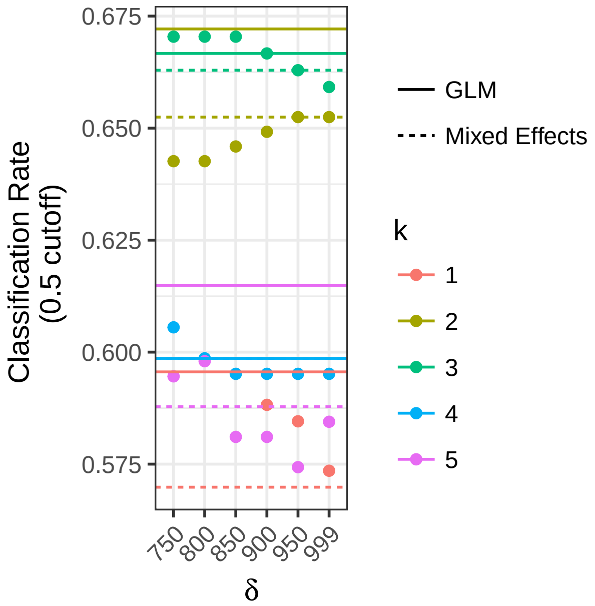
B.1 Home Games Only

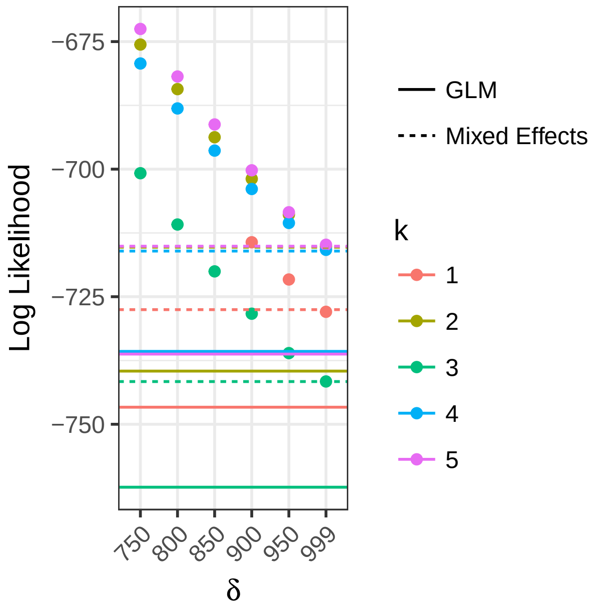


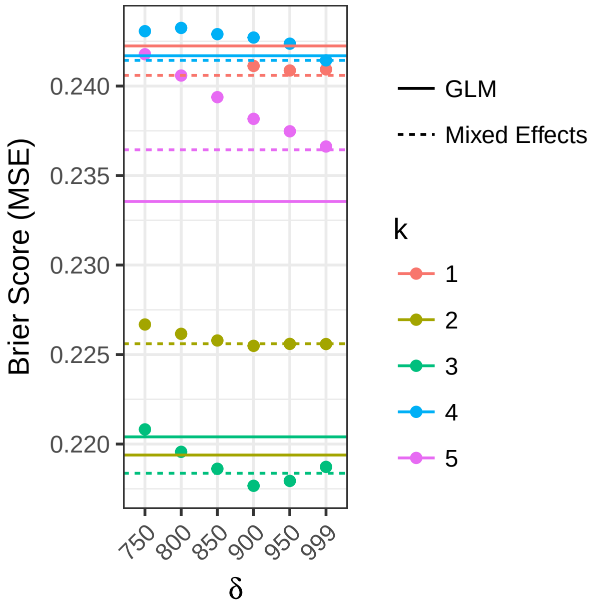




B.2 One Season Only







References

Albert, J. (1993). Statistical analysis of hitting streaks in baseball: Comment. *Journal of the American Statistical Association*, *88*(424), 1184–1188.

Albert, J. (2013). Looking at spacings to assess streakiness. *Journal of Quantitative Analysis in Sports*, *9*(2), 1–13.

Albert, J., & Williamson, P. (1999). Using model/data simulations to detect streakiness. *The American Statistician*, *55*, 41–50.

Ameen, J. R., & Harrison, P. J. (1984). Discount weighted estimation. *Journal of Forecasting*, *3*, 285–296.

Bar-Eli, M., Avugos, S., & Raab, M. (2006). Twenty years of "hot hand" research: Review and critique. *Psychology of Sport and Exercise*, *7*, 525–553.

Gilovich, T., Vallone, R., & Tversky, A. (1985). The hot hand in basketball: On the misperception of random sequences. *Cognitive Psychology*, *17*, 295–314.

Joseph, L. (n.d.). Specifying a new sampling distribution. Retrieved from `http://www.medicine.mcgill.ca/epidemiology/Joseph/courses/common/Tricks.html`

Prado, R., & West, M. (2010). *Time series: Modelling, Computation & Inference.* Chapman & Hall/CRC Press.

Wetzels, R., Tutschkow, D., Dolan, C., Sluis, S. van der, Dutilh, G., & Wagenmakers, E.-J. (2016). A Bayesian test for the hot hand phenomenon. *Journal of Mathematical Psychology*, *72*, 200–209.