10110101
& 1111

00000101

10110101
& 1111

10110000

<<

int a = 255;

a <<= 8;

R

& number

1111111100000000

11111111

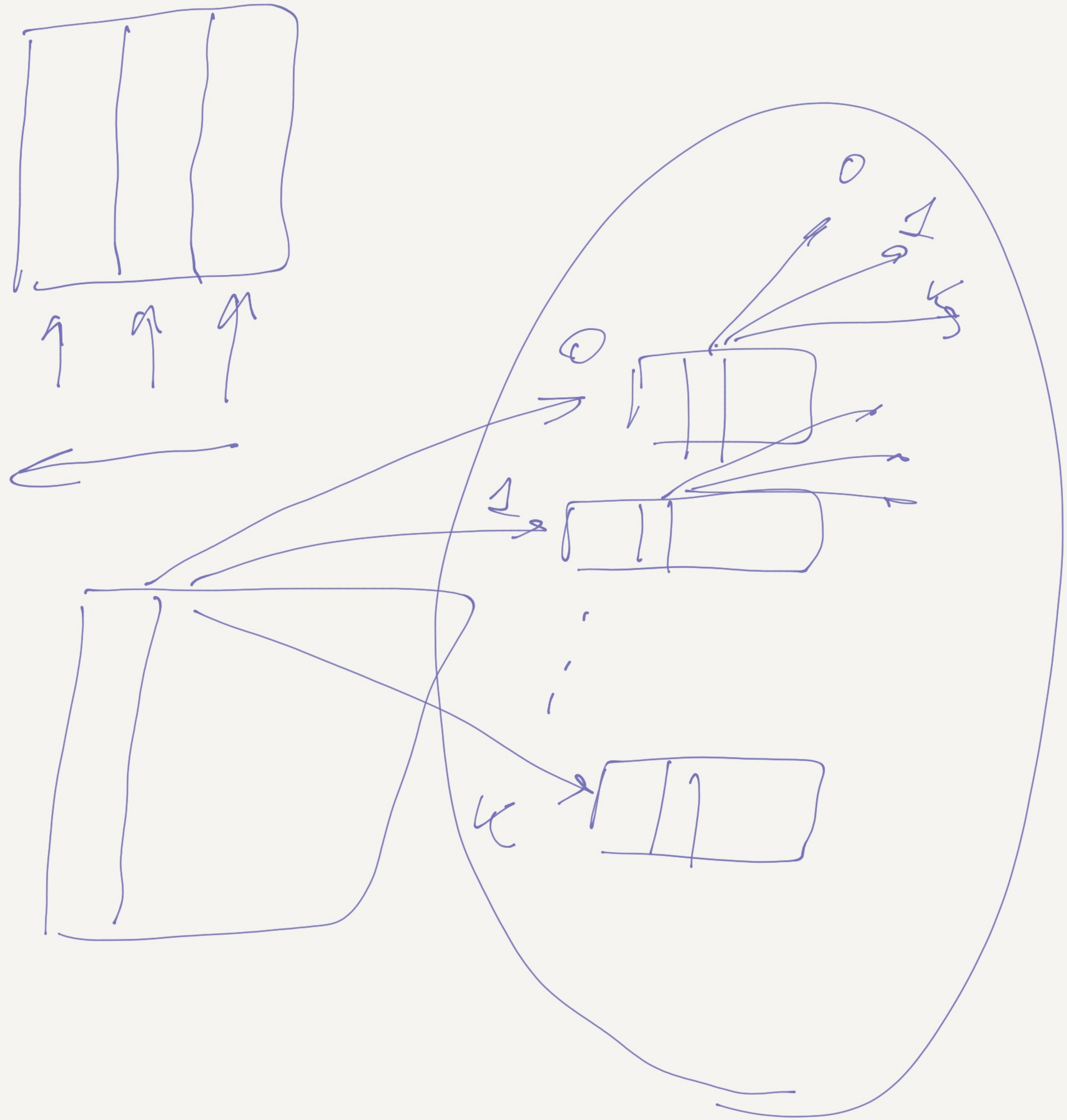
>> 8

R

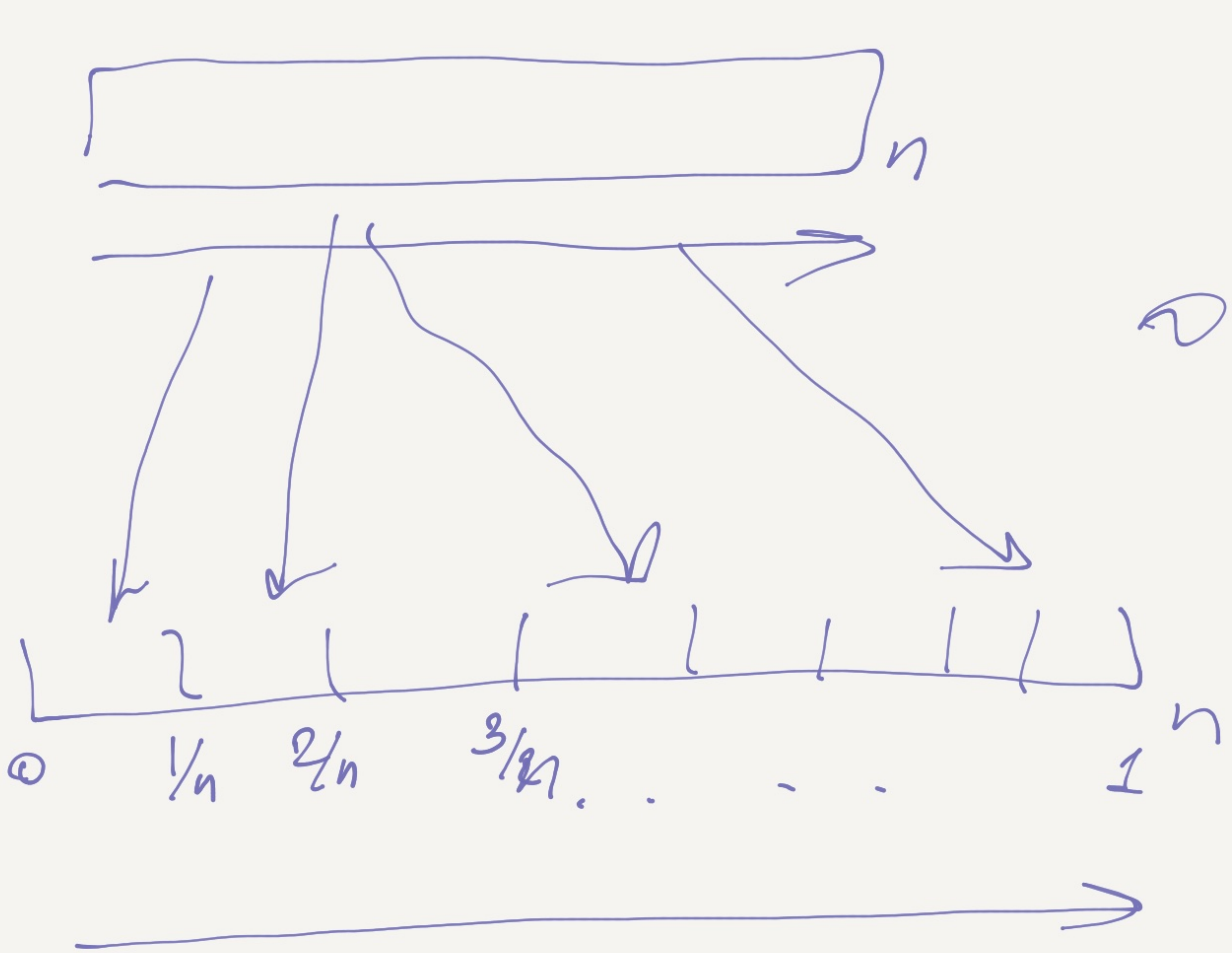
~~++~~ count[a[i]];

++ count[getDigit(a[i], mask, R)];

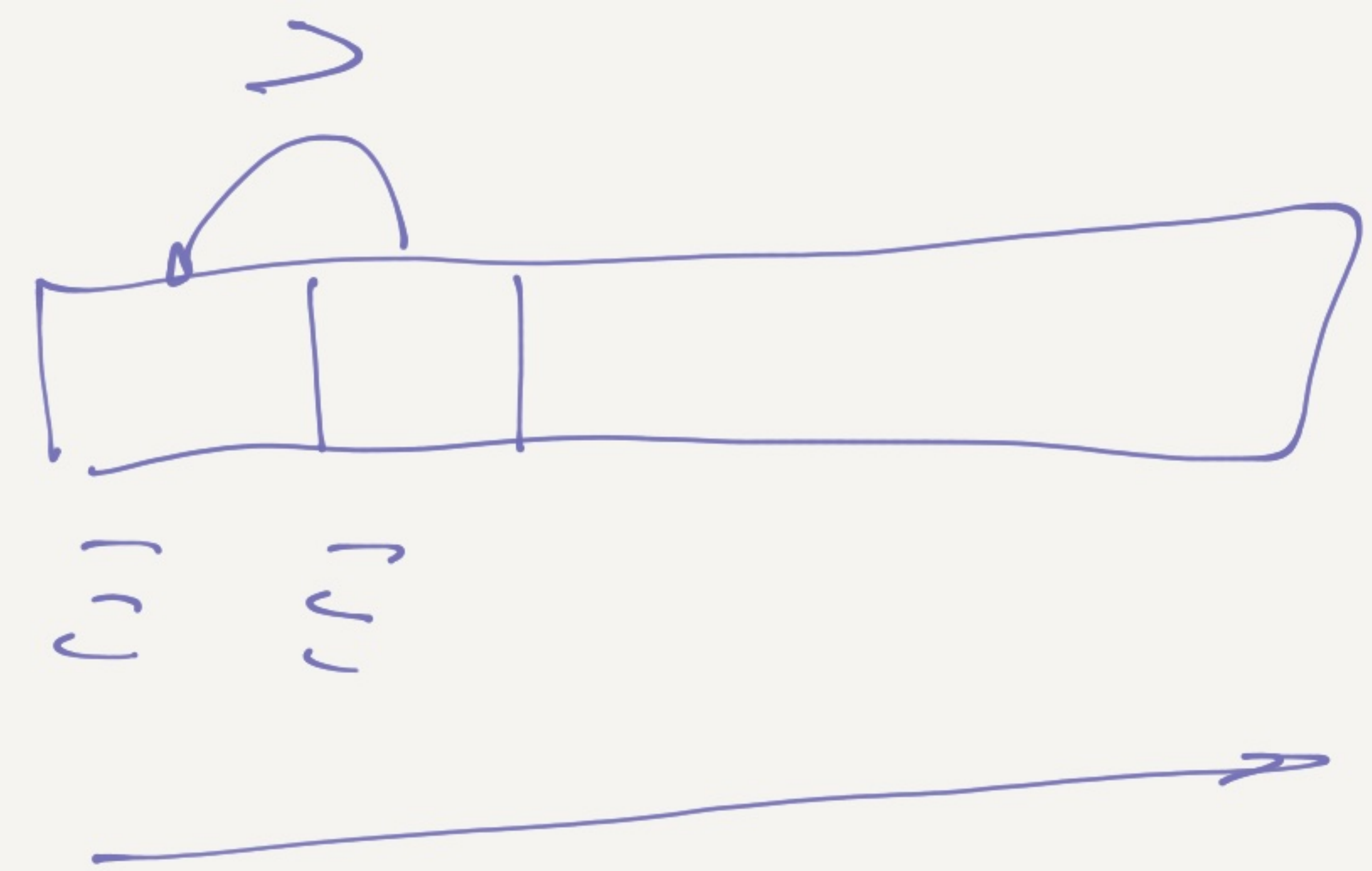
Least Significant Digit



Most Significant Digit



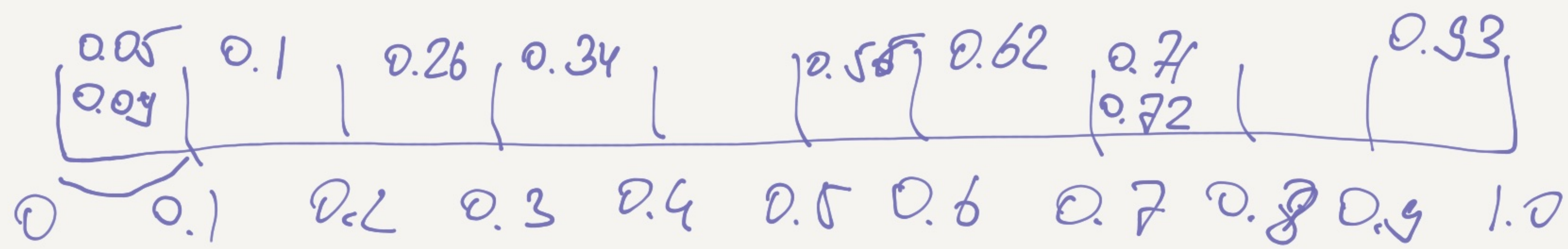
$a_i \in [0;1)$



Insertion Sort

0.1 0.26 0.34 0.05 0.62 0.93 0.09

0.71 0.72 0.56



0.05 0.09 0.1 0.26 0.34 0.56 0.62 0.71 0.72 0.93

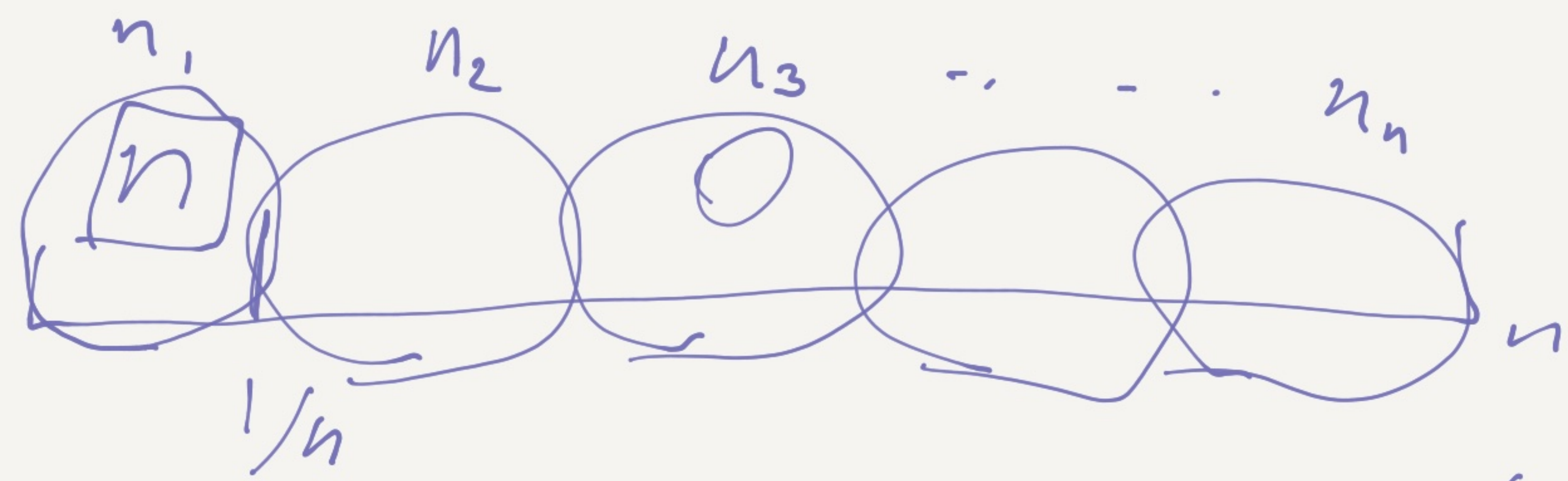
0.001

0.002

0.0011

0.0012

...



$$\boxed{O(n^2)}$$

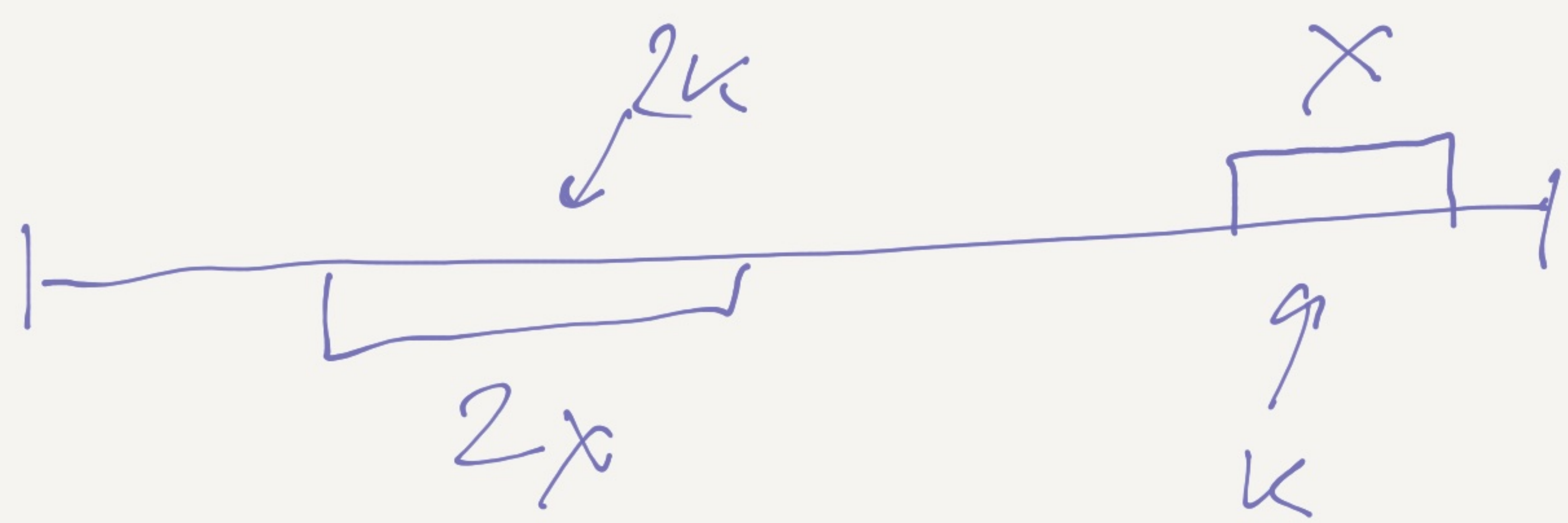
$O(n)$

$$\frac{E(n_i^2)}{} = $$

$$= 2 - \frac{1}{n}$$

$$\begin{cases} n \\ n \cdot 1 \\ n \end{cases}$$

$O(n)$



$O(n)$

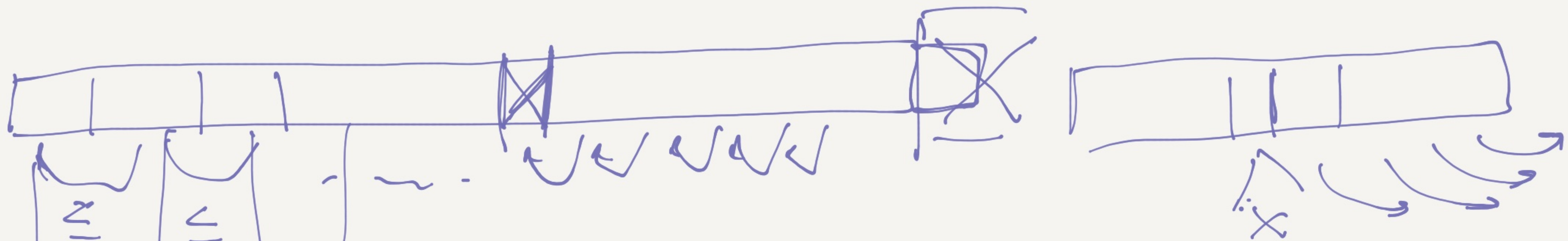
$$\sum_i V_i \cdot P_i = E(V)$$

$$0 \cdot 0{,}5 + 1 \cdot 0{,}5 = 0.5$$

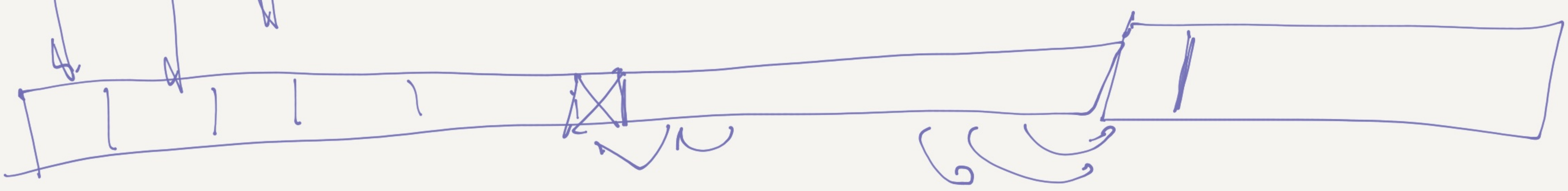
РЕШКА ОРЕЛ

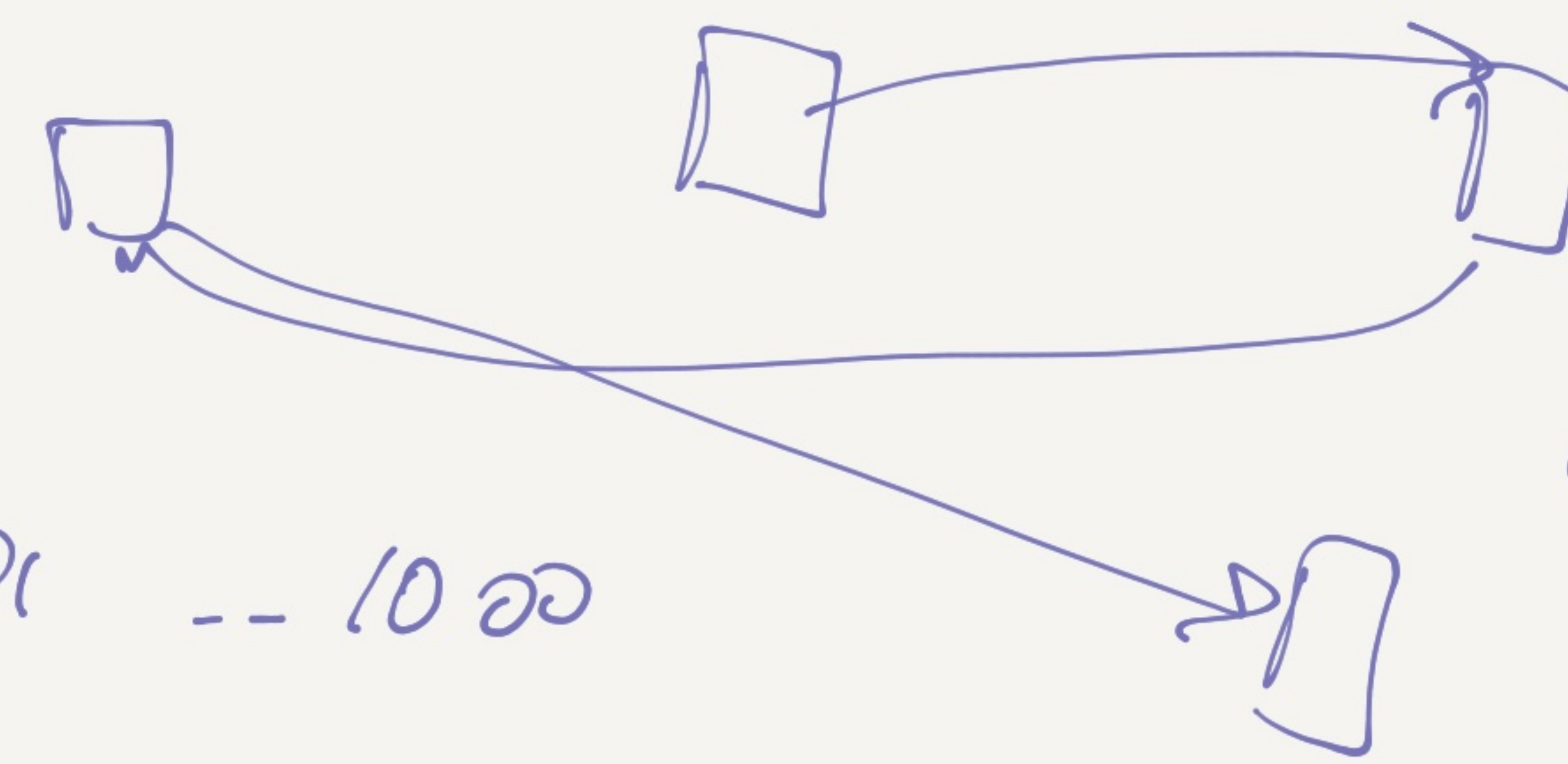
$$1 \cdot \tfrac{1}{6} + 2 \cdot \tfrac{1}{6} + \ldots + 6 \cdot \tfrac{1}{6} = \tfrac{1}{6} \cdot \sum_{i=1}^{6} i = \frac{21}{6} = 3{,}5$$

$O(n_i^2)$



$a_i \le a_j \quad \forall i,j : i<j \qquad O(n)$

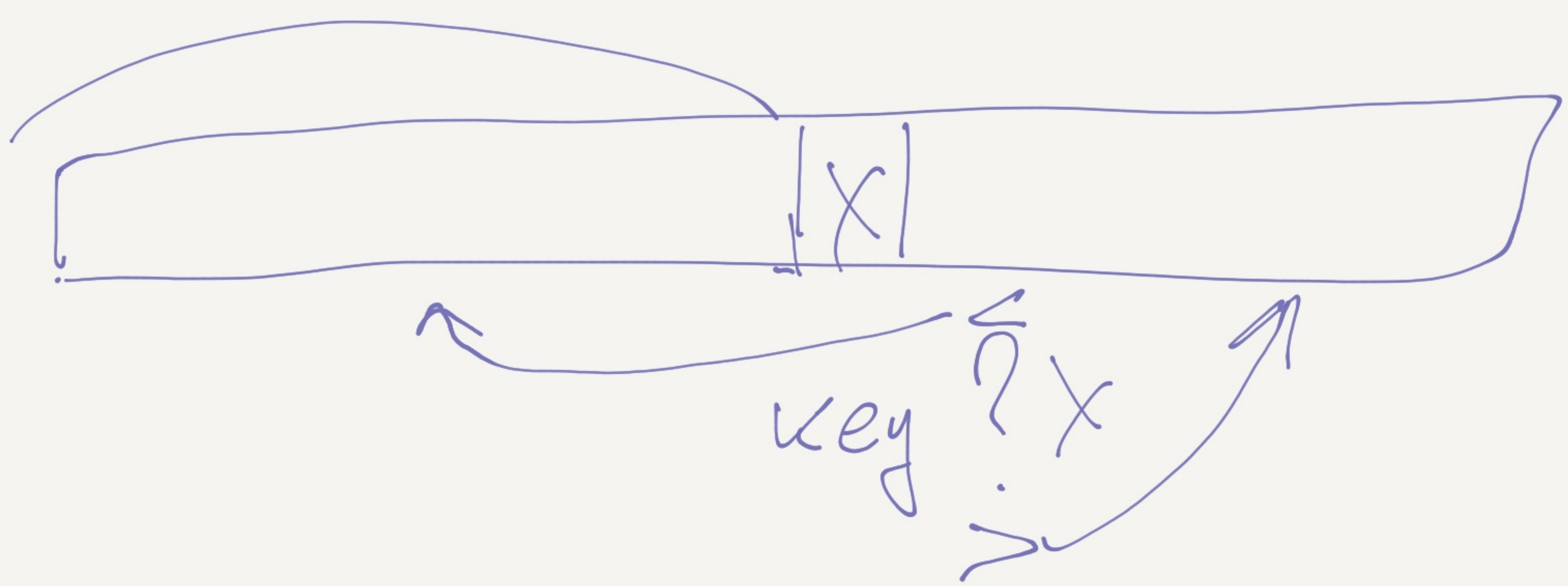




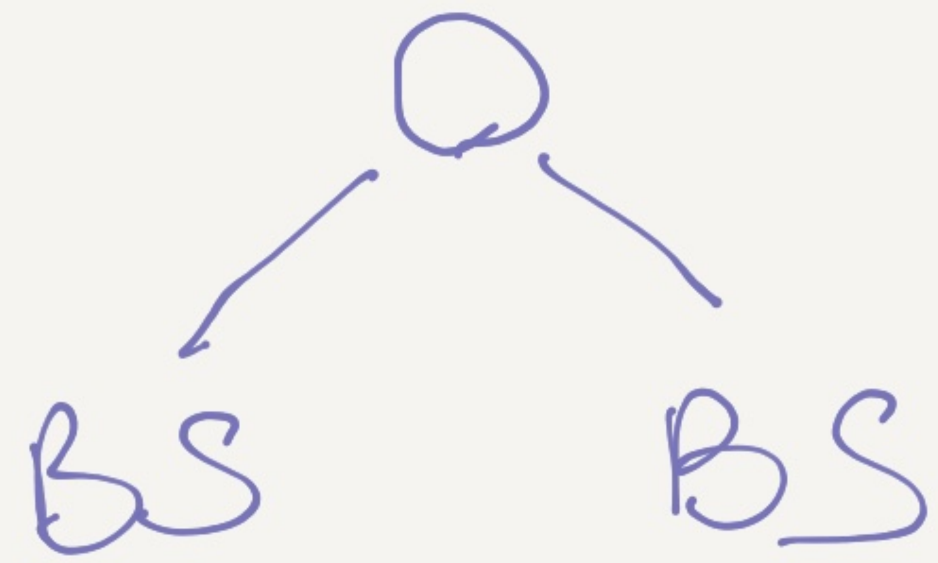
500

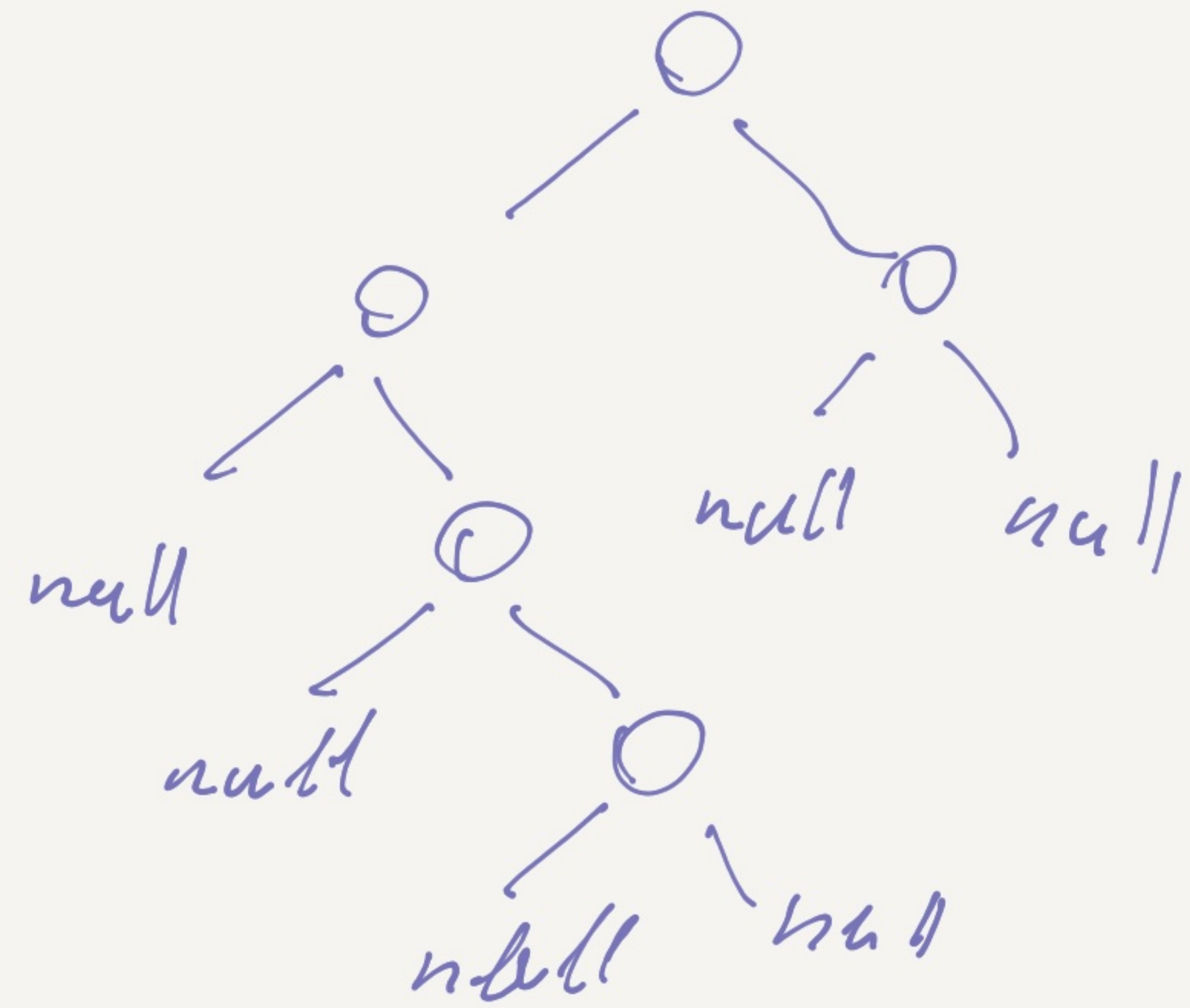
1 .. 499 $\qquad$ 501 ... 1000

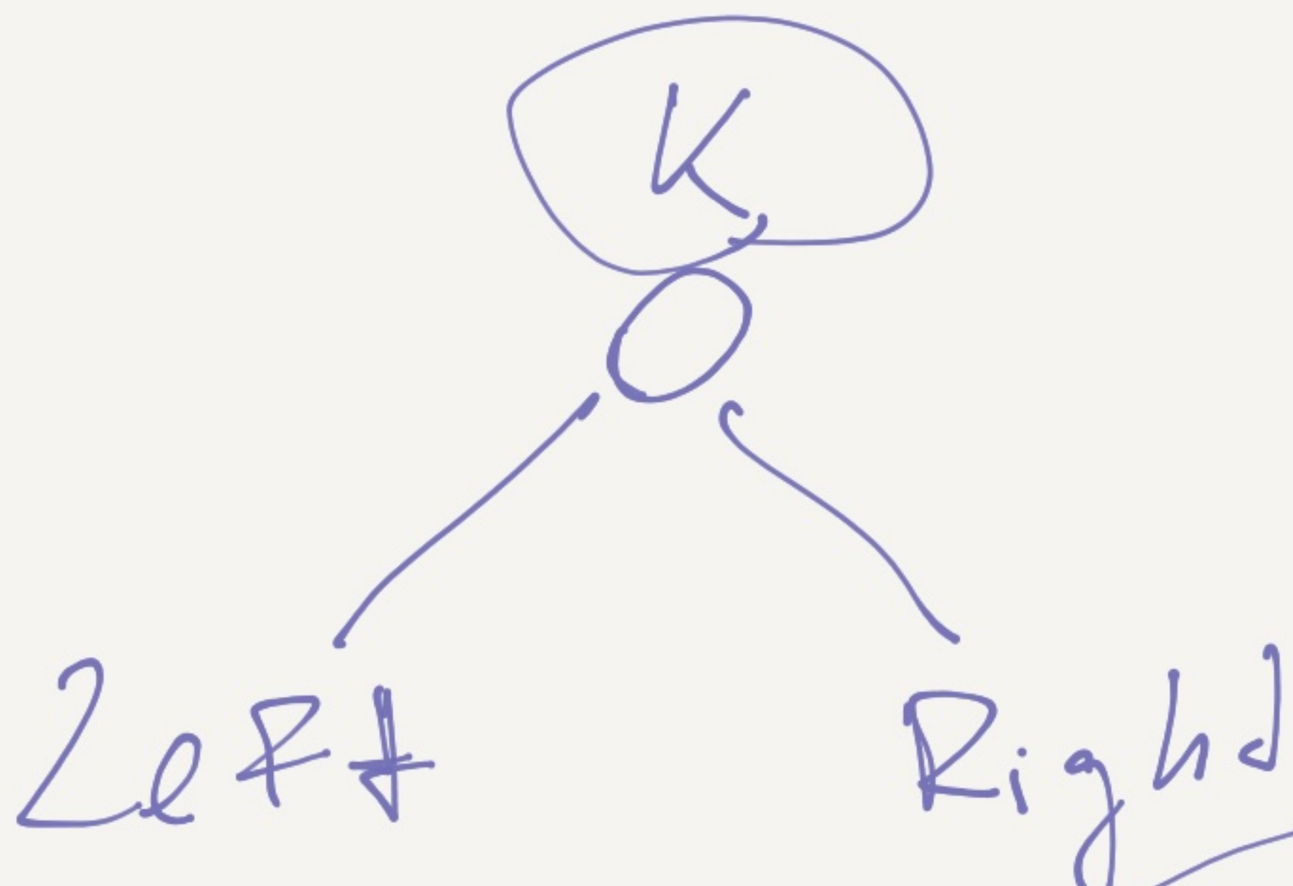
$O(n)$



$O(\log_2 n)$





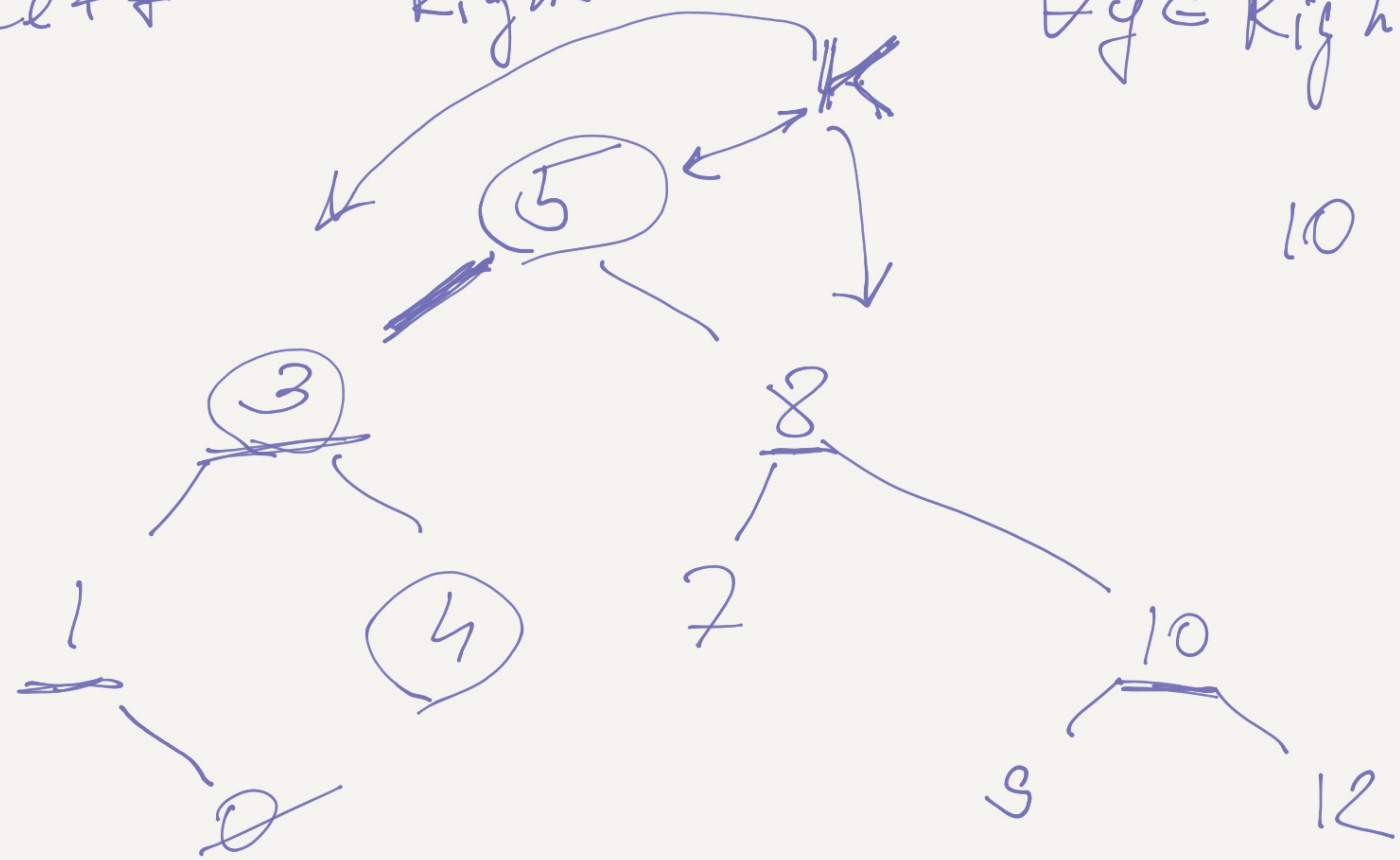


$<$

$\forall x \in Left: x \leq K$

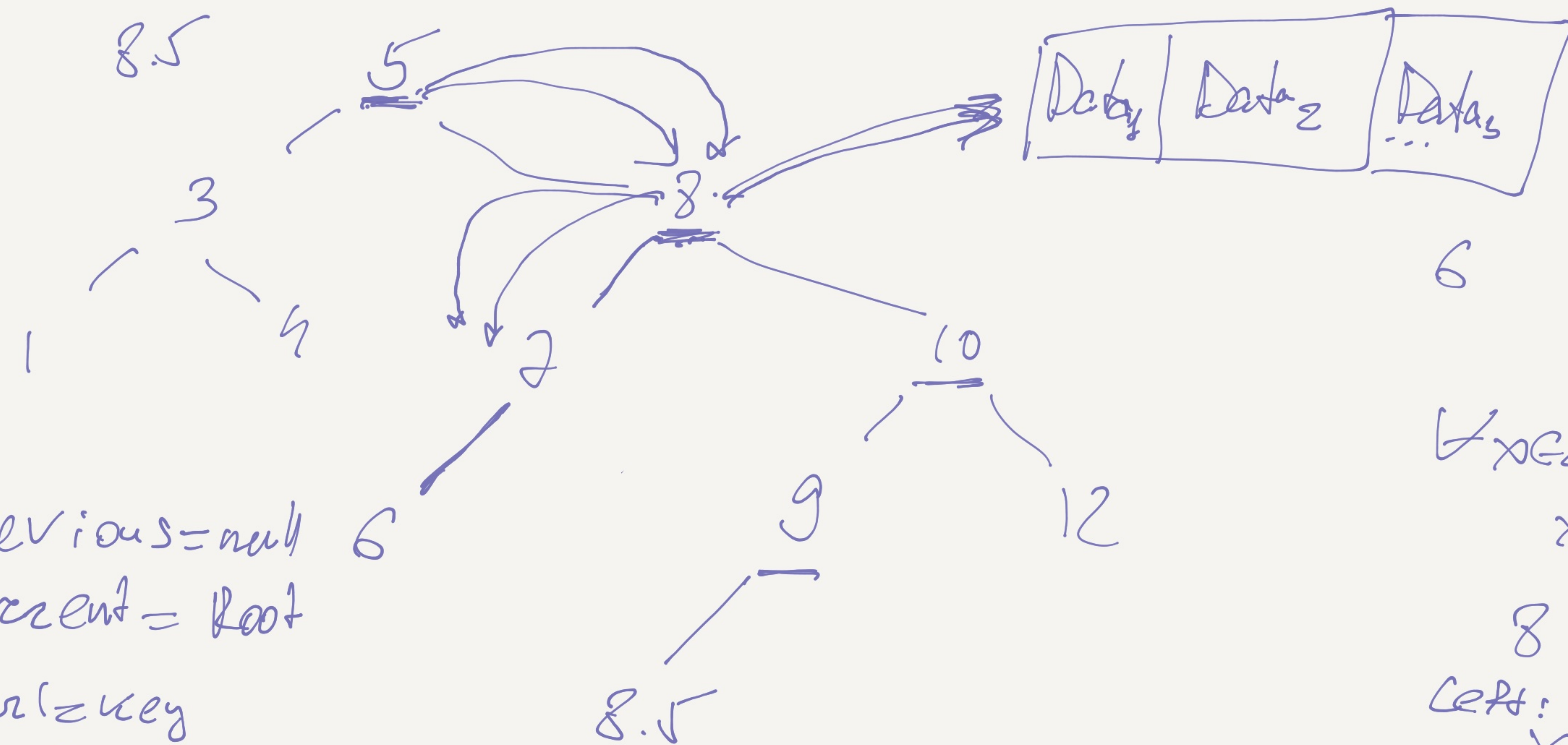
$\forall y \in Right: y > K$

$\geq$



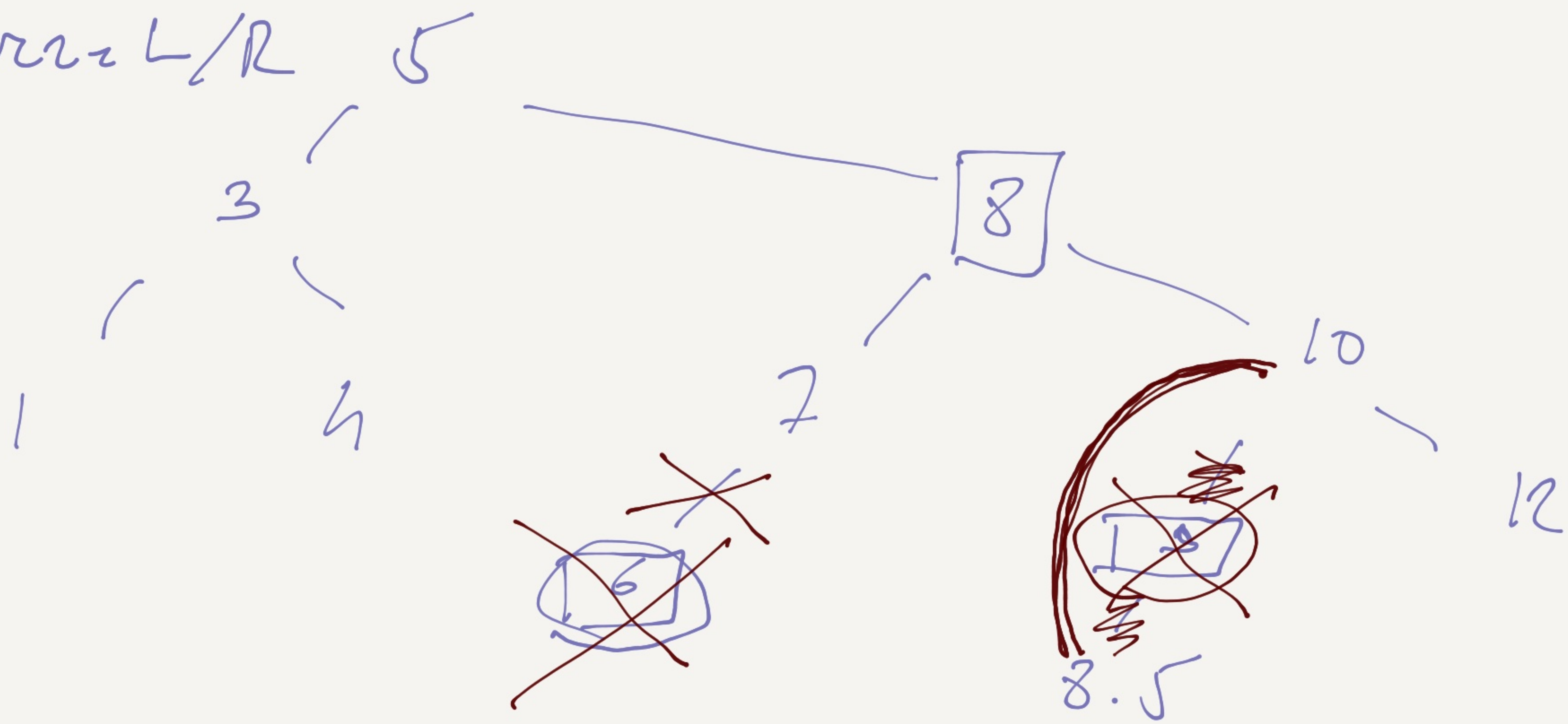
10

2



previous=null
current=Root

curr!=key
prev=curr
curr=L/R



$\forall x \in Left:$
$x \leq key$

$8 \leq 8$

Left:
$x < key$

Right
$y > key$

5
3
8
1
4
7
10
9
12
5
3
9
1
4
7
10
12
9
8
null
≤
10
16
9
12
13
9.5
10
9.5
12

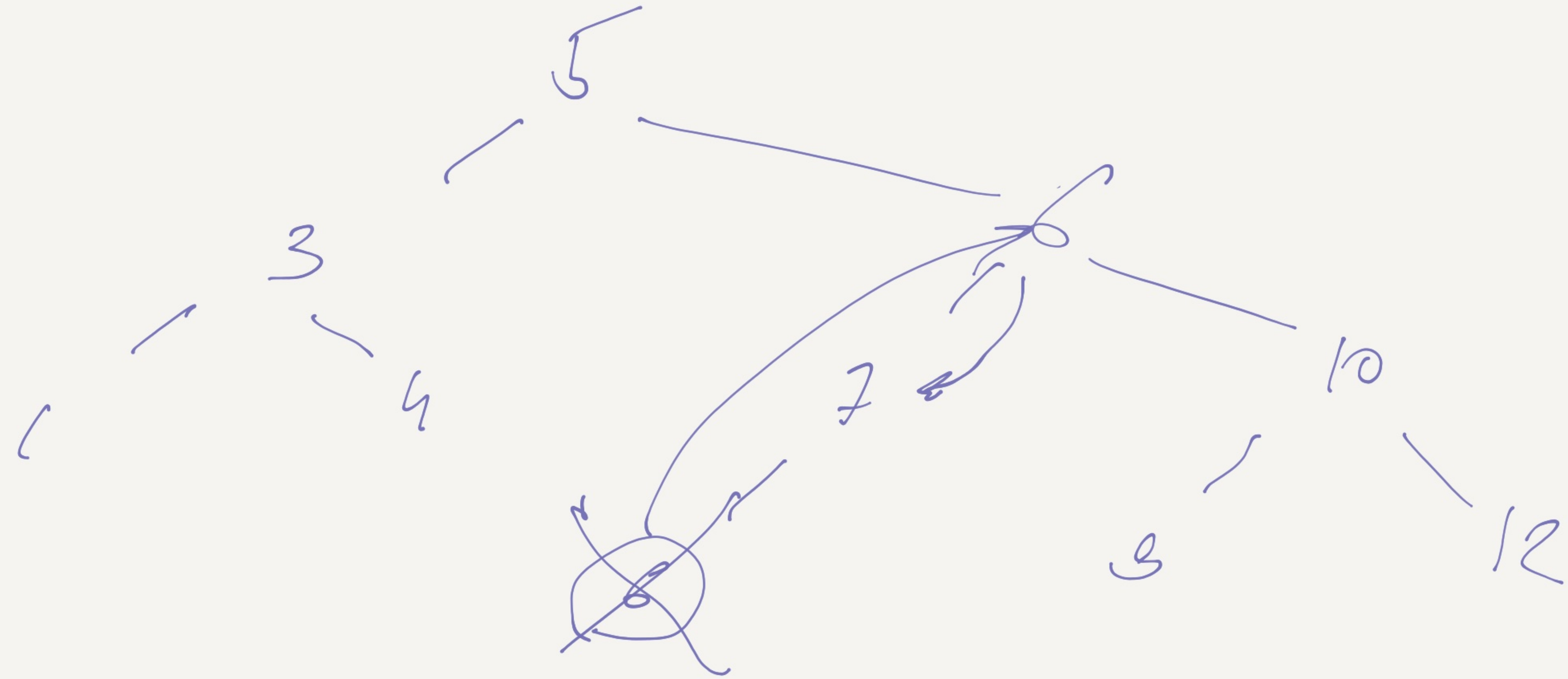
prev = null
curr = 5

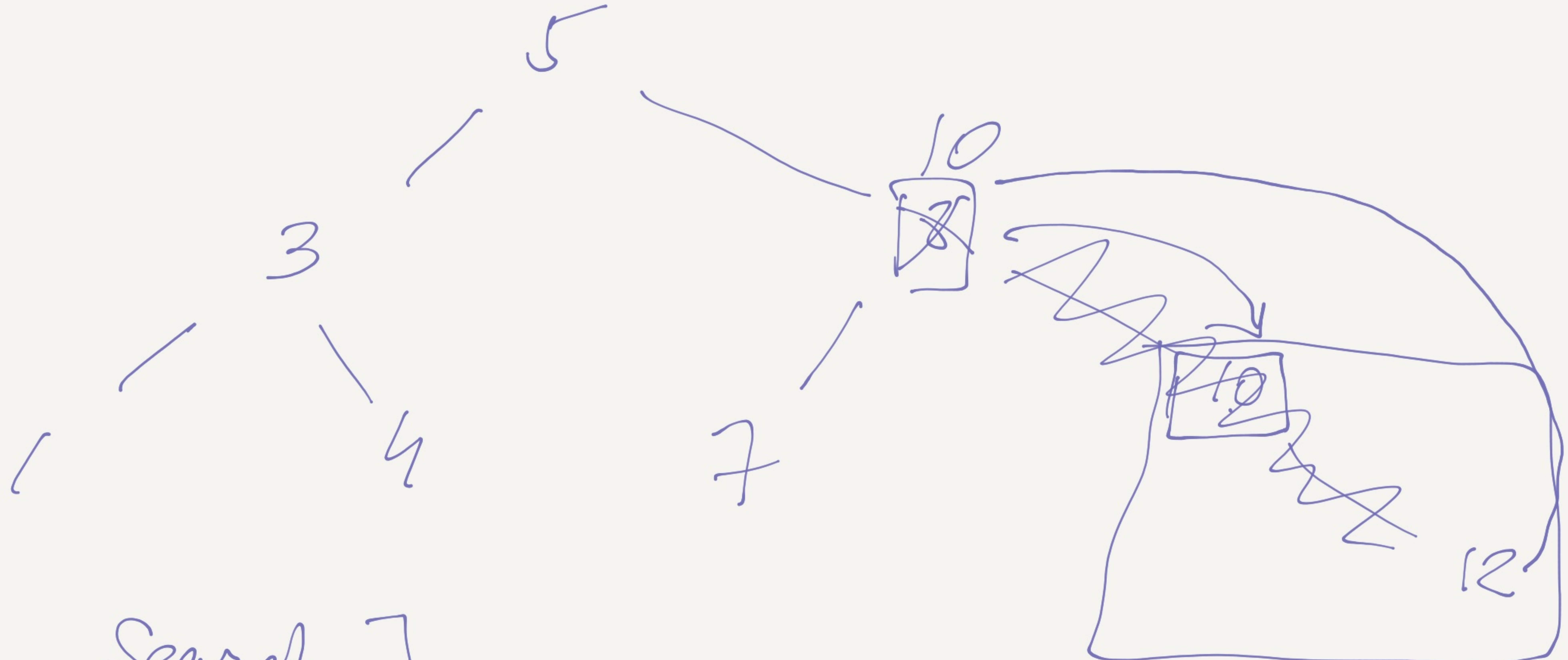
prev = 5
curr = 8

prev = 8
curr = 10

prev = 10
curr = 9

parent





n

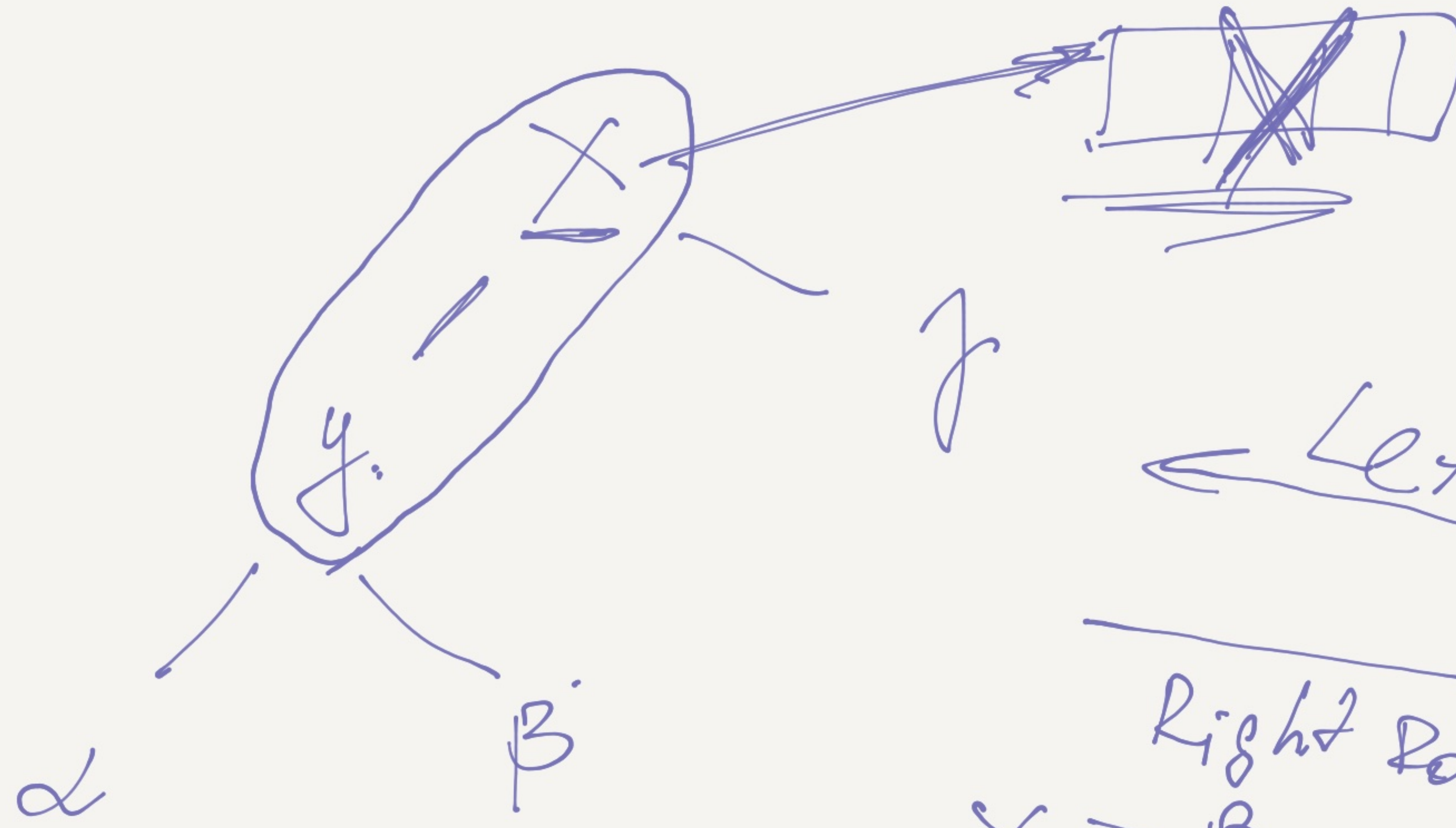
Search
Insert
Delete

$\rightarrow \Theta(n) \leftrightarrow O(h)$

$\log_2 n$

$O(\log_2 n)$

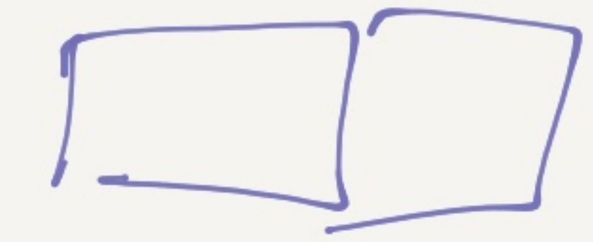
12:00 — продолжаем



x, y - элементы

α, β, γ — BST set: key → ~~value~~

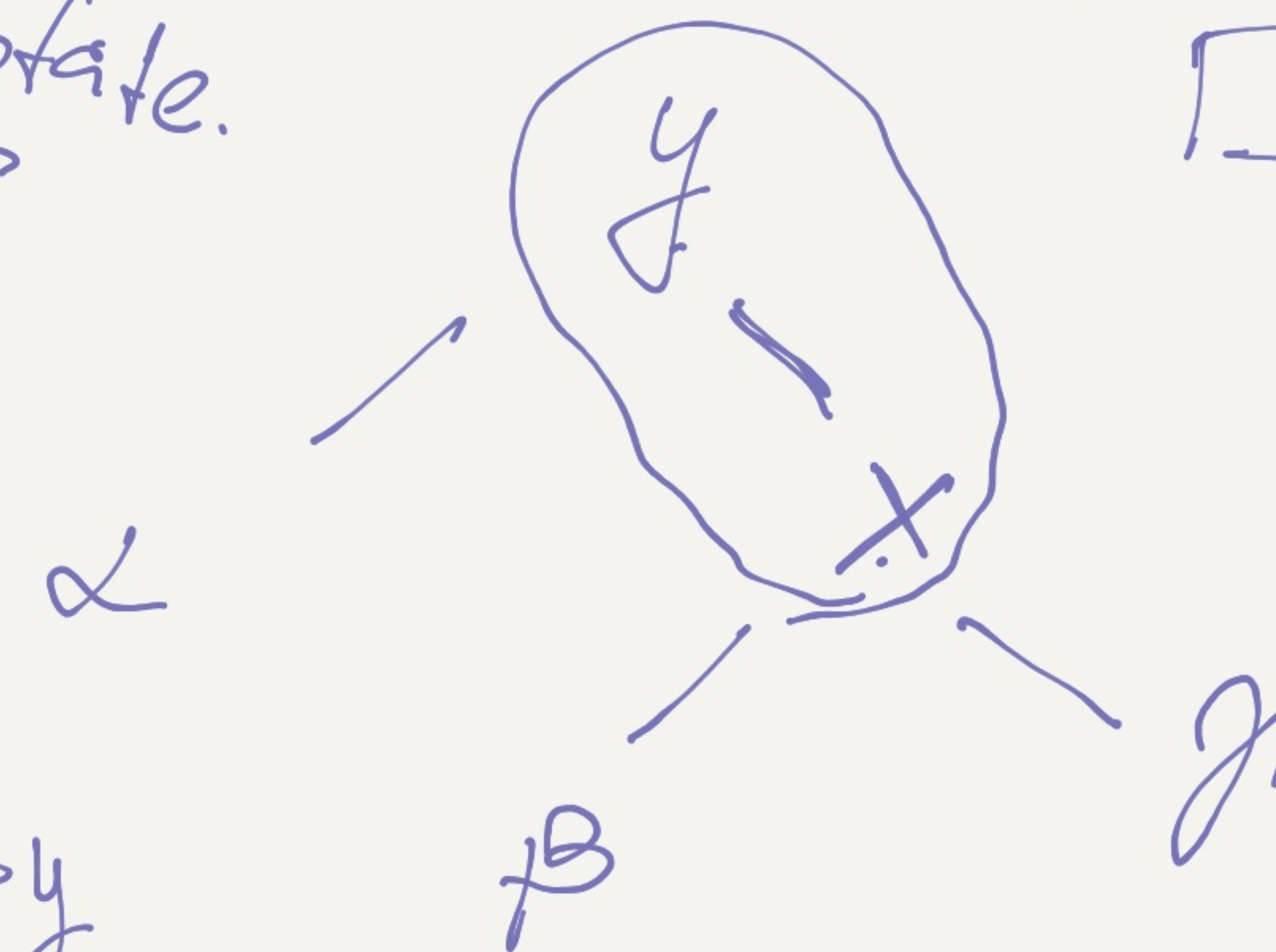
multiset: key → values



Left Rotate

Right Rotate

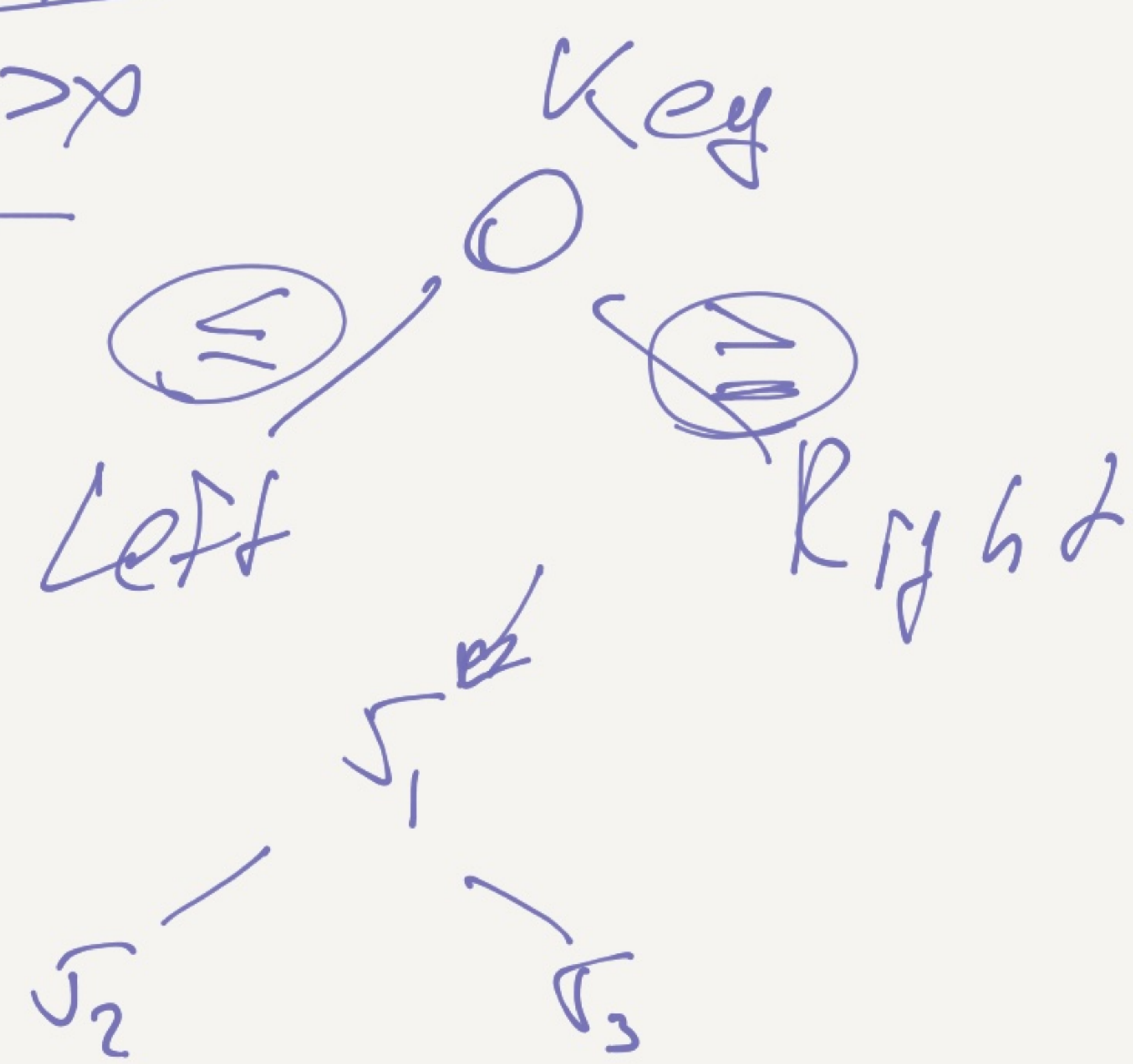
$x > \beta$

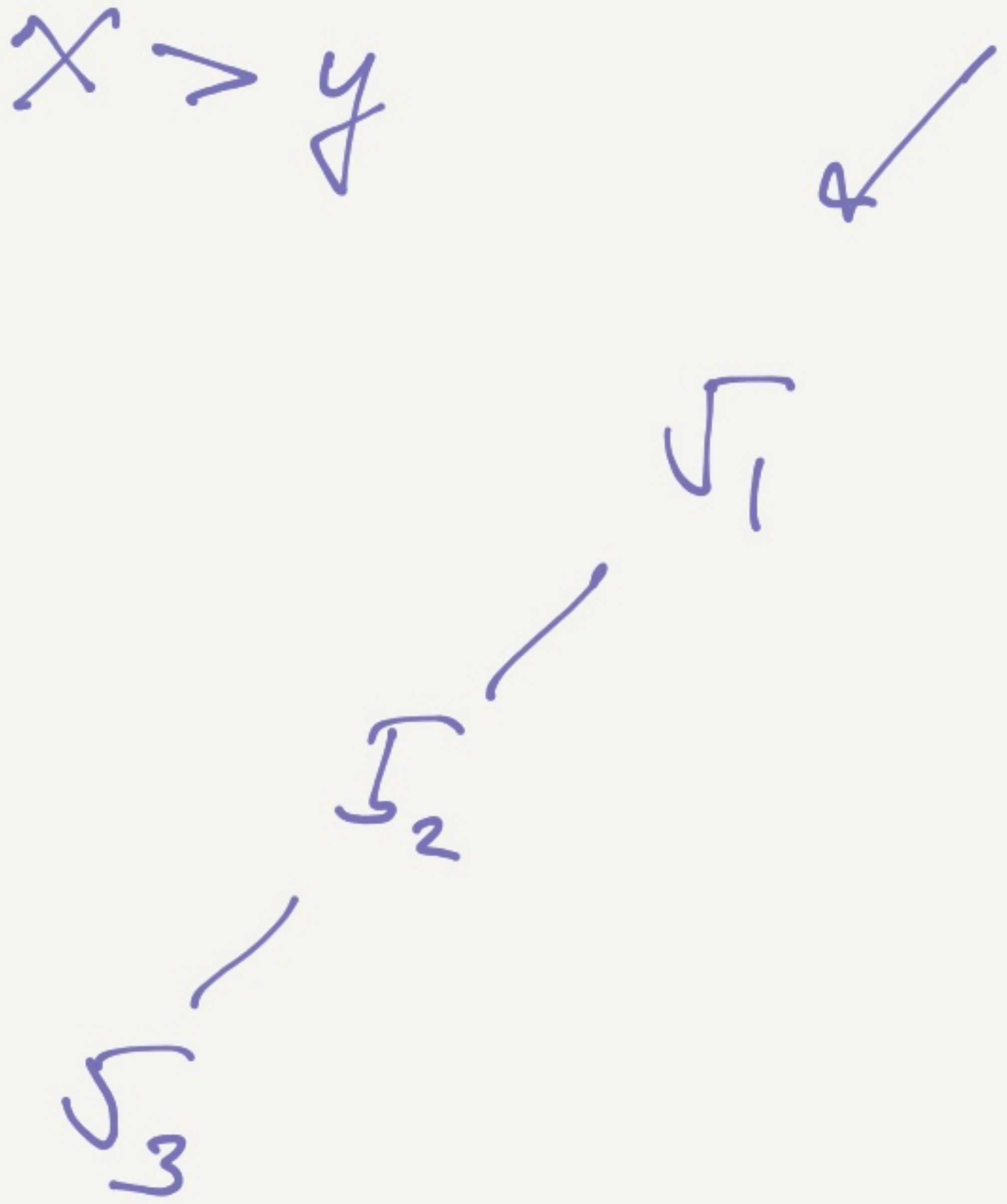


$\beta > y$

$\alpha \le y$	$\alpha < y$
$\beta > y$	$\beta > y$
$y \le x$	$y < x$
$\gamma > x$	$\gamma > x$

$\alpha < y$	$\alpha \le y$
$x > y$	$x > y$
$\gamma > x$	
$\beta < x$	



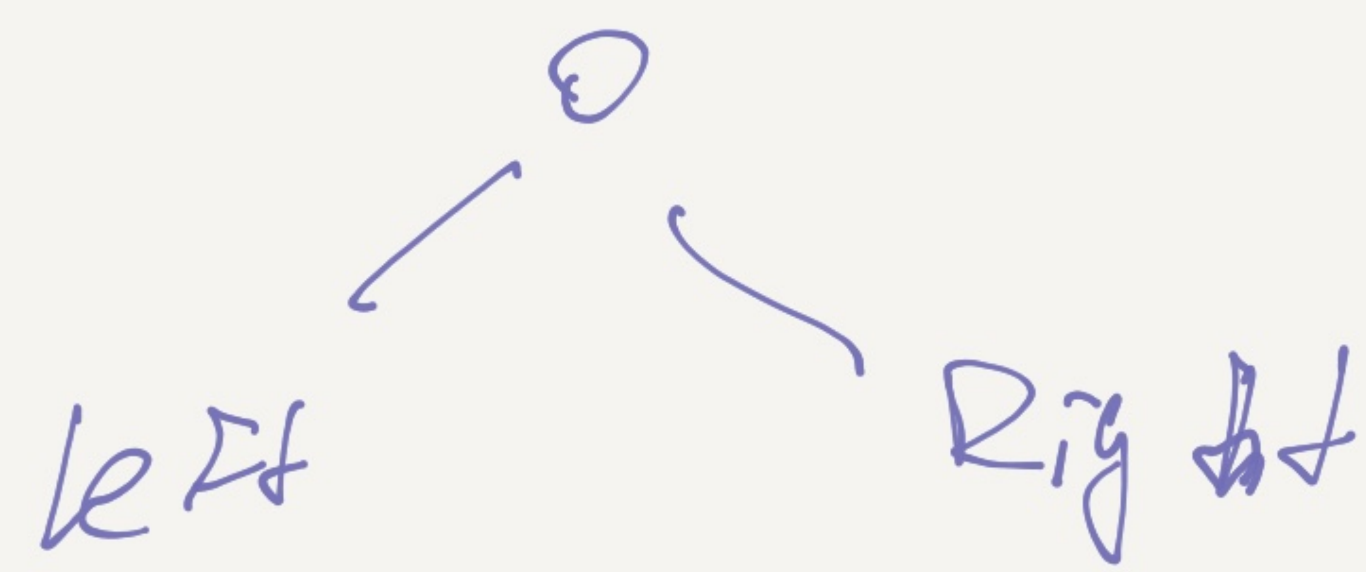


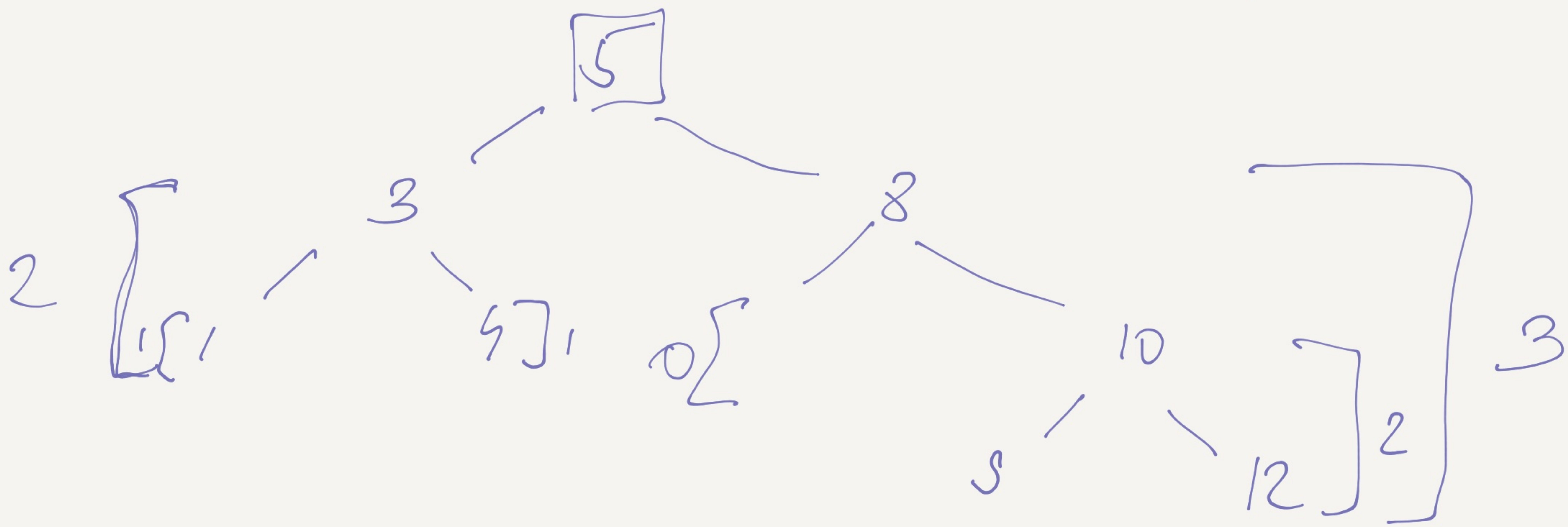
$\sigma_1, \sigma_2, \sigma_3$

AVL - дерево

$O(n) \to O(\log n)$

$$|h(Left) - h(Right)| \le 1$$



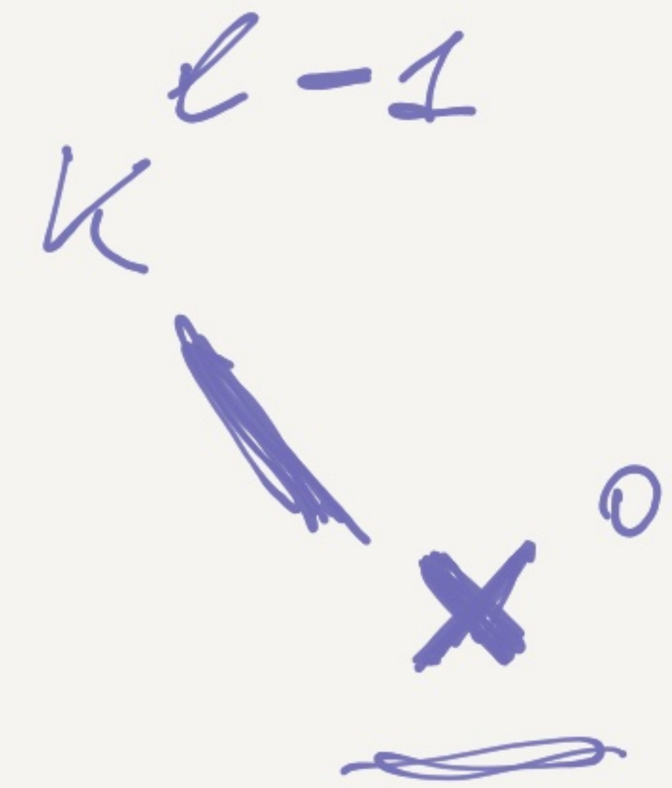


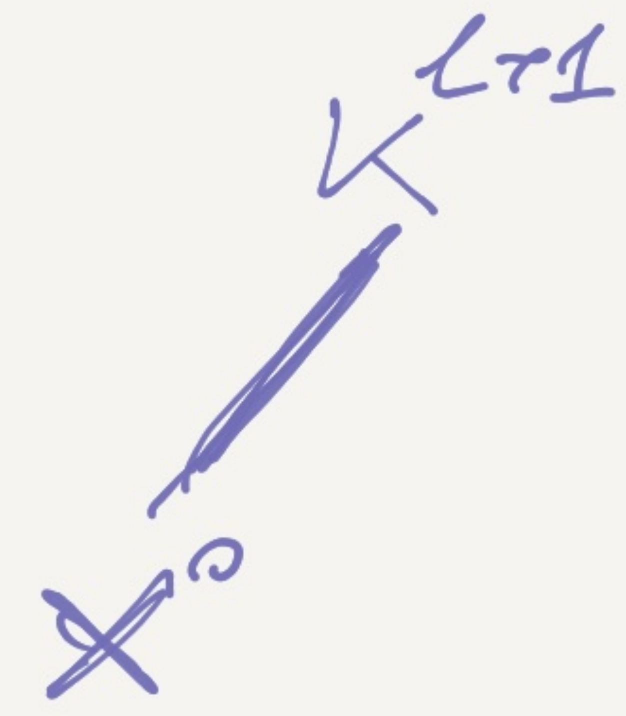
$$|0 - 2| = 2 > 1$$

$|Left - Right| \leq 1$

$balance = Left - Right$

0	00
-1	01
+1	10





$balance_u = Left - Right$

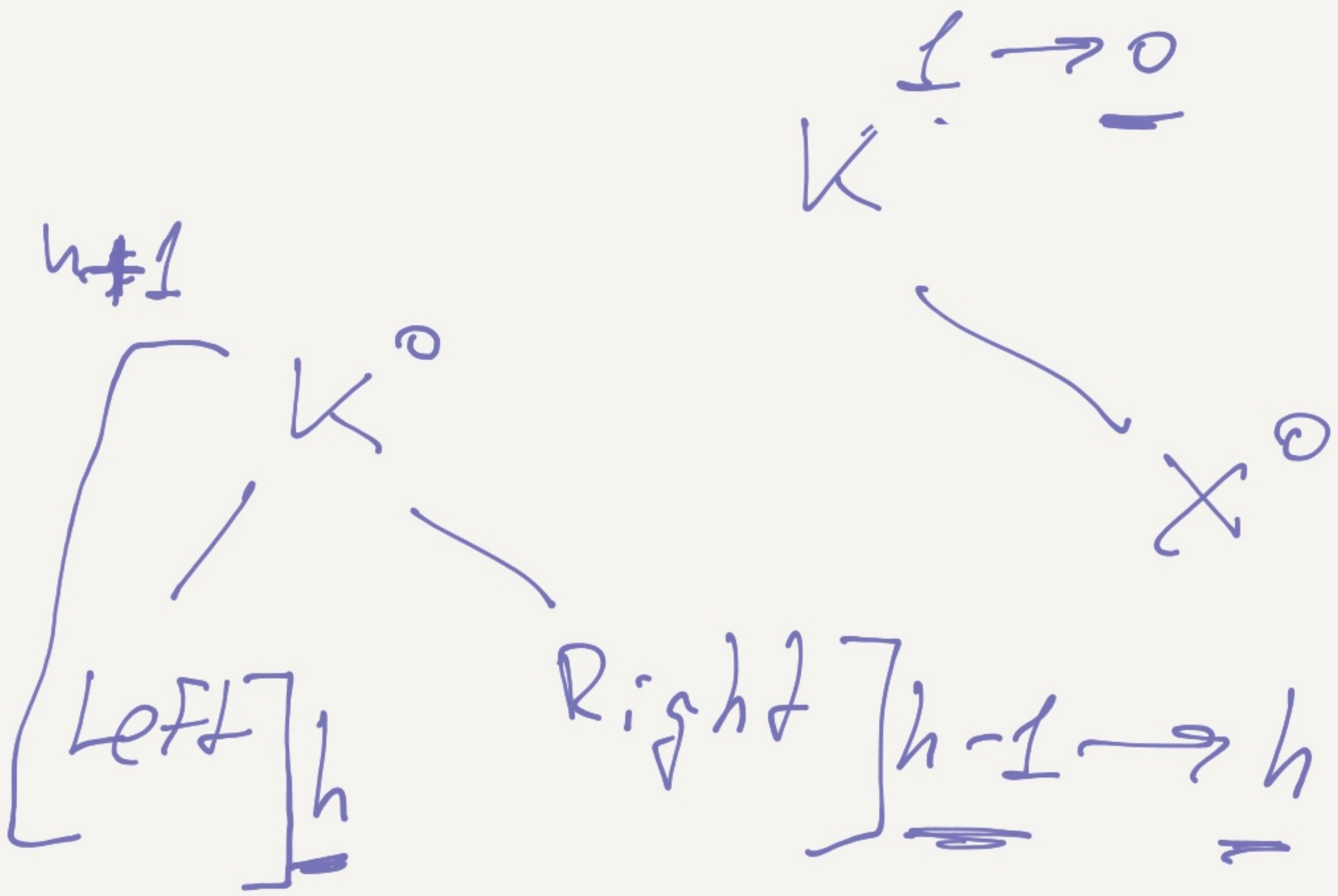
+1   +1

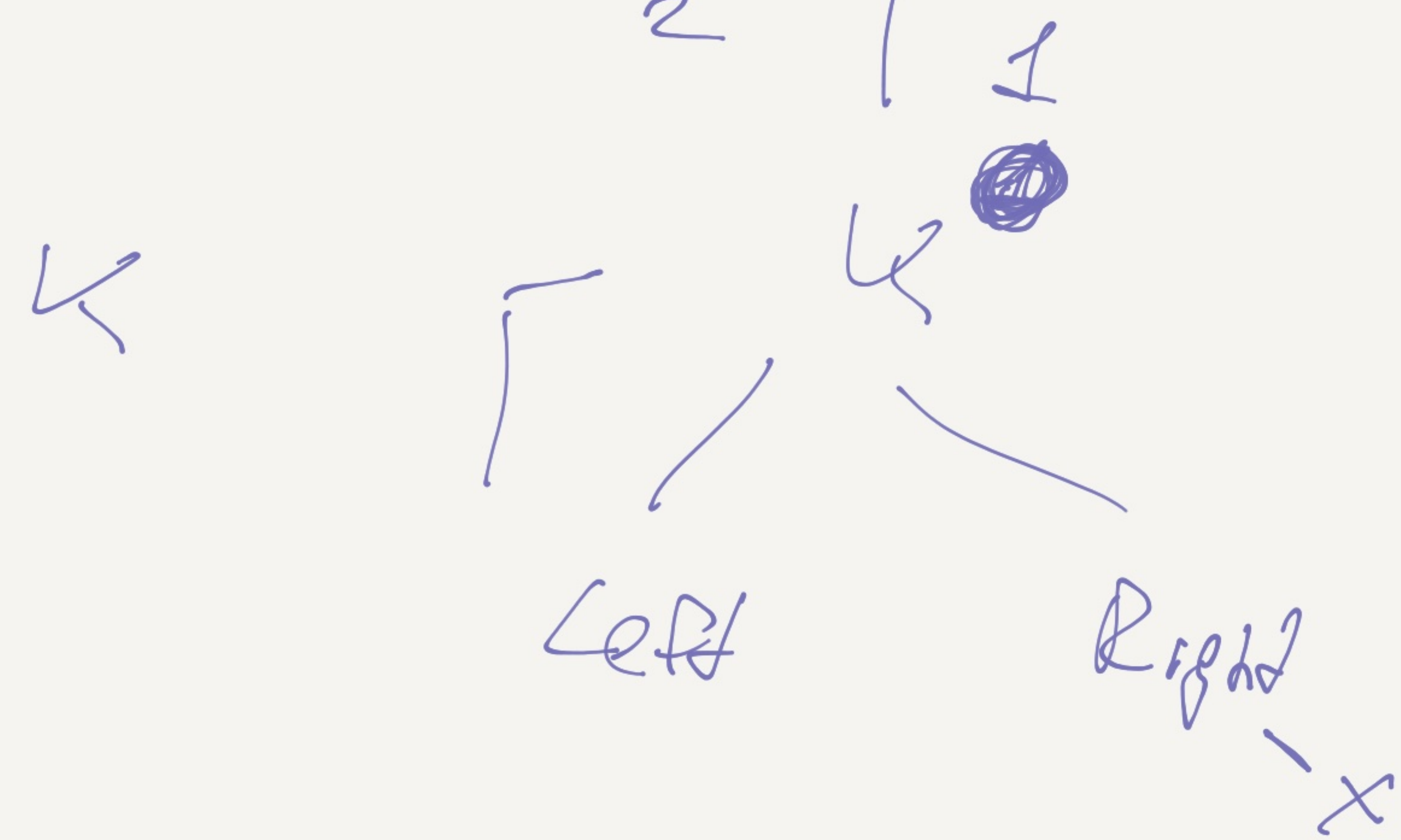
0 → +1, −1

−1 → −2, 0

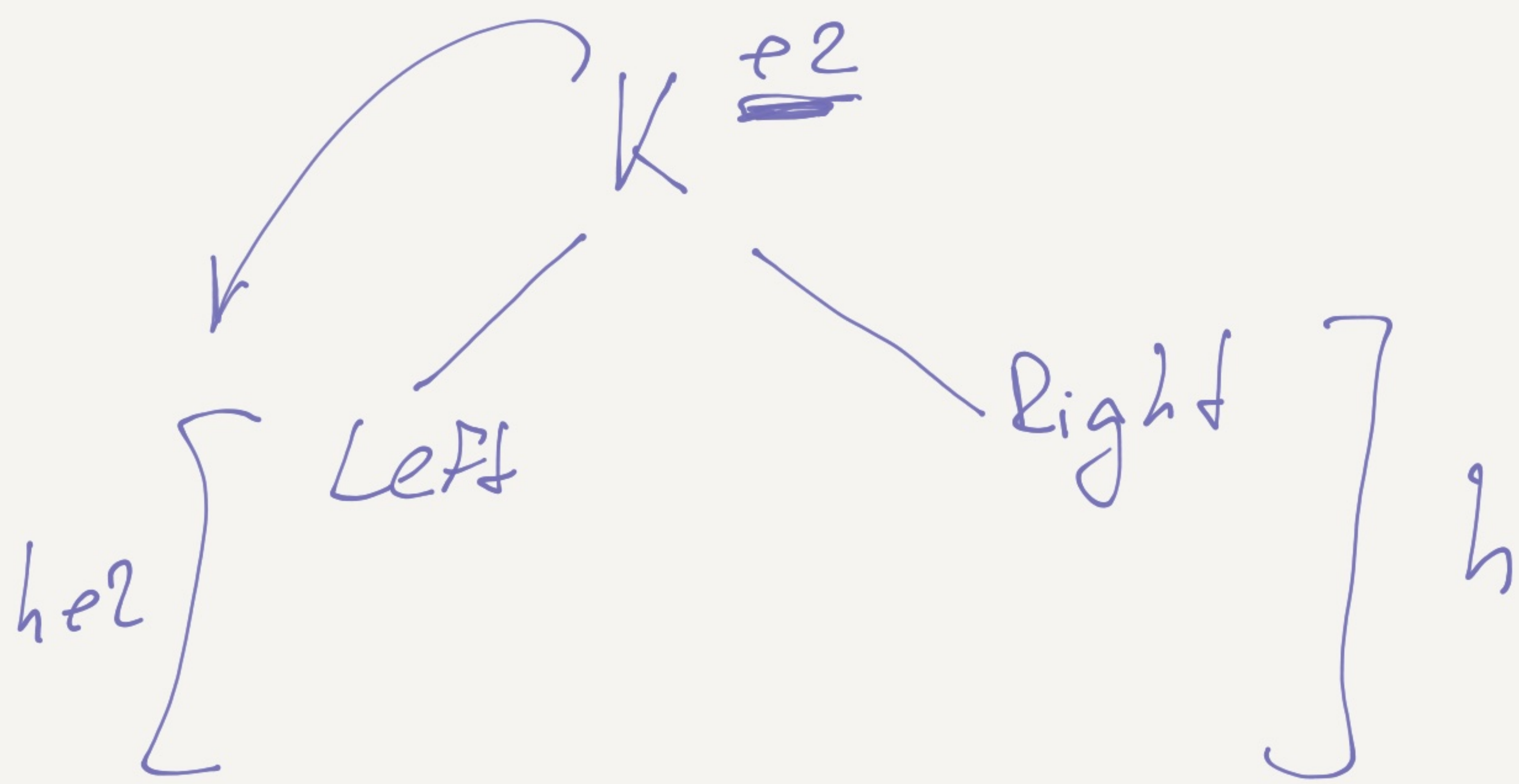
+1 → 0, 2

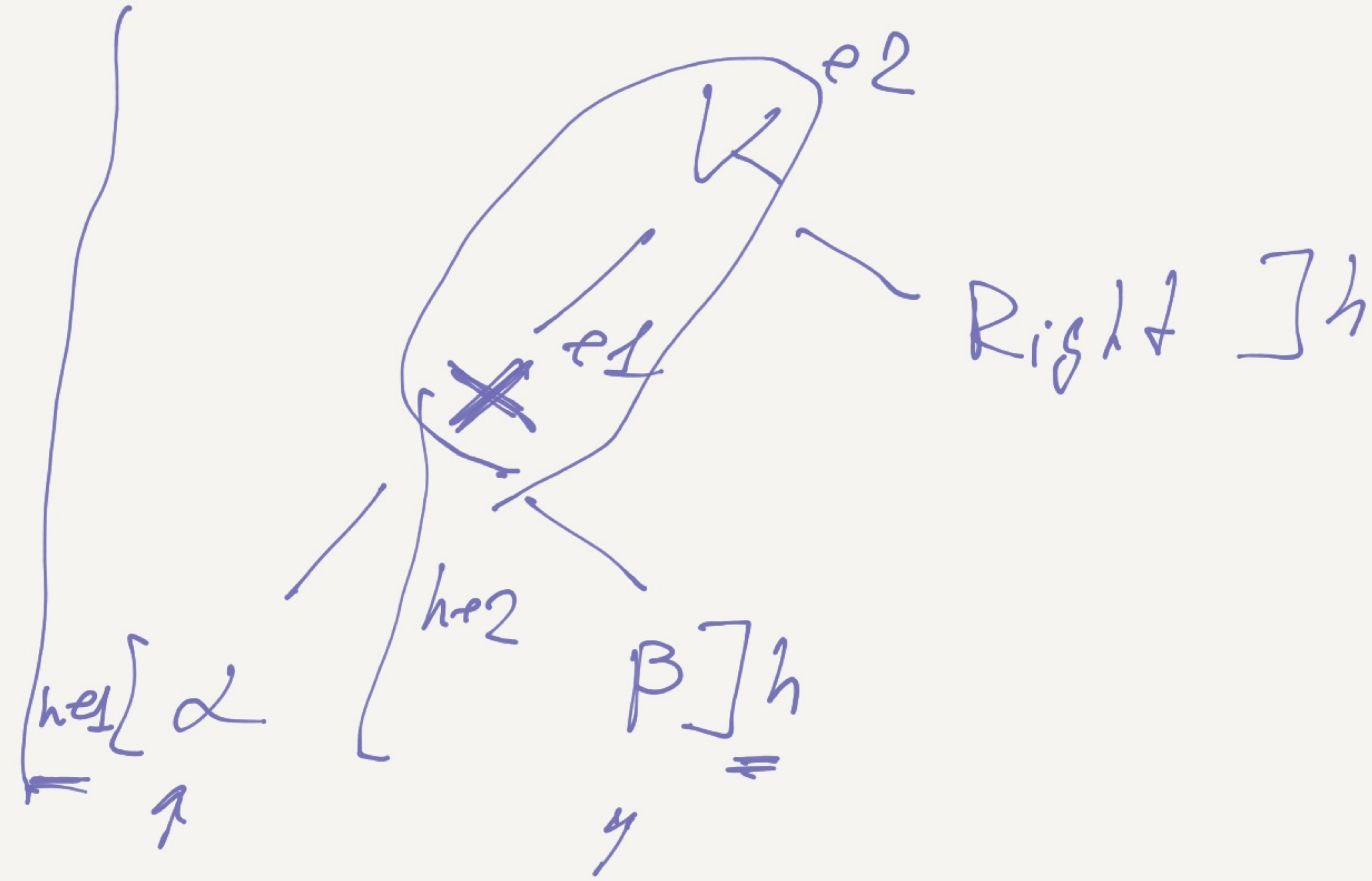
→ −2, −1, 0, +1, +2

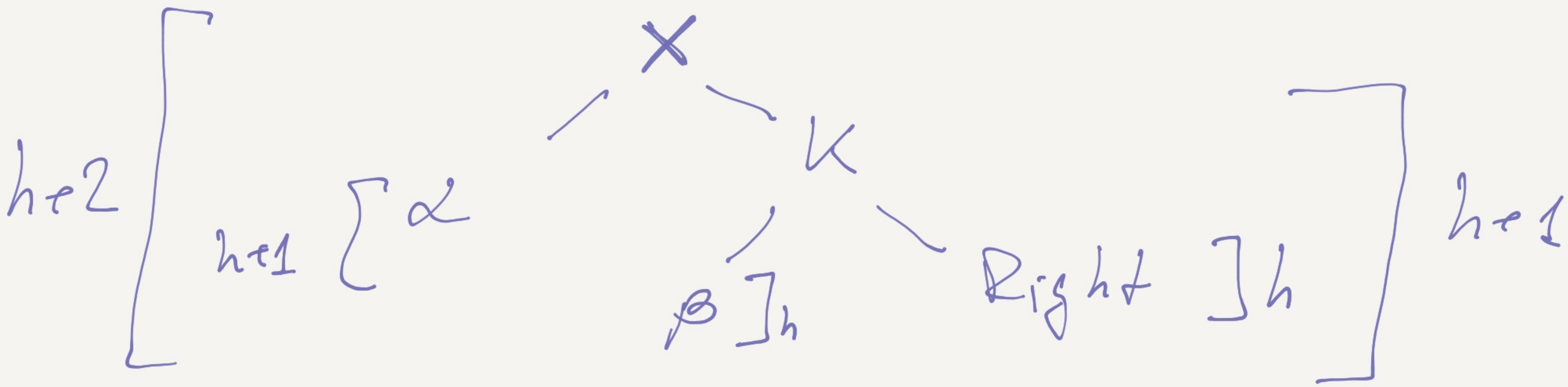




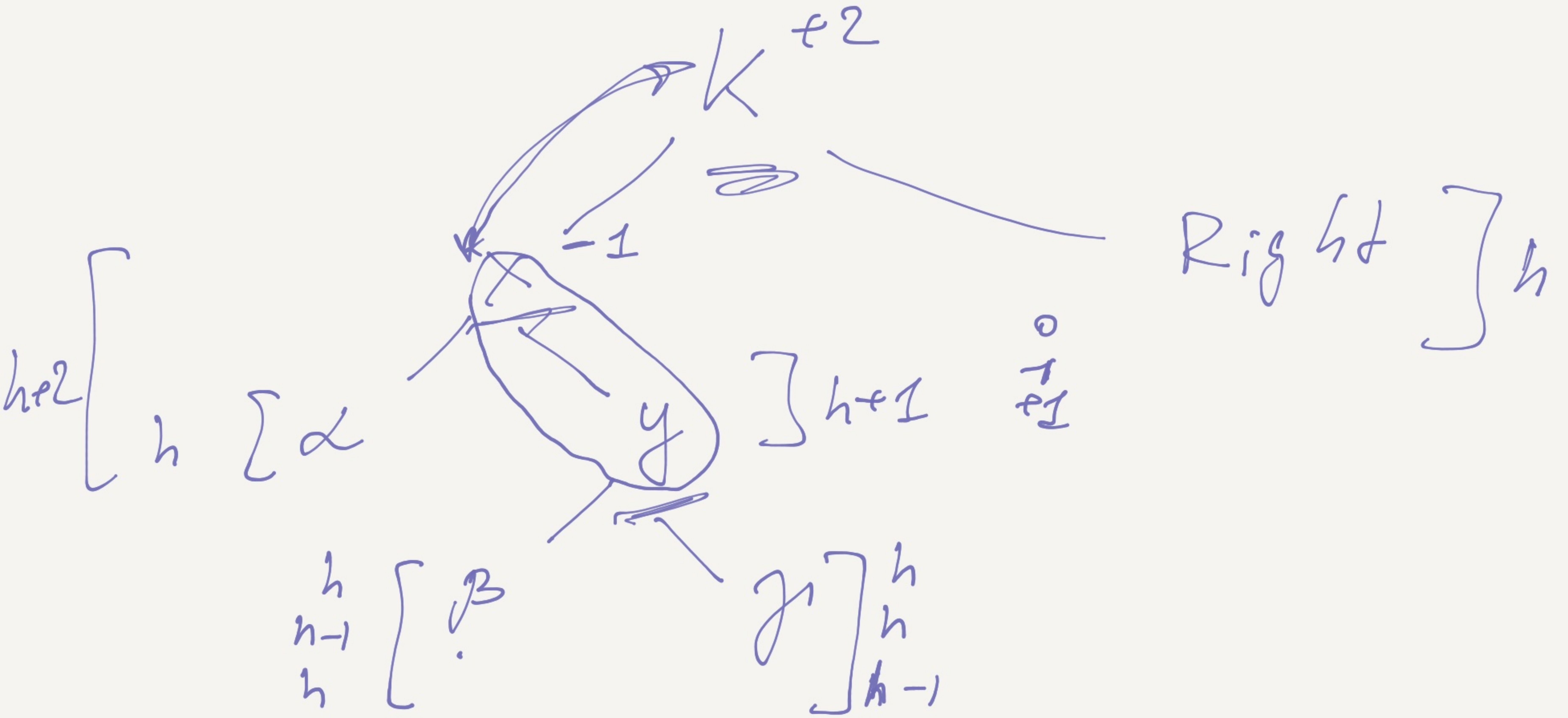
+1
K
Left
Right
h
h+1 ← h
x
0
1
0
-1
P
T_h
K
h
h
h+1
+1
K
h+1
h
h+2


K
e2
Left
Right
he2
h

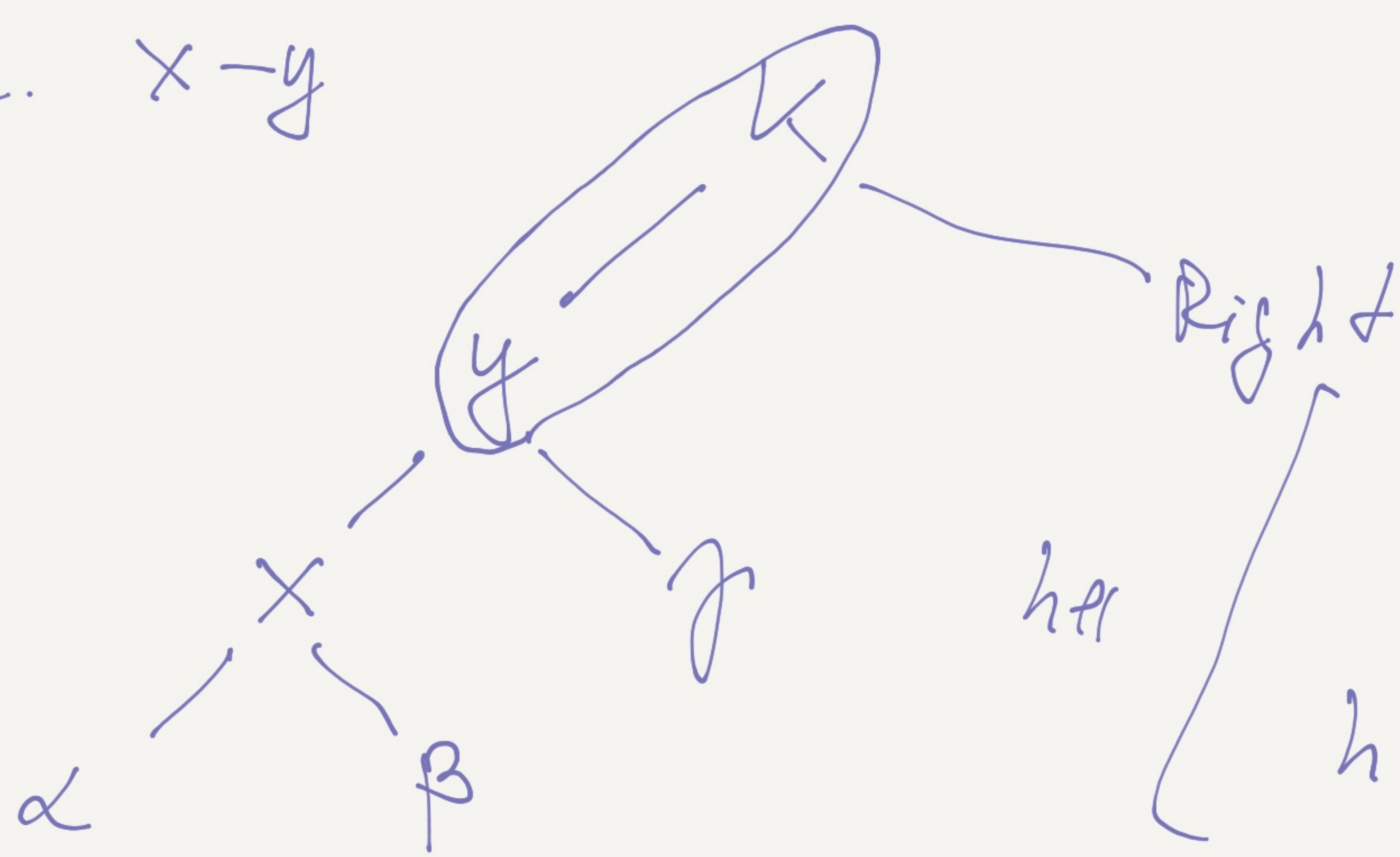

K
e2
e1
Right
h
he1
α
he2
β
h


K
he2
he1
α
β
h
Right
h
he1

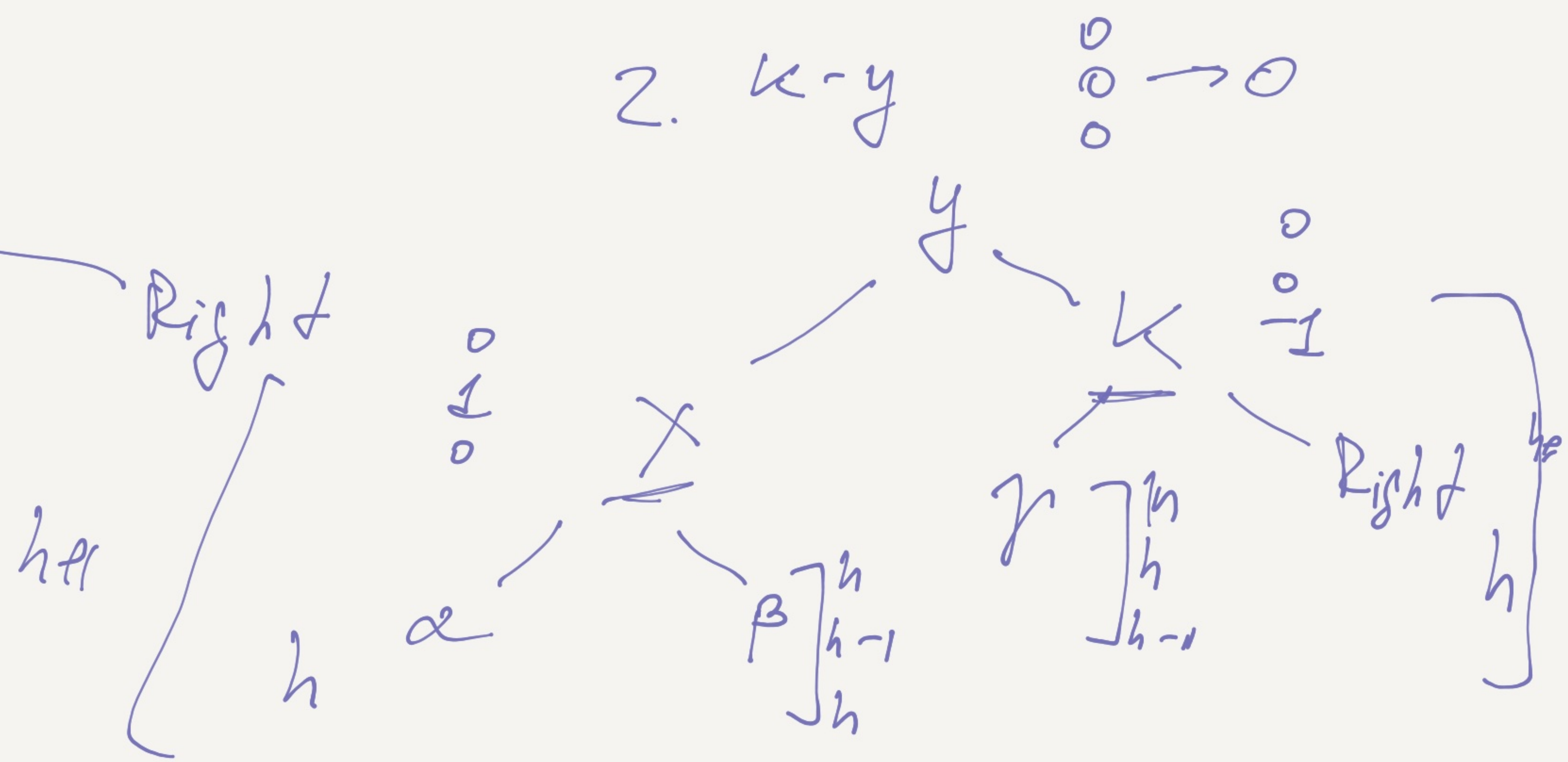
e2
-2
h e1
h e2
K e2
e1
-1
x
α
β
Right
h
he2
h
K -2
x -1 e1
K e2
x +1
Right
h
he2
he1
α
β
h
K e1
x
Right
h
he1
α
β
he1
h/h-1
h/h-1
he1
he2
X 0
K 0
he1
he1
α
Right
h
h
β
X 0 → +1
h
α
β
h
he1

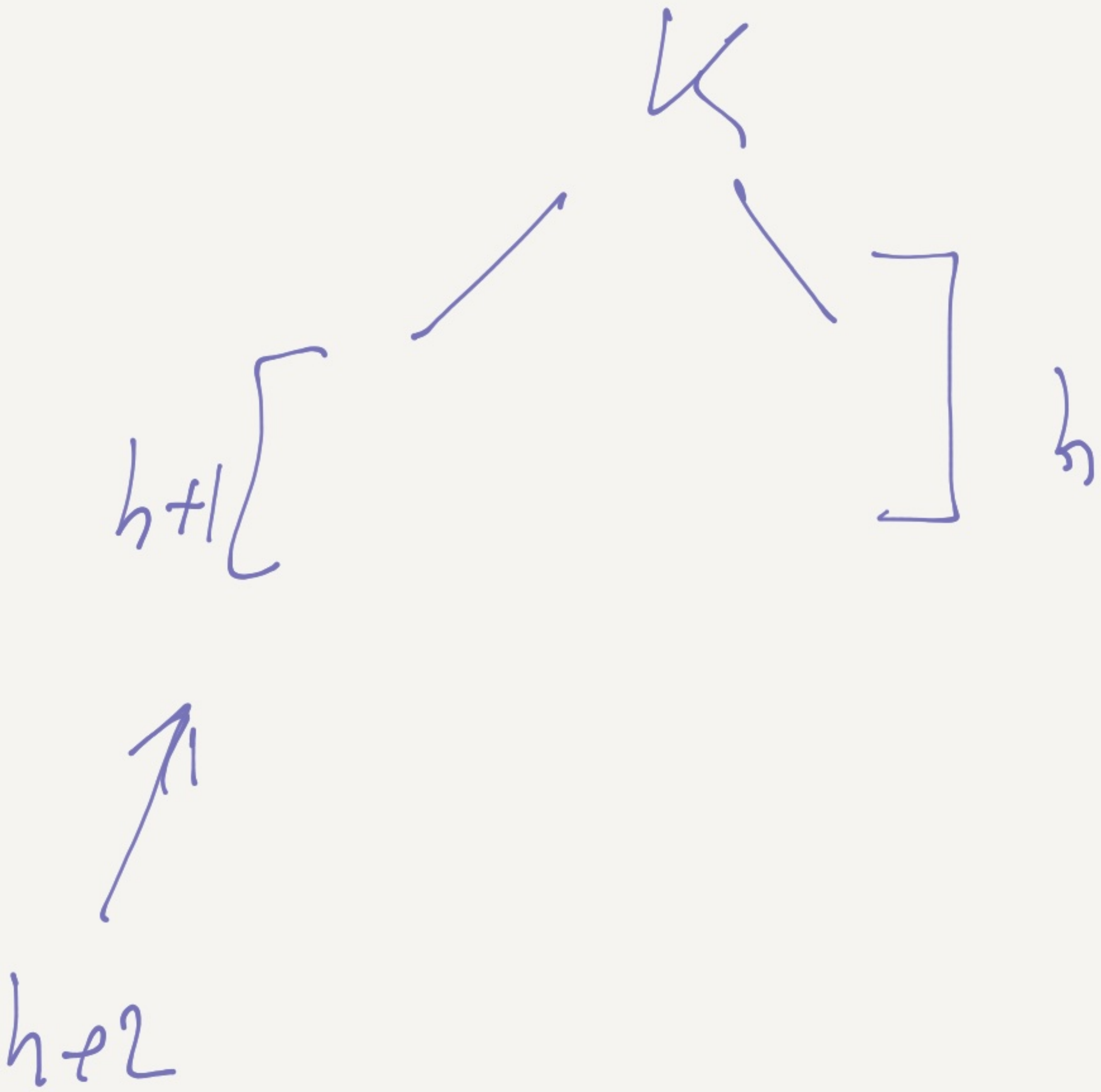

K
+2
-1
Right
h
x
y
h+1
0
-1
+1
h+2
h
α
β
γ
h
h-1
h
h
h
h-1

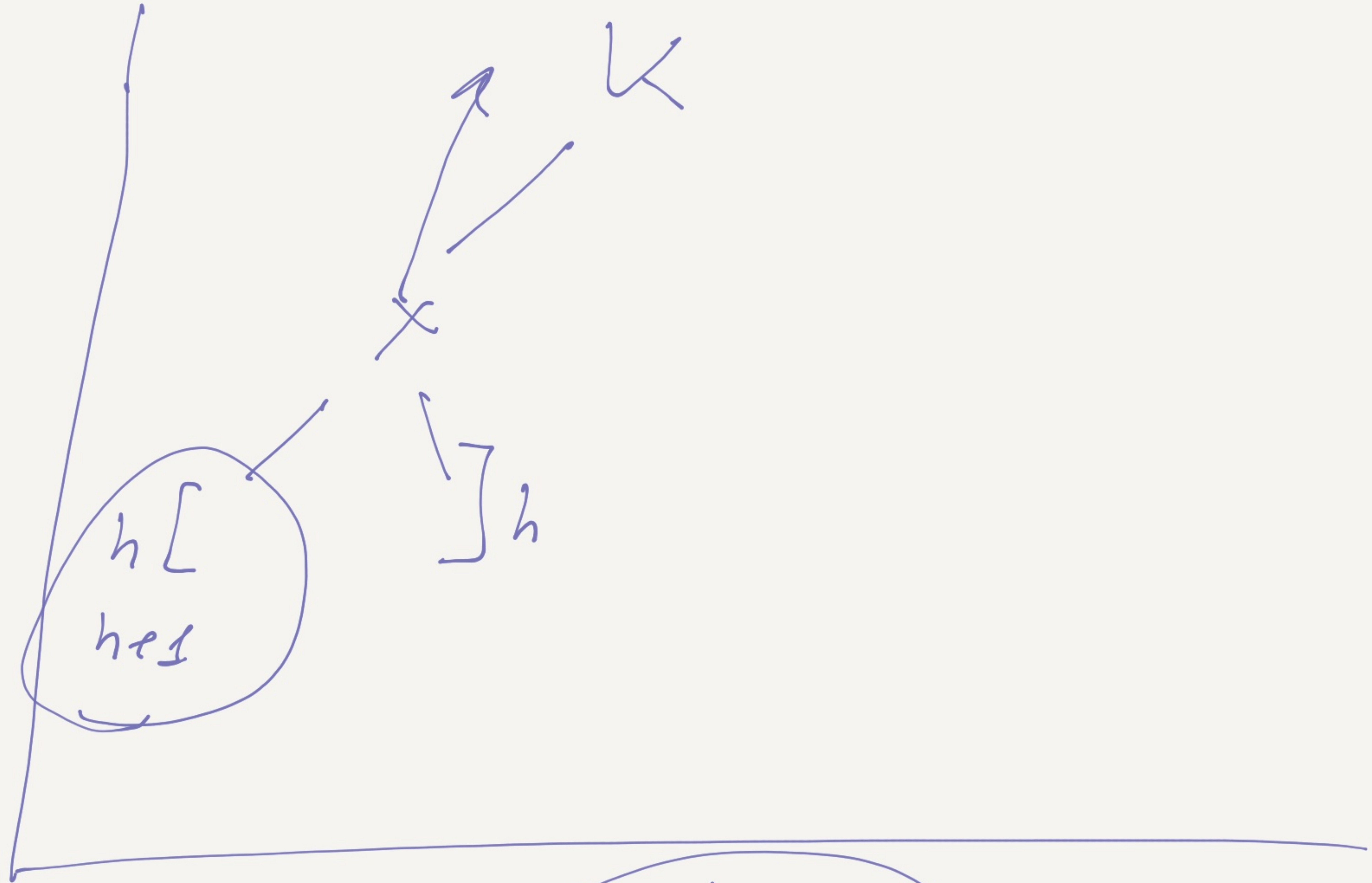
1. x−y

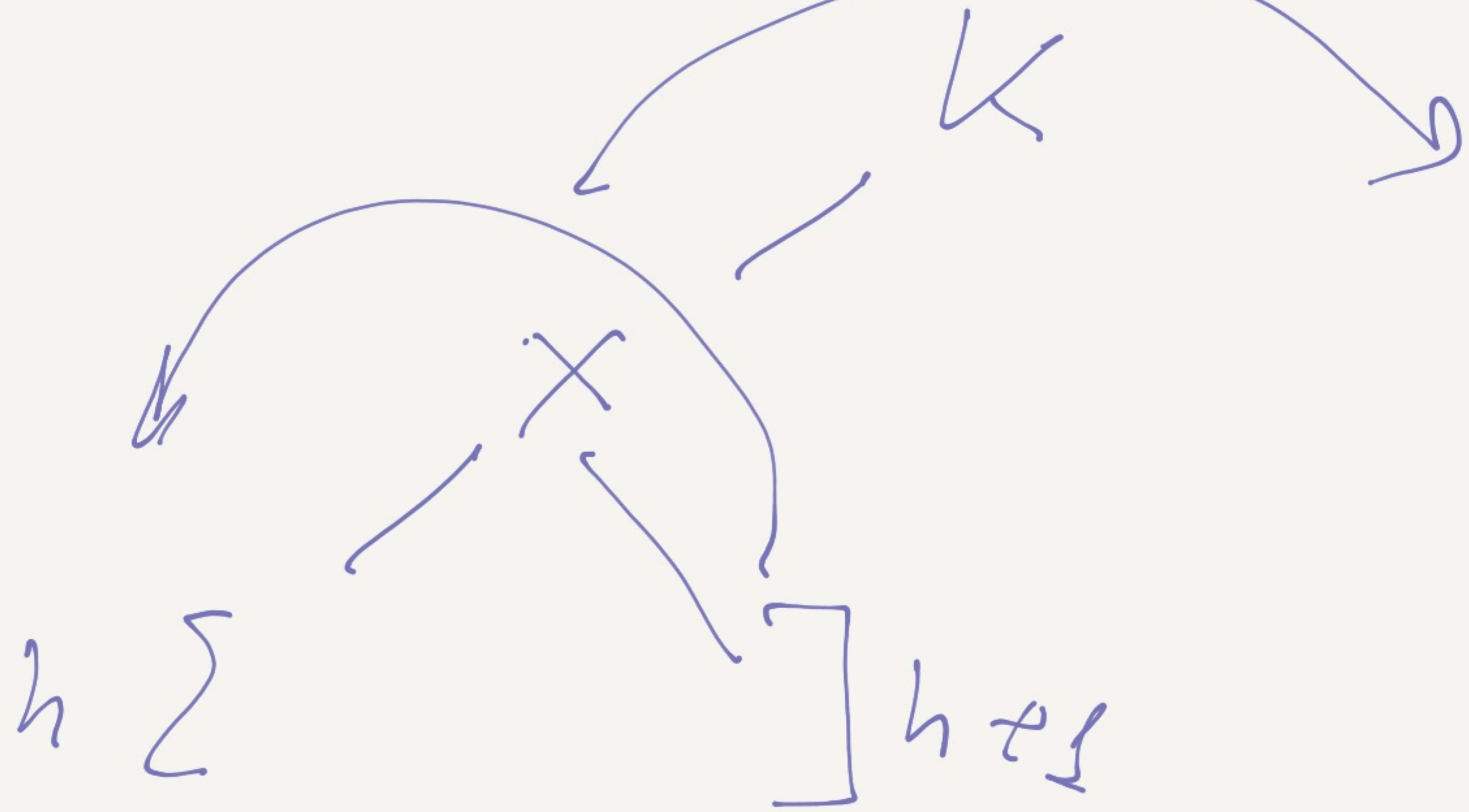

K
Right
y
x
γ
α
β

2. k−y

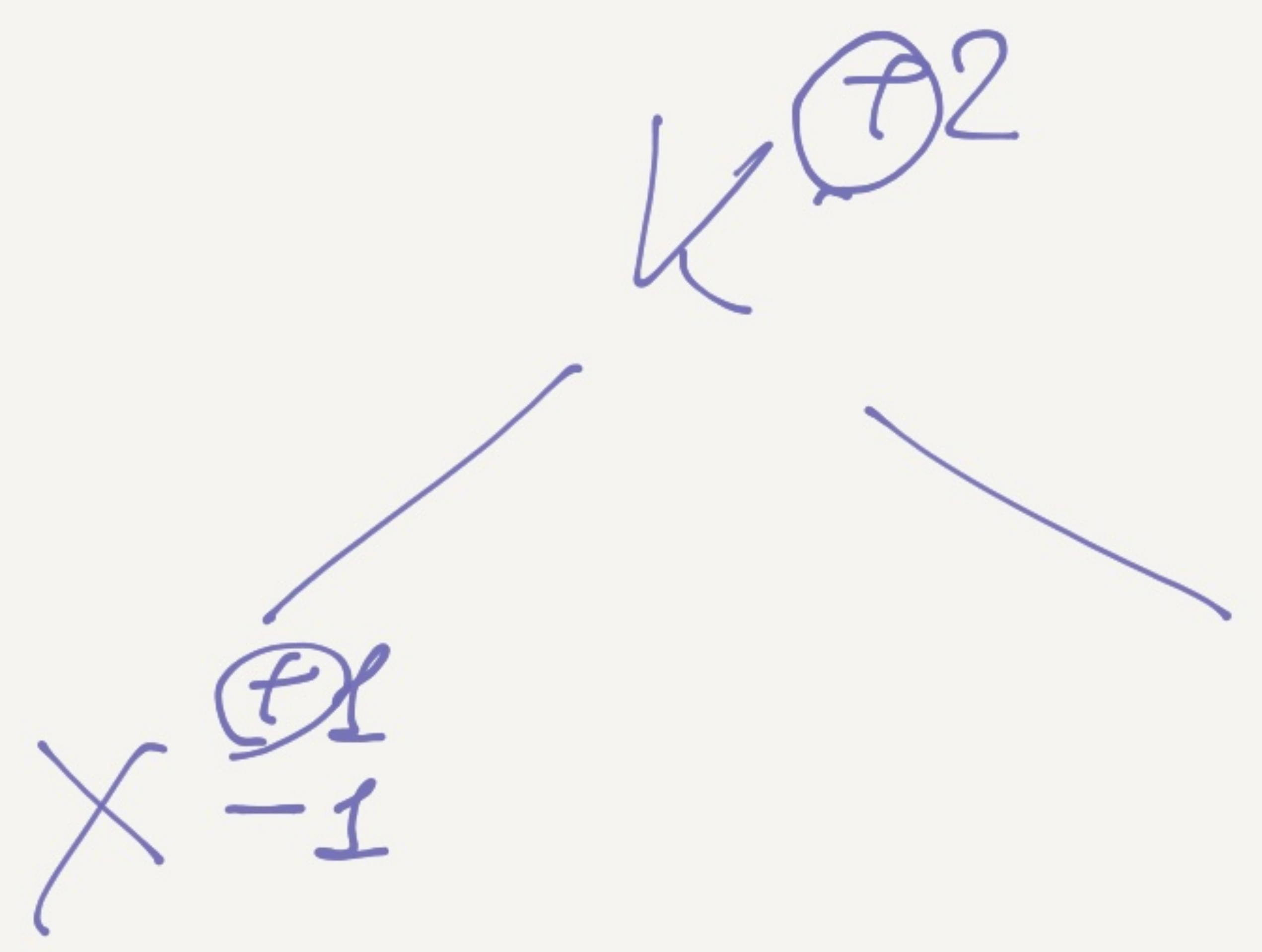

0
0
0
0
y
K
0
0
-1
0
1
0
x
Right
h+1
h
α
β
γ
h
h-1
h
h
h
h-1
h

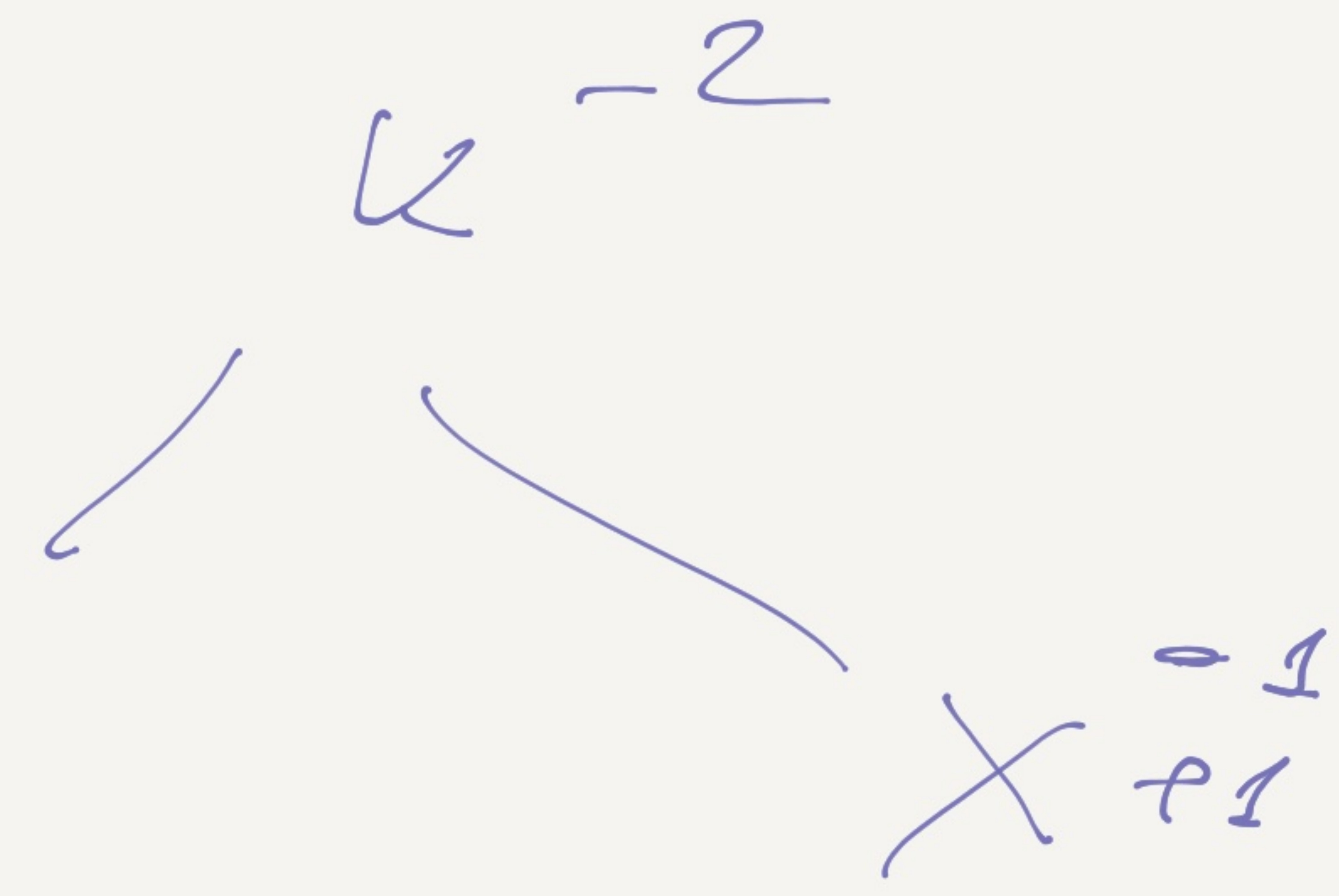

K
h+1
h
h+2


K
X
h
h
h+1


K
X
h
h+1

3 14 15 9 2 6 5 3 5 8 9 7



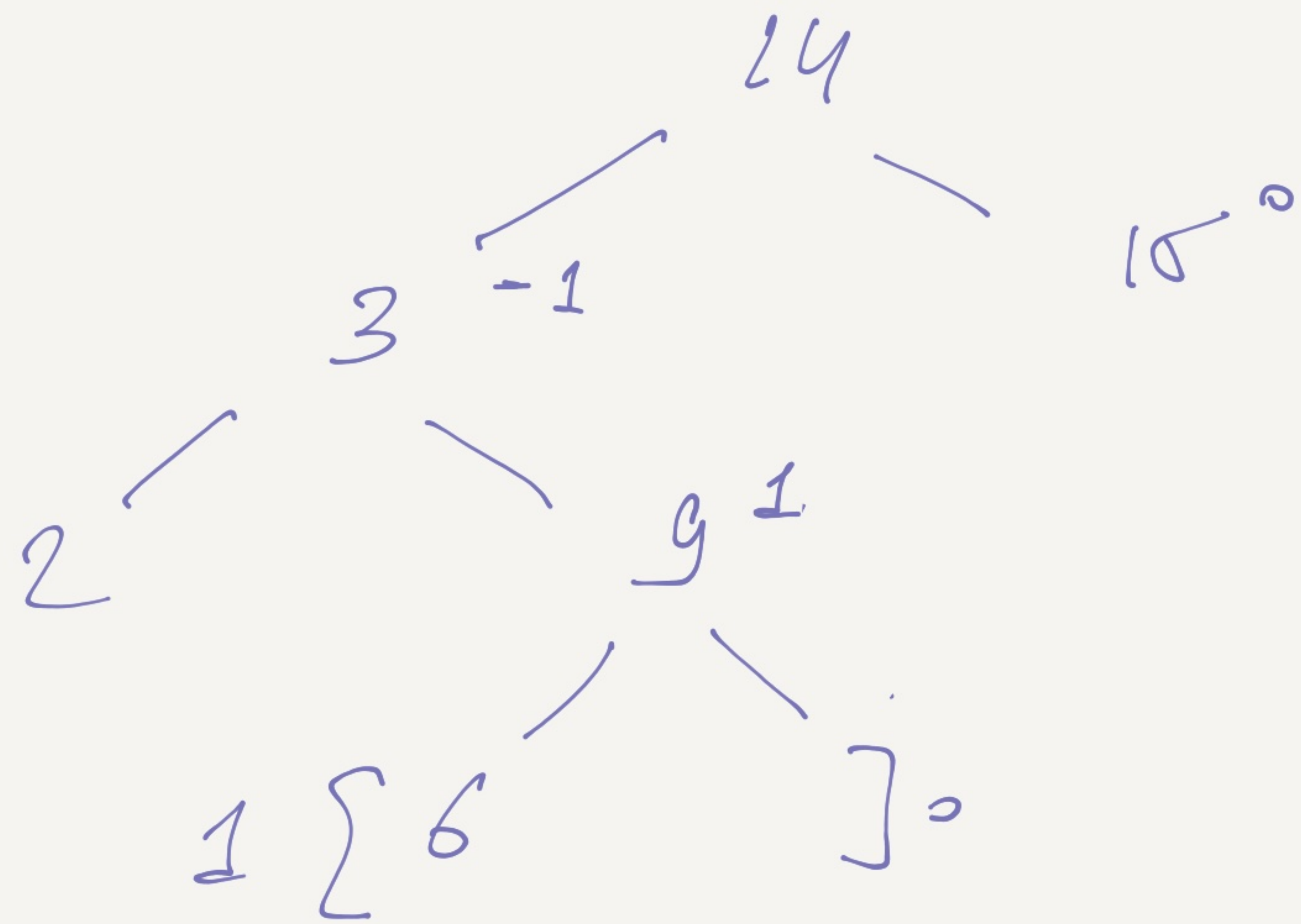


"+" x 2 → 1

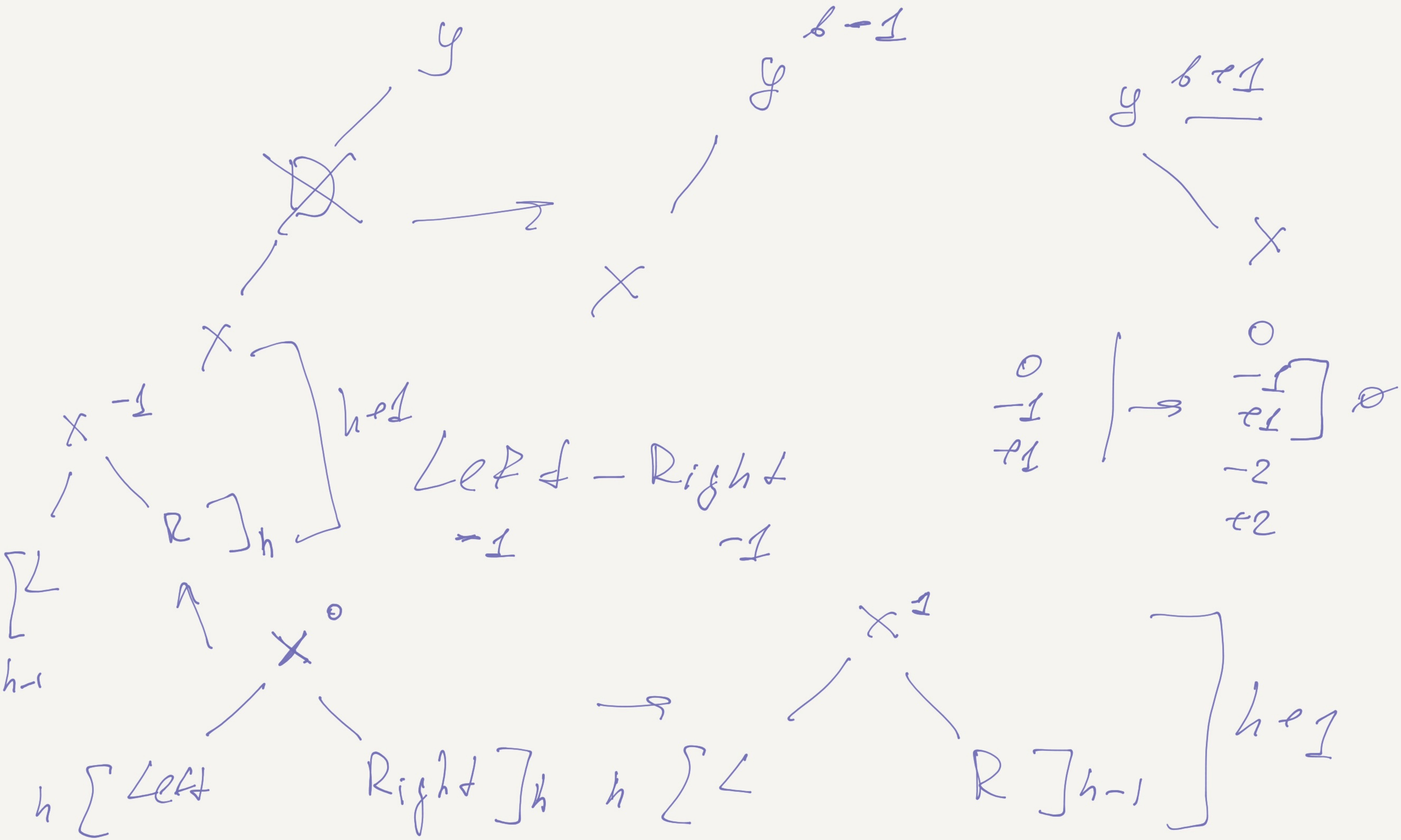
"−" x 2 → 1

"+"/"−" → 2

"−"/"+" → 2

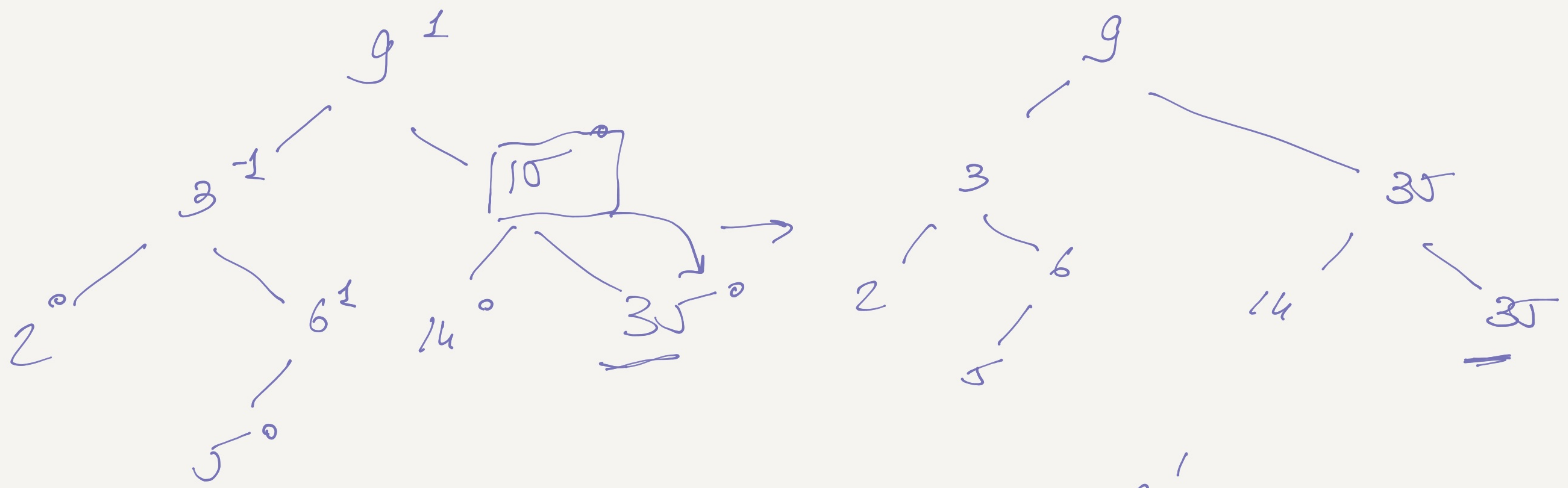


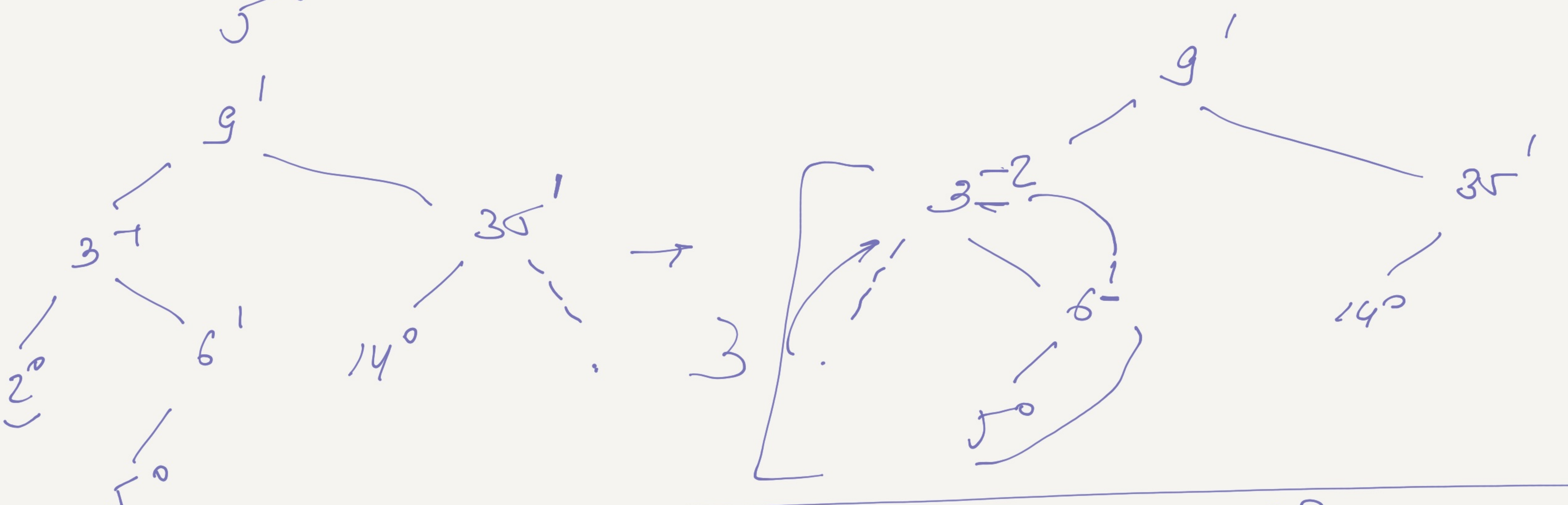
3 5 8
9 0
3 0
14 +
2 0
6 0
15 0
9 1
3 -1
14 -2
15 -1
2 0
6 1
35 0
5 0
3
9 1
3 -1
15 0
2 0
6 1
14 0
35 0
5 0
2
3



Left - Right
-1
-1
h+1
h
h-1
Left
Right
L
R
h+1
0
-1
+1
-2
+2


h+2
n
Left
delete
h-1
Right
h
h+1
h+3
+2
-2
0
+1
-1
L>R
=
L<R
=



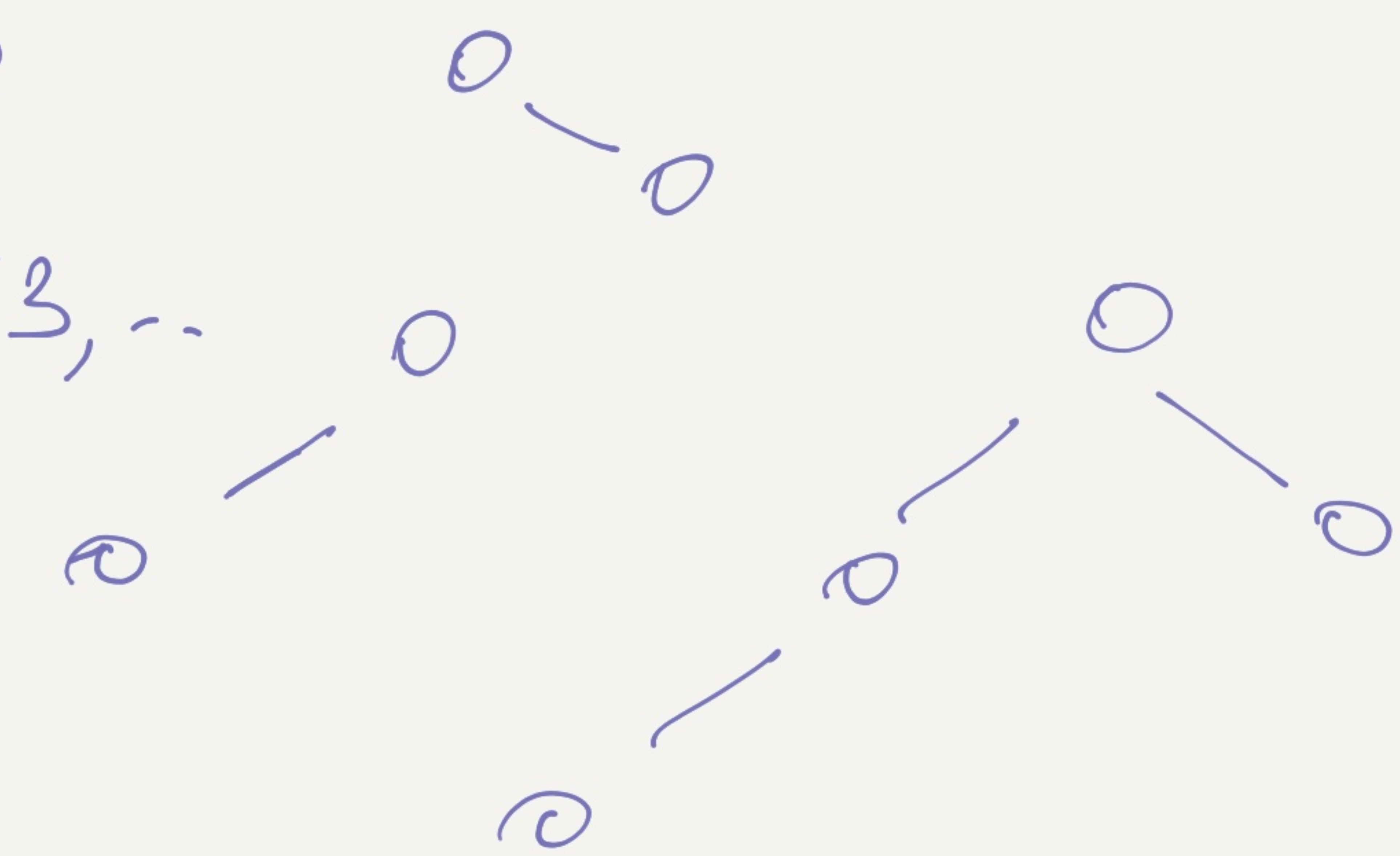


1

AVL, h – высота

m_h – min $1, 1, 2, 3, 5, 8, 13, \ldots$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$



$h=1, \ m_h = 1$

$h=2, \ m_h = 2$

$h=3, \ m_h = 4$

$- \quad - \quad -$

$h=4; \ m_h = 7$

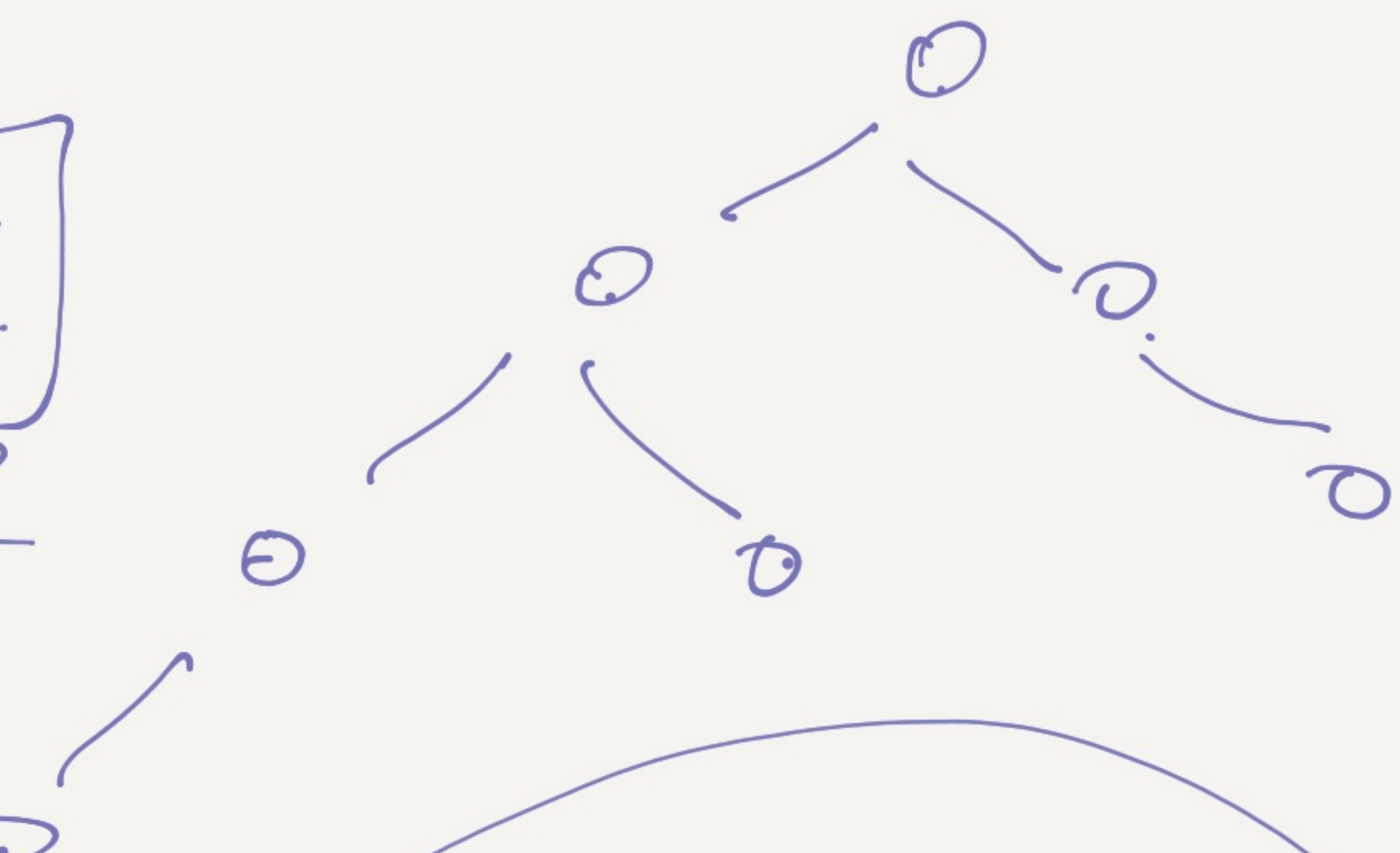
$$\underline{m_h = F_{h+2} - 1}$$

$$\boxed{m_1 = F_3 - 1 = 2 - 1 = 1}$$

$$m_2 = F_4 - 1 = 3 - 1 = 2$$

$$m_3 = F_5 - 1 = 5 - 1 = 4$$

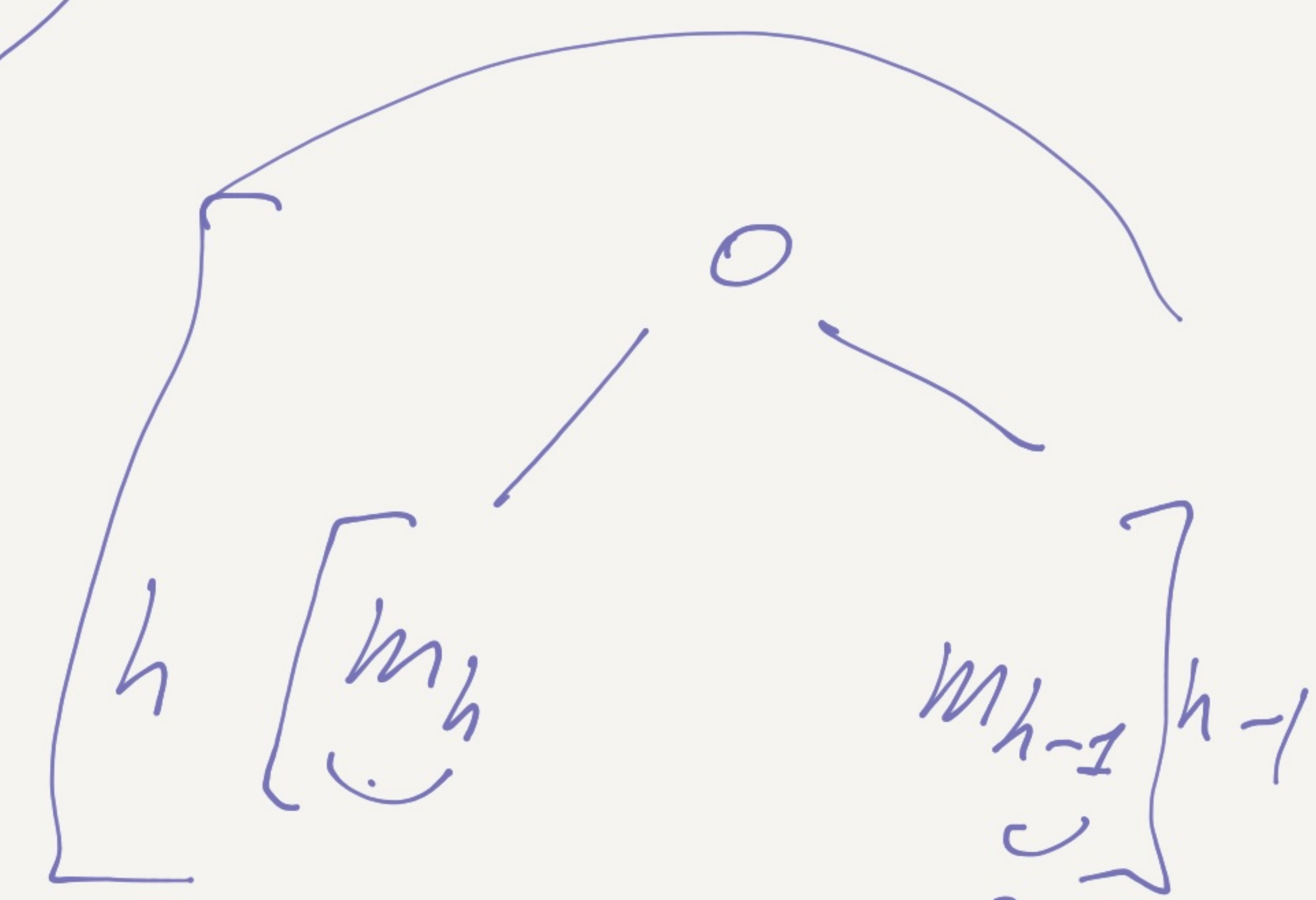
$$m_4 = F_6 - 1 = 8 - 1 = 7$$



m_h..

m_{h+1}

$h+1$



$$m_{n+1} = 1 + m_{n-1} + m_n = \cancel{1} + F_{n+1} \cancel{-1} + F_{n+2} - 1 =$$

$$m_n = \varphi^n$$

$$= F_{n+1} + F_{n+2} - 1 = F_{n+3} - 1$$

$$\frac{m_{n+1}}{m_{n-1}} = \varphi^2 \qquad m_n = F_{n+2} - 1 \qquad \frac{m_{n+1}}{m_n} = \frac{F_{n+3} - 1}{F_{n+2} - 1}$$

$$\frac{m_n}{m_{n-1}} = \varphi$$

$$\frac{F_{n+1}}{F_n} \approx 1.618 = \frac{\sqrt{5} + 1}{2}^{\,n}$$

$$\cancel{n = \varphi}^{\,n}$$

log

$$\frac{m_{n+1}}{m_n} = 1.618 = \varphi \qquad \frac{m_{n+1}}{m_{n-2}} = \varphi^3$$

$$\frac{m_{h+1}}{m_h} = \varphi \Rightarrow m_{h+1} = \varphi \cdot m_h = \varphi^2 \cdot m_{h-1}$$

$$\frac{m_h}{m_{h-1}} = \varphi \Rightarrow m_h = \varphi \cdot m_{h-1}$$

$$\frac{m_{h+1}}{m_{h-1}} = \varphi^2$$

$$\varphi = \frac{\sqrt{5}+1}{2} = 1.618$$

$$m_h = \varphi^h \cdot \cancel{m_1}$$

$$m_{h-1} = \varphi \cdot m_{h-2}$$

$$m_{h+1} = \varphi^3 \cdot m_{h-2}$$

$$\log(m_h) = \log(\varphi^h)$$

$$h = O(\log n)$$

$$m_h = \varphi^h$$

$$\log m_h = h \cdot \cancel{\log \varphi}$$

1.618

$$\boxed{h \approx \log m_h}$$

$$h = O(\log m_h)$$

Кнут, т. 3, п. 6.2.3

Кормен, гл. 12, 13
RBT

Седжвик, гл. 13.