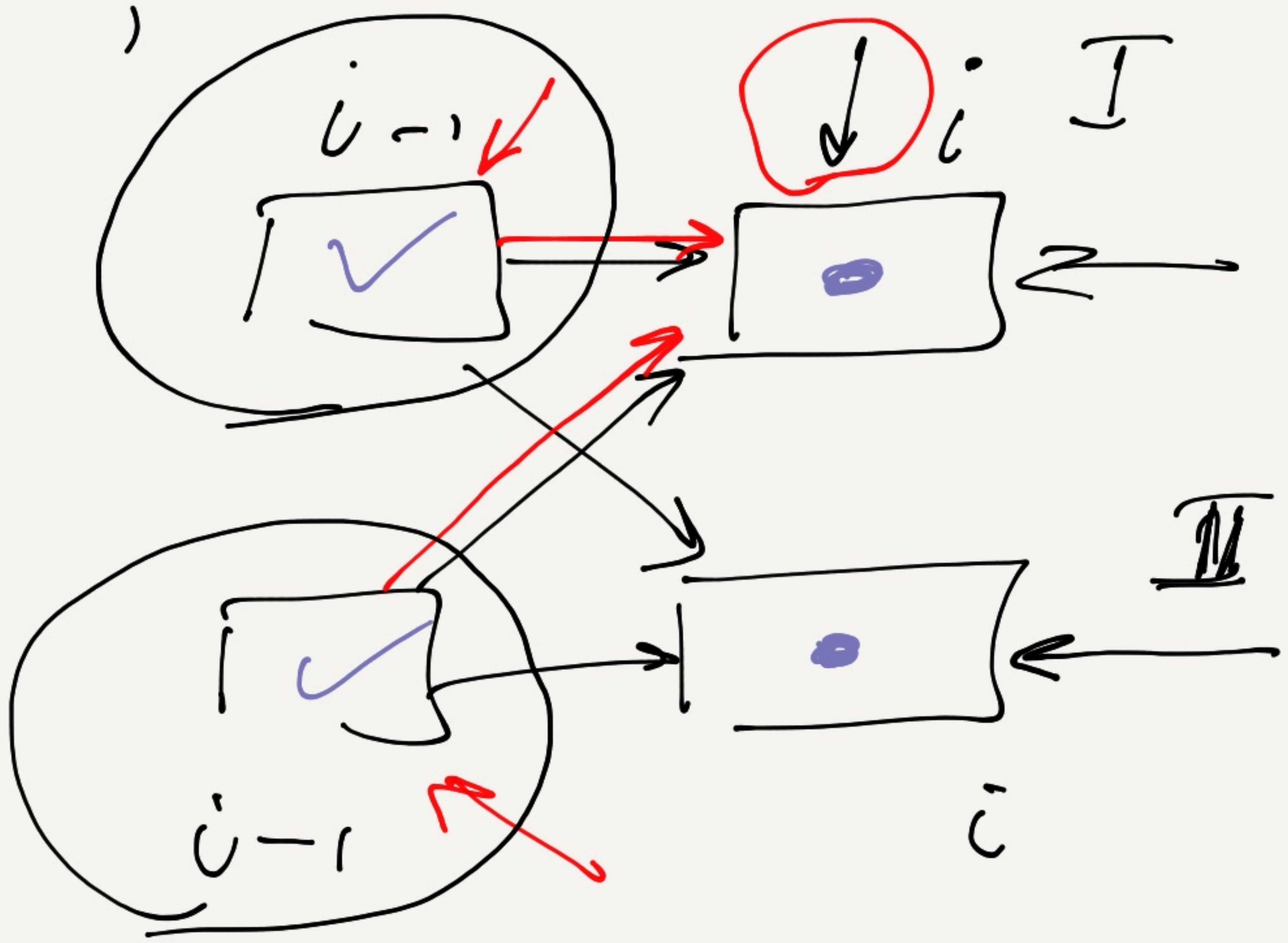
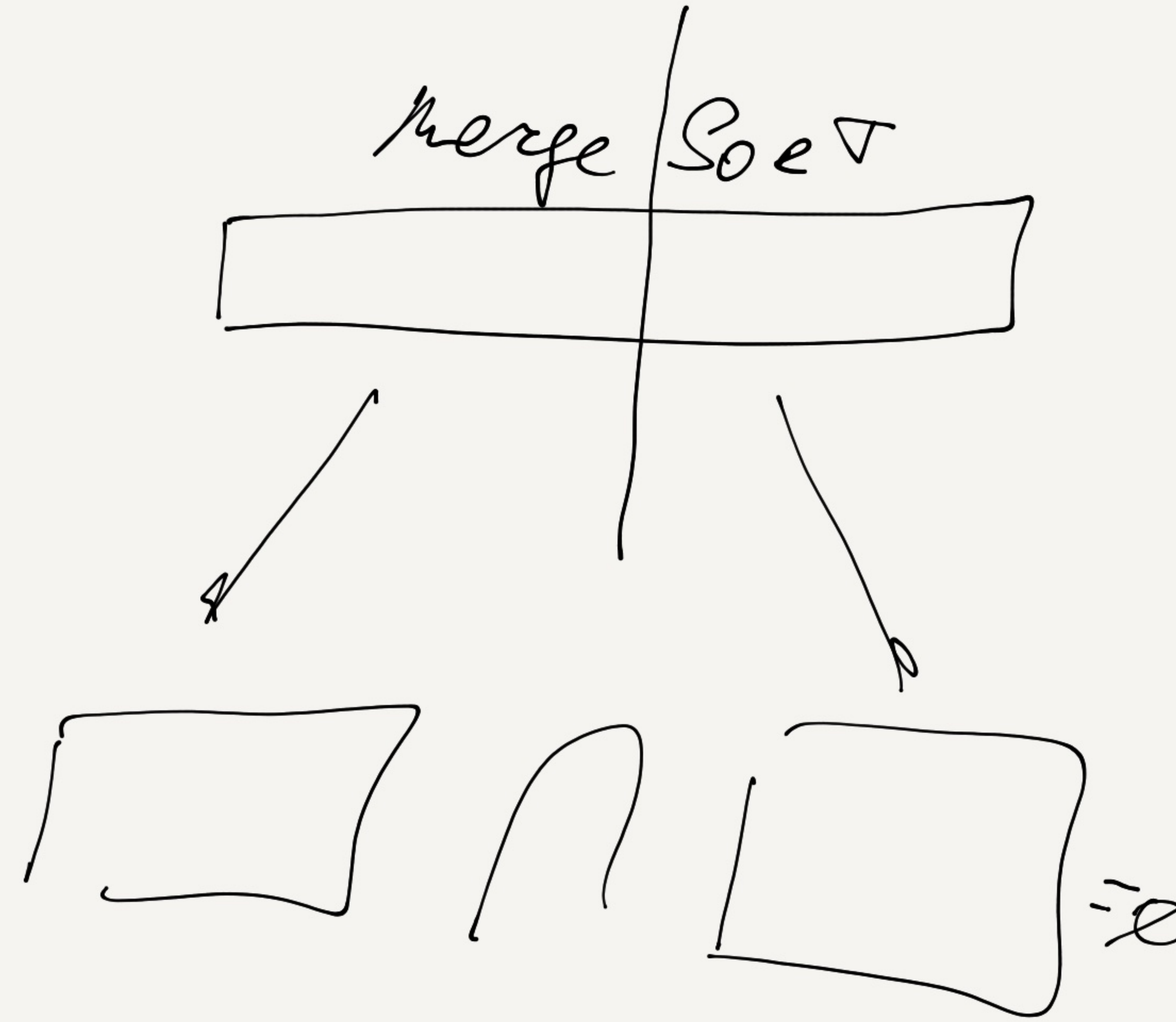
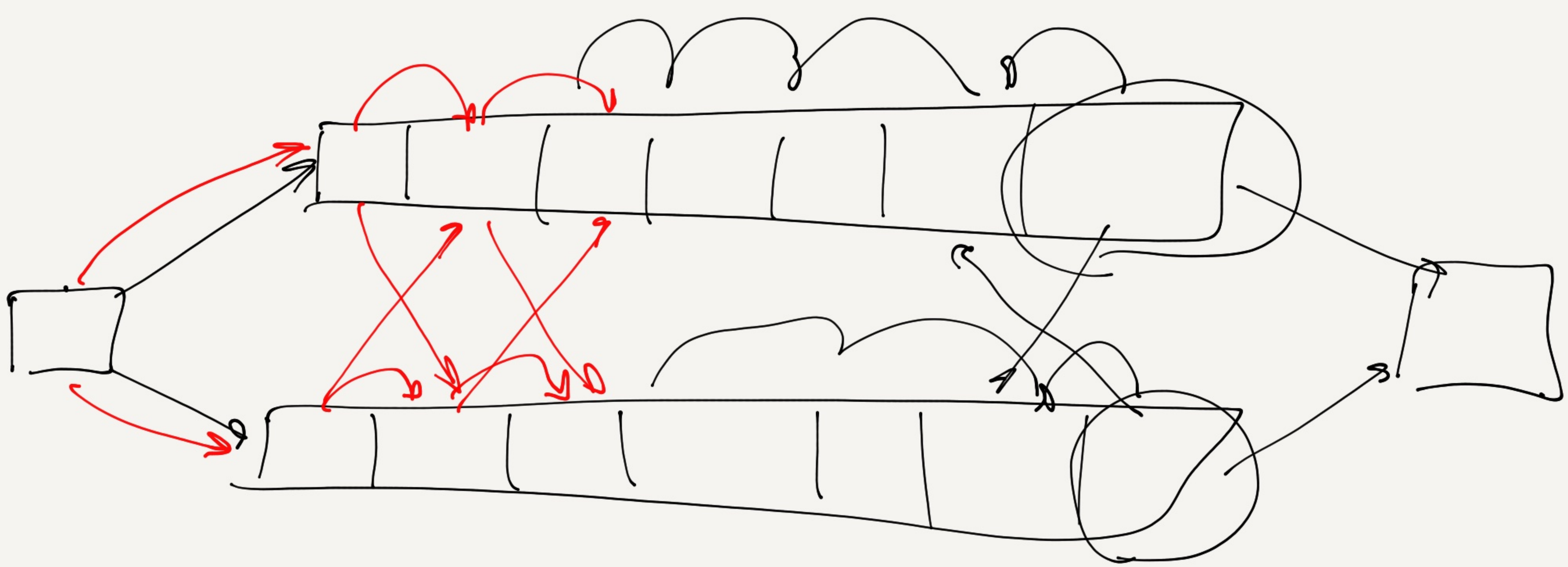
ДП, Кормен, гл. 15

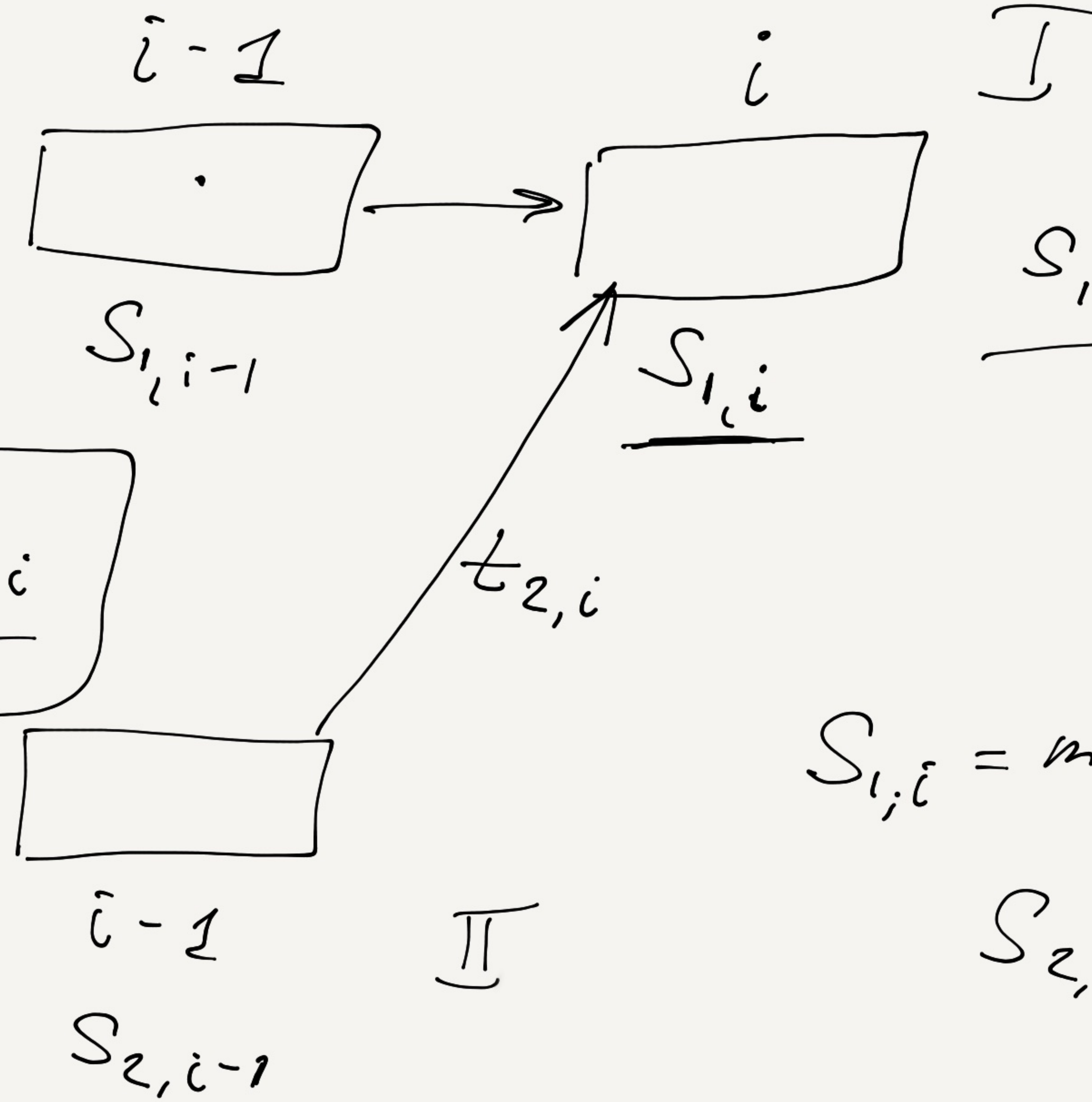






$S_{1,i-1} > \boxed{S'_{1,i-1}}$

$$\boxed{S_{1,i} = \boxed{S_{1,i-1}} + a_i}$$



$S'_{1,i-1} + a_i < S_{1,i-1} + a_i$

$$S_{1,i} = \min\left(S_{1,i-1} + a_i,\ S_{2,i-1} + t_{2,i} + a_i\right).$$

$$S_{1,i} = S_{2,i-1} + t_{2,i} + a_i$$

$$A_1 \quad A_2 \quad A_3 \quad \ldots \quad A_n$$
$$p_0 \times p_1 \quad p_1 \times p_2 \quad p_2 \times p_3 \quad \ldots \quad p_{n-1} \times p_n$$

$$A_i A_{i+1} \qquad p_{i-1} \cdot p_i \cdot p_{i+1}$$
$$p_{i-1} \times p_i \quad p_i \times p_{i+1}$$

$$A \quad B \qquad p \cdot q \cdot R$$
$$p \times q \quad q \times R$$

$$A_1 (A_2 \, A_3)$$
$$50 \cdot 5 \quad 5 \cdot 100 \quad 100 \cdot 10$$

$$A_1 \quad A_2 \quad A_3$$
$$10 \times 100 \quad 100 \times 5 \quad 5 \times 50$$

$$10 \times 5 \quad 5 \times 50$$

1) $(A_1 \, A_2) A_3$

$$10 \cdot 100 \cdot 5 = 5000$$
$$10 \cdot 5 \cdot 50 = 2500$$
$$\boxed{7500}$$

$$10 \times 100 \quad 100 \times 50$$

2) $A_1 (A_2 \, A_3)$

$$100 \cdot 5 \cdot 50 = 25000$$
$$10 \cdot 100 \cdot 50 = 50000$$
$$\boxed{75000}$$

$$A_k \underline{(A_{k+1} A_{k+2})} A_{k+3}$$

$$\downarrow$$

$$A_k \underline{A'} A_{k+3}$$

$$A_1 A_2 \ldots \Big| \; A_n \longrightarrow A_{1\ldots k} \quad A_{k+1\ldots n}$$

$$\downarrow \qquad\qquad \downarrow$$

$$A_1 A_2 \ldots A_k \qquad A_{k+1} \ldots A_n.$$

$$\sum_{k=1}^{n-1} P(k) \cdot P(n-k) \longleftarrow \text{Каталана}$$

$$O(4^n)$$

$A_i \; A_{i+1} \ldots \;\Big|\; A_j$

$m_{i,j}$

$(A_i A_{i+1} \ldots \underline{A_k})(A_{k+1} A_{k+2} \ldots A_j)$

$[\underbrace{A_{i \ldots k}}]$ $\{\underbrace{A_{k+1 \ldots j}}\}$

$\underline{m_{i,k}}$ $\underline{m_{k+1,j}}$

$p_{i-1} \times p_k \; A_k \quad p_k \times p_j$

$(A_{i \ldots k})(A_{k+1 \ldots j})$

$m_{i,k}.$ $m_{k+1,j}$

$$m_{i,j} = m_{i,k} + m_{k+1,j} + p_{i-1} \cdot p_k \cdot p_j$$

$$\boxed{p_{i-1} \cdot p_k \cdot p_j} + \underline{m_{i,k}} + \underline{m_{k+1,j}} = \boxed{m_{i,j}}$$

10x100 100x5 5x50

$A_1 A_2 A_3$

$(A_1 A_2) A_3 \to A_{1 \ldots 2} \quad A_{3 \ldots 3}$

$A_1 (A_2 A_3) \to A_{1 \ldots 1} \quad A_{2 \ldots 3}$

$m_{1,2} = 10 \cdot 100 \cdot 5 = 5000$

$m_{2,3} = 100 \cdot 5 \cdot 50 = 25000$

$m_{1,3} = 7500$

$A_1 A_2 A_3 A_4$

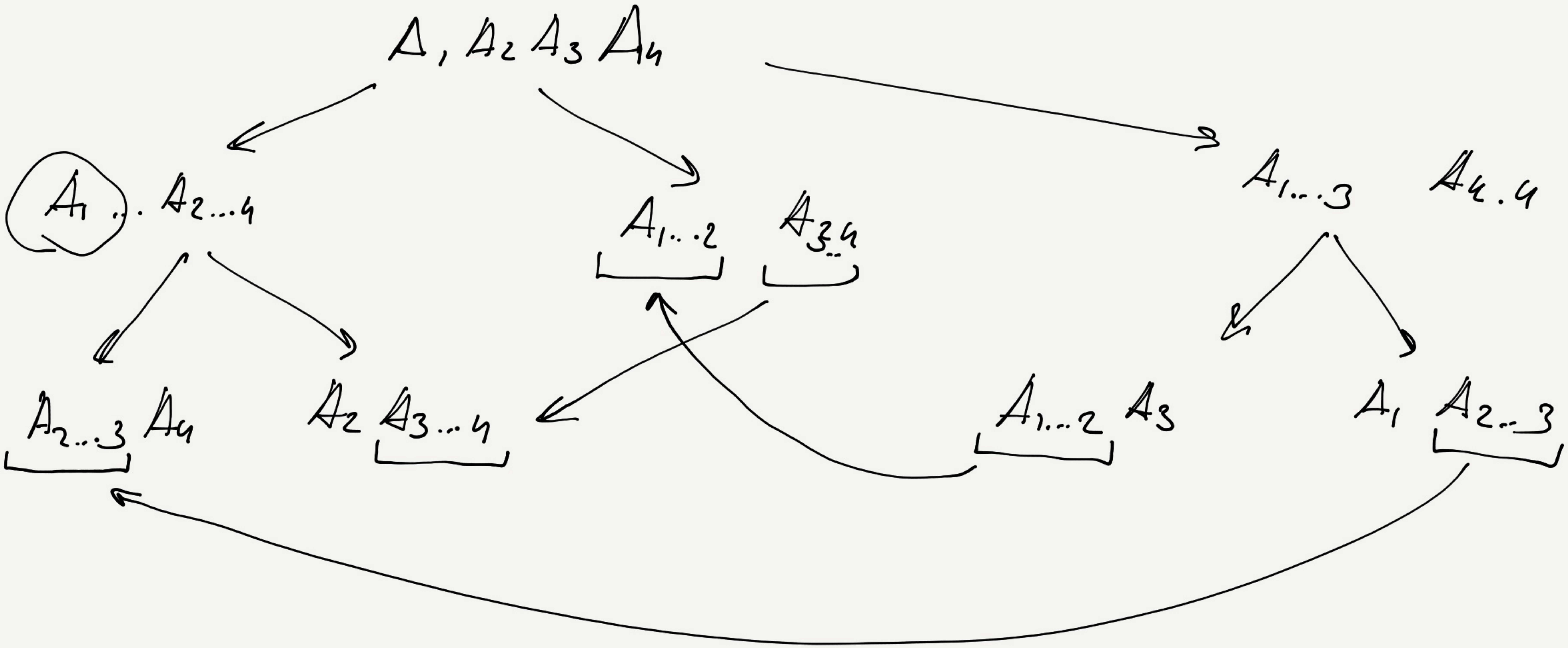
$((A_1 A_2) A_3) A_4 \to A_{1 \ldots 3} \quad A_{4 \ldots 4}$

$(A_1 (A_2 A_3)) A_4$

$A_1 (A_2 (A_3 A_4)) \to A_{1 \ldots 1} \cdot A_{2 \ldots 4}$

$A_1 ((A_2 A_3) A_4)$

$(A_1 A_2)(A_3 A_4) \to A_{1 \ldots 2} \cdot A_{3 \ldots 4}$



$A_1 A_2$ $\quad$ $A_2 A_3$ $\quad$ $A_3 A_4$ . . . $\quad$ $A_{n-1} \; A_n$

$m_{1,2}$ $\quad$ $m_{2,3}$ $\quad$ $m_{3,4}$ . . . $\quad$ $m_{n-1,n}$

$A_1 A_2 A_3$

$(A_1 A_2) A_3$

$A_1 (A_2 A_3)$

$$\sum_{k=1}^{n-1} P(k)\,(P_{n-k})$$

$$m_{i,j} = \min_{k=i}^{j-1} \left(m_{i,k} + m_{k+1,j} + p_{i-1} \cdot p_k \cdot p_j \right)$$

A_1 A_2 A_3 A_4

1×4 4×10 10×100 100×15

i j

	1	2	3	4
1	0	40	1040	2540
2		0	4000	10000
3			0	15000
4				0

$\boxed{40 + 0 + 1\cdot 10\cdot 100}$

$0 + 4000 + 1\cdot 4\cdot 100 = 4400$

$4000 + 0 + 4\cdot 100\cdot 15 = \underline{10000}$

$0 + 15000 + 4\cdot 10\cdot 15 =$
$= 15600$

$A_1 A_2$

$A_2 A_3$

$A_3 A_4$

$A_1 | A_2 | A_3$

$A_{1\ldots 2}\ A_3$

$A_1\ A_{2\ldots 3}$

$A_2 A_3 A_4$

$A_{2\ldots 3} A_4$

$A_2\ A_{3\ldots 4}$

$A_1 A_2 A_3 A_4$

$A_1\ A_{2\ldots n}$

$0 + 10000 +$
$+ 1\cdot 4\cdot 15 =$
$= 10060.$

$A_{1\ldots 2}\ A_{3\ldots 4}$

$40 + 15000 +$
$+ 1\cdot 10\cdot 15 =$
$= \underline{15190}$

$A_{1\ldots 3}\ A_4$

$1040 + 0 +$
$+ 1\cdot 100\cdot 15 =$
$= \underline{2540}$

for $i = 1$ to n:

$m_{i,i} = 0$ $\qquad$ $m_{i,j} = \infty$ $\quad j > i$ $\qquad$ $S_{i,j}$

for $l = 2$ to n: // $2 \to 3 \to 4 \dots \to n$

for $i = 1$ to $n - l + 1$:

$j = i + l - 1$ // $A_i \dots A_j$

for $k = i$ to $j - 1$:

$tmp = m_{i,k} + m_{k+1,j} + P_{i-1} \cdot P_k \cdot P_j$

if $tmp < m_{i,j}$:

$m_{i,j} = tmp.$

$S_{i,j} = K$

$O(n^3)$

$\cancel{O(n^2)}$

$$1 + 2 + \dots + n - 1 = \frac{n(n-1)}{2} = \frac{n^2}{2} - \frac{n}{2} \sim \frac{n^2}{2}.$$

$A_1 \quad A_2 \quad A_3 \quad A_4$

$1\times4 \quad 4\times10 \quad 10\times100 \quad 100\times15$

m

$i \backslash j$	1	2	3	4
1	0	40	1040	~~10060~~ 2540
2		0	4000	10000
3			0	15000
4				0

S

$i \backslash j$	1	2	3	4
1	0	1	2	~~X~~ 3
2		0	2	3
3			0	3
4				0

$(A_1) | A_2 \qquad A_2 | A_3 \qquad A_3 | A_4$

$(A_1 A_2 | A_3$

$A_1 | A_2 A_3 A_4$

$A_1 A_2 A_3 | A_4$

$tmp = m_{i,k} + m_{k+1,j} + P_{i-1} \times P_k \times P_j$

if $m_{i,j} > tmp$:

$m_{i,j} = tmp$

$S_{i,j} = k \longleftarrow m_{j,i} = k.$

$A_1 | A_2 | A_3 | A_4$

$((A_1 A_2) A_3) A_4$

Мемоизация

A_1 A_2 A_3 A_4

1×4 4×10 10×100 100×15

I

$A_1A_2A_3 \cdot A_4$

$A_1A_2 \cdot A_3$ $A_1 \cdot A_2A_3$

$A_1A_2A_3A_4$

II

$\boxed{A_1 \cdot A_2} \cdot A_3A_4$

$A_1 \cdot A_2A_3A_4$

10000

$\boxed{A_2A_3} \cdot A_4$

$4000 + 4 \cdot 100 \cdot 15$

1000

$A_2 \cdot \boxed{A_3A_4}$

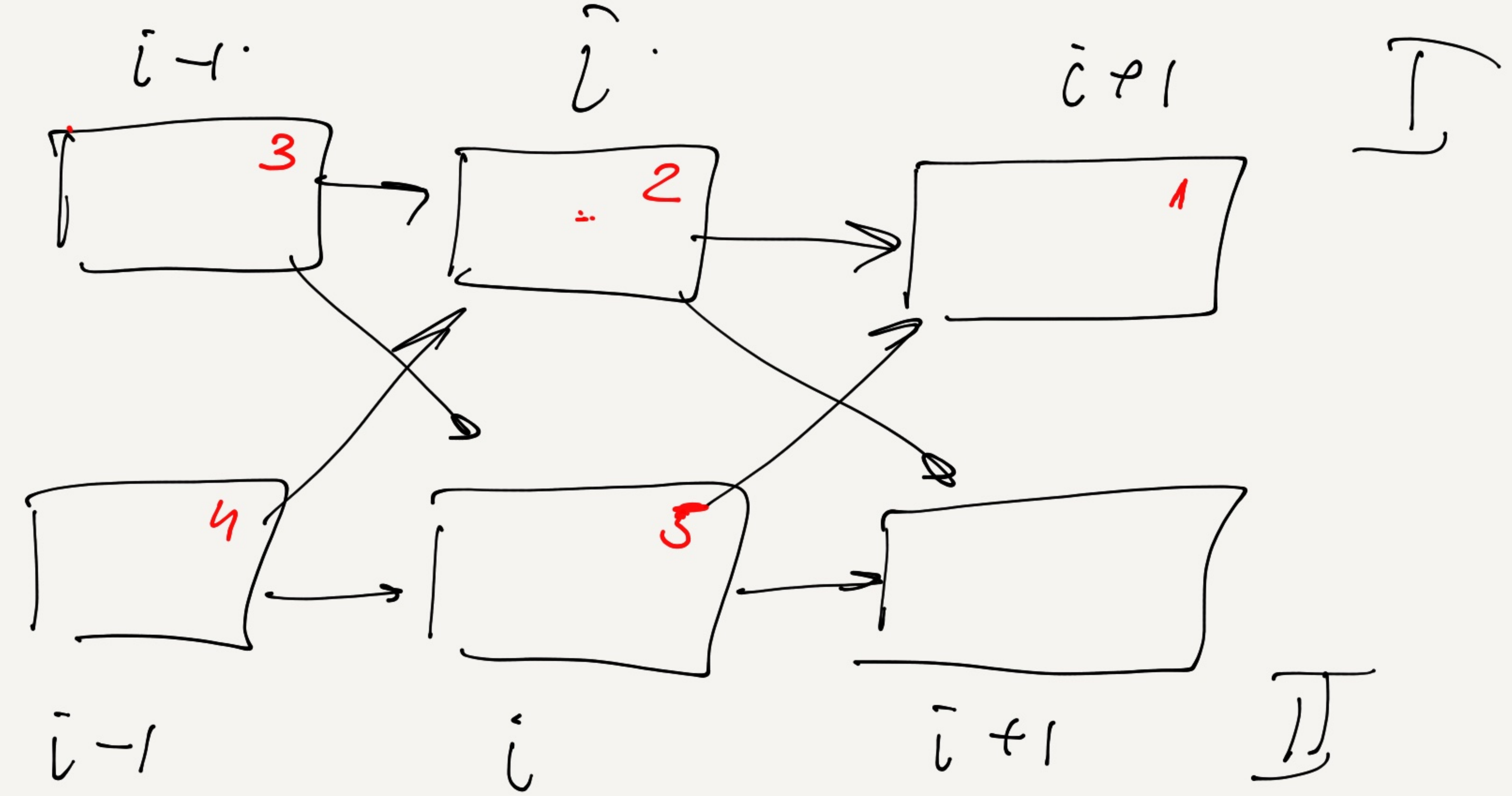
$15000 +$

$+ 4 \cdot 10 \cdot 15$

> 15000

m, j →, i ↓

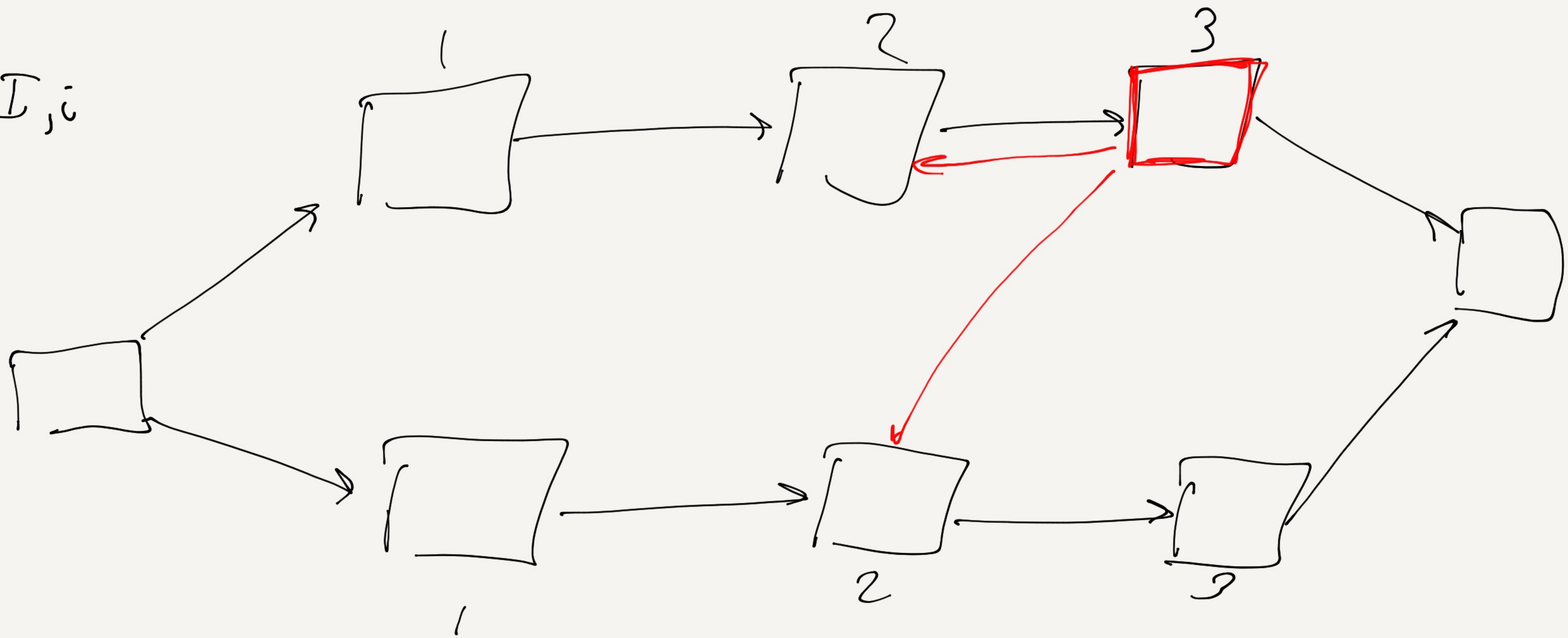
i \ j	1	2	3	4
1	0	40	1040	∞
2		0	4000	10000
3			0	15000
4				0



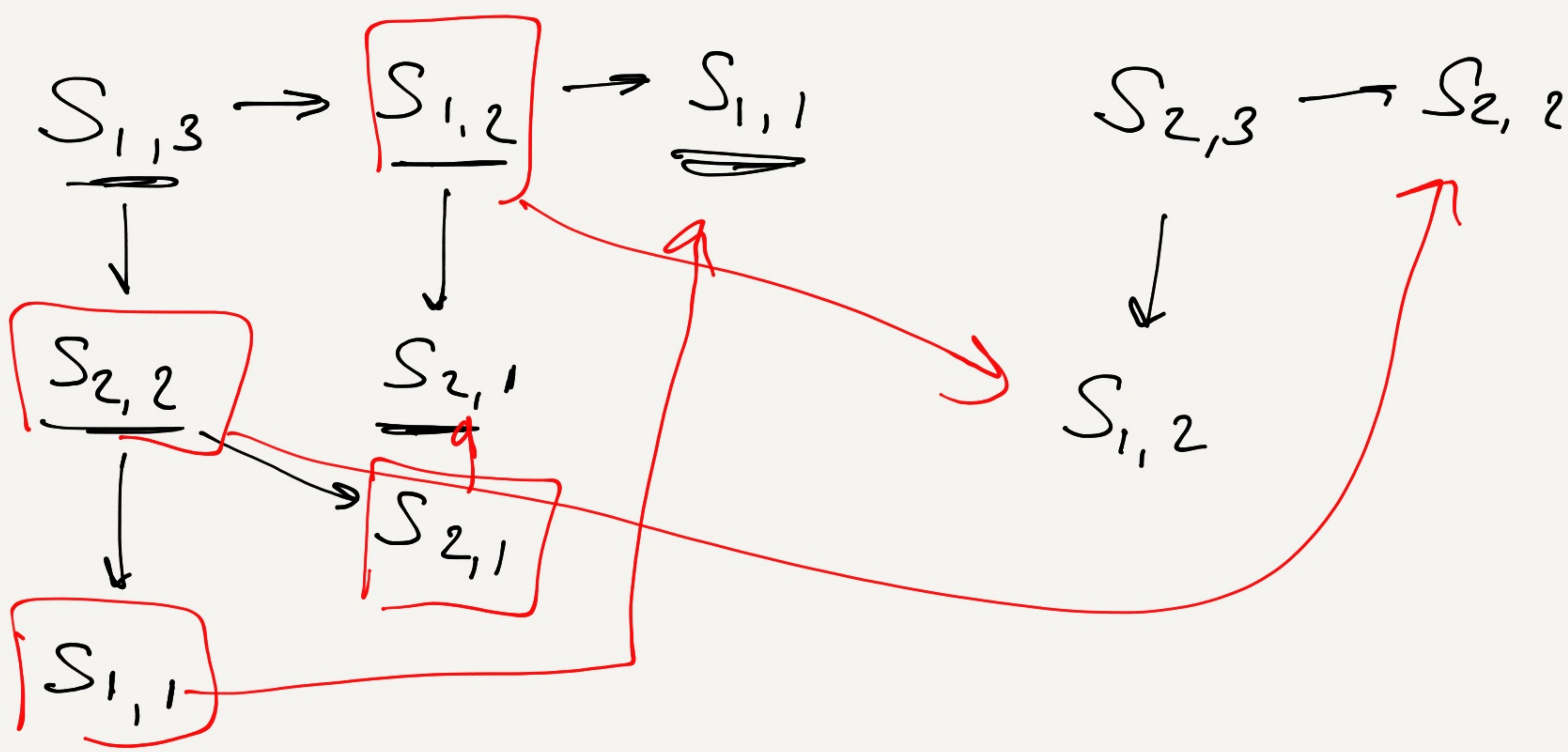
$i-1 \rightarrow i \rightarrow i+1$

$i+1 \rightarrow i \rightarrow i-1.$

$S_{I,i}$



$1 \to 2 \to 3$



```
For i = 1 to n:
    For j = 1 to n:
        m_{i,j} = ∞.
For i = 1 to n:
    m_{i,i} = 0
Solve(1, n).
```

```
Solve(i, j):
    if m_{i,j} != ∞:
        return m_{i,j}
    For k = i to j-1:
        tmp = Solve(i,k) +
              + Solve(k+1, j) +
              + p_{i-1} · p_k · p_j
        if tmp < m_{i,j}:
            m_{i,j} = tmp
    return m_{i,j}.
```

LCS

Largest Common Subsequence

S_1 $\qquad$ P, $\quad P \subset S_1$ $\quad$ $|P| \to \max$ $\qquad$ $i, i+1, \ldots, j-1, j$

S_2 $\qquad\qquad\quad$ $P \subset S_2$ $\qquad\qquad\qquad\qquad$ $i \ldots j$

31

20 - 30, 32 - 34.

20 - 30

20, 21, ~~22~~, ~~23~~, ~~24~~, 25, 26, ~~27~~, ~~28~~, 29, ~~30~~

ABCDE

BCD → substring

BCD → subsequence

BDE → subsequence

ABCBACB

Substring = ABC

BAABCAB

Subsequence = ABCAB

$S_1 \quad |S_1| = n$

$S_2 \quad |S_2| = m$

~~AB BA~~ ~~ACA~~

BAABCAB

$n \approx m$

$O(m 2^n)$

$O(n \cdot 2^m)$

$O(n 2^n)$

$X = \{x_1\ x_2\ \dots\ x_n\}$ $\qquad$ $LCS(X, Y) = Z = \{z_1, z_2, \dots, z_k\}.$

$Y = \{y_1\ y_2\ \dots\ y_m\}$

1) $x_n = y_m \Rightarrow x_n = y_m = z_k$, $\boxed{LCS(X[1\dots n-1], Y[1\dots m-1]) = Z[1\dots k-1].}$

$\cancel{x_n = y_m \neq z_k}$

$\cancel{x_a = z_k,\ a < n}$

$\cancel{y_b = z_k,\ b < m}$

$LCS(X[1\dots m-1], Y[1\dots m-1]) =$

$= F,\ |F| > k-1.$

$F = \{f_1, f_2, \dots, f_k\} + z_k > k$

2) $x_n \neq y_m$

$x_1\ x_2\ \boxed{x_a}\ \quad x_n$

$y_1\ y_2\ \dots\ \boxed{y_b}\ \quad y_m$

$z_1\ z_2\ \dots\ \quad z_k,\ x_n$

$x_n \neq y_m$

$Z = LCS(X, Y)$

z_k.

z_1
z_2
$\vdots$
z_k

$x_1 \quad y_1$
$x_2 \quad y_2$
$\vdots \quad \vdots$
$x_n \quad y_m$

$x_n \neq z_k$

$\underline{y_m} \neq z_k$

$LCS(X, Y) = Z$.

$LCS(X, Y) = Z$

$LCS(X[1 \ldots n-1], Y) = Z$

$LCS(X, Y[1, \ldots m-1]) = Z$

$x_1 \quad x_2 \quad \ldots \quad x_{n-1} \quad x_n$

$y_1 \quad y_2 \quad \ldots \quad y_{m-1} \quad y_m$

$z_1 \quad z_2 \quad \ldots \quad z_{k-1} \quad z_k$

$x_n \neq z_k$

$y_n \neq z_k$

$$LCS(X, Y) = \begin{cases} 0, & |X| = 0 \ || \ |Y| = 0 \\ 1 + LCS(X[1 \ldots n-1], Y[1 \ldots m-1]), & x_n = y_m \\ \max(LCS(X[1 \ldots n-1], Y), LCS(X, Y[1 \ldots m-1])), & x_n \neq y_m \end{cases}$$

$S_1 = ABACBAC$

$S_2 = BAABCA|C$

ABCAC.

A/B B/A

$LCS(S_1, S_2) = 5$

$O(n \cdot m)$

S_1 →

S_2 ↓

	∅	A	B	A	C	B	A	C
∅	0	0	0	0	0	0	0	0
B	0	0	1	1	1	1	1	1
A	0	1	1	2	2	2	2	2
A	0	1	1	2	2	2	3	3
B	0	1	2	2	2	3	3	3
C	0	1	2	2	3	3	3	4
A	0	1	2	3	3	3	4	4
C	0	1	2	3	4	4	4	5

ABA   BA → BABAC

ABACBA   BA

$O(n)$

$O(m)$

Расстояние Левенштейна

Delete : $x_1 x_2 \dots x_{n-1} x_n \to x_1 x_2 \dots x_{n-1}$

Insert : $x_1 x_2 \dots x_n \to x_1 x_2 \dots x_n x_{n+1}$

Replace : $x_1 x_2 \dots x_{n-1} x_n \to x_1 x_2 \dots x_{n-1} z$.

$d \to$ Delete

$i \to$ Insert

$z \to$ Replace

S_1, S_2.

$S_1 \to S_2$

$k \cdot d + m \cdot i + n \cdot z$ = Levenstein Distance $\to$ min

СЛОН → МУХА

4\$ 4 replace

$d = i = R = 1\$$

УЧЕБА ЗНАНИЕ.

$d = 1\$$

$i = 2\$$

$R = 4\$$

16\$

12\$

УЧЕБА

ЗНАНИЕ.

слон → муха

~~слон~~ → сло → муха

слон → мух ← а

[сло]н → мухн → муха

а

$$LD(x_{1\dots n}, y_{1\dots m}) = \begin{cases} 0, & |x| = 0 \;\|\; |y| = 0 \\ i \cdot |y|, & |x| = 0, |y| \neq 0 \\ d \cdot |x|, & |x| \neq 0, |y| = 0 \\ LD(x_{1\dots n-1}, y_{1\dots m-1}), & x_n = y_m \\ \min\big(LD(x_{1,\dots n-1}, y_{1,\dots m}) + d, \\ \quad LD(x_{1\dots n}, y_{1,\dots m-1}) + i, \\ \quad LD(x_{1\dots n-1}, y_{1\dots m-1}) + R\big), & x_n \neq y_m \end{cases}$$

слон, мухан

слон, муха

сло, муха

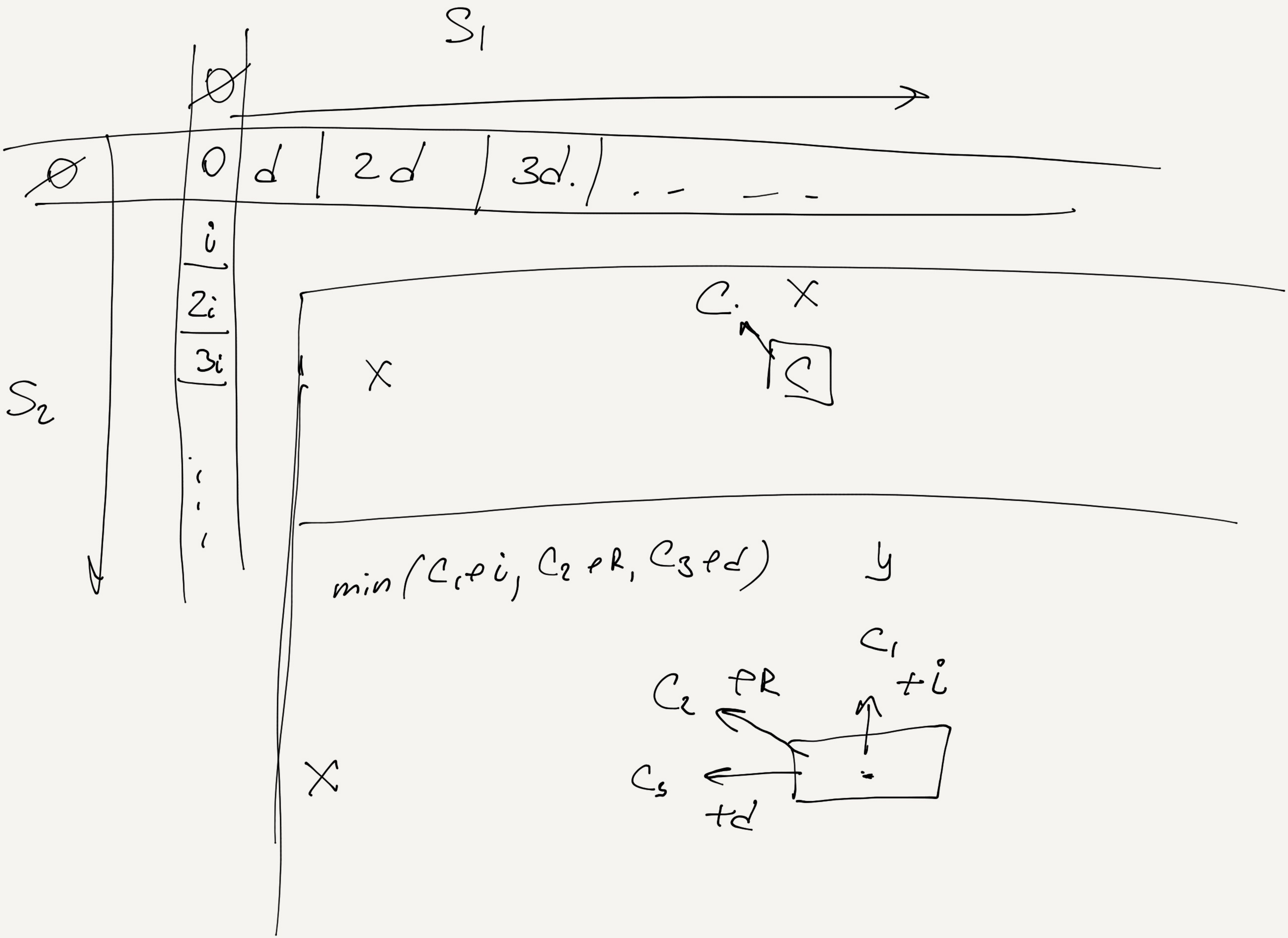
слон, мух ← а

сло, мух

слон

d ↓

муха



S_1
$\emptyset$
$\emptyset$
0
d
2d
3d.
i
2i
3i
S_2
C.
X
X
C
min(c_1+i, c_2+R, c_3+d)
y
c_1
+i
c_2
+R
c_3
+d
X