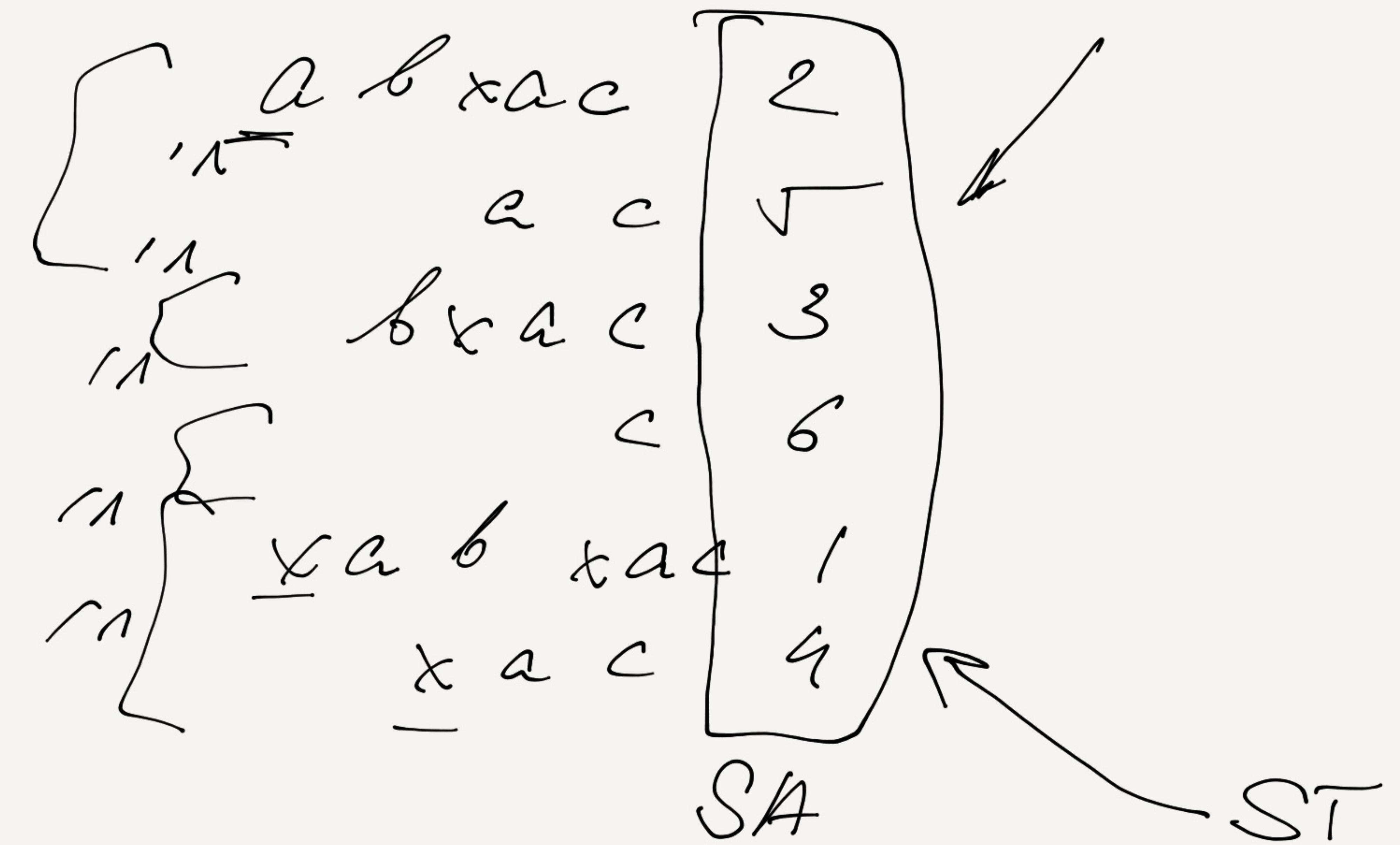


x a b x a c    1  
 a b x a c    2  
 b x a c    3  
 x a c    4  
 a c    5  
 c    6



1 ... n

(0 ... n - 1)

a b a c a b a	1
b a c a b a	2
a c a b a	3
c a b a	4
a b &	5
b c	6
a	7

	a
a b a	5
a b a c a b a	1
a c a b a	3
b a	6
b a c a b a	2
c a b a	4

0 1 2 3 4 5 6 7

$T = \underline{abacaba\$}$  ←

$$8 = 2^3$$

$a, b, c, c, a, b, a, \$$  → 16 =  $2^4$

\$ < a < b < c

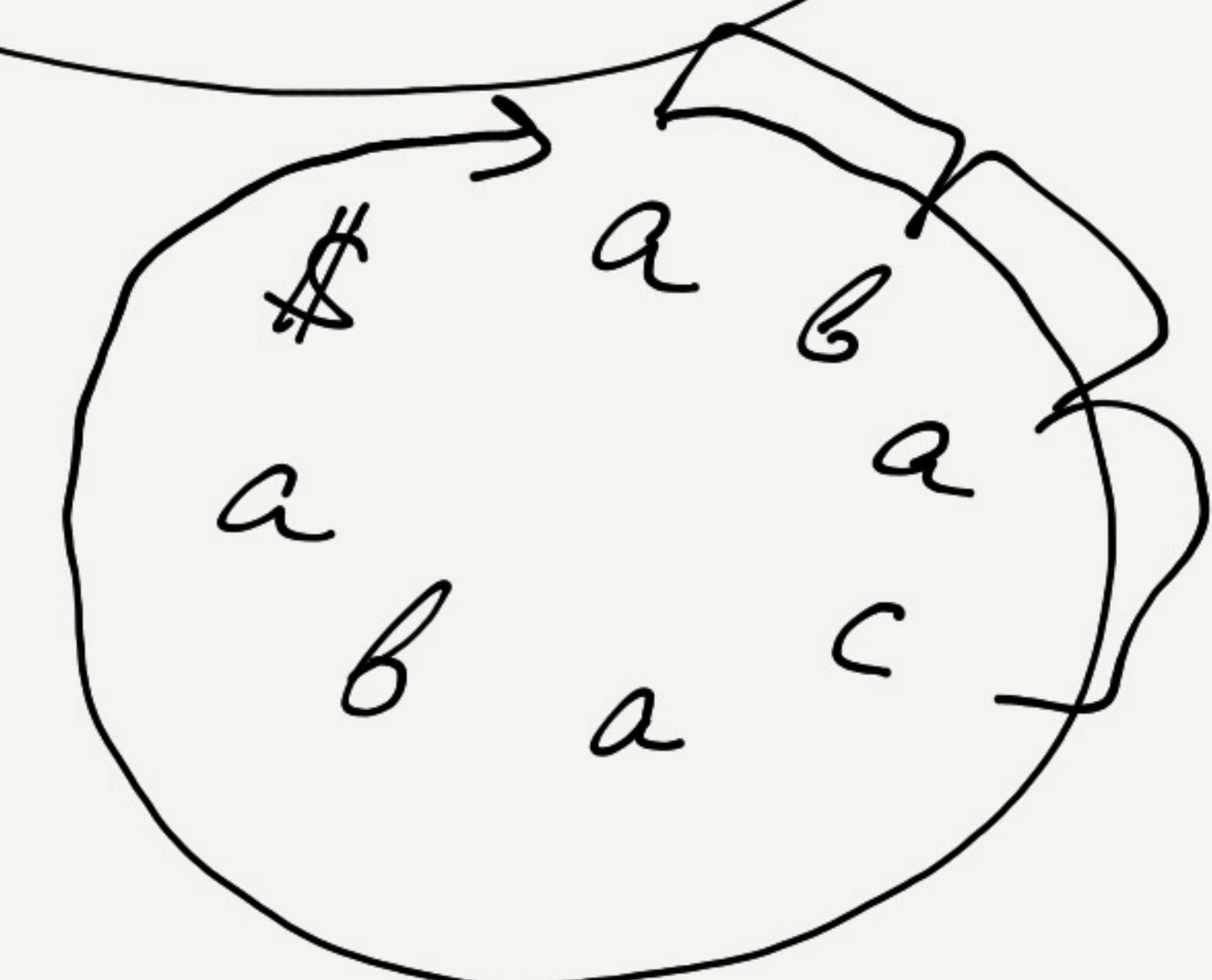
0    1    2    3 ←

indices	7	0	2	4	6	1	5	3
text	\$	a	a	a	a	b	b	c
c	0	1	1	1	1	2	2	3

abaca ba ba \$ ← \$ssss,

abaca ba ba \$

ab  
ba  
ac  
ca  
ab  
ba  
ab  
\$a



~~a b a c a b a \$~~ 0  
~~b a c a b a \$ a~~ 1  
~~a c a b a \$ a b~~ 2  
~~c a b a \$ a b a~~ 3  
~~a b a \$ a b a c~~ 4  
~~b a \$ a b a c a~~ 5  
~~a \$ a b a c a b~~ 6  
~~\$ a b a c a b a~~ 7

<del>\$ a b a c a b a</del>	7	
<del>a \$ a b a c a b</del>	6	7
<del>a b a \$ a b a c</del>	4	5
<del>a b a c a b a \$</del>	0	1
<del>a c a b a \$ a b a c</del>	2	3
<del>b a \$ a b a c a</del>	5	6
<del>b a c a b a \$ a</del>	0	2
<del>c a b a \$ a b a</del>	3	4

\$ c a c b c c

a b a c \$

~~a b a c~~

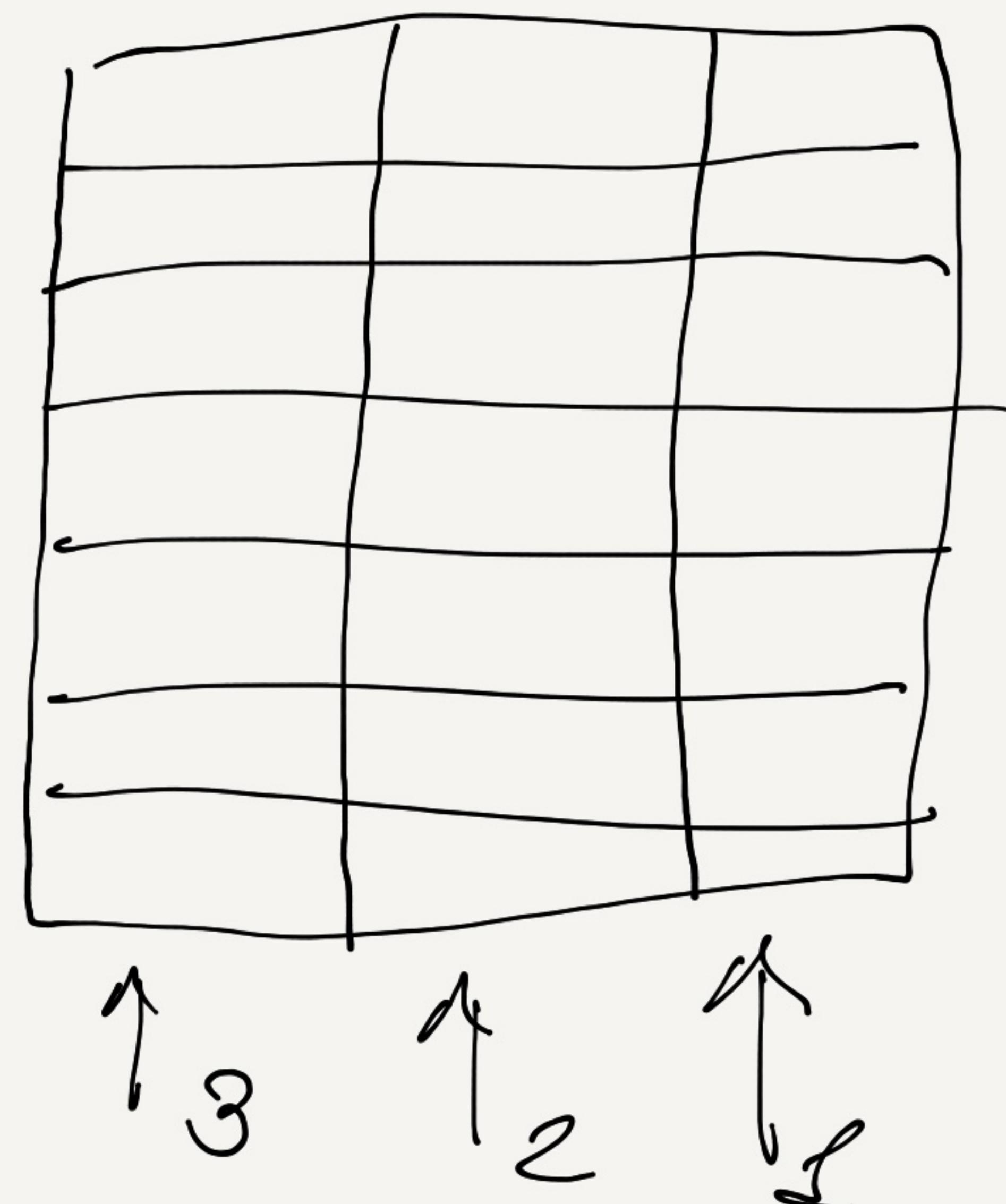
~~a b a c \$~~ 0  
~~a c \$ a b~~ 2  
~~b a c \$ a~~ 1  
~~c \$ a b a .~~ 3

~~a b a c~~  
~~a c~~  
~~b a c~~  
~~c~~

$$n, n \sim O(n \log n) = O(n^2 \log n)$$

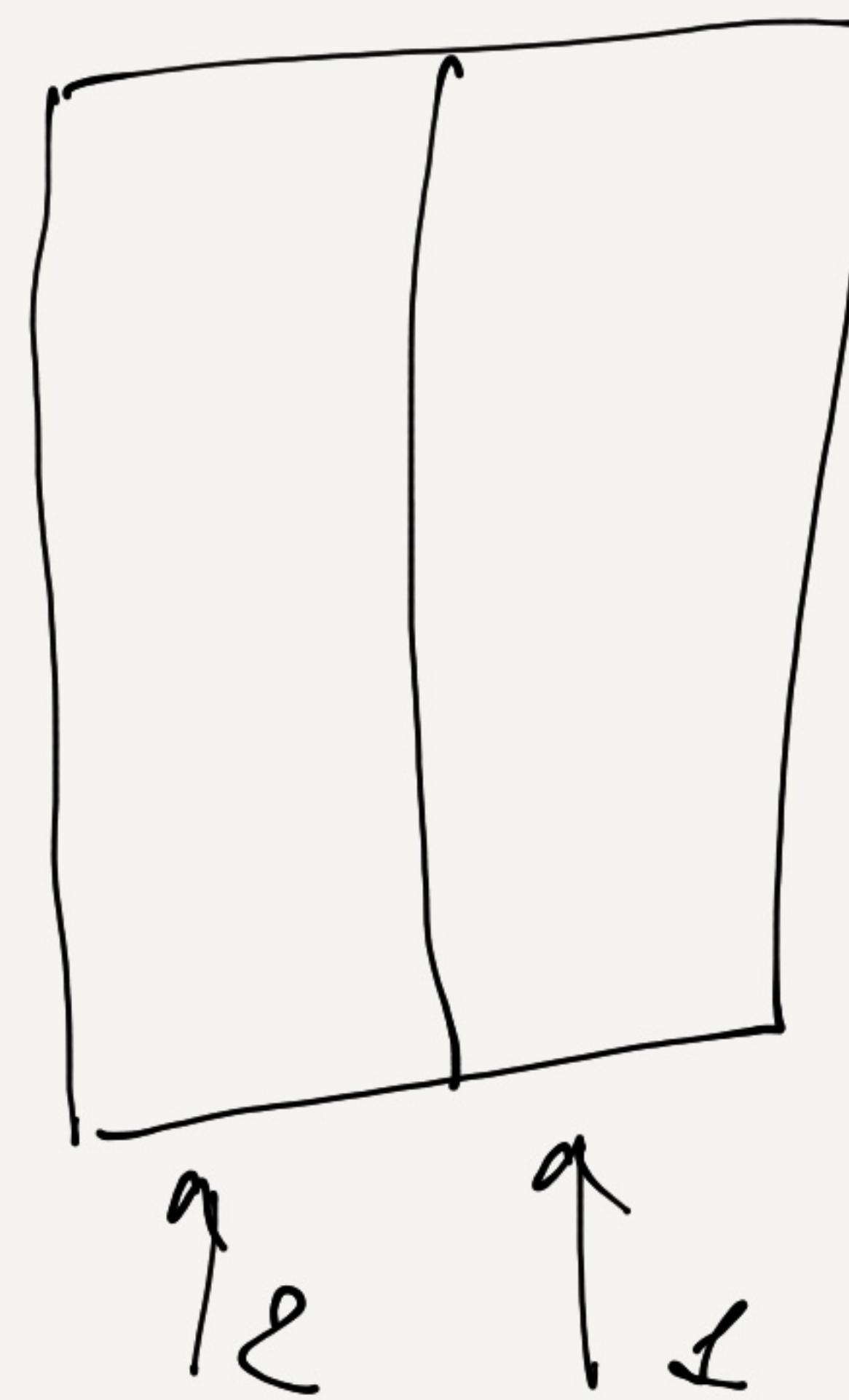
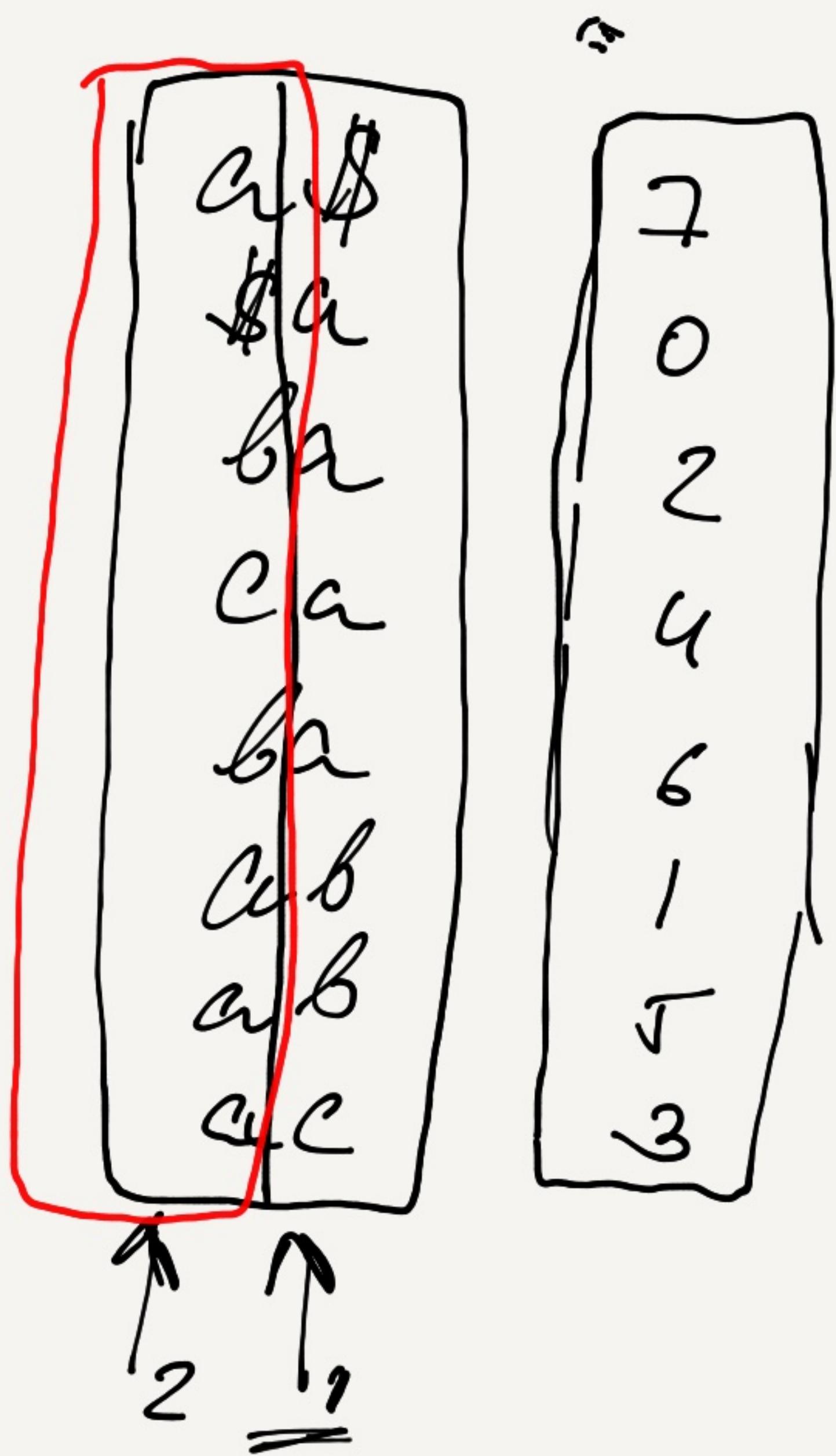
a b a c a b a \$ ←

0 1 2 3 4 5 6 7



ind	7	0	2	4	6	'	5	3
text	\$	a	a	a	a	b	b	c
c	0	1	1	1	2	2	3	

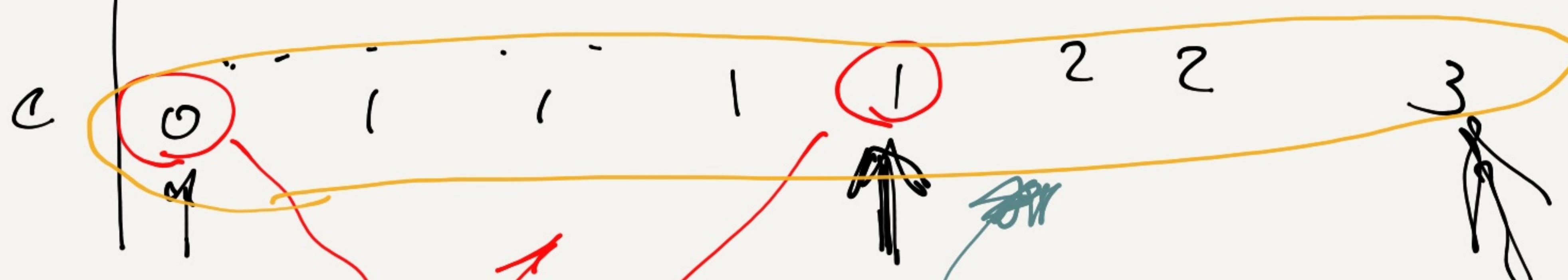
0 a b  
1 b a  
2 a c  
3 c a  
4 a b  
5 b a  
6 a \$  
7 \$ c



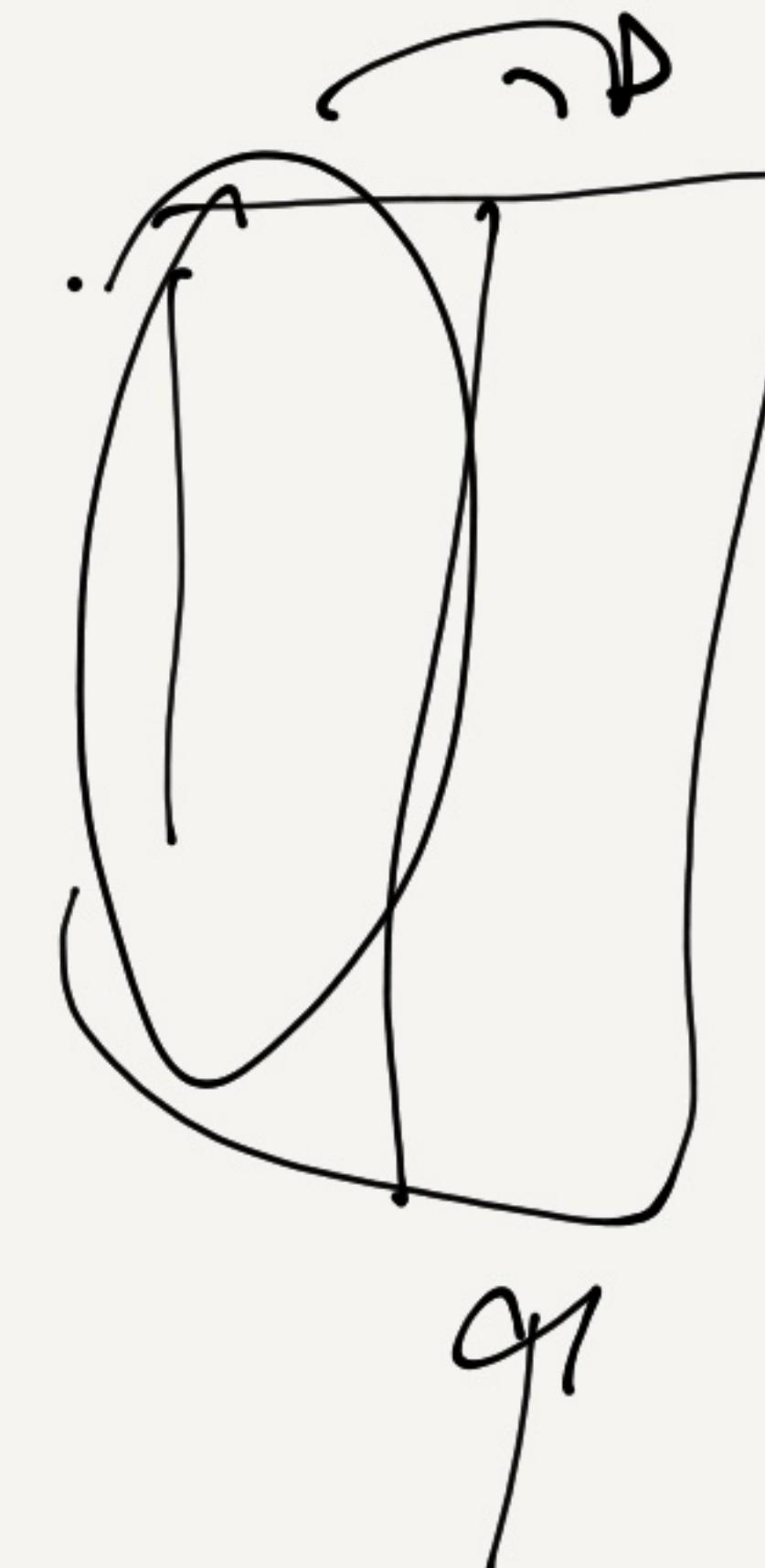
$a b c c a b c$

0 1 2 3 4 5 6 7

ind	7	0	2	a	6	1	5	3
text	\$	a	a	c	a	b	b	c



I.



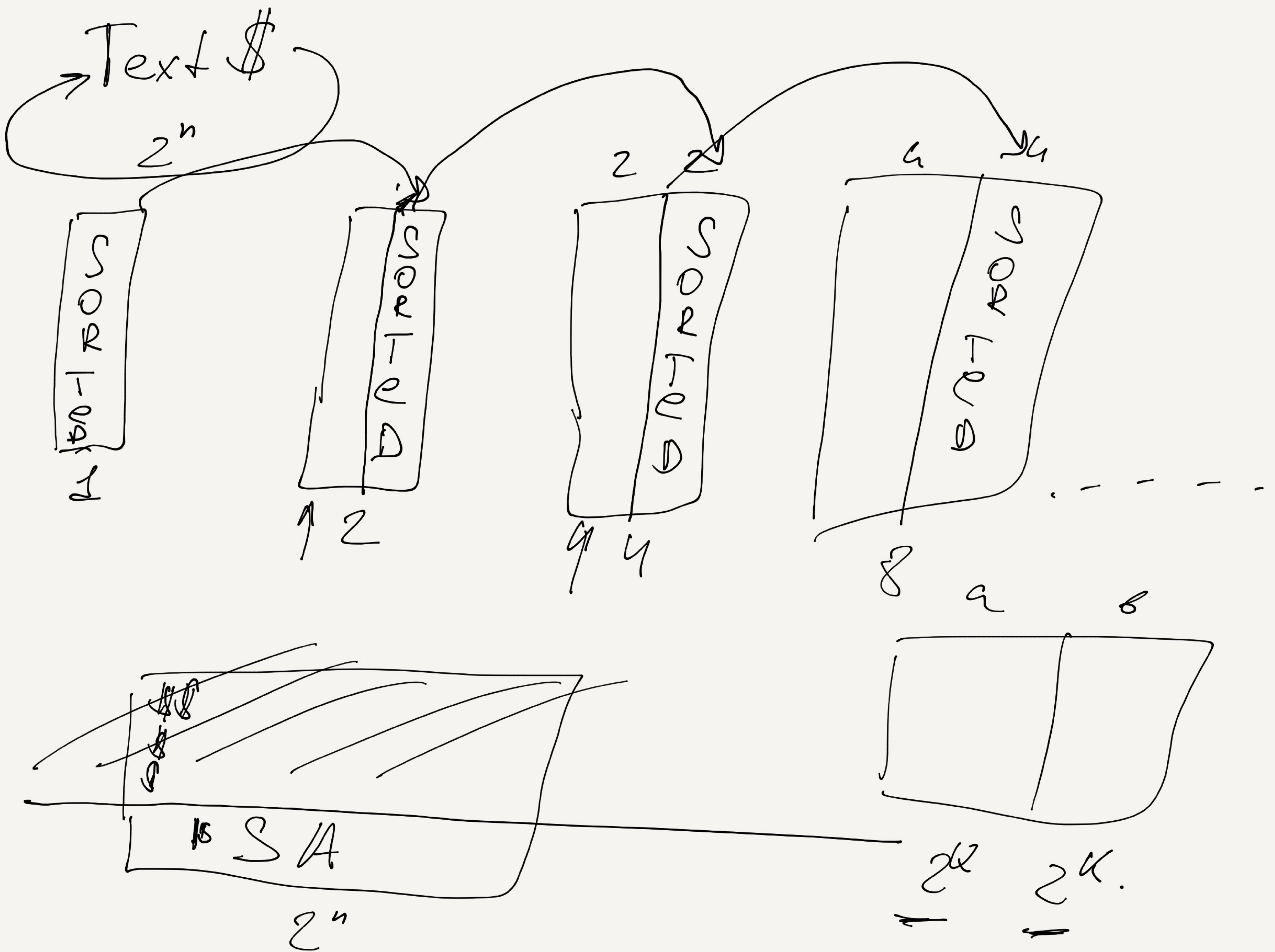
g1

ind	6	7	1	3	5	0	4	2
text	\$a	=	ba	-	ba	-	ab	-
c:	1	0	2	3	2	1	1	1

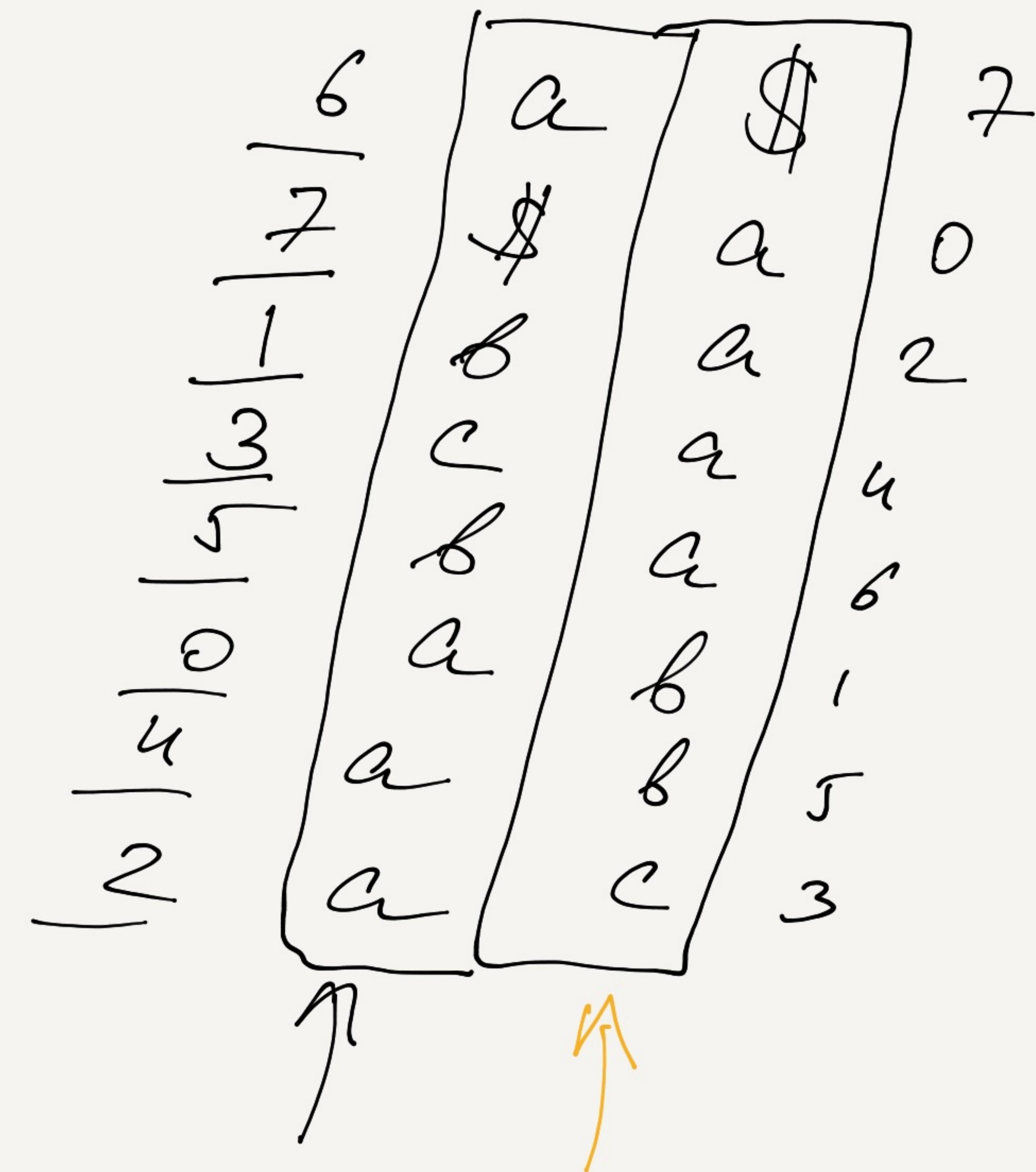
II.

✓

ind	7	6	0	4	2	1	5	3	4
text	\$a	=	ab	-	ab	-	ba	-	ca
c:	0	1	2 = 2	2	3	4 = 4	5	5	



a b a c a b a \$  
0 1 2 3 4 5 6 7



0	\$ c	7
1	a \$	6
2	a b	5
2	a b	4
3	a C	2
4	b a	1
4	b a	5
5	c a	3
C	test	ind
		<del>test</del>

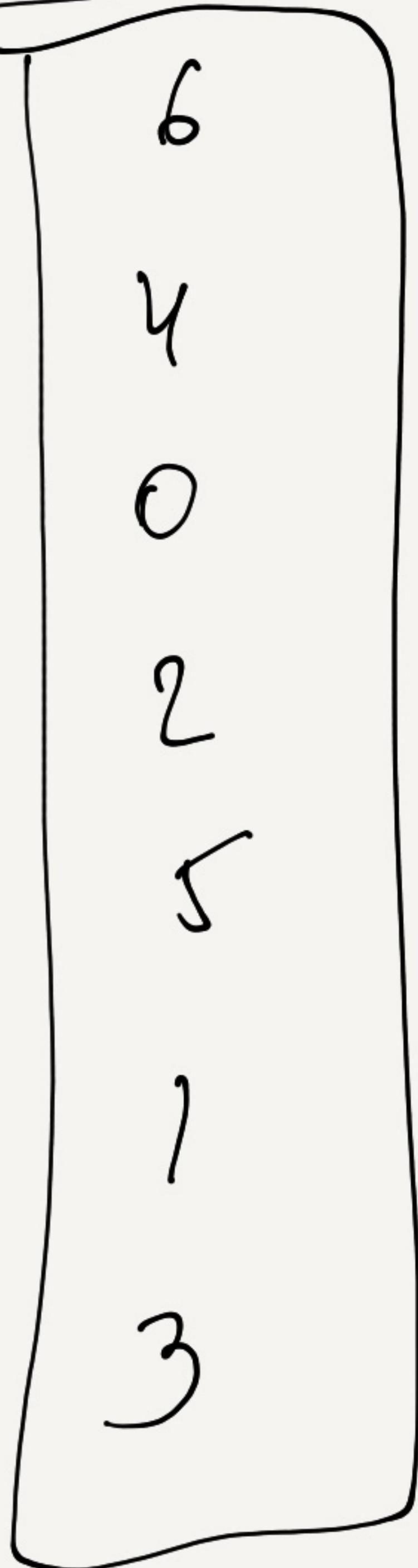
a b a c a b a \$  
0 1 2 3 4 5 6 7

5	4	b a \$ a
4	2	a b a \$
6	1	a \$ a b
2	3	a c a b
0	2	a b a c
7	0	\$ a b a
3	5	c a b a
1	4	b a c a

Y  
3n

ind	7	6	9	0	2	5	1	3
test	\$ a b a	a b a b	a b a \$	a b a c	a c a b	b a \$ a	b a c a	c a b a
C	0	1	2	3	4	5	6	7

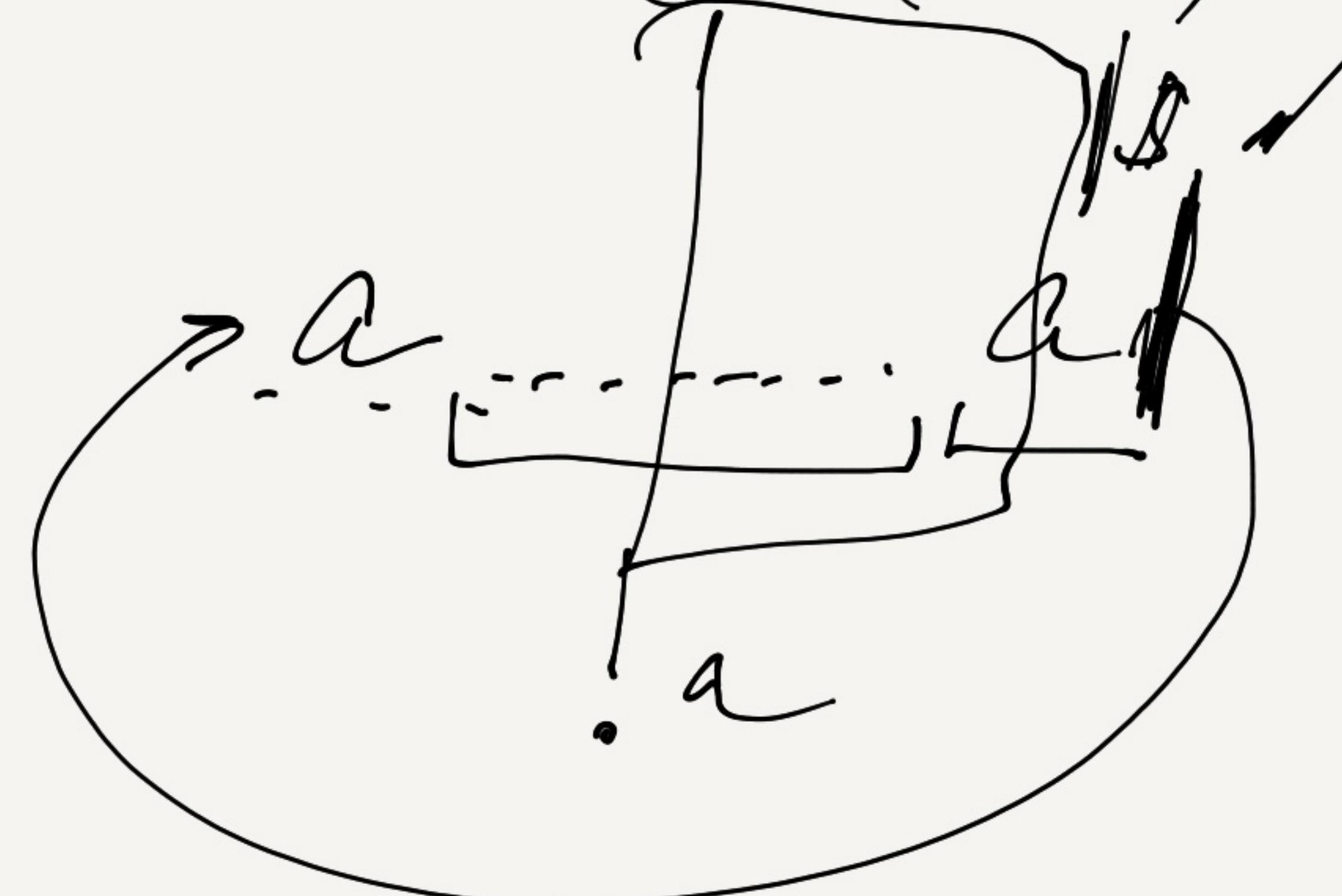
7



$1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \dots \rightarrow 2^n$

$O(n \log n)$

$O(n)$



$T = \frac{dc}{d\$} c a b a c a \frac{c}{d\$} \$ \$ \$ \$ \$ \$ \$$   
 i 2 3 4 5 6 7 8 9 10 11 12 13 14 15  
 . . . . .  
 1

$\frac{dc}{d\$}$   
 $d\$$

ind	8	9	10	11	12	13	14	15	2	4	6	3	1	5	0	2
text	\$	\$	\$	\$	\$	\$	\$	\$	a	a	a	b	c	c	d	d
c	0	0	0	0	0	0	0	1	i	i	2	<u>3</u>	3	<u>4</u>	<u>5</u>	<u>6</u>

2

ind	7	8	9	10	11	12	13	14	1	3	5	2	0	4	15	6
text	d\$	<del>d\$</del>	ca	ba	ca	ab	dc	ac	<del>dc</del>	<del>ad</del>						
c'	4	0	0	0	0	0	0	0	3	2	3	1	4	1	0	,

ind	8	9	10	11	12	13	14	15	2	4	6	3	1	5	7	0
text	<del>d\$</del>	ab	ac	ad	bc	ca	ca	<del>d\$</del>	<del>dc</del>							
c-old	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	1,2	1,3	1,4	2,1	3,1	3,1	4,0	4,3
c	0	0	0	0	0	0	0	1	2	3	4	5	6	6	7	8

8	\$\$	0
9	\$\$	0
10	\$\$	0
11	\$\$	0
12	\$\$	0
13	\$\$	0
14	\$\$	0
15	sd	1
2	ab	2
4	ac	3
6	ad	4
3	ba	5
1	ca	6
5	ca	6
7	da	7
0	dc	8
ind	test	c

4	ad	9
6	ad	9
7	da	7
8	\$\$	0
9	\$\$	0
10	\$\$	0
11	\$\$	0
12	\$\$	0
13	\$\$	0
0	dcab	8
2	abac	2
4	acad	3
1	caba	6
15	adca	1
3	baca	5
5	cada	6
14	\$\$dc	0
ind	test	c

8	\$\$	0, 0	0
9	\$\$	0, 0	0
10	\$\$	0, 0	0
11	\$\$	0, 0	0
12	\$\$	0, 0	0
13	\$\$	0, 1	1
14	\$\$dc	0, 8	2
15	adca	1, 6	3
2	abac	2, 3	4
4	acad	3, 4	5
6	ad\$\$	4, 0	6
3	baca	5, 6	7
1	ca ba	6, 5	8
5	cada	6, 7	9
7	da\$\$	7, 0	10
0	d cab	8, 2	11
ind	test	C-O/I	C

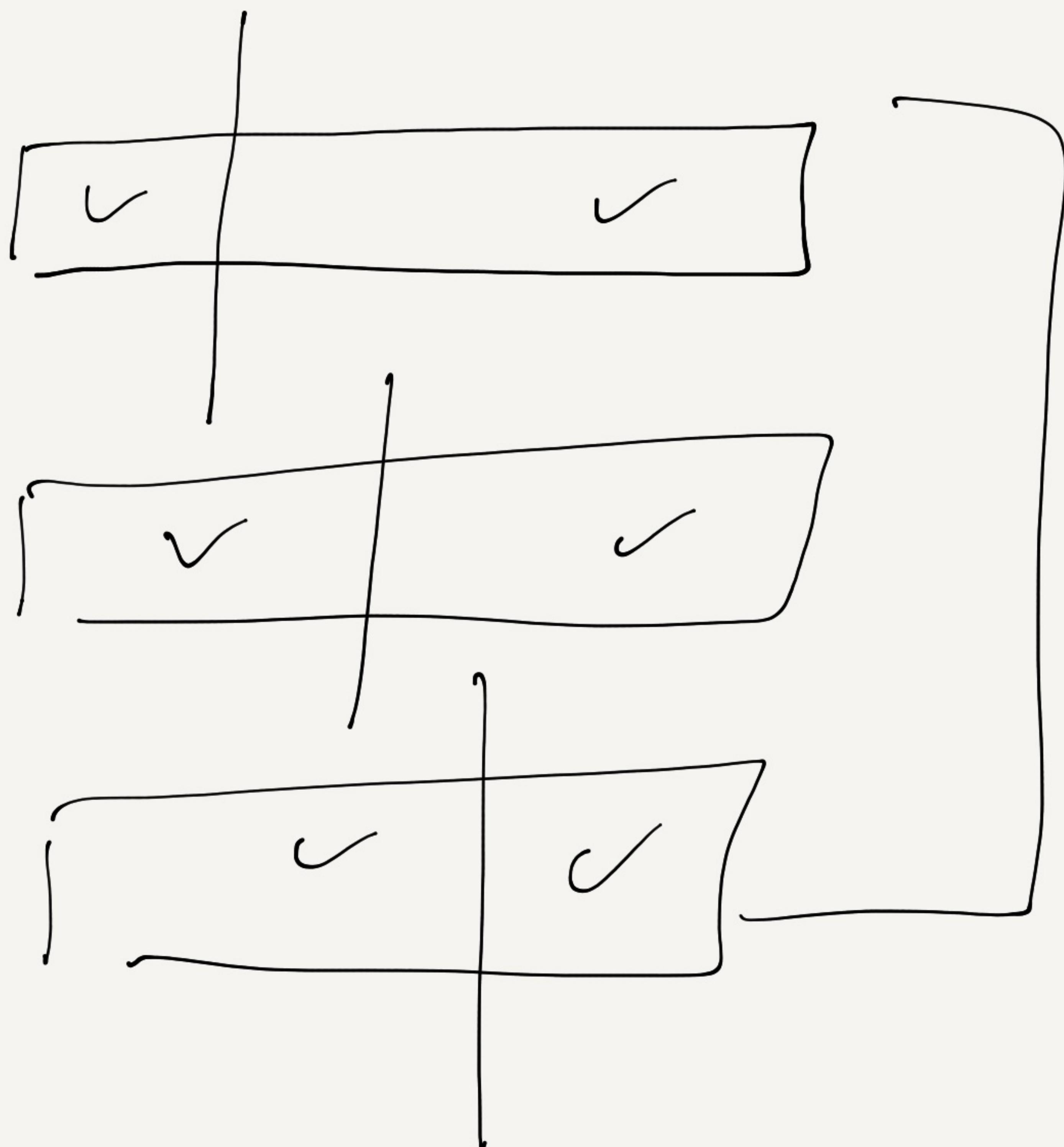
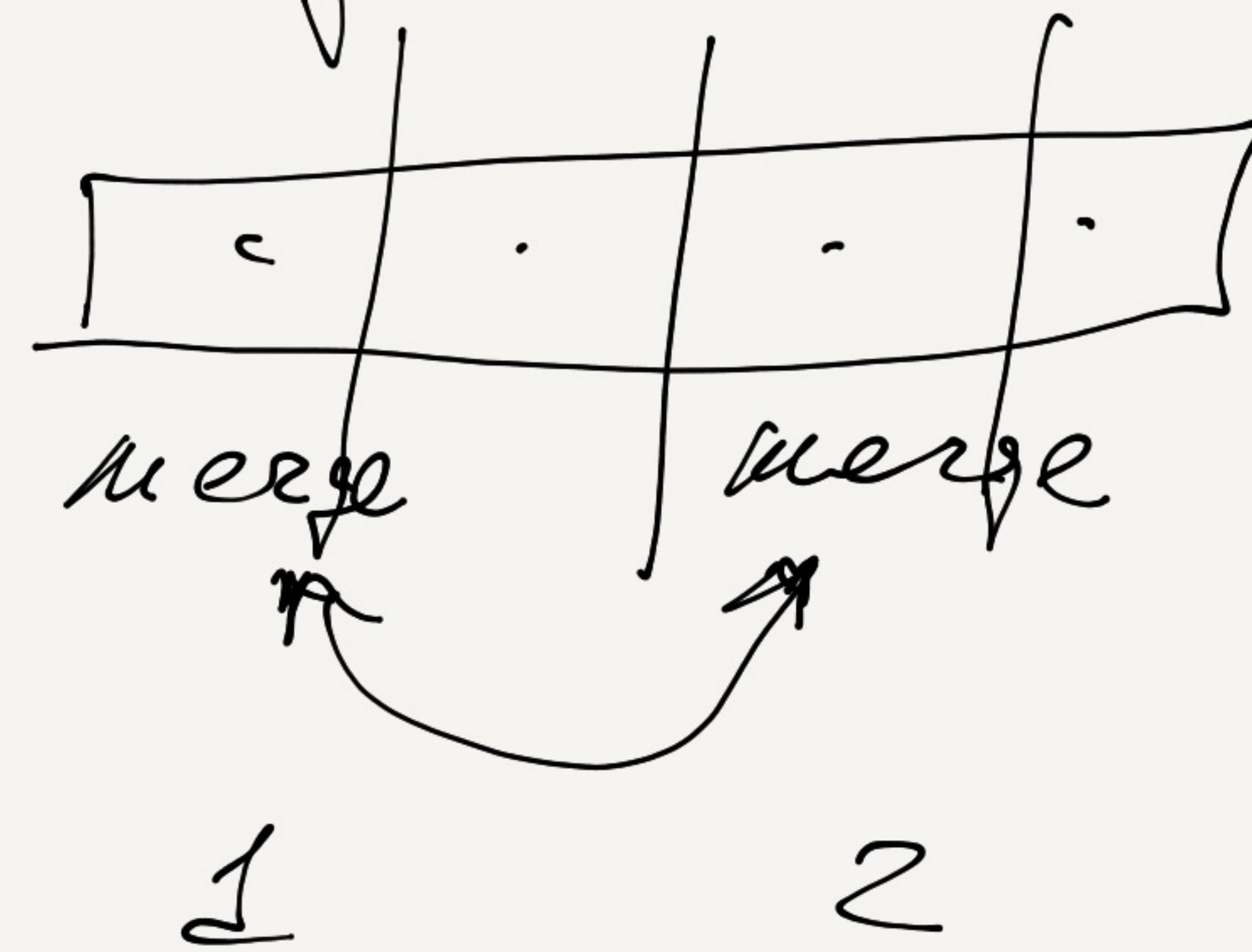
↓  
 1cabacada\$\$  
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

e - maxx. RU  
 O(log n).

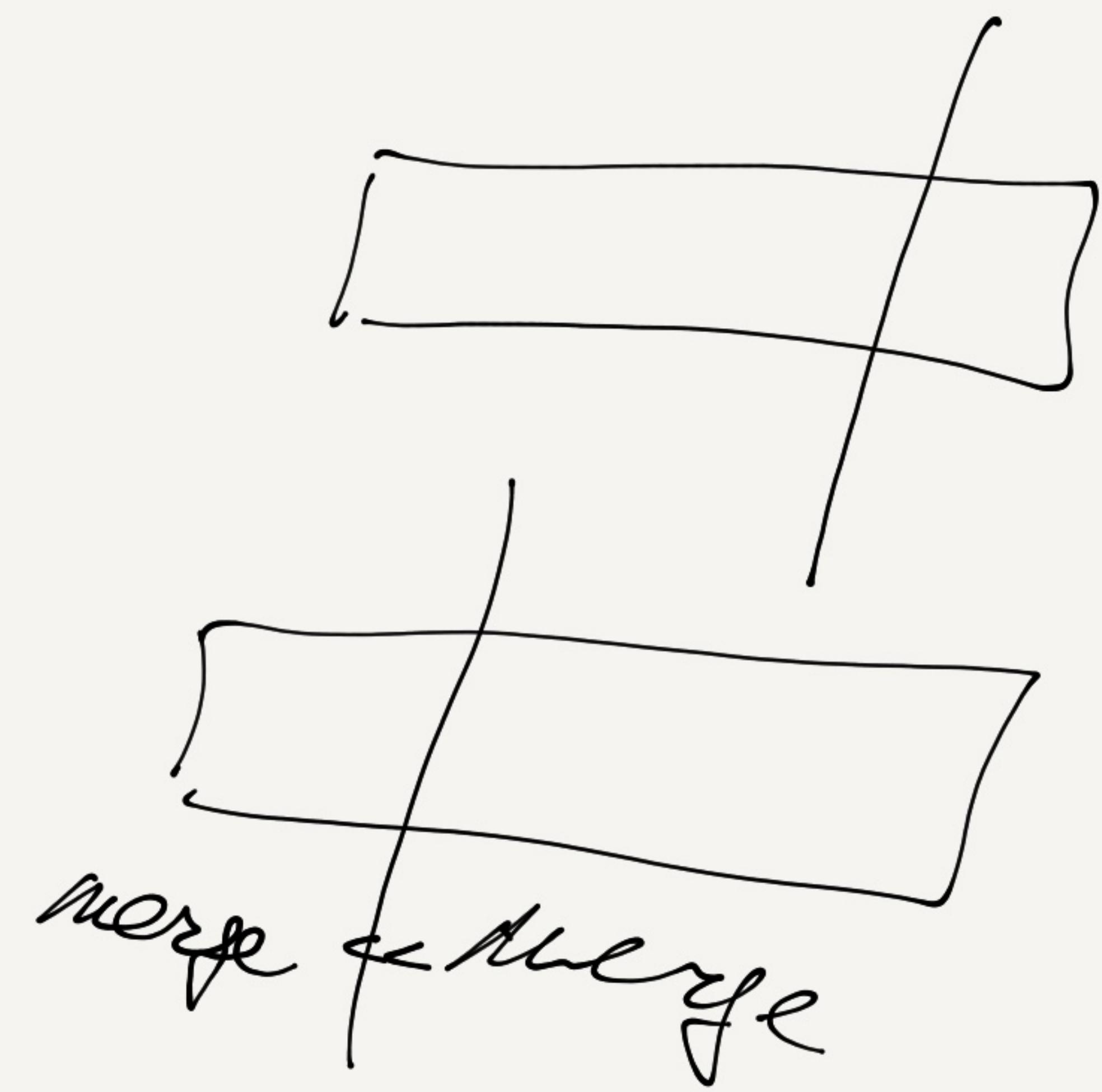
$\Delta \cap$

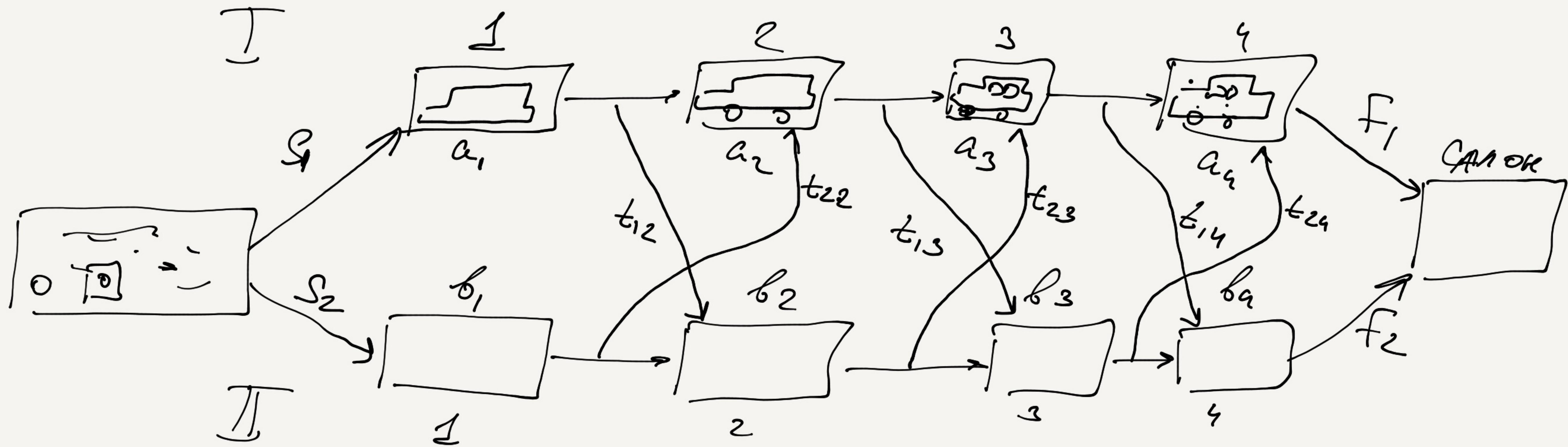
Корней, ГН. 15.

merge sort

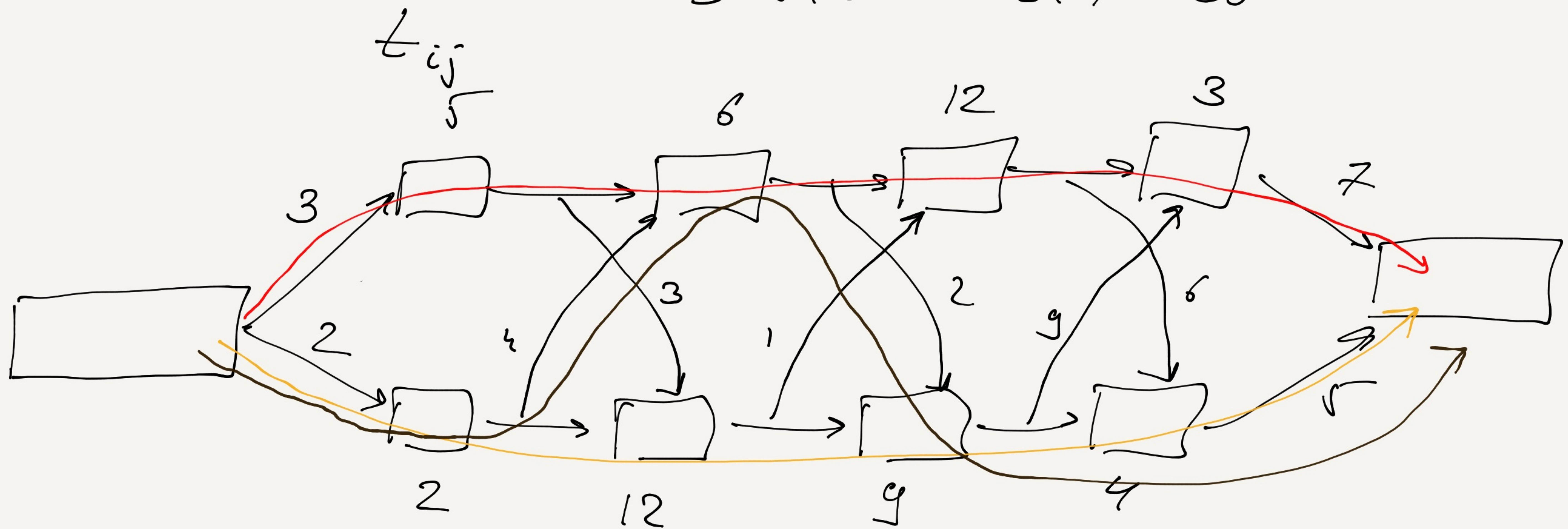


optimal.



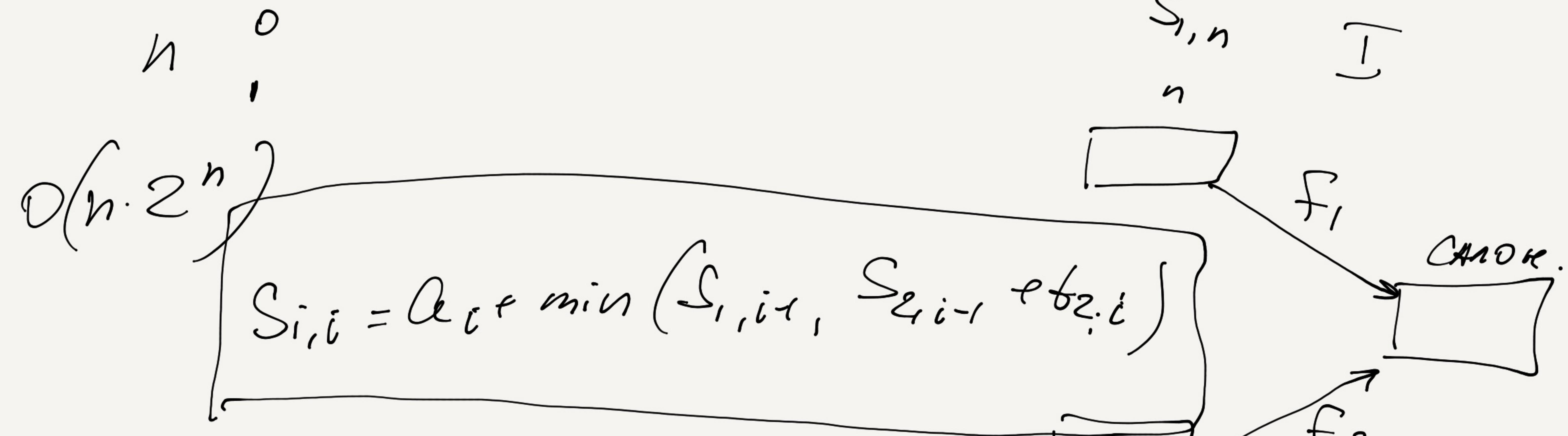


$$3 \times 5 \times 6 \times 12 \times 3 \times 7 = 36$$

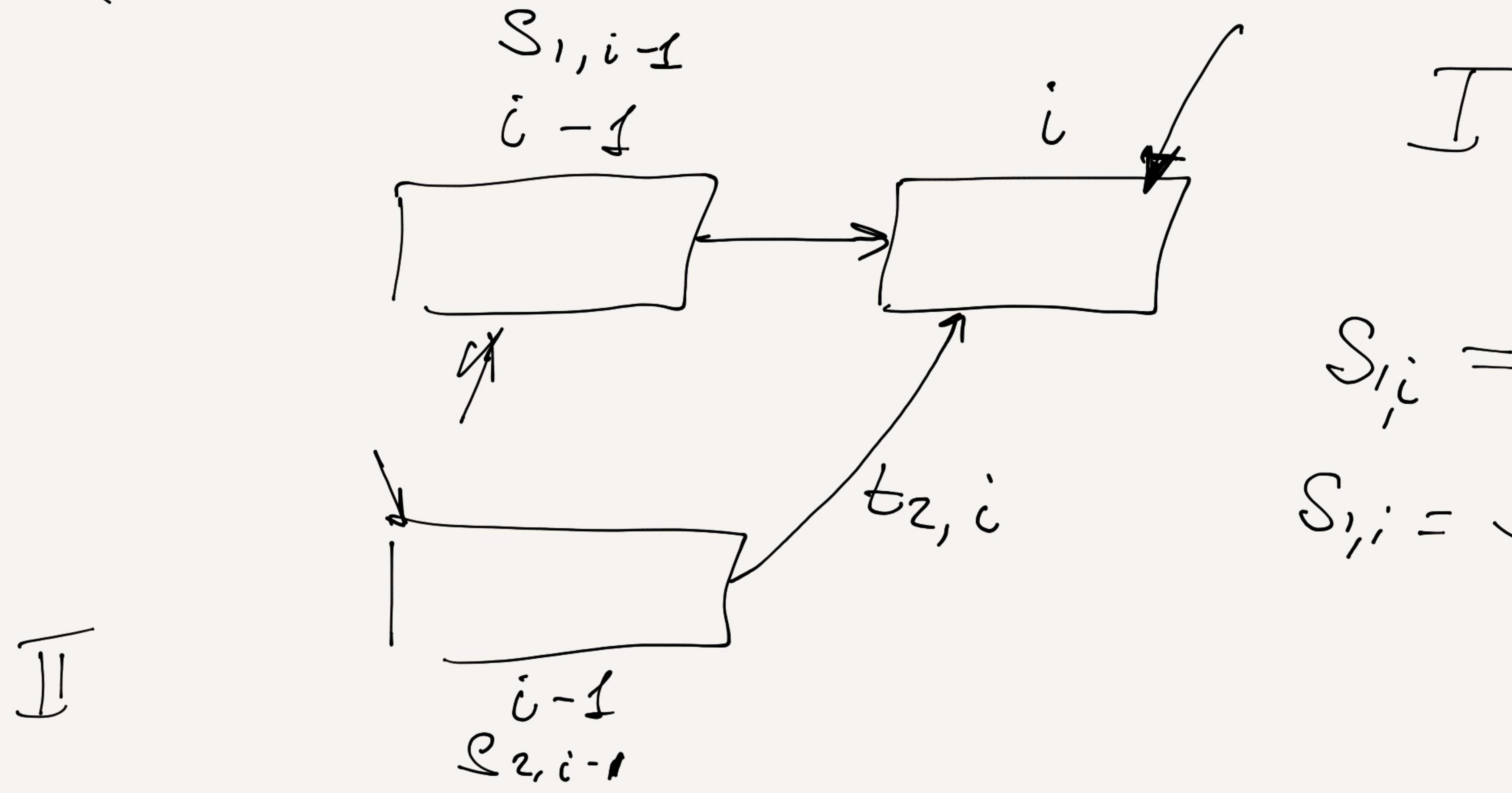


$$2 \times 2 \times 12 \times 3 \times 4 \times 5 = 34.$$

$$2 \times 2 \times 4 \times 6 \times 2 \times 9 \times 4 \times 5 = 34.$$



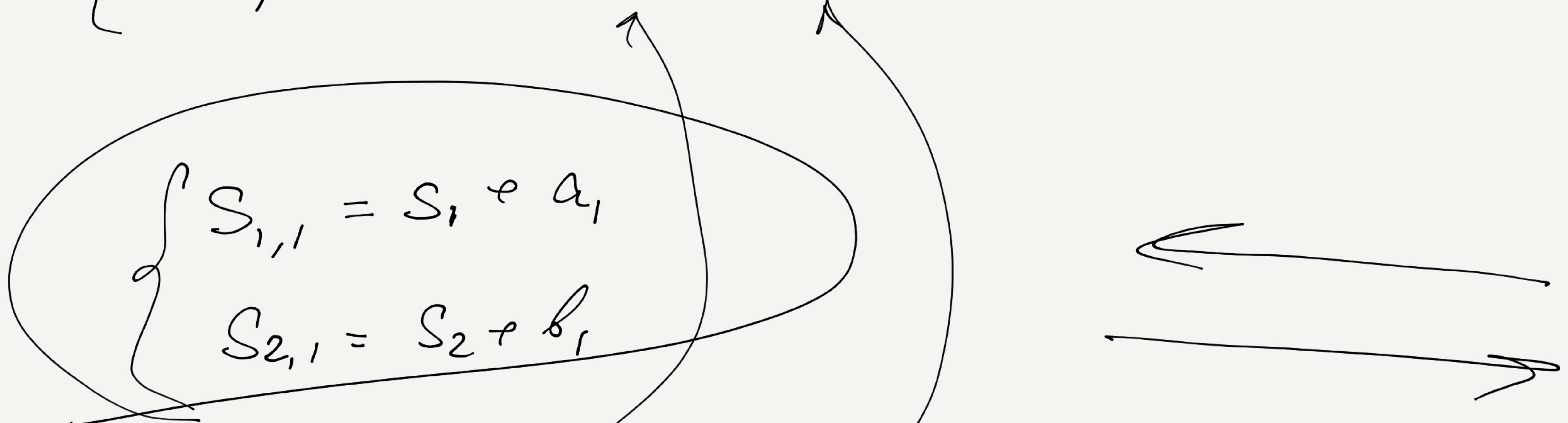
$$\min(S_{1,n} + f_1, S_{2,n} + f_2)$$



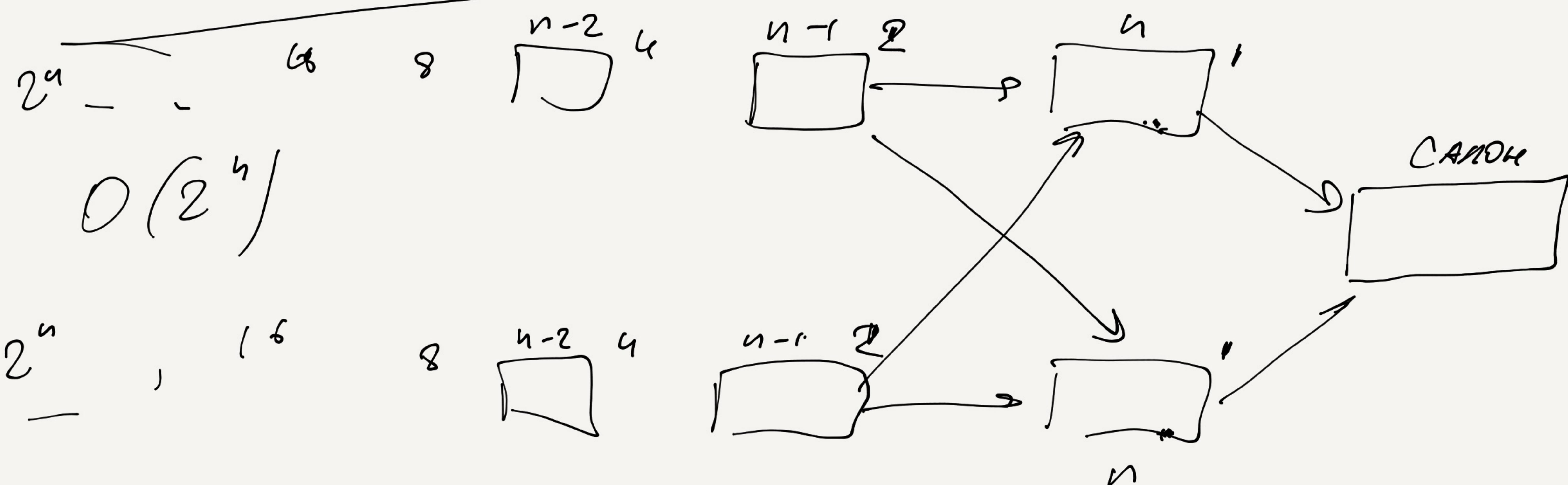
$$S_{1,i} = S_{1,i-1} + a_i$$

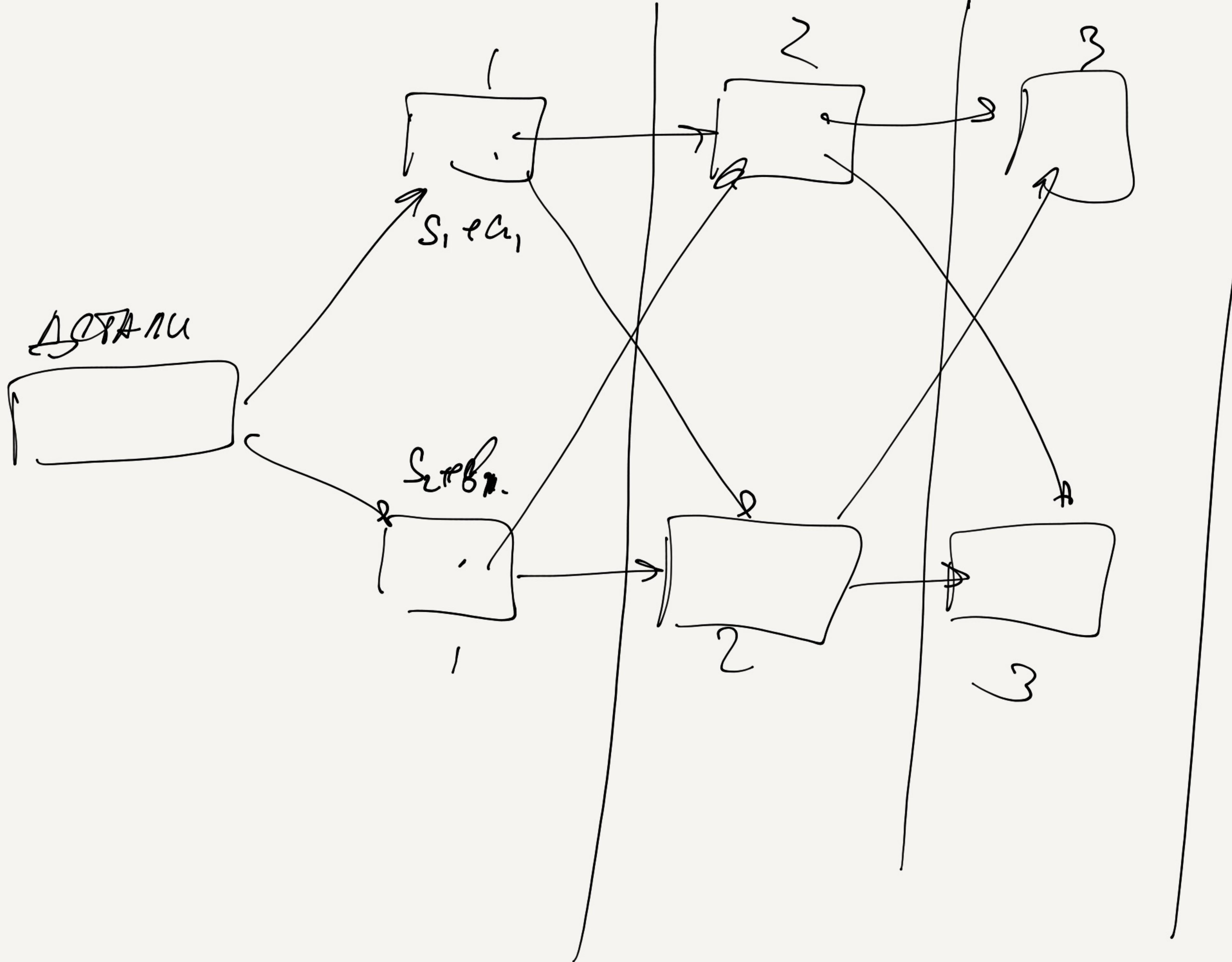
$$S_{1,i} = S_{2,i-1} + t_{2,i} + a_0$$

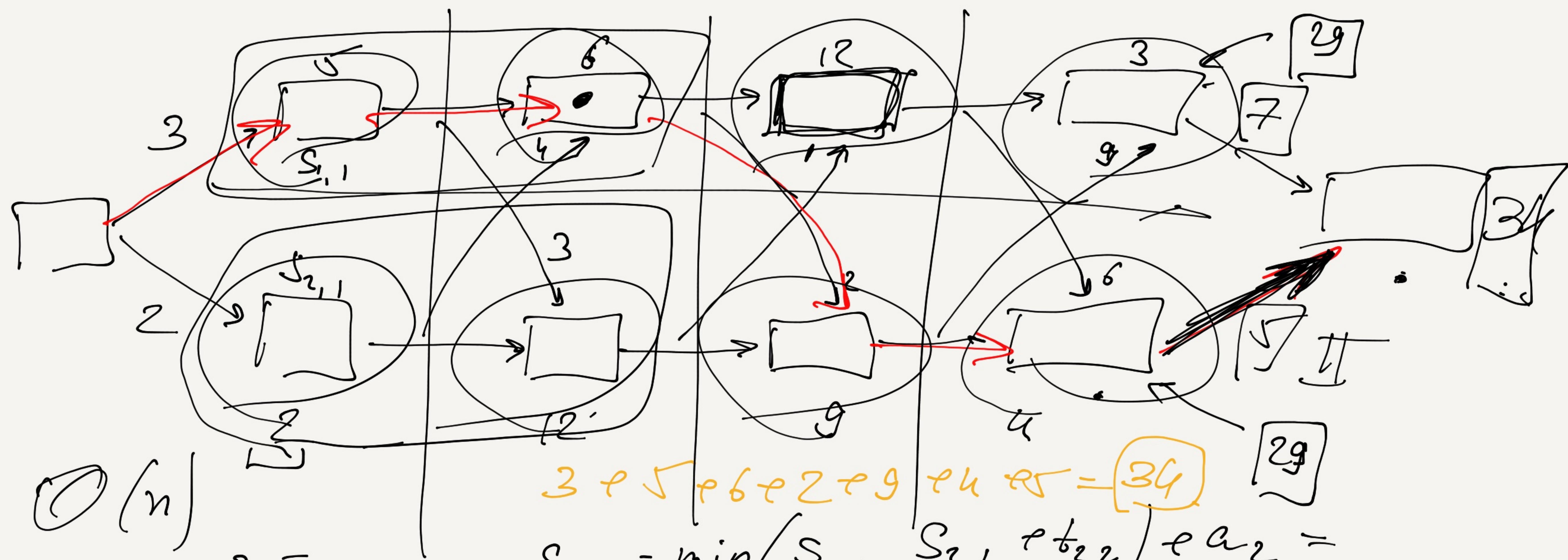
$$\begin{cases} S_{1,i} = a_i + \min(S_{1,i-1}, S_{2,i} + b_{2,i}) \\ S_{2,i} = b_i + \min(S_{2,i-1}, S_{1,i} + b_{1,i}) \end{cases}$$



$$\text{Result} = \min(S_{1,n} + f_1, S_{2,n} + f_2)$$







$O(n)$

$$S_{1,1} = 3 + \sqrt{5} = 8$$

$$S_{2,1} = 2 + 2 = 4$$

$$3 + 5 + 6 + 2 + 9 + 4 = 34$$

$$S_{1,2} = \min(S_{1,1}, S_{2,1} + t_{2,2}) + a_2 =$$

$$= \min(8, 4 + 9) + 6 = 14 \quad \min(\underline{29 + 7}, \underline{29 + 5}) = 34$$

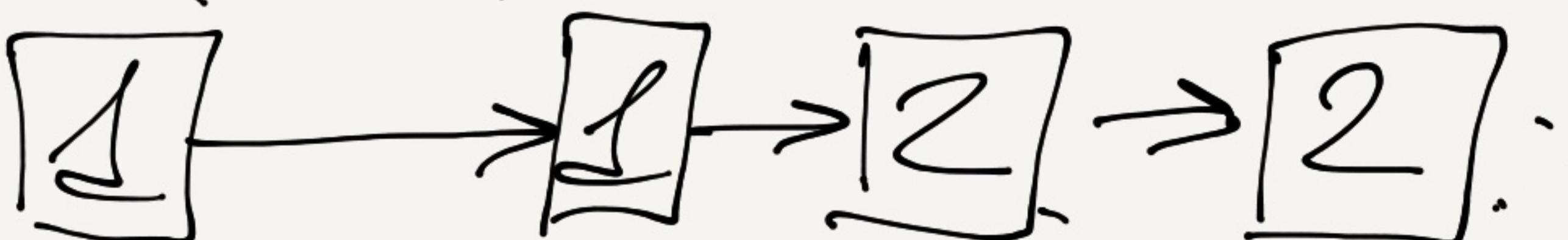
$$S_{2,2} = \min(4, 11) + 12 = 16$$

$$S_{1,3} = \min(14, 16 + 1) + 12 = 26$$

$$S_{2,3} = \min(16, 14 + 2) + 9 = 25$$

$$S_{1,4} = \min(26, 25 + 9) + 3 = 29$$

$$S_{2,4} = \min(25, 26 + 6) + 4 = 29$$



$$\frac{l_{1,2}}{l_{2,2}} = \frac{1}{2}$$

$$l_{1,3} = 1$$

$$\frac{l_{2,3}}{l_{1,4}} = \frac{1}{1}$$

$$\rightarrow \frac{l_{2,4}}{l_{2,4}} = 2$$

$$A_1 \ A_2 \ A_3 \dots \ A_n.$$

$$P_0 \times P_1 \ P_1 \times P_2 \ P_2 \times P_3 \dots \ P_{n-1} \times P_n$$

$$A_i \cdot A_{i+1}$$

$$P_{i-1} \times P_i \quad P_i \times P_{i+1}.$$

$$A_1 \ A_2 \ A_3$$

$$100 \times 10 \quad 10 \times 50 \quad 50 \times 5$$

$$(A_1 \ A_2) A_3$$

$$100 \times 50 \quad 10 \times 5$$

$$100 \cdot 10 \cdot 50 = 50000$$

$$100 \times 50 \cdot 5 = 25000$$

$$A_1 \ A_2 \dots A_{i-1} (A_i A_{i+1}) A_{i+2} \dots A_n$$

$$\downarrow$$

$$A_1 \ A_2 \dots A_{i-1} \overline{A} A_{i+2} \dots A_n.$$

$$P_{i-1} \times P_{i+1}$$

$$P_{i-1} \cdot P_0 \cdot P_{i+1}$$

$$n \rightarrow n-1$$

$$\downarrow$$

$$A_1 \dots i \ A_{i+1} \dots n.$$

$$P_0 \cdot P_i \cdot P_n +$$

$$A_1 (A_2 \ A_3) \quad 10 \times 5$$

$$+ C_1 \dots i \times C_{i+1 \dots n}.$$

$$10 \cdot 50 \cdot 5 = 2500$$

$$100 \cdot 10 \cdot 5 = 5000$$

7500

$A_1 \dots i \ A_{i+1 \dots n}$

$$C_{1,i} + C_{i+1,n} + P_0 \cdot P_i \cdot P_n$$

$$A_1 \cancel{(A_2 \dots A_n)}$$

$$(A_1 A_2) \cancel{(A_3 \dots A_n)}$$

$$(A_1 A_2 A_3) \cancel{(A_4 \dots A_n)}$$

— — — —

$$(A_1 \dots n-1) A_n.$$

Result in

$$\begin{array}{c} \cancel{(C_{1,k})} + \cancel{(C_{k+1,n})} \\ \downarrow \qquad \qquad \qquad \downarrow \\ K \end{array}$$

$\cancel{+ P_0 \cdot P_k \cdot P_n}$

—

$G^G$

$$(A_i A_{i+1}) \cancel{A_{i+2}}$$

$$\left\{ \begin{array}{l} A_1 A_2 , A_2 A_3 , A_3 A_4 \dots A_{n-1} A_n \\ A_1 A_3 , A_2 A_4 , \dots , A_{n-2} A_n \\ A_1 A_4 - A_2 A_5 \\ \hline A_1 A_{n-1} \quad A_2 A_n \end{array} \right.$$

$$\begin{aligned}
 & A, A_2 A_3 A_4 \\
 & ((A, A_2) A_3) / A_4 \\
 & ((A, A_2) / (A_3 A_4)) \cdot \\
 & (A_1 / (A_2 (A_3 A_4))) \\
 & ((A_1 (A_2 A_3)) / A_4) \\
 & (A_1 / (A_2 A_3) A_4)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{A, A_2}{A, A_2 A_3}, \frac{A_2 A_3}{A_2, A_3, A_4}, \frac{A_3 A_4}{A_2, A_3, A_4} \\
 & \frac{A, A_2 A_3}{A, A_2 A_3 A_4}.
 \end{aligned}$$

$$\begin{aligned}
 & \text{min}(-) \quad \text{DPT} \\
 & (A_1 \dots A_i) (A_{i+1} \dots A_n) \\
 & \boxed{P_0 \cdot P_i \cdot P_n} + \frac{C_{i,i}}{\dots} + \frac{C_{i+1,n}}{\dots}
 \end{aligned}$$

$$\begin{aligned}
 & ((A, A_2) A_3) / A_4 \quad (A_1 (A_2 A_3)) / A_4 \\
 & (A_1 \dots 3) A_4.
 \end{aligned}$$

for  $m = 2$  to  $n$ :

for  $i = 1$  to  $n-m+1$ :

for  $j = i$  to  $i+m-1$ :

if  $C_{ij} + C_{j, i+m-1} + P_{i-1} \cdot P_j \cdot P_{i+m-1} < opt$ :  
 $opt = C_{ij} + C_{j, i+m-1} + P_{i-1} \cdot P_j \cdot P_{i+m-1}$ .

else