Vibrational Analysis with a Focus on Phase-Plane Diagrams

[Your Name]

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Contents

1 Part 1: Harmonic Force on the Rocket

1.1 System Sketch and Representation

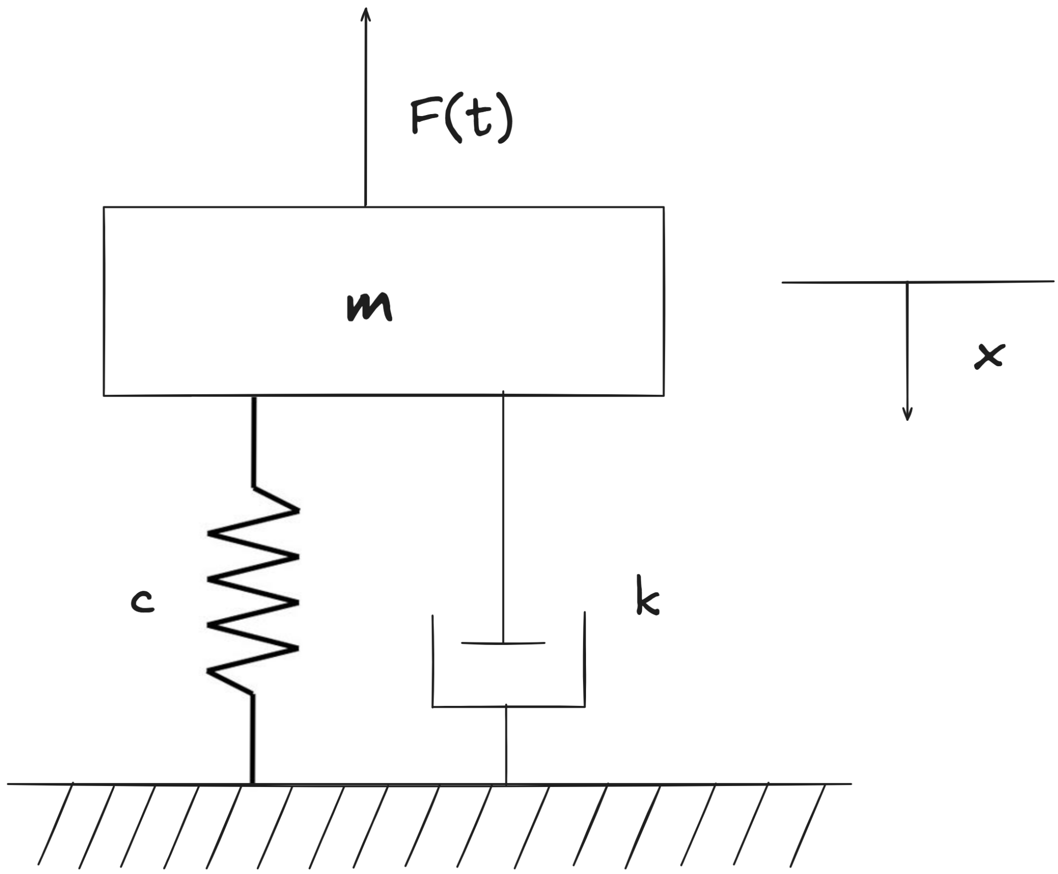

Figure 1: Mass-Spring-Damper System.

Where:

- Spring constant, $c = 20\,\text{N/m}$
- Damping constant, $k = 5\,\text{Ns/m}$
- Mass, $m = 5\,\text{kg}$
- Initial position, $x = 10\,\text{m}$
- Harmonic Force, $F(t) = F_0 \cos(\omega t)$
- Angular frequencies, $\omega = [3, 162, 4.5]\,\text{rad/s}$

1.2 Hand-Drawn Gaussian Plane: Force Estimation

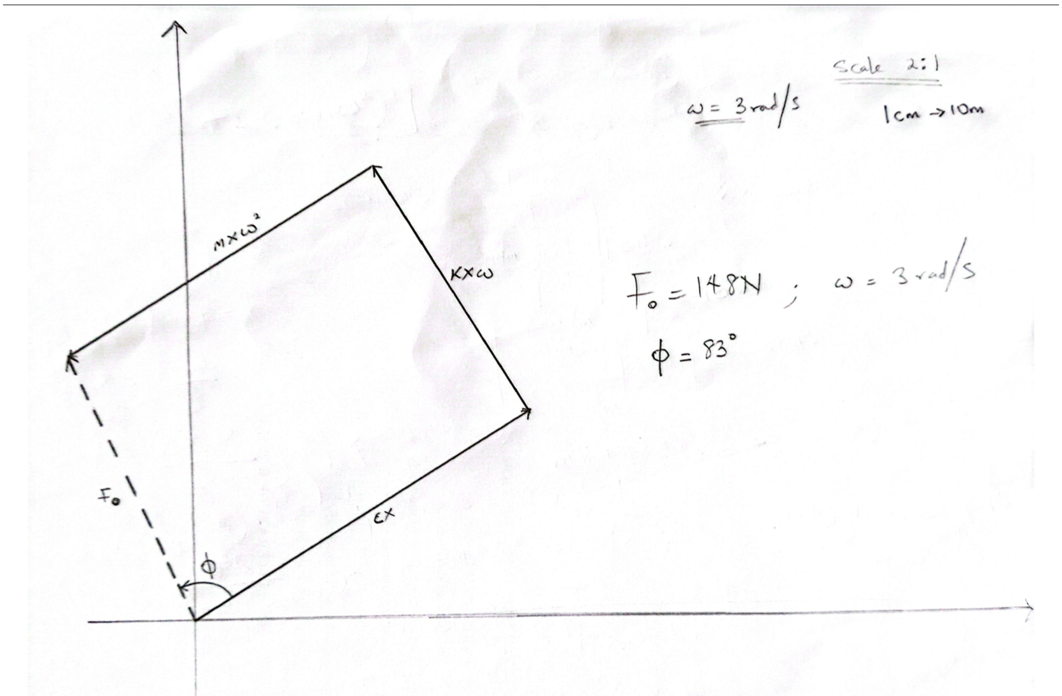

Figure 2: Force estimation for $\omega = 3$ rads/s.

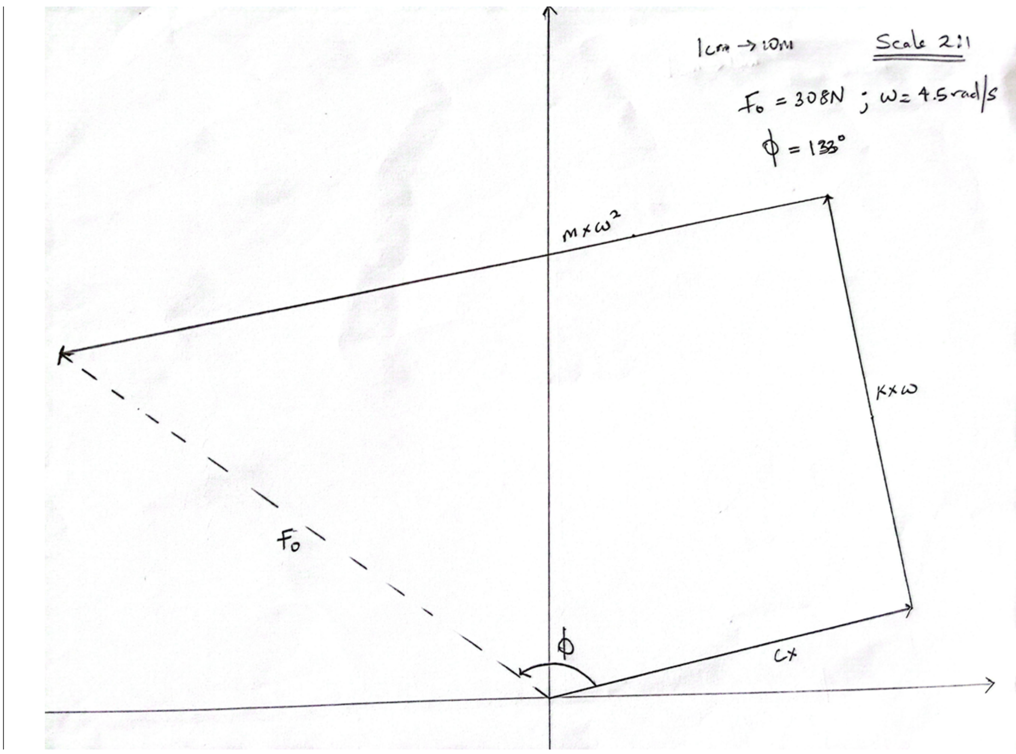

Figure 3: Force estimation for $\omega = 4.5$ rads/s.

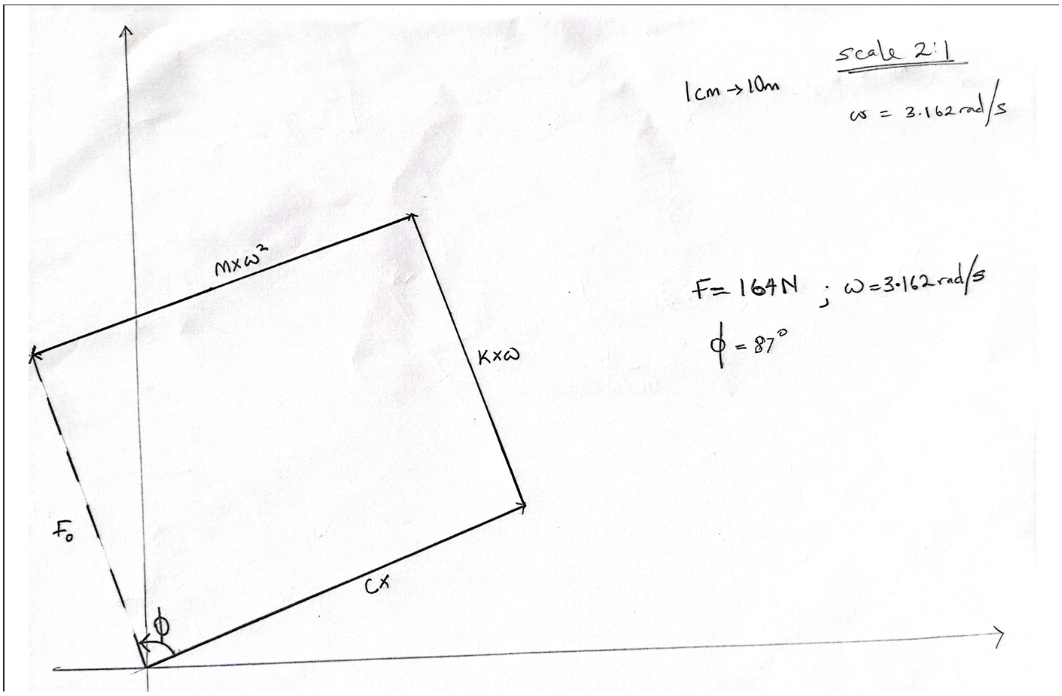

Figure 4: Force estimation for $\omega = 3.162$ rads/s.

Disscusion ...

1.3 Mathematical Comparison of Forces

Parameters:

- Spring constant, $c = 20\,\mathrm{N/m}$
- Damping constant, $k = 5\,\mathrm{Ns/m}$
- Mass, $m = 2\,\mathrm{kg}$
- Initial position, $x = 10\,\mathrm{m}$
- Angular frequencies, $\omega = [3, 162, 4.5]\,\mathrm{rad/s}$

Amplitude:

$$\hat{X} = \frac{F_{\text{exc}}}{\sqrt{(k \cdot \omega)^2 + (c - m \cdot \omega^2)^2}}$$

Phase Angle:

$$\phi = \tan^{-1}\left(\frac{k \cdot \omega}{c - m \cdot (\omega^2)}\right)$$

After substituting:

- $\omega = 3$ rad/s ; F = 151.33N
- $\omega = 3.162$ rad/s ; F = 158.1N
- $\omega = 4.5$ rad/s ; F = 304.38N

Compare results from the calculation to that of the hand-drawn approach ...

1.4 Discussion of the Gaussian Plane Approach

Reflect on how well the graphical estimation matches the mathematical computation ... Discuss the usefulness of the Gaussian plane method in understanding force relationships ...

1.5 Damping Ratio and Curl

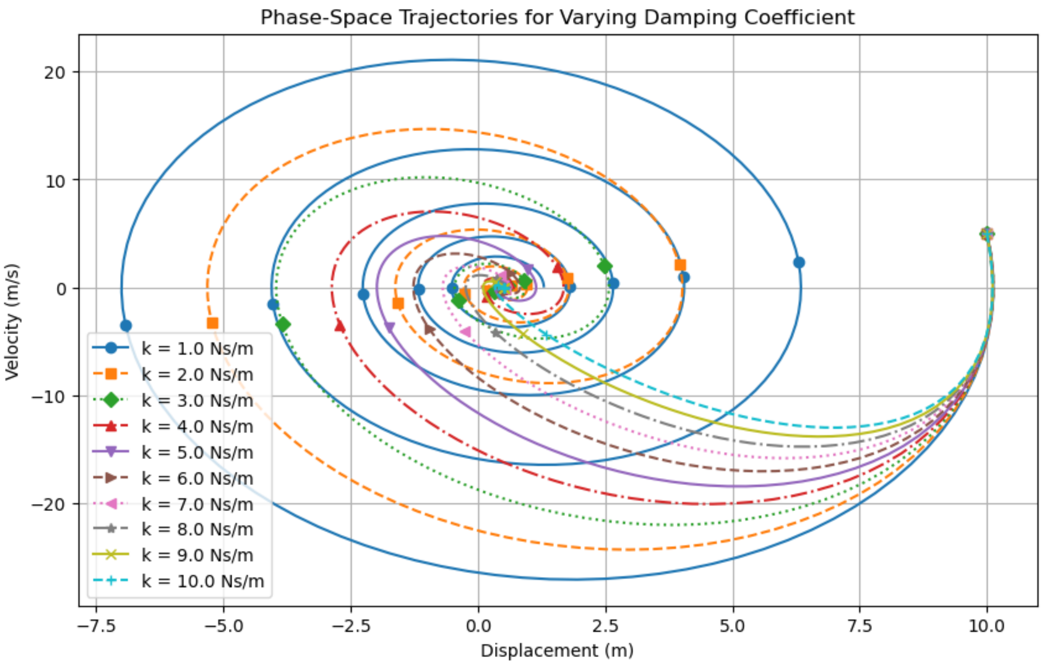

Figure 5: Phase Trajectory for Varying Damping Coefficient

Discussion

1.6 Angular Frequency and Its Effect on the System

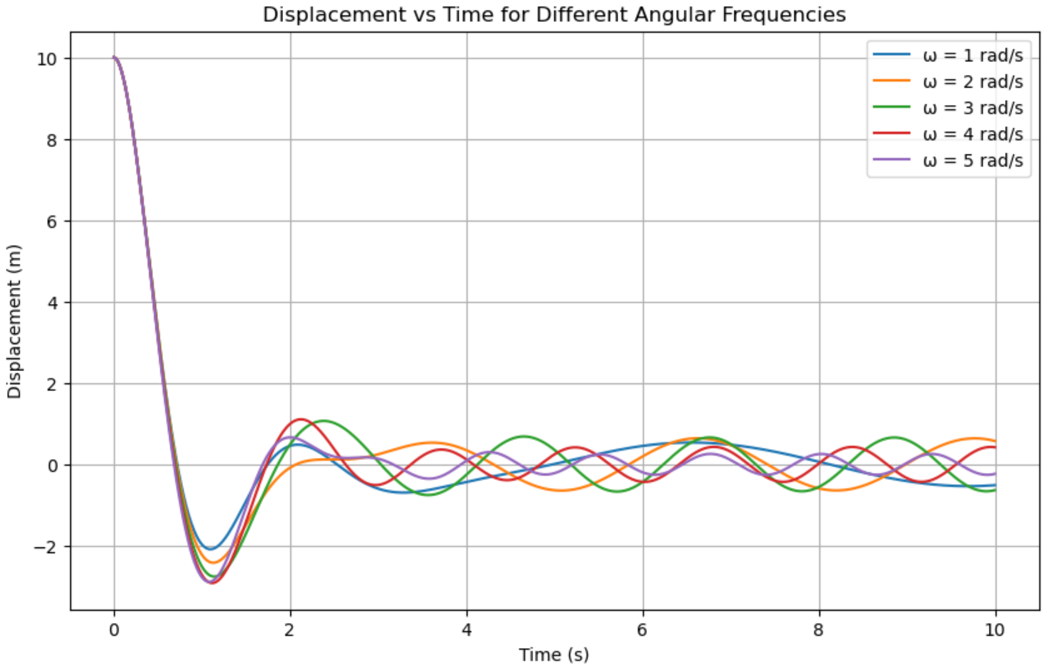

Figure 6: Displacement vs Time for Different Angular Frequencies

Discussion

1.7 Varying the Spring Constant

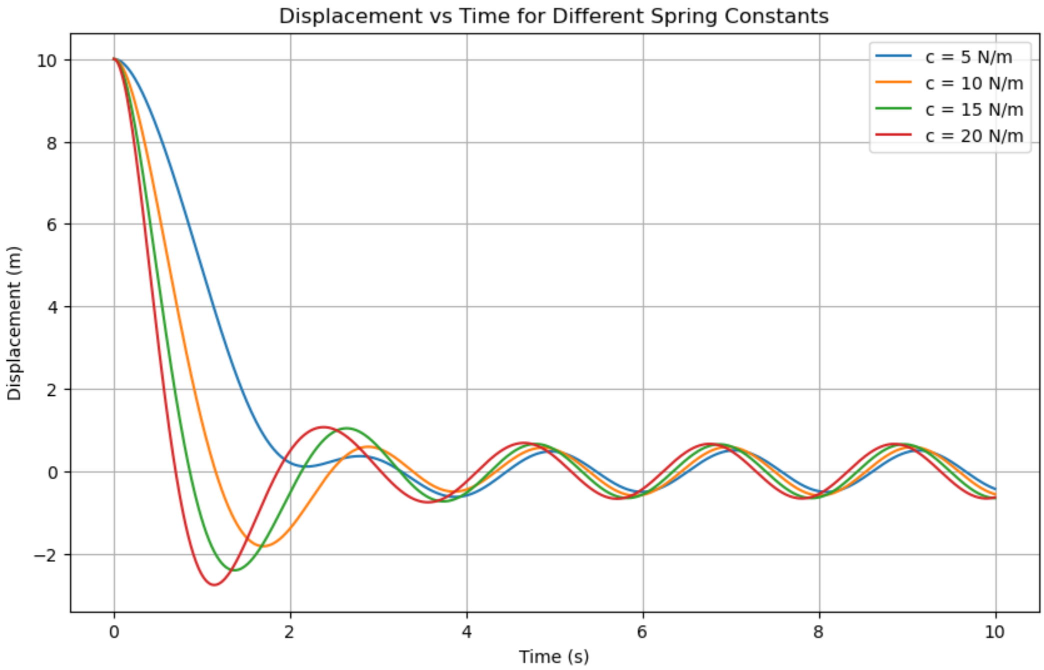

Figure 7: Displacement vs Time for Different Angular Frequencies

Discussion

1.8 Square Wave Analysis

1.8.1 System Responses

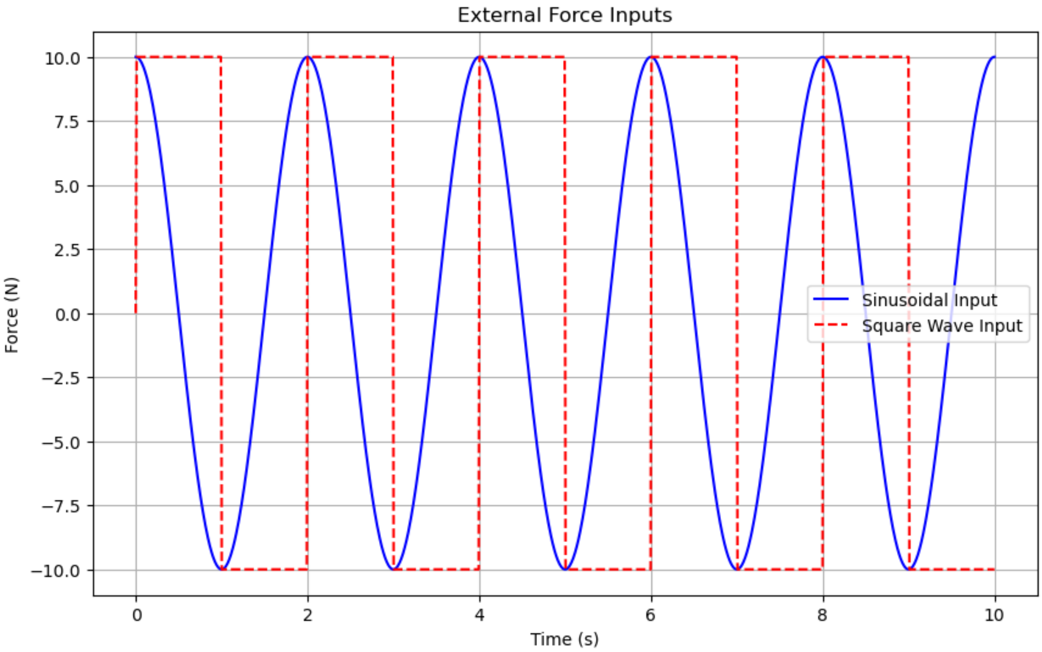

Figure 8: External Force Input

Discussion

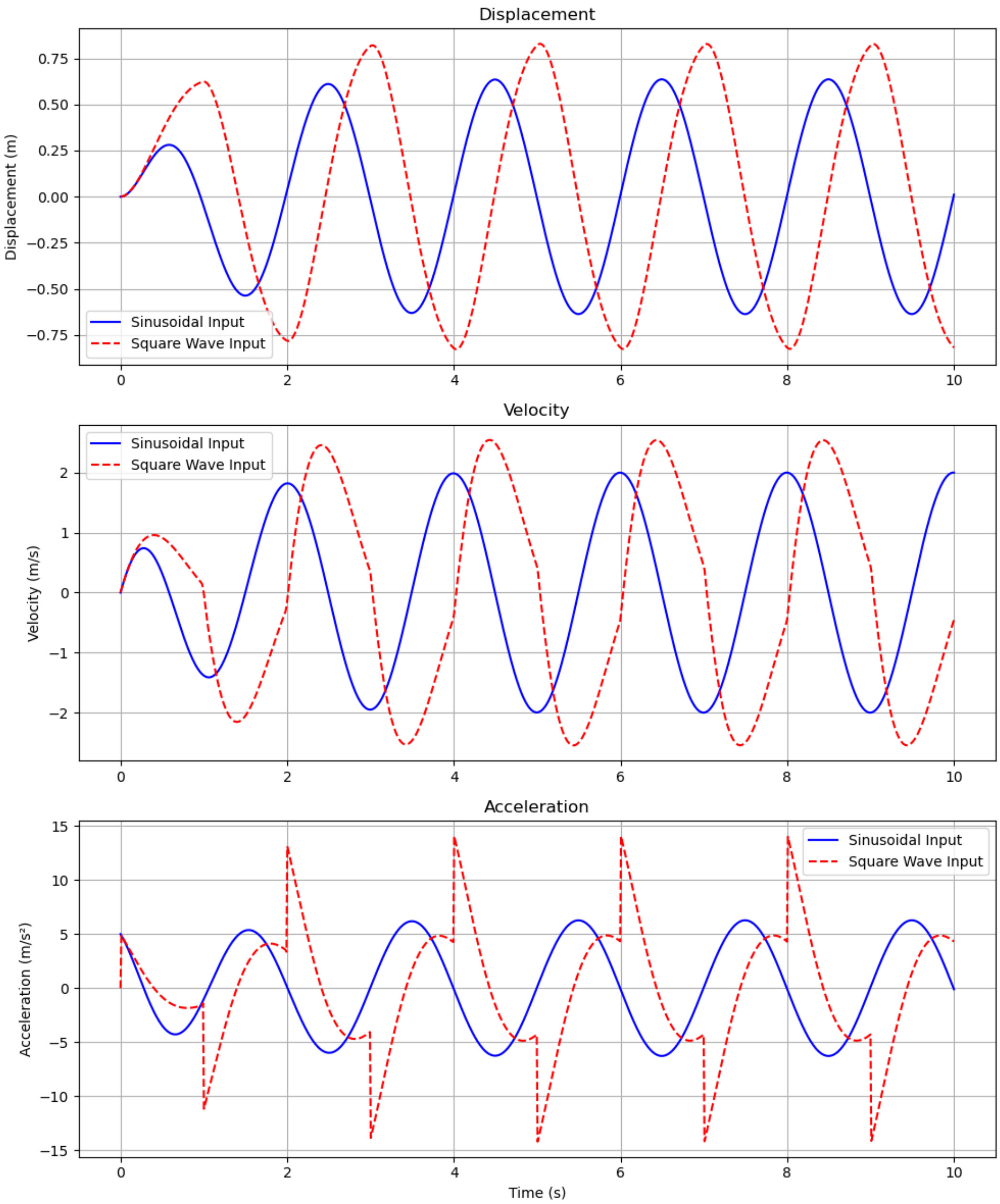

Figure 9: Displacement, Velocity and Acceleration

Discussion

1.8.2 Frequency Domain Analysis

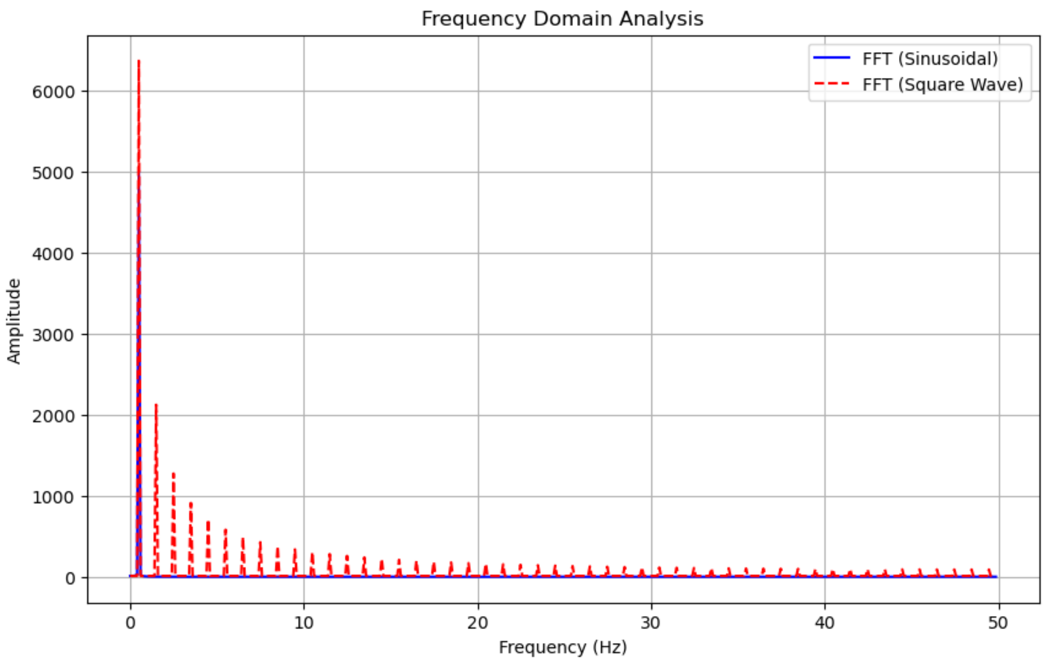

Figure 10: Frequency Domain Analysis

Discussion

2 Part 2: Base Motion in the Rocket

2.1 System Sketch and Representation

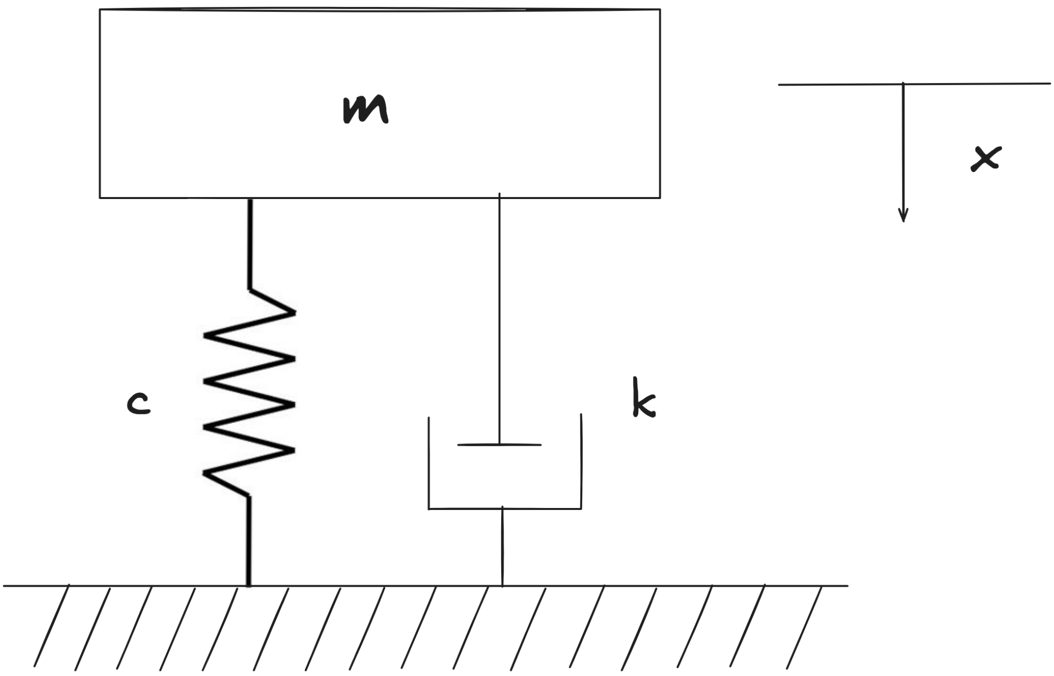

Figure 11: Mass-Spring-Damper System.

Where:

- Spring constant, $c = 20\,\mathrm{N/m}$
- Damping constant, $k = 5\,\mathrm{Ns/m}$
- Mass, $m = 5\,\mathrm{kg}$
- Initial position, $x = 10\,\mathrm{m}$

2.2 Damped and Undamped Responses

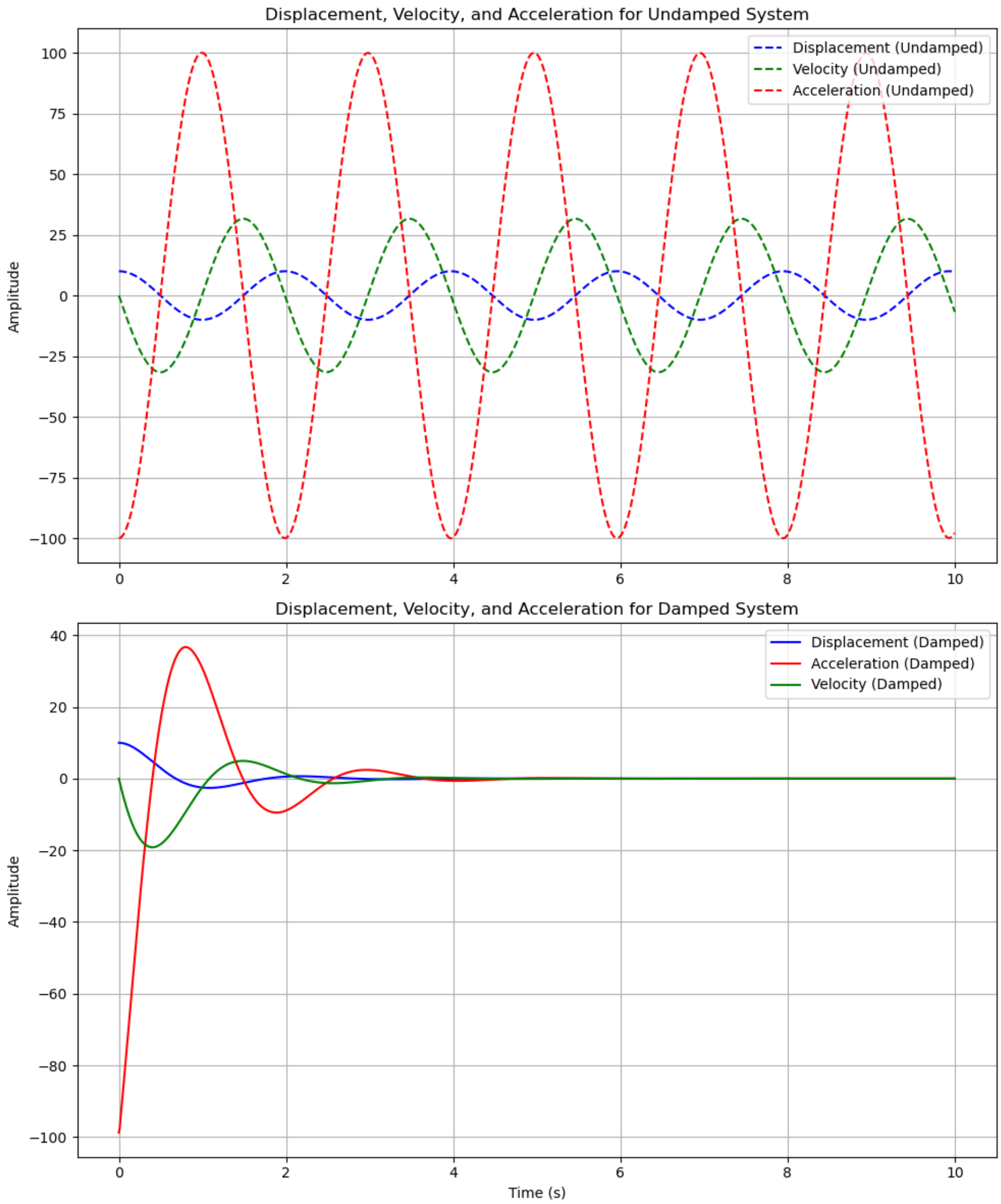

Figure 12: Displacement, Velocity and acceleration for Damped and Undamped system.

Discussion

2.3 Mass Spring Damper with moving Base

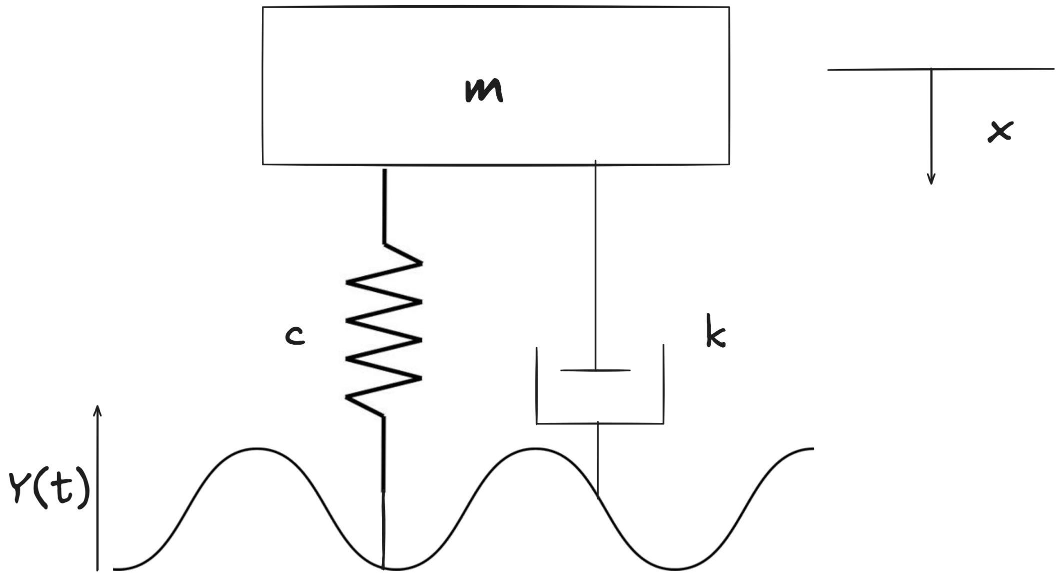

Figure 13: Mass-Spring-Damper system with moving base.

Where:

- Mass, $m = 5\,\mathrm{kg}$
- Spring constant, $c = 20\,\mathrm{N/m}$
- Damping constant, $k = 5\,\mathrm{Ns/m}$
- Initial position, $x = 10\,\mathrm{m}$
- Base Force, $Y(t) = Y_0 \cos(\omega t)$

2.4 Amplitude and Phase Angle

Calculating Amplitude and Phase Angle:

Given:
- Base Excitation Force:

$$Y(t) = Y_0 \cos(\omega t)$$
$$\dot{Y}(t) = -\omega Y_0 \sin(\omega t)$$

- Angular Frequency:

$$\omega = \frac{2\pi}{l}$$

Given:

- $V = 2000\,\mathrm{m/s}$
- $l = 1000\,\mathrm{m}$

$$\omega = 12.566\,\mathrm{rad/s}$$

The system model is given by:

$$m\ddot{x} + k\dot{x} + cx = y(t)$$

where:

- m = Mass
- k = Damping Constant
- c = Spring Constant
- x = Displacement
- $\dot{x}$ = Velocity
- $\ddot{x}$ = Acceleration
- $Y(t) = Y\cos(\omega t)$

From Newton's Second Law, $\sum F = ma$, we have:

$$m\ddot{x} + k(\dot{x} - \dot{Y}) + c(x - Y) = 0$$
$$m\ddot{x} + k\dot{x} - k\dot{Y} + cx - cY = 0$$
$$m\ddot{x} + k\dot{x} + cx = cY + k\dot{Y}$$

Substituting for y and $\dot{y}$, we have:

$$m\ddot{x} + k\dot{x} + cx = cY\cos(\omega t) - k\omega Y\sin(\omega t)$$

We know that:

$$x = X_0\cos(\omega t - \phi) \quad \text{(eq. 1)}$$
$$\dot{x} = -\omega X_0\sin(\omega t - \phi) \quad \text{(eq. 2)}$$
$$\ddot{x} = -\omega^2 X_0\cos(\omega t - \phi) \quad \text{(eq. 3)}$$

Substituting eq. (1), eq. (2), and eq. (3) into the system model, we have:

$$m(-\omega^2 X_0\cos(\omega t - \phi)) + k(-\omega X_0\sin(\omega t - \phi)) + c(X_0\cos(\omega t - \phi)) = cY\cos(\omega t) - k\omega Y\sin(\omega t)$$

After Phasor representation, we have:

$$\begin{aligned} F_{\text{res1}}^2 &= cY^2 + kY\omega^2 \\ &= Y^2(c^2 + k^2\omega^2) \end{aligned}$$

$$\begin{aligned} F_{\text{res2}}^2 &= (kX_0\omega)^2 + (cX_0 - mX_0\omega)^2 \\ &= X_0^2\left[(k\omega)^2 + (c - m\omega^2)^2\right] \end{aligned}$$

Equating F_{res1}^2 and F_{res2}^2, we have:

$$Y^2(c^2 + k^2\omega^2) = X_0^2\left[(k\omega)^2 + (c - m\omega^2)^2\right]$$

Equations for X_0 and ϕ, we have:

$$X_0 = Y_0\sqrt{\frac{c^2 + (k\omega)^2}{(k\omega)^2 + (c - m\omega^2)^2}}$$

$$\phi = \tan^{-1}\left(\frac{m\omega^3 k}{c^2 - m\omega^2 c + k^2\omega^2}\right)$$

Solving for X_0 and ϕ:
Given:

- $m = 2\,\text{kg}$
- $k = 5\,\text{N/m}$
- $c = 20\,\text{Ns/m}$
- $Y_0 = 150\,\text{m}$
- $\omega = 12.566\,\text{rad/s}$

We have:

$$X_0 = 151\sqrt{\frac{20^2 + (5 \times 12.566)^2}{(5 \times 12.566)^2 + (20 - 2 \times (12.566)^2)^2}}$$

$$\phi = \tan^{-1}\left(\frac{2 \times (12.566)^3 \times 5}{20^2 - 2 \times (12.566)^2 \times 20 + 5^2 \times (12.566)^2}\right)$$

Thus:

$$X_0 = 32.92\,\text{m}$$
$$\phi = -84.334\,\text{rad/s} \quad (95.67^\circ)$$

2.5 State Space Representation

State-Space Representation:

$$\dot{X} = \underset{\rightarrow}{A}\vec{x} + \underset{\rightarrow}{B}\vec{y}$$

Introduce the state variables:

$$x_1 = x, \quad x_2 = \dot{x}$$

This gives:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \ddot{x}$$

From the differential equation:

$$m\ddot{x} = -k\dot{x} - cx + k\dot{Y} + cY$$

Re-arranging for $\ddot{x}$:

$$\dot{x}_2 = -\frac{c}{m}x_1 - \frac{k}{m}x_2 + \frac{k}{m}\dot{Y} + \frac{c}{m}Y$$

Now, the state-space form becomes:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{k}{m} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ \frac{c}{m} & \frac{k}{m} \end{bmatrix}}_{B} \begin{bmatrix} Y \\ \dot{Y} \end{bmatrix}$$

Here:

- A is the system matrix.
- B accounts for the influence of $Y(t)$ and $\dot{Y}(t)$, the base motion and its velocity.

2.6 Impact of Base Excitation Amplitude

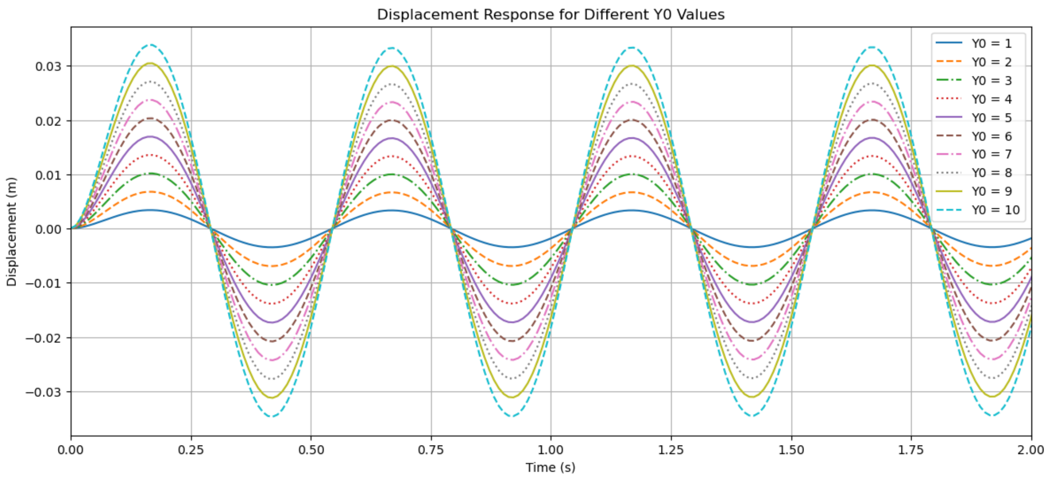

Figure 14: Displacement Response for different Y0 values

Discussion

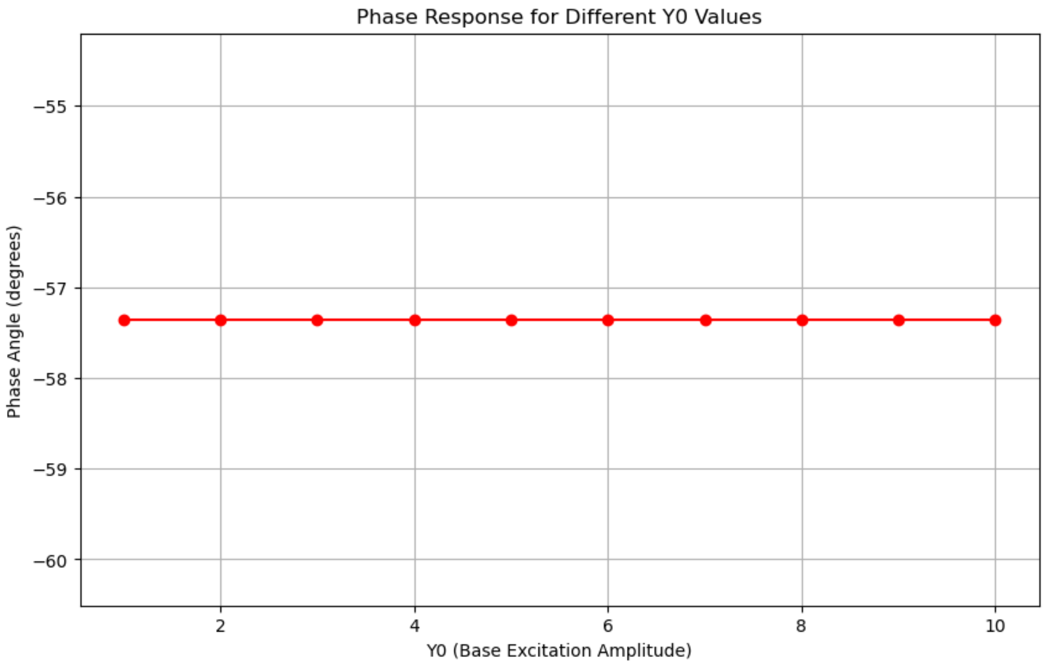

Figure 15: Phase Response for different Y0 values

Discussion ...