HOLIDAY PROJECT

Deadline: January 7th, 2025

<u>Note: All systems are initialized with a velocity of 0m/s only if otherwise stated</u>

Project: Vibrational Analysis with a Focus on Phase-Plane Diagrams (Use graphs for the hand-drawn diagrams)

Part 1: Harmonic Force on the Rocket

Scenario

The rocket experiences periodic oscillations from engine vibrations. The external harmonic force is:

$$F(t) = F_0 cos(\omega t)$$

Tasks

1. **System Sketch and Representation**
   - Draw the mass-spring-damper system. Clearly label:
     - Mass (m) = 2kg
     - Spring (c) = 20N/m
     - Damper (k) = 5Ns/m
     - Initial Position (x) = 10m
     - Harmonic force $F(t)$
2. **Hand-Drawn Gaussian Plane: Force Estimation**
   - Represent the forces as vectors on the Phasor plane
   - Estimate the resultant force (F_0) when the angular frequency is 3 rad/s, 4.5 rad/s and 3.162 rad/s
   - Use trigonometry to estimate the **phase angle (ϕ, phi)** between the resultant force and the spring force amplitudes on the Gaussian plane using a protractor.
   - What do you observe, is there any notable change in the system? Document all observations and changes. Plot the phase angle (ϕ), and the resultant force (F_0) against the angular frequency (ω) and describe the trend.
   - Assume that the system is considered at a unit time.
3. **Mathematical Comparison of Forces**
   - Calculate the Amplitude and phase angle analytically using the mathematical equations for harmonic excitation.
   - Compare the results (phase angle) obtained from the hand-drawn phasor plane with the computed values, how does this broaden your knowledge on precision and accuracy.
4. **Discussion of the Gaussian Plane Approach**
   - Reflect on how well the graphical estimation matches the mathematical computation.
   - Discuss the usefulness of the Gaussian plane method in understanding force relationships.
5. **Damping ratio and curl**
   Let the damper vary from **1 to 10 Ns/m**. Plot the phase-space trajectory for varying the damping coefficient of the system. Preferably code and plot this on any desired software. Using multiple plots here is fine (note the system here is transient). **For this question, the initial velocity is 5 m/s**

Hint: *To plot the phase space system trajectory of the system, export your MATLAB variables and then, let the horizontal axis x(t) be the displacement values and the vertical axis be the velocity*

6. **Angular frequency (ω) and the effect on the system**
   Let the system's angular frequency vary from **1 to 5**, take time steps of your choice, and then show the plot of the different changes this brings to the system. (the system here is under steady conditions)

7. **Varying the spring constant (c)**
   Let the system's spring constant vary from **5 to 20 N/m**, take time steps of your choice, and then show the plot of the different changes this brings to the system. (the system here is under steady conditions)

8. Replace F(t) with a square wave using software, you can even use MATLAB Simulink a block simply works
   Let T = 2 seconds.
   $$f(t) = \begin{cases} +F_0, & x \le t < T/2 \\ -F_0, & \frac{T}{2} \le t < T \end{cases}$$
   Compare the system's displacement, velocity, and acceleration responses to sinusoidal wave inputs. Explain what you understand.
   a. Are there discontinuities in the square wave, what do you think is the reason behind this?
   b. Use the MATLAB fast Fourier transform function (FFT) to analyse the frequency content of the sinusoidal and the square wave or if you are coding in python, there is a function to analyse the frequency domain signals.
   c. What do you observe in the frequency domain

Note that the clarity of your graph is important (5 plots or steps is just fine), I do not need your code too, code can be copied 'after all GPT can be used to write codes' (trust me, if you copy even if chat GPT is used, chances remain that I will find out) and the biggest penalty will be on plagiarism ***. Do not copy anybody, let your explanations be yours***.

Part 2: Base Motion in the Rocket

Scenario

The rocket experiences turbulence-induced base excitation during re-entry, and this is modeled as:

$$Y(t) = Y_0 cos(\omega t)$$

The angular frequency (ω) of the system is obtained from the formulae ($V\frac{2\pi}{l}$)

Where V (speed of the rocket relative to the air) = 2000 m/s and l (characteristic length scale of the disturbances caused by turbulence) = 1000m

Tasks

1. System sketch and Representation (use the values of the mass, spring and damper of the system in Part 1)
2. Do not consider the excitation force, show a plot of the system as when it is damped and undamped
3. Draw the mass-spring-damper system with a moving base Y(t)
4. Find the amplitude and phase angle of the system using the mathematical equations
5. Give a state space representation of the system
   $$\dot{X} = Ax + Bu$$
6. Vary Y_0 of the system from 1 to 10, using a desired time step, how does increasing the amplitude of the base excitation affect the system's displacement and phase response? Plot graphs showing these properties.

Summary:

1. Plagiarism will be gravely penalized
2. Ensure your plots are very clear and properly labelled
3. Give a clear section to detailed explanation of your answers, make them as clear and explanatory as possible
4. Relate your results to real-world scenarios.

All the Best! A Merry, Merry Christmas in advance.