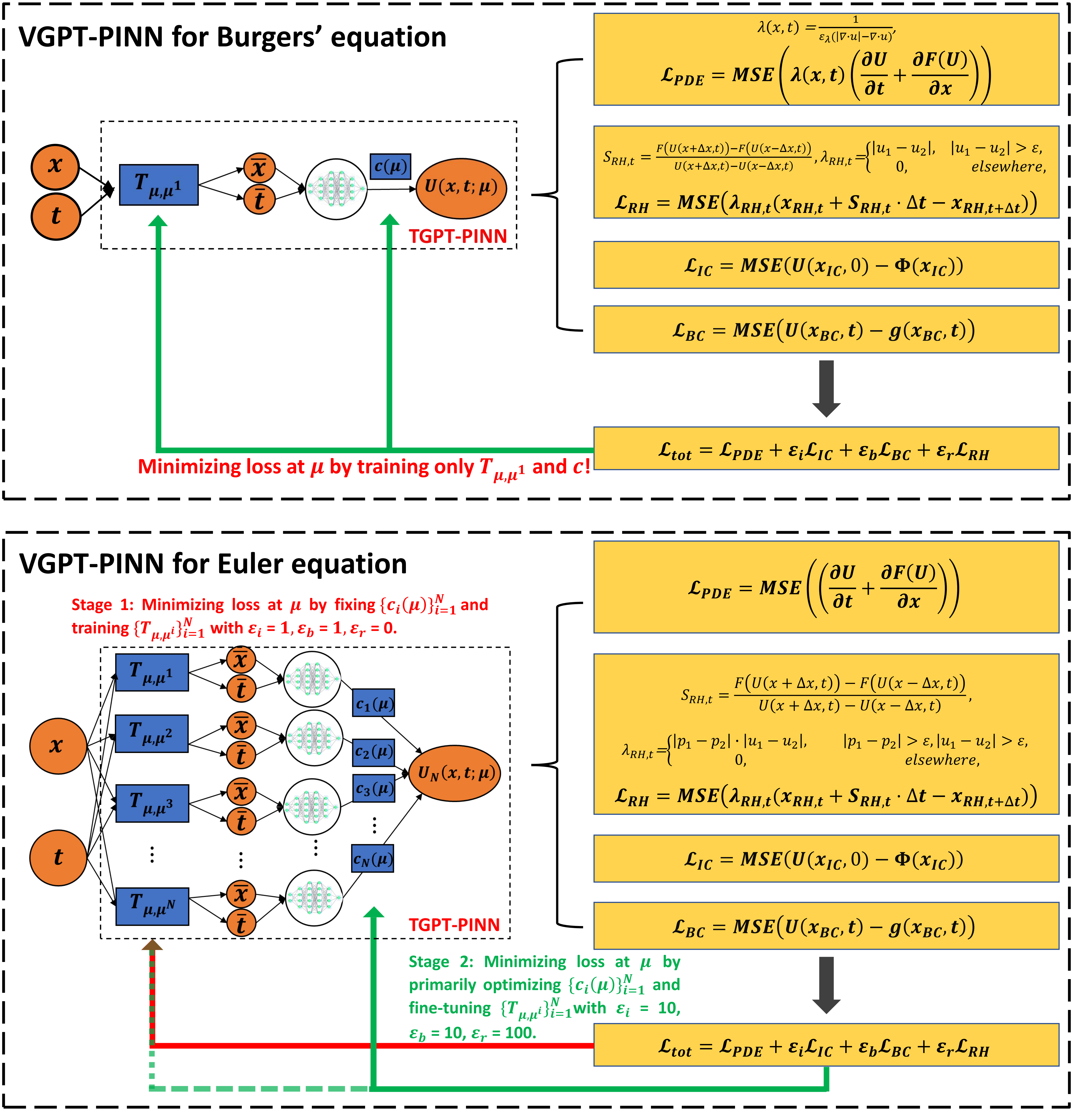

VGPT-PINN for Burgers' equation
$\lambda(x,t) = \frac{1}{\varepsilon_\lambda(|\nabla\cdot u| - \nabla\cdot u)}$,
$\mathcal{L}_{PDE} = MSE\left(\lambda(x,t)\left(\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x}\right)\right)$
$S_{RH,t} = \frac{F(U(x+\Delta x,t)) - F(U(x-\Delta x,t))}{U(x+\Delta x,t) - U(x-\Delta x,t)}$, $\lambda_{RH,t} = \begin{cases} |u_1 - u_2|, & |u_1 - u_2| > \varepsilon, \\ 0, & elsewhere, \end{cases}$
$\mathcal{L}_{RH} = MSE(\lambda_{RH,t}(x_{RH,t} + S_{RH,t} \cdot \Delta t - x_{RH,t+\Delta t}))$
$\mathcal{L}_{IC} = MSE(U(x_{IC}, 0) - \Phi(x_{IC}))$
$\mathcal{L}_{BC} = MSE(U(x_{BC}, t) - g(x_{BC}, t))$
$\mathcal{L}_{tot} = \mathcal{L}_{PDE} + \varepsilon_i \mathcal{L}_{IC} + \varepsilon_b \mathcal{L}_{BC} + \varepsilon_r \mathcal{L}_{RH}$
x
t
T_{μ,μ^1}
$\bar{x}$
$\bar{t}$
$c(\mu)$
$U(x,t;\mu)$
TGPT-PINN
Minimizing loss at μ by training only T_{μ,μ^1} and c!
VGPT-PINN for Euler equation
Stage 1: Minimizing loss at μ by fixing $\{c_i(\mu)\}_{i=1}^N$ and training $\{T_{\mu,\mu^i}\}_{i=1}^N$ with $\varepsilon_i = 1, \varepsilon_b = 1, \varepsilon_r = 0$.
T_{μ,μ^1}
T_{μ,μ^2}
T_{μ,μ^3}
T_{μ,μ^N}
$c_1(\mu)$
$c_2(\mu)$
$c_3(\mu)$
$c_N(\mu)$
$U_N(x,t;\mu)$
TGPT-PINN
$\mathcal{L}_{PDE} = MSE\left(\left(\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x}\right)\right)$
$S_{RH,t} = \frac{F(U(x+\Delta x,t)) - F(U(x-\Delta x,t))}{U(x+\Delta x,t) - U(x-\Delta x,t)}$,
$\lambda_{RH,t} = \begin{cases} |p_1 - p_2| \cdot |u_1 - u_2|, & |p_1 - p_2| > \varepsilon, |u_1 - u_2| > \varepsilon, \\ 0, & elsewhere, \end{cases}$
$\mathcal{L}_{RH} = MSE(\lambda_{RH,t}(x_{RH,t} + S_{RH,t} \cdot \Delta t - x_{RH,t+\Delta t}))$
$\mathcal{L}_{IC} = MSE(U(x_{IC}, 0) - \Phi(x_{IC}))$
$\mathcal{L}_{BC} = MSE(U(x_{BC}, t) - g(x_{BC}, t))$
Stage 2: Minimizing loss at μ by primarily optimizing $\{c_i(\mu)\}_{i=1}^N$ and fine-tuning $\{T_{\mu,\mu^i}\}_{i=1}^N$ with $\varepsilon_i = 10$, $\varepsilon_b = 10$, $\varepsilon_r = 100$.
$\mathcal{L}_{tot} = \mathcal{L}_{PDE} + \varepsilon_i \mathcal{L}_{IC} + \varepsilon_b \mathcal{L}_{BC} + \varepsilon_r \mathcal{L}_{RH}$