

Example of B+ tree

B+ Tree Example

- A balanced tree
- Each node can have **at most m** key fields and **$m+1$** pointer fields
- Half-full must be satisfied (except root node):
- m is even and $m=2d$
 - Leaf node half full: at least d entries
 - Non-leaf node half full: at least d entries
- m is odd and $m = 2d+1$
 - Leaf node half full: at least $d+1$ entries
 - Non-leaf node half full: at least d entries (i.e., $d+1$ pointers)

Show the tree after insertions

- Suppose each B+-tree node can hold up to 4 pointers and 3 keys.
- $m=3$ (odd), $d=1$
- Half-full (for odd m value)
 - Leaf node, at least 2 ($d+1$) entries
 - Non-leaf nodes, at least 2 ($d+1$) pointers (1 entry)
- Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

- Insert 1

1			
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Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

1			
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- Insert 3, 5

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

1		
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- Insert 3, 5

1	3	5	
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Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

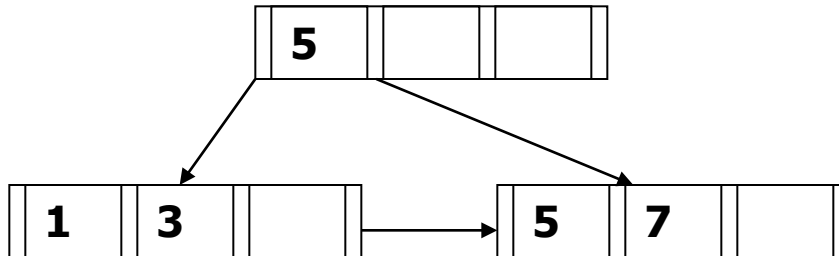
1	3	5	
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- Insert 7

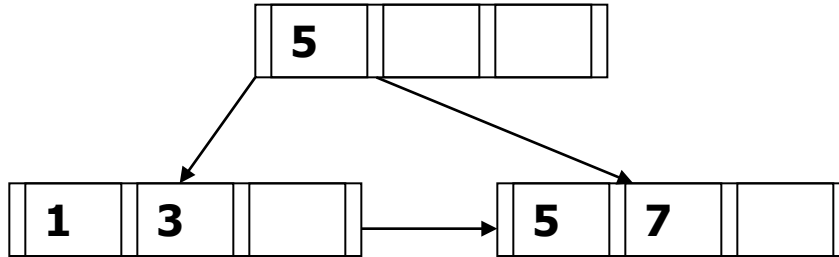
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 7

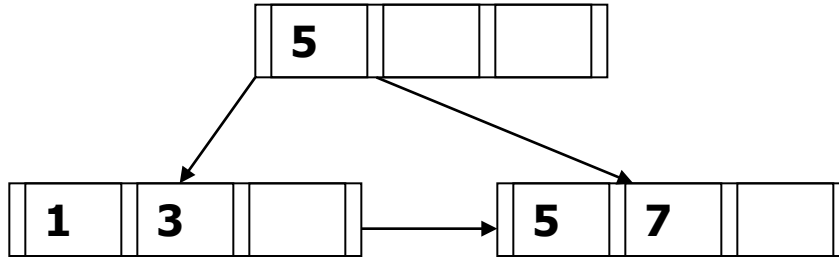


Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

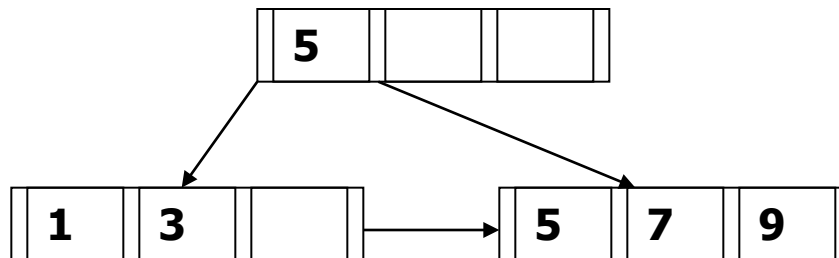


- Insert 9

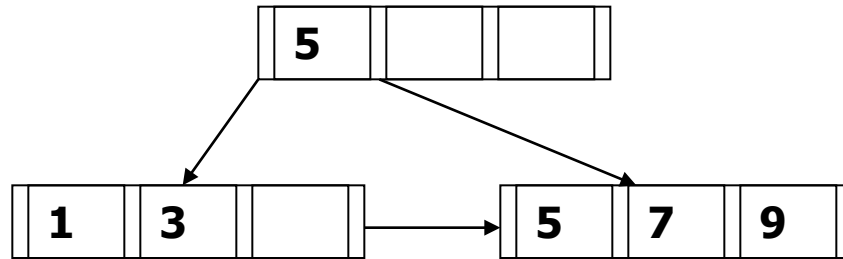
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 9

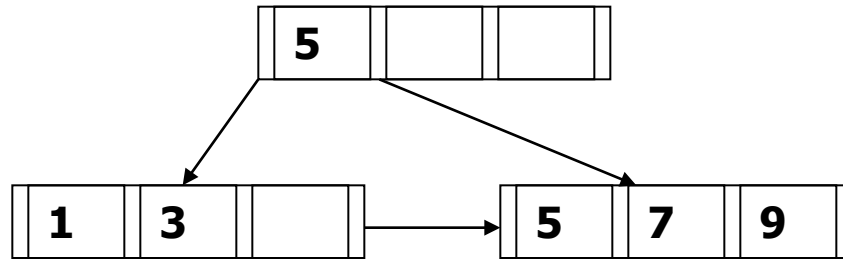


Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

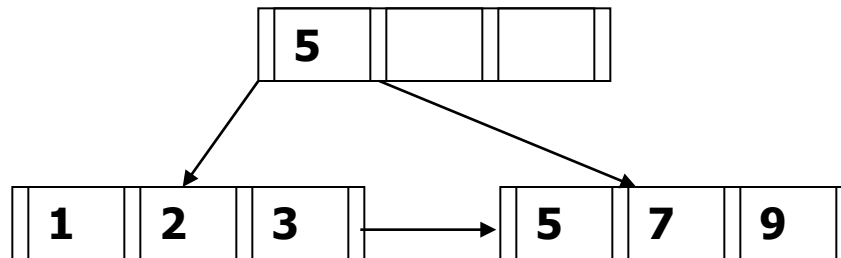


- Insert 2

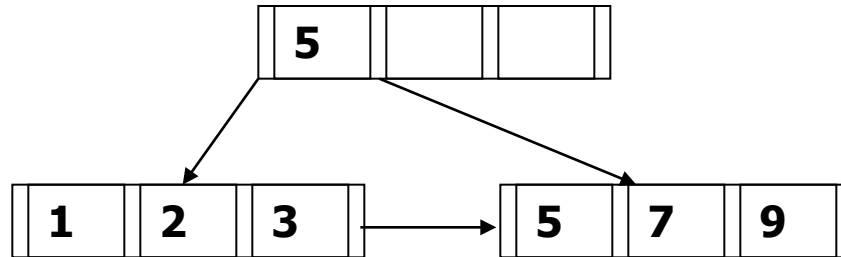
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 2

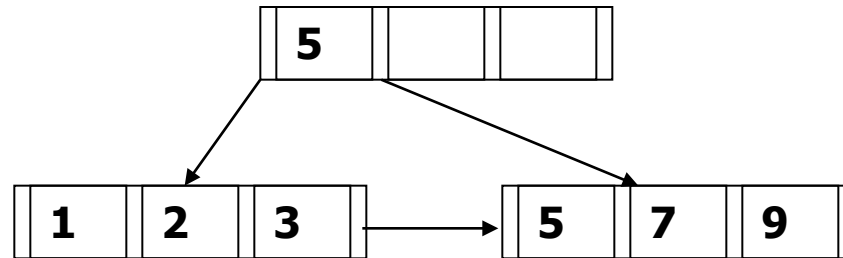


Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

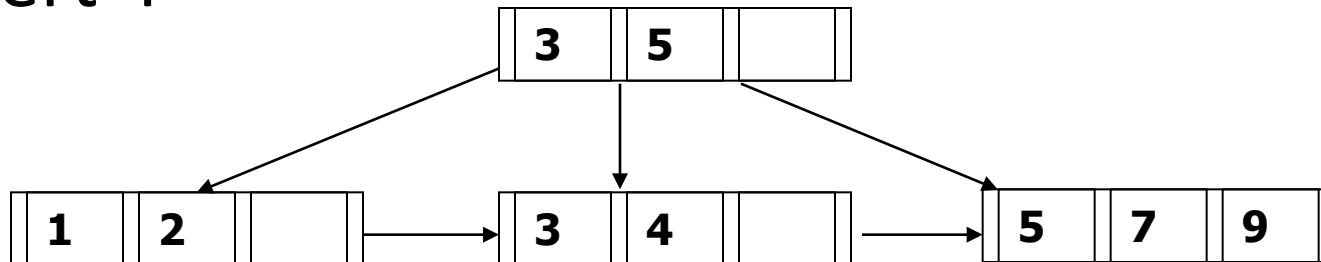


- Insert 4

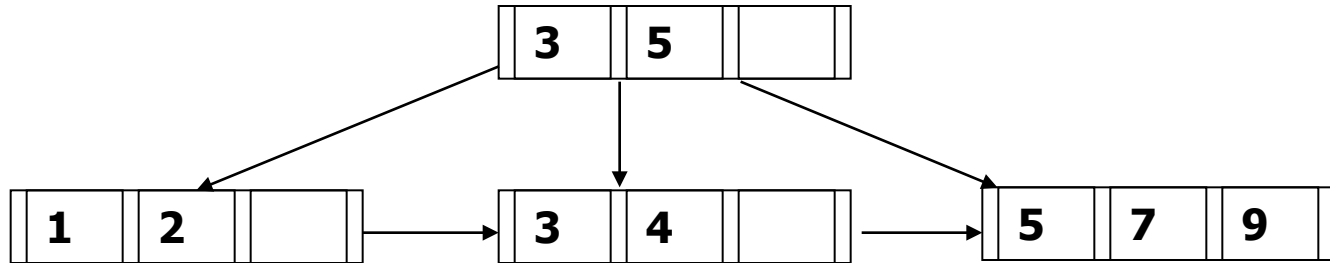
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 4

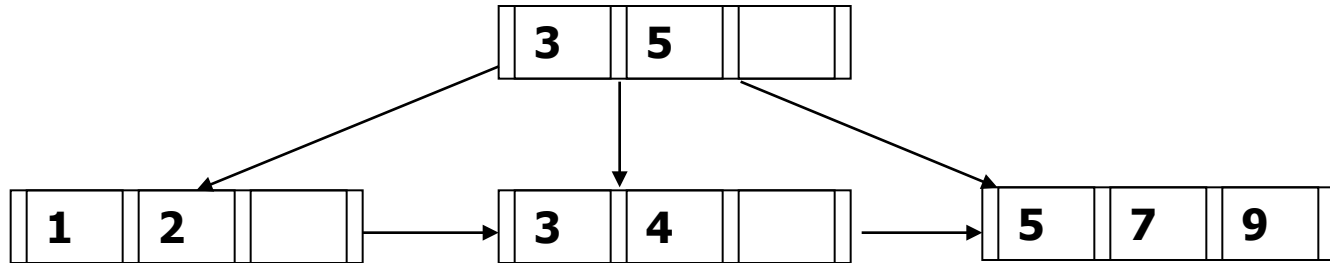


Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

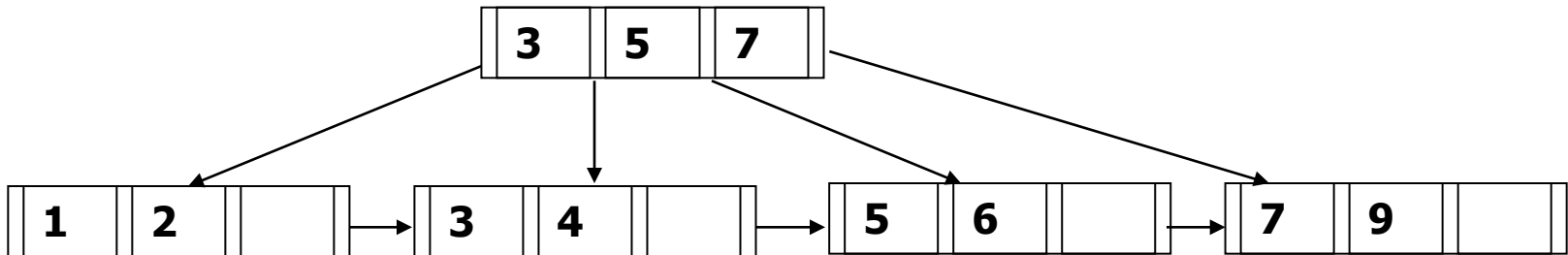


- Insert 6

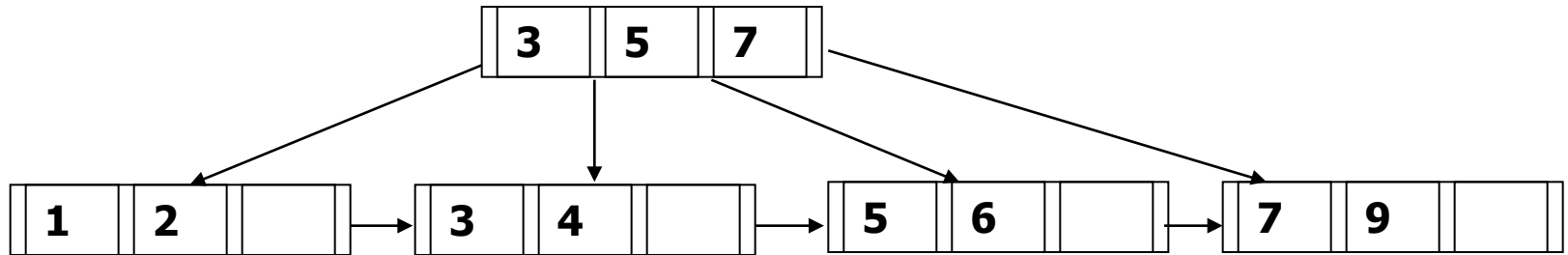
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 6

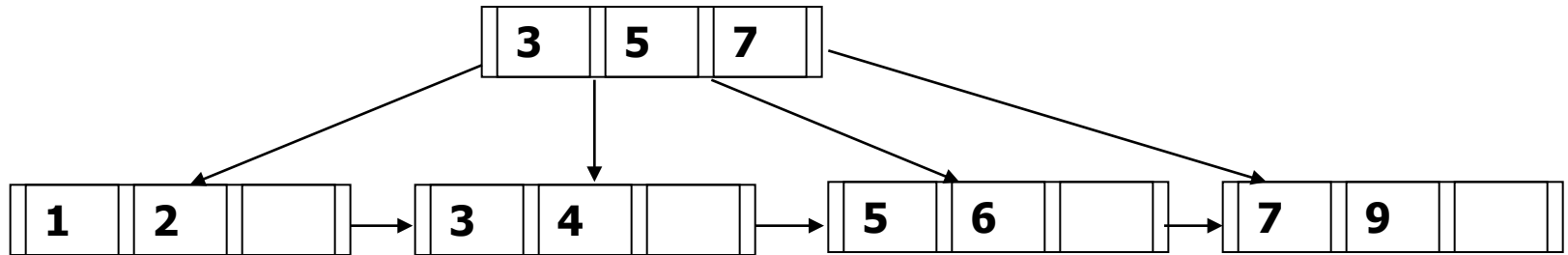


Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

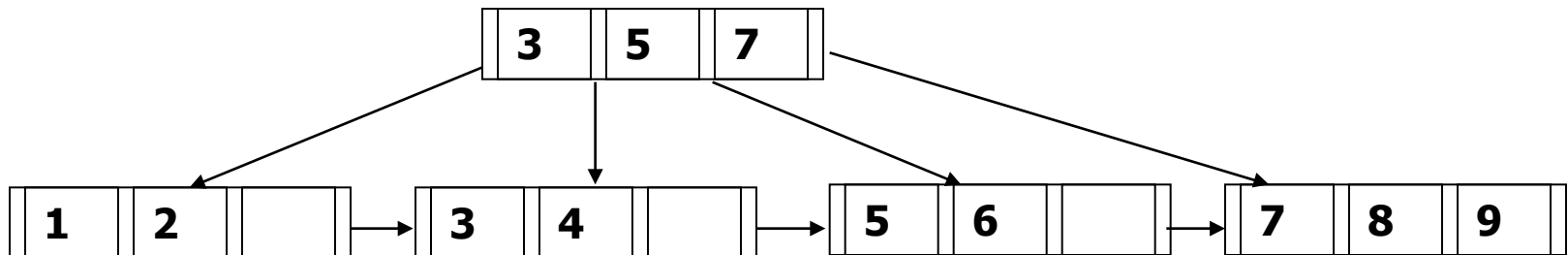


- Insert 8

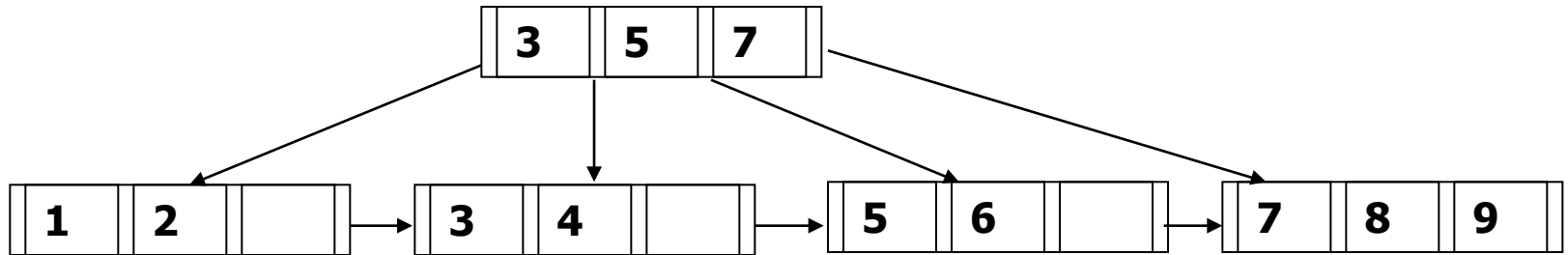
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 8

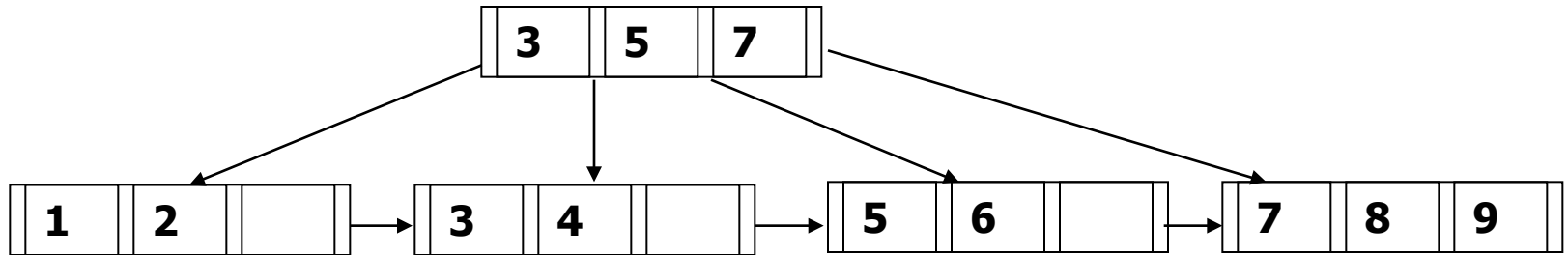


Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

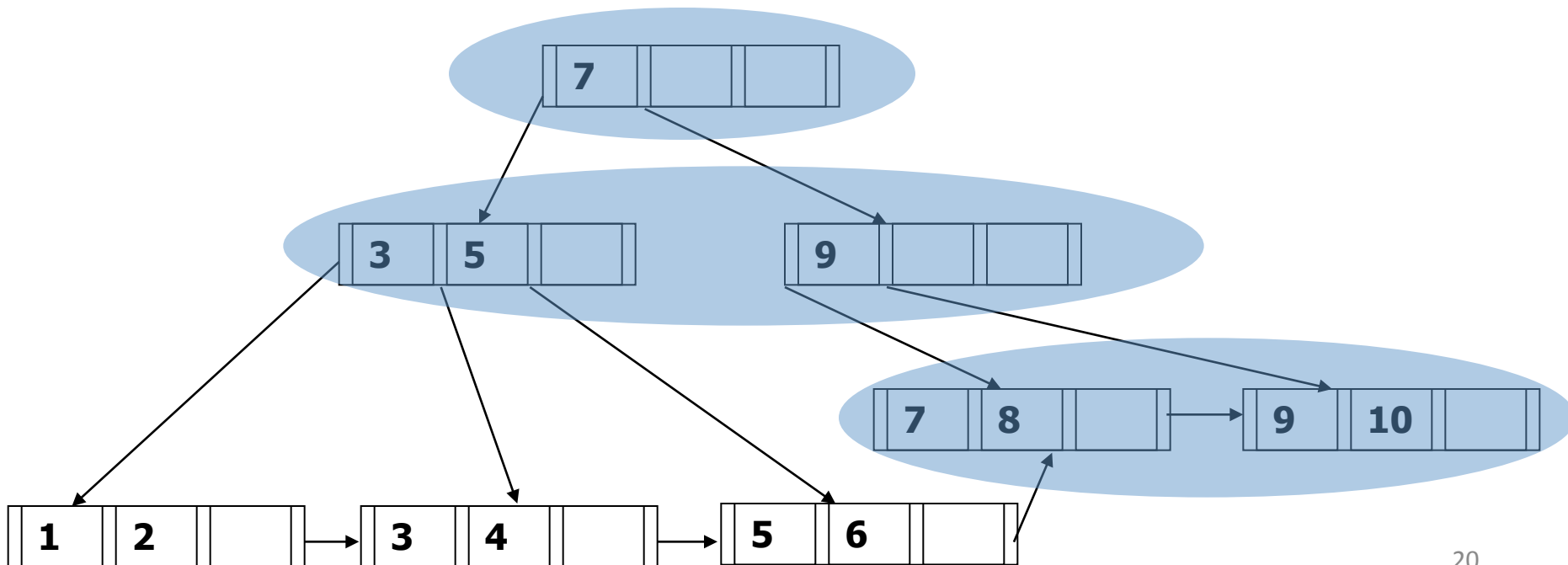


- Insert 10

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

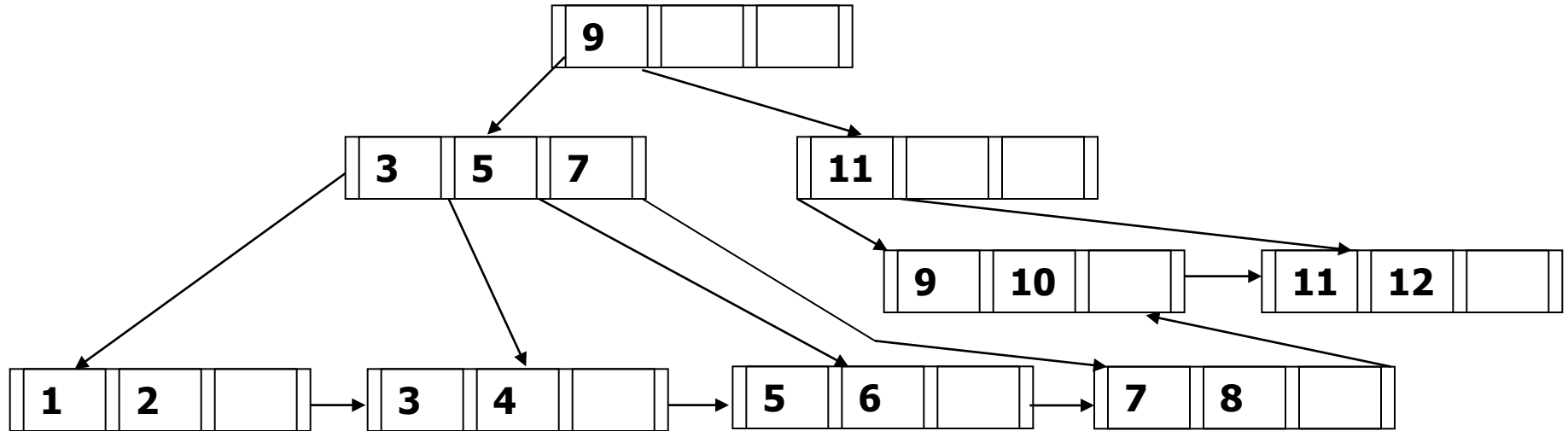


- Insert 10

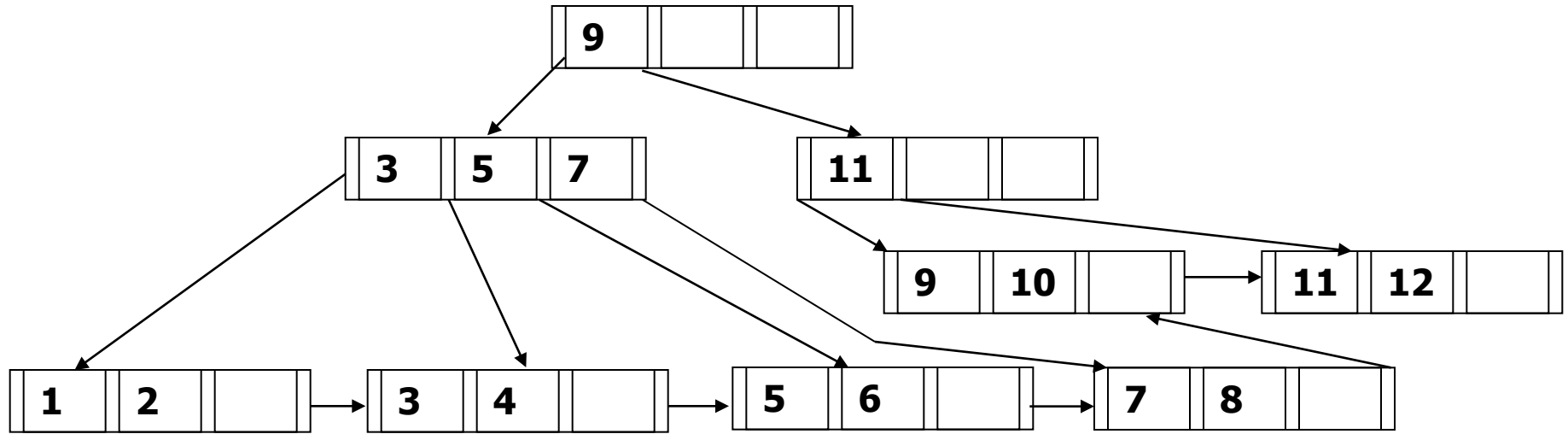


- Deletion

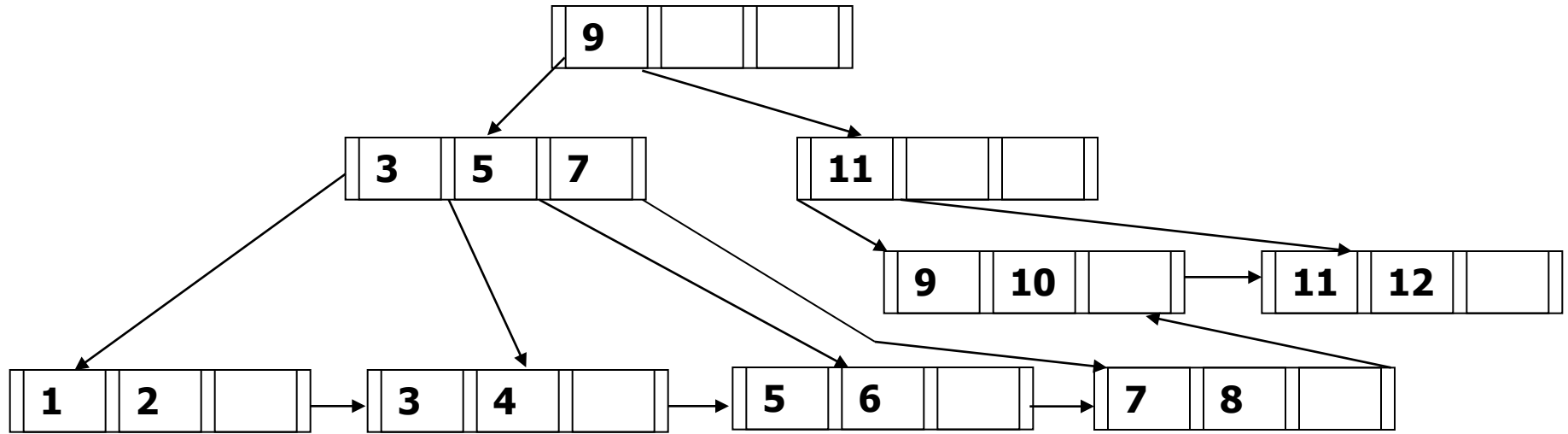
Show the tree after deletions



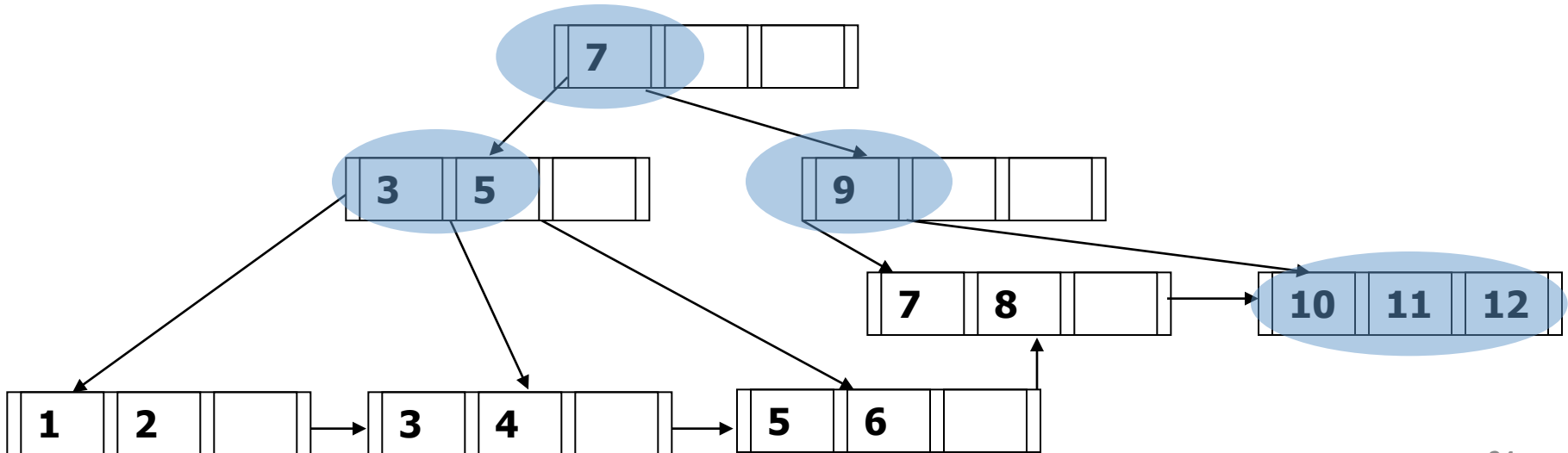
- Remove 9, 7, 8



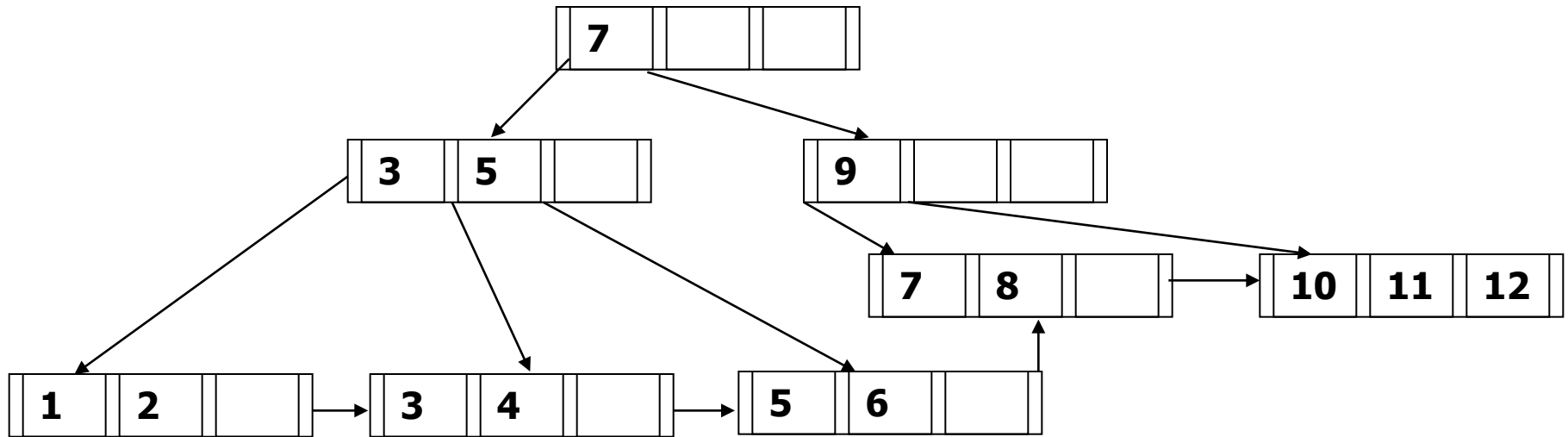
- After removing 9



- After removing 9

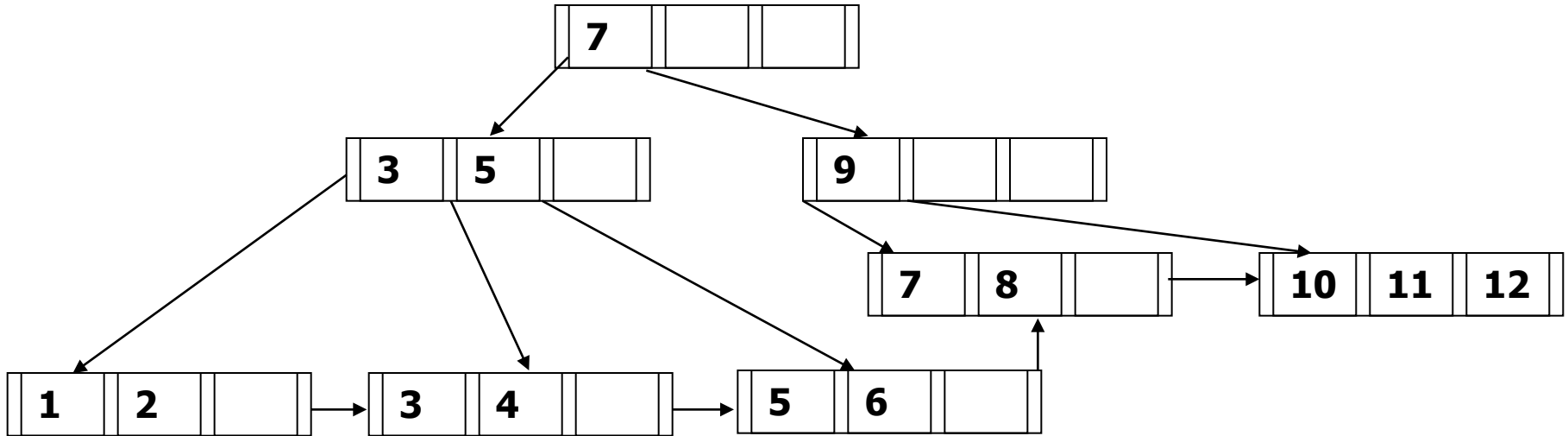


Remove 9, 7, 8

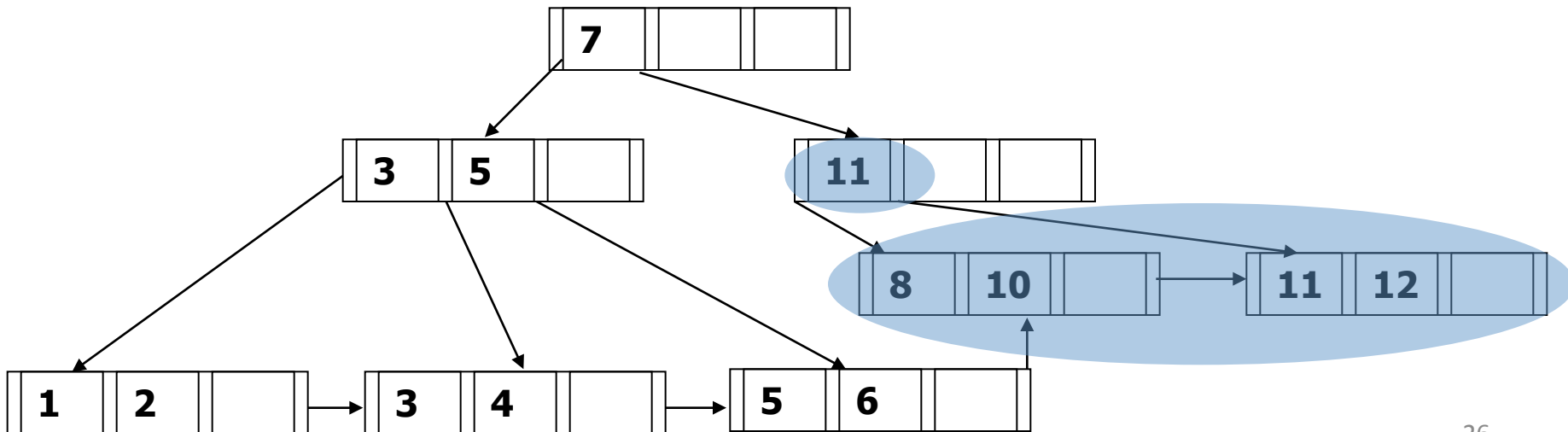


- After removing 7

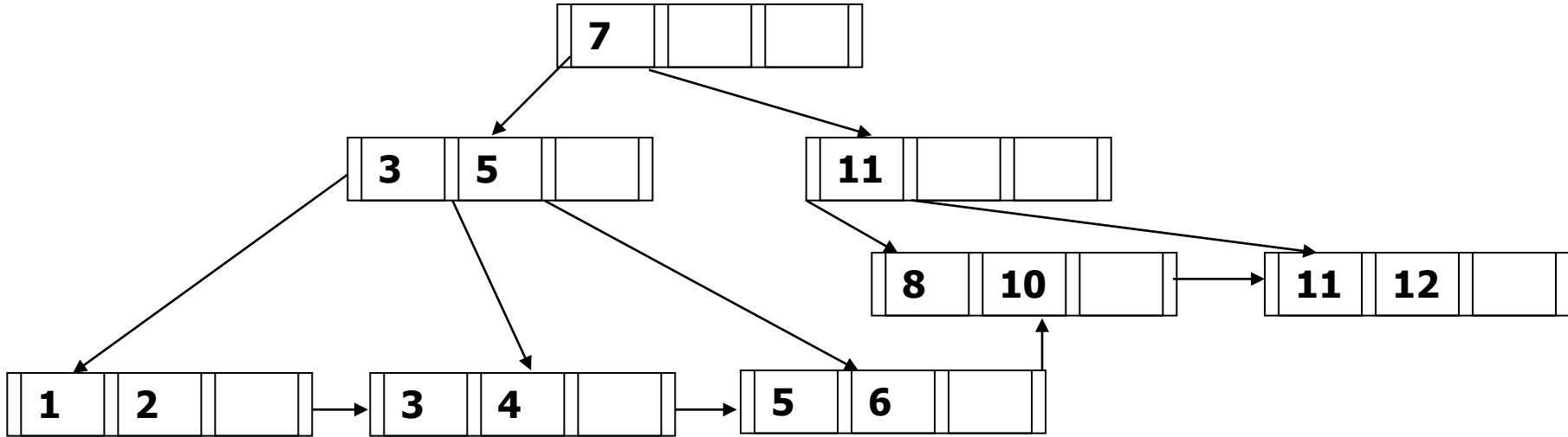
Remove 9, 7, 8



- After removing 7

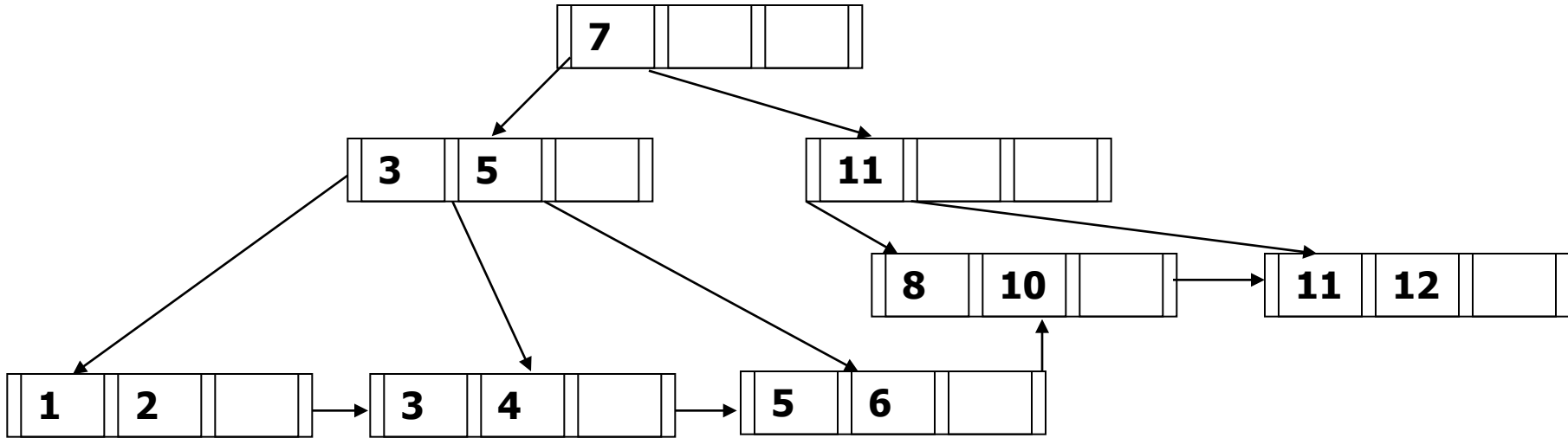


Remove 9, 7, 8



- After removing 8

Remove 9, 7, 8



- After removing 8

