(4) Bernoulli Equation:

The equation, $\frac{dy}{dx} + P(x)y = Q(x)y^n$ is known as Bernoulli equation.

C.W TBT (1) Solve $\frac{dy}{dx} + \frac{y}{x} = y^2$

Solution: Given that,

$$\frac{dy}{dx} + \frac{y}{x} = y^2$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} + \frac{y}{xy^2} = 1$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = 1 \quad \text{——— (i)}$$

Let, $\frac{1}{y} = z$

$$\Rightarrow \frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$$

$$\therefore \frac{1}{y^2}\frac{dy}{dx} = -\frac{dz}{dx}$$

Now putting this value in (i)

$$-\frac{dz}{dx} + \frac{z}{x} = 1$$

$$\Rightarrow \frac{dz}{dx} - \frac{z}{x} = -1 \quad \text{——— (ii)}$$

Here the equation is linear in z. so

$$I.F. = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$$

Now (ii) $\times \frac{1}{x} \Rightarrow$

$$\frac{1}{x}\frac{dz}{dx} - \frac{z}{x^2} = -\frac{1}{x}$$

$$\Rightarrow \frac{d}{dx}\left(z \cdot \frac{1}{x}\right) = -\frac{1}{x}$$

$$\Rightarrow z \cdot \frac{1}{x} = \int -\frac{1}{x}\, dx + c$$

$$\Rightarrow \frac{z}{x} = -\ln x + c$$

$$\Rightarrow z = cx - x\ln x$$

$$\therefore \frac{1}{y} = cx - x\ln x. \quad \text{Ans.}$$

HW 2. $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$

Solution:

Given that,

$$\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} + \frac{y}{x} \times \frac{1}{y^2} = \frac{y^2}{x^2} \cdot \frac{1}{y^2}$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = \frac{1}{x^2} \quad \text{--- (i)}$$

Let $\frac{1}{y} = z$

$$\Rightarrow \frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} = -\frac{dz}{dx}$$

Now Putting the value in (i),

$$-\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$$

$$\Rightarrow \frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2} \quad \text{—(ii)}$$

This equ^n is linear in z, we get

$$I.F. = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = x^{-1} = \frac{1}{x}$$

(ii) $\times \frac{1}{x} \Rightarrow$

$$\frac{1}{x}\cdot\frac{dz}{dx} - \frac{z}{x^2} = -\frac{1}{x^3}$$

$$\Rightarrow \frac{d}{dx}\left(z\frac{1}{x}\right) = -\frac{1}{x^3}$$

$$\Rightarrow z\cdot\frac{1}{x} = -\int\frac{1}{x^3}dx + c$$

$$\Rightarrow \frac{z}{x} = \frac{1}{2x^2} + c$$

$$\Rightarrow \frac{1}{xy} = \frac{1}{2x^2} + c.$$

Ans.

class work Test

3. $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^3\cos^2 y$

Solution:

Given that,

$$\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^3\cos^2 y$$

$$\Rightarrow \frac{1}{\cos^2 y}\frac{dy}{dx} + \frac{1}{x}\frac{2\sin y\cos y}{\cos^2 y} = x^3$$

$$\Rightarrow \sec^2 y\frac{dy}{dx} + \frac{2}{x}\tan y = x^3 \quad \text{—(i)}$$

Let $\tan y = z$

$$\therefore \sec^2 y\frac{dy}{dx} = \frac{dz}{dx}$$

Putting this value in (i) we get,

$$\frac{dz}{dx} + \frac{2}{x}z = x^3 \quad \text{—(ii)}$$

Now (ii) is linear in z.

$$\therefore \text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$$

Now (ii) $\times x^2 \Rightarrow$

$$x^2\frac{dz}{dx} + \frac{2}{x}z\cdot x^2 = x^5$$

$$\Rightarrow x^2\frac{dz}{dx} + 2xz = x^5$$

$$\Rightarrow \frac{d}{dx}(zx^2) = x^5$$

$$\Rightarrow zx^2 = \int x^5 dx + c$$

$$\Rightarrow z x^2 = \frac{x^6}{6} + c$$

$$\Rightarrow z = \frac{x^4}{6} + \frac{c}{x^2}$$

$$\therefore \tan y = \frac{x^4}{6} + \frac{c}{x^2}. \quad \text{Ans.}$$

#40 4. $\frac{dy}{dx} + \frac{y}{x} \ln y = \frac{y}{x^2 (\ln y)^2}$

Solution: Given that,

$$\frac{dy}{dx} + \frac{y}{x} \ln y = \frac{y}{x^2 (\ln y)^2}$$

$$\Rightarrow \frac{dy}{dx} \frac{(\ln y)^2}{y} + \frac{\frac{y \ln y}{x}}{\frac{y}{(\ln y)^2}} = \frac{\frac{y}{x^2 (\ln y)^2}}{\frac{y}{(\ln y)^2}}$$

$$\Rightarrow \frac{(\ln y)^2}{y} \times \frac{dy}{dx} + \frac{y \ln x}{x} \times \frac{(\ln y)^2}{y} = \frac{y}{x^2 (\ln y)^2} \times \frac{(\ln y)^2}{y}$$

$$\Rightarrow \frac{(\ln y)^2}{y} \frac{dy}{dx} + \frac{(\ln y)^3}{x} = \frac{1}{x^2} \quad \text{——} (1)$$

Let, $(\ln y)^3 = z$

$$\Rightarrow 3(\ln y)^2 \frac{1}{y} \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{(\ln y)^2}{y} \frac{dy}{dx} = \frac{1}{3} \frac{dz}{dx}$$

Now putting this value in (1) we get,

$$\frac{1}{3}\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$$

$$\Rightarrow \frac{dz}{dx} + \frac{3z}{x} = \frac{3}{x^2} \quad \text{——(ii)}$$

This is linear in z. So

$$\text{I.F.} = e^{\int \frac{3}{x}dx} = e^{\ln x^3} = x^3$$

Now (ii) $\times\ x^3 \Rightarrow$

$$x^3\frac{dz}{dx} + 3zx^2 = 3x$$

$$\Rightarrow \frac{d}{dx}(zx^3) = 3x$$

$$\Rightarrow zx^3 = \int 3x\,dx + c$$

$$\Rightarrow zx^3 = 3x\frac{x^2}{2} + c$$

$$\Rightarrow (\ln y)^3\ x^3 = \frac{3}{2}x^2 + c$$

$$\therefore (x\ln y)^3 = \frac{3}{2}x^2 + c. \text{ Ans.}$$

5. $$\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$$

Solution:

Given that,

$$\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$$

$$\Rightarrow \frac{1}{\sqrt{y}}\frac{dy}{dx} + \frac{x}{1-x^2}y^{1/2} = x$$

$$\Rightarrow y^{-\frac{1}{2}}\frac{dy}{dx} + \frac{x}{1-x^2}y^{1/2} = x \quad \text{--- (I)}$$

let $y^{1/2} = z$

$$\Rightarrow \frac{1}{2}y^{-1/2}\frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow y^{-1/2}\frac{dy}{dx} = 2\frac{dz}{dx}$$

Putting this value in (I),

$$2\frac{dz}{dx} + \frac{x}{1-x^2}z = x$$

$$\Rightarrow \frac{dz}{dx} + \frac{xz}{2(1-x^2)} = \frac{x}{2} \quad \text{--- (II)}$$

(II) is linear in z: so

$$\text{I.F.} = e^{\int \frac{1}{2}\frac{x}{1-x^2}dx}$$

$$= e^{\frac{1}{2}(-\frac{1}{2})\int\frac{-2x}{1-x^2}dx}$$

$$= e^{-\frac{1}{4}\ln(1-x^2)}$$

$$= e^{\ln(1-x^2)^{-1/4}}$$

$$= (1-x^2)^{-1/4}$$

Now (II) $\times (1-x^2)^{-1/4} \Rightarrow$

$$(1-x^2)^{-1/4}\frac{dz}{dx} + (1-x^2)^{-\frac{1}{4}} \times \frac{zx}{2(1-x^2)} = \frac{x}{2}(1-x^2)^{-1/4}$$

$$\Rightarrow (1-x^2)^{-1/4}\frac{dz}{dx} + \frac{2x}{2}(1-x^2)^{-5/4} = \frac{x}{2}(1-x^2)^{-1/4}$$

$$\Rightarrow \frac{d}{dx}\left\{ z \times (1-x^2)^{-1/4} \right\} = \frac{x}{2} \times (1-x^2)^{-1/4}$$

$$\Rightarrow z(1-x^2)^{-1/4} = \int \frac{x}{2}(1-x^2)^{-1/4}\,dx + c$$

$$\Rightarrow \frac{z}{(1-x^2)^{1/4}} = \frac{1}{2}\int \frac{x}{(1-x^2)^{1/4}}\,dx + c$$

$$\Rightarrow \frac{\sqrt{y}}{(1-x^2)^{1/4}} = -\frac{1}{4}\int \frac{dz}{z^{1/4}} + c$$

$1-x^2 = z$

$\Rightarrow -2x\,dx = dz$

$\Rightarrow x\,dx = -\frac{dz}{2}$

$$= -\frac{1}{4}\left[\frac{z^{-\frac{1}{4}+1}}{-\frac{1}{4}+1}\right] + c$$

$$= -\frac{1}{4}\,\frac{z^{3/4}}{\frac{3}{4}} + c$$

$$= -\frac{1}{4} \times \frac{4}{3} z^{3/4} + c$$

$$\Rightarrow \frac{\sqrt{y}}{(1-x^2)^{1/4}} = -\frac{1}{3}(1-x^2)^{3/4} + c$$

$$\therefore \sqrt{y} = -\frac{1}{3}(1-x^2) + c(1-x^2)^{1/4}.$$

Ans.