Green University of Bangladesh

Class Test 03 Assignment

Course Title: Discrete Mathematics

Course Code: **CSE 101**

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Batch & Section: 213/C

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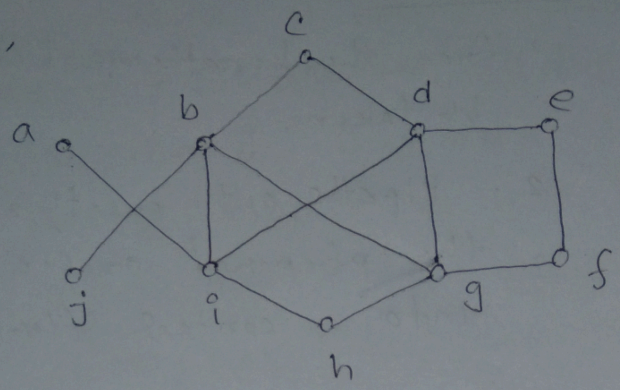
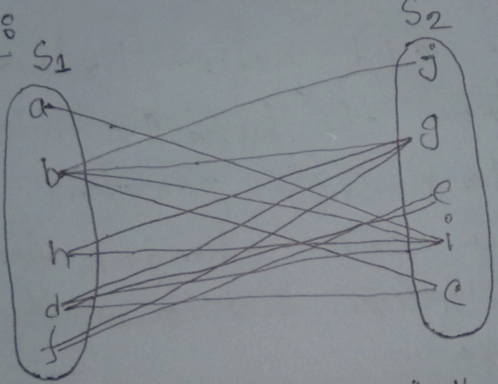
Answer to the question no: 01

~~Let~~ given,



Let : Graph "A"

Solution :



we first consider the graph "A". we will try to assign one of two sets, say S_1 and S_2, to vertex in "A" so that no edge "A" connects a S_1 vertex and a S_2 vertex. without loss of generality we begin by arbitrarily assigning A to a. Then, we must assign S_2 to j, i, g, e, c because each of these vertices is adjacent to a.

To avoid having an edge with two S_2 end points, we must assign S_1 to ~~both b, h, d and f (and means that a must assigand~~ all the vertices adjacent to either j, i, g, e and c. This means that we must assign S_1 to both b, h, d and f (and means that a must assigand S_1, which it already has been). We have now assigned set to all verties, with b, h, d and f is S_1 and j, i, g, e and c is S_2.

Checking all edges, we see that every edge connects a ~~set~~ S_1 vertex and a S_2 vertex.

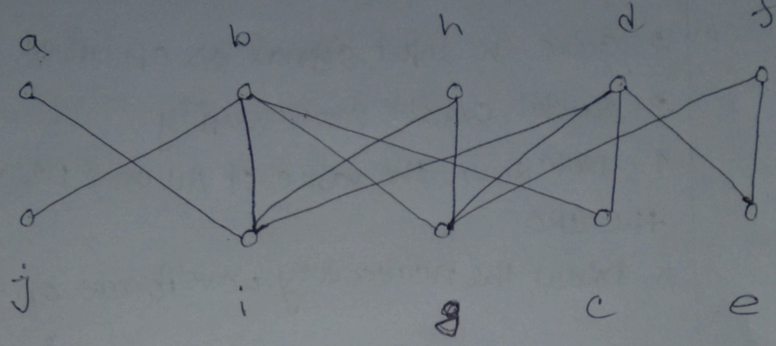
Hence, "A" simple graph is bipartite if and it is possible to assign one of two different set to each vertex of the graph so that no two adjacent vertices are assigned the same sets. So graph "A" is bipartite.

proof: First, suppose that $A = (V, E)$ is a bipartite simple graph. The $V = S_1 \cup S_2$

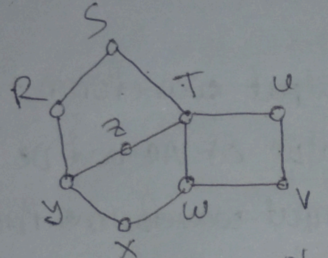
where S_1 and S_2 are disjoint sets and every edge in E connects a vertex in S_1 and a vertex in S_2. If we assign one set to each vertex in S_1 and a second sets to each vertex in S_2, then no two adjacent vertices are assigned the same sets.

Now, suppose that it is prossibe to assign sets to the vertices of the graph using just two ~~er~~ sets so that no two adjacent vertices are assigned the same sets. Let S_1 be the set of vertices assiged one set and S_2 set of the other sets. Then S_1 and S_2 are disjoint and $V = S_1 \cup S_2$. Furthermore, every edge connects a vertex in S_1 and a vertex in S_2 because no two adjancent vertices are either both in S_1 or both in S_2. consequently "A" is bipartite.

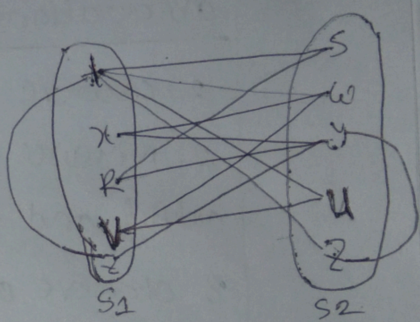
Redraw the graph:



Given:



Let: Graph "B"



again, we will try to assign either S_1 or S_2 to each vertex in "B" so that no edge in "B" connects a S_1 vertex and a S_2 vertex. without loss of generality we arbitrarily assign S_1 to Z. Then we must assign S_2 to s, w, y, u, because each is adjancet to Z. but this is not prossible because Z and t are adjancet.

So both cannot be assigned S_2. The Argument shows that we cannot assign on ~~of~~ two sets to each of the vertices of "B" : so that no adjanent vertices are assigned the same ~~order~~ sets.

we know that, A simple graph is ~~bipartiet~~ bipartite if and only if it is possible to assign on of two different sets to each vertex of the graph. so that no two adjacent vertres are assigned the same sets.

So we can tall that graph "B" is not bipartite.

Answer to the question no: 02

Solution: Let $P(n)$ be the proposition that

$3^n \geq n^3$.

Basis step: To prove the inequality $n \geq 3$ requires that basis step be $P(3)$.

Note that $P(3)$ is true, because

$3^3 \geq 3^3$

$\Rightarrow 27 \geq 27$

inductive step: for the inductive step, we let that $P(x)$ is true for an arbitrary integer x with ~~x≥3~~ $x \geq 3$. That is, we let $3^x \geq x^3$ for the positive integer x with $x \geq 3$. we must show that under this hypothesis, $P(x+1)$ is also true. That is, we must show that if $3^x \geq x^3$ for an arbitrary positive integer x where $x \geq 3$ then

$3^{x+1} \geq (x+1)^3$ we have,

$$3^{x+1} \geq (x+1)^3$$

$\Rightarrow 3 \cdot 3^x \geq (x+1)^3$ [if $3^x \geq x^3$ so 3^x ~~replace~~ replace x^3]

$\Rightarrow 3 \cdot x^3 \geq (x+1)^3$ [$3^x \geq x^3$ by the inductive hypothesis]

$\Rightarrow 3x^3 \geq x^3 + 3x^2 + 3x + 1$ [by binomial expansion]

$\Rightarrow 3x^3 - x^3 - 3x^2 - 3x - 1 \geq 0$

$\Rightarrow 2x^3 - 3x^2 - 3x - 1 \geq 0$

$\Rightarrow 2x^3 - 3(x^2 - x) - 1 \geq 0$

The first term is positive by assomption and second and third terms are both squares So they cannot be negative. If $x \geq 9$ then $3^x \geq x^3$ is possitive. Thus $3^{x+1} - (k+1)^3 \geq 0$ form the principle of induction this proves the result.

again say that $x \geq 3$ this x integer cannot be negative so $2x^3 - 3(x^2 - x) - 1 \geq 0$ it is true

3^n versus n^3

$3^3 = 27 \geq 3^3 = 27$

$3^4 = 81 > 4^3 = 64$

$3^5 = 243 > 5^3 = 125$

$3^6 = 729 > 6^3 = 216$

$3^7 = 2187 > 7^3 = 343$

$3^8 = 6561 > 8^3 = 512$

3^n is much bigger to n^3.

so we tall that,

$3^n \geq n^3$ for $n \geq 3$. it's all time true. and also proved the page numbe: 08.