Graphical Representation

Sketch the region in z-plane represented by the following set of points:

(i) $Re(\bar{z}-1)=2$

Soln: The given expression is

$Re(\bar{z}-1)=2$

Let, $z=x+iy$

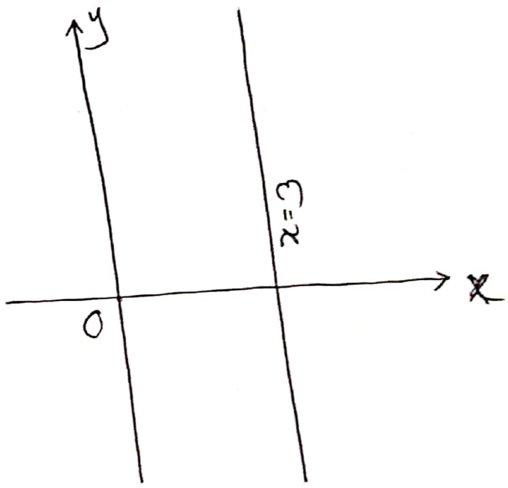
$\bar{z}=x-iy$

$\because Re(x-iy-1)=2$

$\Rightarrow Re[(x-1)-iy]=2$

$\Rightarrow x-1=2$

$\Rightarrow x=3$



(ii) $|z-1|+|z+1|=4$

Soln: The given expression is

$|z-1|+|z+1|=4$

Let $z=x+iy$

$\because |x+iy-1|+|x+iy+1|=4$

$\Rightarrow |(x-1)+iy|+|(x+1)+iy|=4$

$\Rightarrow \sqrt{(x-1)^2+y^2}+\sqrt{(x+1)^2+y^2}=4$

$\Rightarrow (x+1)^2+y^2=\left\{4-\sqrt{(x-1)^2+y^2}\right\}^2$

$$\Rightarrow (x+1)^2+y^2 = 16 - 8\sqrt{(x-1)^2+y^2} + (x-1)^2+y^2$$

$$\Rightarrow (x+1)^2-(x-1)^2 = 16 - 8\sqrt{(x-1)^2+y^2}$$

$$\Rightarrow x^2+2x+1-(x^2-2x+1) = 16 - 8\sqrt{(x-1)^2+y^2}$$

$$\Rightarrow \cancel{x^2}+2x+\cancel{1}-\cancel{x^2}+2x-\cancel{1} = 4\left[4-2\sqrt{(x-1)^2+y^2}\right]$$

$$\Rightarrow 4x = 4\left[4-2\sqrt{(x-1)^2+y^2}\right]$$

$$\Rightarrow x = 4-2\sqrt{(x-1)^2+y^2}$$

$$\Rightarrow x-4 = -2\sqrt{(x-1)^2+y^2}$$

$$\Rightarrow (x-4)^2 = 4\{(x^2-2x+1)+y^2\}$$

$$\Rightarrow x^2-8x+16 = 4x^2-8x+4+4y^2$$

$$\Rightarrow x^2-\cancel{8x}-4x^2+\cancel{8x}-4y^2 = 4-16$$

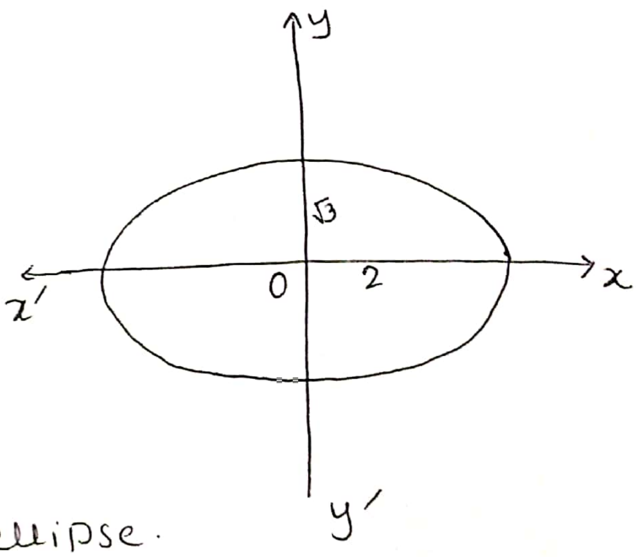
$$\Rightarrow -3x^2-4y^2 = -12$$

$$\Rightarrow 3x^2+4y^2 = 12$$

$$\Rightarrow \frac{3x^2}{12}+\frac{4y^2}{12} = 1$$

$$\Rightarrow \frac{x^2}{4}+\frac{y^2}{3} = 1$$

which represents an ellipse.



$(x-a)^2+(y-b)^2=r^2$
Centre (a,b)
radius r

(iii) $Re\left(\frac{1}{z}\right)=1$

Soln: The given expression is

$$Re\left(\frac{1}{z}\right)=1$$

$$\Rightarrow Re\left(\frac{1}{x+iy}\right)=1, \quad \text{where } z=x+iy$$

$$\Rightarrow Re\left[\frac{x-iy}{(x+iy)(x-iy)}\right]=1$$

$$\Rightarrow Re\left[\frac{x-iy}{x^2+y^2}\right]=1$$

$$\Rightarrow Re\left[\frac{x}{x^2+y^2}-i\frac{y}{x^2+y^2}\right]=1$$

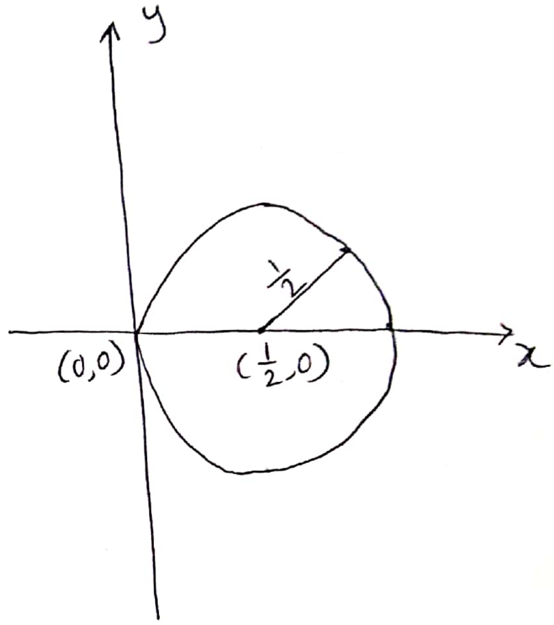
$$\Rightarrow \frac{x}{x^2+y^2}=1$$

$$\Rightarrow x^2+y^2-x=0$$

$$\Rightarrow x^2-x+y^2=0$$

$$\Rightarrow x^2-2\cdot\frac{1}{2}x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+y^2=0$$

$$\Rightarrow \left(x-\frac{1}{2}\right)^2+y^2=\left(\frac{1}{2}\right)^2$$



which represents a circle whose centre is $\left(\frac{1}{2},0\right)$ and radius $\frac{1}{2}$.

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(IV) $|z-i| \leq |z+i|$

Soln: The given expression is

$|z-i| \leq |z+i|$

$\Rightarrow |x+iy-i| \leq |x+iy+i|$ where $z=x+iy$

$\Rightarrow |x+i(y-1)| \leq |x+i(y+1)|$

$\Rightarrow \sqrt{x^2+(y-1)^2} \leq \sqrt{x^2+(y+1)^2}$

$\Rightarrow x^2+(y-1)^2 \leq x^2+(y+1)^2$

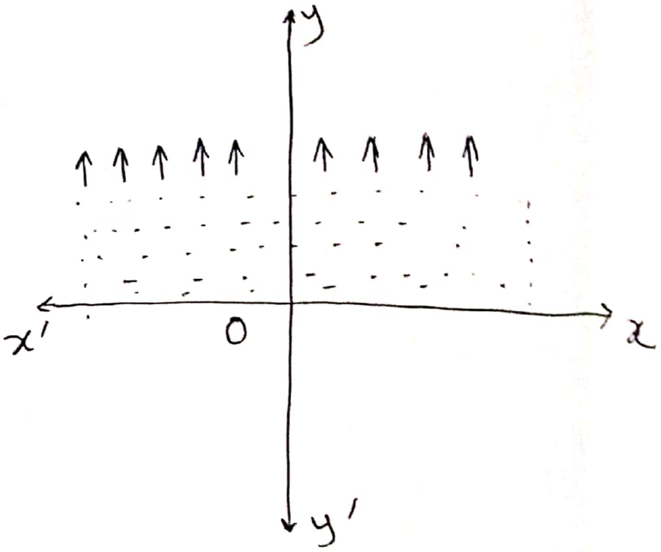
$\Rightarrow (y+1)^2-(y-1)^2 \geq 0$

$\Rightarrow (y^2+2y+1)-(y^2-2y+1) \geq 0$

$\Rightarrow \cancel{y^2}+2y+1-\cancel{y^2}+2y-1 \geq 0$

$\Rightarrow 4y \geq 0$

$\therefore y \geq 0$



Which represents the region of the upper part of the x-axis including the x-axis.

H.W (V) Prove that $|z+3i|+|z-3i|=5$ represents an ellipse.

H.W (VI) $|z-2|-|z+2|=4$

(vii) $|z+1-i| \le |z-1+i|$

Soln: The given expression is

$|z+1-i| \le |z-1+i|$

$\Rightarrow |x+iy+1-i| \le |x+iy-1+i|$

$\Rightarrow |(x+1)+i(y-1)| \le |(x-1)+i(y+1)|$

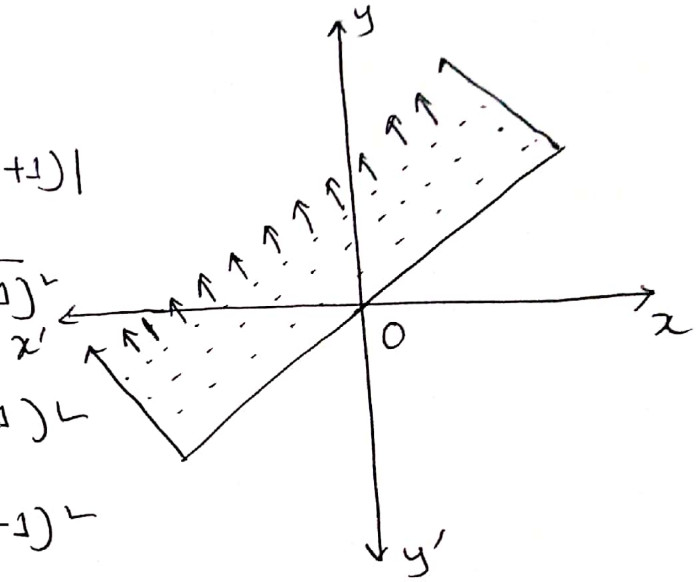
$\Rightarrow \sqrt{(x+1)^2+(y-1)^2} \le \sqrt{(x-1)^2+(y+1)^2}$

$\Rightarrow (x+1)^2+(y-1)^2 \le (x-1)^2+(y+1)^2$

$\Rightarrow (x+1)^2-(x-1)^2 \le (y+1)^2-(y-1)^2$

$\Rightarrow 4x \le 4y$

$\Rightarrow y \geq x$



which represents the region containing whole second quadrant, the upper half of first quadrant and the upper half of third quadrant including the line $y=x$.