Baye's theorem:

Let $\{A_1, A_2, \dots, A_i, \dots, A_k\}$ be a set of mutually exclusive and exhaustive events form a partition of the sample space S such that $A_1 \cup A_2 \cup \dots \cup A_k = S$ and $P(A_i) > 0$. Again Let the event B of S such that $P(B) > 0$ then

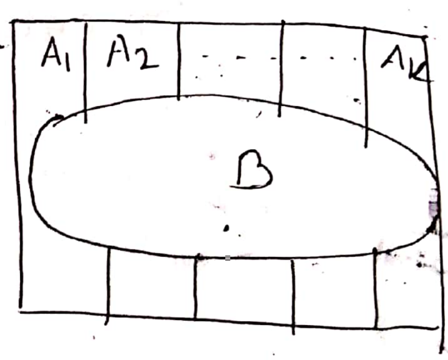
$$P(A_i \mid B) = \frac{P(A_i) P(B \mid A_i)}{\sum_{i=1}^{k} P(A_i) P(B \mid A_i)}$$

$i = 1, 2, \dots, k$

which is Baye's theorem.

Proof: According to the given theorem, A_i and B are dependent.

Then by using multiplication rule of probability for dependent events we have



$$P(A_i \mid B) = \frac{P(A_i \cap B)}{P(B)} \quad \dots (i)$$

$$= \frac{P(A_i) P(B \mid A_i)}{P(B)} \quad \dots (1)$$

$$\left[P(B \mid A_i) = \frac{P(A_i \cap B)}{P(A_i)} \right.$$
$$\left. \Rightarrow P(A_i \cap B) = P(A_i) \cdot P(B \mid A_i) \right]$$

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we have, $B = S \cap B$

$= (A_1 \cup A_2 \cup \dots \cup A_k) \cap B$

$= (A_1 \cap B) \cup (A_2 \cap B) \cup \dots (A_k \cap B)$

$P(B) = P[(A_1 \cap B) \cup (A_2 \cap B) \cup \dots \cup (A_k \cap B)]$

$S = \{1, 2, 3, 4\}$
$B = \{2, 3\}$
$S \cap B = \{2, 3\}$
$= B$

$= P(A_1 \cap B) + P(A_2 \cap B) + \dots + P(A_k \cap B)$

[since $(A_1 \cap B), (A_2 \cap B), \dots (A_k \cap B)$ are mutually exclusive

$$= \sum_{i=1}^{k} P(A_i \cap B)$$

$$= \sum_{i=1}^{k} P(A_i) P(B|A_i)$$

Now putting the value of $P(B)$ in equation (1) we have

$$P(A_i|B) = \frac{P(A_i) P(B|A_i)}{\sum_{i=1}^{k} P(A_i) P(B|A_i)}$$

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problem: Three machines M_1, M_2 and M_3 produce ~~M_1, M_2 and M_3~~ respectively 40%, 25% and 35% of the total number of items of a factory. The percentages of defective items of these machines are 2%, 4% and 5%.

(i) If an item is selected at random, find the probability that the item is defective.

(ii) If an item is selected at random, find the probability that the defective item was produced by machine M_1.

Soln:

Let,

A_1: Machine M_1 produce the item

A_2: Machine M_2 produce the item

A_3: Machine M_3 produce the item.

And event B: The item is defective

According to the question we have

$P(A_1) = 40\% = 0.40$

$P(A_2) = 25\% = 0.25$

$P(A_3) = 35\% = 0.35$

$P(B|A_1) = 2\% = 0.02$

$P(B|A_2) = 4\% = 0.04$

$P(B|A_3) = 5\% = 0.05$

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(i) The probability that the item is defective = $P(B)$

$$= P(A_1)P(B\backslash A_1) + P(A_2)P(B\backslash A_2) + P(A_3)P(B\backslash A_3)$$

$$= 0.40 \times 0.02 + 0.25 \times 0.04 + 0.35 \times 0.05$$

$$= 0.0355$$

(ii) By using Bayes' theorem, the probability that the defective item was produced by machine M_1

$$P(A_1\backslash B) = \frac{P(A_1)P(B\backslash A_1)}{P(A_1)P(B\backslash A_1) + P(A_2)P(B\backslash A_2) + P(A_3)P(B\backslash}$$

$$= \frac{0.40 \times 0.02}{0.40 \times 0.02 + 0.25 \times 0.04 + 0.35 \times 0.05}$$

$$= \frac{0.008}{0.0355}$$

$$= 0.22504$$

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