ASSIGNMENT 1

EM 314 – NUMERICAL METHODS

LIYANAGE D.P

E/15/202

QUESTION 01

$$\sin(\mathrm{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} + E_n(x)$$

$$E_n(x) = (-1)^{2n+3} \frac{x^{2n+3}}{(2n+3)!}$$

Therefore error occurs in using Taylor's series for sin(x) = $(-1)^{2n+3} \frac{x^{2n+3}}{(2n+3)!}$

Error for sin(x) ≈ x ⇒ $(-1)^{2n+3} \frac{x^{2n+3}}{(2n+3)!}$

To become this approximation to be accurate within 10^{-6}

$$\left| \frac{x^{2n+3}}{(2n+3)!} - x \right| \le 10^{-6}$$

QUESTION 02

Number of digits that can be possessed by the number just after the floating point is β-1 (because it can have only numbers from 1 to β-1)

But the other numbers after that number can have numbers from 0 to β-1

So the number of elements it can have is β

Mantissa has t numbers

So the number of different ways that we can keep the numbers from 0 to β-1 in t-1 places is = β^{t-1}

Therefore the total number of different mantissas is = $(\beta-1) \times \beta^{t-1}$

Here the upper bound is U and the lower bound is L

So the different numbers that the exponent can have is U – L + 1

Number of different positive or negative floating point numbers = $(\beta-1)\, \beta^{t-1}(U - L + 1)$

Therefore number of elements in the set **F** is = $2(\beta-1)\, \beta^{t-1}(U - L + 1)$

QUESTION 03

$f(x+h) = f(x) + f^{(1)}(x)h + \frac{f^{(2)}(x)}{2!}h^2$

$$\frac{f(x+h) - f(x)}{h} = f^{(1)}(x) + \frac{f^{(2)}(x)}{2!}h$$

$$\mathrm{f}_h^{(1)}(x) = f^{(1)}(x) + \frac{f^{(2)}(x)}{2!}h$$

$$f^{(1)}(x) - \mathrm{f}_h^{(1)}(x) = -\frac{f^{(2)}(x)}{2!}h$$

$$\left| f^{(1)}(x) - \mathrm{f}_h^{(1)}(x) \right| = \left|\frac{f^{(2)}(x)}{2!}h\right| \leq Ch$$

We can find a constant C such that $\left| f^{(1)}(x) - \mathrm{f}_h^{(1)}(x) \right| \leq Ch$ when x ≥ k

Therefore $E_h(x) = \mathcal{O}(h)$

QUESTION 04

```
%(a)
A = magic(500);
ta1=cputime();
a = det(A)
ta2 = cputime();

B = magic(1000);
tb1 = cputime();
b = det(B)
tb2 = cputime();

C = magic(1500);
tc1 = cputime();
c = det(C)
tc2 = cputime();

D = magic(2000);
td1 = cputime();
d = det(D)
td2 = cputime();

E = magic(2500);
te1 = cputime();
e = det(E)
te2 = cputime();

F = magic(3000);
tf1 = cputime();
f = det(F)
tf2 = cputime();

G = magic(3500);
tg1 = cputime();
g = det(G)
tg2 = cputime();

H = magic(4000);
th1 = cputime();
h = det(H)
th2 = cputime();

I = magic(4500);
ti1 = cputime();
i = det(I)
ti2 = cputime();

J = magic(5000);
tj1 = cputime();
j = det(J)
tj2 = cputime();



%(c)
X = [500 1000 1500 2000 2500 3000 3500 4000 4500 5000]
```

```
Y = [ta2-ta1 tb2-tb1 tc2-tc1 td2-td1 te2-te1 tf2-tf1 tg2-tg1 th2-th1 ti2-
ti1 tj2-tj1]

loglog(X,Y,'o');
hold on

%(d)
p = polyfit(log(X),log(Y),1);
p1 = polyval(p,log(X));
loglog(X,exp(p1),'-')

legend('Without polyfit','With polyfit')

hold off
```

Solution

a = 0

b = 0

c = 0

d = 0

e = 0

f = 0

g = 0

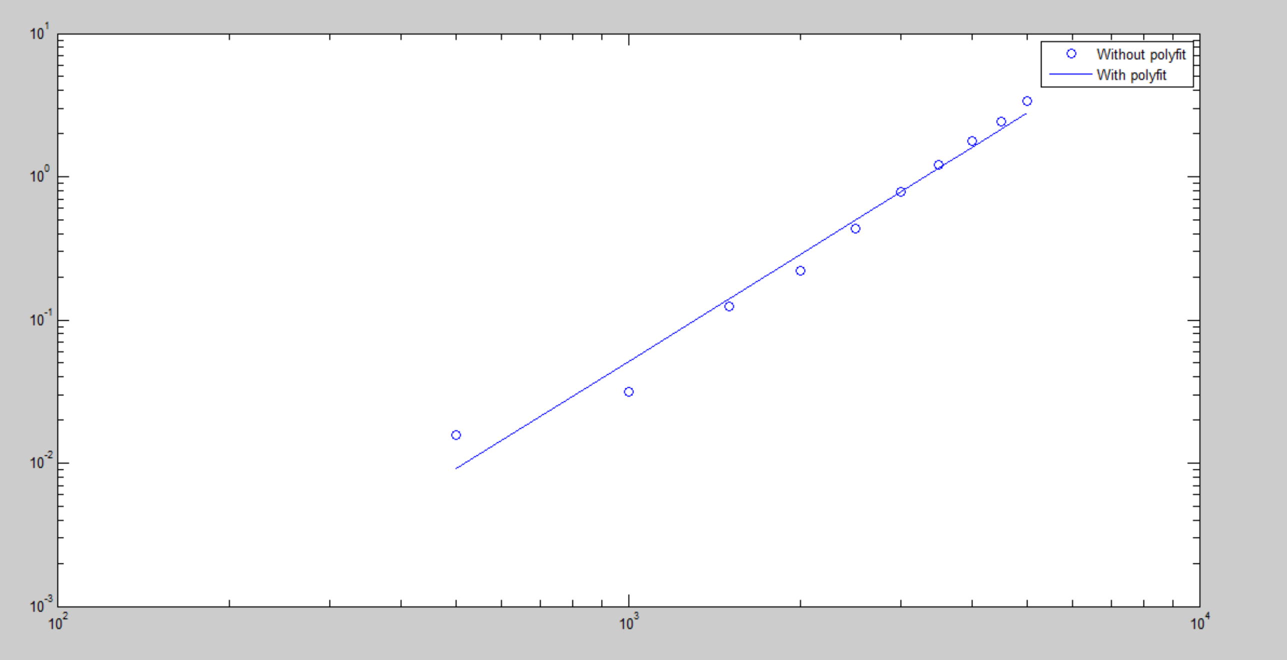
h = 0

i = 0

j = 0

X = 500 1000 1500 2000 2500 3000 3500 4000 4500 5000

Y = 0.0156   0.0313   0.1250   0.2188   0.4375   0.7813 1.2031   1.7813   2.4063   3.3906



(e) $0.0313 \ \alpha \ 1000^{\alpha}$ ———————(1)

$3.3906 \ \alpha \ 5000^{\alpha}$ ———————(2)

$(1)/(2) \Rightarrow \quad 0.0313 / 3.3906 = 0.2^{\alpha}$

$\ln(0.0313 / 3.3906) = \alpha \ln(0.2)$

$\alpha = \ln(0.0313 / 3.3906) / \ln(0.2)$

$\underline{\underline{\alpha = 2.91}}$

(f) $\det(A) = a < 500^3$

$\det(B) = b < 1000^3$

$\det(C) = c < 1500^3$

$\det(D) = d < 2000^3$

$\det(E) = e < 2500^3$

$\det(F) = f < 3000^3$

$\det(G) = g < 3500^3$

$\det(H) = h < 4000^3$

$\det(I) = i < 4500^3$

$\det(J) = j < 5000^3$

Therefore computer results and theoretical results are same

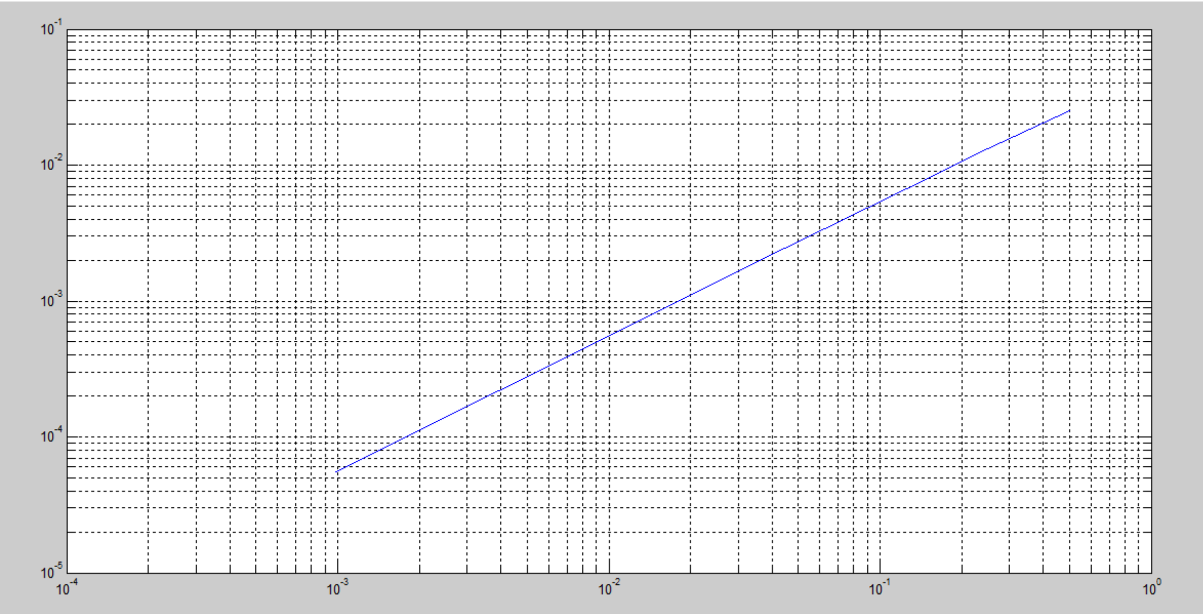
QUESTION 05

(a)
```
K = [1:1:10]
H = 1/2.*(1/2).^(0:9)
x = 3;

F = 1./H.*(log(x+H) - log(x))

E = abs(1/x - F)

loglog(H,E)
grid on
```

Solution

k	h_k	$f'_{h_k}(x)$	E_{h_k}
1	0.5000	0.3083	0.0250
2	0.2500	0.3202	0.0132
3	0.1250	0.3266	0.0068
4	0.0625	0.3299	0.0034
5	0.0313	0.3316	0.0017
6	0.0156	0.3325	0.0009
7	0.0078	0.3329	0.0004
8	0.0039	0.3331	0.0002
9	0.0020	0.3332	0.0001
10	0.0010	0.3333	0.0001

(b)



$0.025 \propto 0.5^{\gamma}$ ———— (1)

$0.0132 \propto 0.25^{\gamma}$ ———— (2)

(1)/(2) $\Rightarrow$ $0.025/0.0132 = (0.5/0.25)^{\gamma}$

$\ln(0.025/0.0132) = \gamma \ln(0.5/0.25)$

$\underline{\underline{\gamma = 0.92}}$

According to the value table we can say that every E_h value is always less than h values when

$h \geq 0.5$

Therefore it satisfy $E_h = O(h)$

(c)

```
K = [1:1:40]
H = 1/2.*(1/2).^(0:39)
x = 3;

F = 1./H.*(log(x+H) - log(x))

E = abs(1/x - F)

loglog(H,E)
grid on
```

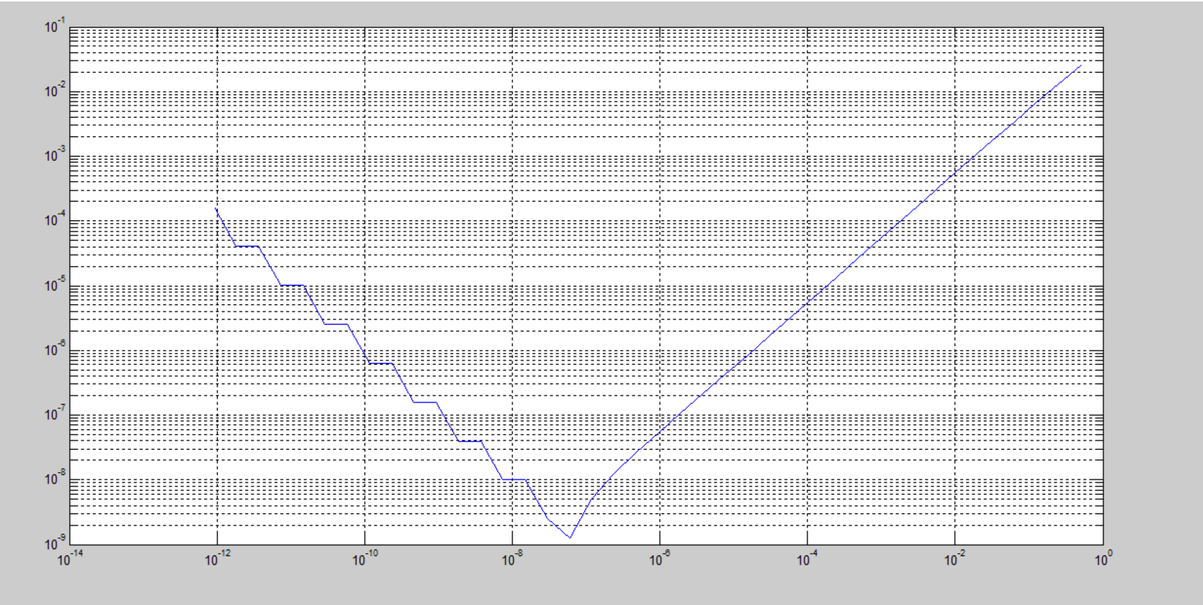
Solution

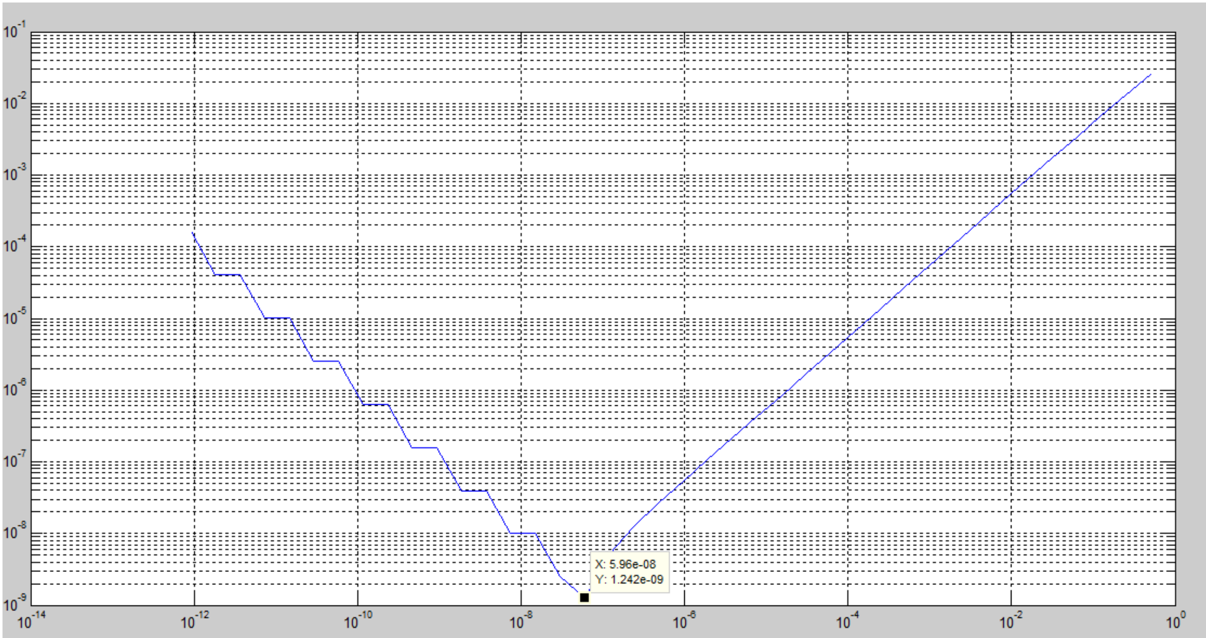
k	h_k	$f'_{h_k}(x)$	E_{h_k}
1	0.5000	0.3083	0.0250
2	0.2500	0.3202	0.0132
3	0.1250	0.3266	0.0068
4	0.0625	0.3299	0.0034
5	0.0313	0.3316	0.0017
6	0.0156	0.3325	0.0009
7	0.0078	0.3329	0.0004
8	0.0039	0.3331	0.0002
9	0.0020	0.3332	0.0001
10	0.0010	0.3333	0.0001
11	0.0005	0.3333	0.0000
12	0.0002	0.3333	0.0000
13	0.0001	0.3333	0.0000
14	0.0001	0.3333	0.0000
15	0.0000	0.3333	0.0000
16	0.0000	0.3333	0.0000
17	0.0000	0.3333	0.0000
18	0.0000	0.3333	0.0000

19	0.0000	0.3333	0.0000
20	0.0000	0.3333	0.0000
21	0.0000	0.3333	0.0000
22	0.0000	0.3333	0.0000
23	0.0000	0.3333	0.0000
24	0.0000	0.3333	0.0000
25	0.0000	0.3333	0.0000
26	0.0000	0.3333	0.0000
27	0.0000	0.3333	0.0000
28	0.0000	0.3333	0.0000
29	0.0000	0.3333	0.0000
30	0.0000	0.3333	0.0000
31	0.0000	0.3333	0.0000
32	0.0000	0.3333	0.0000
33	0.0000	0.3333	0.0000
34	0.0000	0.3333	0.0000
35	0.0000	0.3333	0.0000
36	0.0000	0.3333	0.0000
37	0.0000	0.3333	0.0000
38	0.0000	0.3334	0.0000
39	0.0000	0.3334	0.0000
40	0.0000	0.3335	0.0002

(d) As you can see in the table $f'_{h_k}(x)$ increases after 37. That means the derivative of ln(x) shows a minimum value as it is the graph of a x^{-1}
Therefore the according to the equation of error, the associated error will starts to increase after the minimum value.
This happens because we are doing this in a computer. There are limitations for computer arithmetic. But if we do this manually the value will decrease.

(e)





$\ln(h_{min}) = 5.96*10^{-8}$

$h_{min} = e^{5.96*10^{-8}} = \underline{\underline{1}}$