EM 314 – ASSIGNMENT 4

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E/15/202

QUESTION 01

(a)

```
function [I] = CompositeTrapezoidalRule(a,b,n)
h = (b-a)./n;
sum = 0;
i = 1;

while i < n
    sum = sum + g(a + i*h);
    i = i + 1;
end

I = (h/2)*(g(a) + 2*sum + g(b));
display(I);

p = g1(b) - g1(a);
q = g2(b) - g2(a);
r = g3(b) - g3(a);
s = g4(b) - g4(a);

anlt = p - q - 4*r + 2*s;

error = ((anlt - I)./anlt)*100;

display(error);
```

```
function t = g(x)
t = 1 - x - 4*(x.^3) + 2*(x.^5);

function t = g1(x)
t = x;

function t = g2(x)
t = (x.^2)./2;

function t = g3(x)
t = (x.^4)./4;

function t = g4(x)
t = (x.^6)./6;
```

(b)

Segment	Integral	Percent Relative Error
2	1852	-67.5513
3	1447.7	-30.9759
4	1300	-17.6116

(c)

```
function [I] = mulAplicSimpson13rdRule(a,b,n)
h = (b-a)/n;
sumE = 0;
sumO = 0;
i = 1;
j = 2;

while i < n
    sumO = sumO+ g(a + i*h);
    i = i + 2;
end

while j < n
    sumE = sumE + g(a + j*h);
    j = j + 2;
end

I = (h/3)*(g(a) + 4*sumO + 2*sumE + g(b));
display(I);

p = g1(b) - g1(a);
q = g2(b) - g2(a);
r = g3(b) - g3(a);
s = g4(b) - g4(a);

anlt = p - q - 4*r + 2*s;

error = ((anlt - I)./anlt)*100;

display(error);


function t = g(x)
t = 1 - x - 4*(x.^3) + 2*(x.^5);

function t = g1(x)
t = x;

function t = g2(x)
t = (x.^2)./2;

function t = g3(x)
t = (x.^4)./4;

function t = g4(x)
t = (x.^6)./6;
```

(d)

Segments	Integration	Percent Relative Error
2	1276	-15.4403%
4	1116	-0.9650%
6	1107.4	-0.1906%

(e)

Segments	Error in Composite Trapezoidal Rule	Error in Multiplication Application of Simpson's Rule
2	-67.5513%	-15.4403%
3	-30.9759%	-6.8624%
4	-17.6116%	-0.9650%

Since we cannot apply multiple application of Simpson's 1/3rd rule for 3 segments we apply Simpson's 3/8th rule.

```
function [I] = Simpsons38thRule(a,b)
h = (b-a)/3;
I = (3/8)*(g(a) + 3*g(a+h) + 3*g(a + 2*h) + g(b));

display(I);

p = g1(b) - g1(a);
q = g2(b) - g2(a);
r = g3(b) - g3(a);
s = g4(b) - g4(a);

anlt = p - q - 4*r + 2*s;

error = ((anlt - I)./anlt)*100;

display(error);
```

As you can see in the table percent relative error of multiple application of Simpson's rule has the highest convergence rate. It is because although the Simpson's rule has 3rd accuracy, it has a 4th order polynomial. So at a high convergence rate Simpson's rule will be converged to zero.

QUESTION 02

(a) (i) – Composite Trapezoidal Rule

(ii) – Multiple Application of Simpson's 1/3rd rule

	Segments	(i)	(ii)
β(1,2)	2	0.5000	0.5000
	3	0.5000	0.5000
	4	0.5000	0.5000
β(1.5,2.5)	2	0.1250	0.1667
	3	0.1571	0.1768
	4	0.1708	0.1860
β(2,3)	2	0.0625	0.0833
	3	0.0741	0.0833
	4	0.0781	0.0833

```
syms x
f = x*((1 -x).^2);
anlt = int(f,[0,1]);
```

```
syms x
f = (x.^0.5)*((1-x).^1.5);
anlt = int(f,[0,1]);
```

```
syms x
f = 1 - x;
anlt = int(f,[0,1]);
```

Analytical integration was done by these code segments

(b)

	Segments	(i)-percent relative error	(ii)-percent relative error
β(1,2)	2	0%	0%
	3	0%	0%
	4	0%	0%
β(1.5,2.5)	2	36.3380%	15.1004%
	3	19.9896%	9.9565%
	4	13.0122%	5.2709%

β(2,3)	2	25.0000%	0%
	3	11.1111%	0%
	4	6.2500%	0%

$\beta(1,2) = 1 - x$

$\beta(2,3) = x(1-x)^2$

$\beta(1.5,2.5) = \int_0^1 x^{0.5}(1-x)^{1.5}dx$

$\sqrt{1-x} = t \qquad x \to 0 \quad t \to 1$

$1 - x = t^2 \qquad x \to 1 \quad t \to 0$

$-1 = 2t\frac{dt}{dx}$

$dx = -2tdt$

$x = 1 - t^2$

$\beta(1.5,2.5) = \int_1^0 (1-t^2)^{0.5}t^3(-2t)dt$

$\beta(1.5,2.5) = -2\int_1^0 t^4(1-t^2)^{0.5}d\,t$

$sin\theta = t \qquad t \to 1 \quad \theta \to \frac{\pi}{2}$

$d\theta = \frac{dt}{cos\theta} \qquad t \to 0 \quad \theta \to 0$

$\beta(1.5,2.5) = -2\int_{\frac{\pi}{2}}^0 sin^4\theta\sqrt{1-sin^2\theta}d\,t$

$\beta(1.5,2.5) = -2\int_{\frac{\pi}{2}}^0 sin^4\theta cos\theta cos\theta d\theta$

$\beta(1.5,2.5) = -2\int_{\frac{\pi}{2}}^0 sin^4\theta cos\theta cos\theta d\theta$

$\beta(1.5,2.5) = -2\int_{\frac{\pi}{2}}^0 sin^4\theta(1-sin^2\theta)d\theta$

$\beta(1.5,2.5) = -2\int_{\frac{\pi}{2}}^0 sin^4\theta d\theta + 2\int_{\frac{\pi}{2}}^0 sin^6\theta\, d\theta$

$$\beta(1.5,2.5) = \frac{3\pi}{8} - \frac{5\pi}{16}$$

$$\beta(1.5,2.5) = \frac{\pi}{16}$$

Here I have used Simpson's 3/8th rule for the calculations of 3 segments.

As you can see in the table percent relative error of multiple application of Simpson's rule has the highest convergence rate. It is because although the Simpson's rule has 3rd accuracy, it has a 4th order polynomial. So at a high convergence rate Simpson's rule will be converged to zero.

QUESTION 03

$$\mathrm{erf}(1.5) = \frac{2}{\sqrt{\pi}}\int_0^{1.5} e^{-x^2}dx$$

$x = a_0 + a_1 t$

$x \to 0 \qquad t \to -1$

$x \to 1.5 \qquad t \to 1$

$0 = a_0 - a_1$

$1.5 = a_0 + a_1$

$a_0 = 0.75 \qquad a_1 = 0.75$

$x = 0.75 + 0.75t$

$dx = 0.75dt$

$$\mathrm{erf}(1.5) = \int_{-1}^{1} \frac{2}{\sqrt{\pi}} e^{-(0.75+0.75t)^2} 0.75dt$$

$$g(t) = \frac{2}{\sqrt{\pi}} e^{-(0.75+0.75t)^2} 0.75$$

$$I = g\left(\frac{1}{-\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

$$g\left(\frac{1}{-\sqrt{3}}\right) = 0.76538$$

$$g\left(\frac{1}{\sqrt{3}}\right) = 0.20879$$

$I = 0.76538 + 0.20879 = 0.97417$

Exact solution is = 0.966105

Percent relative is = $\frac{0.966105-0.97417}{0.966105} \times 100\% = -0.8348\%$