EM314 –ASSIGNMENT 03

LIYANAGE D.P

E/15/202

SEMESTER 4

QUESTION 01

i	0	1	2	3
x	1	2	3	4
y	$ln1 = 0$	$ln2 = 0.69315$	$ln3 = 1.09861$	$ln4 = 1.38629$

$$l_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)\left(\frac{x - x_3}{x_0 - x_3}\right)$$

$$= \left(\frac{x - 2}{1 - 2}\right)\left(\frac{x - 3}{1 - 3}\right)\left(\frac{x - 4}{1 - 4}\right)$$

$$= \frac{-1}{6}(x^3 - 9x^2 + 26x - 24)$$

$$l_1(x) = \left(\frac{x - 1}{2 - 1}\right)\left(\frac{x - 3}{2 - 3}\right)\left(\frac{x - 4}{2 - 4}\right)$$

$$= \frac{1}{2}(x^3 - 8x^2 + 19x - 12)$$

$$l_2(x) = \left(\frac{x - 1}{3 - 1}\right)\left(\frac{x - 2}{3 - 2}\right)\left(\frac{x - 4}{3 - 4}\right)$$

$$= \frac{-1}{2}(x^3 - 7x^2 + 14x - 8)$$

$$l_3(x) = \left(\frac{x - 1}{4 - 1}\right)\left(\frac{x - 2}{4 - 2}\right)\left(\frac{x - 3}{4 - 3}\right)$$

$$= \frac{1}{6}(x^3 - 6x^2 + 11x - 6)$$

$$p_3(x) = ln1 \times \frac{-1}{6}(x^3 - 9x^2 + 26x - 24) + ln2 \times \frac{1}{2}(x^3 - 8x^2 + 19x - 12) + ln3 \times \frac{-1}{2}(x^3 - 7x^2 + 14x - 8) + ln4 \times \frac{1}{6}(x^3 - 6x^2 + 11x - 6)$$

$$= ln2 \times \frac{1}{2}(x^3 - 8x^2 + 19x - 12) + ln3 \times \frac{-1}{2}(x^3 - 7x^2 + 14x - 8) + ln4 \times \frac{1}{6}(x^3 - 6x^2 + 11x - 6)$$

$$= x^3\left(\frac{0.69315}{2} - \frac{1.09861}{2} + \frac{1.38629}{6}\right) - x^2\left(4 \times 0.69315 - \frac{7 \times 1.09861}{2} + 1.38629\right) + x\left(\frac{19 \times 0.69315}{2} - 7 \times 0.69315 + \frac{11 \times 1.38629}{6}\right) - 6 \times 0.69315 + 4 \times 1.09861 - 1.38629$$

$$= 0.02832x^3 - 0.313755x^2 + 4.27441x - 1.15075$$

QUESTION 02

When we interpolate the function $f(x) = 1$, the interpolation polynomial in Lagrange form is

$$P(x) = \sum_{i=1}^{n} f(x_i) l_i(x) = \sum_{i=1}^{n} l_i(x)$$

Since $f(x) = 1$ the perfectly interpolated polynomial will be $P(x) = 1$

This is the zeroth-order polynomial

Since $P(x) = 1$

$$P(x) = \sum_{i=1}^{n} l_i(x) = 1$$

QUESTION 03

(a)

```
function [] = LagrangeInterpolant(x,y)
i = 1;
q = 1;
syms a;
pa = 0;

while (i <= length(x))
      j = 1;
      while (j <= length(x))
            if i ~= j
                  q = q*(a - x(j))/(x(i) - x(j));
            end
            j = j + 1;
      end
      pa = pa + q * y(i);
      i = i + 1;
      q = 1;
end

disp(simplify(pa));
ezplot(pa);
hold;
plot(x,y,'ro');
```

(b)

LagrangeInterpolant([0 1/2 1],[0 1/4 1])

a^2

Yes, we get the answer as expected which is $\boldsymbol{a^2}$

$$l_0(x) = \left(\frac{x - {}^1\!/_2}{0 - {}^1\!/_2}\right)\left(\frac{x - 1}{0 - 1}\right)$$

$$= 2x^2 - 3x + 1$$

$$l_1(x) = \left(\frac{x - 0}{{}^1\!/_2 - 0}\right)\left(\frac{x - 1}{{}^1\!/_2 - 1}\right)$$

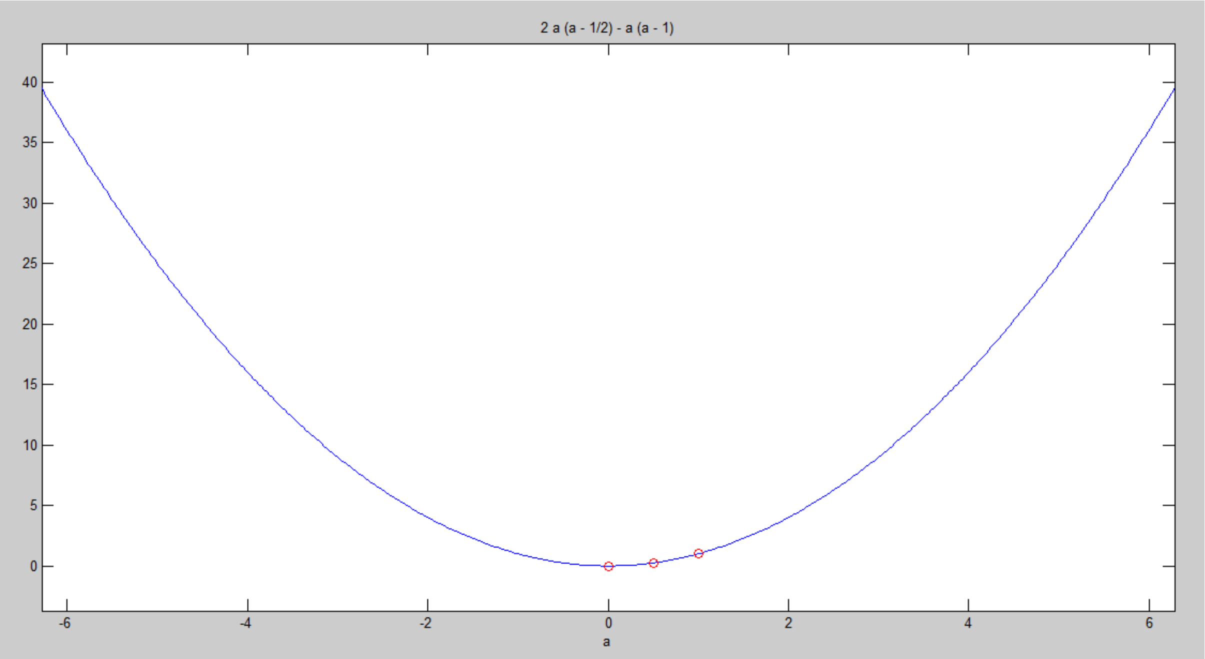
$$= -4x^2 + 4x$$

$$l_1(x) = \left(\frac{x - 0}{1 - 0}\right)\left(\frac{x - {}^1\!/_2}{1 - {}^1\!/_2}\right)$$

$$= 2x^2 - x$$

$$P(x) = 0(2x^2 - 3x + 1) + \frac{1}{4}(-4x^2 + 4x) + 2x^2 - x$$

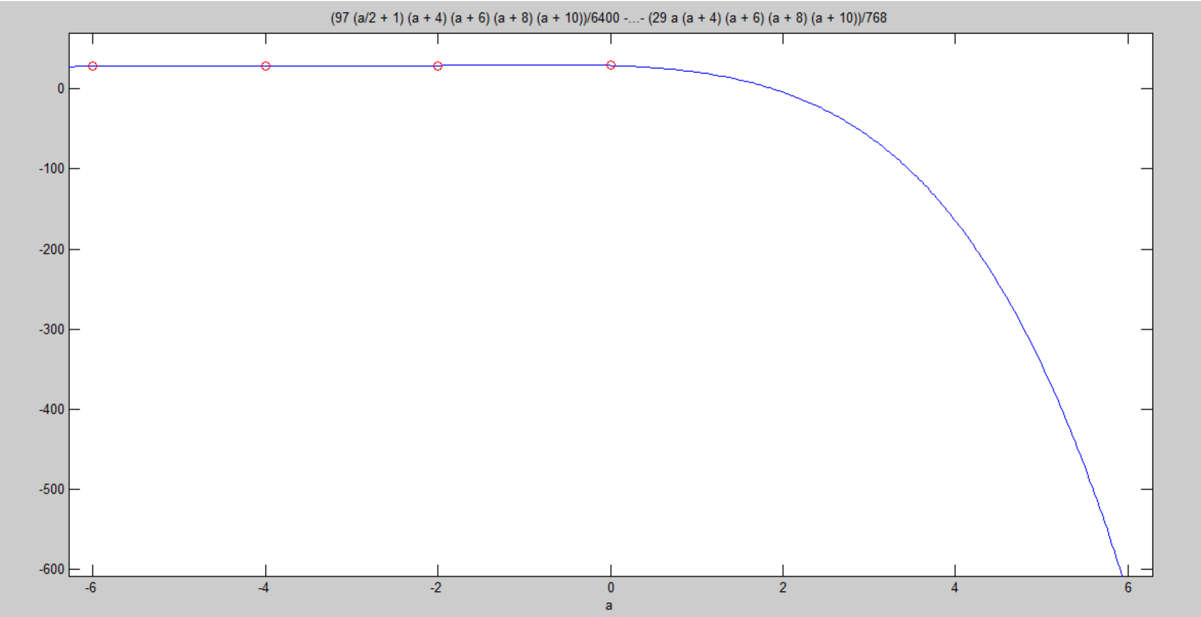
$$P(x) = x^2$$



QUESTION 04

(a)

LagrangeInterpolant([0 -2 -4 -6 -8 -10],[29.1 29 28.7 28.2 20.7 19.1])



(b)

By using the above code with this command `a = sym(-7);`

We can obtain the Temperature as $\frac{12949}{512}$ = 25.3 ℃

But if we obtain this from the given table

$$\frac{6-7}{6-8} = \frac{28.2-x}{28.2-20.7}$$

$$x = 24.45$$

Since the 2 answer are somewhat close, we can say that the answer we got from the Lagrange interpolate code is valid.

(c)

```
function [] = LagrangeInterpolant(x,y)
i = 1;
q = 1;
syms a;
pa = 0;

while (i <= length(x))
      j = 1;
      while (j <= length(x))
            if i ~= j
                  q = q*(a - x(j))/(x(i) - x(j));
            end
            j = j + 1;
      end
      pa = pa + q * y(i);
      i = i + 1;
      q = 1;
end


t = simplify(diff(pa));

eqn = t == 0;

sola = solve(eqn,a);

disp(sola);
```

As answer we get 4 data points,

-0.71219561215024224440289385707655
-3.0902449602122413385089077426 37
-4.9618529705648865831598105669069
-9.2885366457518751169472557579079

So maximum z value would be -9.2885366457518751169472557579079 m