EM314 -ASSIGNMENT 02

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E/15/202

SEMESTER 4

QUESTION 1

According to the convergence theorem of bisection method

$$\tau > \frac{b_k - a_k}{2}$$

$$\tau > \frac{b-a}{2^{k+1}}$$

$$2^{k+1} > \frac{b-a}{\tau}$$

$$(k+1)log_2 2 > log_2\left(\frac{b-a}{\tau}\right)$$

$$k+1 > log_2\left(\frac{b-a}{\tau}\right)$$

$$k > log_2\left(\frac{b-a}{\tau}\right) - 1$$

QUESTION 2

(a) If g is a contraction in the [ln1.1 , ln3] range then $|\ g(p)-g(q)\ | < \ |p-q|$ for

p, q $\in$[ln1.1 , ln3] and p $\neq$ q

Since $|\ g(ln1.1)-g(ln3)\ |$ is the largest value that can have , to be a contraction this value should be smaller than $|ln1.1-ln3|$

$$|\ g(ln1.1)-g(ln3)\ |$$

$$= |e^{-ln1.1}-e^{-ln3}|$$

$$= 0.58 < |ln1.1-ln3| < \ 1.0$$

Therefore g is a contraction in the $[ln1.1\ ,\ ln3]$ closed interval.

(b) Choose p such that $ln1.1 \leq p \ \leq ln3$

$$e^{-p} = q$$

$$-plne = lnq$$

$$-p = \ln(q)$$

$$p = \ -\ln(q)$$

Since p $\in$ G

-ln(q) $\in$ G

Therefore $\ln(1.1) \leq \ -\ln(q) \leq \ln(3)$

The greatest value q can have, when p is in G is $e^{-1.1} = 0.90909$ and the smallest value q can have is $e^{-3} = 0.333$ when p is in G

But ln(1.1) = 0.0953   and ln(3) = 1.0986

$$0.0953 < 0.90909 < 1.0986$$

$$0.0953 < 0.333 < 1.0986$$

Therefore $g\colon G \rightarrow G$

(c) $|x_{k+1} - x_k| \leq L^k |x_1 - x_0|$

$|g(x_k) - x_k| \leq L^k |x_1 - x_0|$

$x_{k+1} = g(x_k)$

$L \in [0,1)$

Since the $0 \leq L < 1$

When k is large

$L^k \to 0$

$g(x_k) - x_k \to 0$

Therefore $g(x_k) \to x_k$

Therefore x_k is the fixed point

QUESTION 3

(a) $x_{k+1} = g(x_k)$ ———— (1)

$x_* = g(x_*)$ ———— (2)

(1) – (2) $\Rightarrow$ $x_{k+1} - x_* = g(x_{k+1}) - g(x_*)$

$x_{k+1} - x_* = g'(c_n)(x_{k+1} - x_*)$ [Using taylor series for $g(x_{k+1})$]

c_n is between x_* and x_k

$|x_{k+1} - x_*| \leq \lambda |x_{k+1} - x_*|$

$\frac{x_* - x_{k+1}}{x_* - x_k} = g'(c_n)$

Since $x_k \to x_*$ and c_n is between x_* and x_k we have $g'(c_n) \to g(x_*)$

Therefore we can write

$x_{k+1} - x_* = g'(x_*)(x_{k+1} - x_*)$

This tell us that near to the root x_* , the errors will decrease by a constant factor $g'(x_*)$

And also this tell what happens when $|g'(x_*)| > 1$

Then errors will increase as we approach to the root rather than decreasing the size.

In this question $x_* = 0$ and $g'(x_*) = \frac{2}{1+4x^2}$, then $g'(0) = 2$

Therefore if we use fixed point iteration, it will diverge instead of converge as the error

get increased by a factor of 2.

(b) (i) $x = 1.16$ (fixed point)

$x_0 = 2$

$\boldsymbol{k = 0}$

$x_1 = \tan^{-1}(2x_0) = 1.3258$

$e_0 = |1.16 - 1.3258| = 0.1658$

$\boldsymbol{k = 1}$

$x_2 = \tan^{-1}(2x_1) = \tan^{-1} 2.6516 = 1.2102$

$e_1 = |1.16 - 1.2102| = 0.0502$

$\boldsymbol{k = 2}$

$x_3 = \tan^{-1}(2x_2) = \tan^{-1} 2.4204 = 1.1790$

$e_2 = |1.16 - 1.1790| = 0.019$

(ii) At fixed point $tan^{-1}(2x) = x$

Therefore let's take $f(x) = tan^{-1}(2x) - x$

$\boldsymbol{k = 0}$

$$x_1 = x_0 - \frac{tan^{-1}(2x_0) - x_0}{\frac{2}{1+4x_0^2} - 1} = 2 - \frac{tan^{-1}(4) - 2}{\frac{2}{1+16} - 1} = 1.2356$$

$e_0 = |1.16 - 1.2356| = 0.0756$

$\boldsymbol{k = 1}$

$$x_2 = x_1 - \frac{tan^{-1}(2x_1) - x_1}{\frac{2}{1+4x_1^2} - 1} = 1.2356 - \frac{tan^{-1}(2.4712) - 1.2356}{\frac{2}{1+6.1068} - 1} = 1.1669$$

$e_1 = |1.16 - 1.1669| = 0.0069$

$\boldsymbol{k = 2}$

$$x_3 = x_2 - \frac{tan^{-1}(2x_2) - x_2}{\frac{2}{1+4x_2^2} - 1} = 1.1669 - \frac{tan^{-1}(2.3338) - 1.1669}{\frac{2}{1+5.4466} - 1} = 1.1656$$

$e_2 = |1.16 - 1.1656| = 0.0056$

QUESTION 4

(a)

```
function[zero,res,niter] = newton(f,df,x0,tol,nmax)
niter = 0;
x = x0 - f(x0)/df(x0);
e = abs(x -0.8284);
while abs(x - x0) >= tol && niter <= nmax
    ne = e;
    x0 = x;
    x = x0 - f(x0)/df(x0);
    res = abs(x - x0)
    e = abs(x - 0.8284);
    q = e/ne.^2
    niter = niter + 1;
end

zero = x
niter

if niter > nmax
    fprintf('Newtons method stop without convergence');
end
```

(b) Yes

The answer is 0.8284

(c)

k	x_k	$e_k = \|x_k - x_*\|$	e_k / e_{k-1}^2
1	23.5980	22.7696	0.0098
2	10.9553	10.1269	0.0195
3	4.7864	3.9580	0.0386
4	1.9826	1.1542	0.0737
5	0.9957	0.1673	0.1256
6	0.8331	0.0047	0.1678
7	0.8284	3.0971e-05	1.4047
8	0.8284	2.7125e-05	2.8278e+04

Yes.

From the last column we can see that the increase is getting higher and higher.

(d)

k	x_k	$e_k = \lvert x_k - x_* \rvert$	e_k / e_{k-1}^2
1	23.5980	22.7696	0.0098
2	10.9553	10.1269	0.0195
3	4.7864	3.9580	0.0386
4	1.9826	1.1542	0.0737
5	0.9957	0.1673	0.1256
6	0.8331	0.0047	0.1678
7	0.8284	3.0971e-05	1.4047
8	0.8284	2.7125e-05	2.8278e+04
9	0.8284	2.7125e-05	3.6867e+04

This shows the quadratic convergence. We can see that near the real solution, the increase of e_k / e_{k-1}^2 is very high.

QUESTION 5

```
f(x) = 0.8*sin(x) - x + 3

df(x) = 0.8*cos(x) - 1
```

The answer is $x = 3.0629$
And number of iterations $niter = 15$

Solution

newton(@f,@df,100,10.^-8,40)

res =

789.6413

res =

351.8600

res =

135.7533

res =

424.8231

res =

211.7088

res =

423.6056

res =

164.1453

```
res =

   83.2772

res =

  111.8004

res =

   46.7327

res =

   17.6963

res =

    0.6260

res =

    0.0265

res =

   1.5076e-05

res =

   3.9759e-12

zero =

    3.0629
```

QUESTION 6

```
t = (3.5.*10.^7 + 0.401.*(1000./x).^2).*(x - 1000.*42.7.*10.^-6) -
1.3806503.*(10.^-23).*1000.*300;
```

The answer is $V = 0.0427m^3$

Solution

bisection(@f,-1,2,10.^-12,100)

zero =

0.0427

res =

-6.6125e-05

ans =

0.0427