EM 314 – ASSIGNMENT - ODE

E/15/202

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QUESTION 01

Here y0 → Initial value of y

h → step size

end_val → y(end value)

start → y(start)

f → is the differentiated function

Since the functions are in x or y syms x y used. And values are calculated for x and y

(A)

```
function [] = ForwardEuler(y0,h,end_val,start)
x0 = start;
n = 0;

syms x y
%y = -0.5*x^4 + 4*x^3 - 10*x^2 + 8.5*x + 1;

%f = diff(y,x);
%f = x + y;
%f = -2*x^3 + 12*x^2 -20*x + 8.5;

Y_Euler = y0;

x1 = x0 + n*h;

while x1 < end_val
   g = subs(f,x,x1);
   Y_Euler = Y_Euler + h * subs(g,Y_Euler);

   a = vpa(Y_Euler,6)

   n = n + 1;
   x1 = x0 + n*h;
end
```

(B)

```
function [] = ImprovedEuler(y0,h,end_val,start)
x0 = start;
n = 0;

syms x y
%y = -0.5*x^4 + 4*x^3 - 10*x^2 + 8.5*x + 1;

%f = diff(y,x);
%f = x + y;
%f = -2*x^3 + 12*x^2 -20*x + 8.5;

b = y0;

x1 = x0 + n*h;

while x1 < end_val
   g = subs(f,x,x1);
   hy = b;
   b = b + h * subs(g,b);

   x2 = x0 + (n+1)*h;
   k = subs(f,x,x2);

   Y_Avg = (subs(g,hy) + subs(k,b))/2;

   Y_ImproveEuler = hy + h * Y_Avg;
   b = Y_ImproveEuler;
   a = vpa(Y_ImproveEuler,6)
   n = n + 1;
   x1 = x0 + n*h;
end
```

(C)

```
function [] = RungeKutta(y0,h,end_val,start)
x0 = start;
n = 0;

syms x y
%y = -0.5*x^4 + 4*x^3 - 10*x^2 + 8.5*x + 1;

%f = diff(y,x);
%f = x + y;
%f = -2*x^3 + 12*x^2 -20*x + 8.5;

Y_RK = y0;

x1 = x0 + n*h;

while x1 < end_val
    g = subs(f,x,x1);
    j = subs(g,Y_RK);

    x2 = (x1 + (x0 + (n+1)*h))/2;

    x3 = x0 + (n+1)*h;

    k0 = h * j;

    g = subs(f,x,x2);
    j = subs(g,Y_RK + 1/2 * k0);

    k1 = h * j;

    g = subs(f,x,x2);
    j = subs(g,Y_RK + 1/2 * k1);

    k2 = h * j;

    g = subs(f,x,x3);
    j = subs(g,Y_RK + k2);

    k3 = h * j;

    Y_Euler = Y_RK + 1/6 * (k0 + 2*k1 + 2*k2 + k3);

    a = vpa(Y_RK,6)

    n = n + 1;
    x1 = x0 + n*h;
end
```

QUESTION 02

Consider $\frac{dh}{dt} = \frac{f_i - f_0}{A}$ and $f_0 = \beta\sqrt{h}$

h → y

t → x

then $\frac{dy}{dx} = \frac{f_i - \beta\sqrt{y}}{A}$

$$\frac{dy}{dx} = 10 - 8\sqrt{y}$$

(A)

```
function [] = ForwardEuler(y0,h,end_val,start)
x0 = start;
n = 0;

syms x y

f = 10 - 8*sqrt(y);


Y_Euler = y0;

x1 = x0 + n*h;

while x1 < end_val
   g = subs(f,x,x1);
   Y_Euler = Y_Euler + h * subs(g,Y_Euler);

   a = vpa(Y_Euler,6)

   n = n + 1;
   x1 = x0 + n*h;
end
```

Output:

ForwardEuler(0,0.2,1.2,0)

y(0)	0
y(0.2)	2.0
y(0.4)	1.73726
y(0.6)	1.62838
y(0.8)	1.58665
y(1.0)	1.57125
y(1.2)	1.56566

ForwardEuler(0,0.1,1.2,0)

y(0)	0
y(0.1)	1.0
y(0.2)	1.2
y(0.3)	1.32364

y(0.4)	1.40325
y(0.5)	1.45558
y(0.6)	1.4904
y(0.7)	1.51374
y(0.8)	1.52947
y(0.9)	1.5401
y(1.0)	1.54729
y(1.1)	1.55217
y(1.2)	1.55548

(B)

```
function [] = ImprovedEuler(y0,h,end_val,start)
x0 = start;
n = 0;

syms x y
f = 10 - 8*sqrt(y);

b = y0;

x1 = x0 + n*h;

while x1 < end_val
   g = subs(f,x,x1);
   hy = b;
   b = b + h * subs(g,b);

   x2 = x0 + (n+1)*h;
   k = subs(f,x,x2);

   Y_Avg = (subs(g,hy) + subs(k,b))/2;

   Y_ImproveEuler = hy + h * Y_Avg;
   b = Y_ImproveEuler;
   a = vpa(Y_ImproveEuler,6)
   n = n + 1;
   x1 = x0 + n*h;
end
```

Output:

ImprovedEuler(0,0.2,1.2,0)

y(0)	0
y(0.2)	0.868629
y(0.4)	1.18412
y(0.6)	1.35257
y(0.8)	1.44506
y(1.0)	1.49652
y(1.2)	1.52535

(C)

```
function [] = RungeKutta(y0,h,end_val,start)
x0 = start;
n = 0;

syms x y
f = 10 - 8*sqrt(y);

Y_RK = y0;

x1 = x0 + n*h;

while x1 < end_val
    g = subs(f,x,x1);
    j = subs(g,Y_RK);

    x2 = (x1 + (x0 + (n+1)*h))/2;

    x3 = x0 + (n+1)*h;

    k0 = h * j;

    g = subs(f,x,x2);
    j = subs(g,Y_RK + 1/2 * k0);

    k1 = h * j;

    g = subs(f,x,x2);
    j = subs(g,Y_RK + 1/2 * k1);

    k2 = h * j;

    g = subs(f,x,x3);
    j = subs(g,Y_RK + k2);

    k3 = h * j;

    Y_RK = Y_RK + 1/6 * (k0 + 2*k1 + 2*k2 + k3);

    a = vpa(Y_RK,6)



    n = n + 1;
    x1 = x0 + n*h;
end
```

Output:

RungeKutta(0,0.2,1.2,0)

y(0)	0
y(0.2)	0.925929

y(0.4)	1.24427
y(0.6)	1.39866
y(0.8)	1.47706
y(1.0)	1.51767
y(1.2)	1.53891

(a)with h = 0.1

```
plot([0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2],[0
1.0 1.2 1.32364 1.40325 1.45558 1.4904 1.51374 1.52947 1.5401
1.54729 1.55217 1.55548],'red')
```

with h = 0.2

```
plot([0 0.2 0.4 0.6 0.8 1.0 1.2],[0 2.0 1.73726 1.62838
1.58665 1.57125 1.56566],'blue')
```

(b)

```
plot([0 0.2 0.4 0.6 0.8 1.0 1.2],[0 0.868629 1.18412
1.35257 1.44506 1.49652 1.52535],'green')
```

(c)

```
plot([0 0.2 0.4 0.6 0.8 1.0 1.2],[0 0.925929 1.24427
1.39866 1.47706 1.51767 1.53891],'black')
```