Assignment 1 EE387

E/15/202

Problem 1

a) $x(t) = 3\cos(10t+1) - \sin(4t-1)$

$x_1(t) = 3\cos(10t+1)$ $x_2(t) = \sin(4t-1)$

$\omega_1 = 10$ $\omega_2 = 4$

$T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$ $T_2 = \frac{2\pi}{4} = \frac{\pi}{2}$

Fundamental period T_0

$$T_0 = LCM\left(\frac{\pi}{5}, \frac{\pi}{2}\right) = \frac{LCM(\pi,\pi)}{HCF(5,2)} = \pi //$$

b) $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$

Period of 1 is undefined.

$x_1(t) = e^{j4\pi n/7}$ $x_2(t) = e^{j2\pi n/5}$

$\omega_1 = \frac{4\pi}{7}$ $\omega_2 = \frac{2\pi}{5}$

$T_1 = 2\pi \times \frac{7}{4\pi} = \frac{7}{2}$ $T_2 = 2\pi \times \frac{5}{2\pi} = 5$

Fundamental period T_0

$$T_0 = LCM\left(\frac{7}{2}, 5\right) = \frac{LCM(7,5)}{HCF(2,1)} = 35 //.$$

Problem 2

a) Assume $x(t)$ - is periodic

Then fundamental period $T_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$x(t+T_0) = x(t+\pi/2) = 2\cos\left(4\left(t+\pi/2\right)+\pi/3\right)$$

$$= 2\cos\left(4t + 2\pi + \pi/3\right)$$

$$= 2\cos\left(4t + \pi/3\right) = x(t).$$

$\therefore$ This signal is periodic.

Fundamental period $= \pi/2$ //.

b) $x(t) = \left[\sin\left(2t - \frac{\pi}{4}\right)\right]^2 = \frac{1 - \sin 2\left(2t - \frac{\pi}{4}\right)}{2}$

$= \frac{1}{2} - \frac{1}{2}\sin\left(4t - \frac{\pi}{2}\right)$

Assume $x(t)$ is periodic

then fundamental period $T_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$x(t+T_0) = x(t + \pi/2) = \frac{1}{2} - \frac{1}{2}\sin\left(4(t+\pi/2) - \pi/2\right)$

$= \frac{1}{2} - \frac{1}{2}\sin\left(4t + 2\pi - \pi/2\right)$

$= \frac{1}{2} - \frac{1}{2}\sin\left(4t - \pi/2\right)$

$= \left[\sin\left(2t - \pi/4\right)\right]^2 = x(t)$

$\therefore$ $x(t)$ is periodic

Period $= \pi/2$ //

2 c)

$x[n] = \sin\left(\frac{6\pi}{7} n + 1\right)$

For $x[n]$ to be periodic $x[n] = x[n+N]$

$$\sin\left(\frac{6\pi}{7} n + 1\right) = \sin\left(\frac{6\pi}{7}(n+N) + 1\right)$$

$$\frac{6\pi n}{7} + 1 + 2\pi k = \frac{6\pi}{7}(n+N) + 1 \qquad k \in \mathbb{Z}$$

$$k = \frac{3}{7} N$$

$$N = \frac{7}{3} k$$

~~$\therefore$ $x[n]$ is periodic with $N = \frac{7}{3}$~~

$\cancel{= \sin\left(\frac{6\pi}{7} + 1\right) = x[n]}$

~~$\therefore x[n]$ is periodic with $N = \frac{7}{3}$~~ //.

Since $x[n]$ is a discrete function
N should be an integer value
$\therefore N = 7$

$$x[n+N] = x[n+7]$$
$$= \sin\left(\frac{6\pi}{7}(n+7) + 1\right)$$
$$= \sin\left(\frac{6\pi}{7} + 6\pi + 1\right) = \sin\left(\frac{6\pi}{7} + 1\right) = x[n]$$

$\therefore x[n]$ is periodic with $N = 7$ //

$\cancel{x[n] = \cos\left(\frac{\pi}{8} n^2\right)}$

② c) $x[n] = \sin\left(6\frac{\pi}{7}n+1\right) = \sin\left(\frac{42+\pi}{7}n+1\right)$

$$= \sin\left(\frac{42n}{7}+\frac{\pi}{7}n+1\right)$$

$$= \sin\left(6n+\frac{\pi}{7}n+1\right)$$

$$= \sin\left(\left(6+\frac{\pi}{7}\right)n+1\right)$$

For $x[n]$ to be periodic $x[n] = x[n+N]$

$$\sin\left(\left(6+\frac{\pi}{7}\right)n+1\right) = \sin\left(\left(6+\frac{\pi}{7}\right)(n+N)+1\right)$$

$$\left(6+\frac{\pi}{7}\right)n+1+2\pi k = \left(6+\frac{\pi}{7}\right)(N+n)+1 \; ; K \in \mathbb{Z}$$

$$2\pi k = \left(6+\frac{\pi}{7}\right)N$$

$$N = \frac{14\pi}{42+\pi}k$$

Since π is a rational number, N would never be an integer value for any K.

$\therefore$ $x[n]$ would never be periodic.

$x[n]$ is aperiodic //

$\therefore$ $x(t)$ is an energy signal.

2) d) $x[n] = \cos\left(\frac{\pi}{8} n^2\right)$

For DT function $x[n]$, we need to find a finite, non-zero integer N s.t $x[n] = x[n+N]$ of all n
The smallest integer N for which this holds is the fundamental period.

$\therefore$ We need. $\cos\left(\frac{\pi}{8} n^2\right) = \cos\left(\frac{\pi}{8}(n+N)^2\right)$

$$\frac{\pi}{8} n^2 + 2\pi k = \frac{\pi}{8}\left(n^2 + 2nN + N^2\right) \quad k \in \mathbb{Z}$$

$$2k = \frac{1}{8}\left(2nN + N^2\right)$$

$$N^2 + 2nN = 16k$$

for any value n, when $N = 8$

$$64 + 16n = 16k$$
$$\underbrace{16(n+4)}_{\mathbb{Z}} = 16k$$

$$\cos\left(\frac{\pi}{8}(n+8)^2\right) = \cos\left(\frac{\pi}{8}\left(n^2 + 16n + 64\right)\right)$$
$$= \cos\left(\frac{\pi}{8}n^2 + 2\pi n + 8\pi\right)$$

Since $n \in \mathbb{Z}$

~~then~~

$$\cos\left(\frac{\pi}{8}n^2 + 2\pi n + 8\pi\right) = x[n]$$

$\therefore$ $x[n]$ is periodic with a period $N = 8$ //

$\therefore x[n]$ is periodic with $N = 1$ //

(~~d~~) ~~$x[n] = \cos\left(\frac{\pi}{8} n^2\right)$~~

(e) For $x(t) = \sin\left(\frac{\pi}{8} t^2\right)$

for $x(t)$ to be periodic $x(t) = x(t+T)$ where T is the fundamental period

For that,

$$\sin\left(\frac{\pi}{8} t^2\right) = \sin\left(\frac{\pi}{8}(t+T)^2\right)$$

$$= \sin\left(\frac{\pi}{8}(t^2 + 2tT + T^2)\right) \longrightarrow 2$$

since t is not an integer, equation 2 will not hold for every t value (What ever the T value is)

$\therefore x(t)$ is not periodic.

$x(t)$ is aperiodic. //

(f) $x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

$= \frac{1}{2}\left\{\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right)n + \cos\left(\frac{\pi}{2}-\frac{\pi}{4}\right)n\right\}$

$x[n] = \frac{1}{2}\left\{\cos\frac{3\pi}{4}n + \cos\frac{\pi}{4}n\right\}$

Assume $x(n)$ is periodic

~~Then~~

$x_1(t) = \cos\frac{3\pi}{4}n$ $\qquad$ $x_2(t) = \cos\frac{\pi}{4}n$

$T_1 = 2\pi \times \frac{4}{3\pi} = \frac{8}{3}$ $\qquad$ $T_2 = 2\pi \times \frac{4}{\pi} = 8$

Then fundamental period would be

$T_0 = \text{LCM}(T_1, T_2) = \text{LCM}\left(\frac{8}{3}, 8\right) = \frac{\text{LCM}(8,8)}{\text{HCF}(3,1)} = 8$

So if $N = 8$

if $x[n]$ is periodic $x[n] = x[n+N]$

$x[n+8] = \cos\left(\frac{\pi}{2}(n+8)\right)\cos\left(\frac{\pi}{4}(n+8)\right)$

$= \cos\left(\frac{\pi}{2}n + 4\pi\right)\cos\left(\frac{\pi}{4}n + 2\pi\right)$

$= \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

$= x[n]$

$\therefore x(n)$ is periodic and period $= 8$ //.

③ (a) $x[n] = \begin{cases} \cos \pi n & n \geq 0 \\ 0 & o/w \end{cases}$

When $n \geq 0$, $x[n]$ is periodic with $n = 2$

$\therefore x[n]$ is a power signal.

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left| \cos \pi n \right|^2$$

Since for every n, $n \in Z^+$

$n = 0, 1, 2, \ldots$

Then $\cos \pi n = \cos 0' = 1$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1^2$$

$$P_\infty = \lim_{N \to \infty} \frac{N-0+1}{2N+1} = \lim_{N \to \infty} \frac{1 + 1/N}{2 + 1/N} = \frac{1}{2}$$

$\therefore x[n]$ is a power signal //.

(b) $x(t) = \begin{cases} \frac{1}{2}(\cos \omega t + 1) & -\pi/\omega < t < \pi/\omega \\ 0 & o/w \end{cases}$

Fundamental period of $\frac{1}{2}(\cos \omega t + 1)$

$T_0 = \frac{2\pi}{\omega}$

$\therefore -\frac{\pi}{\omega} < t < \frac{\pi}{\omega}$ there is only 1 period.

$\therefore \frac{1}{2}(\cos \omega t + 1)$ is not periodic within $-\frac{\pi}{\omega} < t < \pi/\omega$.

$\therefore x(t)$ is an energy signal.

$$E_\infty = \int_{-\pi/\omega}^{\pi/\omega} \left| \frac{1}{2}(\cos\omega t + 1) \right|^2 dt$$

$$= \frac{1}{4} \int_{-\pi/\omega}^{\pi/\omega} \left(\cos^2 \omega t + 2\cos\omega t + 1 \right) dt$$

$$= \frac{1}{4} \int_{-\pi/\omega}^{\pi/\omega} \frac{1+\cos 2\omega t}{2} dt + \frac{1}{2} \int_{-\pi/\omega}^{\pi/\omega} \cos\omega t + \frac{1}{4} \int_{-\pi/\omega}^{\pi/\omega} dt.$$

$$= \frac{1}{8} \left(t \right)_{-\pi/\omega}^{\pi/\omega} + \frac{1}{4} \left(t \right)_{-\pi/\omega}^{\pi/\omega}$$

$$= \frac{1}{8} \left(\frac{\pi}{\omega} + \frac{\pi}{\omega} \right) + \frac{1}{4} \left(\frac{\pi}{\omega} + \frac{\pi}{\omega} \right)$$

$$= \frac{\pi}{4\omega} + \frac{2\pi}{4\omega} = \frac{3\pi}{4\omega} // \quad < \infty.$$

$\therefore$ $x(t)$ is an energy signal //

~~(a) d)~~

(a) d) $x[n] = \cos(\pi n^2)$

Problem 4

$$x(t) = \begin{cases} \sin(\pi t) & 0 \le t \le 2 \\ 0 & 2 < t \le 4 \end{cases} \qquad \text{Period} = 4$$

Fundamental frequency $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$a_k = \frac{1}{4}\int_0^4 x(t)\, e^{-j\frac{\pi}{2}kt}\, dt$$

$$a_k = \frac{1}{4}\left\{ \int_0^2 \sin \pi t\, e^{-j\frac{\pi}{2}kt}\, dt + \int_2^4 0\, dt \right\}$$

$$a_k = \frac{1}{4}\int_0^2 \sin \pi t\, e^{-j\frac{\pi}{2}kt}\, dt$$

When $k=0$

$$a_0 = \frac{1}{4}\int_0^2 \sin \pi t\, dt$$

Fundmental period of $\sin \pi t = 2\pi \times \frac{1}{\pi} = 2$

$\therefore a_0 = 0$

$$\cos \pi t + j \sin \pi t = e^{j\pi t} \longrightarrow ①$$

$$\cos \pi t - j \sin \pi t = e^{-j\pi t} \longrightarrow ②$$

$$① - ② \Rightarrow 2j \sin \pi t = e^{j\pi t} - e^{-j\pi t}$$

$$\sin \pi t = \frac{1}{2j} e^{j\pi t} - \frac{1}{2j} e^{j\pi t}$$

$$= \frac{1}{2j} e^{j2\frac{\pi}{2}t} - \frac{1}{2j} e^{j2\frac{\pi}{2}t}$$

$\therefore a_2 = \frac{1}{2j} \quad a_{-2} = -\frac{1}{2j}$

$a_0 = a_1 = a_{-1} = 0$

$a_2 = \frac{1}{2j}, \quad a_{-2} = -\frac{1}{2j}$

Problem 5

Fundamental period $T = \frac{1}{2}$

$$\omega_0 = \frac{2\pi}{T} = 2\pi \times 2 = 4\pi$$

(a) $x(t) = \cos 4\pi t$

$$a_K = 2 \int_{1/2} \cos 4\pi t \, e^{-j4\pi t k} \, dt$$

$K = 0$

$$a_0 = 2 \int_{1/2} \cos 4\pi t \, dt = 0$$

$$\cos 4\pi t + j\sin 4\pi t = e^{j4\pi t} \longrightarrow ①$$

$$\cos 4\pi t - j\sin 4\pi t = e^{-j4\pi t} \longrightarrow ②$$

$$①+② \Rightarrow 2\cos 4\pi t = e^{j4\pi t} + e^{-j4\pi t}$$

$$\cos(4\pi t) = \frac{1}{2} e^{j4\pi t} + \frac{1}{2} e^{-j4\pi t}$$

$$\therefore a_1 = a_{-1} = \frac{1}{2}$$

$$a_0 = 0 \quad //.$$

(b) $y(t) = \sin(4\pi t)$ $\qquad \omega_0 = 4\pi$

$$a_K = 2 \int_{1/2} \sin(4\pi t) \, e^{-jK4\pi t} \, dt$$

$K = 0 \quad a_0 = 2 \int_{1/2} \sin(4\pi t) \, dt = 0$

$$\cos 4\pi t + j\sin 4\pi t = e^{j4\pi t} \longrightarrow ①$$

$$\cos 4\pi t - j\sin 4\pi t = e^{-j4\pi t} \longrightarrow ②$$

$$①-② \Rightarrow \sin 4\pi t = \frac{1}{2j} e^{j4\pi t} - \frac{1}{2j} e^{-j4\pi t}$$

$$a_1 = \frac{1}{2j} \qquad a_{-1} = -\frac{1}{2j}$$

$$a_0 = 0 \quad //.$$

(c) $z(t) = x(t)\, y(t)$

$$x(t)\,y(t) = \left(\frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t}\right)\left(\frac{1}{2j}e^{j4\pi t} - \frac{1}{2j}e^{-j4\pi t}\right)$$

$$= \frac{1}{4j}e^{j8\pi t} + \frac{1}{4j} - \frac{1}{4j} - \frac{1}{4j}e^{-j8\pi t}$$

$$= \frac{1}{4j}e^{j8\pi t} - \frac{1}{4j}e^{-j8\pi t}$$

$a_1 = \frac{1}{4j}$ $\quad a_{-1} = \cancel{+}-\frac{1}{4j}$

$a_0 = 0$ //

(d) $z(t) = \cos(4\pi t)\sin(4\pi t)$

$z(t) = \frac{1}{2}\sin 8\pi t$

Fundamental period of $z(t)$ ~~x~~ is

$T_0 = 2\pi \times \frac{1}{8\pi} = \frac{1}{4}$

Fundamental frequency $\omega_0 = 8\pi$

$$a_k = 4\int_{1/4} \frac{1}{2}\sin 8\pi t\, e^{-j8\pi k t}\, dt$$

$k = 0$

$$a_0 = 4\int_{1/4} \frac{1}{2}\sin 8\pi t\, dt = 0$$

$$\sin 8\pi t = \frac{1}{2j}e^{j8\pi t} - \frac{1}{2j}e^{-j8\pi t}$$

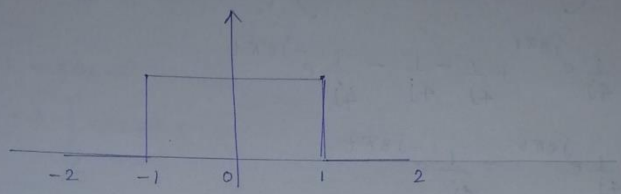
$$\frac{1}{2}\sin 8\pi t = \frac{1}{4j}e^{j8\pi t} - \frac{1}{4j}e^{-j8\pi t}$$

$a_1 = \frac{1}{4j}$ $\quad a_{-1} = -\frac{1}{4j}$ $\quad a_0 = 0$ //.

Same as (c) //.

(6) $x(t) = \begin{cases} 1 & |t| < 1 \\ 0 & 1 < |t| < 2 \end{cases}$ Period = 4

(a)



$$a_k = \frac{1}{4}\int_{-2}^{2} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{4}\left\{ \int_{-2}^{-1} x(t)\, e^{-jk\omega_0 t}\, dt + \int_{-1}^{1} x(t)\, e^{-jk\omega_0 t}\, dt + \int_{1}^{2} x(t)\, e^{-jk\omega_0 t}\, dt \right\}$$

$$= \frac{1}{4}\left\{ 0 + \int_{-1}^{1} e^{-jk\omega_0 t}\, dt + 0 \right\}$$

$$= \frac{1}{4}\int_{-1}^{1} e^{-jk\omega_0 t}\, dt$$

When $k = 0$

$$a_0 = \frac{1}{4}\left(t\right)_{-1}^{1} = \frac{2}{4} = \frac{1}{2}$$

$k \neq 0$

$$a_k = \frac{1}{4}\left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right)_{-1}^{1}$$

$$a_k = \frac{1}{4}\left(\frac{e^{-jk\omega_0} - e^{jk\omega_0}}{-jk\omega_0}\right) = \frac{2}{4k\omega_0}\left(\frac{e^{jk\omega_0} - e^{-jk\omega_0}}{2j}\right)$$

$$a_k = \frac{1}{2k\omega_0}\sin k\omega_0 = \frac{1}{2}\,\frac{\sin k\omega_0}{k\omega_0}$$

$\therefore$ When $k\omega_0 = m\pi$ where $m = \pm 1; \pm 2; \ldots$

$$\frac{\sin k\omega_0}{k\omega_0} = 0$$

$$\boxed{\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \pi/2}$$

$k\omega_0 = m\pi$

$$k = \frac{m\pi}{\omega_0} = m\pi \times \frac{2}{\pi} = 2m \qquad k = 2m$$

$$a_k = \frac{1}{2}\,\frac{\sin\left(k\pi/2\right)}{k(\pi/2)} = \frac{\sin\left(k\pi/2\right)}{k\pi}$$

$k = 0$

$$a_0 = \frac{1}{2}$$

$k \neq 0$

$$a_k = \frac{\sin\left(k\pi/2\right)}{k\pi}$$

<u>Problem 6</u>

b)
```
t=linspace(-2,2);
N = [1,3,7,19,43,79];
asex6(t,N);

function y = asex6(t,N)
    x = 1;
    for g = 1:length(N)
        for n = 1:length(t)
            k = -1*N(g);
            y(n) = 0;
            while k <= N(g)
                if k == 0
                    a = 0.5;
                else
```

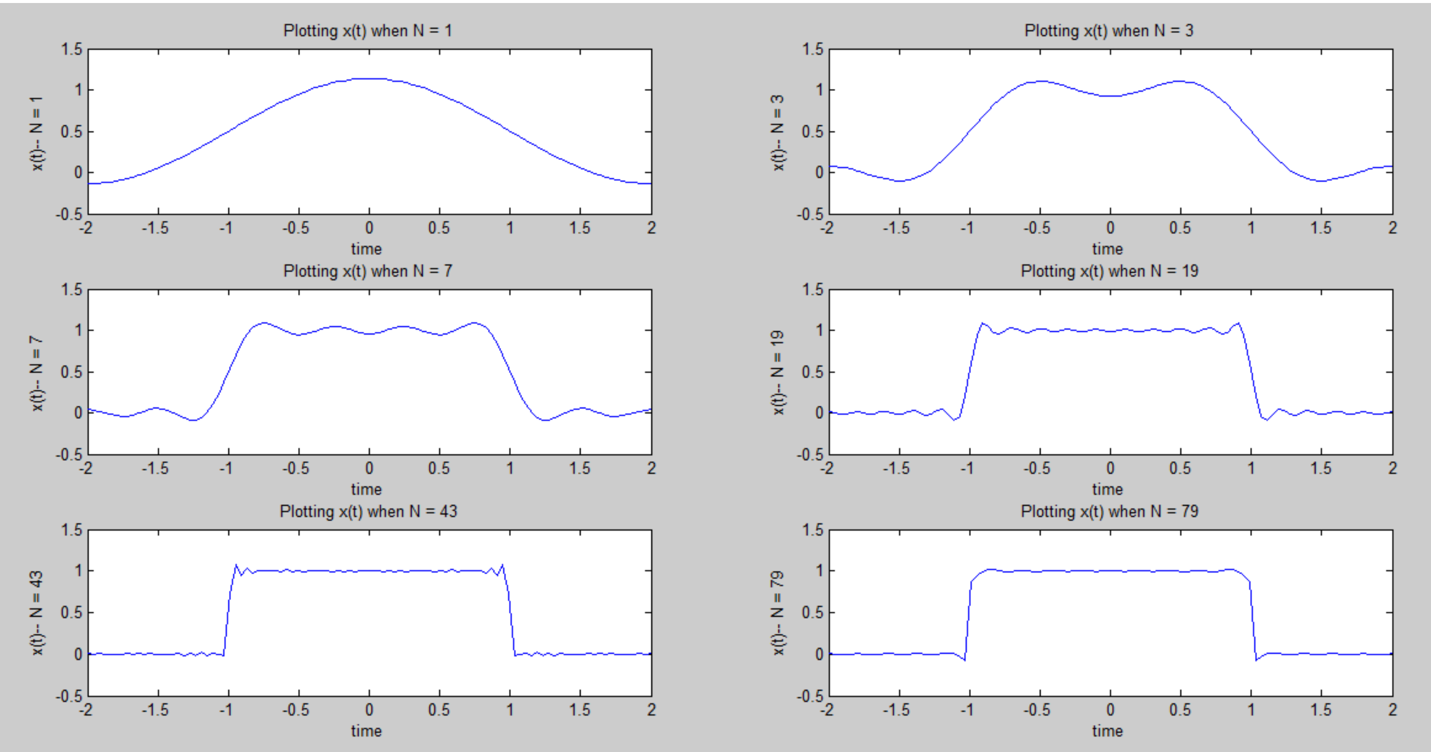
```
                a = sin(k*pi*0.5)/(k*pi);
            end

            y(n) = y(n) + a*exp(1i*pi*0.5*k*t(n));
            k = k + 1;
        end
    end

    %plotting the x(t)
    subplot(3,2,x);
    plot(t,real(y));
    xlabel('time');
    ystr = sprintf('x(t)-- N = %d',N(g));
    ylabel(ystr);
    titstr = sprintf('Plotting x(t) when N = %d',N(g));
    title(titstr);

    %calculate percentage overshoot
    povershoot = real((max(y)-1)*100);
    fprintf('percentage overshoot when N is %d =
  %.2f%%\n',N(g),povershoot);

    x = x+1;
  end
end
```



c) percentage overshoot when N is 1 = 13.63%

percentage overshoot when N is 3 = 10.02%

percentage overshoot when N is 7 = 9.21%

percentage overshoot when N is 19 = 8.55%

percentage overshoot when N is 43 = 8.39%
percentage overshoot when N is 79 = 1.43%

⑦ a) $x_1(t) = x(1-t) + x(-1-t)$

$x(-t) \xleftrightarrow{F} X(-j\omega)$ (time reversal)

$$\begin{array}{ccc} y(t) & = & x(-t) \\ F\downarrow & & \downarrow F \\ Y(j\omega) & & X(-j\omega) \end{array}$$

$y(t-1) = x(-(t-1)) = x(-t+1)$

$F\downarrow$

$Y(j\omega)e^{-j\omega}$ (time shift)

$x(-t+1) \xleftrightarrow{F} X(-j\omega)e^{-j\omega}$

$y(t+1) = x(-(t+1)) = x(-t-1)$

$F\downarrow$

$Y(j\omega)e^{j\omega}$ (time shift)

$x(-t-1) \xleftrightarrow{F} X(-j\omega)e^{j\omega}$

linearity

$x(t) = x(1-t) + x(-1-t) \xleftrightarrow{F} X(-j\omega)e^{-j\omega} + X(-j\omega)e^{j\omega}$ //

(b) $x_2(t) = x(3t-6)$

$$\begin{array}{ccc} y(t) & = & x(3t) \\ \downarrow F & & \downarrow F \\ Y(j\omega) & & \frac{1}{3} X\left(\frac{j\omega}{3}\right) \end{array}$$

$$y(t-2) = x\big(3(t-2)\big) = x(3t-6)$$

$$y(t-2) \xleftrightarrow{F} Y(j\omega)e^{-j2\omega}$$

$$x_2(t) = x(3t-6) \xleftrightarrow{F} \frac{1}{3} X\left(\frac{j\omega}{3}\right) e^{-j2\omega} \quad //.$$

(c) $x_3(t) = \frac{d^2}{dt^2} x(t-1)$

$$x(t) \xleftrightarrow{F} X(j\omega)$$

$$\begin{array}{ccc} y(t) & = & x(t-1) \\ F\downarrow & & \downarrow F \\ Y(j\omega) & & X(j\omega)e^{-j\omega} \end{array}$$

$$y(t) \xleftrightarrow{F} Y(j\omega)$$

$$\frac{d\,y(t)}{dt} \xleftrightarrow{F} j\omega Y(j\omega)$$

$$\begin{array}{ccc} z(t) & = & \frac{d}{dt} y(t) \\ F\downarrow & & \downarrow F \\ Z(j\omega) & & j\omega Y(j\omega) \end{array}$$

$$z(t) \xleftrightarrow{F} Z(j\omega)$$

$$\frac{dz(t)}{dt} \xleftrightarrow{F} j\omega Z(j\omega)$$

$$\frac{d}{dt} z(t) = \frac{d}{dt}\left(\frac{d}{dt} y(t)\right) = \frac{d^2}{dt^2} y(t) = \frac{d^2}{dt^2} x(t-1)$$

$$\downarrow F$$

$$(j\omega)(j\omega)X(j\omega)e^{-j\omega}$$

$$\frac{d^2}{dt^2} x(t-1) \xleftrightarrow{F} -\omega^2 X(j\omega)e^{-j\omega} \quad //.$$

$\frac{d}{dt^2}$ ... //

(8) $e^{-|t|} \xleftrightarrow{F} \frac{2}{1+\omega^2}$

(a) $x(t) = e^{-|t|}$

$x(t) \xleftrightarrow{F} \frac{2}{1+\omega^2} = X(\cancel{j}\omega)$

$\cancel{y(t) = tx(t)}$ $\quad y(t) = t$

$\downarrow F$

$Y(\omega)$ $\quad z(t) = y(t)\, x(t)$

F

Property multiplication with t

$t^k \cdot f(t) \xleftrightarrow{F} j^k \frac{d^k F(\omega)}{d\omega^k}$

$\cancel{f}(t) = e^{-|t|}$

$t e^{-|t|} \xleftrightarrow{F} j \frac{d\left(\frac{2}{1+\omega^2}\right)}{d\omega}$

$= j \frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right)$

$= j \frac{(1+\omega^2)(0) - 2(2\omega)}{(1+\omega^2)^2}$

$= \cancel{j} \frac{-4j\omega}{(1+\omega^2)^2}$

$t e^{-|t|} \xleftrightarrow{F} -\frac{4j\omega}{(1+\omega^2)^2}$ //

(b) Duality Property

if $x(t) \xleftrightarrow{F} X(j\omega)$

$X(t) \xleftrightarrow{F} 2\pi\, x(-j\omega)$

$x(t) = t e^{-|t|} \xleftrightarrow{F} \dfrac{-4j\omega}{(1+\omega^2)^2} = X(j\omega)$

$$t e^{-|t|} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{-4j\omega}{(1+\omega^2)^2} e^{j\omega t}\, d\omega$$

$$2\pi t e^{-|t|} = \int_{-\infty}^{\infty} \frac{-4j\omega}{(1+\omega^2)^2} e^{j\omega t}\, d\omega$$

$$2\pi j t e^{-|t|} = \int_{-\infty}^{\infty} \frac{4\omega}{(1+\omega^2)^2} e^{j\omega t}\, d\omega$$

Now interchanging the names of variables t and ω

$$2\pi \omega j e^{-|\omega|} = \int_{-\infty}^{\infty} \frac{4t}{(1+t^2)^2} e^{j\omega t}\, dt$$

$$\cancel{\frac{4t}{(1+t^2)^2} \xleftrightarrow{F} 2\pi \omega j e^{-|\omega|}}$$

Multiply by ~~ω~~ by -1

$$-2\pi \omega j e^{-|\omega|} = \int_{-\infty}^{\infty} \frac{4t}{(1+t^2)^2} e^{-j\omega t}\, dt$$

$$\frac{4t}{(1+t^2)^2} \xleftrightarrow{F} -2\pi\, \omega j\, e^{-|\omega|}$$

a) ~~$A(0,2)$ $B(1,1)$~~

$$\cancel{\frac{2-1}{0-1} = \frac{y-1}{x-1}}$$

$$\cancel{1-x = y-1}$$

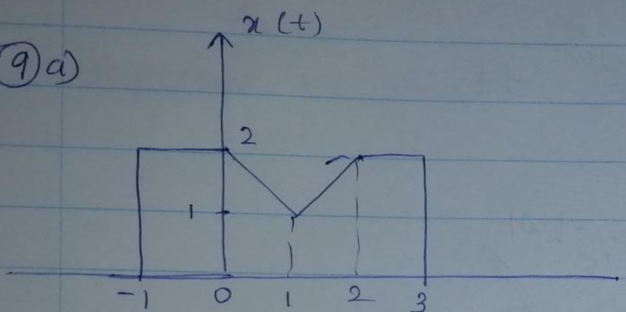
$$\cancel{2-x = y}$$

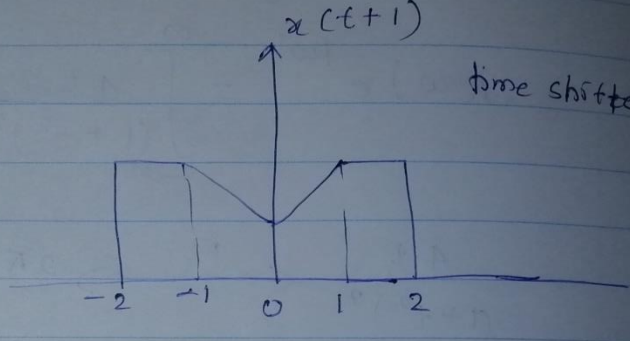
$$\cancel{y\ x(t) = -t+2}$$

$y =$

$x(t) = x_1$

(9) a)





time shifted.

$$y(t) = x(t+1)$$

We can see that $y(t)$ is real and even.
Therefore from the properties of Fourier transform, if the time domain signal $y(t)$ is even and real, it's Fourier transform $Y(j\omega)$ is also even and real.

As $Y(j\omega)$ is real $\angle Y(j\omega) = 0$

~~$x(t+1) = Y(j\omega)$~~ $\quad x(t) \xleftrightarrow{F} X(j\omega)$

$$x(t+1) \xleftrightarrow{F} e^{j\omega} X(j\omega)$$

$$Y(j\omega) = F\{x(t+1)\}$$

$$Y(j\omega) = e^{j\omega} X(j\omega)$$

$$X(j\omega) = e^{-j\omega} Y(j\omega)$$

$$\angle X(j\omega) = \angle e^{-j\omega} + \angle Y(j\omega)$$

$$= -\omega + 0$$

$$\angle X(j\omega) = -\omega$$

(b) $$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$$

consider $\omega = 0$

$$X(j0) = \int_{-\infty}^{\infty} x(t)\, dt$$

$$x(t) = \begin{cases} 2 & -1 \le t \le 0 \\ -t+2 & 0 \le t \le 1 \\ t & 1 \le t \le 2 \\ 2 & 2 \le t \le 3 \\ 0 & o/w. \end{cases}$$

$$X(j0) = \int_{-\infty}^{\infty} x(t)\, dt$$

$$X(j0) = \int_{-1}^{0} 2\,dt + \int_{0}^{1} (-t+2)\,dt + \int_{1}^{2} t\,dt + \int_{2}^{3} 2\,dt.$$

$$= 2\left(t\right)_{-1}^{0} + \left(\frac{-t^2}{2} + 2t\right)_{0}^{1} + \left(\frac{t^2}{2}\right)_{1}^{2} + \left(2t\right)_{2}^{3}$$

$$= 2(0+1) + \left(\frac{-1}{2} + 2\right) + \left(2 - \frac{1}{2}\right) + (6-4)$$

$$= 2 + 2 + 3$$

$$X(j0) = 7$$

(c) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$

Consider $t = 0$

$$2\pi\, x(0) = \int_{-\infty}^{\infty} X(j\omega)\, d\omega$$

$x(0) = 2$

$$\therefore \int_{-\infty}^{\infty} X(j\omega)\, d\omega = 4\pi \;//$$

(d) From the Parseval's relation of Fourier transforms

$$\int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= 2\pi \left\{ \int_{-1}^{0} 4\, dt + \int_{0}^{1} (t^2 - 4t + 4)\, dt + \int_{1}^{2} t^2\, dt + \int_{2}^{3} 4\, dt \right\}$$

$$= 2\pi \left\{ 4(t)\Big|_{-1}^{0} + \left(\frac{t^3}{3} - \frac{4t^2}{2} + 4t\right)\Big|_{0}^{1} + \left(\frac{t^3}{3}\right)\Big|_{1}^{2} + 4(t)\Big|_{2}^{3} \right\}$$

$$= 2\pi \left\{ 4 + \left(\frac{1}{3} - 2 + 4\right) + \frac{8}{3} - \frac{1}{3} + 4(3-2) \right\}$$

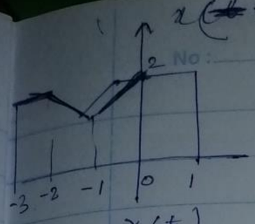
$$= 2\pi \left\{ 10 + \frac{8}{3} \right\}$$

$$\int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega = \frac{76\pi}{3} \;//$$

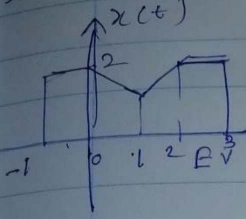
(e) From the properties of Fourier transform inverse of $Re\{X(j\omega)\}$ is $Ev\{x(t)\}$

even part.

$$Re\{X(j\omega)\} \xleftrightarrow{F} Ev\{x(t)\} = \frac{x(t) + x(-t)}{2}$$



$$x(-t) = \begin{cases} 2 & -3 \le t \le 2 \\ -t & -2 \le t \le -1 \\ t+2 & -1 \le t < 0 \\ 2 & 0 \le t \le 1 \end{cases}$$



$$Ev\{x(t)\} = \begin{cases} 1 & -3 \le t \le 2 \\ -t/2 & -2 \le t \le -1 \\ (t+4)/2 & -1 \le t \le 0 \\ (-t+4)/2 & 0 \le t \le 1 \\ t/2 & 1 \le t \le 2 \\ 1 & 2 \le t \le 3 \end{cases}$$