EE387 – SYSTEM FUNCTIONS AND FREQUENCY RESPONSE

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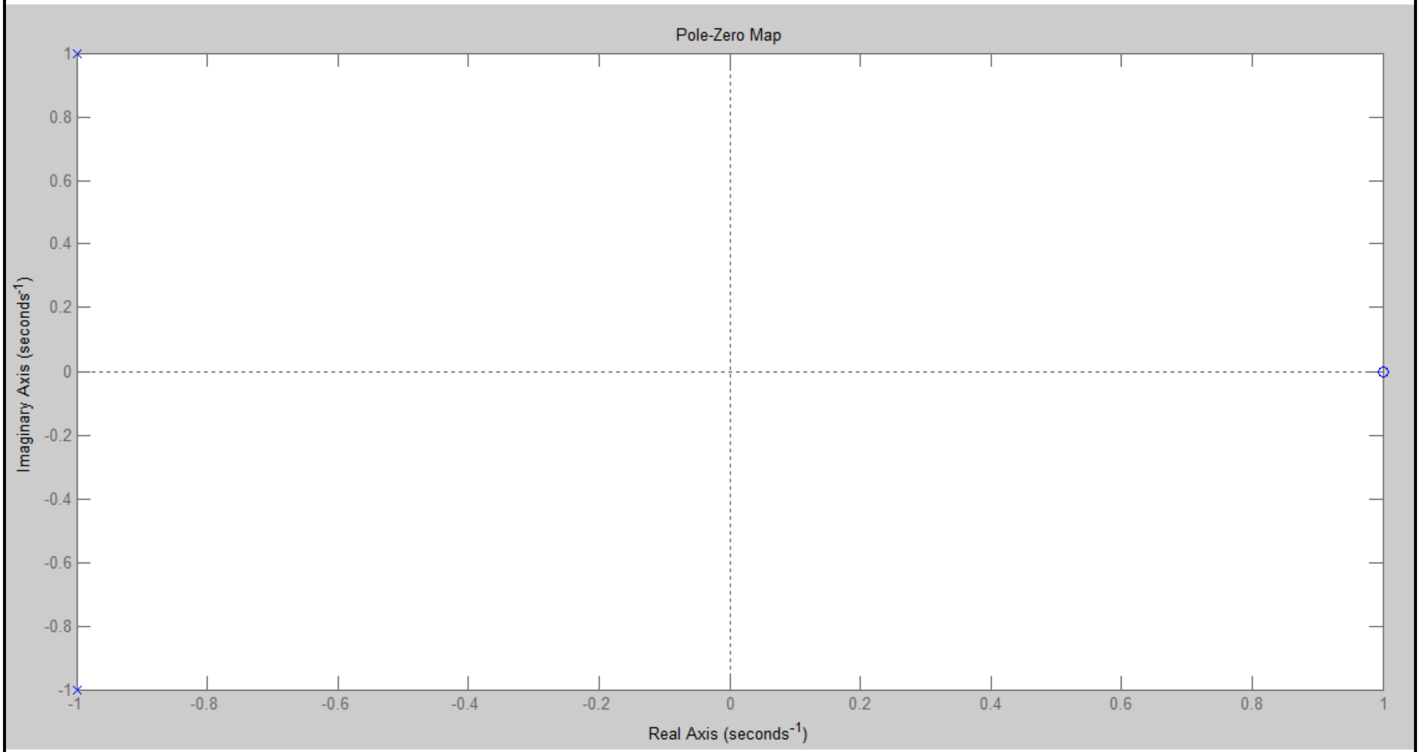
E/15/202
SEMESTER 06

PART 1: Pole-Zero Diagrams in MATLAB

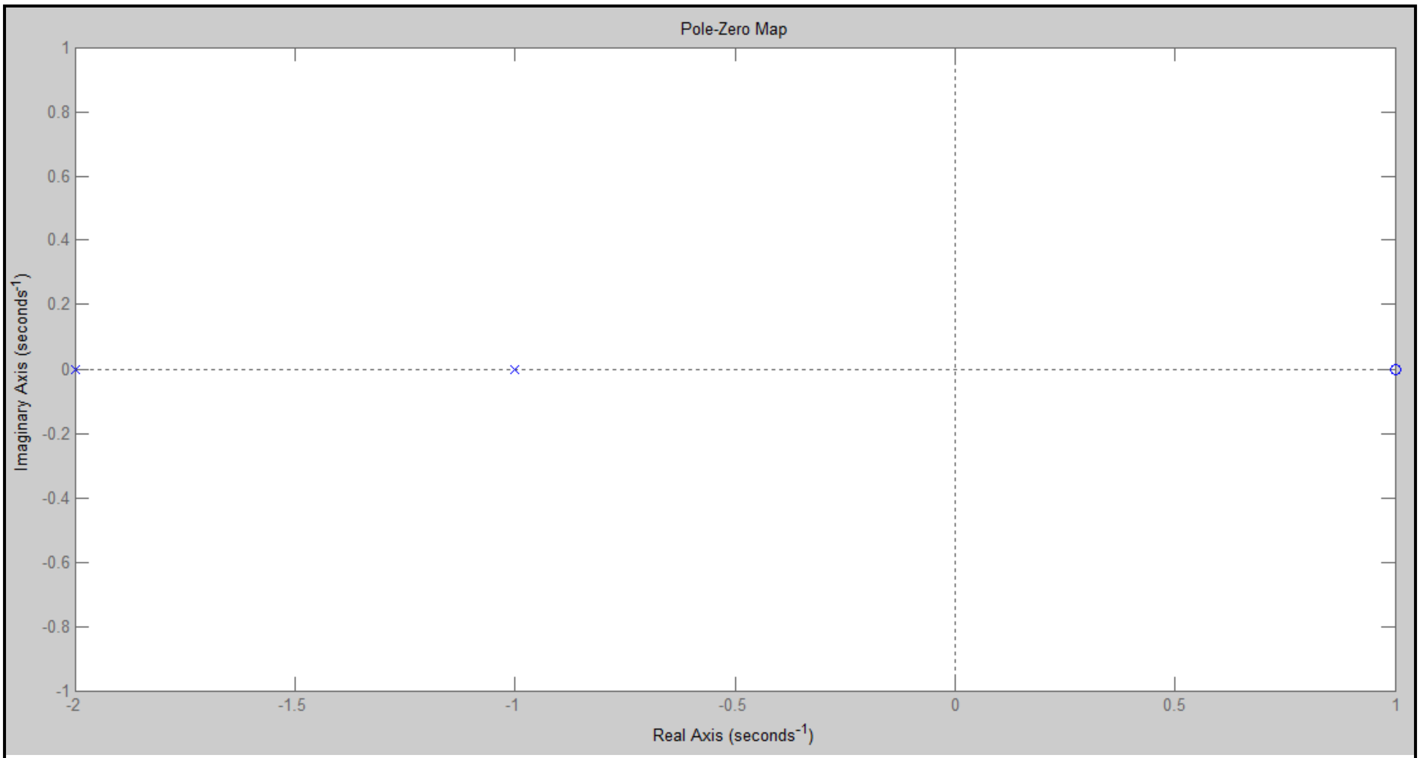
$$H(s) = \frac{s-1}{s^2+2s+2}$$

```
b = [1 -1]; % Numerator coefficients
a = [1 2 2]; % Denominator coefficients
zs  = roots(b); % Generates Zeros
ps = roots(a); % Generates poles
pzmap(ps,zs); % generates pole-zero diagram
```



$$H(s) = \frac{s-1}{s^2+3s+2}$$

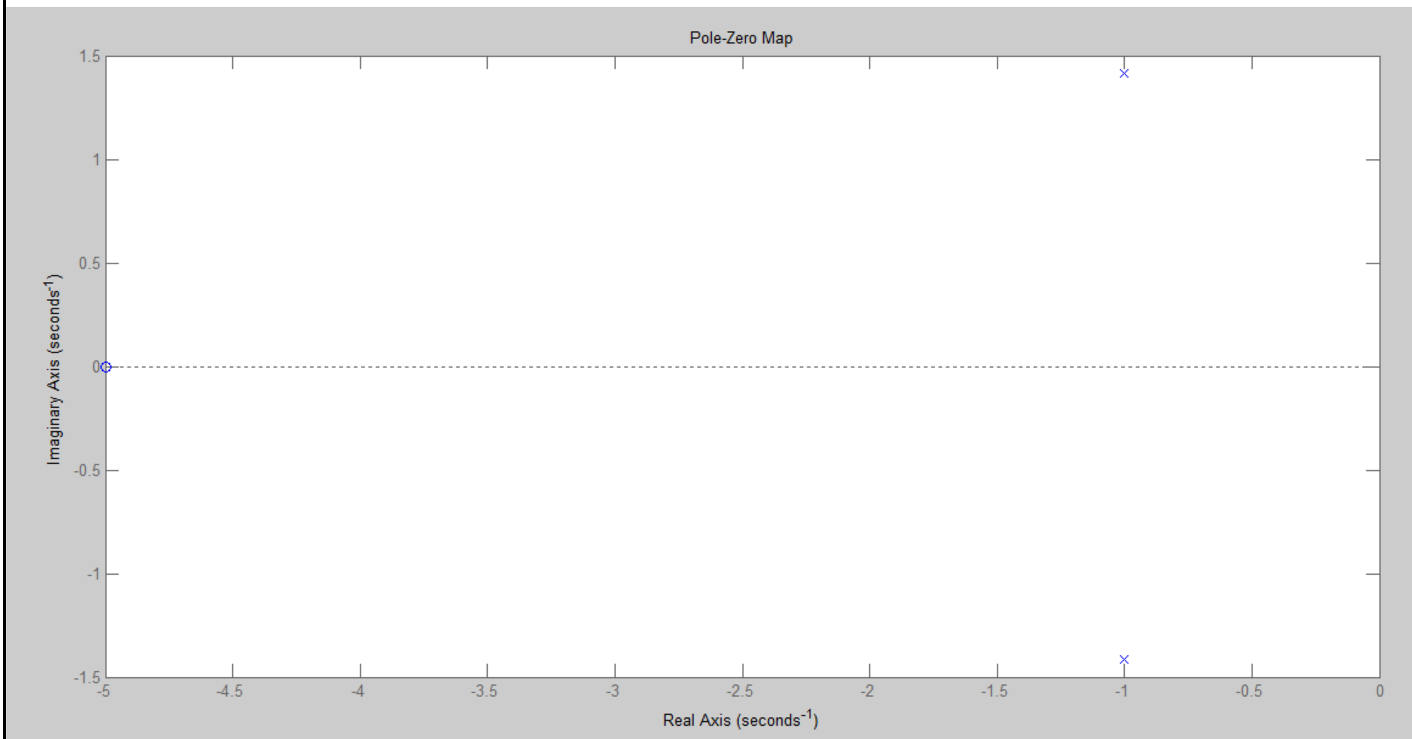
```
b = [1 -1]; % Numerator coefficients
a = [1 3 2]; % Denominator coefficients
zs  = roots(b); % Generates Zeros
ps = roots(a); % Generates poles
pzmap(ps,zs); % generates pole-zero diagram
```



Exercise

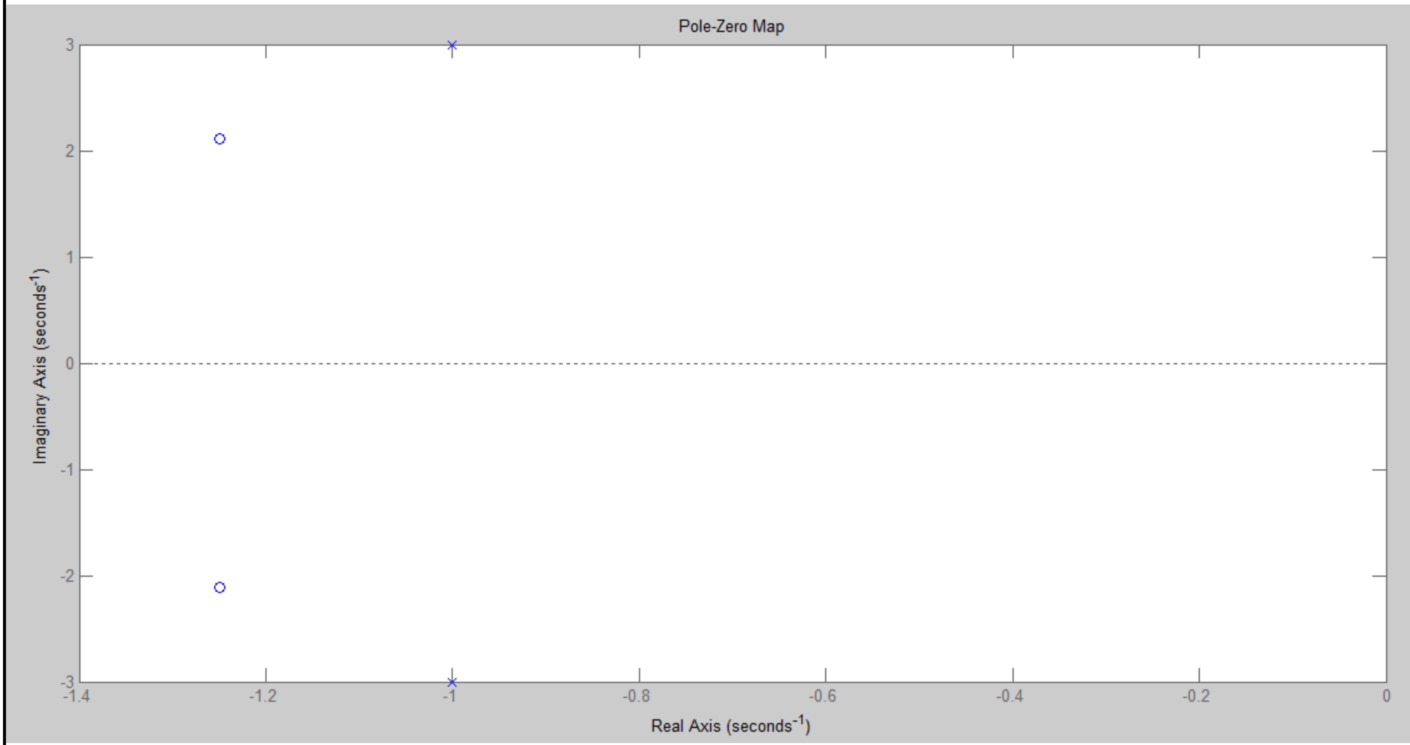
1. $H(s) = \frac{s+5}{s^2+2s+3}$

```
b = [1 5]; % Numerator coefficients
a = [1 2 3]; % Denominator coefficients
zs  = roots(b); % Generates Zeros
ps = roots(a); % Generates poles
pzmap(ps,zs) % generates pole-zero diagram
```



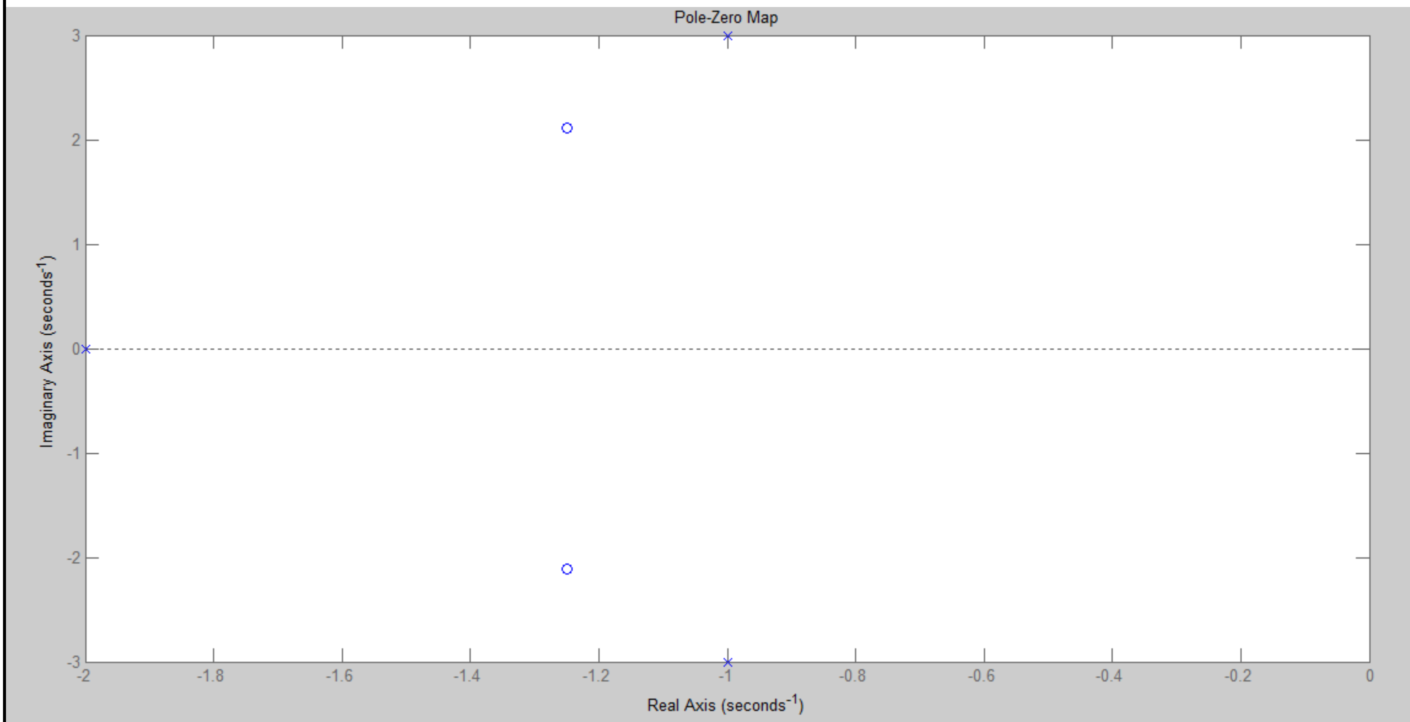
2. $H(s) = \frac{2s^2+5s+12}{s^2+2s+10}$

```
b = [2 5 12]; % Numerator coefficients
a = [1 2 10]; % Denominator coefficients
zs  = roots(b); % Generates Zeros
ps = roots(a); % Generates poles
pzmap(ps,zs) % generates pole-zero diagram
```



3. $H(s) = \frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}$

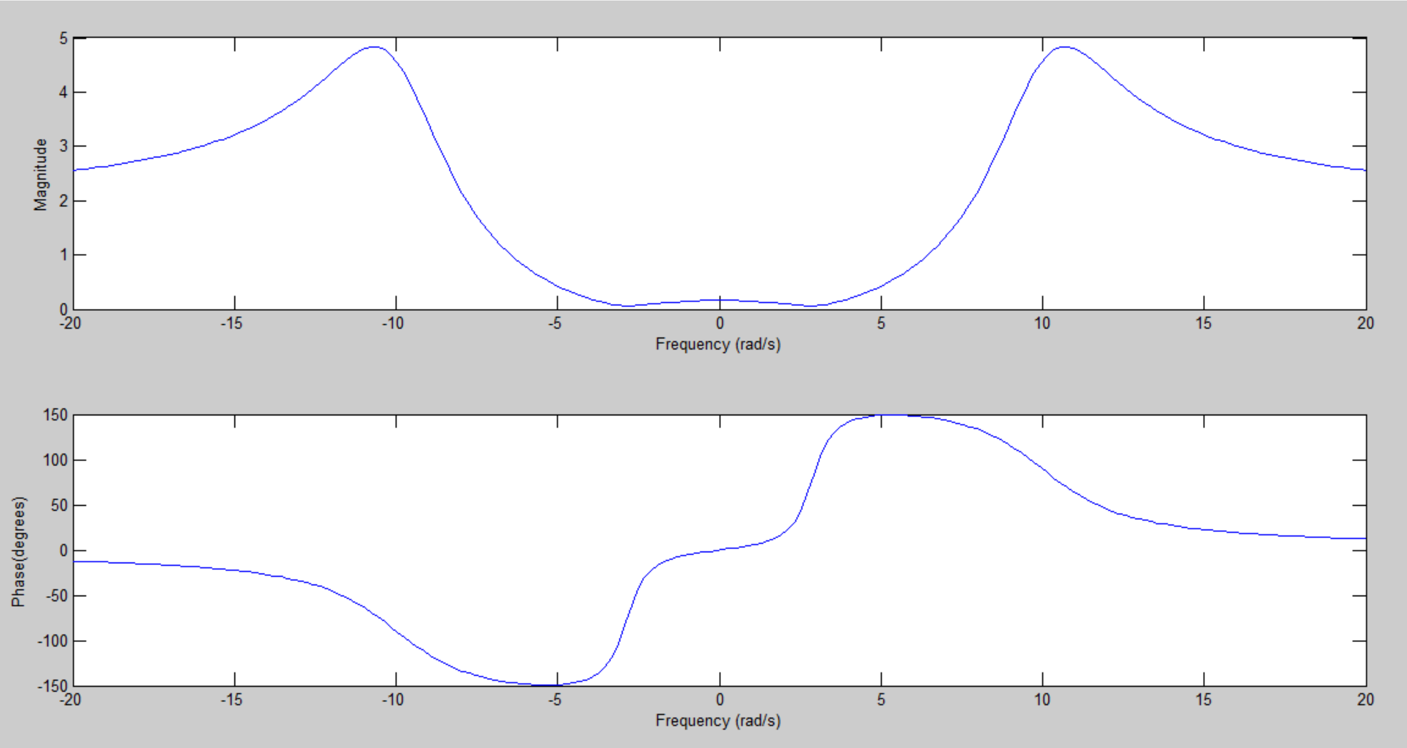
```
b = [2 5 12]; % Numerator coefficients
a = [1 4 14 20]; % Denominator coefficients
zs  = roots(b); % Generates Zeros
ps = roots(a); % Generates poles
pzmap(ps,zs) % generates pole-zero diagram
```



PART 2: Frequency Response and Bode Plots in MATLAB

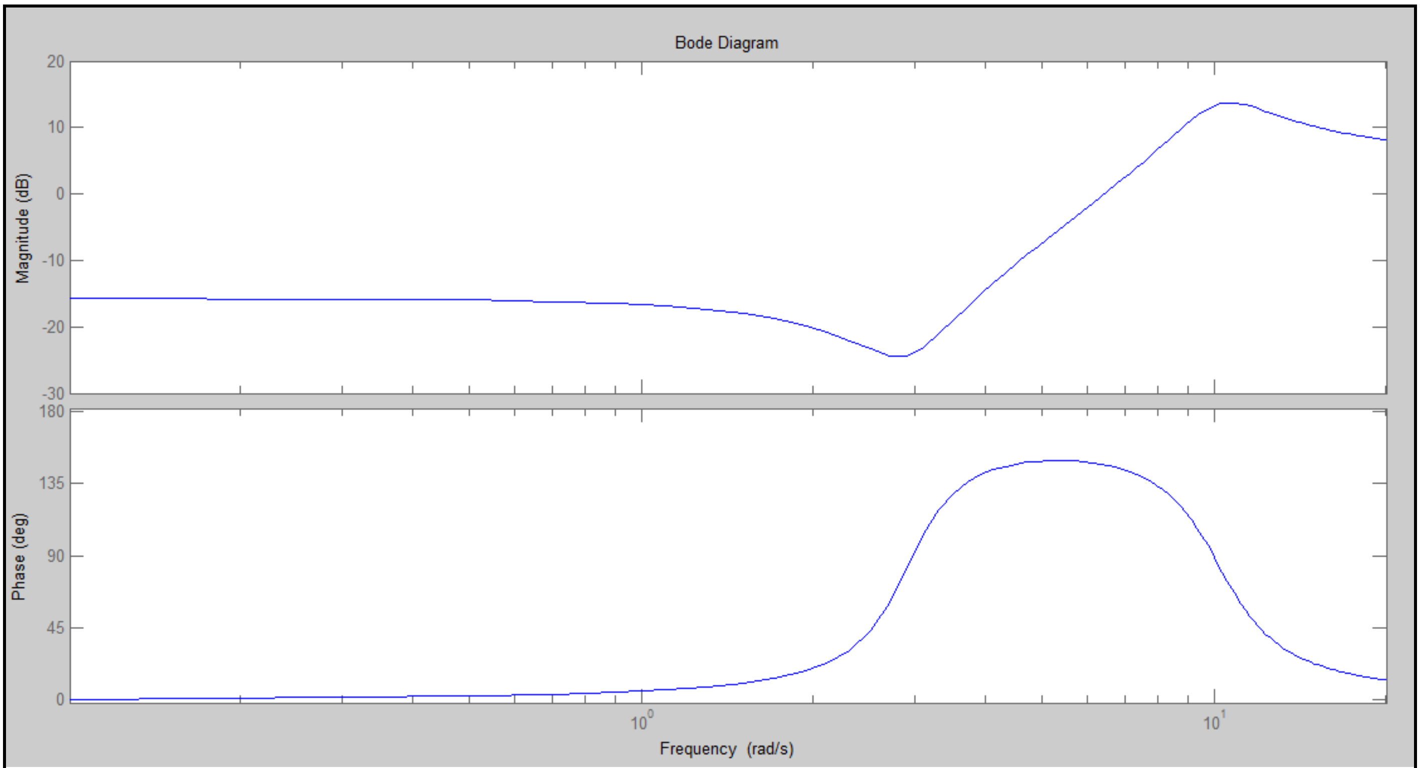
$$H(s) = \frac{2s^2 + 2s + 17}{s^2 + 4s + 104}$$

```
b = [2 2 17]; % Numerator coefficients
a = [1 4 104]; % Denominator coefficients
omega = linspace(-20,20,200);
H = freqs(b,a,omega);
mag = abs(H);
phase = angle(H);
phasedeg = phase*180/pi; %phase in degrees
subplot(2,1,1);
plot(omega,mag)
xlabel('Frequency (rad/s)');
ylabel('Magnitude');
subplot(2,1,2);
plot(omega, phasedeg)
xlabel('Frequency (rad/s)');
ylabel('Phase(degrees)');
```



4. Plot the bode plot of the given by utilizing the results in 2. (Hint: use the definitions of the bode plot)

```
Ht = tf([2 2 17],[1 4 104]);
bode(Ht,omega)
```



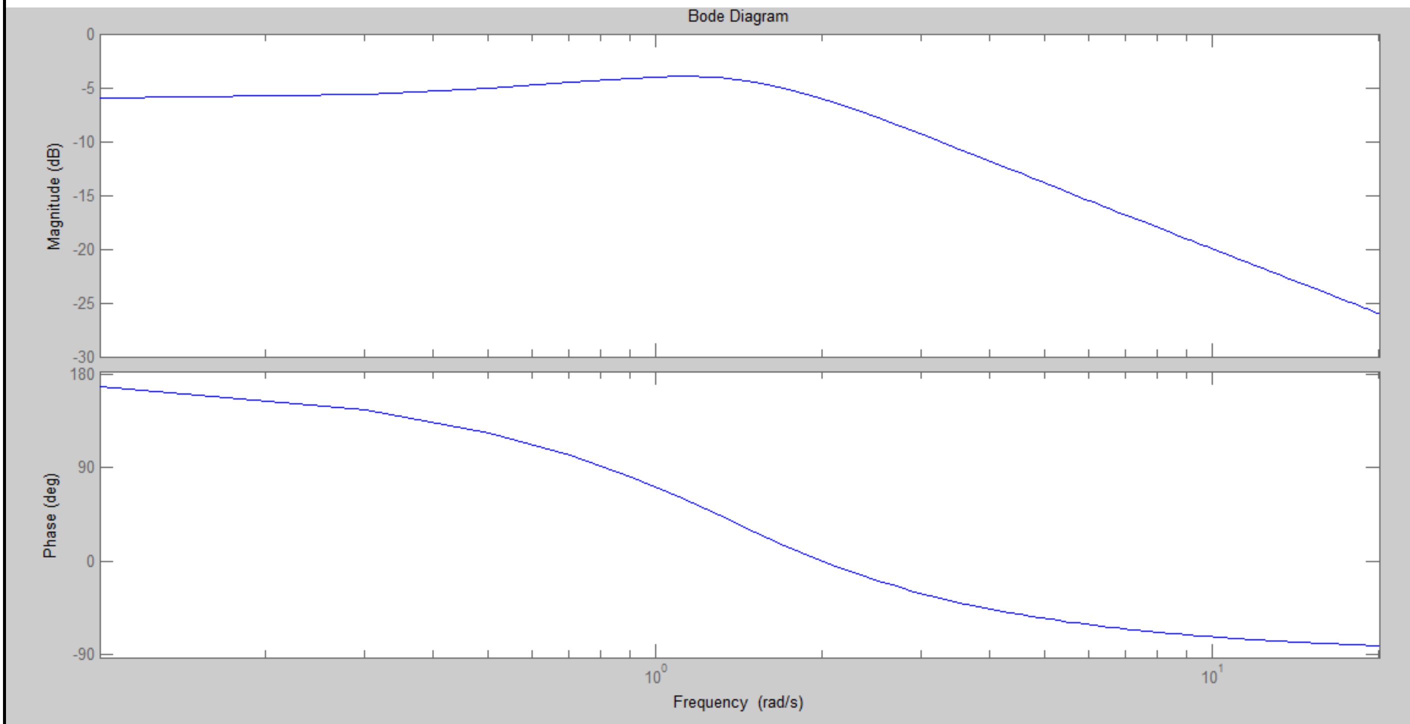
Warning: Negative data ignored

Exercise

Warning: Negative data ignored

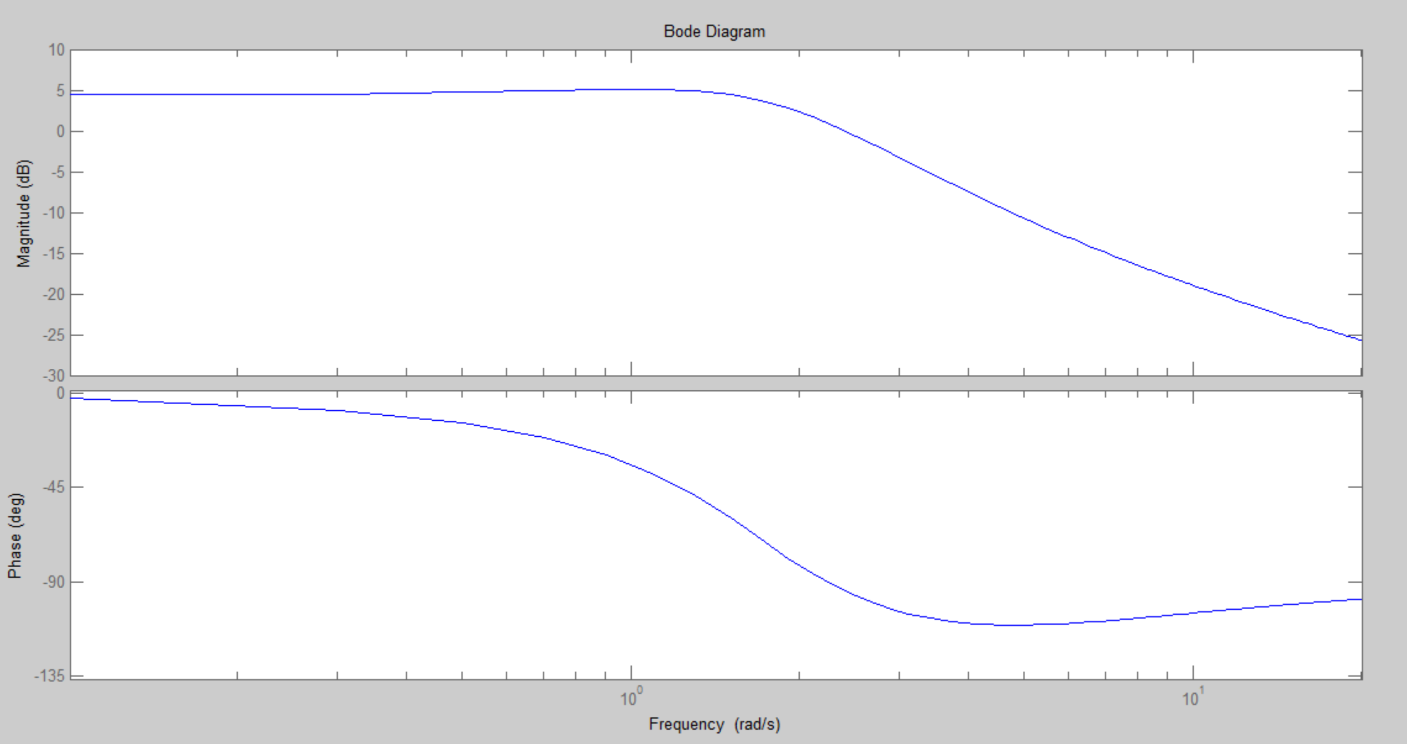
$$H(s) = \frac{s-1}{s^2 + 2s + 2}$$

```
b = [1 -1]; % Numerator coefficients
a = [1 2 2]; % Denominator coefficients
omega = linspace(-20,20,200);

Ht = tf([1 -1],[1 2 2]);
bode(Ht,omega)
```



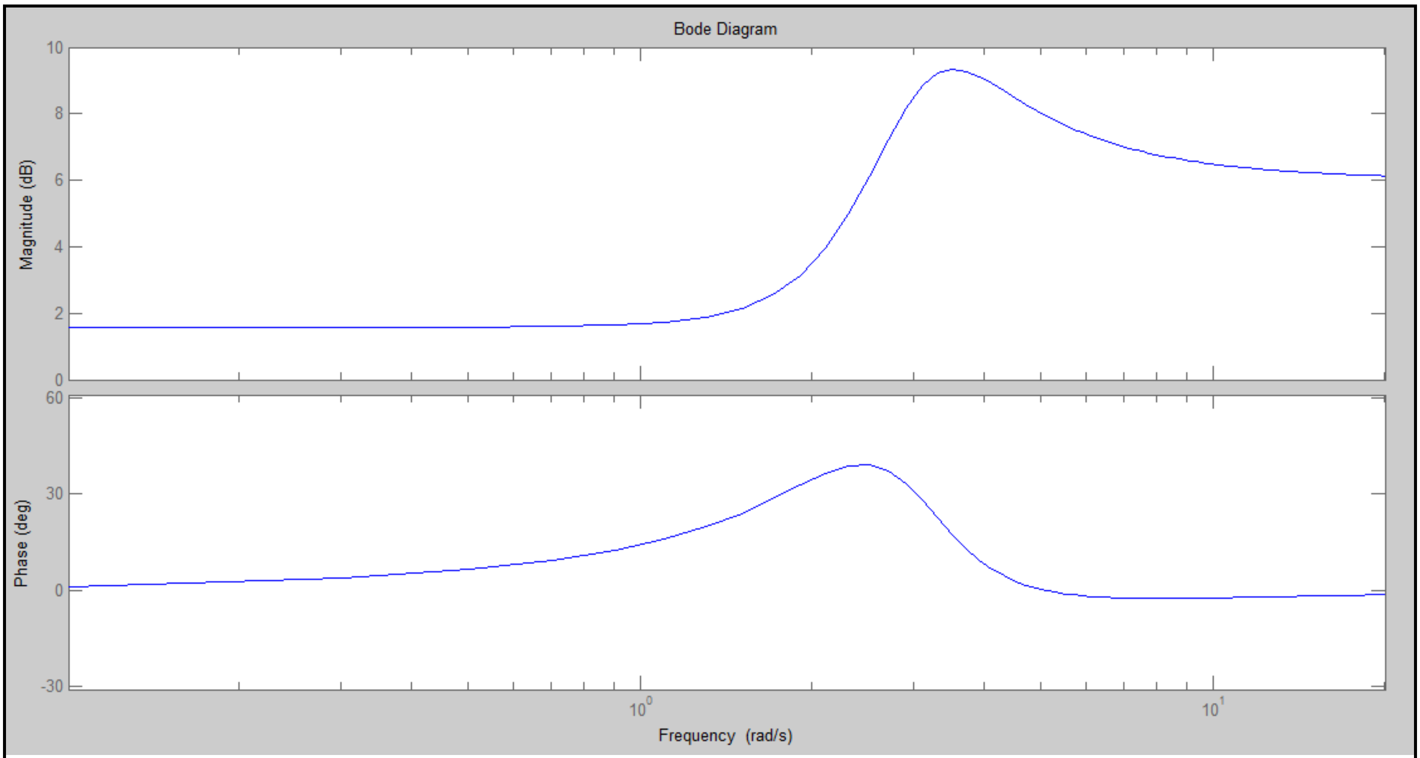
1. $H(s) = \frac{s+5}{s^2+2s+3}$

```
b = [1 5]; % Numerator coefficients
a = [1 2 3]; % Denominator coefficients
omega = linspace(-20,20,200);

Ht = tf([1 5],[1 2 3]);
bode(Ht,omega)
```



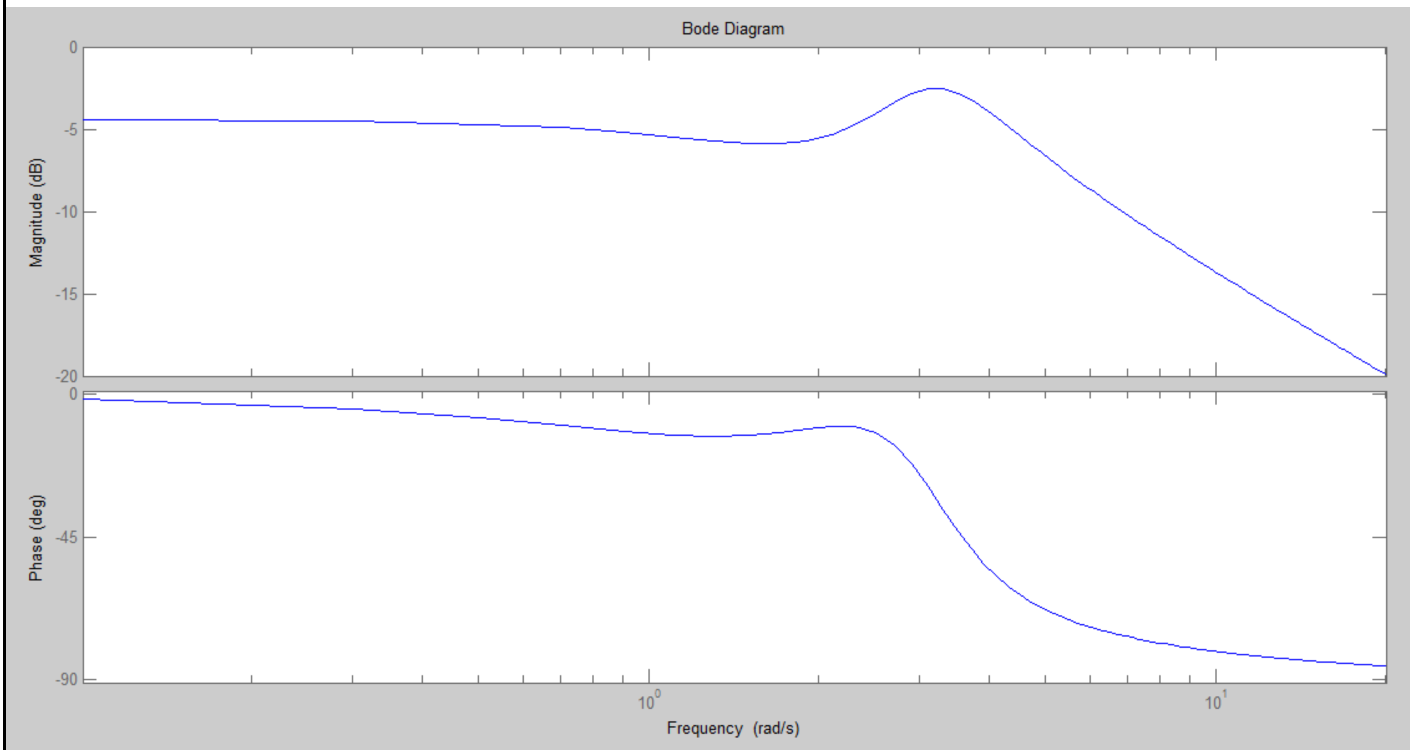
2. $H(s) = \frac{2s^2+5s+12}{s^2+2s+10}$

```
b = [2 5 12]; % Numerator coefficients
a = [1 2 10]; % Denominator coefficients
omega = linspace(-20,20,200);
Ht = tf([2 5 12],[1 2 10]);
bode(Ht,omega)
```



3. $H(s) = \frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}$

```
b = [2 5 12]; % Numerator coefficients
a = [1 4 14 20]; % Denominator coefficients
omega = linspace(-20,20,200);
Ht = tf([2 5 12],[1 4 14 20]);
bode(Ht,omega)
```



2. Select three sinusoidal signals with unit magnitude, zero phase and three different frequencies (f_1, f_2, f_3 *in kHz*, here $f_i = Registration\ number * i$). Assume that they are three inputs for above mentioned four systems. Then find the corresponding three outputs for each system.

$f_1 = 202 * 1 = 202kHz$
$f_2 = 202 * 2 = 404kHz$
$f_3 = 202 * 3 = 606kHz$

$\omega_1 = 2\pi f_1 = 404\pi \times 10^3 = 1269.20 \times 10^3$
$\omega_2 = 2\pi f_2 = 808\pi \times 10^3 = 2538.40 \times 10^3$
$\omega_3 = 2\pi f_3 = 1212\pi \times 10^3 = 3807.61 \times 10^3$

Sinusoidal signals, $\sin \omega_1 t\, u(t)$, $\sin \omega_2 t\, u(t)$, $\sin \omega_3 t\, u(t)$

$$H(s) = \frac{Y(s)}{X(s)}$$
$$Y(s) = H(s)X(s)$$

Laplace transformation of $\sin\omega\, t\ \ u(t) = \frac{\omega}{s^2+\omega^2} = X(s)$

1. $H(s) = \frac{s-1}{s^2+2s+2}$

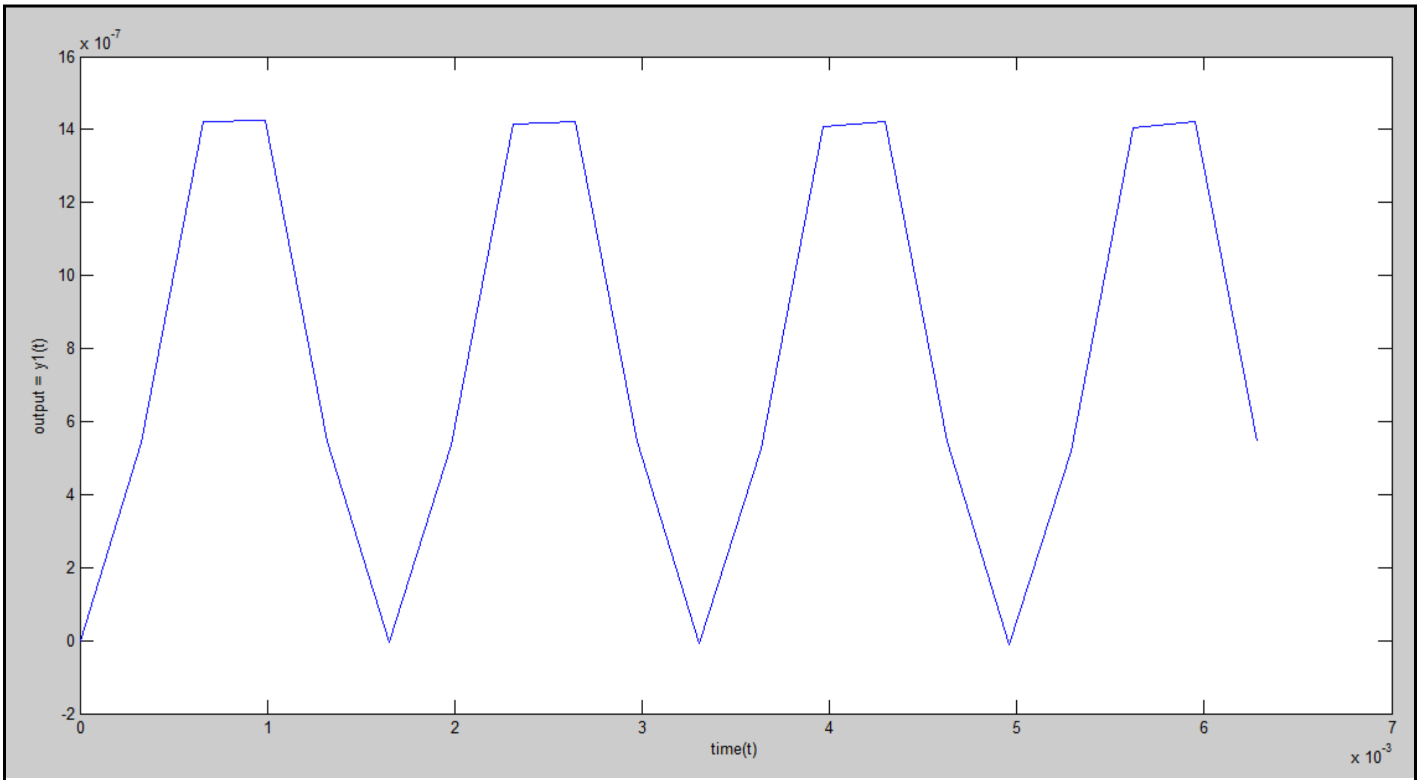
```
syms s a
F1 = ilaplace((s-1)*a/((s^2+2*s+2)*(s^2+a^2))) %laplace inverse

F1=

(- (2*sin(a*t) - 4*a*cos(a*t) + a^3*cos(a*t) - 3*a^2*sin(a*t))/((a^2 - 2*a
+ 2)*(a^2 + 2*a + 2)) - (exp(-t)*(- a^3 + 4*a)*(cos(t) + sin(t)*((a^3 +
6*a)/(- a^3 + 4*a) - 1)))/((a^2 - 2*a + 2)*(a^2 + 2*a + 2))) u(t)
```

$X_1(s) = \sin \omega_1 t\, u(t)$
$Y_1(s) = \left(\frac{s-1}{s^2+2s+2}\right)\left(\frac{\omega_1}{s^2+{\omega_1}^2}\right)$

$output = \ y_1(t)$ = F1

(Where a = ω_1)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 404*pi*10^3;
plot(t,subs(F1))
xlabel('time(t)')
ylabel('output = y1(t)')
```



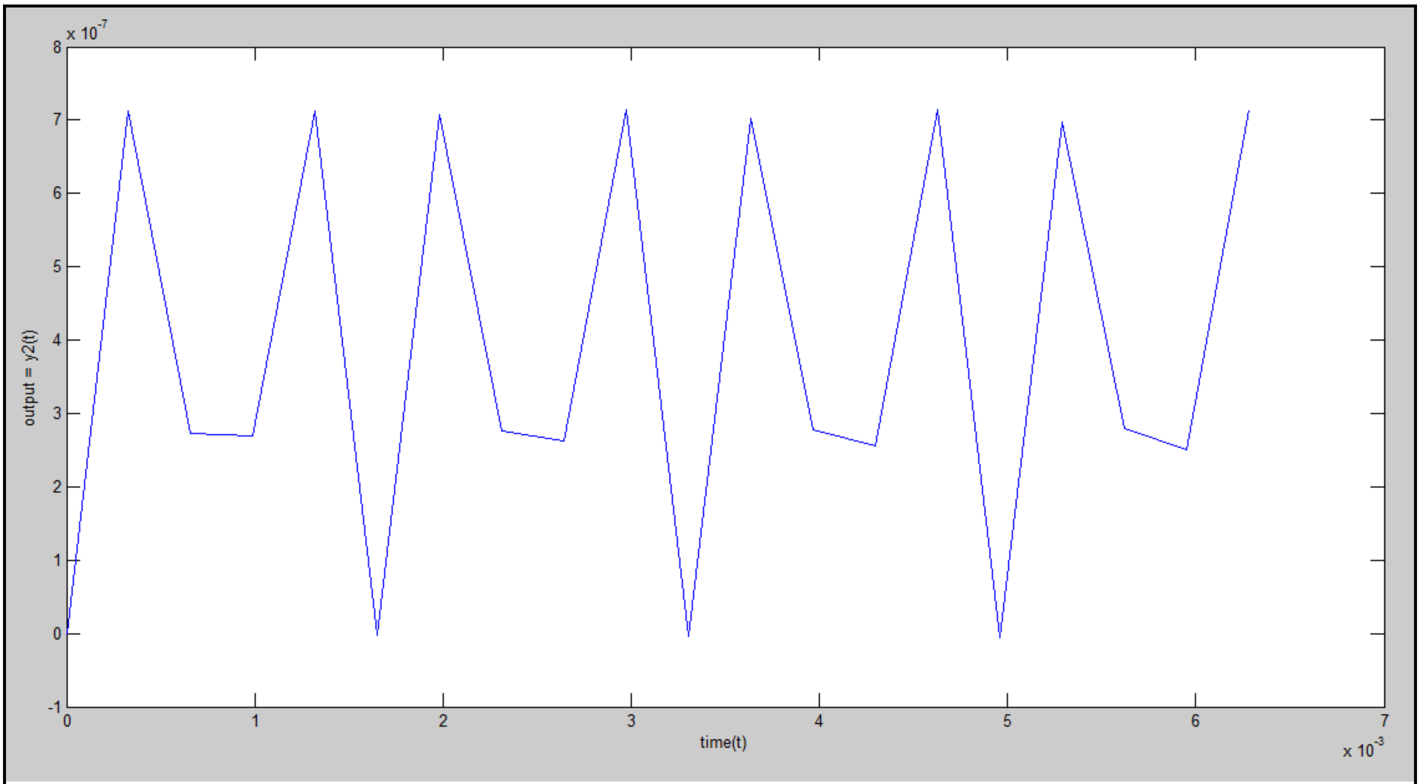
$X_2(s) = \sin \omega_2 t \, u(t)$

$Y_2(s) = \left(\frac{s-1}{s^2+2s+2}\right)\left(\frac{\omega_2}{s^2+{\omega_2}^2}\right)$

$output = \; y_2(t)$ = F1

(Where a = ω_2)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 808*pi*10^3;
plot(t,subs(F1))
xlabel('time(t)')
ylabel('output = y2(t)')
```

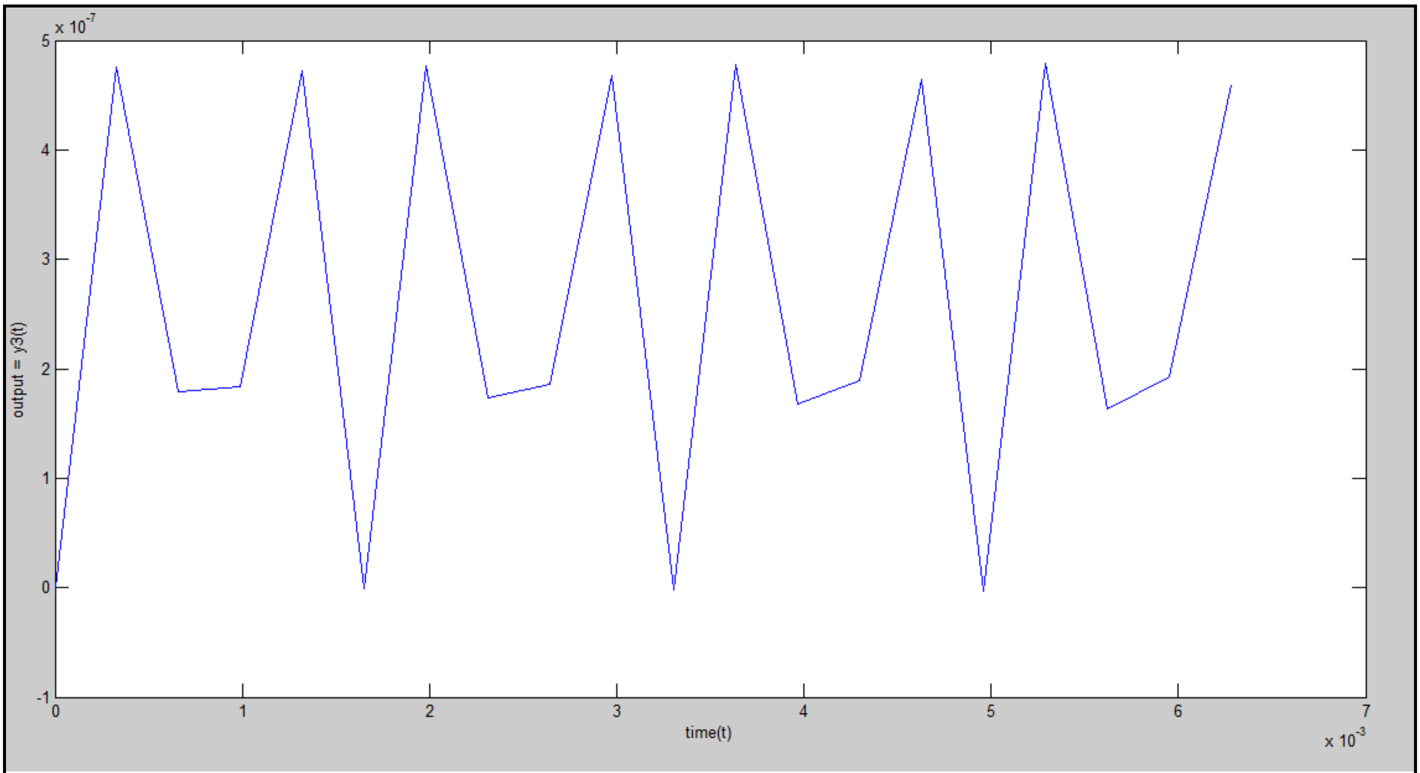


$$X_3(s) = \sin \omega_3 t\, u(t)$$
$$Y_3(s) = \left(\frac{s-1}{s^2+2s+2}\right)\left(\frac{\omega_3}{s^2+{\omega_3}^2}\right)$$

$output = \ y_3(t)$ = F1

(Where a = ω_3)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 1212*pi*10^3;
plot(t,subs(F1))
xlabel('time(t)')
ylabel('output = y3(t)')
```



2. $H(s) = \frac{s+5}{s^2+2s+3}$

```
syms s a
F2 = ilaplace((s+5)*a/((s^2+2*s+3)*(s^2+a^2))) %laplace inverse

F2 =

((exp(-t)*(cos(2^(1/2)*t) - (2^(1/2)*sin(2^(1/2)*t)*((- 5*a^3 + a)/(a^3 +
7*a) + 1))/2)*(a^3 + 7*a))/(a^4 - 2*a^2 + 9) - (7*a*cos(a*t) - 15*sin(a*t)
+ a^3*cos(a*t) + 3*a^2*sin(a*t))/(a^4 - 2*a^2 + 9))
```
$u(t)$

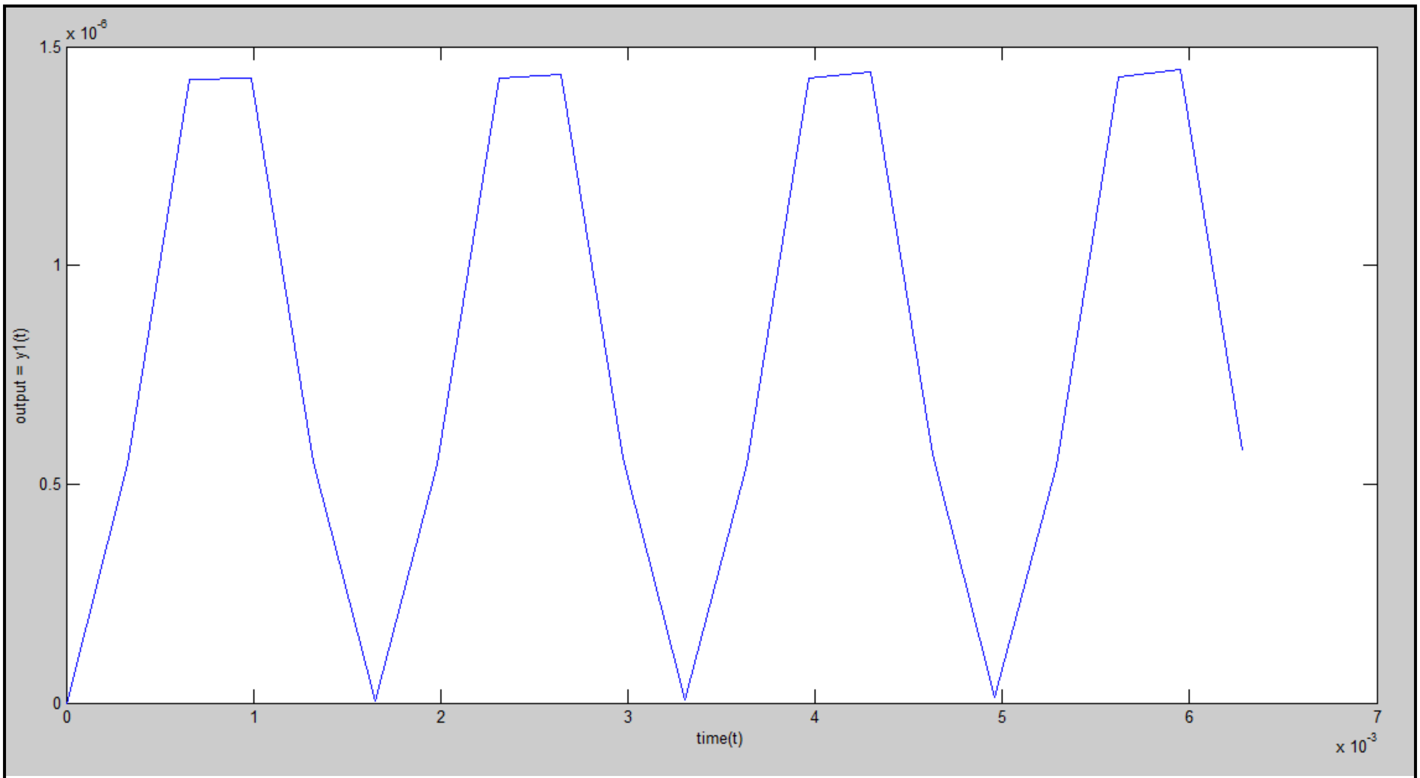
$X_1(s) = \sin \omega_1 t\, u(t)$

$Y_1(s) = \left(\frac{s+5}{s^2+2s+3}\right)\left(\frac{\omega_1}{s^2+\omega_1{}^2}\right)$

$output = y_1(t)$ = F2

(Where a = ω_1)

```
t=linspace(0,0.002*pi,20);
a = 404*pi*10^3;
plot(t,subs(F2))
xlabel('time(t)')
ylabel('output = y1(t)')
```



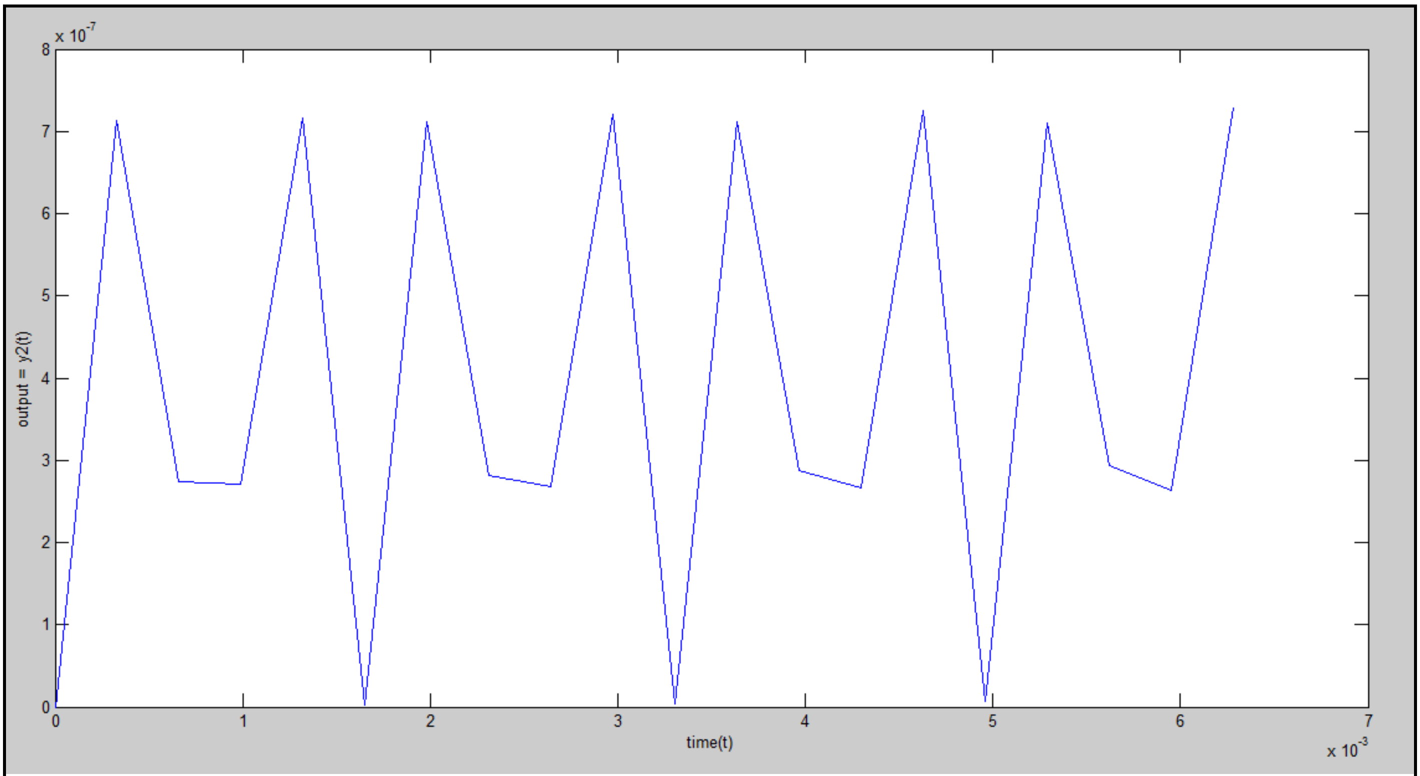
$X_2(s) = \sin\omega_2 t\, u(t)$

$Y_2(s) = \left(\frac{s+5}{s^2+2s+3}\right)\left(\frac{\omega_2}{s^2+{\omega_2}^2}\right)$

$output = y_2(t)$ = F2

(Where a = ω_2)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 808*pi*10^3;
plot(t,subs(F2))
xlabel('time(t)')
ylabel('output = y2(t)')
```



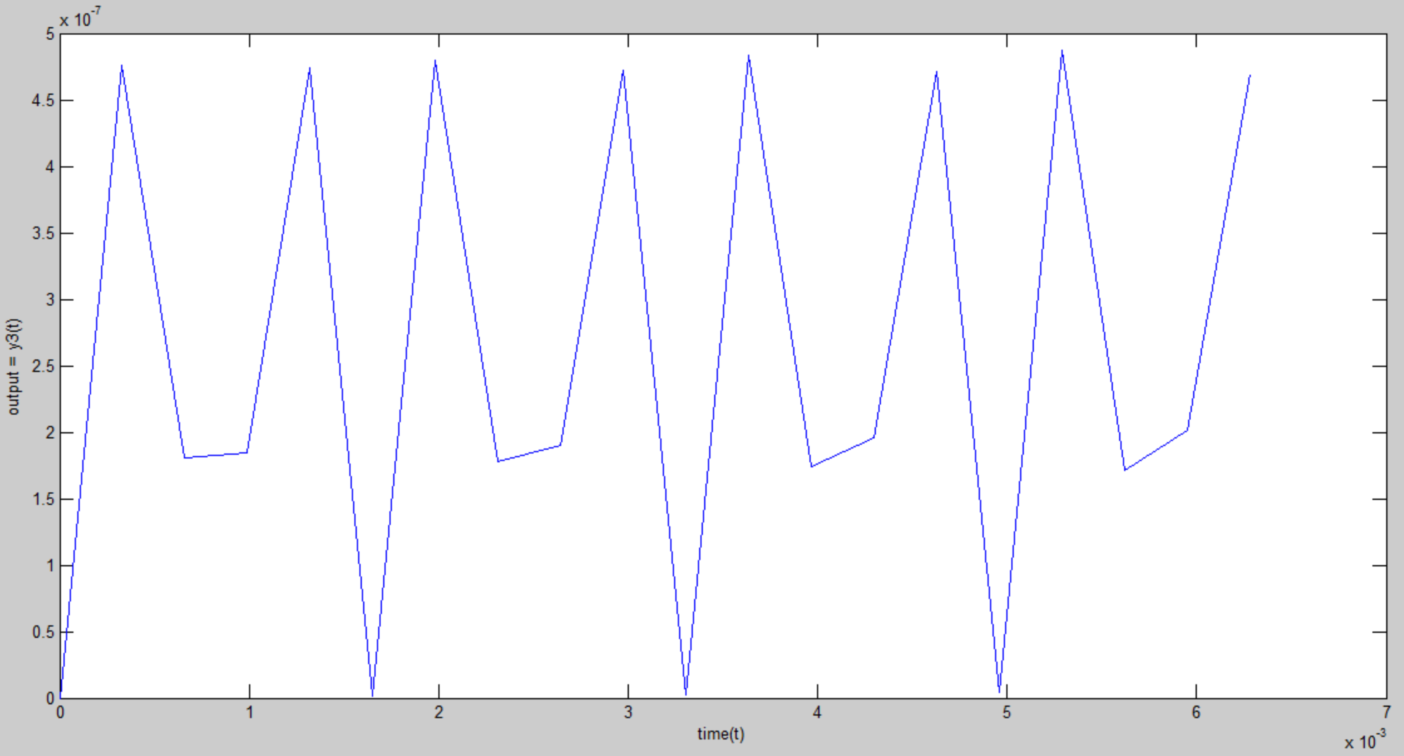
$X_3(s) = \sin \omega_3 t \, u(t)$

$Y_3(s) = \left(\frac{s+5}{s^2+2s+3}\right)\left(\frac{\omega_3}{s^2+{\omega_3}^2}\right)$

$output = \ y_3(t)$ = F2

(Where a = ω_3)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 1212*pi*10^3;
plot(t,subs(F2))
xlabel('time(t)')
ylabel('output = y3(t)')
```



3. $H(s) = \frac{2s^2+5s+12}{s^2+2s+10}$

```
syms s a
F3 = ilaplace((2*(s^2)+5*s+12)*a/((s^2+2*s+10)*(s^2+a^2))) %laplace
inverse

F3 =

((120*sin(a*t) + 26*a*cos(a*t) - a^3*cos(a*t) - 22*a^2*sin(a*t) +
2*a^4*sin(a*t))/((a^2 - 6*a + 10)*(a^2 + 6*a + 10)) - (exp(-t)*(- a^3 +
26*a)*(cos(3*t) - sin(3*t)*((- 8*a^3 + 28*a)/(3*(- a^3 + 26*a)) +
1/3)))/((a^2 - 6*a + 10)*(a^2 + 6*a + 10)))u(t)
```

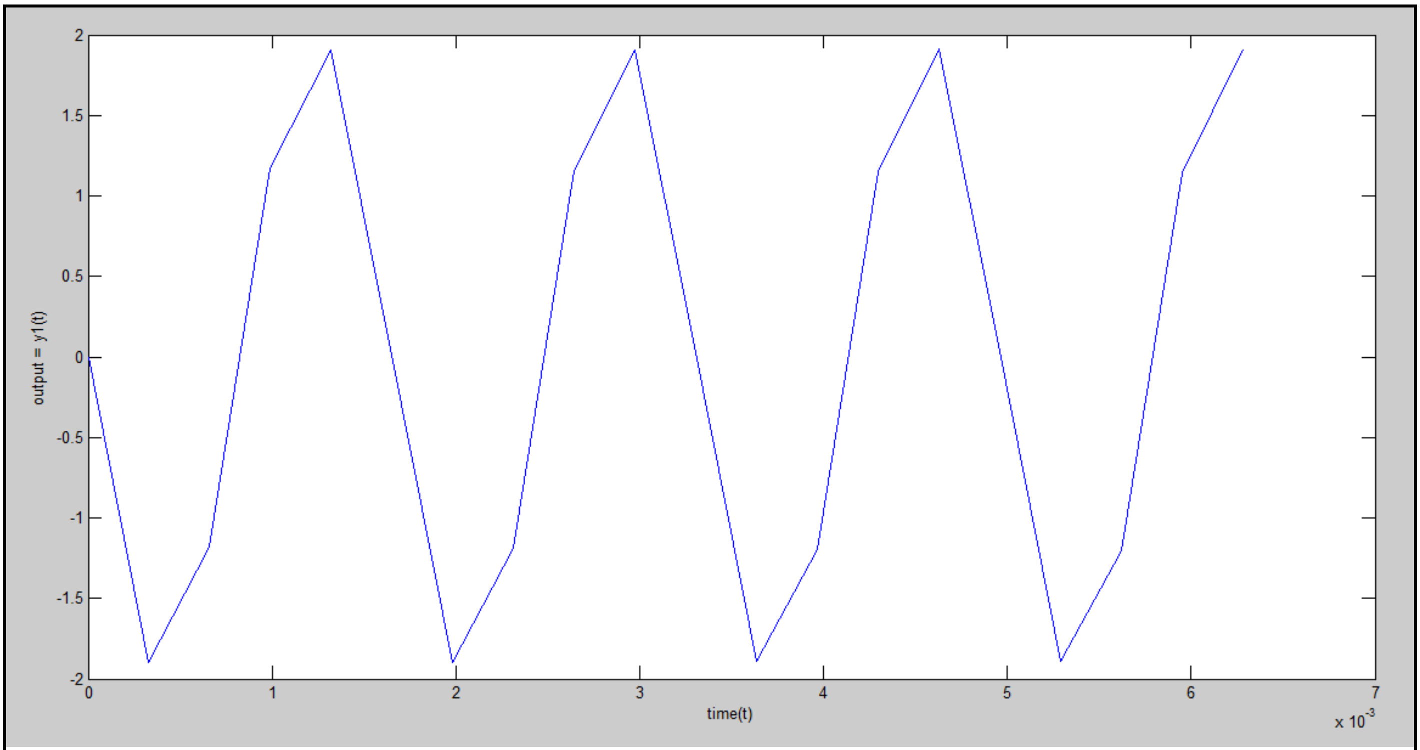
$X_1(s) = \sin \omega_1 t \, u(t)$

$$Y_1(s) = \left(\frac{2s^2+5s+12}{s^2+2s+10}\right)\left(\frac{\omega_1}{s^2+{\omega_1}^2}\right)$$

$output = y_1(t)$ = F3

(Where a = ω_1)

```
t=linspace(0,0.002*pi,20);
a = 404*pi*10^3;
plot(t,subs(F3))
xlabel('time(t)')
ylabel('output = y1(t)')
```



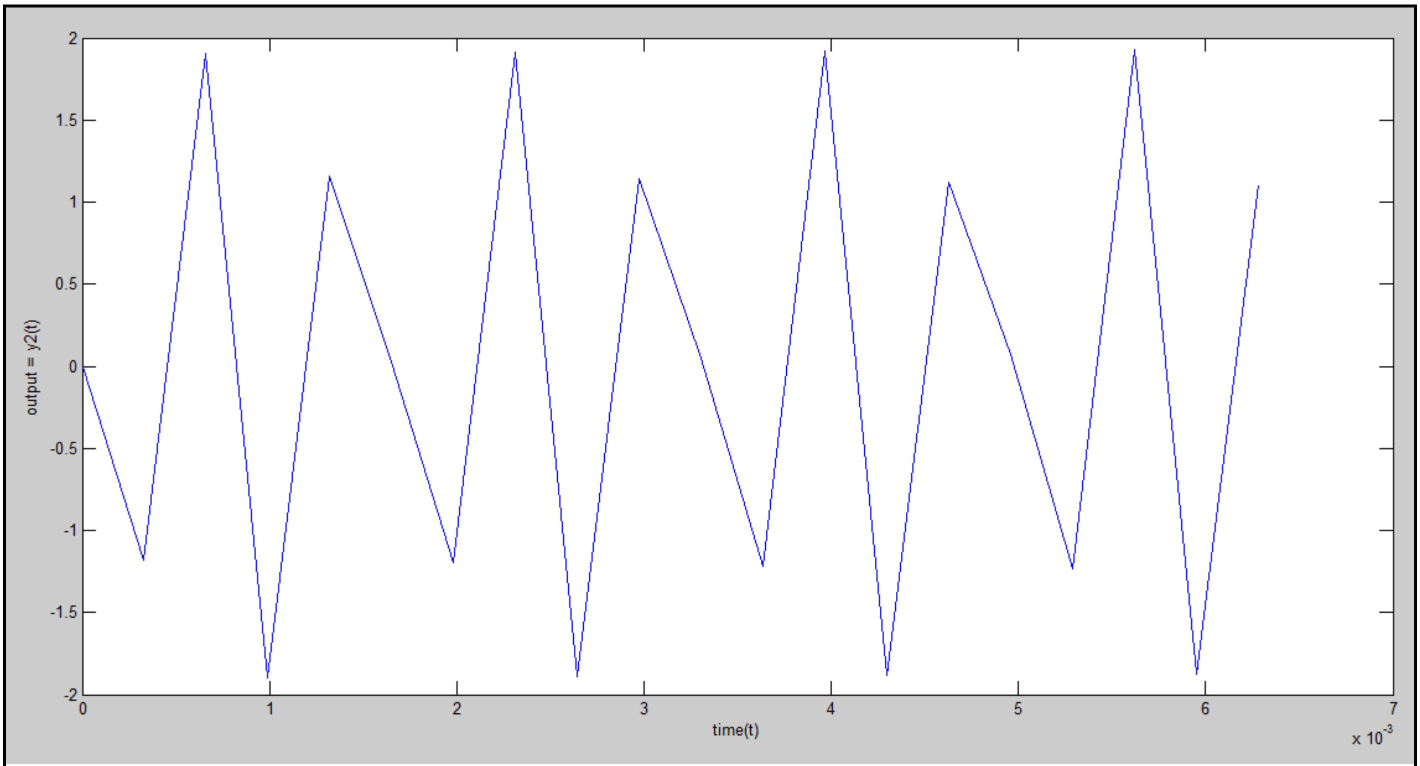
$X_2(s) = \sin \omega_2 t \, u(t)$

$$Y_2(s) = \left(\frac{2s^2 + 5s + 12}{s^2 + 2s + 10}\right)\left(\frac{\omega_2}{s^2 + \omega_2{}^2}\right)$$

$output = \; y_2(t)$ = F3

(Where a = ω_2)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 808*pi*10^3;
plot(t,subs(F3))
xlabel('time(t)')
ylabel('output = y2(t)')
```



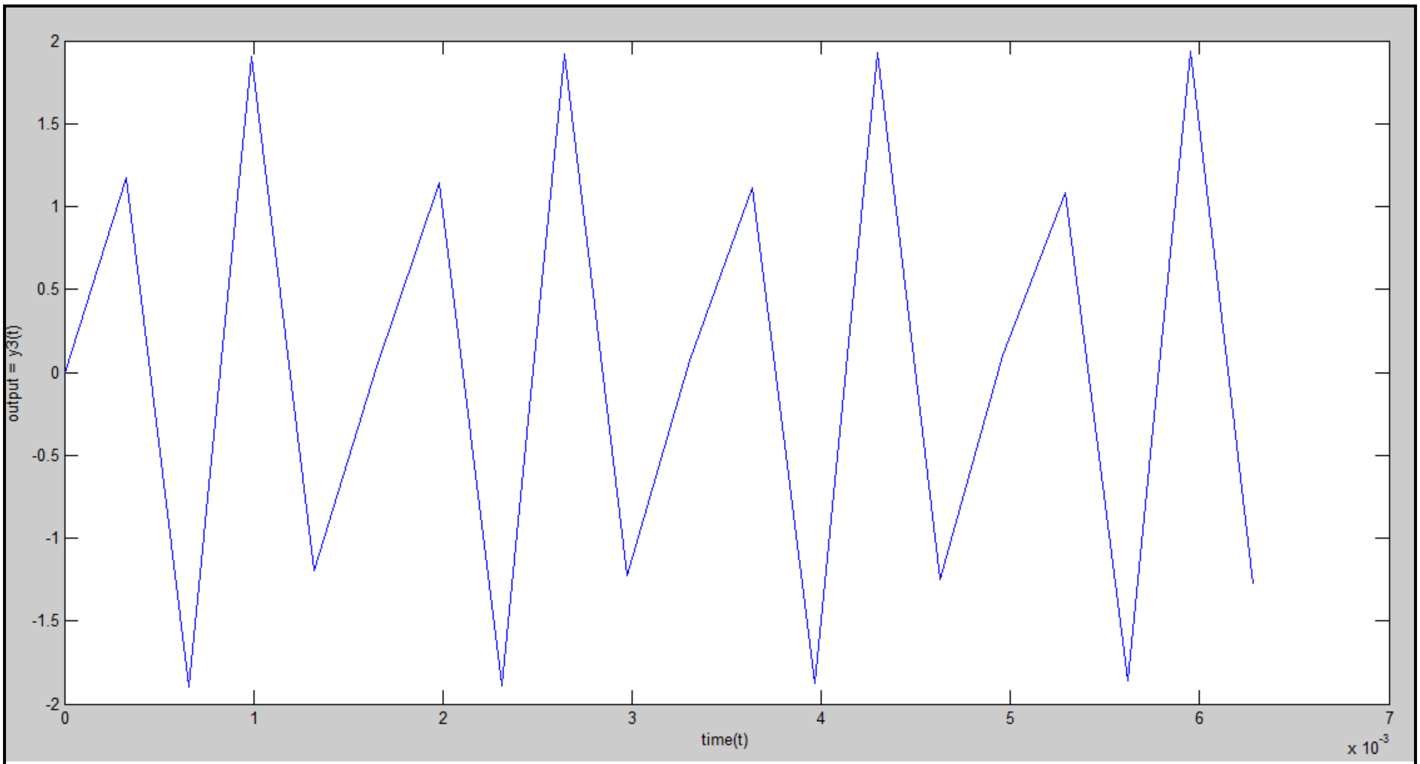
$$X_3(s) = \sin\omega_3 t\, u(t)$$

$$Y_3(s) = \left(\frac{2s^2 + 5s + 12}{s^2 + 2s + 10}\right)\left(\frac{\omega_3}{s^2 + {\omega_3}^2}\right)$$

$$output = y_3(t) = F3$$

(Where a = ω_3)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 1212*pi*10^3;
plot(t,subs(F3))
xlabel('time(t)')
ylabel('output = y3(t)')
```



4. $H(s) = \frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}$

```
syms s a
F4 = ilaplace((2*(s^2)+5*s+12)*a/((s^2+2*s+10)*(s^2+a^2)*(s+2))) %laplace
inverse

F4 =

((a*exp(-2*t))/(a^2 + 4) + (240*sin(a*t) - 68*a*cos(a*t) + 20*a^3*cos(a*t)
- 2*a^5*cos(a*t) - 18*a^2*sin(a*t) + 3*a^4*sin(a*t))/((a^2 + 4)*(a^2 - 6*a
+ 10)*(a^2 + 6*a + 10)) - (exp(-t)*(- a^3 + 8*a)*(cos(3*t) + sin(3*t)*((-
a^3 + 26*a)/(3*(- a^3 + 8*a)) - 1/3)))/((a^2 - 6*a + 10)*(a^2 + 6*a +
10)))
```
$u(t)$

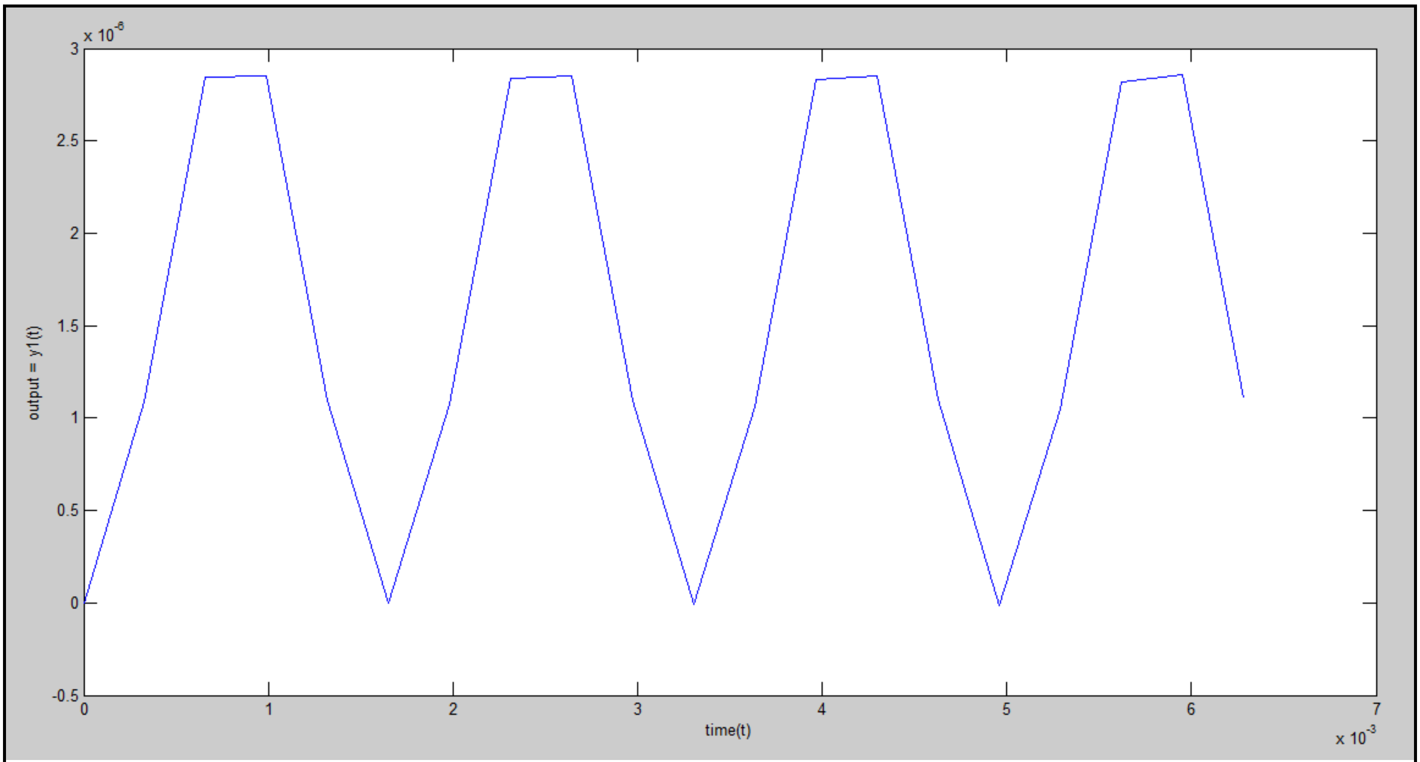
$X_1(s) = \sin \omega_1 t \, u(t)$

$$Y_1(s) = \left(\frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}\right)\left(\frac{\omega_1}{s^2+{\omega_1}^2}\right)$$

$output = y_1(t)$ = F4

(Where a = ω_1)

```
t=linspace(0,0.002*pi,20);
a = 404*pi*10^3;
plot(t,subs(F4))
xlabel('time(t)')
ylabel('output = y1(t)')
```



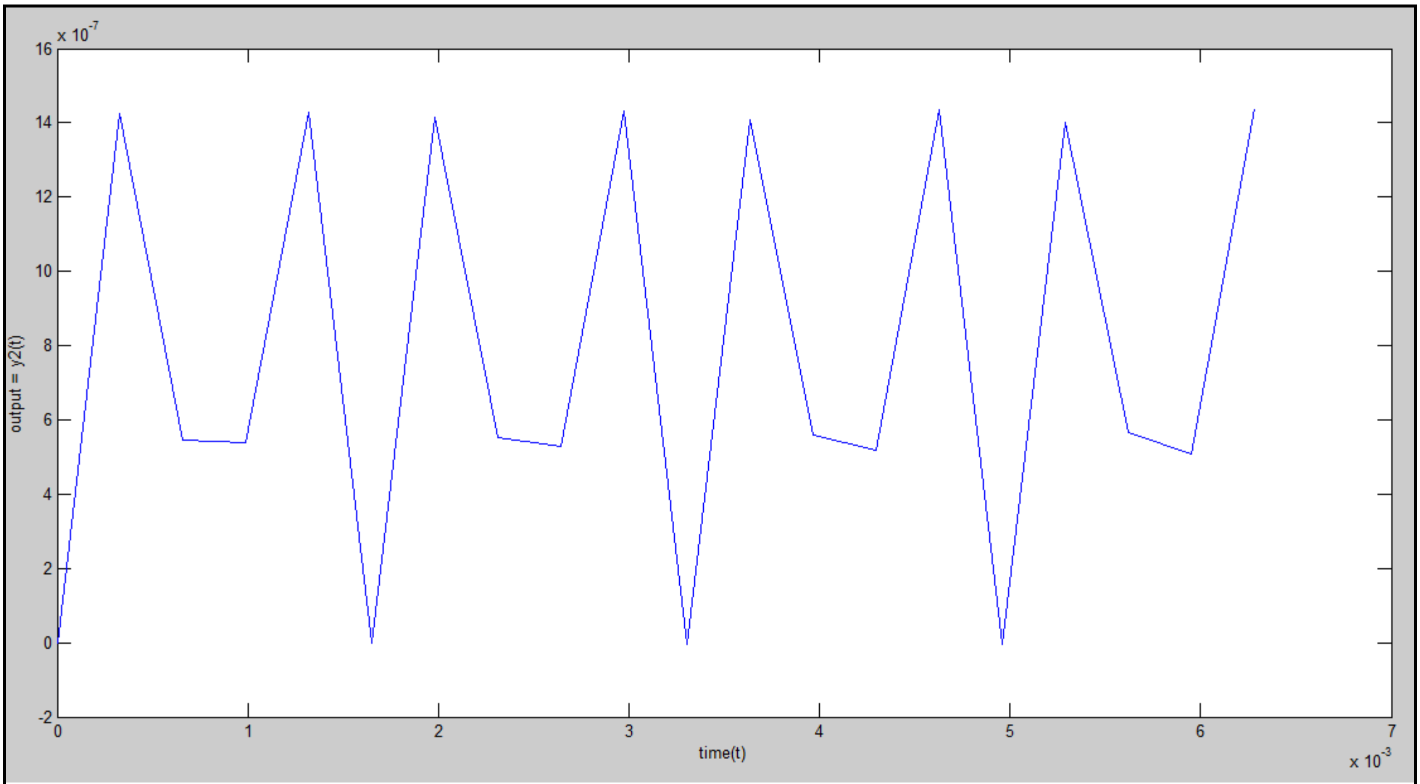
$X_2(s) = \sin \omega_2 t \, u(t)$

$Y_2(s) = \left(\frac{2s^2 + 5s + 12}{(s^2 + 2s + 10)(s + 2)}\right)\left(\frac{\omega_2}{s^2 + {\omega_2}^2}\right)$

$output = y_2(t)$ = F4

(Where a = ω_2)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 808*pi*10^3;
plot(t,subs(F4))
xlabel('time(t)')
ylabel('output = y2(t)')
```



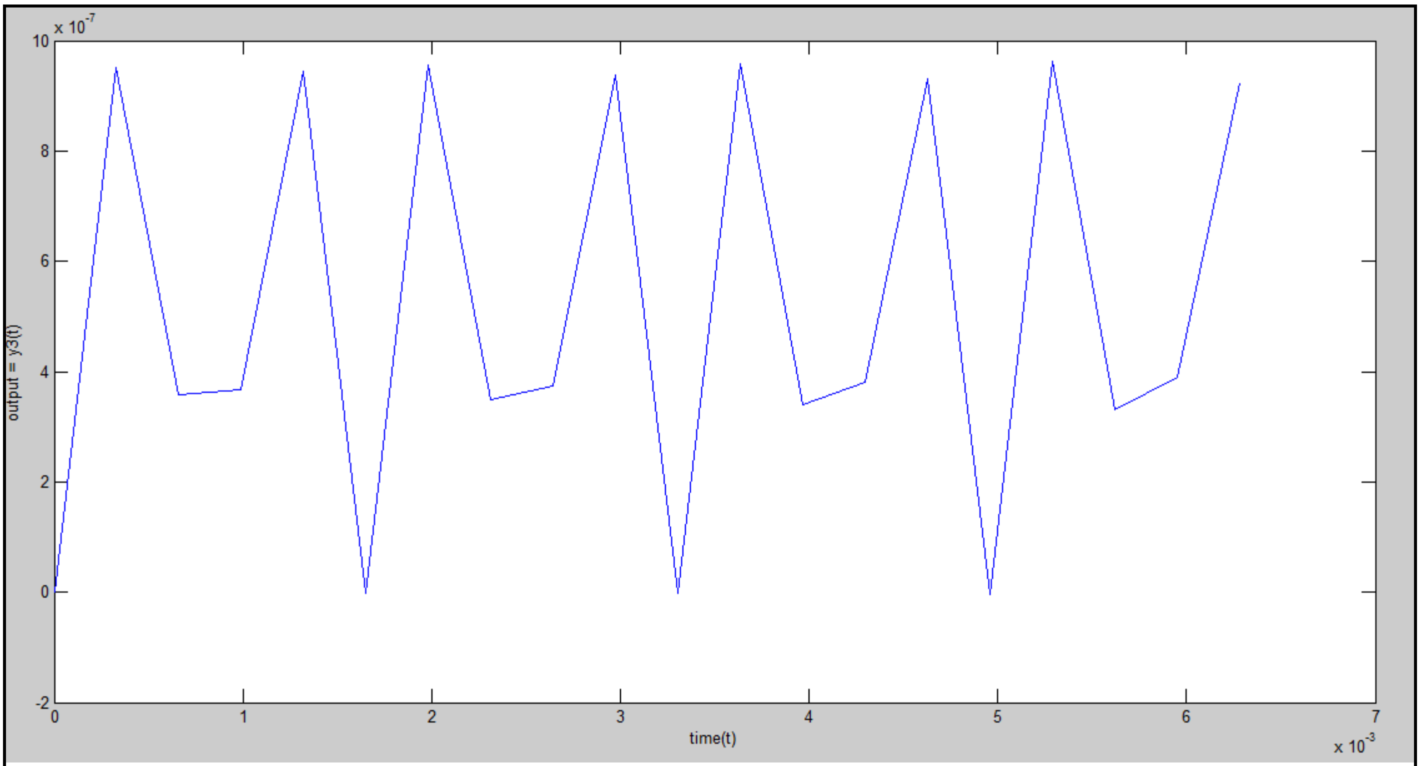
$X_3(s) = \sin \omega_3 t \, u(t)$

$Y_3(s) = \left(\frac{2s^2 + 5s + 12}{(s^2 + 2s + 10)(s + 2)}\right)\left(\frac{\omega_3}{s^2 + \omega_3{}^2}\right)$

$output = \ y_3(t)$ = F4

(Where a = ω_3)

```
%%%%%%%code%%%%%%
t=linspace(0,0.002*pi,20);
a = 1212*pi*10^3;
plot(t,subs(F4))
xlabel('time(t)')
ylabel('output = y3(t)')
```



PART 3: Surface Plots of a System Function in MATLAB

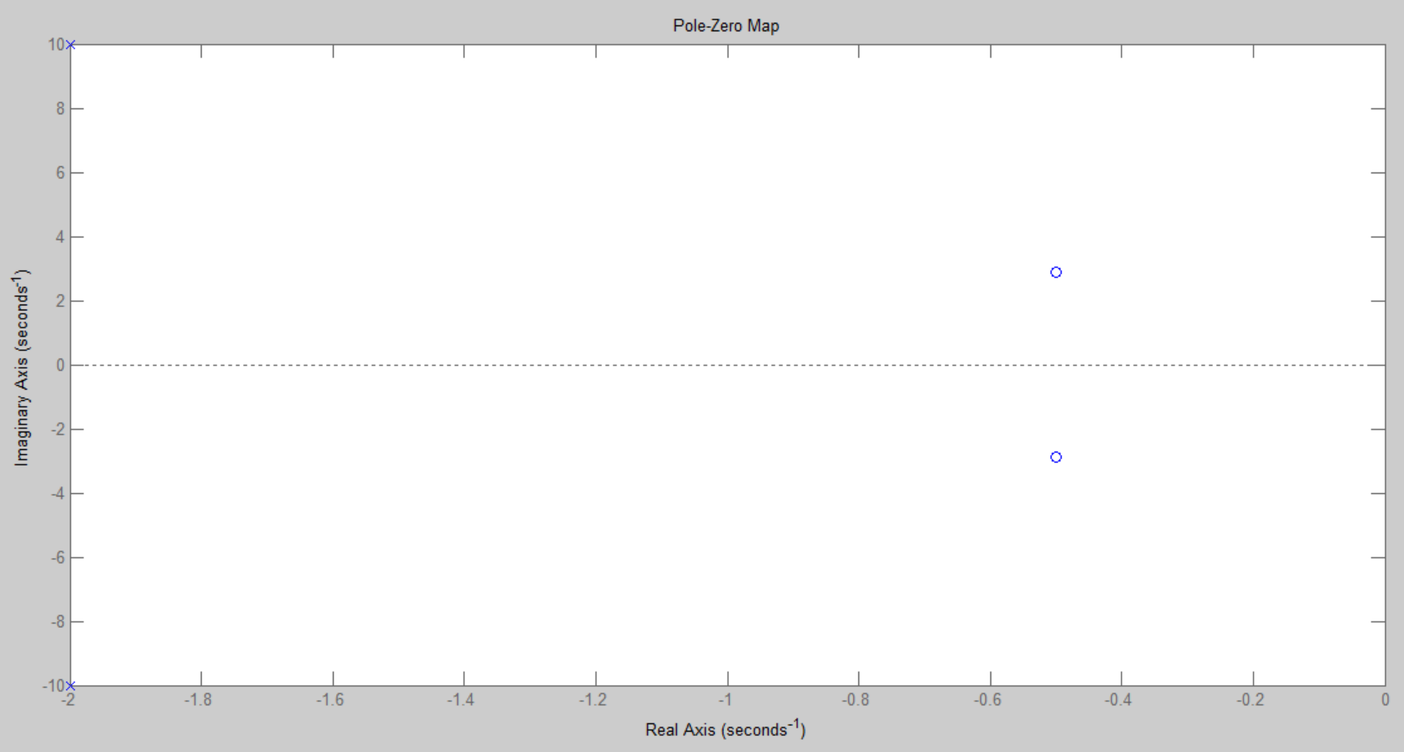
Exercise

For example let's take

$$H(s) = \frac{2s^2 + 2s + 17}{s^2 + 4s + 104}$$

```
b = [2 2 17];
a = [1 4 104];
zs = roots(b);
ps = roots(a);
pzmap(ps,zs);
```

Pole – zero diagram



Values of zeros,

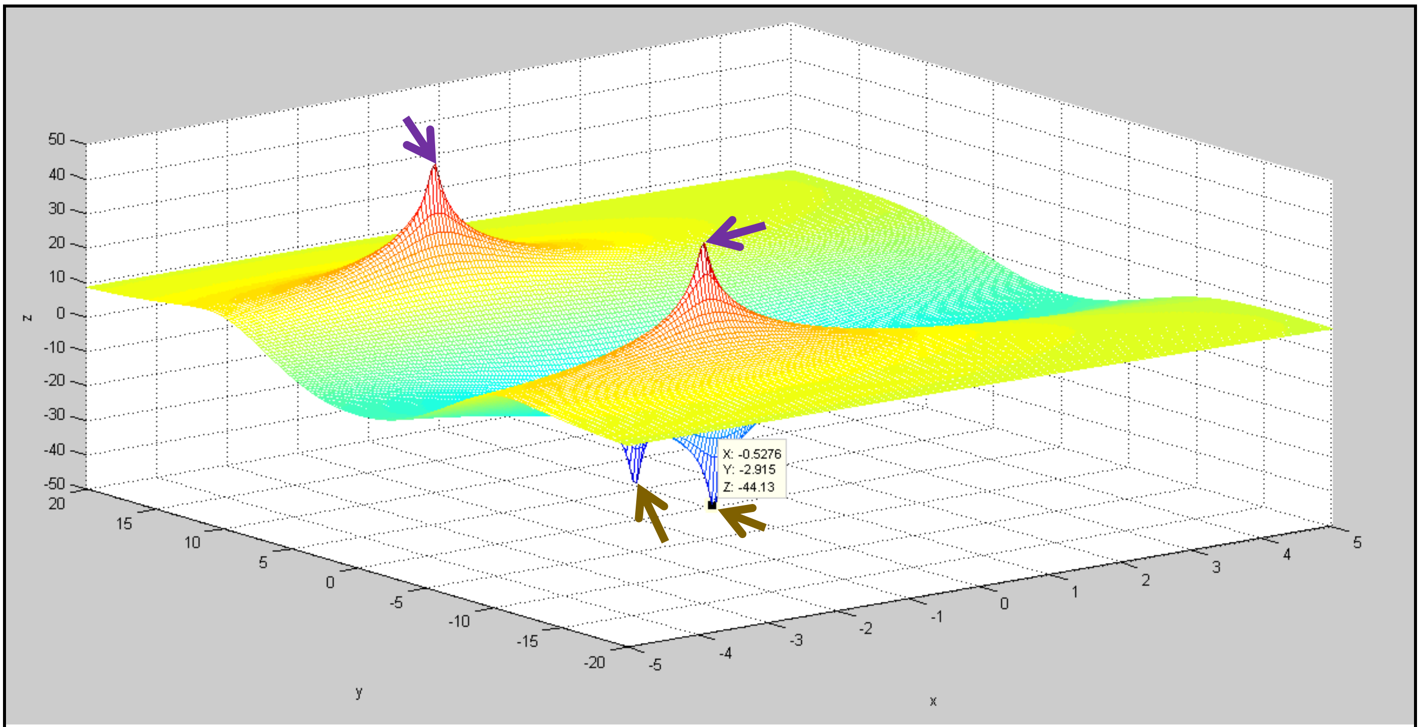
```
zs =
-0.5000 + 2.8723i
-0.5000 - 2.8723i
```

Values of poles,

```
ps =
-2.0000 +10.0000i
-2.0000 -10.0000i
```

Let's look at the surface plot,

```
omega = linspace(-20,20,200);
sigma = linspace(-5,5,200);
[sigmagrid,omegagrid] = meshgrid(sigma,omega);
sgrid =  sigmagrid+j*omegagrid;
H1 = polyval(b,sgrid)./polyval(a,sgrid);
mesh(sigma,omega,20*log10(abs(H1)));
```



As shown in the above mesh purple arrows are pointed towards poles and brown arrows are pointed towards zeros.

Poles are at the peak and zeros are at zeros.

Surface of the surface plot shows the magnitude of the transfer function. Bode plot is the projection of this surface along the $\sigma = 0$ axis.