Department of Computer Engineering
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CO 544 Machine Learning and Data Mining

Tutorial 01

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Answers:

1. *Show that the following perceptron model can be used to achieve an AND gate (Activation function:Threshold function with output threshold value given as 0).*

$$x_1 + x_2 - 1.5 : (w_0 = -1.5, w_1 = 1, w_2 = 1)$$

Ans:
Consider the first row of the data table: (A=0, B=0, Out=0)
Substitiue values for the given model: $v = x_1 + x_2 - 1.5$=$0 + 0 - 1.5 = -1.5 < 0$; $v < 0 \Rightarrow y = 0$

y(predicted value) = output value.

Now consider the second row of the data table: (A=0, B=1, Out=0)
Substitiue values for the given model: $v = x_1 + x_2 - 1.5$=$0 + 1 - 1.5 = -0.5 < 0$; $v < 0 \Rightarrow y = 0$

y(predicted value) = output value.

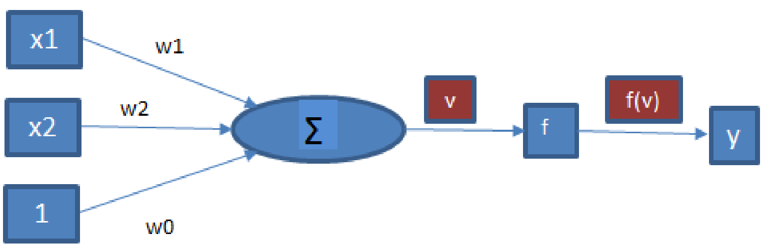
Now the third row: (A=1, B=0, Out=0)
Substitiue values for the given model: $v = x_1 + x_2 - 1.5$=$1 + 0 - 1.5 = -0.5 < 0$; $v < 0 \Rightarrow y = 0$

y(predicted value) = output value.

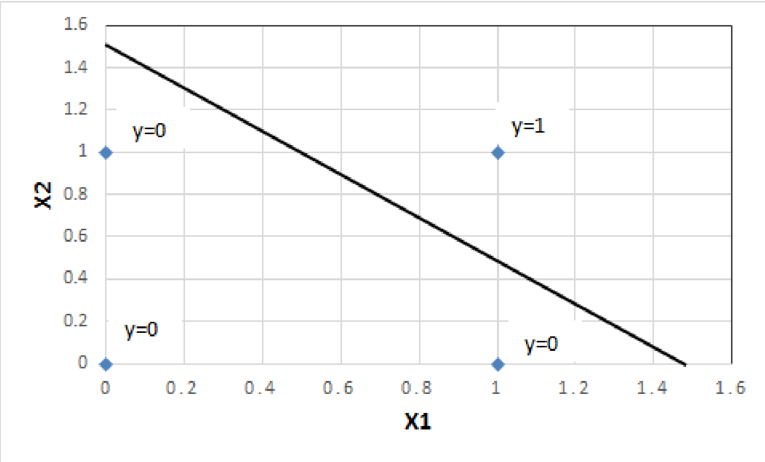
Last row: (A=1, B=1, Out=1)
Substitiue values for the given model: $v = x_1 + x_2 - 1.5$=$1 + 1 - 1.5 = 0.5 \geq 0$; $v \geq 0 \Rightarrow y = 1$
y(predicted value) = output value.

For all the data entries $\Rightarrow$ y(predicted value) = output value.
$\therefore$ the given model can be used to achieve an AND gate.

(a) *Draw the schematic diagram of the perceptron model.*



(b) *Mark the outputs in a 2D plot and draw the model to visualize the separation of two classes.*



2. *Three inputs with values given in the table below used as inputs to a neuron. The corresponding weights are* $w_0 = 0.4, w_1 = 0.1, w_2 = 0.4, w_3 = 0.5$.

x_1	x_2	x_3
1	3	2
2	2	4
3	1	5
2	4	1
3	3	3

If the activation function is the threshold function with output threshold value given as 3, calculate the outputs of this neuron respect to each row of the given table.

Threshold function : $y = \begin{cases} 1 & v \geq 3 \\ 0 & v < 3 \end{cases}$; where v - output of the sigma function.

Ans:

$v = 0.1x_1 + 0.4x_2 + 0.5x_3 + 0.4b = 0.1x_1 + 0.4x_2 + 0.5x_3 + 0.4$

x_1	x_2	x_3	Σ	Activation Func:	y
1	3	2	2.7	< 3	0
2	2	4	3.4	≥ 3	1
3	1	5	3.6	≥ 3	1
2	4	1	2.7	< 3	0
3	3	3	3.4	≥ 3	1

3. *Consider the data set given below.*

x_1	x_2	d
0	0	0
0	1	1
1	0	1
1	1	1

Consider a perceptron with the below activation function. If the threshold value v=0.5 and learning parameter value η *=0.1, Show the updates of the weight parameter values till convergence using* ***Stochastic Gradient Decent(SGD)*** *method for optimisation.Consider the initial weight parameter values as:* $w_1 = 0, w_2 = 0, w_0 = 0$

Note: Equation to update the weights when using SGD : $w_i(n+1) = w_i(n) + \eta * e(n) * x_i(n)$;

where, $e(n)$ is the prediction error for n^{th} instance; $e(n)$= Actual output ($d(n)$) - Predicted output ($y(n)$)

Ans:

Epoch : 1

n=0 ;$(w_1 = 0, w_2 = 0, w_0 = 0)$; $(x_1 = 0, x_2 = 0, d = 0)$

$$v = w_1x_1 + w_2x_2 + w_0b = (0)(0) + (0)(0) + (0)(1) = 0$$

$$y = f(v) = 0$$

$$e(0) = (0 - 0) = 0$$

Now let's update the weights,

$$w_1(1) = w_1(0) + \eta * e(0) * x_1(0) \; = 0 + 0.1 * 0 * 0 = 0$$

$$w_2(1) = w_2(0) + \eta * e(0) * x_2(0) \; = 0 + 0.1 * 0 * 0 = 0$$

$$w_0(1) = w_0(0) + \eta * e(0) * b(0) \; = 0 + 0.1 * 0 * 1 = 0$$

Go to next input.

n=1 ;$(w_1 = 0, w_2 = 0, w_0 = 0)$; $(x_1 = 0, x_2 = 1, d = 1)$

$$v = w_1x_1 + w_2x_2 + w_0b = (0)(0) + (0)(1) + (0)(1) = 0$$

$$y = f(v) = 0$$

$$e(1) = (1 - 0) = 1$$

Updating weights,

$$w_1(2) = w_1(1) + \eta * e(1) * x_1(1) \; = 0 + 0.1 * 1 * 0 = 0$$

$$w_2(2) = w_2(1) + \eta * e(1) * x_2(1) \; = 0 + 0.1 * 1 * 1 = 0.1$$

$$w_0(2) = w_0(1) + \eta * e(1) * b(1) \; = 0 + 0.1 * 1 * 1 = 0.1$$

Go to the next input.

n=2 ;$(w_1 = 0, w_2 = 0.1, w_0 = 0.1)$; $(x_1 = 1, x_2 = 0, d = 1)$

$$v = w_1x_1 + w_2x_2 + w_0b = (0)(1) + (0.1)(0) + (0.1)(1) = 0.1$$

$$y = f(v) = 0$$

$$e(2) = (1 - 0) = 1$$

Updating weights,

$$w_1(3) = w_1(2) + \eta * e(2) * x_1(2) = 0 + 0.1 * 1 * 1 = 0.1$$

$$w_2(3) = w_2(2) + \eta * e(2) * x_2(2) = 0.1 + 0.1 * 1 * 0 = 0.1$$

$$w_0(3) = w_0(2) + \eta * e(2) * b(2) = 0.1 + 0.1 * 1 * 1 = 0.2$$

Go to the next input.

n=3 ;$(w_1 = 0.1, w_2 = 0.1, w_0 = 0.2)$; $(x_1 = 1, x_2 = 1, d = 1)$

$$v = w_1x_1 + w_2x_2 + w_0b = (0.1)(1) + (0.1)(1) + (0.2)(1) = 0.4$$

$$y = f(v) = 0$$

$$e(3) = (1 - 0) = 1$$

Updating weights,

$$w_1(4) = w_1(3) + \eta * e(3) * x_1(3) = 0.1 + 0.1 * 1 * 1 = 0.2$$

$$w_2(4) = w_2(3) + \eta * e(3) * x_2(3) = 0.1 + 0.1 * 1 * 1 = 0.2$$

$$w_0(4) = w_0(3) + \eta * e(3) * b(3) = 0.2 + 0.1 * 1 * 1 = 0.3$$

Now we have completed one **epoch**(one run through all inputs).

Epoch : 2

n=4 ;$(w_1 = 0.2, w_2 = 0.2, w_0 = 0.3)$; $(x_1 = 0, x_2 = 0, d = 0)$

$$v = w_1x_1 + w_2x_2 + w_0b = (0.2)(0) + (0.2)(0) + (0.3)(0) = 0$$

$$y = f(v) = 0$$

$$e(4) = (0 - 0) = 0$$

Updating weights,

$$w_1(5) = w_1(4) + \eta * e(4) * x_1(4) = 0.2 + 0.1 * 0 * 0 = 0.2$$

$$w_2(5) = w_2(4) + \eta * e(4) * x_2(4) = 0.2 + 0.1 * 0 * 0 = 0.2$$

$$w_0(5) = w_0(4) + \eta * e(4) * b(4) = 0.3 + 0.1 * 0 * 1 = 0.3$$

Go to the next input.

n=5 ;$(w_1 = 0.2, w_2 = 0.2, w_0 = 0.3)$; $(x_1 = 0, x_2 = 1, d = 1)$

$$v = w_1x_1 + w_2x_2 + w_0b = (0.2)(0) + (0.2)(1) + (0.3)(1) = 0.5$$

$$y = f(v) = 1$$

$e(5) = (1 - 1) = 0$

Updating weights,

$w_1(6) = w_1(5) + \eta * e(5) * x_1(5) = 0.2 + 0.1 * 0 * 0 = 0.2$

$w_2(6) = w_2(5) + \eta * e(5) * x_2(5) = 0.2 + 0.1 * 0 * 1 = 0.2$

$w_0(6) = w_0(5) + \eta * e(5) * b(5) = 0.3 + 0.1 * 0 * 1 = 0.3$

Go to the next input.

n=6 ;$(w_1 = 0.2, w_2 = 0.2, w_0 = 0.3)$; $(x_1 = 1, x_2 = 0, d = 1)$

$v = w_1x_1 + w_2x_2 + w_0b = (0.2)(1) + (0.2)(0) + (0.3)(1) = 0.5$

$y = f(v) = 1$

$e(6) = (1 - 1) = 0$

Updating weights,

$w_1(7) = w_1(6) + \eta * e(6) * x_1(6) = 0.2 + 0.1 * 0 * 1 = 0.2$

$w_2(7) = w_2(6) + \eta * e(6) * x_2(6) = 0.2 + 0.1 * 0 * 0 = 0.2$

$w_0(7) = w_0(6) + \eta * e(6) * b(6) = 0.3 + 0.1 * 0 * 1 = 0.3$

Go to the next input.

n=7 ;$(w_1 = 0.2, w_2 = 0.2, w_0 = 0.3)$; $(x_1 = 1, x_2 = 1, d = 1)$

$v = w_1x_1 + w_2x_2 + w_0b = (0.2)(1) + (0.2)(1) + (0.3)(1) = 0.7$

$y = f(v) = 1$

$e(7) = (1 - 1) = 0$

Updating weights,

$w_1(8) = w_1(7) + \eta * e(7) * x_1(7) = 0.2 + 0.1 * 0 * 1 = 0.2$

$w_2(8) = w_2(7) + \eta * e(7) * x_2(7) = 0.2 + 0.1 * 0 * 1 = 0.2$

$w_0(8) = w_0(7) + \eta * e(7) * b(7) = 0.3 + 0.1 * 0 * 1 = 0.3$

Now we have completed second epoch.

We have reached to the convergence.

Therefore, the optimum weight parameters for the above data set is: $w_1 = 0.2, w_2 = 0.2, w_0 = 0.3$

And the perceptron model to represent the given data is: $0.2x_1 + 0.2x_2 + 0.3$