CO544

Tutorial 01

$= x_1 w_1 + x_2 w_2 - 1.5 w_0$

$= x_1 \times 1 + x_2 \times 1 - 1.5(-1.5)$

$= x_1 + x_2 + 2.25$

$= x_1 + x_2 + 2.25$

① $x_1 w_1 + x_2 w_2 + w_0$

$= x_1(1) + x_2(1) - 1.5$

$= x_1 + x_2 - 1.5$

Perceptron rule :- $x_1 + x_2 - 1.5 > 0$ & $x_1 + x_2 - 1.5 \leq 0$

Prediction = 1 Prediction = 0

Let's substitute values of AND gate

AND Gate

Row 1

$x_1 = 0$, $x_2 = 0$

$= -1.5 < 0$

$0 + 0 - 1.5 \longrightarrow \therefore$ Prediction = 0

Row 2

$x_1 = 0$, $x_2 = 1$

$0 + 1 - 1.5 = -0.5 < 0 \longrightarrow \therefore$ Prediction = 0

Row 3

$x_1 = 1$, $x_2 = 0$

$1 + 0 - 1.5 = -0.5 < 0 \longrightarrow \therefore$ Prediction = 0

Row 4

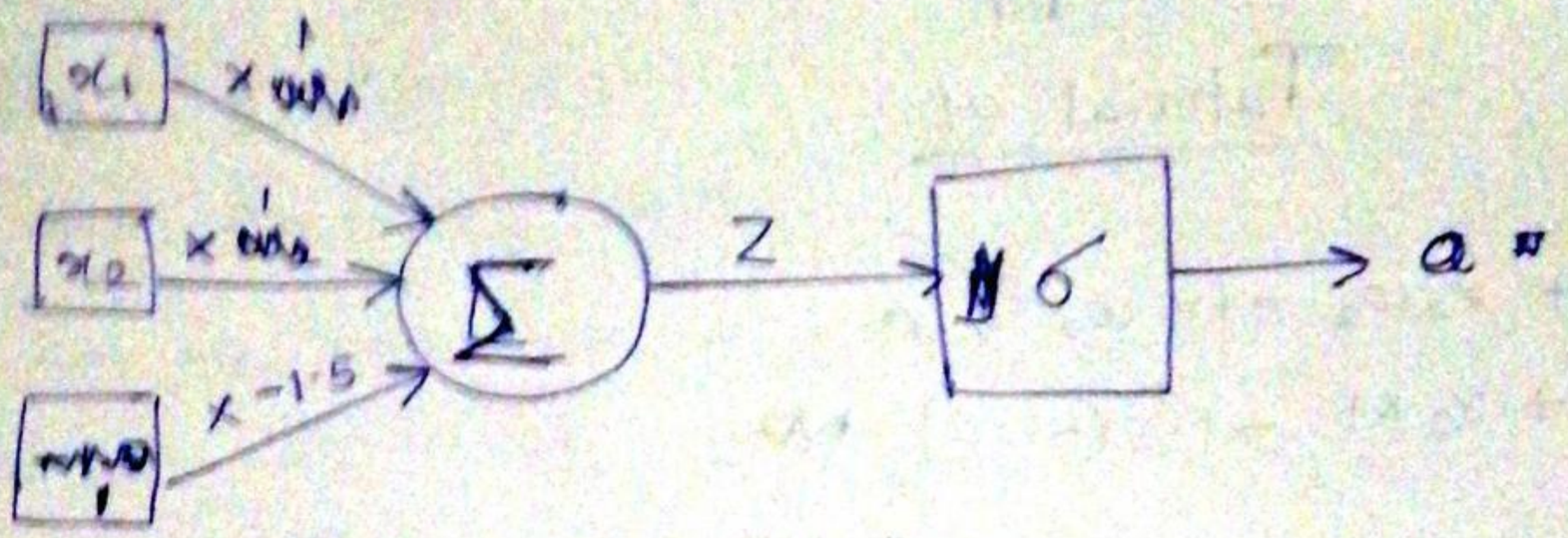
$x_1 = 1$, $x_2 = 1$

$1 + 1 - 1.5 = 0.5 > 0 \longrightarrow \therefore$ Prediction = 1

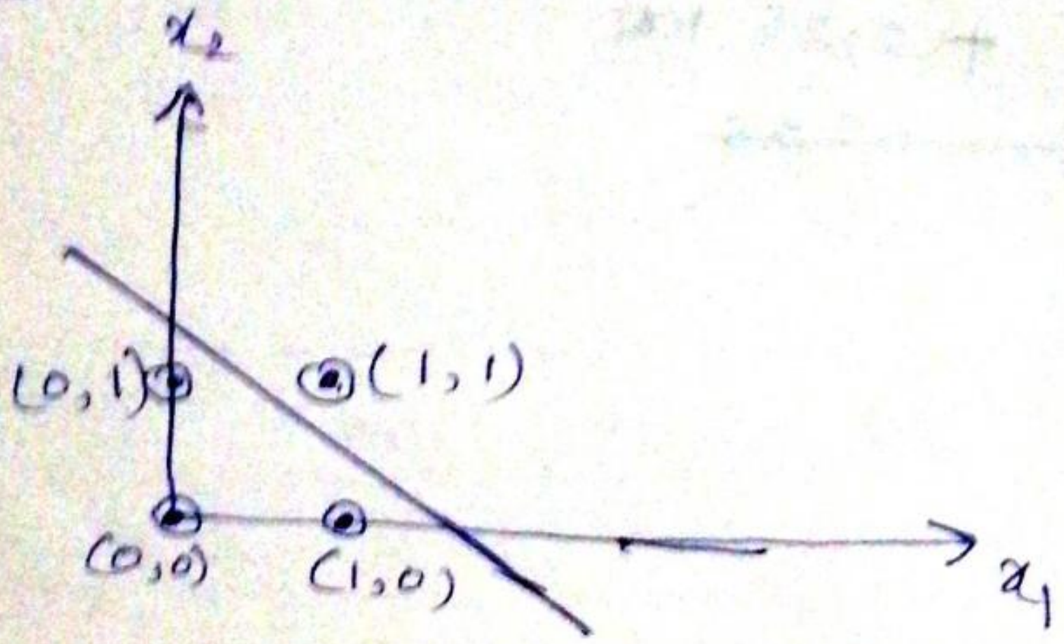
As shown above substituting each inputs of A and B to x_1 & x_2 gives the same output as the AND gate.

So the give perceptron model can be used to achieve an AND gate.

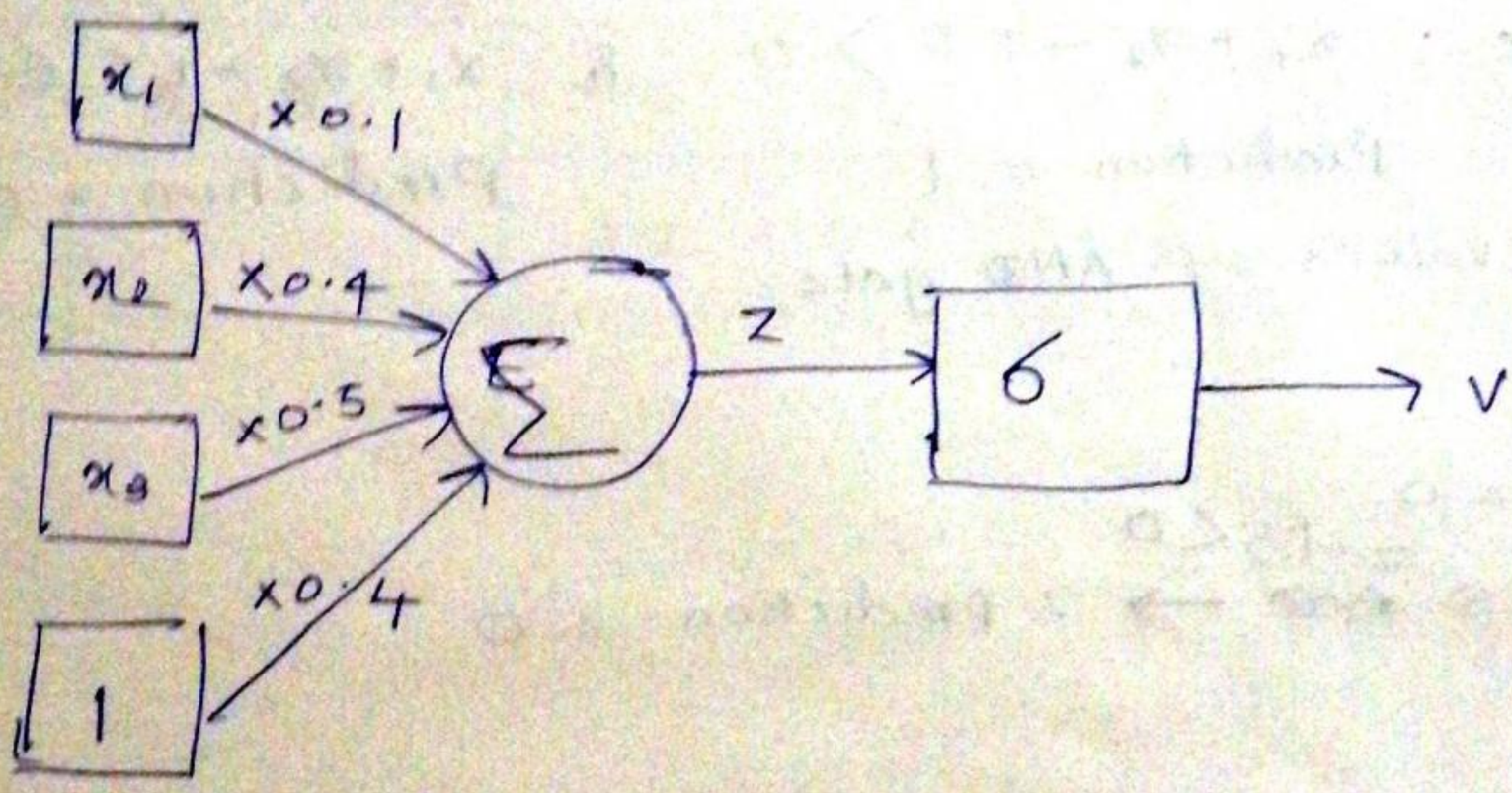
a)



b)



②



$X = 0.1x_1 + 0.4x_2 + 0.5x_3 + 0.4$

x_1	x_2	x_3	X	y
1	3	2	2.7	0
2	2	4	3.4	1
3	1	5	3.6	1
2	4	1	2.7	0
3	3	3	3.4	1

③ When $w_1 = w_2 = w_0 = 0$

$n = 0$

$w_1 x_1 + w_2 x_2 + w_0 < 0.5$

Initial

x_1	x_2	d	
0	0	0	d_1
0	1	1	d_2
1	0	1	d_3
1	1	1	d_4

$d \geq 0$

$w_1 x_1 + w_2 x_2 + w_0 \geq 0.5$

$d = 1$

$d_1 \in (0, 0)$ $d_3 = (1, 0)$

$d_2 \in (0, 1)$ $d_4 = (1, 1)$

Then $d_1 = d_2 = d_3 = d_4 = 0$

∴ only d_2, d_3, d_4 are misclassified.

A21

Consider d_2

$w = (0, 0) + 0.1 \times (1 - 0) \times (0, 1)$

$= (0, 0) + 0.1 \times (0, 1)$

$= (0, 0) + (0, 0.1)$

$= (0, 0.1)$

Consider d_3

$w_0 = 0 + 0.1 \times (1 - 0) = 0.1$

Applying w_0, d_2, d_3 & d_4

$d_2 \Rightarrow 0 \times 0 + 0.1 \times 1 + 0.1 = 0.2 < 0.5$

$y = 0$ ✗

$d_3 \Rightarrow 0 \times 1 + 0.1 \times 0 + 0.1 = 0.1 < 0.5$

$y = 0$ ✗

$d_4 \Rightarrow 0 \times 1 + 0.1 \times 1 + 0.1 = 0.2 < 0.5$

$y = 0$ ✗

$d_4 \Rightarrow 0 \times 0 + 0.1 \times 0 + 0.1 = 0.1 < 0.3$

$y = 0$ ✓

Consider d_3

$w = (0,0) + 0.1 \times (1-0) \times (1,0)$

$= (0,0) + (0.1, 0)$

$= (0.1, 0)$

$w_0 = 0 + 0.1 \times (1,0) = 0.1$

Applying to d_1, d_2, d_3 & d_4

$d_1 \Rightarrow 0.1 \times 0 + 0 \times 0 + 0.1 = 0.1 < 0.5$

$y_d = 0$ ✓

$d_2 \Rightarrow 0.1 \times 0 + 0 \times 1 + 0.1 = 0.1 < 0.5$

$y_d = 0$ ✗

$d_3 \Rightarrow 0.1 \times 1 + 0 \times 0 + 0.1 = 0.2 < 0.5$

$y_d = 0$ ✗

$d_4 \Rightarrow 0.1 \times 1 + 0 \times 1 + 0.1 = 0.2 < 0.5$

$y_d = 0$ ✗

consider d_4

$w = (0,0) + 0.1 \times (1-0) \times (1,1)$

$= (0.1, 0.1)$

$w_0 = 0 + 0.1 \times (1-0) = 0.1$

Applying to d_1, d_2, d_3 & d_4

$d_1 \Rightarrow 0.1 \times 0 + 0.1 \times 0 + 0.1 = 0.1 < 0.5$

$y_d = 0$ ✓

$d_2 \Rightarrow 0.1 \times 0 + 0.1 \times 1 + 0.1 = 0.2 < 0.5$

$y_d = 0$ ✗

$d_3 \Rightarrow 0.1 \times 1 + 0.1 \times 0 + 0.1 = 0.2 < 0.5$

$y_d = 0$ ✗

$d_4 \Rightarrow 0.1 \times 1 + 0.1 \times 1 + 0.1 = 0.3 < 0.5$

$y_d = 0$ ✗

Take this set as the final

$w = (0.1, 0.1)$

$w_0 = 0.1$

n=2 Again d_2, d_3 & d_4 are miss classified.

Consider d_2

$$w = (0.1, 0.1) + 0.1 \times (1-0) \times (0,1)$$
$$= (0.1, 0.1) + (0, 0.1)$$
$$= (0.1, 0.2)$$

$$w_0 = 0.1 + 0.1 \times (1-0) = 0.2$$

Applying to d_1, d_2, d_3 & d_4

$d_1 \Rightarrow 0.1 \times 0 + 0.2 \times 0 + 0.2 = 0.2 < 0.5$
$y = 0$ ✓

$d_2 \Rightarrow 0.1 \times 0 + 0.2 \times 1 + 0.2 = 0.4 < 0.5$
$y = 0$ ✗

$d_3 \Rightarrow 0.1 \times 1 + 0.2 \times 0 + 0.2 = 0.3 < 0.5$
$y = 0$ ✗

$d_4 \Rightarrow 0.1 \times 1 + 0.2 \times 1 + 0.2 = 0.5 \geq 0.5$
$y = 1$ ✓

Consider d_3

$$w = (0.1, 0.1) + 0.1 \times (1-0) \times (1, 0)$$
$$= (0.2, 0.1)$$

$$w_0 = 0.2$$

Applying to d_1, d_2, d_3, & d_4

$d_1 \Rightarrow 0.2 \times 0 + 0.1 \times 0 + 0.2 = 0.2 < 0.5$
$y = 0$ ✓

$d_2 \Rightarrow 0.2 \times 0 + 0.1 \times 1 + 0.2 = 0.3 < 0.5$
$y = 0$ ✗

$d_3 \Rightarrow 0.2 \times 1 + 0.1 \times 0 + 0.2 = 0.4 < 0.5$
$y = 0$ ✗

$d_4 \Rightarrow 0.2 \times 1 + 0.1 \times 1 + 0.2 = 0.5 \geq 0.5$
$y = 1$ ✓

Consider d_4

$$w = (0.1, 0.1) + 0.1 * (1-0) * (1, 1)$$
$$= (0.2, 0.2)$$

$w_0 = 0.2$

Applying to d_1, d_2, d_3 & d_4

$d_1 \Rightarrow 0.2 \times 0 + 0.2 \times 0 + 0.2 = 0.2 < 0.5$

$y = 0$ ✓

$d_2 \Rightarrow 0.2 \times 0 + 0.2 \times 1 + 0.2 = 0.4 < 0.5$

$y = 0$ ✗

$d_3 \Rightarrow 0.2 \times 1 + 0.2 \times 0 + 0.2 = 0.4 < 0.5$

$y = 0$ ✗

$d_4 \Rightarrow 0.2 \times 1 + 0.2 \times 1 + 0.2 = 0.6 > 0.5$

$y = 1$ ✓

Take this set as the final on this step

$$w = (0.2, 0.2)$$

$w_0 = 0.2$

Now only d_2 and d_3 are miss classified.

$n = 3$

Consider d_2

$$w = (0.2, 0.2) + 0.1 \times (1-0) \times (0, 1)$$
$$w = (0.2, 0.3)$$
$$w_0 = 0.2 + 0.1 \times (1-0) = 0.3$$

Applying to d_1, d_2, d_3 & d_4

$d_1 \Rightarrow 0.2 \times 0 + 0.3 \times 0 + 0.3 = 0.3 < 0.5$

$y = 0$ ✓

$d_2 \Rightarrow 0.2 \times 0 + 0.3 \times 1 + 0.3 = 0.6 > 0.5$

$y = 1$ ✓

$d_3 \Rightarrow 0.2 \times 1 + 0.3 \times 0 + 0.3 = 0.5 \geq 0.5$

$y = 1$ ✓

$d_4 \Rightarrow 0.2 \times 1 + 0.3 \times 1 + 0.3 = 0.8 > 0.5$

$y = 1$ ✓

When $w = (0.2, 0.3)$ and $w_0 = 0.3$ all the predicted outputs are correctly classified.

So we can take,

$w_1 = 0.2$, $w_2 = 0.3$ and $w_0 = 0.3$

Round	Missclassified Record	w_1	w_2	w_0	d_1	d_2	d_3	d_4
0	d_2, d_3, d_4	0	0	0	0	0	0	0
1	d_2, d_3, d_4	0.1	0.1	0.1	0.1	0.2	0.2	0.3
2	d_2, d_3	0.2	0.2	0.2	0.2	0.4	0.4	0.6
3		0.2	0.3	0.3	0.3	0.6	0.5	0.8