EE 387 - Signal Processing
Assignment 1

Dr. W. A. N. I. Harischandra
Department of Electrical and Electronic Engineering

June 2, 2020

Problem 1. *Determine the fundamental period of the following signals:*

(a) $x(t) = 3cos(10t + 1) - sin(4t - 1)$

(b) $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$

Problem 2. *Determine whether the following signals are periodic or aperiodic? If periodic, also find the period.*

(a) $x(t) = 2cos(4t + \frac{\pi}{3})$

(b) $x(t) = [sin(2t - \frac{\pi}{4})]^2$

(c) $x[n] = sin(6\frac{\pi}{7}n + 1)$

(d) $x[n] = cos(\frac{\pi}{8}n^2)$

(e) $x(t) = sin(\frac{\pi}{8}t^2)$

(f) $x[n] = cos(\frac{\pi}{2}n)cos(\frac{\pi}{4}n)$

Problem 3. *Classify each of the signals below as a power signal or an energy signal. In addition, find the power or the energy of the signal.*

(a)

$$x[n] = \begin{cases} cos(\pi n) & n \geq 0 \\ 0 & otherwise \end{cases}$$

(b)

$$x(t) = \begin{cases} \frac{1}{2}(cos(\omega t) + 1) & -\pi/\omega \leq t \leq \pi/\omega \\ 0 & otherwise \end{cases}$$

Problem 4. *Determine the Fourier series representation for the following signal, $x(t)$, with period 4:*

$$x(t) = \begin{cases} sin(\pi t) & 0 \leq t \leq 2 \\ 0 & 2 < t \leq 4 \end{cases}$$

Problem 5. *Consider the following three continuous-time signals with a fundamental period of T=1/2:*

$$x(t) = cos(4\pi t),$$
$$y(t) = sin(4\pi t),$$
$$z(t) = x(t)y(t).$$

(a) *Determine the Fourier series coefficients of $x(t)$.*

(b) *Determine the Fourier series coefficients of $y(t)$.*

(c) *Use the results of parts (a) and (b), along with the multiplication property of the continuous-time Fourier series to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.*

(d) *Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part (c).*

Problem 6. *Consider the following periodic square wave pulse (symmetric) with a period of 4,*

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & 1 < |t| < 2 \end{cases}$$

(a) *Determine the Fourier series coefficients of $x(t)$.*

(b) *$x(t)$ can be approximated by a linear combination of a finite number of harmonically related complex exponentials- that is, by a finite series of the form,*

$$x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$$

Plot $x_N(t)$ for N= 1, 3, 7, 19, 43, and 79 over the fundamental period (Hint: you may use any programing language).

(c) *Determine the percentage overshoot of the approximated square pulse for each of the above partial sums.*

Problem 7. *Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may use the Fourier transform properties.*

(a) $x_1(t) = x(1-t) + x(-1-t)$

(b) $x_2(t) = x(3t-6)$

(c) $x_3(t) = \frac{d^2}{dt^2} x(t-1)$

Problem 8. *Consider the Fourier transform pair*

$$e^{-|t|} \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{2}{1+\omega^2}$$

(a) *Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$.*

(b) *Use the above result, along with the duality property, to determine the Fourier transform of*

$$\frac{4t}{(1+t^2)^2}$$

Problem 9. *Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in Figure 1.*

(a) *Find $\angle X(j\omega)$.*

(b) *Find $X(j0)$.*

(c) *Find $\int_{-\infty}^{\infty} X(j\omega) d\omega$.*

(d) *Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.*

(e) *Sketch the inverse Fourier transform of $\Re\{X(j\omega)\}$.*

Note: You should peform all these calculations without explicitly evaluating $X(j\omega)$.

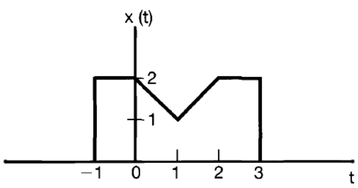

Figure 1