Fortran codes for computing the acoustic field surrounding a vibrating plate by the Rayleigh integral method

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Fortran codes for computing the acoustic field surrounding a vibrating plate by the Rayleigh integral method

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Abstract - This paper describes the author's RIM3 Fortran code. RIM3 delivers a computational solution to the acoustic field produced by a vibrating plate placed on an infinite baffle. The mathematical model is based on the Rayleigh integral. The method is implemented in three-dimensions by subroutine RIM3 in Fortran, which is made freely available for download. The subroutine is applied to test cases and results are given. The software is available as open source.

I INTRODUCTION

Methods based on integral equations, or boundary element methods (BEMs) have played an important role in many areas of science and engineering [eg 1-2]. Boundary Element Methods have been developed in acoustics for several decades. The method continues to be developed and applied more widely [3-20].

In this paper, a method for computing the acoustic field surrounding a vibrating panel, the Rayleigh Integral Method, is developed in a computer code (RIM3). Although the Rayleigh Integral Method is not strictly a boundary element method, it contains the same components and hence it is a useful addition to an acoustics boundary element method library.

   The purpose of this work is to develop a method and Fortran subroutine that delivers a computational solution to the acoustic field surrounding a (set of) flat plate(s) with specified acoustic properties, lying in an infinite baffle. The physical situation is illustrated in figure 1. The acoustic domain is the three-dimensional half-space. The motion of the plate is governed by a given condition on the plate. The baffle is rigid and perfectly reflecting. This model can be applied to a range of acoustic problems. For example it can model the acoustic field around the near-flat surface of an object. The Rayleigh integral [21] directly relates the velocity potential (or sound pressure) in the acoustic domain to the velocity distribution on the plate. Methods obtained in this way have been derived and applied to the problem of computing the properties of the acoustic field exterior to a flat or near-flat plate for some time. These methods are generally derived through applying a direct numerical integration method (see, for example Schenck [22]) to the Rayleigh integral. The resulting method is often termed the *simple source method*. However, the integrand of the Rayleigh integral can be singular or near-singular and it becomes increasingly oscillatory as the wavenumber increases. Hence the direct numerical integration that underlies the simple source method is expected to be a computationally inefficient means of obtaining the solution in many cases.
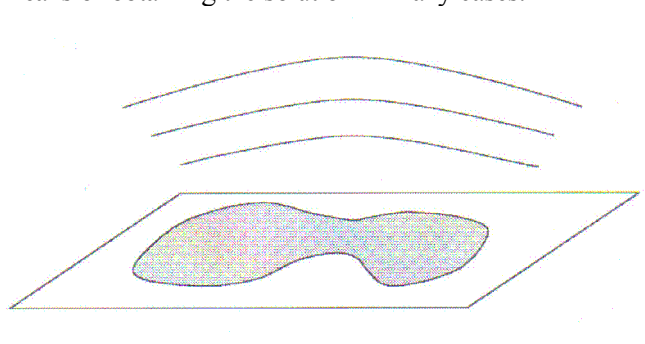

Figure 1: The plate situated in an infinite baffle.

The extension of boundary elements to include the Rayleigh integral method was set out in the author's PhD [23] and later published [24], in which the method is developed through numerical product integration of the Rayleigh integral (for the Neumann condition on the plate). The method is termed the Rayleigh integral method and it is superior to the simple source method since its accuracy is virtually unaffected by the nature of the integrand. The Rayleigh Integral Method is extended to cover similar problems but with a wider range of boundary conditions in this work.

   A Fortran (Fortran 77) subroutine (RIM3) that implements the method for three-dimensional acoustic

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problems is developed. Test problems have been written to demonstrate the method. The results from a range of test problems are given. The subroutine RIM3, the test problems and the other necessary software is provided with this article as *open source* code. The codes can be downloaded from the webpages listed at the bottom of the page.

II. MATHEMATICAL MODEL

The acoustic radiation model consists of a vibrating plate ∏ of arbitrary shape lying in an infinite rigid baffle and surrounded by a semi-infinite acoustic medium E, as shown in figure 1 for the three-dimensional case. The baffle is perfectly rigid and reflecting. The problems covered in this work allow a general Robin boundary condition on the plate; modelling radiation problems with a perfectly reflecting, perfectly absorptive or an impedance boundary condition at the plate. The aim is that of determining the properties of the semi-infinite acoustic medium surrounding the plate.

Acoustic Model

The acoustic field is governed by the wave equation in *E*,

$$\nabla^2 \Psi(\mathbf{p},t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \Psi(\mathbf{p},t)$$

(1)

where Ψ is the scalar time-dependent velocity potential related to the time-dependent particle velocity *V* by

$$V(\mathbf{p},t) = \nabla \Psi(\mathbf{p},t)$$

(2)

and c is the propagation velocity (p and t are the spacial and time variables). The time-dependent sound pressure *Q* is given in terms of the velocity potential by

$$Q(\mathbf{p},t) = -\rho \frac{\partial \Psi}{\partial t}(\mathbf{p},t)$$

(3)

where ρ is the density of the acoustic medium.

Only periodic solutions to the wave equation are of interest, thus it is sufficient to consider time-dependent velocity potentials of the form:

$$\psi(\mathbf{p},t) = \varphi(\mathbf{p})e^{i\omega t}$$

(4)

where ω is the angular frequency (ω=2πη, where η is the frequency in hertz) and *φ(p)* is the (time-independent) velocity potential. The substitution of expression (4) into (1) reduces it to the Helmholtz (reduced wave) equation:

$$\nabla^2 \varphi(\mathbf{p}) + k^2 \varphi(\mathbf{p}) = 0$$

(5)

where *k=ω/c* is the wavenumber.

In general, let it be assumed that we have a Robin condition on the plate of the form

$$a(\mathbf{p})\varphi(\mathbf{p}) + b(\mathbf{p})v(\mathbf{p}) = f(\mathbf{p}) \quad (\mathbf{p} \in \Pi)$$

(6)

where $v(\mathbf{p}) = \frac{\partial \varphi}{\partial n_p}(\mathbf{p})$ and with $f(\mathbf{p})$ given for $\mathbf{p} \in \Pi$ and $\mathbf{n}_p$ is the unit outward normal to the plate at $\mathbf{p}$. The true velocity of the plate can be expressed as $v(\mathbf{p})e^{i\omega t}$ for $\mathbf{p} \in \Pi$.

For the problems we consider in this document, φ must also satisfy the Sommerfeld radiation condition:

$$\lim_{r \to \infty} r(\frac{\partial \varphi}{\partial r} - ik\varphi) = 0$$

(7)

where *r* is the distance between the point **p** and a fixed origin.

Properties of the time-harmonic acoustic field

In this section other important acoustic properties are related to those already defined. The substitution of (4) into equation (1) gives the time-independent sound pressure:

$$P(\mathbf{p}) = i\rho\omega\varphi(\mathbf{p}) \ (\mathbf{p} \in \Pi \cup E).$$

(8)

The sound intensity on the plate with respect to the normal to the plate is given by the following expression:

$$I(\mathbf{p}) = \frac{1}{2} \text{Re}\{P*(\mathbf{p})v(\mathbf{p})\} \quad (\mathbf{p} \in \Pi)$$

(9)

where the asterix denotes the complex conjugate. The sound intensity on the baffle is zero since *v* is zero there. The sound power is given by

$$W = \int_{\Pi} I(\mathbf{p})dS_p .$$

(10)

The radiation ratio (often termed the radiation resistance or radiation efficiency) is given by the following expression:

$$\sigma_{RAD} = \frac{W}{\frac{1}{2}\rho c \int_{\Pi} v*(\mathbf{q})v(\mathbf{q})dS_q} .$$

(11)

III. RAYLEIGH INTEGRAL FORMULATION

A derivation of the Rayleigh integral formulation is given in Pierce [25]. In brief, it relates the velocity potential $\varphi(\mathbf{p})$ at a point p in the exterior E, on the plate $\prod$, or on the baffle to the normal velocity v on the plate $\prod$.

In the standard integral operator notation used in integral equation methods the Rayleigh integral is stated as follows:

$$\varphi(\mathbf{p}) = -2\{L_k v\}_\Pi (\mathbf{p}) \quad (\mathbf{p} \in \Pi \cup E).$$

(12)

In equation (11) the operator L_k is defined by

$$\{L_k \mu\}_\Gamma (\mathbf{p}) \equiv \int_\Gamma G_k(\mathbf{p},\mathbf{q})\mu(\mathbf{q})dS_q \quad (\mathbf{p} \in \Pi \cup E),$$

(13)

where $G_k(\mathbf{p},\mathbf{q})$ is a free-space Greens function for the Helmholtz equation in three dimensions and Γ represents the whole or part of the plate. The Green's function is defined as follows:

$$G_k(\mathbf{p},\mathbf{q}) = \frac{1}{4\pi}\frac{e^{ikr}}{r}.$$

(14)

where $r = |\mathbf{r}|$ and $\mathbf{r} = \mathbf{q} - \mathbf{p}$ and i unit imaginary number (often also denoted by j). The Green's function (14) also satisfies the Sommerfeld radiation condition (6), ensuring that all scattered and radiated waves are outgoing in the farfield.

IV. RAYLEIGH INTEGRAL METHOD

Approximations to the properties of the acoustic medium can be found through applying numerical integration technique to equation (12). This requires us to represent the plate by a set of panels and go on to approximate the integral (12); rewriting it in discrete form.

Representation of the Plate

In order that the resulting computational method is applicable to a class or arbitrary plates there must be a facility for representing the plate as a set of panels. For example a set of triangles can be used to approximate a plate of arbitrary shape. Thus we may write

$$\Pi \approx \widetilde{\Pi} = \sum_{j=1}^n \Delta\Pi_j$$

(15)

where each $\Delta\Pi_j$ is a triangle. Triangulations of circular and square plates are illustrated in sections VII and VIII of this paper.

Simple Source Method

The velocity potential can be evaluated by using a numerical approximation of the integral in (12). Perhaps the most straightforward approach, and the one most often adopted, is to use the mid-point numerical integration rule. This method is otherwise known as the simple source method. The method is explained in Schenck [22], for example. If the mid-points of each element is $\mathbf{q_j}$ for j = 1, 2, ..,m , then the application of the mid-point rule to equation (11) gives

$$\varphi(\mathbf{p}) \approx -\frac{1}{2\pi}\sum_j area(\Delta\Pi_j)\frac{\exp(jkr_j)}{r_j}v_j$$

where $area(\Delta\prod_j)$ is the area of the element $\Delta\prod_j$, $r_j = r(\mathbf{p}, \mathbf{q_j})$ is the distance between $\mathbf{p}$ and $\mathbf{q_j}$ and $v_j = v(\mathbf{q_j})$. Clearly, since the integrand in (12) tends to become more oscillatory with increasing k, the accuracy of this method tends to deteriorate as k increases. Furthermore, if p is near or on the plate then the numerical approximation is likely to be poor. A further difficulty with this approach is that the approximation is only directly applicable in the case of the Neumann condition.

Collocation

A more natural approach is by the use of collocation in which the potentially badly behaved weighting function $\frac{e^{jkr}}{r}$ is incorporated into the quadrature rule. The normal velocity on $\widetilde{\Pi}$ is expressed in the form

$$\widetilde{v}(\mathbf{p}) \approx \sum_{j=1}^n v(\mathbf{p}_j)\zeta_j(\mathbf{p}) = \sum_{j=1}^n v_j\zeta_j(\mathbf{p}).$$

(16)

where $\zeta_1, \zeta_2, ..., \zeta_n$ are basis functions. In this work the basis functions are defined such that

$$\zeta_j(\mathbf{p}) = \begin{cases} 1 & \mathbf{p} \in \Delta\widetilde{\Pi}_j \\ 0 & \mathbf{p} \notin \Delta\widetilde{\Pi}_j \end{cases}$$

and $v_j = v(\mathbf{p}_j)$, the velocity at the jth collocation point.

The replacement of the true plate Π by the approximate plate $\widetilde{\Pi}$ and the substitution of the approximation (16) allows us to write

$$\{L_k v\}_\Pi = \int_\Pi G(\mathbf{p},\mathbf{q})v(\mathbf{q})dS_q \approx \int_{\widetilde{\Pi}} G(\mathbf{p},\mathbf{q})\sum_{j=1}^n v_j \zeta_j(\mathbf{p})dS_q$$

$$= \sum_{j=1}^n v_j \int_{\widetilde{\Pi}} G(\mathbf{p},\mathbf{q})\zeta_j(\mathbf{p})dS_q = \sum_{j=1}^n v_j \ L\zeta_j \Big|_{\Delta\widetilde{\Pi}_j}$$

(17)

For a particular wavenumber and a particular value of p, having calculated the $L_k\zeta_j \big|_{\Delta\widetilde{\Pi}_j}$ an approximation to the velocity potential $\varphi(\mathbf{p})$ can be obtained by the summation (17).

Discretising the Lk Operator

The $\{L_k\zeta_j\}_{\Delta\widetilde{\Pi}_j}$ can be evaluated by mapping the elements of the approximate plate onto the standard triangle with vertices (0,0), (0,1) and (1,0). When the integrands are bounded then a standard numerical integration technique can be used. Efficient methods may be obtained through the use of the Gaussian Quadrature formulae in Laursen and Gellert [26].

When $\mathbf{p} \in \Delta\widetilde{\Pi}_j$ the integrand in is singular and special techniques need to be employed. One of the most effective way of treating these integrals is to change the variables to polar coordinates, transforming them into regular integrals. The method employed for carrying this out is described in Kirkup [27], [28]. The Fortan subroutine for carrying this out is denoted H3LC.

V. IMPLEMENTATION OF THE RAYLEIGH INTEGRAL METHOD

In this section an implementation of the Rayleigh Integral Method is described. The plate may be of any shape and is assumed to be discretised into a set of planar triangles. The normal velocity distribution on the plate is described simply by its value at the centroids of the triangles.

As input, the subroutine accepts a description of the geometry of the plate (made up of triangles), the coordinates of selected points in the exterior (where the sound pressure is required), the wavenumbers under consideration and a description of the boundary condition at each wavenumber. As output, the subroutine gives, for each wavenumber, the velocity potential and surface velocity on the plate and the velocity potential at the selected exterior points.

Subroutine **RIM3**

```
C SUBROUTINE RIM3(K,
C * MAXNV,NV,VERTEX,MAXNPI,NPI,SELV,
C * MAXNPE,NPE,PEXT,
C * PIALPHA,PIBETA,PIF,
C * LSOL,LVALID,EGEOM,
C * PIPHI,PIVEL,PEPHI,
C * WKSPC1,WKSPC2,WKSPC3,WKSPC4,WKSPC5,WKSPC6)
C The parameters to the subroutine
C ===============================
C Wavenumber (input)
C real K: Must be positive.
C Geometry of the plate {\Pi} (input)
C integer MAXNV: The limit on the number of vertices of the triangles
C that defines (approximates) {\Pi}. MAXNV>=3.
C integer NV: The number of vertices on {\Pi}. 3<=NV<=MAXNV.
C real VERTEX(MAXNV,3): The coordinates of the vertices. VERTEX(i,1),
C VERTEX(i,2), VERTEX(i,3) are the x,y,z coordinates of the i-th
C vertex. The vertices must be co-planar.
C integer MAXNPI: The limit on the number of elements describing {\Pi}.
C MAXNPI>=1.
C integer NPI: The number of elements describing {\Pi}.
1<=NPI<=MAXNPI.
C integer SELV(MAXNPI,3): The indices of the three vertices defining
C each element. The i-th element have vertices
C
(VERTEX(SELV(i,1),1),VERTEX(SELV(i,1),2),VERTEX(SELV(i,1),3)),
C (VERTEX(SELV(i,2),1),VERTEX(SELV(i,2),2),VERTEX(SELV(i,2),3))
and
C
(VERTEX(SELV(i,3),1),VERTEX(SELV(i,3),2),VERTEX(SELV(i,3),3)).
C Exterior points at which the solution is to be observed (input)
C integer MAXNPE: Limit on the number of points exterior to the
www.boundary-element-method.com
C plate. MAXNPE>=1.
C integer NPE: The number of exterior points. 0<=NPE<=MAXNPE.
C real PEXT(MAXNPE,3). The coordinates of the exterior point.
C PEXT(i,1),PEXT(i,2),PEXT(i,3) are the x,y,z coordinates of the i-th
C point.
C The plate condition ({\alpha} phi + {\beta} v = f) (input)
C complex PIALPHA(MAXNPI): The values of {\alpha} at the centres
C of the elements.
C complex PIBETA(MAXNPI): The values of {\beta} at the centres
C of the elements.
C complex PIF(MAXNPI): The values of f at the centres of the
C elements.
C Validation and control parameters (input)
C logical LSOL: A switch to control whether the particular solution is
C required.
C logical LVALID: A switch to enable the choice of checking of
C subroutine parameters.
C real EGEOM: The maximum absolute error in the parameters that
C describe the geometry.
C Solution (output)
C complex PIPHI(MAXNPI): The velocity potential ({\phi}) at the
C centres of the plate elements.
C complex PIVEL(MAXNPI): The velocity (v or d{\phi}/dn where n is
C the outward normal to the plate) at the centres of the plate
C elements.
C complex PEPHI(MAXNPE): The velocity potential ({\phi}) at the
C exterior points.
C Working space
C COMPLEX*16 WKSPC1(MAXNPI,MAXNPI)
C COMPLEX*16 WKSPC2(MAXNPE,MAXNPI)
C COMPLEX*16 WKSPC3(MAXNPI,MAXNPI)
C COMPLEX*16 WKSPC4(MAXNPI)
C COMPLEX*16 WKSPC5(MAXNPI)
C LOGICAL WKSPC6(MAXNPI)
```

VI. Test Problems

In this section, the Rayleigh integral method is applied to various test problems.

Sound Pressure Field of a Circular Piston

The test problem is that of a uniformly vibrating circular piston of radius 0.1, centred at (0, 0, 0) at wavenumbers $k = 10$ and $k = 25$. If $v(\mathbf{p}) = -V \quad (\mathbf{p} \in \Pi)$ is the velocity of the piston (uniform over its surface) then the sound pressure $P(\mathbf{p})$ at a point p = (0, 0, p_3) on the axis of the piston is given by

$$P(\mathbf{p}) = \rho c V (\exp(ik(0.01 + p_3^2)^{1/2} - \exp(ikp_3))$$

see, for example, Skudrzyk [29], pp631-633.

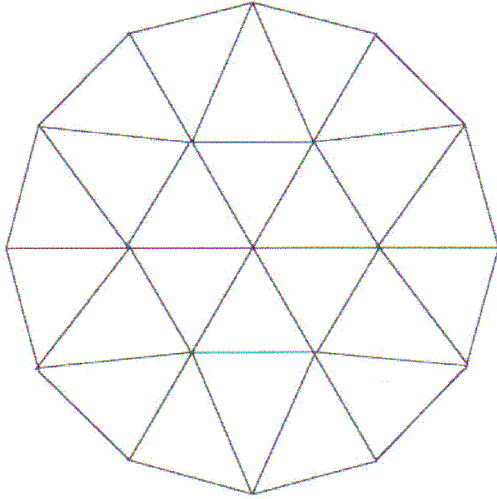The circular piston is divided into 24 triangles, as shown in figure 2.

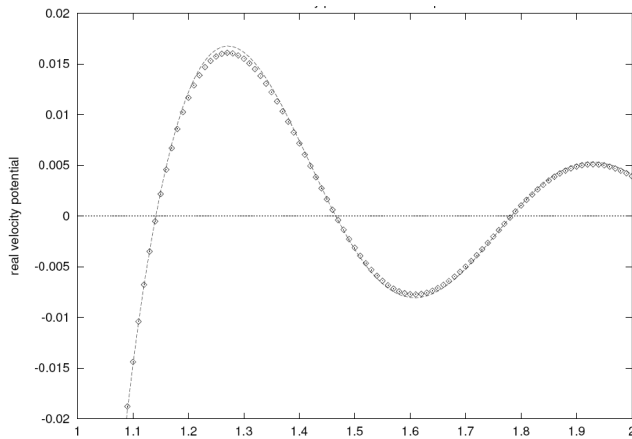Figure 2. The circular plate divided into 24 triangular panels.

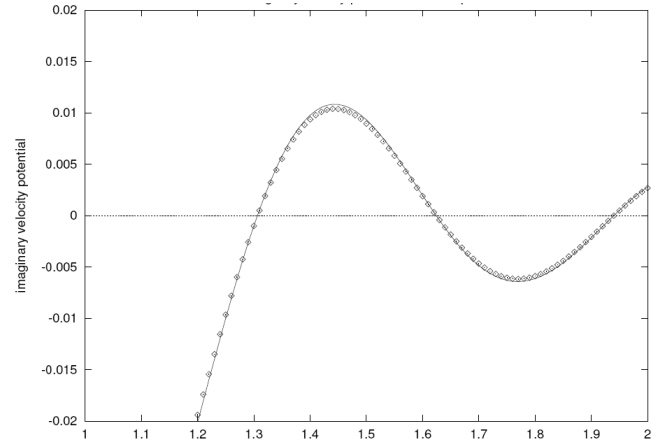Figure 3a. Real on axis sound pressures for circular piston, k=10.

Figure 3b. Imaginary on axis sound pressures for circular piston, k=10.

Figures 3 compare the computed and exact on axis sound pressure obtained from the subroutine at twenty points with exact values for $k = 10$. Figures 4 compare the same but with $k = 25$.
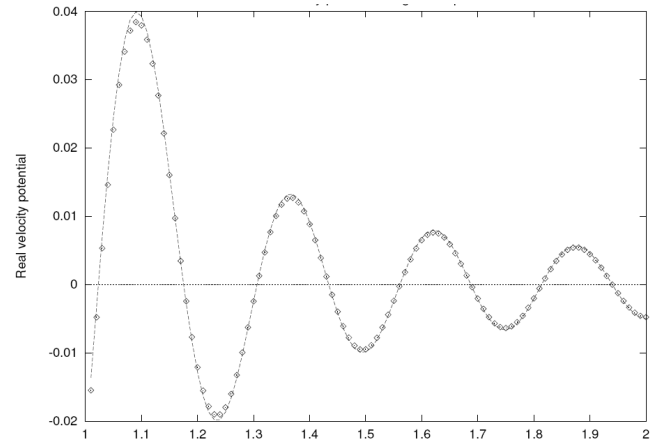

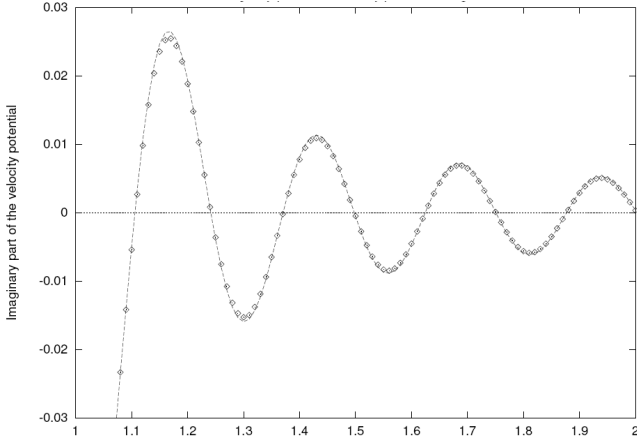Figure 4a. Real on axis sound pressures for circular piston, k=25.

Figure 4b. Imaginary on axis sound pressures for circular piston, k=25.

Square test problem

The second test problem is that of a uniform square plate with its sides hinged onto an infinite rigid baffle. The [0, 1]×[0, 1] square is vibrating in its natural modes which are

$$v(\mathbf{p}) = \sin(lp_1)\sin(mp_2)$$

(18)

where l and m are integers. The property that is of interest is the radiation ratio of the plate vibrating in each of its mode shapes (18).

In order to apply the subroutine to the problem, the square plate is divided into 32 triangles, as shown in figure 5. The mode shapes considered were the sixteen given by putting l = 1 and m = 1 in (18). The wavenumbers at which the radiation ratio is computed are k = 0.0, 0.2, 0.4, ..., 34.8, 20.0 . Figures 6 show the radiation ratio curves constructed from the results of the subroutine run.
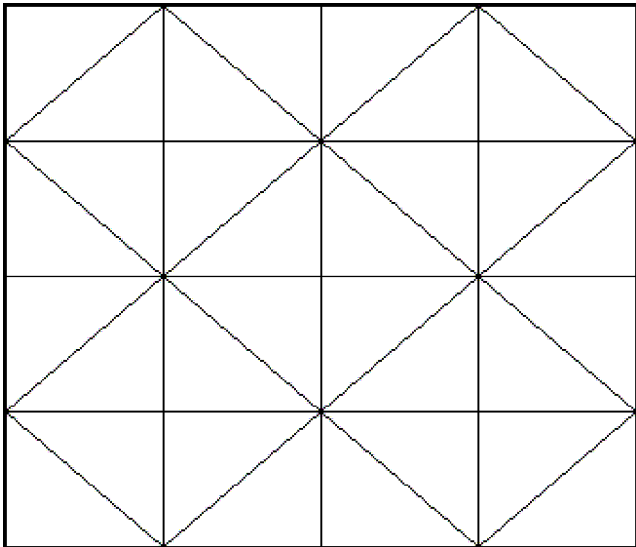

Figure 5. The square plate divided into 32 triangular panels.

The final test problem is that of the same square plate as in the previous example, having the same discretisation but with it vibrating uniformly. The radiation ratio curve for this is shown in figure 7.
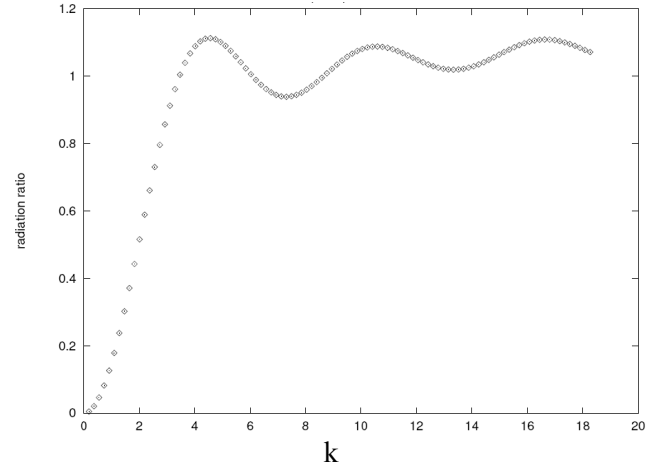

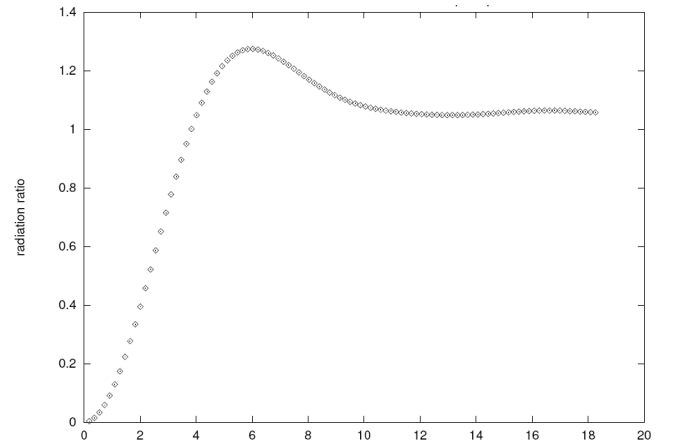Figure 6. The radiation ratio curve for a square panel in the uniform mode.

Figure 7. The radiation ratio curve for a square panel in the sin*sin mode.

VII. CONCLUDING DISCUSSION

The computational solution of a given acoustic radiation problem first involves the selection of an appropriate acoustic radiation model which underlies the choice of method. For example the model of a closed surface in an infinite acoustic medium underlies the boundary element method (see, for example, Kirkup [28]). Several practical acoustic problems are suitably represented by the acoustic

radiation model of a vibrating plate lying in an infinite baffle and thus a computational solution can be obtained via methods based on the Rayleigh integral. For example the Rayleigh integral model can be applied to the problem of predicting the noise radiated by the faces of an in-line engine block (see Yorke [30] or Kirkup and Tyrrell [31], for example). Computational methods based on the Rayleigh integral have been applied to certain classes of acoustic problems for some time. However, such methods have generally been based on direct numerical integration and hence they have poor numerical properties.

In this paper product integration has been applied to the Rayleigh integral to derive a more robust method, that is more in line boundary element methodology.

A particular implementation of the RIM is described in this paper. In figures 3 and 4, computed and exact sound pressures along the axis of the circular piston are compared. The results appear to be in good agreement. In figure 6 and 7 the computed radiation ratio curves for the a square plate in a simple motion are given. These may be compared with similar results given in Wallace [32].

In Kirkup and Thompson [12] a hybrid of the Boundary Element Method and the Rayleigh Integral method is introduced. This method, termed BERIM, can be used to computed the properties of a radiating open cavity.

The Rayleigh integral method is applicable to acoustic problems that include of a flat or nearly flat plate exposed to an acoustic medium on one side. Since the method requires the description of the plate as a set of elements (triangles) then the plate may be of arbitrary shape. Hence the Rayleigh integral method should serve as a useful addition to an acoustic software library.

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