Lab 4: "for" loops applications

Exponential Smoothing

Recall: Exponential smoothing with α:

Given data: $\{x_1, x_2, \dots x_n\}$. Then the smoothed data is $\{\overline{x_1}, \overline{x_2}, \dots \overline{x_n}\}$ where

$$\overline{x_1} = x_1$$
$$\overline{x_i} = \alpha x_i + (1-\alpha)\overline{x_{i-1}}, \quad i = 2, 3, 4, \dots$$

We will use y variable for smoothed data as typing $\bar{x}$ is tedious.

Example 1: Find the smoothed data using the exponential smoothing with $\alpha = 0.25$.

$$36, 46, 47, 50, 51, 60, 50, 46, 52, 45, 53$$

```
a = 0.25; # given smoothing parameter
x = [36, 46, 47, 50, 51, 60, 50, 46, 52, 45, 53]; # given raw data
y = [0]*len(x); # placeholder for the smoothed data, y
y[0]=x[0] # first data is same

for i in range(1, len(x)):
    y[i]=a*x[i]+(1-a)*y[i-1]
print(y)
```

To print the smoothed data in two decimal places, we write

```
for value in y:
   print(f'{value: .2f}', end='') #.2f to display 2 decimal places
# "end" for printing the data horizontally
```

You should get the following smoothed data

```
36.00 38.50 40.62 42.97 44.98 48.73 49.05 48.29 49.22 48.16 49.37
```

The Fibonacci Numbers

The Fibonacci numbers are defined as follows:

$$F_1 = 1, \quad F_2 = 1,$$
$$F_i = F_{i-1} + F_{i-2}.$$

With this formula, we have the first 10 Fibonacci numbers are as follows:

```
1     1     2     3     5     8     13     21     34     55
```

Example 2: Write a for loop that determines the first 12 Fibonacci numbers, and displays them as

```
The xxth Fibonacci number is xx.
```

```
F=[0]*12 #placeholder
F[0]=1 # first term
F[1]=1; # second term
print(f'The 1st term of the sequence is {F[0]}')
print(f'The 2nd term of the sequence is {F[1]}')
for i in range(2,12):
    F[i]=F[i-1]+F[i-2] # current term is sum of the previous two
    print(f'The {i + 1}th term of the sequence is {F[i]}')
```

```
Run this script to get the following:

The 1st term of the sequence is 1
The 2nd term of the sequence is 1
The 3th term of the sequence is 2
The 4th term of the sequence is 3
The 5th term of the sequence is 5
The 6th term of the sequence is 8
The 7th term of the sequence is 13
The 8th term of the sequence is 21
The 9th term of the sequence is 34
The 10th term of the sequence is 55
The 11th term of the sequence is 89
The 12th term of the sequence is 144
```

Newton's Method:

Formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n \geq 0$$

which is an iterative formula. (We will derive this in class on Tuesday.)

Example 3: Use the Newton's Method to perform 10 iterations (that is, find x_{10}) to approximate the solution of the following equation. Round your answer to six decimal places.

$$x + e^x = 2, \qquad \text{with} \quad x_0 = 1.$$

(Solution is given on the next page.)

```
import numpy as np
x = [0]*11 # placeholder
x[0]=1 # initial approximation
for i in range(10):
    x[i+1] = x[i] -(x[i]+np.exp(x[i])-2)/(1+np.exp(x[i]))
print(f'The {i + 1}th iterative approximate solution is{x[i]: .6f}')
```

Run this script to get

```
The 10th iterative approximate solution is 0.442854.
```

Exercises:

Use a for loop to answer the following questions.

1. Find the smoothed data using the exponential smoothing with $\alpha = 0.2$. Round your answer to two decimal places.

$$32, 46, 39, 50, 41, 60, 50, 47, 51, 42, 33, 28, 45, 55, 42, 67$$

2. Write a script that calculates the 25th Fibonacci number (only 25th number), and display it.
3. **The Lucas Numbers** are given by $2, 1, 3, 4, 7, 11, \ldots$, (Fibonacci's sibling). Determine and display (as in example 2) the first 15 Lucas numbers.
4. Amy makes an initial investment of \$5000. The investment loses 7.5% each year. Find the amount Amy has at the end of 8 years. (Do not use a formula, use a "for loop".)
5. A certain type of fish at birth is of 5 mm. It grows about 20% each day for the first 10 days, and about 15% each day for the next 15 days. How long will it be after 20 days from the birth? (Do not use a formula, use "for loops".)
6. Use the Newton's method with the specified initial approximation x_0 to find x_{10}. Round your answer to 6 decimal places.
   (a) $x^3 - 2 = 0, \qquad x_0 = 2$
   (b) $x^5 - 17 = 0, \qquad x_0 = 2.1$
   (c) $\sin x + x - 1 = 0, \qquad x_0 = 1.5$
   (d) $\ln(x+1) = 1, \qquad x_0 = 1.7$