Homework: Population Growth

1. A population of a small city had 7,000 people in the year 2012 and has grown at a rate proportional to its size. In the year 2018 the population was 7,500.
   (a) Estimate the population of the city in 2025.
   (b) Assuming the growth continues at the same rate, when (the Year) will the city have 8,800 people?

2. Suppose a student carrying a flu virus returns to an isolated college campus of 2600 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number P of infected students but also to the number of students not infected, determine the number of infected students after 10 days if it is further observed that after 4 days there are 50 infected people in the campus.

(a)

$P_0 = 7000$
$t_0 = 2012$
$P_e = 7500$
$t_e = 2018$

$P_0 e^{kt}$

$7000 e^{k(6)} = \frac{7500}{7000}$

$\cancel{\ln e}^{k6} = \ln \frac{15}{14}$

$\frac{k\cancel{6}}{\cancel{6}} = \frac{\ln(\frac{15}{14})}{6}$

$k = .0115$

$7000 e^{(.0115)13} = 8129$ ppl

1 b) $\cancel{7000} e^{.0115t} = \frac{8800}{7000}$

$\cancel{\ln e}^{.0115t} = \ln\left(\frac{44}{35}\right)$

$\frac{.0115t}{\cancel{.0115}} = \frac{\ln(\frac{44}{35})}{.0115}$

$t \approx 19.90$ yrs

2)

$P_{max} = 2600$
$P_0 = 1$
$t = 10$

$P_e = 50$
$t_e = 4$

$\frac{dP}{dt} = kP\left(1 - \frac{P}{k}\right)$, $P = \frac{k}{1 + Ae^{-kt}}$, $A = \frac{k - P_0}{P_0}$

$A = \frac{2600 - 1}{1} = 2599$

$50 = \frac{2600}{1 + 2599e^{-k(4)}} \cdot \cancel{1 + 2599e^{-k4}}$

$\frac{\cancel{50}(1 + 2599e^{-k4})}{\cancel{50}} = \frac{2600}{50}$

$\cancel{1} + 2599e^{-k4} = 52 - 1$

$\frac{\cancel{2599}e^{-k4}}{\cancel{2599}} = \frac{51}{2599}$

$\cancel{\ln e}^{-k4} = \ln\left(\frac{51}{2599}\right)$

$\frac{-k\cancel{4}}{\cancel{-4}} = \frac{\ln(\frac{51}{2599})}{-4}$

$k = .9828$

$\frac{2600}{1 + 2599e^{-(.9828)(10)}} = 2281$ students