

CSE 847 (Spring 2016): Machine Learning— Homework 1

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Due on Thursday, Jan 28, before class.

1 Introduction

Questions in the textbook Pattern Recognition and Machine Learning:

1. Page 58, Question 1.3
2. Page 59, Question 1.6
3. Page 59, Question 1.11

2 Linear Algebra I

1. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, E = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}.$$

If possible, compute the following:

- (a) $(2A)^T$
 - (b) $(A - B)^T$
 - (c) $(3B^T - 2A)^T$
 - (d) $(-A)^T E$
 - (e) $(C + D^T + E)^T$
2. Which of the following are subspace of \mathbb{R}^2 ? Justify your answer.
 - (a) $\{(x, y) \in \mathbb{R}^2 | x^2 + y^2 = 0\}$
 - (b) $\{(x, y) \in \mathbb{R}^2 | x^2 - y^2 = 0\}$
 - (c) $\{(x, y) \in \mathbb{R}^2 | x^2 - y = 0\}$
 - (d) $\{(x, y) \in \mathbb{R}^2 | x - y = 0\}$
 - (e) $\{(x, y) \in \mathbb{R}^2 | x - y = 1\}$

3. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}.$$

Is $AB = BA$? Justify your answer.

4.
 - (a) Let A be an $m \times n$ matrix with a row consisting entirely of zeros. Show that if B is an $n \times p$ matrix, then AB has a row of zeros.
 - (b) Let A be an $m \times n$ matrix with a column consisting entirely of zeros, and let B be $p \times m$. Show that BA has a column of zeros.
5. Let x be a vector in \mathbb{R}^n . Show that $\lim_{p \rightarrow \infty} \|x\|_p = \|x\|_\infty$.
6. Let $A \in \mathbb{R}^{m \times r}$ and $B \in \mathbb{R}^{r \times n}$ be two matrices. Show that $\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}$.