CSE 847 (Spring 2016): Machine Learning— Homework 2

Instructor: Jiayu Zhou

Due on Tuesday, Feb 16, before class.

1 Linear Algebra II

1. Compute (by hand) the eigenvalues and the eigenvectors of the following matrix:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

2. Given the three vectors $v_1 = (2, 0, -1), v_2 = (0, -1, 0)$ and $v_3 = (2, 0, 4)$ in $\mathbb{R}^3$.
   - Show that they form an orthogonal set under the standard Euclidean inner product for $\mathbb{R}^3$ but not an orthonormal set.
   - Turn them into a set of vectors that will form an orthonormal set of vectors under the standard Euclidean inner product for $\mathbb{R}^3$.

3. Let A and B be two matrices of size n by n and let $\|.\|_p$ be the matrix p-norm with $p \geq 1$. Show that $\|AB\|_p \leq \|A\|_p \|B\|_p$.

4. Given formulae for the eigenvalues and eigenvectors of $B = \begin{pmatrix} 0 & A^T \\ A & 0 \end{pmatrix}$ in terms of singular values and singular vectors of $A \in \mathbb{R}^{m \times n}$ where $m \geq n$.

5. Suppose that A is an $n \times m$ matrix with linearly independent columns. Show that $A^T A$ is an invertible matrix.

6. Suppose that A is an $n \times m$ matrix with linearly independent columns. Let $\bar{x}$ be a least squares solution to the system of equations $Ax = b$ (the solution of $\min_x \|Ax - b\|_2^2$). Show that $\bar{x}$ is the **unique** solution to the associated normal system $A^T A \bar{x} = A^T b$.

2 Linear Regression I

Questions in the textbook Pattern Recognition and Machine Learning:

1. Page 174, Question 3.2
2. Page 175, Question 3.7
3. Page 175, Question 3.10
4. Page 175, Question 3.11