

10.4.4.3

EE24BTECH11017 - D.Karthik

Question:

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2 ? If so, find its length and breadth.

Solution:

Let the breadth of the rectangular mango grove be b meters. Then the length is $2b$ meters (as it is twice the breadth). The area of the rectangle is given as 800 m^2 .

The relationship between the length, breadth, and area can be expressed as:

$$\text{Area} = \text{Length} \times \text{Breadth} \quad (0.1)$$

$$800 = 2b \cdot b \quad (0.2)$$

This simplifies to a quadratic equation:

$$2b^2 = 800 \quad (0.3)$$

$$b^2 = 400 \quad (0.4)$$

$$b^2 - 400 = 0 \quad (0.5)$$

Using the Newton-Raphson Method

To solve $B^2 - 400 = 0$ using the Newton-Raphson method:

- Define $f(B) = B^2 - 400$, so $f'(B) = 2B$.
- The iterative formula is:

$$B_{n+1} = B_n - \frac{f(B_n)}{f'(B_n)} = B_n - \frac{B_n^2 - 400}{2B_n} \quad (0.6)$$

Let the initial guess be $B_0 = 10$:

$$B_1 = B_0 - \frac{B_0^2 - 400}{2B_0} = 10 - \frac{10^2 - 400}{2 \times 10} = 25 \quad (0.7)$$

$$B_2 = B_1 - \frac{B_1^2 - 400}{2B_1} = 25 - \frac{25^2 - 400}{2 \times 25} = 20.4 \quad (0.8)$$

$$B_3 = B_2 - \frac{B_2^2 - 400}{2B_2} \approx 20.0001 \quad (0.9)$$

Thus, $B \approx 20 \text{ m}$ and $L = 40 \text{ m}$.

Conclusion

It is **possible** to design such a mango grove with:

$$\text{Length} = 40 \text{ m}, \quad \text{Breadth} = 20 \text{ m}. \quad (0.10)$$

Using Eigenvalue Approach:

For this approach, we can model the problem as solving a quadratic equation using matrix methods. The general quadratic equation is given as:

$$ax^2 + bx + c = 0$$

The corresponding companion matrix for this equation is:

$$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ -\frac{c}{a} & -\frac{b}{a} \end{pmatrix}$$

For the quadratic equation $2x^2 - 800 = 0$, we have $a = 2, b = 0, c = -800$. The companion matrix becomes:

$$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ 400 & 0 \end{pmatrix}$$

The eigenvalues of the companion matrix correspond to the roots of the quadratic equation. Using the determinant of the matrix, we solve for the eigenvalues:

$$\text{Determinant} = \lambda^2 - 400 = 0 \implies \lambda = \pm 20$$

Hence, the eigenvalues are 20 and -20, representing the breadth and length (ignoring the negative sign for length, we consider the breadth as 20 meters, and length as 40 meters).

Graphical Representation:

To visualize this, the rectangle can be plotted with the given dimensions, as shown below:

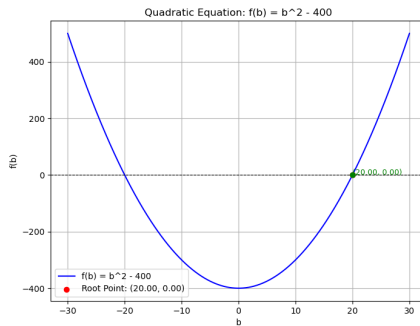


Fig. 0.1: A rectangular mango grove with length 40 m and breadth 20 m.