12.9.5.2.3

EE24BTECH11017 - D.Karthik

Exercise Verify that the given functions are solutions of the corresponding differential equation:

$$\text{Differential Equation: } y' = \frac{x+y}{x} \tag{1}$$

Theoretical Solution

Rewriting the equation:

$$y' = 1 + \frac{y}{x} \tag{2}$$

This is a linear differential equation. Rearranging:

$$\frac{dy}{dx} - \frac{y}{x} = 1 \tag{3}$$

Here, the equation is in the standard form:

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{4}$$

where $P(x) = -\frac{1}{x}$ and $Q(x) = 1$.
The integrating factor (IF) is:

$$IF = e^{\int P(x)dx} = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x} \tag{5}$$

Multiply through by the integrating factor:

$$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = \frac{1}{x} \tag{6}$$

The left-hand side becomes the derivative of $\frac{y}{x}$:

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{1}{x} \tag{7}$$

Integrate both sides:

$$\frac{y}{x} = \ln x + C \tag{8}$$

Multiply through by x to find y:

$$y = x \ln x + Cx \tag{9}$$

Thus, the general solution is:

$$y = x \ln x + Cx \tag{10}$$

Numerical Solution

To numerically verify the solution, we use the Improved Euler's Method as follows: Definition of derivative,

$$f'(y) \approx \lim_{h\to 0} \frac{f(y+h) - f(y)}{h} \tag{11}$$

$$f(y+h) \approx f(y) + hf'(y) \tag{12}$$

$$\implies x_{n+1} \approx x_n + h\frac{dx}{dy}\Big|_{x=x_n, y=y_n} \tag{13}$$

Difference equation,

$$y_{n+1} \approx y_n + h\left(1 + \frac{y_n}{x_n}\right), \quad x_{n+1} = x_n + h. \tag{14}$$