# 9.2.6

EE24BTECH11017 - D.Karthik

**Exercise 9.2** In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

$$\text{Given Function: } y = x\sin x \tag{1}$$

$$\text{Differential Equation: } xy' = y + x\sqrt{x^2 - y^2}, (x \neq 0, x > y \text{ or } x < -y). \tag{2}$$

*Theoretical Solution*

Rewriting the equation:

$$y' = \frac{y}{x} + \sqrt{x^2 - y^2} \tag{3}$$

We introduce a substitution to simplify the problem. Let:

$$y = x\sin\theta \tag{4}$$

where $\theta$ is a function of $x$. Then:

$$y' = \frac{d}{dx}(x\sin\theta) = \sin\theta + x\cos\theta\frac{d\theta}{dx} \tag{5}$$

Substitute $y = x\sin\theta$ and $y'$ into the differential equation:

$$x\sin\theta + x\cos\theta\frac{d\theta}{dx} = x\sin\theta + x\sqrt{x^2 - (x\sin\theta)^2} \tag{6}$$

Simplify the square root term:

$$x\sqrt{x^2 - (x\sin\theta)^2} = x\sqrt{x^2(1 - \sin^2\theta)} = x^2\cos\theta \tag{7}$$

Substitute this back into the equation:

$$x\sin\theta + x^2\cos\theta\frac{d\theta}{dx} = x\sin\theta + x^2\cos\theta \tag{8}$$

Cancel $x(\sin\theta)$ from both sides:

$$x^2\cos\theta\frac{d\theta}{dx} = x^2\cos\theta \tag{9}$$

Divide through by $x^2\cos\theta$ (valid as $x \neq 0$ and $\cos\theta \neq 0$):

$$\frac{d\theta}{dx} = 1 \tag{10}$$

Integrate both sides:

$$\theta = x + C \tag{11}$$

where $C$ is the constant of integration.

Using the substitution $y = x \sin \theta$, we have:

$$y = x \sin(x + C) \tag{12}$$

Thus, the general solution to the differential equation is:

$$y = x \sin(x + C), x \neq 0, \text{ and } x > y \text{ or } x < -y \tag{13}$$

*Numerical Solution*

To numerically verify the solution, we use the Improved Euler's Method as follows: Definition of derivative,

$$f'(y) \approx \lim_{h \to 0} \frac{f(y + h) - f(y)}{h} \tag{14}$$

$$f(y + h) \approx f(y) + hf'(y) \tag{15}$$

$$\implies x_{n+1} \approx x_n + h\frac{dx}{dy}\Big|_{x=x_n, y=y_n} \tag{16}$$

Difference equation,

$$y_{n+1} \approx y_n + h\left(\frac{y_n}{x_n} + \sqrt{x_n^2 - y_n^2}\right), \quad x_{n+1} = x_n + h. \tag{17}$$