11.16.4.7.4

EE24BTECH11017 - D.Karthik

Problem Statement :
Given two events A and B such that:
$P(A) = 0.54, \quad P(B) = 0.69, \quad P(AB) = 0.35$
Find $P(BA')$.

Solution Using Boolean Logic

Given:

$$P(A) = 0.54, \quad P(B) = 0.69, \quad P(AB) = 0.35$$

We need to find:

$$P(BA')$$

Step 1: Boolean Logic Decomposition

$$A + A = A \tag{0.1}$$
$$AA = A \tag{0.2}$$
$$A + A' = 1 \tag{0.3}$$
$$AA' = 0 \tag{0.4}$$
$$AB = BA \tag{0.5}$$
$$A + B = B + A \tag{0.6}$$
$$(A + B) + C = A + (B + C) \tag{0.7}$$
$$(AB)\,C = A\,(BC) \tag{0.8}$$
$$A\,(B + C) = AB + AC \tag{0.9}$$
$$A + BC = (A + B)\,(A + C) \tag{0.10}$$
$$P\,(1) = 1 \tag{0.11}$$
$$P\,(A + B) = P\,(A) + P\,(B)\,, \text{ if } P\,(AB) = 0 \tag{0.12}$$

De Morgan's Theorems:

$$(A + B)' = A'B' \tag{0.13}$$
$$(AB)' = A' + B' \tag{0.14}$$

Using these axioms,

$$B = (A + A')\,B \tag{0.15}$$
$$= AB + A'B \tag{0.16}$$
$$P(B) = P(AB) + P(A'B) \tag{0.17}$$
$$P(A'B) = P(B) - P(AB) \tag{0.18}$$
$$P(A'B) = 0.69 - 0.35 \tag{0.19}$$
$$P(A'B) = 0.34 \tag{0.20}$$

1 Solution Using Random Variables and Conditional Probability

Let X be an indicator random variable such that:

$$X = \begin{cases} 1, & \text{if } B \text{ occurs,} \\ 0, & \text{otherwise.} \end{cases}$$

Using conditional probability,

$$P(B) = P(B|A)P(A) + P(B|A')P(A')$$

Rearranging to find $P(B|A')P(A')$:

$$P(B|A')P(A') = P(B) - P(B|A)P(A)$$

We compute $P(B|A)$:

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{0.35}{0.54} \approx 0.6481$$

Similarly, we compute $P(A')$:

$$P(A') = 1 - P(A) = 1 - 0.54 = 0.46$$

Now, solving for $P(B|A')$:

$$P(B|A') = \frac{P(B) - P(AB)}{P(A')}$$

$$= \frac{0.69 - 0.35}{0.46} = \frac{0.34}{0.46} \approx 0.7391$$

Thus,

$$P(B \cap A') = P(B|A')P(A') = 0.7391 \times 0.46 = 0.34$$

Monte Carlo Simulation Approach: We can verify this result by simulating a large number of trials where:

1) Generate a random value to determine if A occurs.
2) Generate a random value to determine if B occurs, conditioned on A or A'.

3) Count occurrences where B and A' happens.

Running a simulation with one million trials will produce an estimate close to 0.34, confirming our calculations.