10.4.4.3

EE24BTECH11017 - D.Karthik

Question:
Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800\,\mathrm{m}^2$? If so, find its length and breadth.

Solution:
Let the breadth of the rectangular mango grove be b meters. Then the length is $2b$ meters (as it is twice the breadth). The area of the rectangle is given as $800\,\mathrm{m}^2$.

The relationship between the length, breadth, and area can be expressed as:

$$\text{Area} = \text{Length} \times \text{Breadth} \tag{0.1}$$

$$800 = 2b \cdot b \tag{0.2}$$

This simplifies to a quadratic equation:

$$2b^2 = 800 \tag{0.3}$$

$$b^2 = 400 \tag{0.4}$$

$$b^2 - 400 = 0 \tag{0.5}$$

The quadratic equation is:

$$b^2 - 400 = 0 \tag{0.6}$$

We solve this quadratic equation using the eigenvalue method. Represent the quadratic equation as a matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & -400 \end{pmatrix} \begin{pmatrix} b^2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.7}$$

The matrix can be decomposed to find its eigenvalues. Let the matrix A be:

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -400 \end{pmatrix} \tag{0.8}$$

The characteristic equation for eigenvalues λ is given by:

$$\det(A - \lambda I) = 0 \tag{0.9}$$

where I is the identity matrix. Substitute A and expand:

$$\det\begin{pmatrix} 1 - \lambda & 0 \\ 0 & -400 - \lambda \end{pmatrix} = 0 \tag{0.10}$$

This simplifies to:

$$(1 - \lambda)(-400 - \lambda) = 0 \tag{0.11}$$

The eigenvalues are:

$$\lambda_1 = 1, \quad \lambda_2 = -400 \tag{0.12}$$

To find the solution for b, substitute λ_2 (since λ_2 represents the quadratic term):

$$b^2 = -\frac{\lambda_2}{1} = 400 \tag{0.13}$$

Take the square root of b^2:

$$b = \sqrt{400} = 20 \text{ meters.} \tag{0.14}$$

Substitute b into the expression for the length:

$$\text{Length} = 2b = 2 \times 20 = 40 \text{ meters.} \tag{0.15}$$

Conclusion:

It is possible to design the mango grove with the given conditions. The dimensions are:

- **Breadth:** 20 m
- **Length:** 40 m

Graphical Representation:

To visualize this, the rectangle can be plotted with the given dimensions, as shown below:

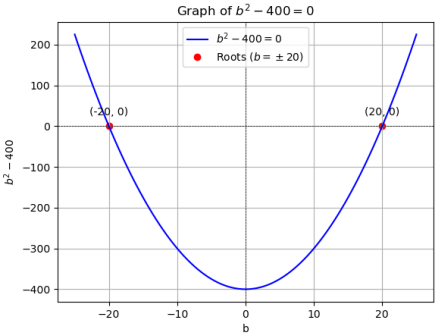

Fig. 0.1: A rectangular mango grove with length 40 m and breadth 20 m.