11.16.4.7.4

EE24BTECH11017 - D.Karthik

Problem Statement :
Given two events A and B such that:
$P(A) = 0.54, \quad P(B) = 0.69, \quad P(A \cap B) = 0.35$
Find $P(B \cap A^c)$.

1 Solution Using Set Properties and Probability Axioms

From the definition of probability and set relations, we know:

$$P(B) = P(A \cap B) + P(A^c \cap B)$$

Rearranging,

$$P(A^c \cap B) = P(B) - P(A \cap B)$$

Substituting the given values,

$$P(A^c \cap B) = 0.69 - 0.35 = 0.34$$

Thus, the required probability is:

$$\boxed{0.34}$$

2 Solution Using Random Variables and Conditional Probability

Let X be an indicator random variable such that:

$$X = \begin{cases} 1, & \text{if } B \text{ occurs,} \\ 0, & \text{otherwise.} \end{cases}$$

Using conditional probability,

$$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$$

Rearranging to find $P(B|A^c)P(A^c)$:

$$P(B|A^c)P(A^c) = P(B) - P(B|A)P(A)$$

We compute $P(B|A)$:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.35}{0.54} \approx 0.6481$$

Similarly, we compute $P(A^c)$:

$$P(A^c) = 1 - P(A) = 1 - 0.54 = 0.46$$

Now, solving for $P(B|A^c)$:

$$P(B|A^c) = \frac{P(B) - P(A \cap B)}{P(A^c)}$$

$$= \frac{0.69 - 0.35}{0.46} = \frac{0.34}{0.46} \approx 0.7391$$

Thus,

$$P(B \cap A^c) = P(B|A^c)P(A^c) = 0.7391 \times 0.46 = 0.34$$

Monte Carlo Simulation Approach: We can verify this result by simulating a large number of trials where:

1) Generate a random value to determine if A occurs.
2) Generate a random value to determine if B occurs, conditioned on A or A^c.
3) Count occurrences where $B \cap A^c$ happens.

Running a simulation with one million trials will produce an estimate close to 0.34, confirming our calculations.