12.6.5.2.3

EE24BTECH11017 - D.Karthik

Question 6.5.2.3 Find the maximum and minimum values, if any, of the following functions given by

$$\text{Given Function : } h(x) = \sin(2x) + 5 \tag{0.1}$$

Theoretical Solution :

Understanding the range of $\sin(2x)$

The sine function, $\sin(2x)$, oscillates between -1 and 1:

$$-1 \leq \sin(2x) \leq 1 \tag{0.2}$$

Adding 5 to $\sin(2x)$

Adding 5 shifts the range of the function vertically:

$$h(x) = \sin(2x) + 5 \implies -1 + 5 \leq \sin(2x) + 5 \leq 1 + 5 \tag{0.3}$$

$$4 \leq h(x) \leq 6 \tag{0.4}$$

Maximum and Minimum Values

- The maximum value of $h(x)$ occurs when $\sin(2x) = 1$:

$$h_{\max} = 1 + 5 = 6 \tag{0.5}$$

- The minimum value of $h(x)$ occurs when $\sin(2x) = -1$:

$$h_{\min} = -1 + 5 = 4 \tag{0.6}$$

Final Answer:

$$\text{Maximum Value: 6,} \quad \text{Minimum Value: 4} \tag{0.7}$$

Solution:

$$h'(x_n) = 2\cos(2x_n) \tag{0.8}$$

Gradient descent to find local minimum,

$$x_{n+1} = x_n - \eta h'(x_n) \tag{0.9}$$

$$x_{n+1} = x_n - 2\eta\cos(2x_n) \tag{0.10}$$

Gradient ascent to find local maximum,

$$x_{n+1} = x_n + \eta h'(x_n) \tag{0.11}$$

$$x_{n+1} = x_n + 2\eta\cos(2x_n) \tag{0.12}$$

Assuming,

$$\eta = 0.1 \text{ Where } \eta \text{ is the learning rate.} \tag{0.13}$$

$$\text{tolerance} = 1e - 6 \tag{0.14}$$

$$x_0 = 0.0 \tag{0.15}$$

We get,

$$x_{min} = -0.7853968861361207, \quad y_{min} = 4.000000000003263 \tag{0.16}$$

$$x_{max} = 0.7853968861361207, \quad y_{max} = 5.999999999996737 \tag{0.17}$$

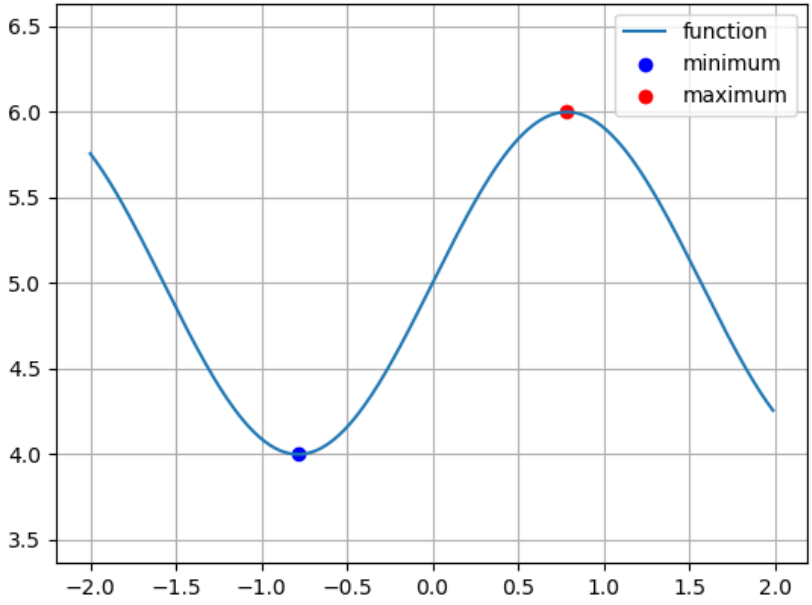

Fig. 0.1: local maximum and minimum