

- 31) The Lagrangian of a particle of mass m is $L = \frac{m}{2} \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 + \left(\frac{dz}{dt} \right)^2 \right] - \frac{V}{2} (x^2 + y^2) + W \sin \omega t$, where V, W and ω are constants. The conserved quantities are GATE 2007
- energy and z -component of linear momentum only.
 - energy and z -component of angular momentum only.
 - z -components of both linear and angular momenta only.
 - energy and z -component of both linear and angular momenta.
- 32) Three particles of mass m each situated at $x_1(t), x_2(t)$, and $x_3(t)$ respectively are connected by two springs of spring constant k and un-stretched length l . The system is free to oscillate only in one direction along straight line joining all the three particles. The Lagrangian of the system is GATE 2007
- $L = \frac{m}{2} \left[\left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 \right] - \frac{k}{2} (x_1 - x_2 - l)^2 + \frac{k}{2} (x_3 - x_2 - l)^2$
 - $L = \frac{m}{2} \left[\left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 \right] - \frac{k}{2} (x_1 - x_3 - l)^2 + \frac{k}{2} (x_3 - x_2 - l)^2$
 - $L = \frac{m}{2} \left[\left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 \right] - \frac{k}{2} (x_1 - x_2 + l)^2 - \frac{k}{2} (x_3 - x_2 + l)^2$
 - $L = \frac{m}{2} \left[\left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 \right] - \frac{k}{2} (x_1 - x_2 - l)^2 - \frac{k}{2} (x_3 - x_2 - l)^2$
- 33) The Hamiltonian of a particle is $H = \frac{p^2}{2m} + pq$, where q is the generalized coordinate and p is the corresponding canonical momentum. The Lagrangian is GATE 2007
- $\frac{m}{2} \left(\frac{dq}{dt} + q \right)^2$
 - $\frac{m}{2} \left(\frac{dq}{dt} - q \right)^2$
 - $\frac{m}{2} \left[\left(\frac{dq}{dt} \right)^2 + q \frac{dq}{dt} - q^2 \right]^2$
 - $\frac{m}{2} \left[\left(\frac{dq}{dt} \right)^2 - q \frac{dq}{dt} + q^2 \right]^2$
- 34) A toroidal coil has N closely-wound turns. Assume the current through the coil to be I and the toroid is filled with a magnetic material of relative permittivity μ_r . The magnitude of magnetic induction \vec{B} inside the toroid, at a radial distance r from the axis, is given by GATE 2007
- $\mu_r \mu_0 N I r$
 - $\frac{\mu_r \mu_0 N I}{r}$
 - $\frac{\mu_r \mu_0 N I}{2\pi r}$
 - $2\pi \mu_r \mu_0 N I r$
- 35) An electromagnetic wave with $\vec{E}(z, t) = E_0 \cos(\omega t - kz) \hat{i}$ is traveling in free space and crosses a disc of radius $2m$ placed perpendicular to the z -axis. If $E_0 = 60 \text{ Vm}^{-1}$, the average power, in Watt, crossing the disc along z -direction is GATE 2007
- 30

- b) 60
- c) 120
- d) 270

36) Can the following scalar and vector potentials describe an electromagnetic field?

$$\phi(\vec{x}, t) = 3xyz - 4t$$

$$\vec{A}(\vec{x}, t) = (2x - \omega t)\hat{i} + (y - 2z)\hat{j} + (z - 2xe^{i\omega t})\hat{k}$$

where ω is a constant.

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- a) Yes, in the Coulomb gauge.
- b) Yes, in the Lorentz gauge.
- c) Yes, provided $\omega = 0$.
- d) No.

37) For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V(x) = \frac{1}{2}m\omega^2 x^2$, the first excited energy eigenstate is $\psi(x) = xe^{-ax^2}$. The value of a is

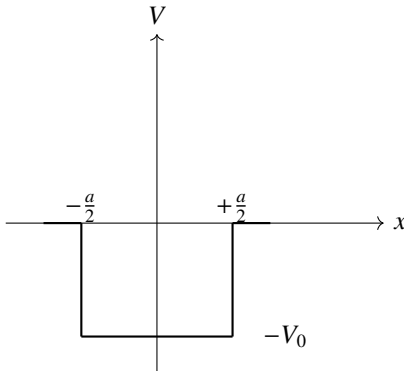
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- a) $\frac{m\omega}{4\hbar}$
- b) $\frac{m\omega}{3\hbar}$
- c) $\frac{m\omega}{2\hbar}$
- d) $\frac{2m\omega}{3\hbar}$

38) If $[x, p] = i\hbar$, the value of $[x^3, p]$ is

- a) $2i\hbar x^2$
- b) $-2i\hbar x^2$
- c) $3i\hbar x^2$
- d) $-3i\hbar x^2$

39) There are only three bound states for a particle of mass m in one-dimensional potential well of the form shown in the figure. The depth V_o of the potential satisfies



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- a) $\frac{2\pi^2\hbar^2}{9mq^2} < V_o < \frac{9\pi^2\hbar^2}{2mq^2}$
- b) $\frac{\pi^2\hbar^2}{mq^2} < V_o < \frac{2\pi^2\hbar^2}{ma^2}$
- c) $\frac{2\pi^2\hbar^2}{mq^2} < V_o < \frac{8\pi^2\hbar^2}{2mq^2}$
- d) $\frac{2\pi^2\hbar^2}{ma^2} < V_o < \frac{50\pi^2\hbar^2}{2ma^2}$

40) An atomic state of hydrogen is represented by the following wavefunction:

$$\psi(r, \theta, \phi) = \frac{1}{\sqrt{2}} \left(\frac{1}{a_0} \right)^{\frac{3}{2}} \left(1 - \frac{r}{2a_0} \right) e^{-\frac{r}{2a_0}} \cos \theta.$$

where a_0 is a constant. The quantum numbers of the state are

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- a) $l = 0, m = 0, n = 1$
- b) $l = 1, m = 1, n = 2$
- c) $l = 1, m = 0, n = 2$
- d) $l = 2, m = 0, n = 3$

- 41) Three operators X, Y and Z satisfy the commutation relations $[X, Y] = i\hbar Z$, $[Y, Z = i\hbar X]$ and $[Z, X] = i\hbar Y$.

The set of all possible eigenvalues of the operator Z , in units of \hbar , is GATE 2007

- a) $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
- b) $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \dots\right\}$
- c) $\left\{0, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 2, \pm \frac{5}{2}, \dots\right\}$
- d) $\left\{-\frac{1}{2}, +\frac{1}{2}\right\}$

- 42) A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22°C by supplying 450kJ s^{-1} . If the outside temperature is 0°C , the heat taken, in kJ s^{-1} , from the outside air is approximately GATE 2007

- a) 487
- b) 470
- c) 467
- d) 417

- 43) The vapour pressure p (in mm of Hg) of a solid, at temperature T , is expressed by $\ln p = 23 - 3863/T$ and that of its liquid phase by $\ln p = 19 - 3063/T$. The triple point (in Kelvin) of the material is GATE 2007

- a) 185
- b) 190
- c) 195
- d) 200

- 44) The free energy for a photon gas is given by $F = -\left(\frac{a}{3}\right)VT^4$, where a is a constant. The entropy S and the pressure P of the photon gas are GATE 2007

- a) $S = \frac{4}{3}aVT^3, P = \frac{a}{3}T^4$
- b) $S = \frac{1}{3}aVT^4, P = \frac{4a}{3}T^3$
- c) $S = \frac{4}{3}aVT^4, P = \frac{a}{3}T^3$
- d) $S = \frac{1}{3}aVT^3, P = \frac{4a}{3}T^4$

- 45) A system has energy levels $E_0, 2E_0, 3E_0, \dots$, where the excited states are triply degenerate. Four non-interacting bosons are placed in the system. If the total energy of the bosons is $5E_0$, the number of microstates is GATE 2007

- a) 2
- b) 3
- c) 4
- d) 5

- 46) In the accordance with the selection rules for the electric dipole transitions, the 4^3P_1 state of helium can decay by photon emission to the states GATE 2007

- a) $2^1S_o, 2^1P_1$ and 3^1D_2
- b) $3^1P_1, 3^1D_2$ and 3^1S_o
- c) $3^3P_2, 3^3D_3$ and 3^3P_o
- d) $2^3S_1, 3^3D_2$ and 3^3D_1

47) If an atom is in the 3D_3 state, the angle between its orbital and spin angular momentum vectors (\vec{L} and \vec{S}) is GATE 2007

- a) $\cos^{-1} \frac{1}{\sqrt{3}}$
- b) $\cos^{-1} \frac{2}{\sqrt{3}}$
- c) $\cos^{-1} \frac{1}{\sqrt{2}}$
- d) $\cos^{-1} \frac{\sqrt{3}}{2}$

48) The hyperfine structure of $Na(3^2P_{\frac{3}{2}})$ with nuclear spin $i = \frac{3}{2}$ has GATE 2007

- a) 1 state
- b) 2 state
- c) 3 state
- d) 4 state

49) The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecules are expressed as $\epsilon_J = BJ(J+1)$, where B is the rotational constant and J is a rotational quantum number.

In a system of such diatomic molecules of reduced mass μ , some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass is changed to 1.05μ . In the rotational spectrum of the system, the shift in the spectral line, corresponding to a transition $J = 4 \rightarrow J = 5$, is GATE 2007

- a) $0.475B$
- b) $0.50B$
- c) $0.95B$
- d) $1.0B$

50) The number of fundamental vibrational modes of CO_2 molecules is GATE 2007

- a) four : 2 are Raman active 2 are infrared active.
- b) four : 1 are Raman active 3 are infrared active.
- c) four : 1 are Raman active 2 are infrared active.
- d) four : 2 are Raman active 1 are infrared active.

51) A piece of paraffin is placed in a uniform magnetic field H_o . The sample contains hydrogen nuclei of mass m_p , which interact only with external magnetic field. An additional oscillating magnetic field is applied to observe resonance absorptions takes place, is given by GATE 2007

- a) $\frac{3g_1eH_o}{2\pi m_p}$
- b) $\frac{3g_1eH_o}{4\pi m_p}$
- c) $\frac{g_1eH_o}{2\pi m_p}$
- d) $\frac{g_1eH_o}{4\pi m_p}$