GATE-2007-PH

EE24BTECH11017-D.KARTHIK

31) The Lagrangian of a particle of mass m is $L = \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2\right] - \frac{V}{2}\left(x^2 + y^2\right) + W\sin\omega t$, where$V, W$ and ω are constants. The conserved quantities are
    a) energy and $z-$component of linear momentum only.
    b) energy and $z-$component of angular momentum only.
    c) $z-$components of both linear and angular momenta only.
    d) energy and $z-$component of both linear and angular momenta.
32) Three particles of mass m each situated at $x_1(t), x_2(t)$, and $x_3(t)$ respectively are connected by two springs of spring constant k and un-stretched length l. The system is free to oscillate only in one direction along straight line joining all the three particles. The Lagrangian of the system is
    a) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_1}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - l)^2 + \frac{k}{2}(x_3 - x_2 - l)^2$
    b) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_1}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_3 - l)^2 + \frac{k}{2}(x_3 - x_2 - l)^2$
    c) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_1}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 + l)^2 - \frac{k}{2}(x_3 - x_2 + l)^2$
    d) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_1}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - l)^2 - \frac{k}{2}(x_3 - x_2 - l)^2$
33) The Hamiltonian of a particle is $H = \frac{p^2}{2m} + pq$, where q is the generalized coordinate and p is the corresponding canonical momentum. The Lagrangian is
    a) $\frac{m}{2}\left(\frac{dq}{dt} + q\right)^2$
    b) $\frac{m}{2}\left(\frac{dq}{dt} - q\right)^2$
    c) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2 + q\frac{dq}{dt} - q^2\right]^2$
    d) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2 - q\frac{dq}{dt} + q^2\right]^2$
34) A toroidal coil has N closely-wound turns. Assume the current through the coil to be I and the toroid is filled with a magnetic material of relative permittivity μ_r. The magnitude of magnetic induction $\overrightarrow{B}$ inside the toriod, at a radial distance r from the axis, is given by
    a) $\mu_r\mu_0 NIr$
    b) $\frac{\mu_r\mu_0 NI}{r}$
    c) $\frac{\mu_r\mu_0 NI}{2\pi r}$
    d) $2\pi\mu_r\mu_0 NIr$
35) An electromagnetic wave with $\overrightarrow{E}(z,t) = E_o cos(\omega t - kz)\hat{i}$ is traveling in free space and crosses a disc of radius $2m$ placed perpendicular to the $z-$axis. If $E_o = 60Vm^-1$, the average power, in Watt, crossing the disc along $z-$direction is
    a) 30

b) 60
c) 120
d) 270

36) Can the following scalar and vector potentials describe an electromagnetic field?
$\phi\left(\overrightarrow{x}, t\right) = 3xyz - 4t$
$\overrightarrow{A}\left(\overrightarrow{x}, t\right) = (2x = \omega t)\,\hat{i} + (y - 2z)\,\hat{j} + \left(z - 2xe^{i\omega t}\right)\hat{k}$
where ω is a constant.

a) Yes, in the Coulomb gauge.
b) Yes, in the Lorentz gauge.
c) Yes,provided $\omega = 0$.
d) No.

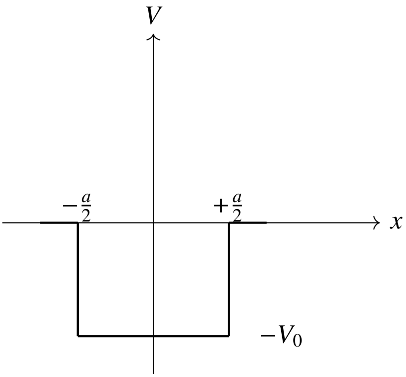
37) For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V(x) = \frac{1}{2}m\omega^2 x^2$, the first excited energy eigenstate is $\psi(x) = xe^{-ax^2}$. The value if a is

a) $\frac{m\omega}{4\hbar}$
b) $\frac{m\omega}{3\hbar}$
c) $\frac{m\omega}{2\hbar}$
d) $\frac{2m\omega}{3\hbar}$

38) If $[x, p] = i\hbar$, the value of $\left[x^3, p\right]$ is

a) $2i\hbar x^2$
b) $-2i\hbar x^2$
c) $3i\hbar x^2$
d) $-3i\hbar x^2$

39) There are only three bound states for a particle of mass m in one-dimensional potential well of the form shown in the figure. The depth V_o of the potential satisfies



a) $\frac{2\pi^2\hbar^2}{ma^2} < V_o < \frac{9\pi^2\hbar^2}{2ma^2}$
b) $\frac{\pi^2\hbar^2}{ma^2} < V_o < \frac{2\pi^2\hbar^2}{ma^2}$
c) $\frac{2\pi^2\hbar^2}{ma^2} < V_o < \frac{8\pi^2\hbar^2}{2ma^2}$
d) $\frac{2\pi^2\hbar^2}{ma^2} < V_o < \frac{50\pi^2\hbar^2}{2ma^2}$

40) An atomic state of hydrogen is represented by the following wavefunction:

$\psi(r,\theta,\phi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_o}\right)^{\frac{3}{2}}\left(1 - \frac{r}{2a_o}\right)e^{-\frac{r}{2a_o}}\cos\theta.$

where a_o is a constant. The quantum numbers of the state are

a) $l = 0, m = 0, n = 1$
b) $l = 1, m = 1, n = 2$
c) $l = 1, m = 0, n = 2$
d) $l = 2, m = 0, n = 3$

41) Three operators X, Y and Z satisfy the commutation relations $[X, Y] = i\hbar Z$, $[Y, Z = i\hbar X]$ and $[Z, X] = i\hbar Y$.
The set of all possible eigenvalues of the operator Z, in units of $\hbar$, is

a) $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
b) $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots\right\}$
c) $\left\{0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm\frac{5}{2}, \ldots\right\}$
d) $\left\{-\frac{1}{2}, +\frac{1}{2}\right\}$

42) A heat pump working on the Carnot cycle maintains the inside temperature of a house at $22°C$ by supplying $450kJs^{-1}$. If the outside temperature is $0°C$,the heat taken, in kjs^{-1},from the outside air is approximately

a) 487
b) 470
c) 467
d) 417

43) The vapour pressure $p(in mm of Hg)$of a solid, at temperature T, is expressed by $\ln p = 23 - 3863/T$ and that of its liquid phase by $\ln p = 19 - 3063/T$. The triple point $(in Kelvin)$ of the material is

a) 185
b) 190
c) 195
d) 200

44) The free energy for a photon gas is given by $F = -\left(\frac{a}{3}\right)VT^4$, where a is a constant.The entropy S and the pressure P of the photon gas are

a) $S = \frac{4}{3}aVT^3, P = \frac{a}{3}T^4$
b) $S = \frac{1}{3}aVT^4, P = \frac{4a}{3}T^3$
c) $S = \frac{4}{3}aVT^4, P = \frac{a}{3}T^3$
d) $S = \frac{1}{3}aVT^3, P = \frac{4a}{3}T^4$

45) A system has energy levels $E_o, 2E_o, 3E_o\ldots$, where the excited states are triply degenerate. Four non- interacting bosons are placed in the system.If the total energy of the bosons is $5E_o$, the number of microstates is

a) 2
b) 3
c) 4
d) 5

46) In the accordance with the selection rules for the electric dipole transitions, the 4^3P_1 state of helium can decay by photon emission to the states

a) $2^1S_o, 2^1P_1$ and 3^1D_2
b) $3^1P_1, 3^1D_2$ and 3^1S_o
c) $3^3P_2, 3^3D_3$ and 3^3P_o
d) $2^3S_1, 3^3D_2$ and 3^3D_1

47) If an atom is in the $^3D_3$ state,the angle between its orbital and spin angular momentum vectors $\left(\overrightarrow{L} and \overrightarrow{S}\right)$ is
a) $\cos^{-1}\frac{1}{\sqrt{3}}$
b) $\cos^{-1}\frac{2}{\sqrt{3}}$
c) $\cos^{-1}\frac{1}{\sqrt{2}}$
d) $\cos{-1}\frac{\sqrt{3}}{2}$

48) The hyperfine structure of $Na\left(3^2P_{\frac{3}{2}}\right)$ with nuclear spin $i = \frac{3}{2}$ has
a) 1 state
b) 2 state
c) 3 state
d) 4 state

49) The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecules are expressed as $\epsilon_J = BJ(J+1)$, where B is the rotational constant and J is a rotational quantum number.
In a system of such diatomic molecules of reduced mass μ, some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass is changed to 1.05μ. In the rotational spectrum of the system, the shift in the spectral line, corresponding to a transition $J = 4 \rightarrow J = 5$, is
a) $0.475B$
b) $0.50B$
c) $0.95B$
d) $1.0B$

50) The number of fundamental vibrational modes of CO_2 molecules is
a) four : 2 are Raman active 2 are infrared active.
b) four : 1 are Raman active 3 are infrared active.
c) four : 1 are Raman active 2 are infrared active.
d) four : 2 are Raman active 1 are infrared active.

51) A piece of paraffin is placed in a uniform magnetic field H_o. The sample contains hydrogen nuclei of mass m_p. which interact only with external magnetic field. An additional oscillating magnetic field is applied to observe resonance absorptions takes place, is given by
a) $\frac{3g_1eH_o}{2\pi m_p}$
b) $\frac{3g_1eH_o}{4\pi m_p}$
c) $\frac{g_1eH_o}{2\pi m_p}$
d) $\frac{g_1eH_o}{4\pi m_p}$