EE24BTECH11017-D.KARTHIK

Question:

Find the coordinates of the point which divides the line segment joining the points (1, -2, 3) and (3, 4, -5) in the ratio 2:3:

- 1) internally, and
- 2) externally.

Solution: Let
$$\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$
 and $\mathbf{B} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$. The ratio is $m : n = 2 : 3$.

1) Internal Division:

The coordinates of the point dividing the line internally are given by:

$$\mathbf{P} = \frac{m\mathbf{B} + n\mathbf{A}}{m+n} \tag{1.1}$$

1

$$= \frac{2\binom{3}{4} + 3\binom{1}{-2}}{2+3}$$
 (1.2)

$$= \frac{\binom{6}{8} + \binom{3}{-6}}{5}$$
 (1.3)

$$=\frac{\begin{pmatrix} 9\\2\\-1 \end{pmatrix}}{5} \tag{1.4}$$

$$\mathbf{P} = \begin{pmatrix} \frac{9}{5} \\ \frac{2}{5} \\ -\frac{1}{5} \end{pmatrix} \tag{1.5}$$

Therefore, the required point dividing the line internally is $\begin{pmatrix} \frac{9}{5} \\ \frac{1}{5} \\ -\frac{1}{5} \end{pmatrix}$.

2) External Division:

The coordinates of the point dividing the line externally are given by:

$$\mathbf{Q} = \frac{m\mathbf{B} - n\mathbf{A}}{m - n}$$

$$2 \begin{pmatrix} 3 \\ 4 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$
(2.1)

$$= \frac{2 \begin{pmatrix} 3\\4\\-5 \end{pmatrix} - 3 \begin{pmatrix} 1\\-2\\3 \end{pmatrix}}{2 - 3}$$

$$= \frac{\begin{pmatrix} 6\\0\\0 \end{pmatrix} \begin{pmatrix} 3\\6\\0 \end{pmatrix}}{\begin{pmatrix} 3\\6\\0 \end{pmatrix}}$$
(2.2)

$$=\frac{\begin{pmatrix} 8\\-10 \end{pmatrix} - \begin{pmatrix} -6\\9 \end{pmatrix}}{-1} \tag{2.3}$$

$$= \frac{\begin{pmatrix} 3\\14\\-19 \end{pmatrix}}{-1} \tag{2.4}$$

$$\mathbf{Q} = \begin{pmatrix} -3 \\ -3 \\ -14 \\ 19 \end{pmatrix} \tag{2.5}$$

Therefore, the required point dividing the line externally is $\begin{pmatrix} -3 \\ -14 \\ 19 \end{pmatrix}$.

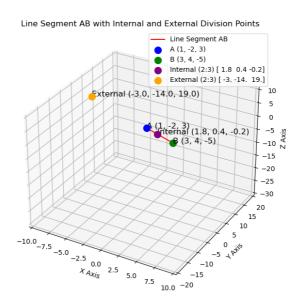


Fig. 2.1: Internal and External Division of Line Segment