1.4.9m

EE24BTECH11017-D.KARTHIK

Question:
Find the coordinates of the point which divides the line segment joining the points $(1, -2, 3)$ and $(3, 4, -5)$ in the ratio $2 : 3$:

1) internally, and
2) externally.

Solution: : Let $\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$. The ratio is $m : n = 2 : 3$.

1) **Internal Division:**
The coordinates of the point dividing the line internally are given by:

$$\mathbf{P} = \frac{m\mathbf{B} + n\mathbf{A}}{m+n} \tag{1.1}$$

$$= \frac{2\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} + 3\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}}{2+3} \tag{1.2}$$

$$= \frac{\begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}}{5} \tag{1.3}$$

$$= \frac{\begin{pmatrix} 9 \\ 2 \\ -1 \end{pmatrix}}{5} \tag{1.4}$$

$$\mathbf{P} = \begin{pmatrix} \frac{9}{5} \\ \frac{2}{5} \\ -\frac{1}{5} \end{pmatrix} \tag{1.5}$$

Therefore, the required point dividing the line internally is $\begin{pmatrix} \frac{9}{5} \\ \frac{2}{5} \\ -\frac{1}{5} \end{pmatrix}$.

2) **External Division:**

The coordinates of the point dividing the line externally are given by:

$$\mathbf{Q} = \frac{m\mathbf{B} - n\mathbf{A}}{m - n} \tag{2.1}$$

$$= \frac{2\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} - 3\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}}{2 - 3} \tag{2.2}$$

$$= \frac{\begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix} - \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}}{-1} \tag{2.3}$$

$$= \frac{\begin{pmatrix} 3 \\ 14 \\ -19 \end{pmatrix}}{-1} \tag{2.4}$$

$$\mathbf{Q} = \begin{pmatrix} -3 \\ -14 \\ 19 \end{pmatrix} \tag{2.5}$$

Therefore, the required point dividing the line externally is $\begin{pmatrix} -3 \\ -14 \\ 19 \end{pmatrix}$.

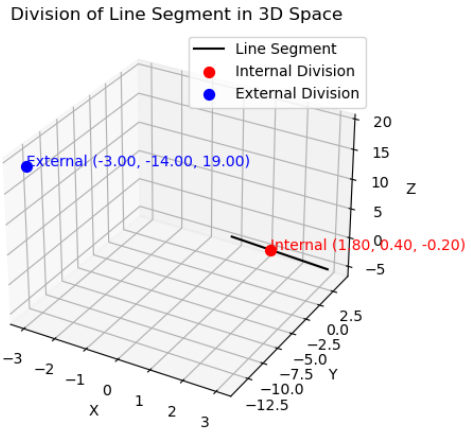

Fig. 2.1: Internal and External Division of Line Segment