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Assignment One

Question One

Homogenous coordinates are used to preform easy transformation such as Rotation, Translation, and Projection.

Question Two

Yes pin-hole cameras have infinite depth of field because the point to receptor ratio is one to one meaning that the light is reflected directly from the object to the receptor.

Question Three

No a camera with a lens does not have a infinite depth of field. The lens equation dictates that only one distance can be in focus. This is because the lens bends light so multiple points map onto a single receptor

Question Four

3D rotations are not commutative as multiplication of matrix are not commutative. Also rotating a cube Left, Left, Up results in the top most face shown and Up, Left, Left results in the bottom most face.

Question Five

As the Field of view increases the focal length decreases

Question Six

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix}$$
$$x = \frac{fX}{Z} y = \frac{fY}{Z}$$

By solving the previous equations we are given the values of (x, y) as an expression of both X/Y and Z. This means that there can exist 2 different values of X/Y and Z that produce the same coordinate

$$\begin{bmatrix}1\\1\\2\end{bmatrix} x = \frac{f1}{2} y = \frac{f1}{2}$$

$$\begin{bmatrix}5\\5\\10\end{bmatrix} x = \frac{f5}{10} = \frac{f1}{2} y = \frac{f5}{10} = \frac{f1}{2}$$

From this we can easily see that any multiple of the (X,Y,Z) coordinate (on the same line) will map to the same (x,y) Coordinate on the image

Question Seven

2 dimensional rotation has 1 degree of freedom. 3 dimensional rotation has 3 degree of freedom. This implies that the entries in matrices are not independent.

Question Eight

Question Nine

A.

$$f = 500 \quad s_x = s_y = 1 \quad (o_x, o_y) = (320, 240)$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}-1 & 0 & 320\\0 & -1 & 240\\0 & 0 & 1\end{bmatrix} \begin{bmatrix}500 & 0 & 0 & 0\\0 & 500 & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix} \begin{bmatrix}1 & 0 & 0 & -170\\0 & 1 & 0 & -105\\0 & 0 & 1 & -70\\0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}X_w\\Y_w\\Z_w\\1\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}-500 & 0 & 320 & 0\\0 & -500 & 240 & 0\\0 & 0 & 1 & 0\end{bmatrix} \begin{bmatrix}1 & 0 & 0 & -170\\0 & 1 & 0 & -105\\0 & 0 & 1 & -70\\0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}X_w\\Y_w\\Z_w\\1\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}-500 & 0 & 320 & 62600\\0 & -500 & 240 & 35700\\0 & 0 & 1 & -70\end{bmatrix} \begin{bmatrix}X_w\\Y_w\\Z_w\\1\end{bmatrix}$$

B.

$$\begin{bmatrix}-500 & 0 & 320 & 62600\\0 & -500 & 240 & 35700\\0 & 0 & 1 & -70\end{bmatrix} \begin{bmatrix}X_w\\Y_w\\Z_w\\1\end{bmatrix} = \begin{bmatrix}-64400\\-38300\\80\end{bmatrix}$$

$$(\frac{-64400}{80}, \frac{-38300}{80}) = (-805, -478.75)$$

10.

Python 3.6.2 OpenCv 3.4 `[ 350. 220. 150.] maps to [-805. -478.75]`

11.

$$RegularConvolution \quad O(n^2)$$
$$SeperableKernal \quad O(n^2)$$

12. False, False, False, False

13. (a, b) represents the largest (x, y) coordinate in the rectangle and the height and width of the rectangle are described by H and W respectively. The formula for computing the pixel values of a given rectangle is

$$(x, y) - (((x - W, y) + (x, y - H)) - (W, H))$$