

MA301.3 – Advanced Mathematics for Computing
Mid Exam Paper

Answer all questions

Question-1 (Vectors)

Mark the correct response with a pen on the answer sheet provided.

- (i) Let the vectors u and v be $(-1, 1, 3)$ and $(-1, -2, 3)$. Then the vector $3u - 2v$ is given by:
- (a) $(-1, 7, -3)$
 - (b) $(-1, 7, 3)$
 - (c) $(1, 7, -3)$
 - (d) $(1, 7, 3)$
 - (e) $(-1, -1, -3)$
- (ii) Find x, y, z such that $(x - y - z, x + 2y - z, 2x - y + z) = (6, 0, 1)$
- (a) $x = 1, y = -2, z = 3$
 - (b) $x = 1, y = -2, z = -3$
 - (c) $x = -1, y = -2, z = 3$
 - (d) $x = 1, y = 2, z = 3$
 - (e) $x = -2, y = 1, z = 3$
- (iii) For any vectors u, v, w in \mathbb{R}^n and any scalars k, k' in \mathbb{R} , which of the following statements are correct?
- I. $(u + v) + w \neq u + (v + w)$
 - II. $u + v \neq v + u$
 - III. $u \cdot v = v \cdot u$
 - IV. $k(u + v) = ku + kv$ and $(k + k')u = ku + k'u$
- (a) III only (b) IV only (c) I and II (d) III and IV (e) All of the above
- (iv) Suppose u and v are two vectors where $u = (k, 17, k, -5)$ and $v = (-5, -k, 3k, -7)$. Find value/s of k such that these vectors are orthogonal?
- (a) 5 only (b) 7 only (c) 5 $\frac{-7}{3}$ and $\frac{7}{3}$ (d) 5 and (e) None of the above

(v) Suppose u and v are two vectors where $u = (1, 2, -1)$ and $v = (3, -1, 2)$. The projection of the vector u onto v is given by:

- (a) $\frac{1}{14}(3, -1, 2)$ (b) $\frac{-1}{\sqrt{14}}(3, -1, 2)$ (c) $\frac{-1}{14}(3, -1, 2)$ (d) $\frac{-1}{14}(1, 2, -1)$ (e) $\frac{-1}{6}(1, 2, -1)$

(vi) Which of the following are unit vectors?

- I. $(\frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}})$
 II. $\frac{1}{\sqrt{2}}(1, 0, 0, 1)$
 III. $(\frac{4}{\sqrt{50}}, \frac{5}{\sqrt{50}}, \frac{9}{\sqrt{50}})$
 IV. $(0, 1, 0, 1)$

- (a) I and II only (b) I and IV only (c) I, II and III (d) I, III and IV (e) All of the above

(vii) Which of the following are true statements for the vectors $u = (2, 1, 1)$ and $v = (1, 1, 2)$

- I. For the above vectors u, v in R^3 $|u \cdot v| \geq \|u\| \|v\|$
 II. Dot product between the two vectors u and v is 6.
 III. Cross product between the vectors u, v in R^3 is $i - 3j - k$

- (a) I and II only (b) II and III only (c) I, II and III (d) I and III only (e) None of the above

Question 2 (Differentiation)

1. Find the limits of the following expressions

i. $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - 1}$ (2 marks)

ii. $\lim_{x \rightarrow \infty} \frac{2x^2 - 1}{3x^2 + 1}$ (2 marks)

2. Differentiate \sqrt{x} using the first principles. (3 marks)

3. Find the derivatives of the following.

I. $y = (x^3 - 3)^3$

II. $y = (x^2 + 5x - 3)^5$

III. $y = (4x^3 - 3)^{-\frac{1}{2}}$

4. Find the derivatives of the following.

1. $y = (x + 1)(x^4 + 5)$

2. $y = (3x^2 - 5x + 2)(2x^2 - 4x + 1)$

3. $y = (x^2 + 1)(x^2 + 3)$

4. $y = x(x - 1)(x + 1)$

5. Find the derivatives of the following.

$$y = \frac{(x + 1)}{(x^4 + 5)}$$

$$y = \frac{(3x^2 - 5)}{(\sqrt{x} - 4)}$$

$$y = \frac{(5x^2)}{(x^2 - 3)}$$

$$y = \frac{(x^3 - 1)}{(x - 1)}$$

6. Prove following.

I. $F'(x) = f(x)g'(x) + g(x)f'(x)$

II. $\frac{d}{dx}(e^x) = e^x$