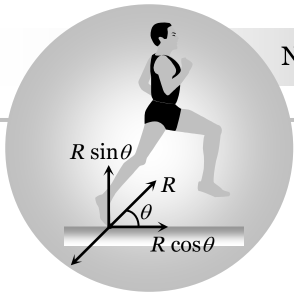


Newton's Laws of Motion

4.1 Point Mass

(1) An object can be considered as a point object if during motion in a given time, it covers distance much greater than its own size.

(2) Object with zero dimension considered as a point mass.

(3) Point mass is a mathematical concept to simplify the problems.

4.2 Inertia

(1) Inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line by their own is called inertia.

(2) Inertia is not a physical quantity, it is only a property of the body which depends on mass of the body.

(3) Inertia has no units and no dimensions

(4) Two bodies of equal mass, one in motion and another is at rest, possess same inertia because it is a factor of mass only and does not depend upon the velocity.

4.3 Linear Momentum

(1) Linear momentum of a body is the quantity of motion contained in the body.

(2) It is measured in terms of the force required to stop the body in unit time.

(3) It is measured as the product of the mass of the body and its velocity *i.e.*, Momentum = mass × velocity.

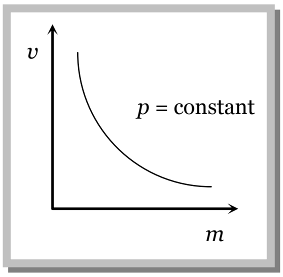
If a body of mass m is moving with velocity $\vec{v}$ then its linear momentum $\vec{p}$ is given by $\vec{p} = m\vec{v}$

(4) It is a vector quantity and it's direction is the same as the direction of velocity of the body.

(5) Units : *kg-m/sec* [S.I.], *g-cm/sec* [C.G.S.]

(6) Dimension : $[MLT^{-1}]$

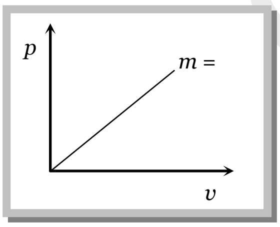
(7) If two objects of different masses have same momentum, the lighter body possesses greater velocity.



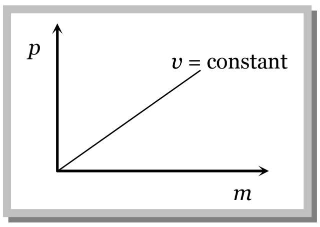
$$p = m_1 v_1 = m_2 v_2 = \text{constant}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1} \quad i.e. \quad v \propto \frac{1}{m} \quad \text{[As } p \text{ is constant]}$$

(8) For a given body $p \propto v$



(9) For different bodies at same velocities $p \propto m$



4.4 Newton's First Law

A body continue to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

(1) If no net force acts on a body, then the velocity of the body cannot change *i.e.* the body cannot accelerate.

(2) Newton's first law defines inertia and is rightly called the law of inertia. Inertia are of three types :

Inertia of rest, Inertia of motion, Inertia of direction

(3) **Inertia of rest :** It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

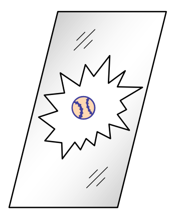
Example : (i) A person who is standing freely in bus, thrown backward, when bus starts suddenly.

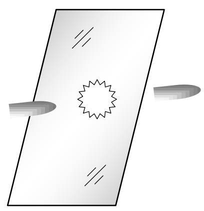
When a bus suddenly starts, the force responsible for bringing bus in motion is also transmitted to lower part of body, so this part of the body comes in motion along with the bus. While the upper half of body (say above the waist) receives no force to overcome inertia of rest and so it stays in its original position. Thus there is a relative displacement between the two parts of the body and it appears as if the upper part of the body has been thrown backward.

Note : ❑ If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk.

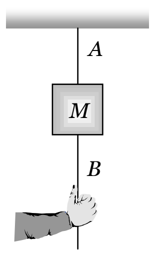
(ii) When a horse starts suddenly, the rider tends to fall backward on account of inertia of rest of upper part of the body as explained above.

(iii) A bullet fired on a window pane makes a clean hole through it while a stone breaks the whole window because the bullet has a speed much greater than the stone. So its time of contact with glass is small. So in case of bullet the motion is transmitted only to a small portion of the glass in that small time. Hence a clear hole is created in the glass window, while in case of ball, the time and the area of contact is large. During this time the motion is transmitted to the entire window, thus creating the cracks in the entire window.


Cracks by the ball


Hole by the bullet

(iv) In the arrangement shown in the figure :



(a) If the string B is pulled with a sudden jerk then it will experience tension while due to inertia of rest of mass M this force will not be transmitted to the string A and so the string B will break.

(b) If the string B is pulled steadily the force applied to it will be transmitted from string B to A through the mass M and as tension in A will be greater than in B by Mg (weight of mass M) the string A will break.

(v) If we place a coin on smooth piece of card board covering a glass and strike the card board piece suddenly with a finger. The cardboard slips away and the coin falls into the glass due to inertia of rest.

(vi) The dust particles in a durree falls off when it is beaten with a stick. This is because the beating sets the durree in motion whereas the dust particles tend to remain at rest and hence separate.

(4) **Inertia of motion :** It is the inability of a body to change itself its state of uniform motion *i.e.,* a body in uniform motion can neither accelerate nor retard by its own.

Example : (i) When a bus or train stops suddenly, a passenger sitting inside tends to fall forward. This is because the lower part of his body comes to rest with the bus or train but the upper part tends to continue its motion due to inertia of motion.

(ii) A person jumping out of a moving train may fall forward.

(iii) An athlete runs a certain distance before taking a long jump. This is because velocity acquired by running is added to velocity of the athlete at the time of jump. Hence he can jump over a longer distance.

(5) **Inertia of direction :** It is the inability of a body to change by itself direction of motion.

Example : (i) When a stone tied to one end of a string is whirled and the string breaks suddenly, the stone flies off along the tangent to the circle. This is because the pull in the string was forcing the stone to move in a circle. As soon as the string breaks, the pull vanishes. The stone in a bid to move along the straight line flies off tangentially.

(ii) The rotating wheel of any vehicle throw out mud, if any, tangentially, due to directional inertia.

(iii) When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

Sample problem based on Newton's first law

***Problem* 1.** When a bus suddenly takes a turn, the passengers are thrown outwards because of **[AFMC 1999; CPMT 2000, 2001]**

(a) Inertia of motion (b) Acceleration of motion
(c) Speed of motion (d) Both (b) and (c)

Solution : (a)

***Problem* 2.** A person sitting in an open car moving at constant velocity throws a ball vertically up into air. The ball fall **[EAMCET (Med.) 1995]**

(a) Outside the car (b) In the car ahead of the person
(c) In the car to the side of the person (d) Exactly in the hand which threw it up

Solution : (d) Because the horizontal component of velocity are same for both car and ball so they cover equal horizontal distances in given time interval.

4.5 Newton's Second Law

(1) The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.

(2) If a body of mass m, moves with velocity $\vec{v}$ then its linear momentum can be given by $\vec{p} = m\vec{v}$ and if force $\vec{F}$ is applied on a body, then

$$\vec{F} \propto \frac{d\vec{p}}{dt} \Rightarrow F = K\frac{d\vec{p}}{dt}$$

or $\vec{F} = \frac{d\vec{p}}{dt}$ (K = 1 in C.G.S. and S.I. units)

or $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$ (As $a = \frac{d\vec{v}}{dt}$ = acceleration produced in the body)

$\therefore$ $\vec{F} = m\vec{a}$

Force = mass × acceleration

Sample problem based on Newton's second law

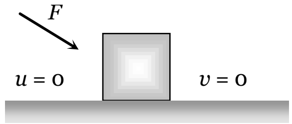
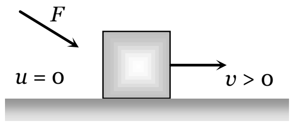
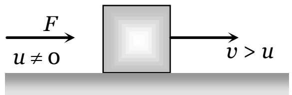
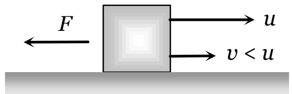
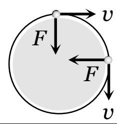
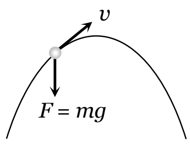
***Problem* 3.** A train is moving with velocity 20 m/sec. on this, dust is falling at the rate of 50 kg/min. The extra force required to move this train with constant velocity will be **[RPET 1999]**

(a) 16.66 N (b) 1000 N (c) 166.6 N (d) 1200 N

Solution : (a) Force $F = v\frac{dm}{dt} = 20 \times \frac{50}{60} = 16.66\,N$

***Problem* 4.** A force of 10 Newton acts on a body of mass 20 kg for 10 seconds. Change in its momentum is **[MP PET 2002]**

(a) 5 *kg m/s* (b) 100 *kg m/s* (c) 200 *kg m/s* (d) 1000 *kg m/s*

	Body remains at rest. Here force is trying to change the state of rest.
	Body starts moving. Here force changes the state of rest.
	In a small interval of time, force increases the magnitude of speed and direction of motion remains same.
	In a small interval of time, force decreases the magnitude of speed and direction of motion remains same.
	In uniform circular motion only direction of velocity changes, speed remains constant. Force is always perpendicular to velocity.
	In non-uniform circular motion, elliptical, parabolic or hyperbolic motion force acts at an angle to the direction of motion. In all these motions. Both magnitude and direction of velocity changes.

Solution : (b)

***Problem* 5.**

(a)

Solution : (a)

Force = $\frac{mc}{d}$

$= \frac{100(0-5}{0.1}$

$F = -5000$

4.6 Force

(

1) Force is an external effect in the form of a push or pulls which

(i) Produces or tries to produce motion in a body at rest.

(ii) Stops or tries to stop a moving body.

(iii) Changes or tries to change the direction of motion of the body.

(2) Dimension : Force = mass × acceleration

$[F] = [M][LT^{-2}] = [MLT^{-2}]$

(3) Units : Absolute units : (i) *Newton* (S.I.) (ii) *Dyne* (C.G.S)

Gravitational units : (i) Kilogram-force (M.K.S.) (ii) Gram-force (C.G.S)

Newton : One Newton is that force which produces an acceleration of $1 m/s^2$ in a body of mass 1 *Kilogram*. $\therefore 1\,Newton = 1 kg\, m/s^2$

Dyne : One dyne is that force which produces an acceleration of $1 cm/s^2$ in a body of mass 1 *gram*. $\therefore$ 1 *Dyne* $= 1 gm\ cm/\sec^2$

Relation between absolute units of force $1\,Newton = 10^5\,Dyne$

Kilogram-force : It is that force which produces an acceleration of $9.8 m/s^2$ in a body of mass 1 *kg*. $\therefore$ 1 *kg-f* = 9.81 *Newton*

Gram-force : It is that force which produces an acceleration of $980\, cm/s^2$ in a body of mass 1*gm*. $\therefore$ 1 *gm-f* = 980 *Dyne*

Relation between gravitational units of force : 1 *kg-f* = 10^7 *gm-f*

(4) $\vec{F} = m\vec{a}$ formula is valid only if force is changing the state of rest or motion and the mass of the body is constant and finite.

(5) If m is not constant $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

(6) If force and acceleration have three component along x, y and z axis, then

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \text{ and } \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

From above it is clear that $F_x = ma_x$, $F_y = ma_y$, $F_z = ma_z$

(7) No force is required to move a body uniformly along a straight line.

$$\vec{F} = ma \qquad \therefore \vec{F} = 0 \qquad (\text{As } a = 0)$$

(8) When force is written without direction then positive force means repulsive while negative force means attractive.

Example : *Positive force* – Force between two similar charges

Negative force – Force between two opposite charges

(9) Out of so many natural forces, for distance 10^{-15} *metre*, nuclear force is strongest while gravitational force weakest. $F_{\text{nuclear}} > F_{\text{electromagnetic}} > F_{\text{gravitational}}$

(10) Ratio of electric force and gravitational force between two electron $F_e / F_g = 10^{43}$ $\therefore F_e >> F_g$

(11) Constant force : If the direction and magnitude of a force is constant. It is said to be a constant force.

(12) Variable or dependent force :

(i) *Time dependent force* : In case of impulse or motion of a charged particle in an alternating electric field force is time dependent.

(ii) *Position dependent force* : Gravitational force between two bodies $\frac{Gm_1m_2}{r^2}$

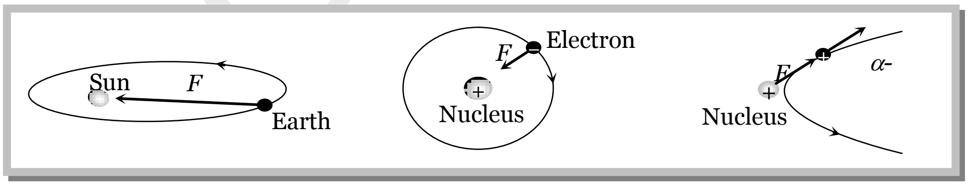
or Force between two charged particles $= \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$.

(iii) *Velocity dependent force* : Viscous force $(6\pi\eta rv)$

Force on charged particle in a magnetic field $(qvB\sin\theta)$

(13) Central force : If a position dependent force is always directed towards or away from a fixed point it is said to be central otherwise non-central.

Example : Motion of earth around the sun. Motion of electron in an atom. Scattering of α-particles from a nucleus.



(14) Conservative or non conservative force : If under the action of a force the work done in a round trip is zero or the work is path independent, the force is said to be conservative otherwise non conservative.

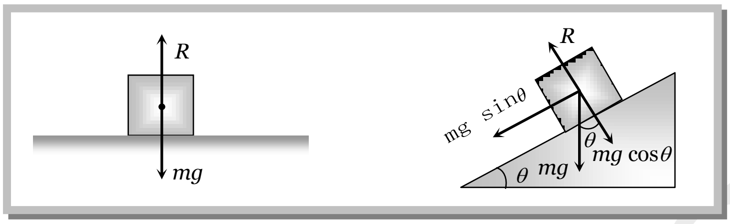
Example : Conservative force : Gravitational force, electric force, elastic force.

Non conservative force : Frictional force, viscous force.

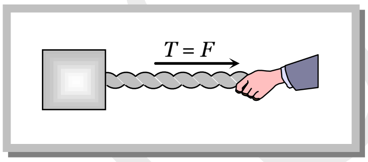
(15) Common forces in mechanics :

(i) *Weight* : Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

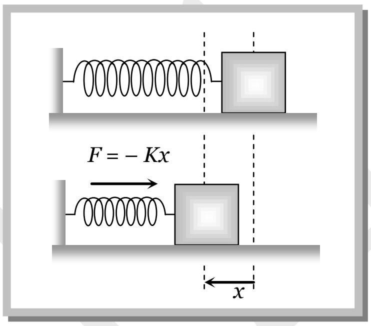
(ii) *Reaction or Normal force :* When a body is placed on a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. Then force is called 'Normal force' or 'Reaction'.



(iii) *Tension* : The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is so as to pull the body.



(iv) *Spring force :* Every spring resists any attempt to change its length. This resistive force increases with change in length. Spring force is given by $F = -Kx$; where x is the change in length and K is the spring constant (unit N/m).



4.7 Equilibrium of Concurrent Force

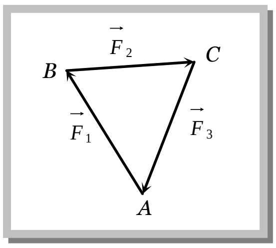
(1) If all the forces working on a body are acting on the same point, then they are said to be concurrent.

(2) A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.

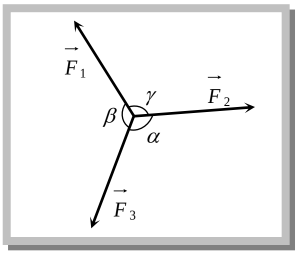
(3) The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.

(4) Mathematically for equilibrium $\sum F_{\text{net}} = 0$ or $\sum F_x = 0$; $\sum F_y = 0$; , $\sum F_z = 0$

(5) Three concurrent forces will be in equilibrium, if they can be represented completely by three sides of a triangle taken in order.

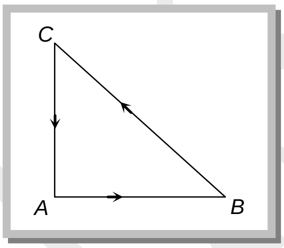


(6) Lami's Theorem : For concurrent forces $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$



Sample problem based on force and equilibrium

Problem 6. Three forces starts acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle *ABC* (as shown). The particle will now move with velocity



(a) $\vec{v}$ remaining unchanged

(b) Less than $\vec{v}$

(c) Greater than $\vec{v}$

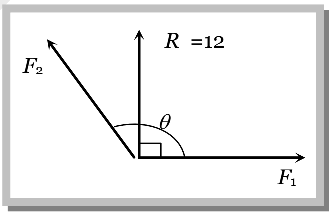
(d) $\vec{v}$ in the direction of the largest force *BC*

Solution : (a) Given three forces are in equilibrium *i.e.* net force will be zero. It means the particle will move with same velocity.

Problem 7. Two forces are such that the sum of their magnitudes is 18 *N* and their resultant is perpendicular to the smaller force and magnitude of resultant is 12. Then the magnitudes of the forces are **[AIEEE 2002]**

(a) 12 *N*, 6 *N* (b) 13 *N*, 5*N* (c) 10 *N*, 8 *N* (d) 16 *N*, 2 *N*

Solution : (b) Let two forces are F_1 and $F_2 (F_1 < F_2)$.



According to problem: $F_1 + F_2 = 18$ .....(i)

Angle between F_1 and resultant (R) is 90°

$\therefore \quad \tan 90 = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} = \infty$

$\Rightarrow F_1 + F_2 \cos\theta = 0$

$\Rightarrow \cos\theta = -\frac{F_1}{F_2}$ .....(ii)

and $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$144 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$ .....(iii)

by solving (i), (ii) and (iii) we get $F_1 = 5N$ and $F_2 = 13N$

Problem 8. The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is **[KCET (Engg./Med.) 2002]**

(a) 60^o (b) 120^o (c) 150^o (d) 90^o

Solution: (b) Let forces are *F* and 2*F* and angle between them is θ and resultant makes an angle α with the force *F*.

$\tan\alpha = \frac{2F\sin\theta}{F + 2F\cos\theta} = \tan 90 = \infty$

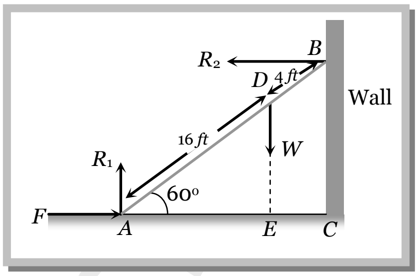
$\Rightarrow F + 2F\cos\theta = 0 \qquad \therefore \cos\theta = -1/2$ or $\theta = 120°$

Problem 9. A weightless ladder, 20 *ft* long rests against a frictionless wall at an angle of 60° with the horizontal. A 150 pound man is 4 *ft* from the top of the ladder. A horizontal force is needed to prevent it from slipping. Choose the correct magnitude from the following **[CBSE PMT 1998]**

(a) 175 *lb* (b) 100 *lb* (c) 70 *lb* (d) 150 *lb*

Solution: (c) Since the system is in equilibrium therefore $\sum F_x = 0$ and $\sum F_y = 0$ $\therefore$ $F = R_2$ and $W = R_1$

Now by taking the moment of forces about point *B*.



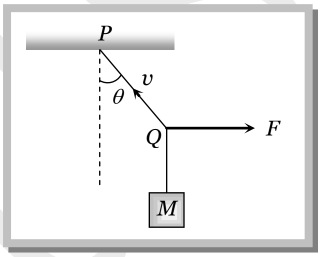
$F.(BC) + W.(EC) = R_1(AC)$ [from the figure $EC = 4\cos 60$]

$F.(20\sin 60) + W(4\cos 60) = R_1(20\cos 60)$

$10\sqrt{3}F + 2W = 10R_1$ [As $R_1 = W$]

$\therefore\ F = \frac{8W}{10\sqrt{3}} = \frac{8\times 150}{10\sqrt{3}} = 70\,lb$

Problem 10. A mass *M* is suspended by a rope from a rigid support at *P* as shown in the figure. Another rope is tied at the end *Q*, and it is pulled horizontally with a force *F*. If the rope *PQ* makes angle θ with the vertical then the tension in the string *PQ* is



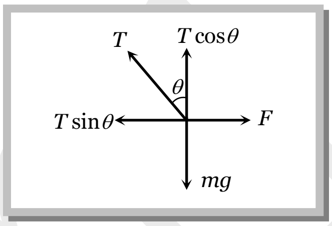
(a) $F\sin\theta$

(b) $F/\sin\theta$

(c) $F\cos\theta$

(d) $F/\cos\theta$

Solution: (b) From the figure

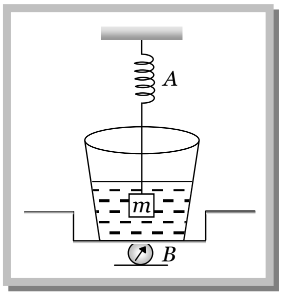


For horizontal equilibrium

$T\sin\theta = F$

$\therefore\ T = \frac{F}{\sin\theta}$

Problem 11. A spring balance A shows a reading of 2 *kg*, when an aluminium block is suspended from it. Another balance *B* shows a reading of 5 *kg*, when a beaker full of liquid is placed in its pan. The two balances are arranged such that the *Al* – block is completely immersed inside the liquid as shown in the figure. Then **[IIT-JEE**



(a) The reading of the balance *A* will be more than 2 *kg*

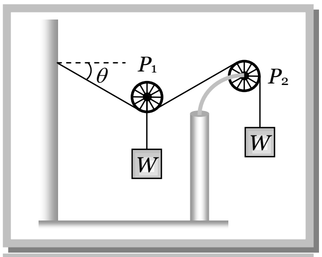
(b) The reading of the balance *B* will be less than 5 *kg*

(c) The reading of the balance *A* will be less than 2 *kg*. and that of *B* will be more than 5 *kg*

(d) The reading of balance A will be 2 *kg*. and that of *B* will be 5 *kg*.

Solution: (c) Due to buoyant force on the aluminium block the reading of spring balance *A* will be less than 2 kg but it increase the reading of balance *B*.

Problem 12. In the following diagram, pulley P_1 is movable and pulley P_2 is fixed. The value of angle θ will be



(a) 60°

(b) 30°

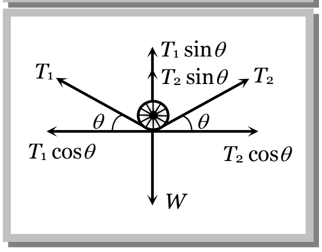
(c) 45°

(d) 15°

Solution: (b) Free body diagram of pulley P_1 is shown in the figure

For horizontal equilibrium $T_1\cos\theta = T_2\cos\theta$ $\therefore$ $T_1 = T_2$

and $T_1 = T_2 = W$

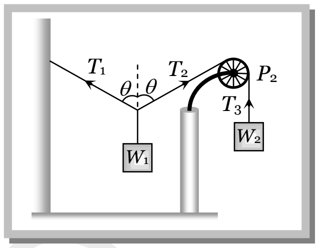


For vertical equilibrium

$T_1 \sin\theta + T_2 \sin\theta = W \Rightarrow W\sin\theta + W\sin\theta = W$

$\therefore \ \sin\theta = \frac{1}{2}$ or $\theta = 30°$

***Problem* 13.** In the following figure, the pulley is massless and frictionless. The relation between T_1, T_2 and T_3 will be



(a) $T_1 = T_2 \neq T_3$

(b) $T_1 \neq T_2 = T_3$

(c) $T_1 \neq T_2 \neq T_3$

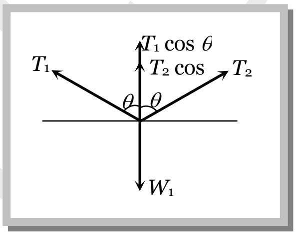
(d) $T_1 = T_2 = T_3$

Solution : (d) Since through a single string whole system is attached so $W_2 = T_3 = T_2 = T_1$

***Problem* 14.** In the above problem (13), the relation between W_1 and W_2 will be

(a) $W_2 = \frac{W_1}{2\cos\theta}$ (b) $2W_1\cos\theta$ (c) $W_2 = W_1$ (d) $W_2 = \frac{2\cos\theta}{W_1}$

Solution : (a) For vertical equilibrium

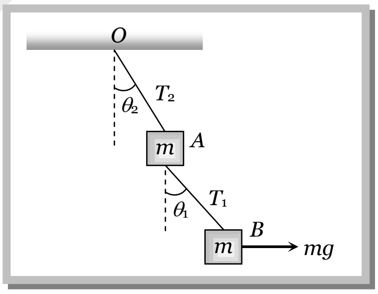


$T_1\cos\theta + T_2\cos\theta = W_1$ $\quad [\text{As } T_1 = T_2 = W_2]$

$2W_2\cos\theta = W_1$

$\therefore W_2 = \frac{W_1}{2\cos\theta}$.

***Problem* 15.** In the following figure the masses of the blocks *A* and *B* are same and each equal to *m*. The tensions in the strings *OA* and *AB* are T_2 and T_1 respectively. The system is in equilibrium with a constant horizontal force mg on *B*. The T_1 is



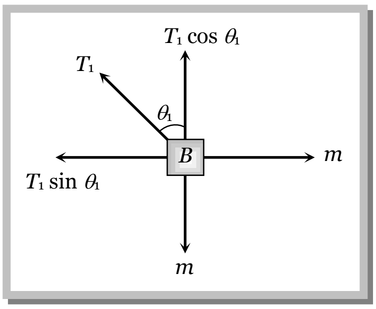
(a) mg

(b) $\sqrt{2}\,mg$

(c) $\sqrt{3}\,mg$

(d) $\sqrt{5}\,mg$

Solution : (b) From the free body diagram of block *B*



$T_1\cos\theta_1 = mg$ ......(i)

$T_1\sin\theta_1 = -mg$ .....(ii)

by squaring and adding $T_1^2\left(\sin^2\theta_1 + \cos^2\theta_1\right) = 2(mg)^2$

$\therefore \ T_1 = \sqrt{2}mg$

***Problem* 16.** In the above problem (15), the angle θ_1 is

(a) 30° (b) 45° (c) 60° (d) $\tan^{-1}\left(\frac{1}{2}\right)$

Solution : (b) From the solution (15) by dividing equation(ii) by equation (i)

$$\frac{T_1\sin\theta_1}{T_1\cos\theta_1} = \frac{mg}{mg}$$

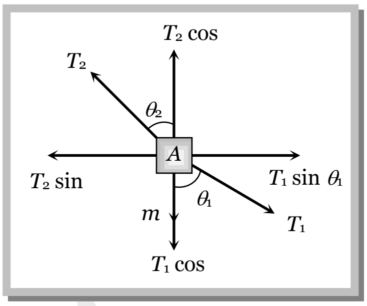
$\therefore \tan\theta_1 = 1$ or $\theta_1 = 45°$

Problem **17.** In the above problem (15) the tension T_2 will be

(a) mg (b) $\sqrt{2}\,mg$ (c) $\sqrt{3}\,mg$ (d) $\sqrt{5}\,mg$

Solution : (d) From the free body diagram of block A

For vertical equilibrium $T_2\cos\theta_2 = mg + T_1\cos\theta_1$



$$T_2\cos\theta_2 = mg + \sqrt{2}mg\cos 45°$$

$$T_2\cos\theta_2 = 2mg \quad .....(i)$$

For horizontal equilibrium $T_2\sin\theta_2 = T_1\sin\theta_1 = \sqrt{2}mg\sin 45°$

$$T_2\sin\theta_2 = mg \quad .....(ii)$$

by squaring and adding (i) and (ii) equilibrium

$$T_2^2 = 5(mg)^2 \text{ or } T_2 = \sqrt{5}mg$$

Problem **18,** In the above problem (15) the angle θ_2 will be

(a) 30° (b) 45° (c) 60° (d) $\tan^{-1}\left(\frac{1}{2}\right)$

Solution : (d) From the solution (17) by dividing equation(ii) by equation (i)

$$\frac{\sin\theta_2}{\cos\theta_2} = \frac{mg}{2mg} \Rightarrow \tan\theta_2 = \frac{1}{2} \qquad \therefore \theta_2 = \tan^{-1}\left[\frac{1}{2}\right]$$

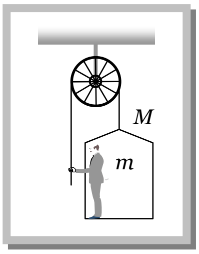
Problem **19.** A man of mass m stands on a crate of mass M. He pulls on a light rope passing over a smooth light pulley. The other end of the rope is attached to the crate. For the system to be in equilibrium, the force exerted by the men on the rope will be

(a) $(M+m)g$

(b) $\frac{1}{2}(M+m)g$

(c) Mg

(d) mg

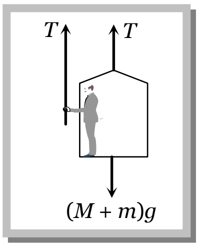


Solution : (b) From the free body diagram of man and crate system:

For vertical equilibrium

$$2T = (M+m)g$$

$$\therefore T = \frac{(M+m)g}{2}$$



Problem **20.** Two forces, with equal magnitude F, act on a body and the magnitude of the resultant force is $\frac{F}{3}$. The angle between the two forces is

(a) $\cos^{-1}\left(-\frac{17}{18}\right)$ (b) $\cos^{-1}\left(-\frac{1}{3}\right)$ (c) $\cos^{-1}\left(\frac{2}{3}\right)$ (d) $\cos^{-1}\left(\frac{8}{9}\right)$

Solution : (a) Resultant of two vectors A and B, which are working at an angle θ, can be given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \qquad [\text{As } A = B = F \text{ and } R = \frac{F}{3}]$$

$$\left(\frac{F}{3}\right)^2 = F^2 + F^2 + 2F^2\cos\theta$$

$$\frac{F^2}{9} = 2F^2 + 2F^2\cos\theta \quad \Rightarrow \frac{-17}{9}F^2 = 2F^2\cos\theta \Rightarrow \cos\theta = \left(\frac{-17}{18}\right) \text{ or } \theta = \cos^{-1}\left(\frac{-17}{18}\right)$$

***Problem* 21.** A cricket ball of mass 150 *gm* is moving with a velocity of 12 *m/s* and is hit by a bat so that the ball is turned back with a velocity of 20 *m/s*. The force of blow acts for 0.01*s* on the ball. The average force exerted by the bat on the ball is

(a) 480 *N* (b) 600 *N* (c) 500 *N* (d) 400 *N*

Solution : (a) $v_1 = -12 m/s$ and $v_2 = +20 m/s$ [because direction is reversed]

$m = 150 gm = 0.15 kg$, $t = 0.01$ sec

Force exerted by the bat on the ball $F = \frac{m[v_2 - v_1]}{t} = \frac{0.15[20-(-12)]}{0.01}$ = 480 *Newton*

4.8 Newton's Third Law

To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

(1) When a body exerts a force on any other body, the second body also exerts an equal and opposite force on the first.

(2) Forces in nature always occurs in pairs. A single isolated force is not possible.

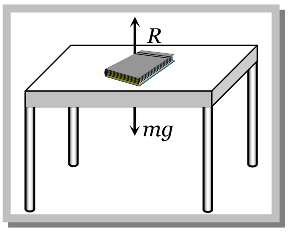
(3) Any agent, applying a force also experiences a force of equal magnitude but in opposite direction. The force applied by the agent is called '*Action*' and the counter force experienced by it is called '*Reaction*'.

(4) Action and reaction never act on the same body. If it were so the total force on a body would have always been zero *i.e.* the body will always remain in equilibrium.

(5) If $\vec{F}_{AB}$ = force exerted on body *A* by body *B* (Action) and $\vec{F}_{BA}$ = force exerted on body *B* by body *A* (Reaction)

Then according to Newton's third law of motion $\vec{F}_{AB} = -\vec{F}_{BA}$

(6) Example : (i) A book lying on a table exerts a force on the table which is equal to the weight of the book. This is the force of action.



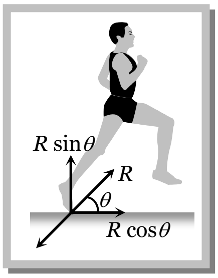
The table supports the book, by exerting an equal force on the book. This is the force of reaction.

As the system is at rest, net force on it is zero. Therefore force of action and reaction must be equal and opposite.

(ii) Swimming is possible due to third law of motion.

(iii) When a gun is fired, the bullet moves forward (action). The gun recoils backward (reaction)

(iv) Rebounding of rubber ball takes place due to third law of motion.



(v) While walking a person presses the ground in the backward direction (action) by his feet. The ground pushes the person in forward direction with an equal force (reaction). The component of reaction in horizontal direction makes the person move forward.

(vi) It is difficult to walk on sand or ice.

(vii) Driving a nail into a wooden block without holding the block is difficult.

Sample problem based on Newton's third law

***Problem* 22.** You are on a frictionless horizontal plane. How can you get off if no horizontal force is exerted by pushing against the surface

(a) By jumping (b) By splitting or sneezing
(c) By rolling your body on the surface (d) By running on the plane

Solution : (b) By doing so we can get push in backward direction in accordance with *Newton's* third law of motion.

4.9 Frame of Reference

(1) A frame in which an observer is situated and makes his observations is known as his 'Frame of reference'.

(2) The reference frame is associated with a co-ordinate system and a clock to measure the position and time of events happening in space. We can describe all the physical quantities like position, velocity, acceleration etc. of an object in this coordinate system.

(3) Frame of reference are of two types : (i) Inertial frame of reference (ii) Non-inertial frame of reference.

(i) **Inertial frame of reference :**

(a) A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.

(b) In inertial frame of reference Newton's laws of motion holds good.

(c) Inertial frame of reference are also called unaccelerated frame of reference or Newtonian or Galilean frame of reference.

(d) Ideally no inertial frame exist in universe. For practical purpose a frame of reference may be considered as inertial if it's acceleration is negligible with respect to the acceleration of the object to be observed.

(e) To measure the acceleration of a falling apple, earth can be considered as an inertial frame.

(f) To observe the motion of planets, earth can not be considered as an inertial frame but for this purpose the sun may be assumed to be an inertial frame.

Example : The lift at rest, lift moving (up or down) with constant velocity, car moving with constant velocity on a straight road.

(ii) **Non inertial frame of reference :**

(a) Accelerated frame of references are called non-inertial frame of reference.

(b) Newton's laws of motion are not applicable in non-inertial frame of reference.

Example : Car moving in uniform circular motion, lift which is moving upward or downward with some acceleration, plane which is taking off.

4.10 Impulse

(1) When a large force works on a body for very small time interval, it is called impulsive force.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. In such case we measure the total effect of force.

(2) Impulse of a force is a measure of total effect of force.

(3) $\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt$.

(4) Impulse is a vector quantity and its direction is same as that of force.

(5) Dimension : $[MLT^{-1}]$

(6) Units : *Newton-second* or $Kg\text{-}m\text{-}s^{-1}$ (S.I.) and *Dyne-second* or $gm\text{-}cm\text{-}s^{-1}$ (C.G.S.)

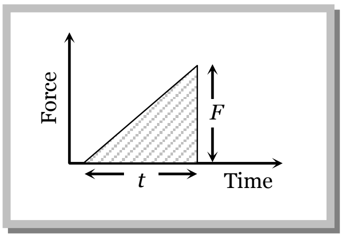
(7) Force-time graph : Impulse is equal to the area under *F-t* curve.

If we plot a graph between force and time, the area under the curve and time axis gives the value of impulse.

$I =$ Area between curve and time axis

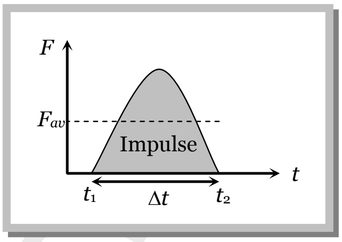
$= \frac{1}{2} \times$ Base $\times$ Height

$= \frac{1}{2} F\, t$



(8) If F_{av} is the average magnitude of the force then

$$I = \int_{t_1}^{t_2} F\,dt = F_{av}\int_{t_1}^{t_2} dt = F_{av}\Delta t$$



(9) From Newton's second law $\vec{F} = \frac{d\vec{p}}{dt}$

or $\int_{t_1}^{t_2} \vec{F}\,dt = \int_{p_1}^{p_2} d\vec{p} \Rightarrow \vec{I} = \vec{p}_2 - \vec{p}_1 = \overrightarrow{\Delta p}$

i.e. The impulse of a force is equal to the change in momentum.

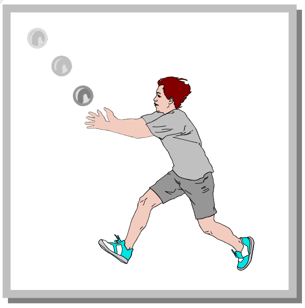
This statement is known as *Impulse momentum theorem*.

(10) *Examples* : Hitting, kicking, catching, jumping, diving, collision *etc*.

In all these cases an impulse acts. $I = \int F\,dt = F_{av}.\Delta t = \Delta p =$ constant

So if time of contact Δt is increased, average force is decreased (or diluted) and vice-versa.

(i) In hitting or kicking a ball we decrease the time of contact so that large force acts on the ball producing greater acceleration.



(ii) In catching a ball a player by drawing his hands backwards increases the time of contact and so, lesser force acts on his hands and his hands are saved from getting hurt.

(iii) In jumping on sand (or water) the time of contact is increased due to yielding of sand or water so force is decreased and we are not injured. However if we jump on cemented floor the motion stops in a very short interval of time resulting in a large force due to which we are seriously injured.

(iv) An athlete is advised to come to stop slowly after finishing a fast race. So that time of stop increases and hence force experienced by him decreases.

(v) China wares are wrapped in straw or paper before packing.

Sample problem based on Impulse

***Problem* 23.** A ball of mass 150*g* moving with an acceleration $20 m/s^2$ is hit by a force, which acts on it for 0.1 *sec*. The impulsive force is **[AFMC 1999]**

(a) 0.5 *N-s* (b) 0.1 *N-s* (c) 0.3 *N-s* (d) 1.2 *N-s*

Solution : (c) Impulsive force $=$ force $\times$ time $= m\,a \times t = 0.15 \times 20 \times 0.1 =$ 0.3 *N-s*

***Problem* 24.** A force of 50 *dynes* is acted on a body of mass 5 *g* which is at rest for an interval of 3 *seconds*, then impulse is

(a) $0.15 \times 10^{-3}\, N\text{-}s$ (b) $0.98 \times 10^{-3}\, N\text{-}s$ (c) $1.5 \times 10^{-3}\, N\text{-}s$ (d) $2.5 \times 10^{-3}\, N\text{-}s$

Solution : (c) Impulse $=$ force $\times$ time $= 50 \times 10^{-5} \times 3 = 1.5 \times 10^{-3}\, N\text{-}s$

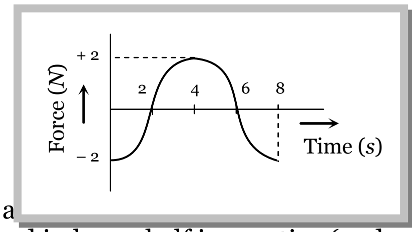
***Problem* 25.** The force-time ($F - t$) curve of a particle executing linear motion is as shown in the figure. The momentum acquired by the particle in time interval from zero to 8 *second* will be

(a) -2 *N-s*

(b) $+4$ *N-s*

(c) 6 *N-s*

(d) Zero



Solution : (d) Momentum acquired by the particle is numerically equal to the a ve and time Axis. For the given diagram area in a upper half is positive and in lower half is negative (and equal to the upper half). So net area is zero. Hence the momentum acquired by the particle will be zero.

4.11 Law of Conservation of Linear Momentum

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

(1) According to this law for a system of particles $\vec{F} = \frac{d\vec{p}}{dt}$

In the absence of external force $\vec{F} = 0$ then $\vec{p} =$ constant

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ... =$ constant.

or $m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + ... =$ constant

This equation shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

(2) Law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.

(3) Conservation of linear momentum is equivalent to Newton's third law of motion.

For a system of two particles in absence of external force by law of conservation of linear momentum.

$\vec{p}_1 + \vec{p}_2 =$ constant. $\quad\therefore\quad$ $m_1\vec{v}_1 + m_2\vec{v}_2 =$ constant.

Differentiating above with respect to time

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0 \Rightarrow m_1\vec{a}_1 + m_2\vec{a}_2 = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$$

$\therefore$ $\vec{F}_2 = -\vec{F}_1$

i.e. for every action there is equal and opposite reaction which is Newton's third law of motion.

(4) Practical applications of the law of conservation of linear momentum

(i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.

(ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

(iii) **Recoiling of a gun :** For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffected.

Let $m_G =$ mass of gun, $m_B =$ mass of bullet,

$v_G =$ velocity of gun, $v_B =$ velocity of bullet

Initial momentum of system = 0

Final momentum of system $= m_G\vec{v}_G + m_B\vec{v}_B$

By the law of conservation linear momentum

$$m_G\vec{v}_G + m_B\vec{v}_B = 0$$

So recoil velocity $\vec{v}_G = -\dfrac{m_B}{m_G}\vec{v}_B$

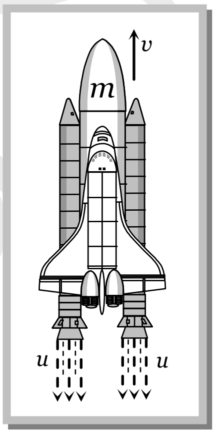
(a) Here negative sign indicates that the velocity of recoil $\vec{v}_G$ is opposite to the velocity of the bullet.

(b) $v_G \propto \dfrac{1}{m_G}$ *i.e.* higher the mass of gun, lesser the velocity of recoil of gun.

(c) While firing the gun must be held tightly to the shoulder, this would save hurting the shoulder because in this condition the body of the shooter and the gun behave as one body. Total mass become large and recoil velocity becomes too small.

$$v_G \propto \frac{1}{m_G + m_{\text{man}}}$$

(iv) **Rocket propulsion :** The initial momentum of the rocket on its launching pad is zero. When it is fired from the launching pad, the exhaust gases rush downward at a high speed and to conserve momentum, the rocket moves upwards.



Let m_0 = initial mass of rocket,

m = mass of rocket at any instant 't' (instantaneous mass)

m_r = residual mass of empty container of the rocket

u = velocity of exhaust gases,

v = velocity of rocket at any instant 't' (instantaneous velocity)

$\dfrac{dm}{dt}$ = rate of change of mass of rocket = rate of fuel consumption

= rate of ejection of the fuel.

(a) Thrust on the rocket : $F = -u\dfrac{dm}{dt} - mg$

Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

$$F = -u\frac{dm}{dt} \text{ (if effect of gravity is neglected)}$$

(b) Acceleration of the rocket : $a = \dfrac{u}{m}\dfrac{dm}{dt} - g$

and if effect of gravity is neglected $a = \dfrac{u}{m}\dfrac{dm}{dt}$

(c) Instantaneous velocity of the rocket : $v = u\log_e\left(\dfrac{m_0}{m}\right) - gt$

and if effect of gravity is neglected $v = u\log_e\left(\dfrac{m_0}{m}\right) = 2.303\,u\log_{10}\left(\dfrac{m_0}{m}\right)$

(d) Burnt out speed of the rocket : $v_b = v_{\max} = u\log_e\left(\dfrac{m_0}{m_r}\right)$

The speed attained by the rocket when the complete fuel gets burnt is called burnt out speed of the rocket. It is the maximum speed acquired by the rocket.

Sample Problem based on conservation of momentum

Problem **26.** A wagon weighing 1000 *kg* is moving with a velocity 50 *km/h* on smooth horizontal rails. A mass of 250 *kg* is dropped into it. The velocity with which it moves now is

(a) 12.5 *km/hour* (b) 20 *km/hour* (c) 40 *km/hour* (d) 50 *km/hour*

Solution : (c) Initially the wagon of mass 1000 *kg* is moving with velocity of 50 *km/h*

So its momentum $= 1000 \times 50 \frac{kg \times km}{h}$

When a mass 250*kg* is dropped into it. New mass of the system $= 1000 + 250 = 1250 kg$

Let v is the velocity of the system.

By the conservation of linear momentum : Initial momentum = Final momentum $\Rightarrow$ $1000 \times 50 = 1250 \times v$

$\therefore$ $v = \frac{50,000}{1250} = 40 km/h.$

***Problem* 27.** The kinetic energy of two masses m_1 and m_2 are equal. The ratio of their linear momentum will be **[RPET 1988]**

(a) m_1/m_2 (b) m_2/m_1 (c) $\sqrt{m_1/m_2}$ (d) $\sqrt{m_2/m_1}$

Solution : (c) Relation between linear momentum (P), man (m) and kinetic energy (E)

$P = \sqrt{2mE}$ $\Rightarrow P \propto \sqrt{m}$ [as E is constant] $\therefore \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$

***Problem* 28.** Which of the following has the maximum momentum

(a) A 100 *kg* vehicle moving at 0.02 ms^{-1} (b) A 4 *g* weight moving at 10000 cms^{-1}

(c) A 200 *g* weight moving with kinetic energy 10^{-6} *J* (d) A 20 *g* weight after falling 1 *kilometre*

Solution : (d) Momentum of body for given options are :

(a) $P = mv = 100 \times 0.02 = 2 kgm/\sec$ (b) $P = mv = 4 \times 10^{-3} \times 100 = 0.4 kgm/\sec$

(c) $P = \sqrt{2mE} = \sqrt{2 \times 0.2 \times 10^{-6}} = 6.3 \times 10^{-4} kgm/\sec$

(d) $P = m\sqrt{2gh} = 20 \times 10^{-3} \times \sqrt{2 \times 10 \times 10^{3}} = 2.82 kgm/\sec$

So for option (d) momentum is maximum.

***Problem* 29.** A rocket with a lift-off mass 3.5×10^4 *kg* is blasted upwards with an initial acceleration of $10 m/s^2$. Then the initial thrust of the blast is

(a) $1.75 \times 10^5 N$ (b) $3.5 \times 10^5 N$ (c) $7.0 \times 10^5 N$ (d) $14.0 \times 10^5 N$

Solution : (c) Initial thrust on the rocket $F = m(g + a) = 3.5 \times 10^4 (10 + 10) = 7.0 \times 10^5 N$

***Problem* 30.** In a rocket of mass 1000 *kg* fuel is consumed at a rate of 40 *kg/s*. The velocity of the gases ejected from the rocket is $5 \times 10^4 m/s$. The thrust on the rocket is

(a) $2 \times 10^3 N$ (b) $5 \times 10^4 N$ (c) $2 \times 10^6 N$ (d) $2 \times 10^9 N$

Solution : (c) Thrust on the rocket $F = \frac{udm}{dt} = 5 \times 10^4 (40) = 2 \times 10^6 N$

***Problem* 31.** If the force on a rocket moving with a velocity of 300 *m/s* is 210 *N*, then the rate of combustion of the fuel is

(a) 0.7 *kg/s* (b) 1.4 *kg/s* (c) 0.07 *kg/s* (d) 10.7 *kg/s*

Solution : (a) Force on the rocket $= \frac{udm}{dt}$ $\therefore$ Rate of combustion of fuel $\left(\frac{dm}{dt}\right) = \frac{F}{u} = \frac{210}{300} = 0.7 kg/s.$

***Problem* 32.** A rocket has a mass of 100 *kg*. 90% of this is fuel. It ejects fuel vapours at the rate of 1 *kg/sec* with a velocity of 500 *m/sec* relative to the rocket. It is supposed that the rocket is outside the gravitational field. The initial upthrust on the rocket when it just starts moving upwards is **[NCERT 1978]**

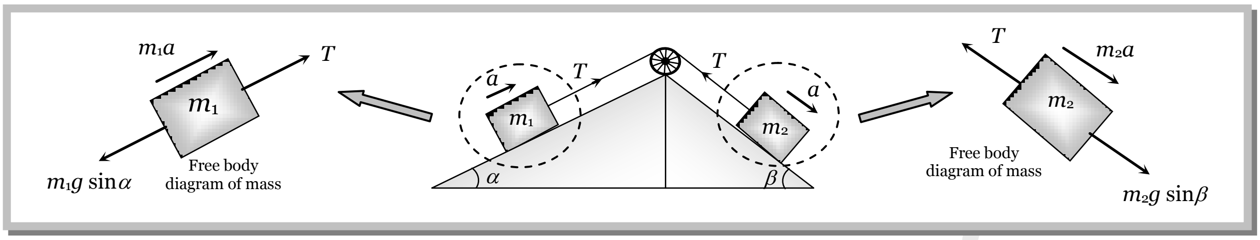
(a) Zero (b) 500 *N* (c) 1000 *N* (d) 2000 *N*

Solution : (b) Up thrust force $F = u\left(\frac{dm}{dt}\right) = 500 \times 1 = 500\ N$

4.12 Free Body Diagram

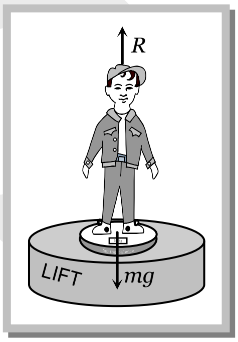
In this diagram the object of interest is isolated from its surroundings and the interactions between the object and the surroundings are represented in terms of forces.

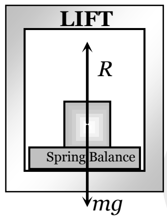
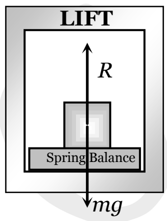
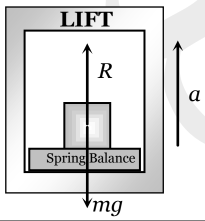
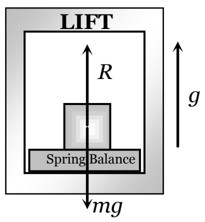
Example :

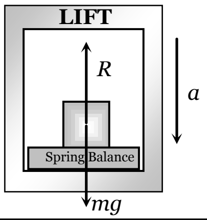
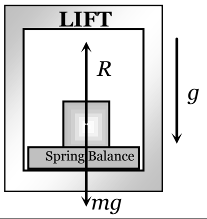
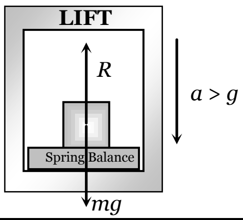


4.13 Apparent Weight of a Body in a Lift

When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg. This acts on a weighing machine which offers a reaction R given by the reading of weighing machine. This reaction exerted by the surface of contact on the body is the *apparent weight* of the body.



Condition	Figure	Velocity	Acceleration	Reaction	Conclusion
Lift is at rest		$v = 0$	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift moving upward or downward with constant velocity		v = constant	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift accelerating upward at the rate of 'a'		v = variable	$a < g$	$R - mg = ma$ $\therefore R = m(g + a)$	Apparent weight > Actual weight
Lift accelerating upward at the rate of 'g'		v = variable	$a = g$	$R - mg = mg$ $R = 2mg$	Apparent weight = 2 Actual weight

Lift accelerating downward at the rate of 'a'		v = variable	$a < g$	$mg - R = ma$ $\therefore R = m(g - a)$	Apparent weight < Actual weight
Lift accelerating downward at the rate of 'g'		v = variable	$a = g$	$mg - R = mg$ $R = 0$	Apparent weight = Zero (weightlessness)
Lift accelerating downward at the rate of a(>g)		v = variable	$a > g$	$mg - R = ma$ $R = mg - ma$ $R = -ve$	Apparent weight negative means the body will rise from the floor of the lift and stick to the ceiling of the lift.

Sample problems based on lift

***Problem* 33.** A man weighs $80kg$. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of $5m/s^2$. What would be the reading on the scale. $(g = 10m/s^2)$

(a) 400 N (b) 800 N (c) 1200 N (d) Zero

Solution : (c) Reading of weighing scale $= m(g+a) = 80\,(10+5) = 1200\,N$

***Problem* 34.** A body of mass 2 kg is hung on spring balance mounted vertically in a lift. If the lift descends with an acceleration equal to the acceleration due to gravity 'g', the reading on the spring balance will be

(a) 2 kg (b) $(4 \times g)$ kg (c) $(2 \times g)$ kg (d) Zero

Solution : (d) $R = m(g-a) = (g-g) = 0$ [because the lift is moving downward with $a = g$]

***Problem* 35.** In the above problem, if the lift moves up with a constant velocity of 2 m/sec, the reading on the balance will

Be (a) 2 kg (b) 4 kg (c) Zero (d) 1 kg

Solution : (a) $R = mg = 2g$ *Newton* or $2\,kg$ [because the lift is moving with the zero acceleration]

***Problem* 36.** If the lift in problem, moves up with an acceleration equal to the acceleration due to gravity, the reading on the spring balance will be

(a) 2 kg (b) $(2 \times g)$ kg (c) $(4 \times g)$ kg (d) 4 kg

Solution : (d) $R = m(g+a) = m\,(g+g)$ [because the lift is moving upward with $a = g$]

$= 2mg$ $R = 2 \times 2g\,N = 4g\,N$ or $4\,kg$

***Problem* 37.** A man is standing on a weighing machine placed in a lift, when stationary, his weight is recorded as 40 kg. If the lift is accelerated upwards with an acceleration of $2\,m/s^2$, then the weight recorded in the machine will be $(g = 10\,m/s^2)$ **[MP PMT 1994]**

(a) 32 kg (b) 40 kg (c) 42 kg (d) 48 kg

Solution : (d) $R = m(g+a) = 40(10+2) = 480\ N$ or $48 kg$

***Problem* 38.** An elevator weighing 6000 *kg* is pulled upward by a cable with an acceleration of $5ms^{-2}$. Taking *g* to be $10ms^{-2}$, then the tension in the cable is **[Manipal MEE 1995]**

(a) 6000 *N* (b) 9000 *N* (c) 60000 *N* (d) 90000 *N*

Solution : (d) $T = m(g+a) = 6000(10+5)$ $T = 90{,}000\ N$

***Problem* 39.** The ratio of the weight of a man in a stationary lift and when it is moving downward with uniform acceleration '*a*' is 3 : 2. The value of '*a*' is (*g*- Acceleration due to gravity on the earth)

(a) $\frac{3}{2}g$ (b) $\frac{g}{3}$ (c) $\frac{2}{3}g$ (d) g

Solution : (b) $\frac{\text{weight of a man in stationary lift}}{\text{weight of a man in downward moving lift}} = \frac{mg}{m(g-a)} = \frac{3}{2}$

$\therefore \frac{g}{g-a} = \frac{3}{2} \Rightarrow 2g = 3g - 3a$ or $a = \frac{g}{3}$

***Problem* 40.** A 60 *kg* man stands on a spring scale in the lift. At some instant he finds, scale reading has changed from 60 *kg* to 50 *kg* for a while and then comes back to the original mark. What should we conclude

(a) The lift was in constant motion upwards

(b) The lift was in constant motion downwards

(c) The lift while in constant motion upwards, is stopped suddenly

(d) The lift while in constant motion downwards, is suddenly stopped

Solution : (c) For retarding motion of a lift $R = m(g+a)$ for downward motion

$R = m(g-a)$ for upward motion

Since the weight of the body decrease for a while and then comes back to original value it means the lift was moving upward and stops suddenly.

Note : ❑ Generally we use $R = m(g+a)$ for upward motion

$R = m(g-a)$ for downward motion

here *a*= acceleration, but for the given problem *a*= retardation

***Problem* 41.** A bird is sitting in a large closed cage which is placed on a spring balance. It records a weight placed on a spring balance. It records a weight of 25 *N*. The bird (mass = 0.5*kg*) flies upward in the cage with an acceleration of $2m/s^2$. The spring balance will now record a weight of **[MP PMT 1999]**

(a) 24 *N* (b) 25 *N* (c) 26 *N* (d) 27 *N*

Solution : (b) Since the cage is closed and we can treat bird cage and air as a closed (Isolated) system. In this condition the force applied by the bird on the cage is an internal force due to this reading of spring balance will not change.

***Problem* 42.** A bird is sitting in a wire cage hanging from the spring balance. Let the reading of the spring balance be W_1. If the bird flies about inside the cage, the reading of the spring balance is W_2. Which of the following is true

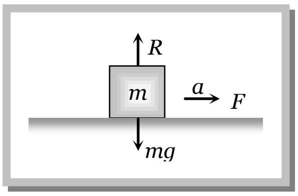
(a) $W_1 = W_2$ (b) $W_1 > W_2$

(c) $W_1 < W_2$ (d) Nothing definite can be predicted

Solution : (b) In this problem the cage is wire-cage the momentum of the system will not be conserved and due to this the weight of the system will be lesser when the bird is flying as compared to the weight of the same system when bird is resting is $W_2 < W_1$.

4.14 Acceleration of Block on Horizontal Smooth Surface

(1) **When a pull is horizontal**

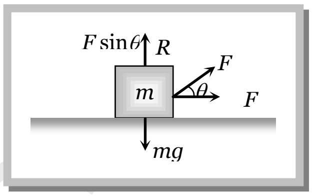


$R = mg$

and $F = ma$

$\therefore$ $a = F/m$

(2) **When a pull is acting at an angle (θ) to the horizontal (upward)**



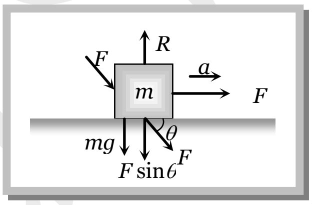
$R + F\sin\theta = mg$

$\Rightarrow$ $R = mg - F\sin\theta$

and $F\cos\theta = ma$

$$\therefore \quad a = \frac{F\cos\theta}{m}$$

(3) **When a push is acting at an angle (θ) to the horizontal (downward)**



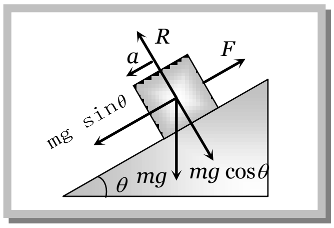
$R = mg + F\sin\theta$

and $F\cos\theta = ma$

$$a = \frac{F\cos\theta}{m}$$

4.15 Acceleration of Block on Smooth Inclined Plane

(1) **When inclined plane is at rest**



Normal reaction $R = mg\cos\theta$

Force along a inclined plane

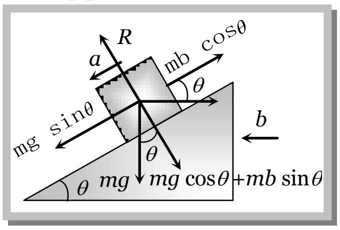
$F = mg\sin\theta$

$ma = mg\sin\theta$

$\therefore$ $a = g\sin\theta$

(2) **When a inclined plane given a horizontal acceleration '*b*'**

Since the body lies in an accelerating frame, an inertial force (mb) acts on it in the opposite direction.



Normal reaction $R = mg\cos\theta + mb\sin\theta$

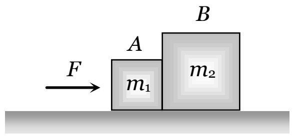
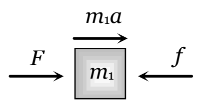
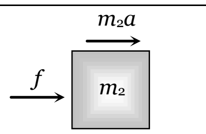
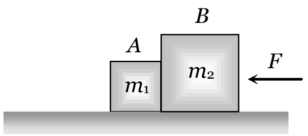
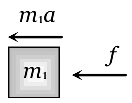
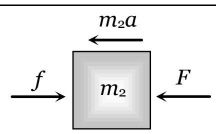
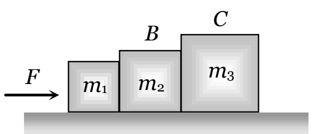
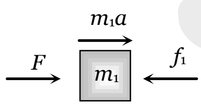
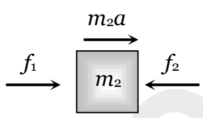
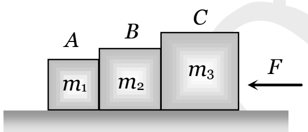
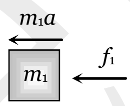
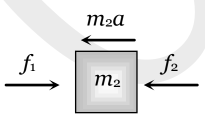
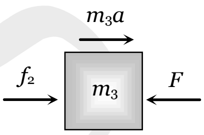
and $ma = mg\sin\theta - mb\cos\theta$

$\therefore$ $a = g\sin\theta - b\cos\theta$

Note : ❑The condition for the body to be at rest relative to the inclined plane : $a = g\sin\theta - b\cos\theta = 0$

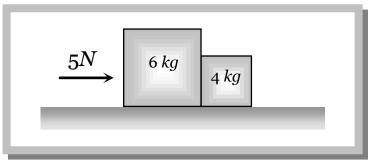
$\therefore$ $b = g\tan\theta$

4.16 Motion of Blocks in Contact

Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$
		$f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_2 - f_1 = m_2 a$	$f_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - f_2 = m_3 a$	$f_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

Sample problems based on motion of blocks in contact

***Problem* 43.** Two blocks of mass 4 *kg* and 6 *kg* are placed in contact with each other on a frictionless horizontal surface. If we apply a push of 5 *N* on the heavier mass, the force on the lighter mass will be



(a) 5 N

(b) 4 N

(c) 2 N

(d) None of the above

Solution : (c) Let $m_1 = 6kg, m_2 = 4kg$ and $F = 5N$ (given)

Force on the lighter mass = $\dfrac{m_2 \times F}{m_1 + m_2} = \dfrac{4 \times 5}{6 + 4} = 2N$

***Problem* 44.** In the above problem, if a push of 5 N is applied on the lighter mass, the force exerted by the lighter mass on the heavier mass will be

(a) 5 N (b) 4 N (c) 2 N (d) None of the above

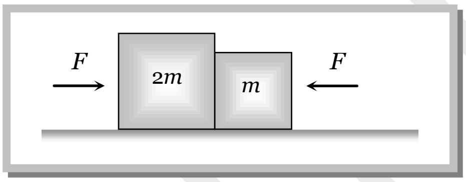
Solution : (d) Force on the heavier mass $= \dfrac{m_1 F}{m_1 + m_2} = \dfrac{6 \times 5}{6 + 4} = 3N$

***Problem* 45.** In the above problem, the acceleration of the lighter mass will be

(a) $0.5\, ms^{-2}$ (b) $\dfrac{5}{4}\, ms^{-2}$ (c) $\dfrac{5}{6}\, ms^{-2}$ (d) None of the above

Solution : (a) Acceleration $= \dfrac{\text{Net force on the system}}{\text{Total mass of the system}} = \dfrac{5}{10} = 0.5\, m/s^2$

***Problem* 46.** Two blocks are in contact on a frictionless table one has a mass m and the other 2 m as shown in figure. Force F is applied on mass $2m$ then system moves towards right. Now the same force F is applied on m. The ratio of force of contact between the two blocks will be in the two cases respectively.



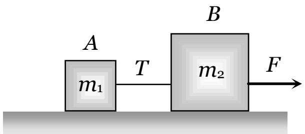
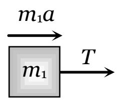
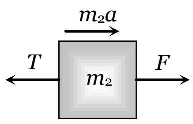
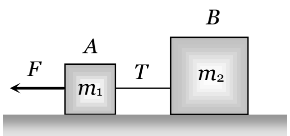
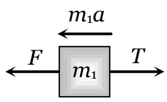
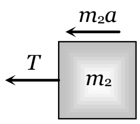
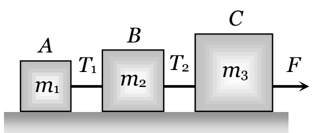
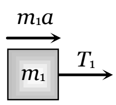
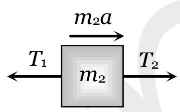
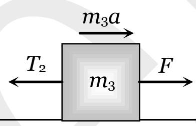
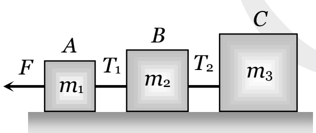
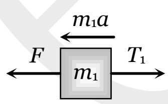
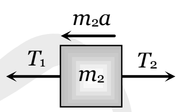
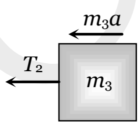
(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

Solution : (b) When the force is applied on mass $2m$ contact force $f_1 = \dfrac{m}{m + 2m} g = \dfrac{g}{3}$

When the force is applied on mass m contact force $f_2 = \dfrac{2m}{m + 2m} g = \dfrac{2}{3} g$

Ratio of contact forces $\dfrac{f_1}{f_2} = \dfrac{1}{2}$

4.17 Motion of Blocks Connected by Mass Less String

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$
		$F - T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_1 - T_2 = m_2 a$	$T_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$T_2 = m_3 a$	$T_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

Sample problems based on motion of blocks connected by mass less string

***Problem* 47.** A monkey of mass 20 *kg* is holding a vertical rope. The rope will not break when a mass of 25 *kg* is suspended from it but will break if the mass exceeds 25 *kg*. What is the maximum acceleration with which the monkey can climb up along the rope $(g = 10 m/s^2)$

(a) $10\, m/s^2$ (b) $25\, m/s^2$ (c) $2.5 m/s^2$ (d) $5 m/s^2$

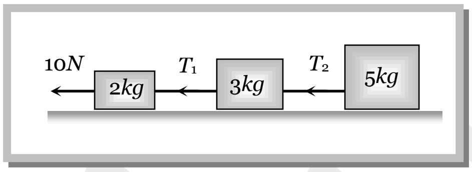
Solution : (c) Maximum tension that string can bear (T_{max}) = $25 \times g\, N = 250\, N$

Tension in rope when the monkey climb up $T = m(g + a)$

For limiting condition $T = T_{\max} \Rightarrow m(g+a) = 250 \Rightarrow 20\,(10 + a) = 250 \quad \therefore\ a = 2.5\, m/s^2$

***Problem* 48.** Three blocks of masses 2 *kg*, 3 *kg* and 5 *kg* are connected to each other with light string and are then placed on a frictionless surface as shown in the figure. The system is pulled by a force $F = 10N$, then tension $T_1 =$

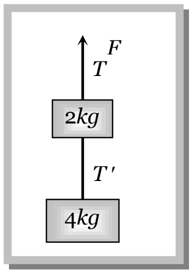
[Orissa JEE 2002]



(a) 1*N* (b) 5 *N* (c) 8 *N* (d) 10 *N*

Solution : (c) By comparing the above problem with general expression. $T_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3} = \dfrac{(3+5)10}{2+3+5} = 8\ Newton$

***Problem* 49.** Two blocks are connected by a string as shown in the diagram. The upper block is hung by another string. A force *F* applied on the upper string produces an acceleration of $2m/s^2$ in the upward direction in both the blocks. If *T* and T' be the tensions in the two parts of the string, then **[AMU (Engg.) 2000]**



(a) $T = 70.8N$ and $T' = 47.2N$

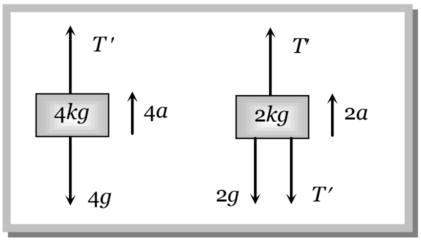
(b) $T = 58.8N$ and $T' = 47.2N$

(c) $T = 70.8N$ and $T' = 58.8N$

(d) $T = 70.8N$ and $T' = 0$

Solution : (a) From F.B.D. of mass 4 *kg* $4a = T' - 4g$ .....(i)

From F.B.D. of mass 2 *kg* $2a = T - T' - 2g$ .....(ii)



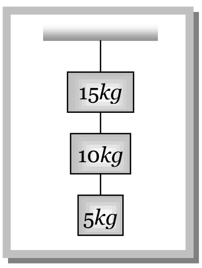
For total system upward force

$F = T = (2+4)(g+a) = 6\,(18 + 2)N$ = 70.8 N

by substituting the value of T in equation (i) and (ii)

and solving we get $T' = 47.2N$

***Problem* 50.** Three masses of 15 *kg*. 10 *kg* and 5 *kg* are suspended vertically as shown in the fig. If the string attached to the support breaks and the system falls freely, what will be the tension in the string between 10 *kg* and 5 *kg* masses. Take $g = 10\, ms^{-2}$. It is assumed that the string remains tight during the motion



(a) 300 *N* (b) 250 *N* (c) 50 *N* (d) Zero

Solution : (d) In the condition of free fall, tension becomes zero.

***Problem* 51.** A sphere is accelerated upwards with the help of a cord whose breaking strength is five times its weight. The maximum acceleration with which the sphere can move up without cord breaking is

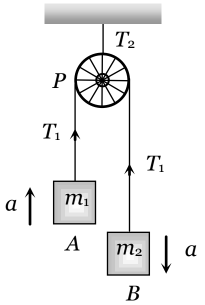
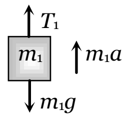
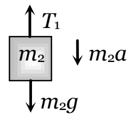
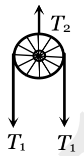
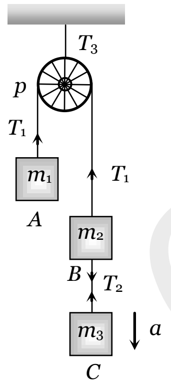
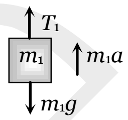
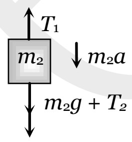
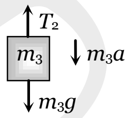
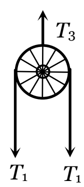
(a) $4g$ (b) $3g$ (c) $2g$ (d) g

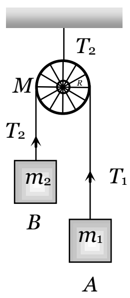
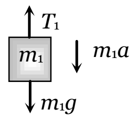
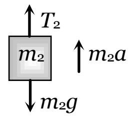
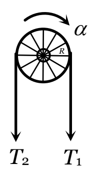
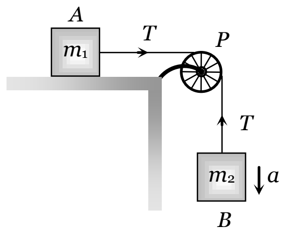
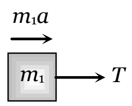
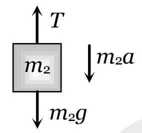
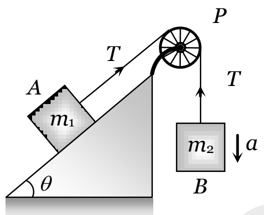
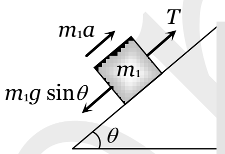
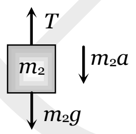
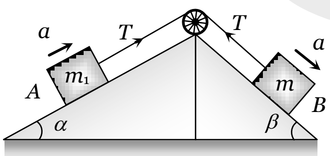
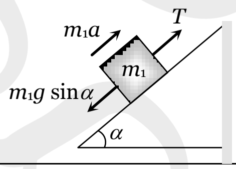
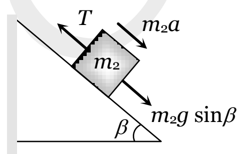
Solution : (a) Tension in the cord = $m(g + a)$ and breaking strength = 5 mg

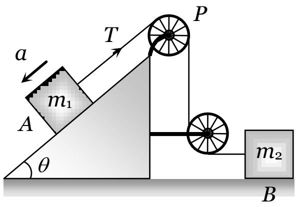
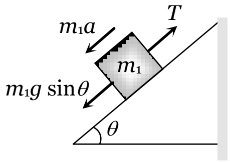
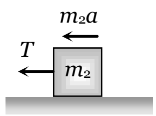
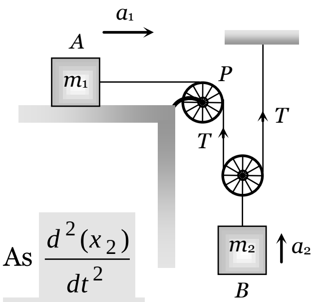
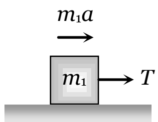
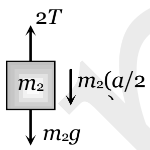
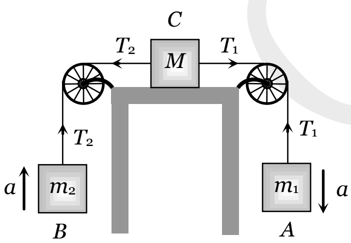
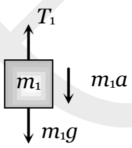
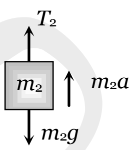
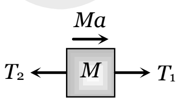
For critical condition $m(g+a)=5mg \Rightarrow a=4g$

This is the maximum acceleration with which the sphere can move up with cord breaking.

4.18 Motion of Connected Block Over a Pulley

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1a = T_1 - m_1g$	$T_1 = \frac{2m_1m_2}{m_1+m_2}g$
		$m_2a = m_2g - T_1$	$T_2 = \frac{4m_1m_2}{m_1+m_2}g$
		$T_2 = 2T_1$	$a = \left[\frac{m_2-m_1}{m_1+m_2}\right]g$
		$m_1a = T_1 - m_1g$	$T_1 = \frac{2m_1[m_2+m_3]}{m_1+m_2+m_3}g$
		$m_2a = m_2g + T_2 - T_1$	$T_2 = \frac{2m_1m_3}{m_1+m_2+m_3}g$
		$m_3a = m_3g - T_2$	$T_3 = \frac{4m_1[m_2+m_3]}{m_1+m_2+m_3}g$
		$T_3 = 2T_1$	$a = \frac{[(m_2+m_3)-m_1]g}{m_1+m_2+m_3}$

Condition	Free body diagram	Equation	Tension and acceleration
When pulley have a finite mass M and radius R then tension in two segments of string are different 		$m_1 a = m_1 g - T_1$	$a = \dfrac{m_1 - m_2}{m_1 + m_2 + \dfrac{M}{2}}$
		$m_2 a = T_2 - m_2 g$	$T_1 = \dfrac{m_1\left[2m_2 + \dfrac{M}{2}\right]}{m_1 + m_2 + \dfrac{M}{2}} g$
		Torque $= (T_1 - T_2)R = I\alpha$ $(T_1 - T_2)R = I\dfrac{a}{R}$ $(T_1 - T_2)R = \dfrac{1}{2}MR^2\dfrac{a}{R}$ $T_1 - T_2 = \dfrac{Ma}{2}$	$T_2 = \dfrac{m_2\left[2m_1 + \dfrac{M}{2}\right]}{m_1 + m_2 + \dfrac{M}{2}} g$
		$T = m_1 a$	$a = \dfrac{m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T$	$T = \dfrac{m_1 m_2}{m_1 + m_2} g$
		$m_1 a = T - m_1 g \sin\theta$	$a = \left[\dfrac{m_2 - m_1 \sin\theta}{m_1 + m_2}\right] g$
		$m_2 a = m_2 g - T$	$T = \dfrac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g$
		$T - m_1 g \sin\alpha = m_1 a$	$a = \dfrac{(m_2 \sin\beta - m_1 \sin\alpha)}{m_1 + m_2} g$
		$m_2 a = m_2 g \sin\beta - T$	$T = \dfrac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1 + m_2} g$

Condition	**Free body diagram**	**Equation**	**Tension and acceleration**
		$m_1 g \sin\theta - T = m_1 a$	$a = \frac{m_1 g \sin\theta}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{2m_1 m_2}{4m_1 + m_2} g$
 As $\frac{d^2(x_2)}{dt^2} = \frac{1}{2}\frac{d^2(x_1)}{dt^2}$ $\therefore a_2 = \frac{a_1}{2}$ $a_1 =$ acceleration of block A $a_2 =$ acceleration of block B		$T = m_1 a$	$a_1 = a = \frac{2m_2 g}{4m_1 + m_2}$
		$m_2 \frac{a}{2} = m_2 g - 2T$	$a_2 = \frac{m_2 g}{4m_1 + m_2}$ $T = \frac{2m_1 m_2 g}{4m_1 + m_2}$
		$m_1 a = m_1 g - T_1$	$a = \frac{(m_1 - m_2)}{[m_1 + m_2 + M]} g$
		$m_2 a = T_2 - m_2 g$	$T_1 = \frac{m_1(2m_2 + M)}{[m_1 + m_2 + M]} g$
		$T_1 - T_2 = Ma$	$T_2 = \frac{m_2(2m_1 + M)}{[m_1 + m_2 + M]} g$

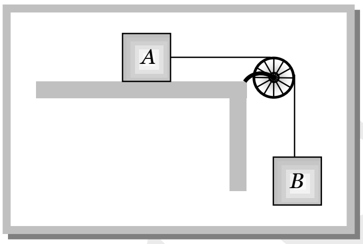
Sample problems based on motion of blocks over pulley

Problem 52. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$ then the ratio of the masses is **[AIEEE 2002]**

(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3

Solution : (b) $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g = \frac{g}{8}$; by solving $\frac{m_2}{m_1} = 9/7$

Problem 53. A block *A* of mass 7 *kg* is placed on a frictionless table. A thread tied to it passes over a frictionless pulley and carries a body *B* of mass 3 *kg* at the other end. The acceleration of the system is (given $g = 10\, ms^{-2}$) **[Kerala (Engg.) 2000]**



(a) $100\, ms^{-2}$

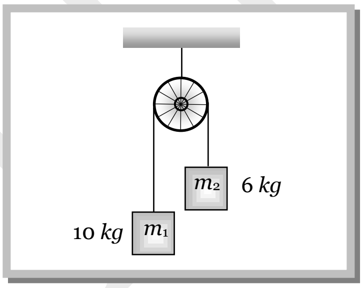
(b) $3 ms^{-2}$

(c) $10 ms^{-2}$

(d) $30 ms^{-2}$

Solution : (b) $a = \left(\frac{m_2}{m_1 + m_2}\right) g = \left(\frac{3}{7+3}\right) 10 = 3 m/s^2$

Problem 54. Two masses m_1 and m_2 are attached to a string which passes over a frictionless smooth pulley. When $m_1 = 10\, kg$, $m_2 = 6\, kg$, the acceleration of masses is **[Orissa JEE 2002]**



(a) $20\, m/s^2$

(b) $5 m/s^2$

(c) $2.5\, m/s^2$

(d) $10\, m/s^2$

Solution : (c) $a = \frac{m_1 - m_2}{m_1 + m_2} g = \left(\frac{10-6}{10+6}\right) 10 = 2.5\, m/s^2$

Problem 55. Two weights W_1 and W_2 are suspended from the ends of a light string passing over a smooth fixed pulley. If the pulley is pulled up with an acceleration *g*, the tension in the string will be

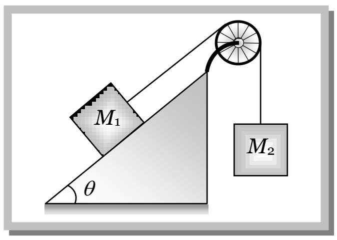
(a) $\frac{4W_1W_2}{W_1 + W_2}$ (b) $\frac{2W_1W_2}{W_1 + W_2}$ (c) $\frac{W_1W_2}{W_1 + W_2}$ (d) $\frac{W_1W_2}{2(W_1 + W_2)}$

Solution : (a) When the system is at rest tension in string $T = \frac{2m_1m_2}{(m_1 + m_2)} g$

If the system moves upward with acceleration *g* then $T = \frac{2m_1m_2}{m_1 + m_2}(g + g) = \frac{4m_1m_2}{m_1 + m_2} g$ or $T = \frac{4w_1w_2}{w_1 + w_2}$

Problem 56. Two masses M_1 and M_2 are attached to the ends of a string which passes over a pulley attached to the top of an inclined plane. The angle of inclination of the plane in θ. Take $g = 10\ ms^{-2}$.

If $M_1 = 10\ kg$, $M_2 = 5\ kg$, $\theta = 30^o$, what is the acceleration of mass M_2



(a) $10 ms^{-2}$ (b) $5 ms^{-2}$

(c) $\frac{2}{3} ms^{-2}$ (d) Zero

Solution : (d) Acceleration $= \frac{m_2 - m_1 \sin\theta}{m_1 + m_2} g = \frac{5 - 10 . \sin 30}{5 + 10} g = \frac{5-5}{5+10} g = 0$

Problem 57. In the above problem, what is the tension in the string

(a) 100 *N* (b) 50 *N* (c) 25 *N* (d) Zero

Solution : (b) $T = \frac{m_1 m_2(1+\sin\theta)}{m_1+m_2} g = \frac{10\times 5(1+\sin 30).10}{10+5} = 50N$

Problem 58. In the above problem, given that $M_2 = 2M_1$ and M_2 moves vertically downwards with acceleration a. If the position of the masses are reversed the acceleration of M_2 down the inclined plane will be

(a) $2a$ (b) a (c) $a/2$ (d) None of the above

Solution : (d) If $m_2 = 2m_1$, then m_2 moves vertically downward with acceleration

$$a = \frac{m_2 - m_1 \sin\theta}{m_1+m_2} g = \frac{2m_1 - m_1 \sin 30}{m_1 + 2m_1} g = g/2$$

If the position of masses are reversed then m_2 moves downward with acceleration

$$a' = \frac{m_2 \sin\theta - m_1}{m_1+m_2} g = \frac{2m_1 \sin 30 - m_1}{m_1+2m_1} . g = 0 \qquad [\text{As } m_2 = 2m_1]$$

i.e. the m_2 will not move.

Problem 59. In the above problem, given that $M_2 = 2M_1$ and the tension in the string is T. If the positions of the masses are reversed, the tension in the string will be

(a) $4T$ (b) $1T$ (c) T (d) $T/2$

Solution : (c) Tension in the string $T = \frac{m_1 m_2(1+\sin\theta)}{m_1+m_2} g$

If the position of the masses are reversed then there will be no effect on tension.

Problem 60. In the above problem, given that $M_1 = M_2$ and $\theta = 30^o$. What will be the acceleration of the system

(a) $10ms^{-2}$ (b) $5ms^{-2}$ (c) $2.5ms^{-2}$ (d) Zero

Solution : (c) $a = \frac{m_2 - m_1\sin\theta}{m_1+m_2} g = \frac{1-\sin 30}{2} g = \frac{g}{4} = 2.5\, m/s^2$ [As $m_1 = m_2$]

Problem 61. In the above problem, given that $M_1 = M_2 = 5\,kg$ and $\theta = 30^o$. What is tension in the string

(a) $37.5\,N$ (b) $25\,N$ (c) $12.5\,N$ (d) Zero

Solution : (a) $T = \frac{m_1 m_2(1+\sin\theta)}{m_1+m_2} g = \frac{5\times 5(1+\sin 30)}{5+5} \times 10 = 37.5\,N$

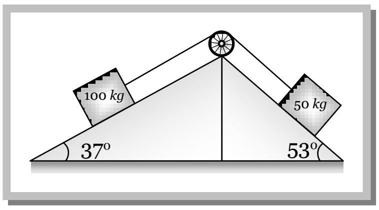
Problem 62. Two blocks are attached to the two ends of a string passing over a smooth pulley as shown in the figure. The acceleration of the block will be (in m/s^2) ($\sin 37^o = 0.60$, $\sin 53^o = 0.80$)

(a) 0.33

(b) 0.133

(c) 1

(d) 0.066

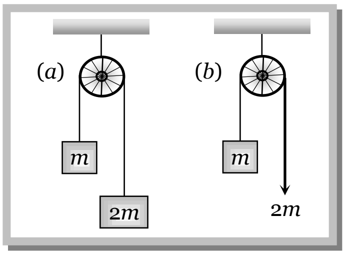


Solution : (b) $a = \frac{m_2 \sin\beta - m_1 \sin\alpha}{m_1+m_2} g = \frac{50 \sin 53° - 100 \sin 37°}{100+50} g = -0.133\, m/s^2$

Problem 63. The two pulley arrangements shown in the figure are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In (b). m is lifted up by pulling the other end of the rope with a constant downward force of $2mg$. The ratio of accelerations in two cases will be

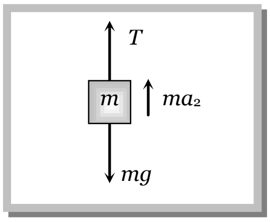
(a) 1 : 1 (b) 1 : 2

(c) 1 : 3 (d) 1 : 4



Solution : (c) For first case $a_1 = \frac{m_2 - m_1}{m_1+m_2} g = \frac{2m-m}{m+2m} = \frac{g}{3}$ .....(i)

For second case
from free body diagram of m



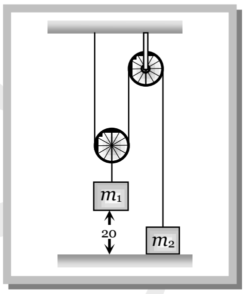
$ma_2 = T - mg$

$ma_2 = 2mg - mg$ [As $T = 2mg$]

$a_2 = g$ ....(ii)

From (i) and (ii) $\frac{a_1}{a_2} = \frac{g/3}{g} = 1/3$

***Problem* 64.** In the adjoining figure $m_1 = 4m_2$. The pulleys are smooth and light. At time $t = 0$, the system is at rest. If the system is released and if the acceleration of mass m_1 is a, then the acceleration of m_2 will be



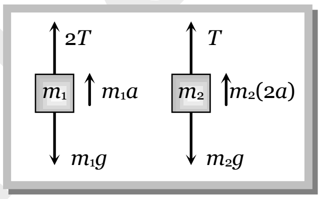
(a) g
(b) a
(c) $\frac{a}{2}$
(d) $2a$

Solution : (d) Since the mass m_2 travels double distance in comparison to mass m_1 therefore its acceleration will be double *i.e.* $2a$

***Problem* 65.** In the above problem (64), the value of a will be

(a) g (b) $\frac{g}{2}$ (c) $\frac{g}{4}$ (d) $\frac{g}{8}$

Solution : (c) By drawing the FBD of m_1 and m_2



$m_1 a = m_1 g - 2T$ .....(i)

$m_2(2a) = T - m_2 g$ .....(ii)

by solving these equation $a = g/4$

***Problem* 66.** In the above problem, the tension T in the string will be

(a) $m_2 g$ (b) $\frac{m_2 g}{2}$ (c) $\frac{2}{3} m_2 g$ (d) $\frac{3}{2} m_2 g$

Solution : (d) From the solution (65) by solving equation

$$T = \frac{3}{2} m_2 g$$

***Problem* 67.** In the above problem, the time taken by m_1 in coming to rest position will be

(a) 0.2 *s* (b) 0.4 *s* (c) 0.6 *s* (d) 0.8 *s*

Solution : (b) Time taken by mass m_2 to cover the distance 20 *cm*

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \times 0.2}{g/4}} = \sqrt{\frac{2 \times 0.2}{2.5}} = 0.4 \text{ sec}$$

***Problem* 68.** In the above problem, the distance covered by m_2 in 0.4 *s* will be

(a) 40 *cm* (b) 20 *cm* (c) 10 *cm* (d) 80 *cm*

Solution : (a) Since the m_2 mass cover double distance therefore $S = 2 \times 20 = 40$ *cm*

***Problem* 69.** In the above problem, the velocity acquired by m_2 in 0.4 second will be

(a) 100 *cm/s* (b) 200 *cm/s* (c) 300 *cm/s* (d) 400 *cm/s*

Solution : (b) Velocity acquired by mass m_2 in 0.4 sec

From $v = u + at$ [As $a = g/2 = \frac{10}{2} = 5\, m/s^2$]

$v = 0 + 5 \times 0.4 = 2 m/s = 200\ cm/sec$.

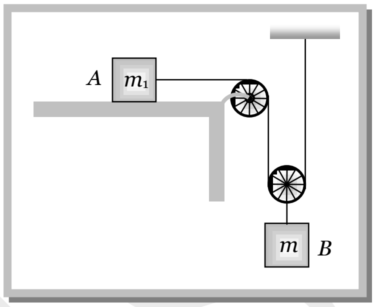
***Problem* 70.** In the above problem, the additional distance traversed by m_2 in coming to rest position will be

(a) 20 *cm* (b) 40 *cm* (c) 60 *cm* (d) 80 *cm*

Solution : (a) When m_2 mass acquired velocity 200 cm/sec it will move upward till its velocity becomes zero.

$$H = \frac{u^2}{2g} = \frac{(200)^2}{2 \times 100} = 20\ cm$$

***Problem* 71.** The acceleration of block *B* in the figure will be



(a) $\frac{m_2 g}{(4m_1 + m_2)}$

(b) $\frac{2m_2 g}{(4m_1 + m_2)}$

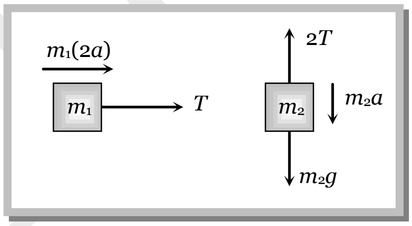
(c) $\frac{2m_1 g}{(m_1 + 4m_2)}$

(d) $\frac{2m_1 g}{(m_1 + m_2)}$

Solution : (a) When the block m_2 moves downward with acceleration *a*, the acceleration of mass m_1 will be $2a$ because it covers double distance in the same time in comparison to m_2.

Let *T* is the tension in the string.

By drawing the free body diagram of *A* and *B*



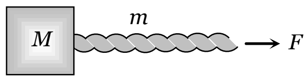
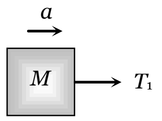
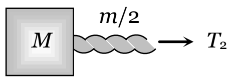
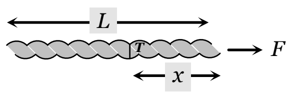
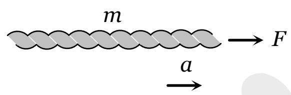
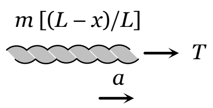
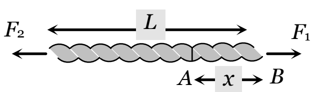
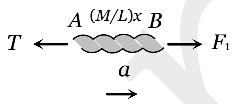
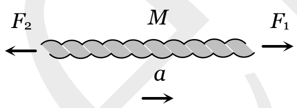
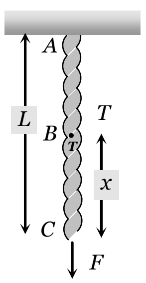
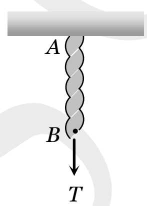
$T = m_1 2a$ ....(i)

$m_2 g - 2T = m_2 a$ ....(ii)

by solving (i) and (ii)

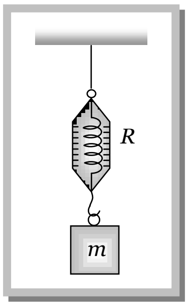
$$a = \frac{m_2 g}{(4m_1 + m_2)}$$

4.19 Motion of Massive String

Condition	Free body diagram	Equation	Tension and acceleration
	 T_1 = force applied by the string on the block	$F=(M+m)a$ $T_1=Ma$	$a=\dfrac{F}{M+m}$ $T_1=M\dfrac{F}{(M+m)}$ $T_2=\dfrac{(2M+m)}{2(M+m)}F$
	 T_2 = Tension at mid point of the rope	$T_2=\left(M+\dfrac{m}{2}\right)a$	
 m = Mass of string T = Tension in string at a distance x from the end where the force is applied		$F=ma$	$a=F/m$ $T=\left(\dfrac{L-x}{L}\right)F$
		$T=m\left(\dfrac{L-x}{L}\right)a$	
 M = Mass of uniform rod L = Length of rod		$F_1-T=\dfrac{Mxa}{L}$	$a=\dfrac{F_1-F_2}{M}$ $T=F_1\left(1-\dfrac{x}{L}\right)+F_2\left(\dfrac{x}{L}\right)$
		$F_1-F_2=Ma$	
 Mass of segment BC $=\left(\dfrac{M}{L}\right)x$		$T=\left(\dfrac{L-x}{L}\right)F$	$T=\left(\dfrac{L-x}{L}\right)F$

4.20 Spring Balance and Physical Balance

(1) **Spring balance :** When its upper end is fixed with rigid support and body of mass m hung from its lower end. Spring is stretched and the weight of the body can be measured by the reading of spring balance $R = W = mg$

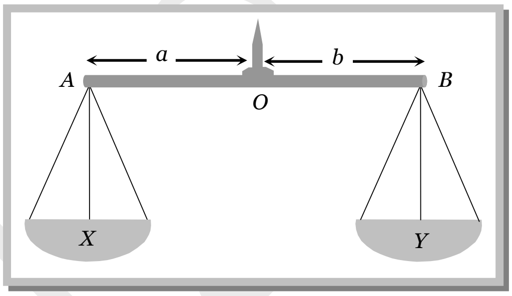


The mechanism of weighing machine is same as that of spring balance.

Effect of frame of reference : In inertial frame of reference the reading of spring balance shows the actual weight of the body but in non-inertial frame of reference reading of spring balance increases or decreases in accordance with the direction of acceleration

[for detail refer Article (4.13)]

(2) **Physical balance :** In physical balance actually we compare the mass of body in both the pans. Here we does not calculate the absolute weight of the body.



Here X and Y are the mass of the empty pan.

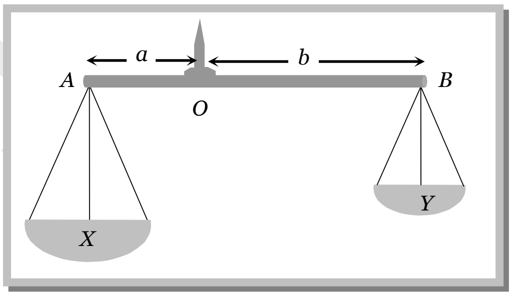
(i) Perfect physical balance :

Weight of the pan should be equal *i.e.* $X = Y$

and the needle must in middle of the beam *i.e.* $a = b$.

Effect of frame of reference : If the physical balance is perfect then there will be no effect of frame of reference (either inertial or non-inertial) on the measurement. It is always errorless.

(ii) False balance : When the masses of the pan are not equal then balance shows the error in measurement. False balance may be of two types



(a) If the beam of physical balance is horizontal (when the pans are empty) but the arms are not equal

$X > Y$ and $a < b$

For rotational equilibrium about point 'O'

$$Xa = Yb \qquad ......(i)$$

In this physical balance if a body of weight W is placed in pan X then to balance it we have to put a weight W_1 in pan Y.

For rotational equilibrium about point 'O'

$$(X + W)a = (Y + W_1)b \qquad .....(ii)$$

Now if the pans are changed then to balance the body we have to put a weight W_2 in pan X.

For rotational equilibrium about point 'O'

$$(X + W_2)a = (Y + W)b \qquad ......(iii)$$

From (i), (ii) and (iii)

True weight $W = \sqrt{W_1 W_2}$

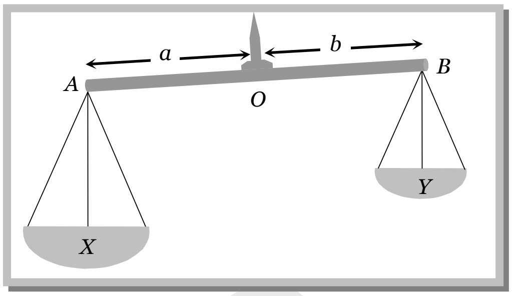
(b) If the beam of physical balance is not horizontal (when the pans are empty) and the arms are equal

i.e. $X > Y$ and $a = b$

In this physical balance if a body of weight W is placed in X Pan then to balance it.

We have to put a weight W_1 in Y Pan

For equilibrium $X + W = Y + W_1$ .....(i)



Now if pans are changed then to balance the body we have to put a weight W_2 in X Pan.

For equilibrium $X + W_2 = Y + W$ .....(ii)

From (i) and (ii)

True weight $W = \dfrac{W_1 + W_2}{2}$

Sample problems (Miscellaneous)

Problem 72. A body weighs 8 *gm*, when placed in one pan and 18*gm*, when placed in the other pan of a false balance. If the beam is horizontal (when both the pans are empty), the true weight of the body is

(a) 13 *gm* (b) 12 *gm* (c) 15.5 *gm* (d) 15 *gm*

Solution : (b) For given condition true weight = $\sqrt{W_1 W_2} = \sqrt{8 \times 18}$ =12 *gm*.

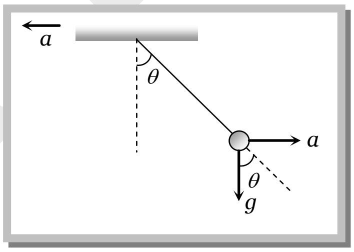
Problem 73. A plumb line is suspended from a ceiling of a car moving with horizontal acceleration of *a*. What will be the angle of inclination with vertical **[Orissa JEE 2003]**

(a) $\tan^{-1}(a/g)$ (b) $\tan^{-1}(g/a)$ (c) $\cos^{-1}(a/g)$ (d) $\cos^{-1}(g/a)$

Solution : (a) From the figure

$$\tan\theta = \frac{a}{g}$$

$$\theta = \tan^{-1}(a/g)$$



Problem 74. A block of mass $5\,kg$ is moving horizontally at a speed of 1.5 *m/s*. A perpendicular force of 5 *N* acts on it for 4 *sec*. What will be the distance of the block from the point where the force started acting **[Pb PMT 2002]**

(a) 10 *m* (b) 8 *m* (c) 6 *m* (d) 2 *m*

Solution : (a) In the given problem force is working in a direction perpendicular to initial velocity. So the body will move under the effect of constant velocity in horizontal direction and under the effect of force in vertical direction.

$$S_x = u_x \times t = 1.5 \times 4 = 6m$$

$$S_y = u_y t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(F/m)t^2 = \frac{1}{2}(5/5)(4)^2 = 8m$$

$$\therefore\ S = \sqrt{S_x^2 + S_y^2} = \sqrt{36+64} = \sqrt{100} = 10m$$

Problem 75. The velocity of a body of rest mass m_0 is $\dfrac{\sqrt{3}}{2}c$ (where *c* is the velocity of light in vacuum). Then mass of this body is **[Orissa JEE 2002]**

(a) $(\sqrt{3}/2)m_0$ (b) $(1/2)m_0$ (c) $2m_0$ (d) $(2/\sqrt{3})m_0$

Solution : (c) From Einstein's formula $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{m_0}{\sqrt{1-\dfrac{\left(\sqrt{\dfrac{3}{2}}C\right)^2}{C^2}}} = \dfrac{m_0}{\sqrt{1-\dfrac{3}{4}}} = 2m_0$

Problem 76. Three weights W, $2W$ and $3W$ are connected to identical springs suspended form rigid horizontal rod. The assembly of the rod and the weights fall freely. The positions of the weights from the rod are such that

[Roorkee 1999]

(a) 3 W will be farthest (b) W will be farthest

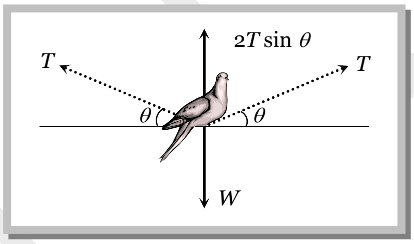
(c) All will be at the same distance (d) 2 W will be farthest

Solution : (c) For W, $2W$, $3W$ apparent weight will be zero because the system is falling freely. So there will be no extension in any spring *i.e.* the distances of the weight from the rod will be same.

Problem 77. A bird is sitting on stretched telephone wires. If its weight is W then the additional tension produced by it in the wires will be

(a) $T = W$ (b) $T > W$ (c) $T < W$ (d) $T = 0$

Solution : (b) For equilibrium



$2T\sin\theta = W$

$T = \dfrac{W}{2\sin\theta}$

θ lies between 0 to 90° *i.e.* $\sin\theta < 1$

$\therefore\ T > W$

Problem 78. With what minimum acceleration can a fireman slides down a rope while breaking strength of the rope is $\dfrac{2}{3}$ his weight **[CPMT 1979]**

(a) $\dfrac{2}{3}g$ (b) g (c) $\dfrac{1}{3}g$ (d) Zero

Solution : (c) When fireman slides down, Tension in the rope $T = m(g-a)$

For critical condition $m(g-a) = 2/3\ mg \Rightarrow mg - ma = \dfrac{2}{3}mg \quad \therefore\ a = \dfrac{g}{3}$

So, this is the minimum acceleration by which a fireman can slides down on a rope.

Problem 79. A car moving at a speed of 30 *kilo meters* per hour's is brought to a halt in 8 *metres* by applying brakes. If the same car is moving at 60 *km*. per hour, it can be brought to a halt with same braking power in

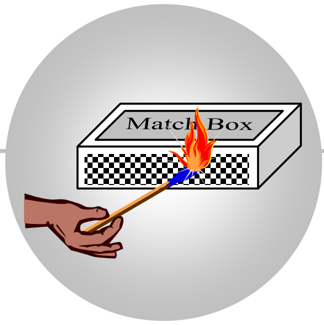
(a) 8 *metres* (b) 16 *metres* (c) 24 *metres* (d) 32 *metres*

Solution : (d) From $v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$s = \dfrac{u^2}{2a} \Rightarrow s \propto u^2$ (if a = constant)

$\dfrac{s_2}{s_1} = \left(\dfrac{u_2}{u_1}\right)^2 = \left(\dfrac{60}{30}\right)^2 = 4 \Rightarrow s_2 = 4s_1 = 4\times 8 = 32$ *metres.*



Friction

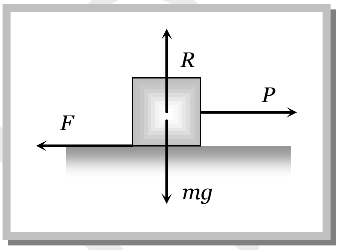
5.1 Introduction

If we slide or try to slide a body over a surface the motion is resisted by a bonding between the body and the surface. This resistance is represented by a single force and is called friction.

The force of friction is parallel to the surface and opposite to the direction of intended motion.

5.2 Types of Friction

(1) **Static friction :** The opposing force that comes into play when one body tends to move over the surface of another, but the actual motion has yet not started is called static friction.



(i) If applied force is P and the body remains at rest then static friction $F = P$.

(ii) If a body is at rest and no pulling force is acting on it, force of friction on it is zero.

(iii) Static friction is a self-adjusting force because it changes itself in accordance with the applied force.

(2) **Limiting friction :** If the applied force is increased the force of static friction also increases. If the applied force exceeds a certain (maximum) value, the body starts moving. This maximum value of static friction upto which body does not move is called limiting friction.

(i) The magnitude of limiting friction between any two bodies in contact is directly proportional to the normal reaction between them.

$$F_l \propto R \text{ or } F_l = \mu_s R$$

(ii) Direction of the force of limiting friction is always opposite to the direction in which one body is at the verge of moving over the other

(iii) Coefficient of static friction : (a) μ_s is called coefficient of static friction and defined as the ratio of force of limiting friction and normal reaction $\mu_s = \frac{F}{R}$

(b) Dimension : $[M^0 L^0 T^0]$

(c) Unit : It has no unit.

(d) Value of μ_s lies in between 0 and 1

(e) Value of μ depends on material and nature of surfaces in contact that means whether dry or wet ; rough or smooth polished or non-polished.

(f) Value of μ does not depend upon apparent area of contact.

(3) **Kinetic or dynamic friction :** If the applied force is increased further and sets the body in motion, the friction opposing the motion is called kinetic friction.

(i) Kinetic friction depends upon the normal reaction.

$$F_k \propto R \text{ or } F_k = \mu_k R \text{ where } \mu_k \text{ is called the coefficient of kinetic friction}$$

(ii) Value of μ_k depends upon the nature of surface in contact.

(iii) Kinetic friction is always lesser than limiting friction $F_k < F_l \quad \therefore \; \mu_k < \mu_s$

i.e. coefficient of kinetic friction is always less than coefficient of static friction. Thus we require more force to start a motion than to maintain it against friction. This is because once the motion starts actually ; inertia of rest has been overcome. Also when motion has actually started, irregularities of one surface have little time to get locked again into the irregularities of the other surface.

(iv) Types of kinetic friction

(a) **Sliding friction** : The opposing force that comes into play when one body is actually sliding over the surface of the other body is called sliding friction. *e.g.* A flat block is moving over a horizontal table.

(b) **Rolling friction** : When objects such as a wheel (disc or ring), sphere or a cylinder rolls over a surface, the force of friction comes into play is called rolling friction.

❑ Rolling friction is directly proportional to the normal reaction (R) and inversely proportional to the radius (r) of the rolling cylinder or wheel.

$$F_{rolling} = \mu_r \frac{R}{r}$$

μ_r is called coefficient of rolling friction. It would have the dimensions of length and would be measured in *metre.*

❑ Rolling friction is often quite small as compared to the sliding friction. That is why heavy loads are transported by placing them on carts with wheels.

❑ In rolling the surfaces at contact do not rub each other.

❑ The velocity of point of contact with respect to the surface remains zero all the times although the centre of the wheel moves forward.

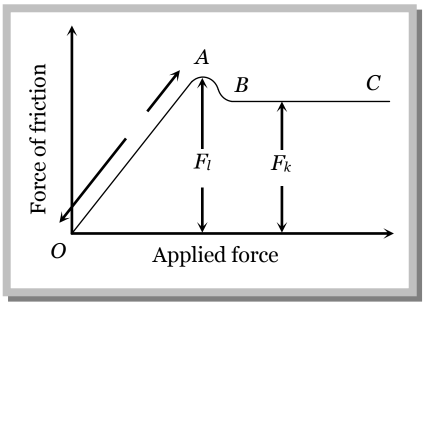
5.3 Graph Between Applied Force and Force of Friction

(1) Part OA of the curve represents static friction (F_s). Its value increases linearly with the applied force

(2) At point A the static friction is maximum. This represent limiting friction (F_l).

(3) Beyond A, the force of friction is seen to decrease slightly. The portion BC of the curve therefore represents the kinetic friction (F_k).

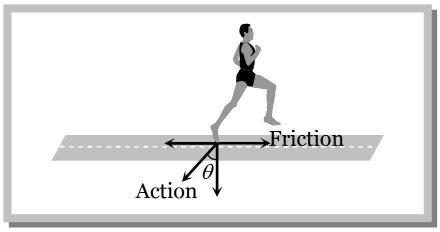
(4) As the portion BC of the curve is parallel to x-axis therefore kinetic friction does not change with the applied force, it remains constant, whatever be the applied force.



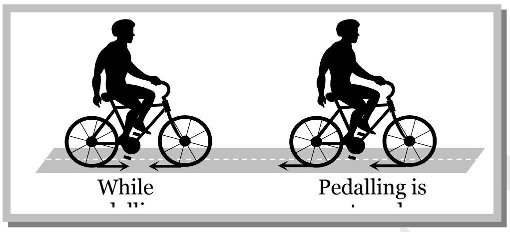
5.4 Friction is a Cause of Motion

It is a general misconception that friction always opposes the motion. No doubt friction opposes the motion of a moving body but in many cases it is also the cause of motion. For example :

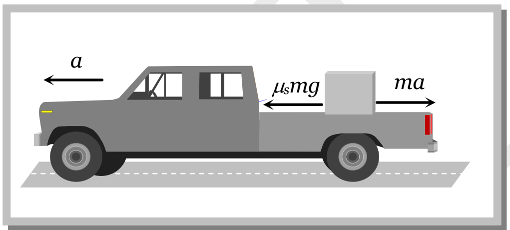
(1) In moving, a person or vehicle pushes the ground backwards (action) and the rough surface of ground reacts and exerts a forward force due to friction which causes the motion. If there had been no friction there will be slipping and no motion.



(2) In cycling, the rear wheel moves by the force communicated to it by pedalling while front wheel moves by itself. So, when pedalling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moving by itself experience force of friction in backward direction (like rolling of a ball). [However, if pedalling is stopped both wheels move by themselves and so experience force of friction in backward direction.]



(3) If a body is placed in a vehicle which is accelerating, the force of friction is the cause of motion of the body along with the vehicle (*i.e.*, the body will remain at rest in the accelerating vehicle until $ma < \mu_s mg$). If there had been no friction between body and vehicle the body will not move along with the vehicle.



From these examples it is clear that without friction motion cannot be started, stopped or transferred from one body to the other.

Sample problems based on fundamentals of friction

Problem 1. If a ladder weighing 250*N* is placed against a smooth vertical wall having coefficient of friction between it and floor is 0.3, then what is the maximum force of friction available at the point of contact between the ladder and the floor **[AIIMS 2002]**

(a) 75 *N* (b) 50 *N* (c) 35 *N* (d) 25 *N*

Solution : (a) Maximum force of friction $F_l = \mu_s R = 0.3 \times 250 = 75 N$

Problem 2. On the horizontal surface of a truck (μ = 0.6), a block of mass 1 *kg* is placed. If the truck is accelerating at the rate of 5*m/sec*^{2} then frictional force on the block will be

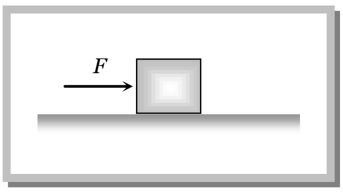
(a) 5 *N* (b) 6 *N* (c) 5.88 *N* (d) 8 *N*

Solution : (a) Limiting friction $= \mu_s R = \mu_s mg = 0.6 \times 1 \times 9.8 = 5.88 N$

When truck accelerates in forward direction at the rate of $5 m/s^2$ a pseudo force (*ma*) of 5*N* works on block in back ward direction. Here the magnitude of pseudo force is less than limiting friction So, static friction works in between the block and the surface of the truck and as we know, static friction = Applied force = 5*N*.

Problem 3. A block of mass 2 *kg* is kept on the floor. The coefficient of static friction is 0.4. If a force *F* of 2.5 *N* is applied on the block as shown in the figure, the frictional force between the block and the floor will be **[MP PET 2**

(a) 2.5 *N*

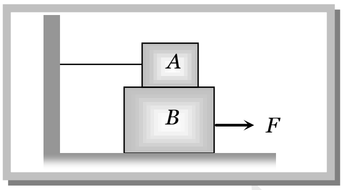


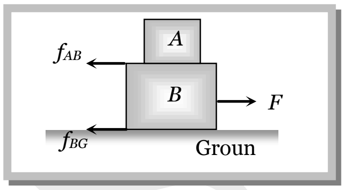
(b) 5 N
(c) 7.84 N
(d) 10 N

Solution : (a) Applied force = 2.5 N and limiting friction = μmg = 0.4 × 2 × 9.8 = 7.84 N

As applied force is less than limiting friction. So, for the given condition static friction will work.
Static friction on a body = Applied force = 2.5 N.

***Problem* 4.** A block A with mass 100 kg is resting on another block B of mass 200 kg. As shown in figure a horizontal rope tied to a wall holds it. The coefficient of friction between A and B is 0.2 while coefficient of friction between B and the ground is 0.3. The minimum required force F to start moving B will be
(a) 900 N
(b) 100 N
(c) 1100 N
(d) 1200 N





Solution : (c) Two frictional force will work on block B.

$F = f_{AB} + f_{BG} = \mu_{AB} m_a g + \mu_{BG}(m_A + m_B)g$

= 0.2 × 100 × 10 + 0.3 (300) × 10

= 200 + 900 = 1100N. (This is the required minimum force)

***Problem* 5.** A 20 kg block is initially at rest on a rough horizontal surface. A horizontal force of 75 N is required to set the block in motion. After it is in motion, a horizontal force of 60 N is required to keep the block moving with constant speed. The coefficient of static friction is
(a) 0.38 (b) 0.44 (c) 0.52 (d) 0.60

Solution : (a) Coefficient of static friction $\mu_S = \frac{F_l}{R} = \frac{75}{20 \times 9.8} = 0.38$.

***Problem* 6.** A block of mass M is placed on a rough floor of a lift. The coefficient of friction between the block and the floor is μ. When the lift falls freely, the block is pulled horizontally on the floor. What will be the force of friction
(a) μMg (b) $\mu Mg/2$ (c) $2\mu Mg$ (d) None of these

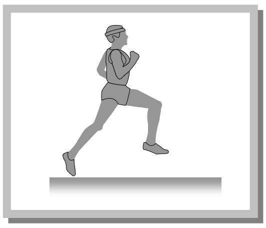
Solution : (d) When the lift moves down ward with acceleration 'a' then effective acceleration due to gravity

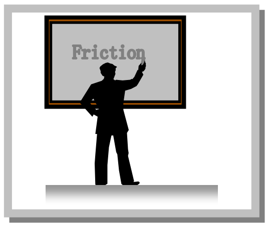
$g' = g - a$

$\therefore g' = g - g = 0$ [As the lift falls freely, so $a = g$]

So force of friction $= \mu mg' = 0$

5.5 Advantages and Disadvantages of Friction





(1) **Advantages of friction**
(i) Walking is possible due to friction.
(ii) Two body sticks together due to friction.
(iii) Brake works on the basis of friction.
(iv) Writing is not possible without friction.
(v) The transfer of motion from one part of a machine to other part through belts is possible by friction.

(2) **Disadvantages of friction**

(i) Friction always opposes the relative motion between any two bodies in contact. Therefore extra energy has to be spent in over coming friction. This reduces the efficiency of machine.

(ii) Friction causes wear and tear of the parts of machinery in contact. Thus their lifetime reduces.

(iii) Frictional force result in the production of heat, which causes damage to the machinery.

5.6 Methods of Changing Friction

We can reduce friction

(1) By polishing.

(2) By lubrication.

(3) By proper selection of material.

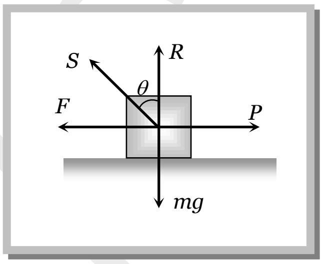
(4) By streamlining the shape of the body.

(5) By using ball bearing.

Also we can increase friction by throwing some sand on slippery ground. In the manufacturing of tyres, synthetic rubber is preferred because its coefficient of friction with the road is larger.

5.7 Angle of Friction

Angle of friction may be defined as the angle which the resultant of limiting friction and normal reaction makes with the normal reaction.



By definition angle θ is called the angle of friction

$$\tan\theta = \frac{F}{R}$$

$\therefore$ $\tan\theta = \mu$ [As we know $\frac{F}{R} = \mu$]

or $\theta = \tan^{-1}(\mu)$

Hence coefficient of limiting friction is equal to tangent of the angle of friction.

5.8 Resultant Force Exerted by Surface on Block

In the above figure resultant force $S = \sqrt{F^2 + R^2}$

$$S = \sqrt{(\mu mg)^2 + (mg)^2}$$

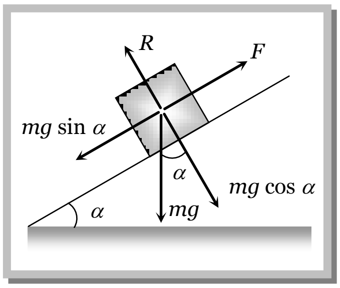
$$S = mg\sqrt{\mu^2 + 1}$$

when there is no friction $(\mu = 0)$ S will be minimum *i.e.*, $S = mg$

Hence the range of S can be given by, $mg \le S \le mg\sqrt{\mu^2 + 1}$

5.9 Angle of Repose

Angle of repose is defined as the angle of the inclined plane with horizontal such that a body placed on it is just begins to slide.



By definition α is called the angle of repose.

In limiting condition $F = mg\sin\alpha$

and $R = mg\cos\alpha$

So $\frac{F}{R} = \tan\alpha$

$\therefore$ $\frac{F}{R} = \mu = \tan\theta = \tan\alpha$ [As we know $\frac{F}{R} = \mu = \tan\theta$]

Thus the coefficient of limiting friction is equal to the tangent of angle of repose.

As well as $\alpha = \theta$ *i.e.* angle of repose = angle of friction.

Sample problems based on angle of friction and angle of repose

***Problem* 7.** A body of 5 *kg* weight kept on a rough inclined plane of angle 30^o starts sliding with a constant velocity. Then the coefficient of friction is (assume g = 10 m/s^2)

(a) $1/\sqrt{3}$ (b) $2/\sqrt{3}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

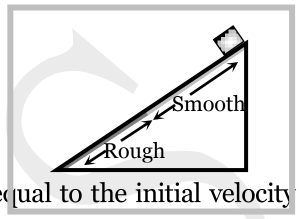
Solution : (a) Here the given angle is called the angle of repose

So, $\mu = \tan 30^o = \dfrac{1}{\sqrt{3}}$

***Problem* 8.** The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A body starting from the rest at top comes back to rest at the bottom if the coefficient of friction for the lower half is given **[Pb PMT 2000]**

(a) $\mu = \sin\theta$ (b) $\mu = \cot\theta$ (c) $\mu = 2\cos\theta$ (d) $\mu = 2\tan\theta$

Solution : (d) For upper half by the equation of motion $v^2 = u^2 + 2as$



$v^2 = 0^2 + 2(g\sin\theta)l/2 = gl\sin\theta$ [As $u = 0, s = l/2, a = g\sin\theta$]

For lower half

$0 = u^2 + 2g(\sin\theta - \mu\cos\theta)\ l/2$ [As $v = 0, s = l/2, a = g(\sin\theta - \mu\cos\theta)$]

$\Rightarrow\ 0 = gl\sin\theta + gl(\sin\theta - \mu\cos\theta)$ [As final velocity of upper half will be equal to the initial velocity of lower half]

$\Rightarrow\ 2\sin\theta = \mu\cos\theta \Rightarrow \mu = 2\tan\theta$

5.10 Calculation of Necessary Force in Different Conditions

If W = weight of the body, θ = angle of friction, $\mu = \tan\theta$ = coefficient of friction

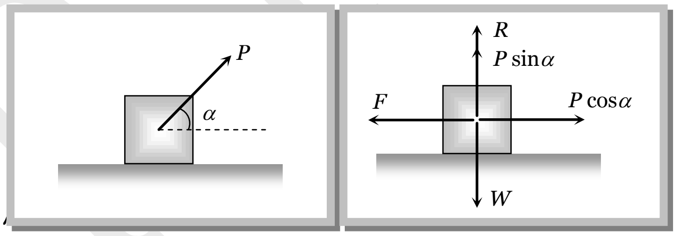
then we can calculate necessary force for different condition in the following manner :

(1) **Minimum pulling force P at an angle α from the horizontal**

By resolving P in horizontal and vertical direction (as shown in figure)

For the condition of equilibrium

$F = P\cos\alpha$ and $R = W - P\sin\alpha$



By substituting these value in $F = \mu R$

$P\cos\alpha = \mu(W - P\sin\alpha)$

$\Rightarrow\quad P\cos\alpha = \dfrac{\sin\theta}{\cos\theta}(W - P\sin\alpha)$ [As

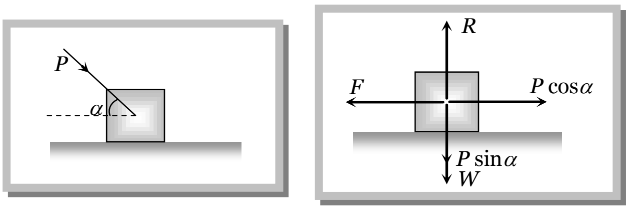
$\Rightarrow\quad P = \dfrac{W\sin\theta}{\cos(\alpha - \theta)}$

(2) **Minimum pushing force P at an angle α from the horizontal**

By Resolving P in horizontal and vertical direction (as shown in the figure)

For the condition of equilibrium

$F = P\cos\alpha$ and $R = W + P\sin\alpha$



By substituting these value in $F = \mu R$

$\Rightarrow\quad P\cos\alpha = \mu(W + P\sin\alpha)$

$\Rightarrow\quad P\cos\alpha = \dfrac{\sin\theta}{\cos\theta}(W + P\sin\alpha)$ [As $\mu = \tan\theta$]

$\Rightarrow\quad P = \dfrac{W\sin\theta}{\cos(\alpha + \theta)}$

(3) **Minimum pulling force P to move the body up an inclined plane**

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

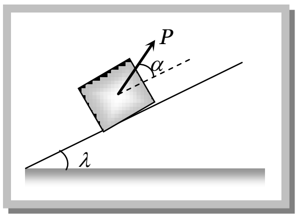
For the condition of equilibrium

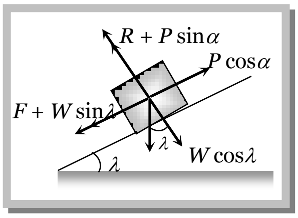
$$R + P\sin\alpha = W\cos\lambda$$

$\therefore$ $\quad R = W\cos\lambda - P\sin\alpha$

and $\quad F + W\sin\lambda = P\cos\alpha$

$\therefore$ $\quad F = P\cos\alpha - W\sin\lambda$





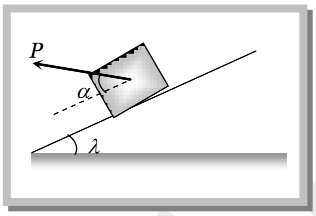
By substituting these values in $F = \mu R$ and solving we get

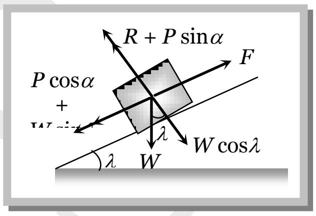
$$P = \frac{W\sin(\theta + \lambda)}{\cos(\alpha - \theta)}$$

(4) **Minimum force on body in downward direction along the surface of inclined plane to start its motion**

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

For the condition of equilibrium





$$R + P\sin\alpha = W\cos\lambda$$

$\therefore$ $\quad R = W\cos\lambda - P\sin\alpha$

and $\quad F = P\cos\alpha + W\sin\lambda$

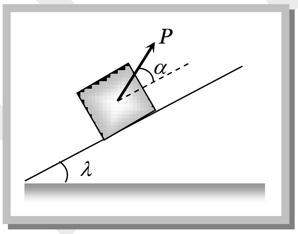
By substituting these values in $F = \mu R$ and solving we get

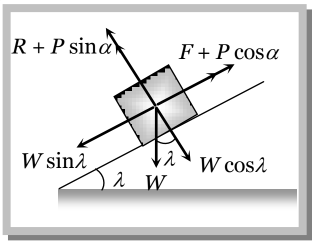
$$P = \frac{W\sin(\theta - \lambda)}{\cos(\alpha - \theta)}$$

(5) **Minimum force to avoid sliding a body down an inclined plane**

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

For the condition of equilibrium





$$R + P\sin\alpha = W\cos\lambda$$

$\therefore$ $\quad R = W\cos\lambda - P\sin\alpha$

and $\quad P\cos\alpha + F = W\sin\lambda$

$\therefore$ $\quad F = W\sin\lambda - P\cos\alpha$

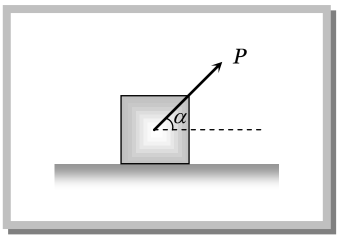
By substituting these values in $F = \mu R$ and solving we get

$$P = W\left[\frac{\sin(\lambda - \theta)}{\cos(\theta + \alpha)}\right]$$

(6) **Minimum force for motion and its direction**

Let the force P be applied at an angle α with the horizontal.

By resolving P in horizontal and vertical direction (as shown in figure)



For vertical equilibrium

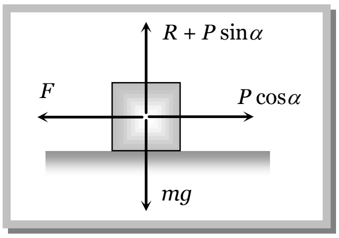
$$R + P\sin\alpha = mg$$

$\therefore$ $\quad R = mg - P\sin\alpha \quad$ ....(i)

and for horizontal motion

$$P\cos\alpha \geq F$$

i.e. $\quad P\cos\alpha \geq \mu R \quad$ ....(ii)



Substituting value of R from (i) in (ii)

$$P\cos\alpha \ge \mu(mg - P\sin\alpha)$$

$$P \ge \frac{\mu mg}{\cos\alpha + \mu\sin\alpha} \qquad \text{....(iii)}$$

For the force P to be minimum $(\cos\alpha + \mu\sin\alpha)$ must be maximum *i.e.*

$$\frac{d}{d\alpha}[\cos\alpha + \mu\sin\alpha] = 0 \;\Rightarrow\; -\sin\alpha + \mu\cos\alpha = 0$$

$\therefore$ $\tan\alpha = \mu$

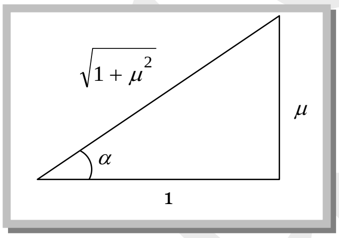
or $\alpha = \tan^{-1}(\mu)$ = angle of friction

i.e. For minimum value of P its angle from the horizontal should be equal to angle of friction

As $\tan\alpha = \mu$ so from the figure $\sin\alpha = \dfrac{\mu}{\sqrt{1+\mu^2}}$ and $\cos\alpha = \dfrac{1}{\sqrt{1+\mu^2}}$

By substituting these value in equation (iii)

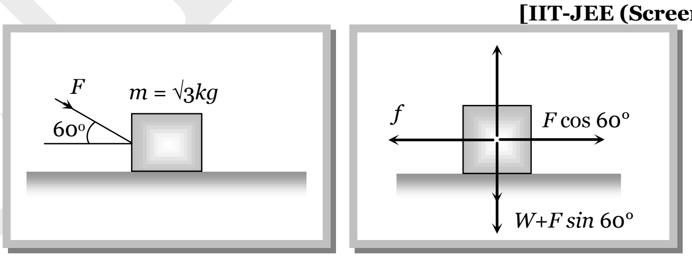
$$P \ge \frac{\mu mg}{\dfrac{1}{\sqrt{1+\mu^2}} + \dfrac{\mu^2}{\sqrt{1+\mu^2}}} \ge \frac{\mu mg}{\sqrt{1+\mu^2}}$$



$\therefore$ $$P_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Sample problems based on force against friction

Problem 9. What is the maximum value of the force F such that the block shown in the arrangement, does not move ($\mu = 1/2\sqrt{3}$) **[IIT-JEE (Screening) 2003]**



(a) 20 N

(b) 10 N

(c) 12 N

(d) 15 N

Solution : (a) Frictional force $f = \mu R$

$\Rightarrow F\cos 60 = \mu(W + F\sin 60)$

$\Rightarrow F\cos 60 = \dfrac{1}{2\sqrt{3}}\left(\sqrt{3}g + F\sin 60\right)$

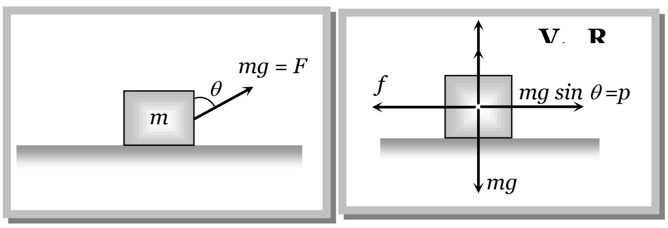
$\Rightarrow F = 20N$.

Problem 10. A block of mass m rests on a rough horizontal surface as shown in the figure. Coefficient of friction between the block and the surface is μ. A force $F = mg$ acting at angle θ with the vertical side of the block pulls it. In which of the following cases the block can be pulled along the surface

(a) $\tan\theta \ge \mu$

(b) $\cot\theta \ge \mu$

(c) $\tan\theta/2 \ge \mu$



(d) $\cot\theta/2 \geq \mu$

Solution : (d) For pulling of block $P \geq f$

$\Rightarrow mg\sin\theta \geq \mu R \Rightarrow mg\sin\theta \geq \mu(mg - mg\cos\theta)$

$\Rightarrow \sin\theta \geq \mu(1-\cos\theta)$

$\Rightarrow 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu\left(2\sin^2\frac{\theta}{2}\right) \Rightarrow \cot\left(\frac{\theta}{2}\right) \geq \mu$

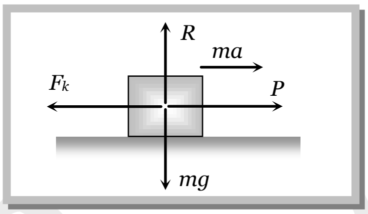
5.11 Acceleration of a Block Against Friction

(1) **Acceleration of a block on horizontal surface**

When body is moving under application of force P, then kinetic friction opposes its motion.

Let a is the net acceleration of the body

From the figure

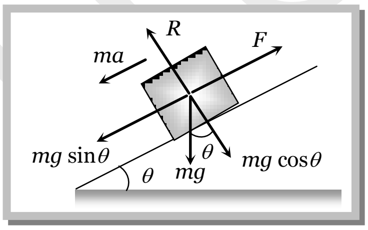


$$ma = P - F_k$$

$$\therefore \quad a = \frac{P - F_k}{m}$$

(2) **Acceleration of a block down a rough inclined plane**

When angle of inclined plane is more than angle of repose, the body placed on the inclined plane slides down with an acceleration a.



From the figure $ma = mg\sin\theta - F$

$\Rightarrow \quad ma = mg\sin\theta - \mu R$

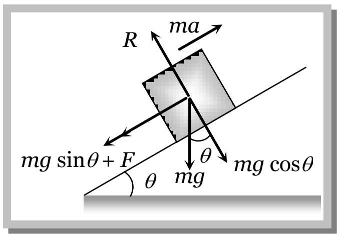
$\Rightarrow \quad ma = mg\sin\theta - \mu mg\cos\theta$

$\therefore$ Acceleration $a = g[\sin\theta - \mu\cos\theta]$

Note : ❑ For frictionless inclined plane $\mu = 0 \;\therefore a = g\sin\theta$.

(3) **Retardation of a block up a rough inclined plane**

When angle of inclined plane is less than angle of repose, then for the upward motion



$ma = mg\sin\theta + F$

$ma = mg\sin\theta + \mu mg\cos\theta$

Retardation $a = g[\sin\theta + \mu\cos\theta]$

Note : ❑ For frictionless inclined plane $\mu = 0 \;\therefore a = g\sin\theta$

Sample problems based on acceleration against friction

***Problem* 11.** A body of mass 10 *kg* is lying on a rough plane inclined at an angle of 30^o to the horizontal and the coefficient of friction is 0.5. The minimum force required to pull the body up the plane is

(a) 914 *N* (b) 91.4 *N* (c) 9.14 *N* (d) 0.914 *N*

Solution : (b) $F = mg(\sin\theta + \mu\cos\theta) = 10\times 9.8(\sin 30 + 0.5\cos 30) = 91.4\,N$

***Problem* 12.** A block of mass 10 *kg* is placed on a rough horizontal surface having coefficient of friction μ = 0.5. If a horizontal force of 100 *N* is acting on it, then acceleration of the block will be **[AIIMS 2002]**

(a) 0.5 m/s^2 (b) 5 m/s^2 (c) 10 m/s^2 (d) 15 m/s^2

Solution : (b) $a = \frac{\text{Applied force} - \text{kinetic friction}}{\text{mass}} = \frac{100 - 0.5\times 10\times 10}{10} = 5m/s^2$.

***Problem* 13.** A body of weight 64 *N* is pushed with just enough force to start it moving across a horizontal floor and the same force continues to act afterwards. If the coefficients of static and dynamic friction are 0.6 and 0.4 respectively, the acceleration of the body will be (Acceleration due to gravity = g) **[EAMCET 2001]**

(a) $\frac{g}{6.4}$ (b) $0.64\,g$ (c) $\frac{g}{32}$ (d) $0.2\,g$

Solution : (d) Limiting friction = $F_l = \mu_s R \Rightarrow 64 = 0.6\,m\,g \Rightarrow m = \frac{64}{0.6g}$.

Acceleration = $\frac{\text{Applied force} - \text{Kinetic friction}}{\text{Mass of the body}} = \frac{64 - \mu_K mg}{m} = \frac{64 - 0.4 \times \frac{64}{0.6}}{\frac{64}{0.6g}} = 0.2g$

***Problem* 14.** If a block moving up at $\theta = 30^o$ with a velocity 5 m/s, stops after 0.5 *sec*, then what is μ

(a) 0.5 (b) 1.25 (c) 0.6 (d) None of these

Solution : (c) From $v = u - at \Rightarrow 0 = u - at \;\; \therefore t = \frac{u}{a}$

for upward motion on an inclined plane $a = g(\sin\theta + \mu\cos\theta)$ $\quad \therefore t = \frac{u}{g(\sin\theta + \mu\cos\theta)}$

Substituting the value of $\theta = 30^o, t = 0.5$ sec and $u = 5m/s$, we get $\mu = 0.6$

5.12 Work Done Against Friction

(1) **Work done over a rough inclined surface**

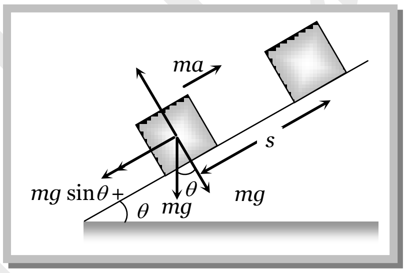
If a body of mass m is moved up on a rough inclined plane through distance s, then

Work done = force × distance

$= ma \times s$

$= mg\,[\sin\theta + \mu\cos\theta]s$

$= mg\,s[\sin\theta + \mu\cos\theta]$



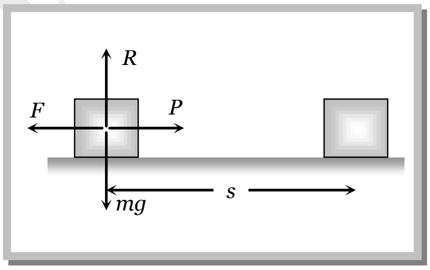
(2) **Work done over a horizontal surface**

In the above expression if we put $\theta = 0$ then

Work done = force × distance

$= F \times s$

$= \mu\,mg\,s$



It is clear that work done depends upon

(i) Weight of the body.

(ii) Material and nature of surface in contact.

(iii) Distance moved.

Sample problems based on work done against friction

***Problem* 15.** A body of mass 5*kg* rests on a rough horizontal surface of coefficient of friction 0.2. The body is pulled through a distance of 10*m* by a horizontal force of 25 *N*. The kinetic energy acquired by it is (g = 10 ms^2) **[EAMCET (Med.) 2000]**

(a) 330 *J* (b) 150 *J* (c) 100 *J* (d) 50 *J*

Solution : (b) Kinetic energy acquired by body = Total work done on the body – Work done against friction

$= F \times S - \mu mgS = 25 \times 10 - 0.2 \times 5 \times 10 \times 10 = 250 - 100 = 150\,J$.

***Problem* 16.** 300 *Joule* of work is done in sliding a 2 *kg*. block up an inclined plane to a height of 10 *meters*. Taking value of acceleration due to gravity 'g' to be 10 m/s^2, work done against friction is **[MP PMT 2002]**

(a) 100 *J* (b) 200 *J* (c) 300 *J* (d) Zero

Solution : (a) Work done against gravity = $mgh = 2 \times 10 \times 10 = 200J$

Work done against friction = Total work done – Work done against gravity = 300 – 200 = 100*J*.

***Problem* 17.** A block of mass 1 *kg* slides down on a rough inclined plane of inclination 60° starting from its top. If the coefficient of kinetic friction is 0.5 and length of the plane is 1 *m*, then work done against friction is (Take g = 9.8 m/s^2) **[AFMC 2000; KCET (Engg./Med.) 2001]**

(a) 9.82 *J* (b) 4.94 *J* (c) 2.45*J* (d) 1.96 *J*

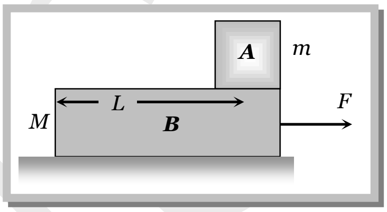
Solution : (c) $W = \mu mg\cos\theta .S = 0.5\times 1\times 9.8\times \frac{1}{2} = 2.45\,J.$

***Problem* 18.** A block of mass 50 *kg* slides over a horizontal distance of 1 *m*. If the coefficient of friction between their surfaces is 0.2, then work done against friction is **[CBSE PMT 1999, 2000; AIIMS 2000; BHU 2001]**

(a) 98 *J* (b) 72*J* (c) 56 *J* (d) 34 *J*

Solution : (a) $W = \mu mgS = 0.2\times 50\times 9.8\times 1 = 98\,J$.

5.13 Motion of Two Bodies One Resting on the Other

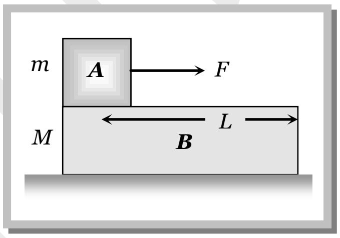


When a body A of mass m is resting on a body B of mass M then two conditions are possible

(1) A force F is applied to the upper body, (2) A force F is applied to the lower body

We will discuss above two cases one by one in the following manner :

(1) **A force *F* is applied to the upper body, then following four situations are possible**



(i) **When there is no friction**

(a) The body A will move on body B with acceleration (F/m).

$$a_A = F/m$$

(b) The body B will remain at rest

$$a_B = 0$$

(c) If L is the length of B as shown in figure A will fall from B after time t

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{F}} \qquad \left[\text{As } s = \frac{1}{2}a\,t^2 \text{ and } a = F/m\right]$$

(ii) **If friction is present between *A* and *B* only and applied force is less than limiting friction ($F < F_l$)**

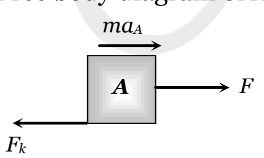
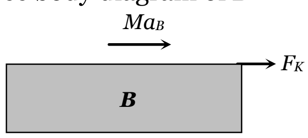
(F = Applied force on the upper body, F_l = limiting friction between A and B, F_k = Kinetic friction between A and B)

(a) The body A will not slide on body B till $F < F_l$ *i.e.* $F < \mu_s mg$

(b) Combined system ($m + M$) will move together with common acceleration $a_A = a_B = \frac{F}{M+m}$

(iii) **If friction is present between *A* and *B* only and applied force is greater than limiting friction ($F > F_l$)**

In this condition the two bodies will move in the same direction (*i.e.* of applied force) but with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of A while will cause the motion of B.

$F - F_k = m\,a_A$	Free body diagram of A	$F_k = M\,a_B$	Free body diagram of B
i.e. $a_A = \frac{F - F_k}{m}$		*i.e.* $a_B = \frac{F_k}{M}$	
$a_A = \frac{(F - \mu_k mg)}{m}$		$\therefore$ $a_B = \frac{\mu_k mg}{M}$	

Note : ❑ As both the bodies are moving in the same direction.

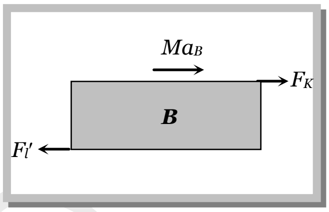
Acceleration of body A relative to B will be $a = a_A - a_B = \frac{MF - \mu_k mg\,(m+M)}{mM}$

So, A will fall from B after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2\,m\,ML}{MF - \mu_k mg\,(m+M)}}$

(iv) **If there is friction between B and floor**

(where $F_l' = \mu'(M+m)g$ = limiting friction between B and floor, F_k = kinetic friction between A and B)

B will move only if $F_k > F_l'$ and then $F_k - F_l' = M\,a_B$



However if B does not move then static friction will work (not limiting friction) between body B and the floor *i.e.* friction force = applied force (= F_k) not F_l'.

(2) **A force F is applied to the lower body, then following four situations are possible**

(i) **When there is no friction**

(a) B will move with acceleration (F/M) while A will remain at rest (relative to ground) as there is no pulling force on A.

$$a_B = \left(\frac{F}{M}\right) \text{ and } a_A = 0$$

(b) As relative to B, A will move backwards with acceleration (F/M) and so will fall from it in time t.

$$\therefore\ t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F}}$$

(ii) **If friction is present between A and B only and $F' < F_l$**

(where F' = Pseudo force on body A and F_l = limiting friction between body A and B)

(a) Both the body will move together with common acceleration $a = \frac{F}{M+m}$

(b) Pseudo force on the body A, $F' = ma = \frac{mF}{m+M}$ and $F_l = \mu_s mg$

(c) $F' < F_l \Rightarrow \frac{mF}{m+M} < \mu_s mg \Rightarrow F < \mu_s(m+M)g$

So both bodies will move together with acceleration $a_A = a_B = \frac{F}{m+M}$ if $F < \mu_s[m+M]g$

(iii) **If friction is present between A and B only and $F > F_l'$**

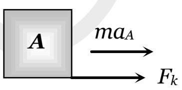
(where $F_l' = \mu_s\,(m+M)g$ = limiting friction between body B and surface)

Both the body will move with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of B while will cause the motion of A.

$ma_A = \mu_k mg$

i.e. $a_A = \mu_k g$

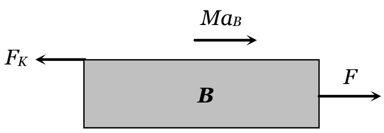
Free body diagram of A



$F - F_k = Ma_B$

i.e. $a_B = \frac{[F - \mu_k mg]}{M}$

Free body diagram of B



Note : ❑ As both the bodies are moving in the same direction

Acceleration of body A relative to B will be

$$a = a_A - a_B = -\left[\frac{F - \mu_k g(m+M)}{M}\right]$$

Negative sign implies that relative to B, A will move backwards and will fall it after time

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F - \mu_k g(m+M)}}$$

(iv) **If there is friction between B and floor :** The system will move only if $F > F_l'$ then replacing F by $F - F_l'$. The entire case (iii) will be valid.

However if $F < F_l'$ the system will not move and friction between B and floor will be F while between A and B is zero.

Sample problems based on body resting on another

Problem 19. A 4 *kg* block *A* is placed on the top of a 8 *kg* block *B* which rests on a smooth table. *A* just slips on *B* when a force of 12 *N* is applied on *A*. Then the maximum horizontal force on B to make both *A* and *B* move together, is

(a) 12 *N* (b) 24 *N* (c) 36 *N* (d) 48 *N*

Solution : (c) Maximum friction *i.e.* limiting friction between *A* and *B*, $F_l = 12\,N$.

If *F* is the maximum value of force applied on lower body such that both body move together

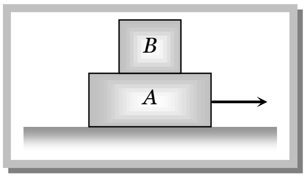
It means Pseudo force on upper body is just equal to limiting friction

$$F' = F_l \Rightarrow m\left(\frac{F}{m+M}\right) = \left(\frac{4}{4+8}\right)F = 12 \quad \therefore F = 36N.$$

Problem 20. A body *A* of mass 1 *kg* rests on a smooth surface. Another body *B* of mass 0.2 *kg* is placed over *A* as shown. The coefficient of static friction between *A* and *B* is 0.15. *B* will begin to slide on *A* if *A* is pulled with a force greater than

(a) 1.764 *N* (b) 0.1764 *N*

(c) 0.3 *N* (d) It will not slide for any *F*



Solution : (a) *B* will begin to slide on *A* if Pseudo force is more than limiting friction

$$F' > F_l \Rightarrow m\left(\frac{F}{m+M}\right) > \mu_s R \Rightarrow m\left(\frac{F}{m+M}\right) > 0.15mg \quad \therefore F > 1.764\,N$$

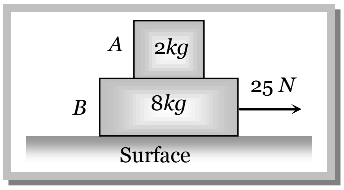
Problem 21. A block *A* of mass 2 *kg* rests on another block *B* of mass 8 *kg* which rests on a horizontal floor. The coefficient of friction between *A* and *B* is 0.2, while that between *B* and floor is 0.5. When a horizontal force of 25 *N* is applied on the block *B*, the force of friction between *A* and *B* is **[IIT-JEE 1993]**

(a) Zero (b) 3.9 *N* (c) 5.0 *N* (d) 49 *N*

Solution : (a) Limiting friction between the block *B* and the surface

$$F_{BS} = \mu_{BS}.R = 0.5(m+M)g = 0.5(2+8)10 = 50N$$

but the applied force is 25 *N* so the lower block will not move *i.e.* there is no pseudo force on upper block *A*. Hence there will be no force of friction between *A* and *B*.

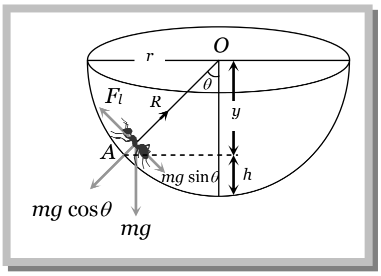


5.14 Motion of an Insect in the Rough Bowl

The insect crawl up the bowl up to a certain height h only till the component of its weight along the bowl is balanced by limiting frictional force.

Let m = mass of the insect, r = radius of the bowl, μ = coefficient of friction

for limiting condition at point A



$R = mg\cos\theta$ ......(i) and $F_l = mg\sin\theta$ ......(ii)

Dividing (ii) by (i)

$$\tan\theta = \frac{F_l}{R} = \mu \qquad [\text{As } F_l = \mu R]$$

$$\therefore \quad \frac{\sqrt{r^2 - y^2}}{y} = \mu \quad \text{or} \quad y = \frac{r}{\sqrt{1+\mu^2}}$$

So $$h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right], \therefore\ h = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$$

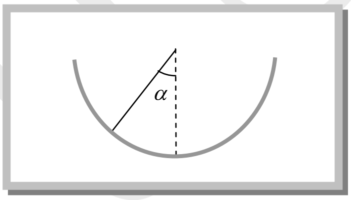
***Problem* 22.** An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by **[IIT-JEE (Screening) 2001]**

(a) $\cot\alpha = 3$

(b) $\tan\alpha = 3$

(c) $\sec\alpha = 3$

(d) cosec $\alpha = 3$



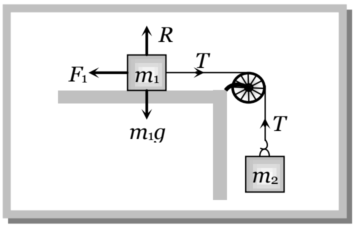
Solution : (a) From the above expression, for the equilibrium $R = mg\cos\alpha$ and $F = mg\sin\alpha$.

Substituting these value in $F = \mu R$ we get $\tan\alpha = \mu$ or $\cot\alpha = \frac{1}{\mu} = 3$.

5.15 Minimum Mass Hung From the String to Just Start the Motion

(1) **When a mass m_1 placed on a rough horizontal plane :** Another mass m_2 hung from the string connected by pulley, the tension (T) produced in string will try to start the motion of mass m_1.

At limiting condition



$T = F_l$

$\Rightarrow \quad m_2 g = \mu R$

$\Rightarrow \quad m_2 g = \mu m_1 g$

$\therefore\ m_2 = \mu m_1$ this is the minimum value of m_2 to start the motion.

Note : ❑ In the above condition Coefficient of friction $\mu = \frac{m_2}{m_1}$

(2) **When a mass m_1 placed on a rough inclined plane :** Another mass m_2 hung from the string connected by pulley, the tension (T) produced in string will try to start the motion of mass m_1.

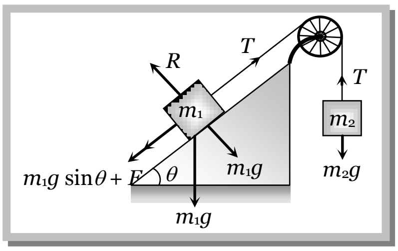
At limiting condition

For m_2 $\quad T = m_2 g$ ...... (i)

For m_1 $\quad T = m_1 g\sin\theta + F \Rightarrow \quad T = m_1 g\sin\theta + \mu R$

$\Rightarrow \quad T = m_1 g\sin\theta + \mu m_1 g\cos\theta$ ......(ii)

From equation (i) and (ii) $\quad m_2 = m_1[\sin\theta + \mu\cos\theta]$



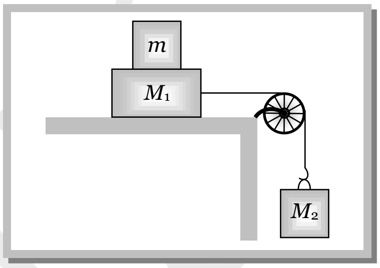
this is the minimum value of m_2 to start the motion

Note : ❑ In the above condition Coefficient of friction

$$\mu = \left[\frac{m_2}{m_1 \cos\theta} - \tan\theta\right]$$

Sample problems based on hung mass

***Problem* 23.** Two blocks of mass M_1 and M_2 are connected with a string passing over a pulley as shown in the figure. The block M_1 lies on a horizontal surface. The coefficient of friction between the block M_1 and horizontal surface is μ. The system accelerates. What additional mass m should be placed on the block M_1 so that the system does not accelerate



(a) $\frac{M_2 - M_1}{\mu}$

(b) $\frac{M_2}{\mu} - M_1$

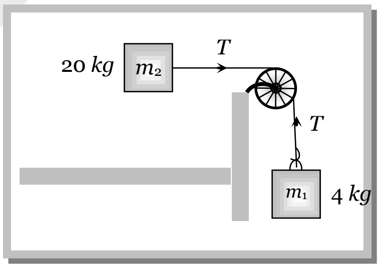
(c) $M_2 - \frac{M_1}{\mu}$

(d) $(M_2 - M_1)\mu$

Solution : (b) By comparing the given condition with general expression

$$\mu = \frac{M_2}{m + M_1} \Rightarrow m + M_1 = \frac{M_2}{\mu} \Rightarrow m = \frac{M_2}{\mu} - M_1$$

***Problem* 24.** The coefficient of kinetic friction is 0.03 in the diagram where mass $m_2 = 20\,kg$ and $m_1 = 4\,kg$. The acceleration of the block shall be $(g = 10 ms^{-2})$



(a) $1.8\, ms^{-2}$

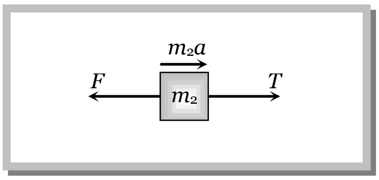
(b) $0.8\, ms^{-2}$

(c) $1.4\, ms^{-2}$

(d) $0.4\, ms^{-2}$

Solution : (c) Let the acceleration of the system is a

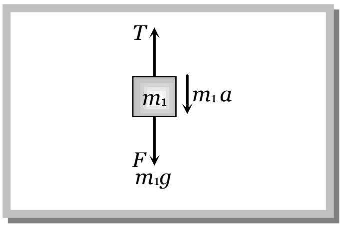
From the F.B.D. of m_2



$T - F = m_2 a \Rightarrow T - \mu m_2 g = m_2 a$

$\Rightarrow T - 0.03 \times 20 \times 10 = 20a \Rightarrow T - 6 = 20a$ .....(i)

From the FBD of m_1



$m_1 g - T = m_1 a$

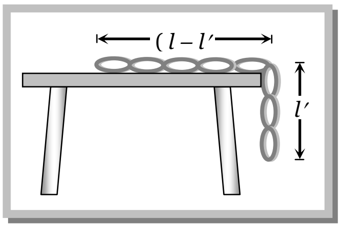
$\Rightarrow 4 \times 10 - T = 4a \Rightarrow 40 - T = 4a$ ....(ii)

Solving (i) and (ii) $a = 1.4 m/s^2$.

5.16 Maximum Length of Hung Chain

A uniform chain of length l is placed on the table in such a manner that its l' part is hanging over the edge of table with out sliding. Since the chain have uniform linear density therefore the ratio of mass or ratio of length for any part of the chain will be equal.

We know $\mu = \frac{m_2}{m_1} = \frac{\text{mass hanging from the table}}{\text{mass lying on the table}}$ [From article 5.15]



$\therefore$ For this expression we can rewrite above expression in the following manner

$\mu = \frac{\text{length hanging from the table}}{\text{length lying on the table}}$ [As chain have uniform linear density]

$\therefore\ \mu = \frac{l'}{l - l'}$

by solving $l' = \frac{\mu l}{(\mu + 1)}$

Problem **25.** A heavy uniform chain lies on a horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is **[CBSE PMT 1990]**

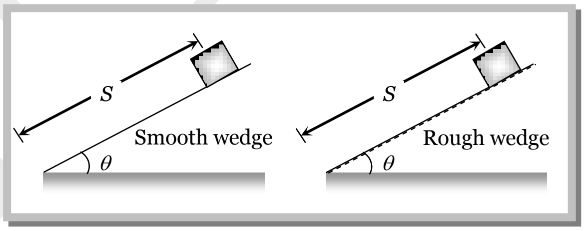
(a) 20% (b) 25% (c) 35% (d) 15%

Solution : (a) From the expression $l' = \left(\frac{\mu}{\mu + 1}\right) l = \left(\frac{0.25}{0.25 + 1}\right) l$ [As $\mu = 0.25$]

$\Rightarrow\ l' = \frac{0.25}{1.25} l = \frac{l}{5}$ = 20% of the length of the chain.

5.17 Coefficient of Friction Between Body and Wedge

A body slides on a smooth wedge of angle θ and its time of descent is t.



If the same wedge made rough then time taken by it to come down becomes n times more (*i.e.* nt) The length of path in both the cases are same.

For smooth wedge	For rough wedge
$S = ut + \frac{1}{2} at^2$	$S = ut + \frac{1}{2} at^2$
$S = \frac{1}{2}(g \sin\theta) t^2$ .....(i)	$S = \frac{1}{2} g(\sin\theta - \mu\cos\theta)(nt)^2$ .....(ii)
[As $u = 0$ and $a = g \sin\theta$]	[As $u = 0$ and $a = g(\sin\theta - \mu\cos\theta)$]

From equation (i) and (ii) $\frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2} g(\sin\theta - \mu\cos\theta)(nt)^2$

$\Rightarrow\ \sin\theta = (\sin\theta - \mu\cos\theta) n^2$

$$\Rightarrow \quad \mu = \tan\theta\left[1-\frac{1}{n^2}\right]$$

***Problem* 26.** A body takes just twice the time as long to slide down a plane inclined at 30° to the horizontal as if the plane were frictionless. The coefficient of friction between the body and the plane is **[JIPMER 1999]**

(a) $\frac{\sqrt{3}}{4}$ (b) $\sqrt{3}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

Solution : (a) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right) = \tan 30\left(1-\frac{1}{2^2}\right) = \frac{\sqrt{3}}{4}$.

5.18 Stopping of Block Due to Friction

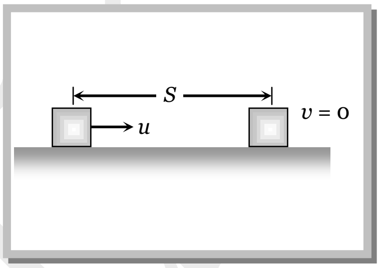
(1) **On horizontal road**

(i) **Distance travelled before coming to rest :** A block of mass m is moving initially with velocity u on a rough surface and due to friction it comes to rest after covering a distance S.

Retarding force $F = ma = \mu R$

$\Rightarrow \quad ma = \mu mg$

$\therefore \quad a = \mu g$.



From $v^2 = u^2 - 2aS \Rightarrow 0 = u^2 - 2\mu g S$ [As $v = 0$, $a = \mu g$]

$\therefore \quad S = \frac{u^2}{2\mu g}$

or $\quad S = \frac{P^2}{2\mu m^2 g}$ [As momentum $P = mu$]

(ii) **Time taken to come to rest**

From equation $v = u - at \Rightarrow 0 = u - \mu g t$ [As $v = 0, a = \mu g$]

$\therefore t = \frac{u}{\mu g}$

(iii) **Force of friction acting on the body**

We know, $F = ma$

So, $F = m\frac{(v-u)}{t}$

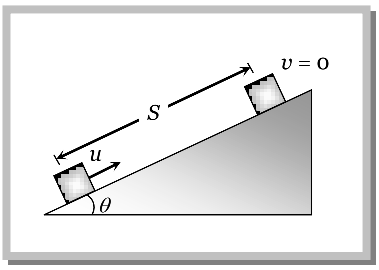
$F = \frac{mu}{t}$ [As $v = 0$]

$F = \mu mg$ $\left[\text{As } t = \frac{u}{\mu g}\right]$

(2) **On inclined road :** When block starts with velocity u its kinetic energy will be converted into potential energy and some part of it goes against friction and after travelling distance S it comes to rest *i.e.* v = 0.

And we know that retardation $a = g[\sin\theta + \mu\cos\theta]$

By substituting the value of v and a in the following equation



$v^2 = u^2 - 2aS$

$\Rightarrow \quad 0 = u^2 - 2g[\sin\theta + \mu\cos\theta]S$

$\therefore S = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$

Sample problems based on motion of body on rough surface

Problem 27. A marble block of mass 2 *kg* lying on ice when given a velocity of 6 *m/s* is stopped by friction in 10*s*. Then the coefficient of friction is

(a) 0.01 (b) 0.02 (c) 0.03 (d) 0.06

Solution : (d) $v = u - at = u - \mu g\,t = 0$ $\therefore \mu = \frac{u}{gt} = \frac{6}{10 \times 10} = 0.06$.

Problem 28. A 2 *kg* mass starts from rest on an inclined smooth surface with inclination 30° and length 2 *m*. How much will it travel before coming to rest on a surface with coefficient of friction 0.25

(a) 4 *m* (b) 6 *m* (c) 8 *m* (d) 2 *m*

Solution : (a) $v^2 = u^2 + 2aS = 0 + 2 \times g \sin 30 \times 2$ $v = \sqrt{20}$

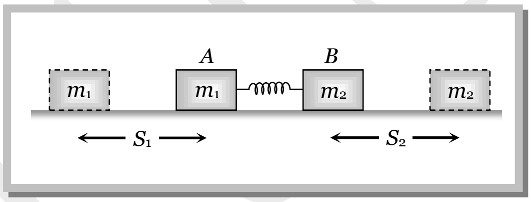
Let it travel distance '*S*' before coming to rest

$$S = \frac{v^2}{2\mu g} = \frac{20}{2 \times 0.25 \times 10} = 4m.$$

5.19 Stopping of Two Blocks Due to Friction

When two masses compressed towards each other and suddenly released then energy acquired by each block will be dissipated against friction and finally block comes to rest

i.e., $F \times S = E$ [Where F = Friction, S = Distance covered by block, E = Initial kinetic energy of the block]



$\Rightarrow F \times S = \frac{P^2}{2m}$ [Where P = momentum of block]

$\Rightarrow \mu mg \times S = \frac{P^2}{2m}$ [As $F = \mu mg$]

$\Rightarrow S = \frac{P^2}{2\mu m^2 g}$

In a given condition P and μ are same for both the blocks.

So $S \propto \frac{1}{m^2}$ $\therefore \frac{S_1}{S_2} = \left[\frac{m_2}{m_1}\right]^2$

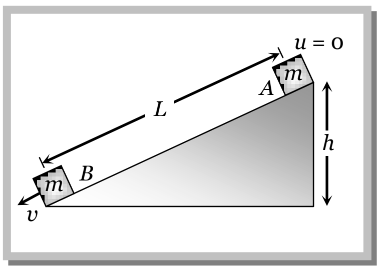
5.20 Velocity at the Bottom of Rough Wedge

A body of mass m which is placed at the top of the wedge (of height h) starts moving downward on a rough inclined plane.

Loss of energy due to friction = FL (Work against friction)

PE at point $A = mgh$

KE at point $B = \frac{1}{2}mv^2$



By the law of conservation of energy

i.e. $\frac{1}{2}mv^2 = mgh - FL$

$$v = \sqrt{\frac{2}{m}(mgh - FL)}$$

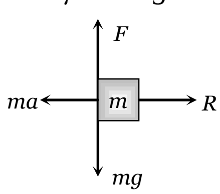
5.21 Sticking of a Block With Accelerated Cart

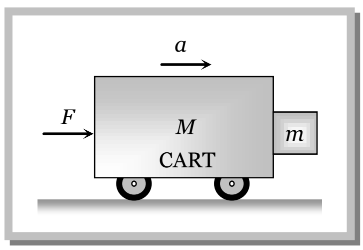
When a cart moves with some acceleration toward right then a pseudo force (ma) acts on block toward left.

This force (ma) is action force by a block on cart.

Now block will remain static *w.r.t.* block. If friction force $\mu R \geq mg$

$\Rightarrow$ $\mu ma \geq mg$ [As $R = ma$]





$\Rightarrow \quad a \geq \frac{g}{\mu}$

$\therefore \quad a_{\min} = \frac{g}{\mu}$

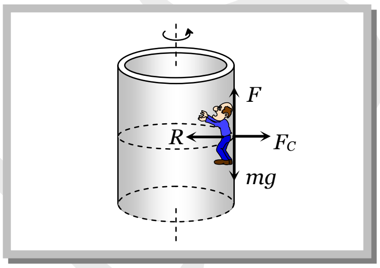
This is the minimum acceleration of the cart so that block does not fall.
and the minimum force to hold the block together

$F_{\min} = (M+m)a_{\min}$ OR $F_{\min} = (M+m)\frac{g}{\mu}$

5.22 Sticking of a Person With the Wall of Rotor

A person with a mass m stands in contact against the wall of a cylindrical drum (rotor). The coefficient of friction between the wall and the clothing is μ.

If Rotor starts rotating about its axis, then person thrown away from the centre due to centrifugal force at a particular speed w, the person stuck to the wall even the floor is removed,
because friction force balances its weight in this condition.



From the figure.

Friction force (F) = weight of person (mg)

$\Rightarrow \quad \mu R = mg$

$\Rightarrow \quad \mu F_c = mg$ [Here, F_c= centrifugal force]

$\Rightarrow \quad \mu m\omega_{\min}^2 r = mg$

$\therefore \quad \omega_{\min} = \sqrt{\frac{g}{\mu r}}$

Sample problems (Miscellaneous)

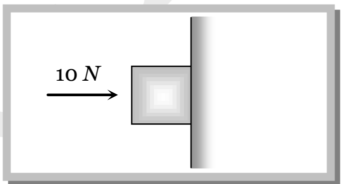
Problem 29. A motorcycle is travelling on a curved track of radius 500m if the coefficient of friction between road and tyres is 0.5. The speed avoiding skidding will be **[MH CET (Med.) 2001]**

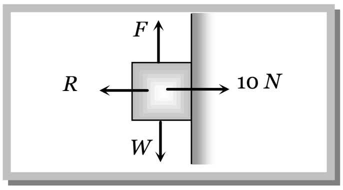
(a) 50 m/s (b) 75 m/s (c) 25 m/s (d) 35 m/s

Solution : (a) $v = \sqrt{\mu rg} = \sqrt{0.5 \times 500 \times 10} = 50\,m/s.$

Problem 30. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is **[AIEEE 2003]**

(a) 2 N
(b) 20 N
(c) 50 N
(d) 100 N





Solution : (a) For equilibrium

Weight (W) = Force of friction (F)

$W = \mu R = 0.2 \times 10 = 2N$

Problem 31. A body of mass 2 kg is kept by pressing to a vertical wall by a force of 100 N. The friction between wall and body is 0.3. Then the frictional force is equal to **[Orissa JEE 2003]**

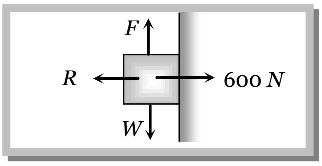
(a) 6 N (b) 20 N (c) 600 N (d) 700 N

Solution : (b) For the given condition Static friction = Applied force = Weight of body = 2 × 10 = 20 N.

Problem 32. A fireman of mass 60kg slides down a pole. He is pressing the pole with a force of 600 N. The coefficient of friction between the hands and the pole is 0.5, with what acceleration will the fireman slide down (g = 10 m/s^2) **[Pb. PMT 2002]**

(a) 1 m/s^2 (b) 2.5 m/s^2 (c) 10 m/s^2 (d) 5 m/s^2

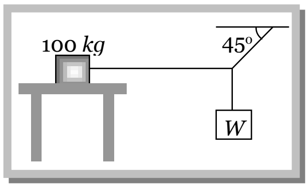
Solution : (d) Friction $= \mu R = 0.5 \times 600 = 300\,N$, Weight $= 600\,N$



$ma = W - F \Rightarrow a = \frac{W-F}{m} = \frac{600-300}{60}$

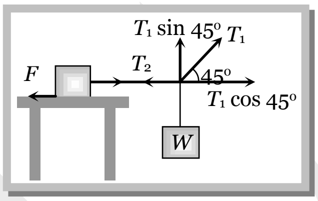
$\therefore \; a = 5\,m/s^2$

***Problem* 33.** The system shown in the figure is in equilibrium. The maximum value of W, so that the maximum value of static frictional force on 100 *kg*. body is 450 *N*, will be



(a) 100 *N*
(b) 250 *N*
(c) 450 *N*
(d) 1000 *N*

Solution : (c) For vertical equilibrium $T_1 \sin 45^o = W$ $\quad \therefore \; T_1 = \frac{W}{\sin 45^o}$



For horizontal equilibrium $T_2 = T_1 \cos 45^o = \frac{W}{\sin 45^o} \cos 45^o = W$

and for the critical condition $T_2 = F$ $\;\therefore\; W = T_2 = F = 450\,N$

Practice Problems

▶ *Basic level*

1. When a body is moving on a surface, the force of friction is called **[MP PET 2002]**

(a) Static friction (b) Dynamic friction (c) Limiting friction (d) Rolling friction

2. Which one of the following is not used to reduce friction **[Kerala (Engg.) 2001]**

(a) Oil (b) Ball bearings (c) Sand (d) Graphite

3. A block of mass 10 *kg* is placed on an inclined plane. When the angle of inclination is 30°, the block just begins to slide down the plane. The force of static friction is

(a) 10 *kg wt* (b) 89 *kg wt* (c) 49 *kg wt* (d) 5 *kg wt*

4. A vehicle of mass m is moving on a rough horizontal road with momentum P. If the coefficient of friction between the tyres and the road be μ, then the stopping distance is **[CBSE PMT 2001]**

(a) $\frac{P}{2\mu m g}$ (b) $\frac{P^2}{2\mu m g}$ (c) $\frac{P}{2\mu m^2 g}$ (d) $\frac{P^2}{2\mu m^2 g}$

5. A box is lying on an inclined plane what is the coefficient of static friction if the box starts sliding when an angle of inclination is 60°

[KCET (Engg./Med.) 2000]

(a) 1.173 (b) 1.732 (c) 2.732 (d) 1.677

6. A brick of mass 2 *kg* begins to slide down on a plane inclined at an angle of 45° with the horizontal. The force of friction will be

[CPMT 2000]

(a) 19.6 sin 45° (b) 19.6 cos 45° (c) 9.8 sin 45° (d) 9.8 cos 45°

7. To avoid slipping while walking on ice, one should take smaller steps because of the **[BHU 1999]**

(a) Friction of ice is large (b) Larger normal reaction

(c) Friction of ice is small (d) Smaller normal reaction

8. Two bodies having the same mass, 2 *kg* each have different surface areas 50 m^2 and 100 m^2 in contact with a horizontal plane. If the coefficient of friction is 0.2, the forces of friction that come into play when they are in motion will be in the ratio **[EAMCET (Med.) 1999]**

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

9. Starting from rest, a body slides down a 45^o inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is **[CBSE PMT 1990]**

(a) 0.33 (b) 0.25 (c) 0.75 (d) 0.80

10. Brakes of very small contact area are not used although friction is independent of area, because friction

(a) Resists motion (b) Causes wear and tear

(c) Depends upon the nature of materials (d) Operating in this case is sliding friction

11. The angle between frictional force and the instantaneous velocity of the body moving over a rough surface is

(a) Zero (b) $\pi/2$

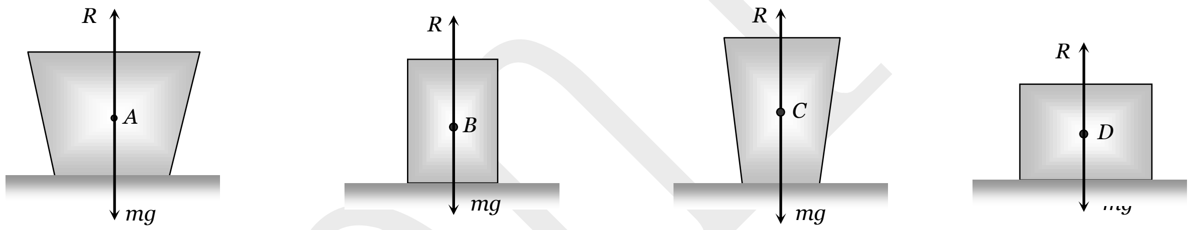
(c) π (d) Equal to the angle of friction

12. What happens to the coefficient of friction, when the normal reaction is halved

(a) Halved (b) Doubled

(c) No change (d) Depends on the nature of the surface

13. What can be inferred regarding the limiting frictional force in the following four figures



(a) $F_A = F_B = F_C = F_D$ (b) $F_A > F_B > F_C > F_D$ (c) $F_A < F_B < F_C < F_D$ (d) $F_A = F_B < F_C < F_D$

14. A force of 98 Newton is required to drag a body of mass 100 *kg* on ice. The coefficient of friction will be

(a) 0.98 (b) 0.89 (c) 0.49 (d) 0.1

15. A 60 *kg* body is pushed with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficients of static and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is

(a) $6\, m/\sec^2$ (b) $4.9\, m/\sec^2$ (c) $3.92\, m/\sec^2$ (d) $1\, m/\sec^2$

16. A particle is projected along a line of greatest slope up a rough plane inclined at an angle of 45^o with the horizontal. If the coefficient of friction is $\frac{1}{2}$, then the retardation is

(a) $\frac{g}{\sqrt{2}}$ (b) $\frac{g}{2\sqrt{2}}$ (c) $\frac{g}{\sqrt{2}}\left[1+\frac{1}{2}\right]$ (d) $\frac{g}{\sqrt{2}}\left[1-\frac{1}{2}\right]$

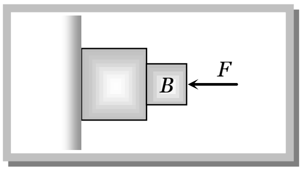
17. A block moves down a smooth inclined plane of inclination θ. Its velocity on reaching the bottom is v. If it slides down a rough inclined plane of same inclination its velocity on reaching the bottom is v/n, where n is a number greater than 0. The coefficient of friction μ is given by

18. (a) $\mu = \tan\theta\left[1-\frac{1}{n^2}\right]$ (b) $\mu = \cot\theta\left[1-\frac{1}{n^2}\right]$ (c) $\mu = \tan\theta\left[1-\frac{1}{n^2}\right]^{\frac{1}{2}}$ (d) $\mu = \cot\theta\left[1-\frac{1}{n^2}\right]^{\frac{1}{2}}$

19. Consider a car moving along a straight horizontal road with a speed of 72 *km/hr*. If the coefficient of static friction between the tyres and the road is 0.5, the shortest distance in which the car can be stopped is $(g = 10 m/s^2)$

(a) 30 *m* (b) 40 *m* (c) 72 *m* (d) 20 *m*

20. All the surfaces shown in the figure are rough. The direction of friction on *B* due to *A* is



(a) Zero

(b) To the left

(c) Upwards

(d) Downwards

21. A body of mass *M* just starts sliding down an inclined plane (rough) with inclination θ, such that $\tan\theta = 1/3$. The force acting on the body down the plane in this position is

(a) Mg (b) $\frac{Mg}{3}$ (c) $\frac{2}{3}Mg$ (d) $\frac{Mg}{\sqrt{10}}$

▶▶ *Advance level*

22. Consider the following statements

Assertion (*A*) : It is difficult to move a cycle along the road with its brakes on.

Reason (*R*) : Sliding friction is greater than rolling friction.

Of these statements **[AIIMS 2002]**

(a) Both *A* and *R* are true and the *R* is a correct explanation of the *A*

(b) Both *A* and *R* are true but the *R* is not a correct explanation of the *A*

(c) *A* is true but the *R* is false

(d) Both *A* and *R* are false

(e) *A* is false but the *R* is true

23. A body is sliding down an inclined plane having coefficient of friction 0.5. If the normal reaction is twice that of the resultant downward force along the incline, the angle between the inclined plane and the horizontal is **[EAMCET (Engg.) 2000]**

(a) 15° (b) 30° (c) 45° (d) 60°

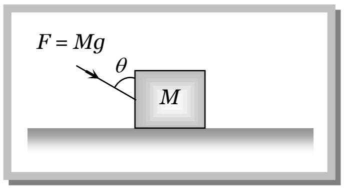
24. A block of mass 2 *kg* rests on a rough inclined plane making an angle of 30^o with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is **[IIT-JEE 1980]**

(a) $9.8\,N$ (b) $0.7 \times 9.8 \times \sqrt{3}\,N$ (c) $9.8 \times \sqrt{3}\,N$ (d) $0.7 \times 9.8\,N$

25. A body of weight *W* is lying at rest on a rough horizontal surface. If the angle of friction is θ, then the minimum force required to move the body along the surface will be

(a) $W\tan\theta$ (b) $W\cos\theta$ (c) $W\sin\theta$ (d) $W\cos\theta$

26. A block of mass *M* is placed on a rough horizontal surface as shown in the figure. A force $F = Mg$ acts on the block. It is inclined to the vertical at an angle θ. The coefficient of friction is μ. The block can be pushed along the surface only when



(a) $\tan\theta \geq \mu$

(b) $\cot\theta \geq \mu$

(c) $\tan\theta/2 \geq \mu$

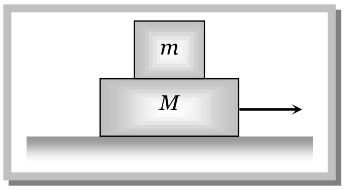
(d) $\cot\theta/2 \geq \mu$

27. A plane is inclined at an angle θ with the horizontal. A body of mass *m* rests on it. If the coefficient of friction is μ, then the minimum force that has to be applied parallel to the inclined plane to make the body just move up the inclined plane is

(a) $mg\sin\theta$ (b) $\mu\,mg\cos\theta$

(c) $\mu\,mg\cos\theta - mg\sin\theta$ (d) $\mu\,mg\cos\theta + mg\sin\theta$

28. A block of mass m is placed on another block of mass M which itself is lying on a horizontal surface. The coefficient of friction between the two blocks is μ_1 and that between the block of mass M and horizontal surface is μ_2. What maximum horizontal force can be applied to the lower block so that the two blocks move without separation



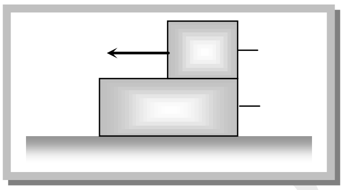
(a) $(M+m)(\mu_2-\mu_1)g$

(b) $(M-m)(\mu_2-\mu_1)g$

(c) $(M-m)(\mu_2+\mu_1)g$

(d) $(M+m)(\mu_2+\mu_1)g$

29. A block of mass M_1 is placed on a slab of mass M_2. The slab lies on a frictionless horizontal surface. The coefficient of static friction between the block and slab is μ_1 and that of dynamic friction is μ_2. A force F acts on the block M_1. Take $g = 10\ ms^{-2}$. If $M_1 = 10\ kg$, $M_2 = 30\ kg$, $\mu_1 = 0.5$, $\mu_2 = 0.15$ and $F = 40\ N$, what will be the acceleration with which the slab will move



(a) $5\ ms^{-2}$

(b) $2\ ms^{-2}$

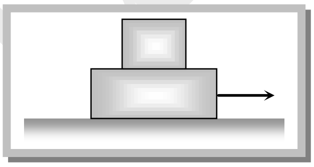
(c) $1\ ms^{-2}$

(d) Zero

30. In the above problem if $F = 100\ N$, what will be the acceleration with which the slab will move

(a) $5\ ms^{-2}$ (b) $2\ ms^{-2}$ (c) $1\ ms^{-2}$ (d) None of these

31. A block X of mass 4 kg is lying on another block Y of mass 8 kg. As shown in the figure. When the force acting on X is 12N, block X is on the verge of slipping on Y. The force F in Newton necessary to make both X and Y move simultaneously will be



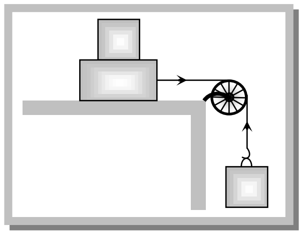
(a) 36

(b) 3.6

(c) 0.36

(d) 3.6

32. Two masses $10\ kg$ and $5\ kg$ are connected by a string passing over a pulley as shown. If the coefficient of friction be 0.15, then the minimum weight that may be placed on 10 kg to stop motion is



(a) 18.7 kg

(b) 23.3 kg

(c) 32.5 kg

(d) 44.3 kg

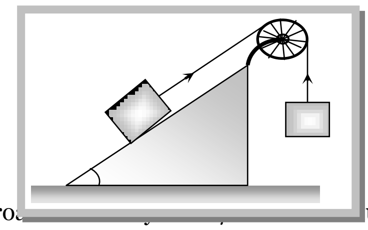
33. Two blocks of mass M_1 and M_2 are connected with a string which passes over a smooth pulley. The mass M_1 is placed on a rough inclined plane as shown in the figure. The coefficient of friction between the block and the inclined plane is μ. What should be the maximum mass M_2 so that block M_1 slides downwards

(a) $M_2 = M_1(\sin\theta + \mu\cos\theta)$

(b) $M_2 = M_1(\sin\theta - \mu\cos\theta)$

(c) $M_2 = M_1/(\sin\theta + \mu\cos\theta)$

(d) $M_2 = M_1/(\sin\theta - \mu\cos\theta)$



34. A car starts from rest to cover a distance s. the coefficient of friction between the ro… um time in which the car can cover the distance is proportional to

(a) μ (b) $\sqrt{\mu}$ (c) $\frac{1}{\mu}$ (d) $\frac{1}{\sqrt{\mu}}$

35. An engine of mass 50,000 kg pulls a coach of mass 40,000 kg. If there is a resistance of 1 N per 100 kg acting on both the engine and the coach, and if the driving force of the engine be 4,500 N, then the acceleration of the engine is

(a) $0.08\, m/s^2$ (b) Zero (c) $0.04\, m/s^2$ (d) None of these

36. In the above question, then tension in the coupling is

(a) 2,000 N (b) 1,500 N (c) 500 N (d) 1000 N

37. An aeroplane requires for take off a speed of 72 km/h. The run of the ground is 100m.The mass of the plane is 10^4 kg and the coefficient of friction between the plane and the ground is 0.2. The plane accelerates uniformly during take off. What is the acceleration of the plane

(a) 1 m/s^2 (b) 2 m/s^2 (c) 3 m/s^2 (d) 4 m/s^2

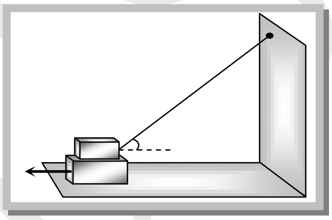
38. The force required to just move a body up an inclined plane is double the force required to just prevent it from sliding down. If ϕ is angle of friction and θ is the angle which incline makes with the horizontal then

(a) $\tan\theta = \tan\phi$ (b) $\tan\theta = 2\tan\phi$ (c) $\tan\theta = 3\tan\phi$ (d) $\tan\phi = 3\tan\theta$

39. A body is on a rough horizontal plane. A force is applied to the body direct towards the plane at an angle ϕ with the vertical. If θ is the angle of friction then for the body to move along the plane

(a) $\phi > \theta$ (b) $\phi < \theta$ (c) $\phi = \theta$ (d) ϕ can take up any value

40. In the arrangement shown $W_1 = 200\, N$, $W_2 = 100\, N$, $\mu = 0.25$ for all surfaces in contact. The block W_1 just slides under the block W_2



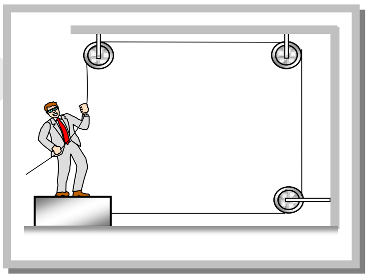
(a) A pull of $50\, N$ is to be applied on W_1

(b) A pull of $90\, N$ is to be applied on W_1

(c) Tension in the string AB is $10\sqrt{2}\, N$

(d) Tension in the string AB is $20\sqrt{2}\, N$

41. A board of mass m is placed on the floor and a man of mass M is standing on the board as shown. The coefficient of friction between the board and the floor is μ. The maximum force that the can exert on he rope so that the board does not slip on the floor is



(a) $F = \mu(M+m)g$

(b) $F = \mu mg$

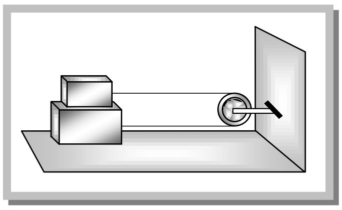
(c) $F = \dfrac{\mu Mg}{\mu + 1}$

(d) $F = \dfrac{\mu(M+m)g}{\mu+1}$

42. A body slides over an inclined plane forming an angle of 45° with the horizontal. The distance x travelled by the body in time t is described by the equation $x = kt^2$, where $k = 1.732$. The coefficient of friction between the body and the plane has a value

(a) $\mu = 0.5$ (b) $\mu = 1$ (c) $\mu = 0.25$ (d) $\mu = 0.75$

43. Two blocks A and B of masses m and M respectively are placed on each other and their combination rests on a fixed horizontal surface C. A light string passing over the smooth light pulley is used to connect A and B as shown. The coefficient of sliding friction between all surfaces in contact is μ. If A is dragged with a force F then for both A and B to move with a uniform speed we have



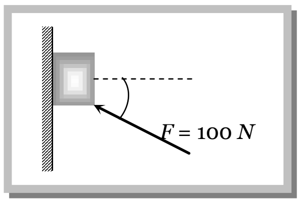
(a) $F = \mu(M+m)g$

(b) $F = \mu mg$

(c) $F = \mu(3M+m)g$

(d) $F = \mu(3m+M)g$

44. A force of 100 N is applied on a block of mass 3 kg as shown in figure. The coefficient of friction between the surface of the block is 1/4. The friction force acting on the block is



(a) 15 N downwards (b) 25 N upwards (c) 20 N downwards (d) 20 N upwards

Answer Sheet (Practice problems)

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
b	c	d	d	b	a	c	a	c	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	c	a	d	d	c	a	b	c	d
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
a	c	a	a	c	d	d	c	d	a
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
b	b	d	c	a	b	c	a	b, d	d
41.	**42.**	**43.**	**44**						
a	d	c	b						