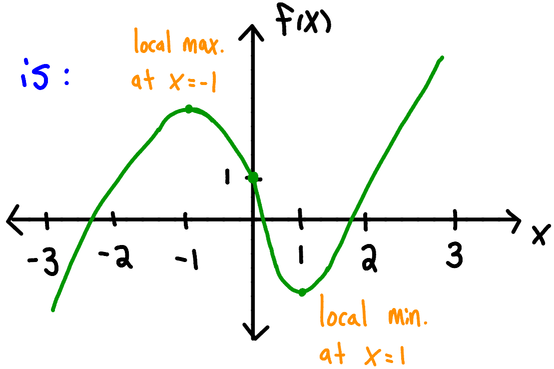
Test #2: VII

Written Portion

$\boxed{a}$ We know that f is differentiable for all values of x and $f'(-1) = f'(1) = 0$. Hence $x=-1$ and $x=1$ are critical points of f. Since $f'(x) > 0$ for all x in $(-\infty, -1)$ and $f'(x) < 0$ for all x in $(-1, 1)$, by the First Derivative Test we can conclude that f has a <u>local maximum at $x=-1$</u>. Similarly, since $f'(x) < 0$ on $(-1, 1)$ and $f'(x) > 0$ on $(1, \infty)$, by the First Derivative Test we can conclude that f has a <u>local minimum at $x=1$</u>.

$\boxed{b}$ A possible graph for f on $(-3, 3)$ is:



$\boxed{c}$ Since f is differentiable for all values of x, it's also continuous for all values of x. In particular, f is continuous on $[6, 11]$ and differentiable on $(6, 11)$. Hence, by the Mean Value Theorem, there exists a number c in $(6, 11)$ such that $f'(c) = \dfrac{f(11) - f(6)}{11 - 6} = \dfrac{f(11) - 4}{5}$.

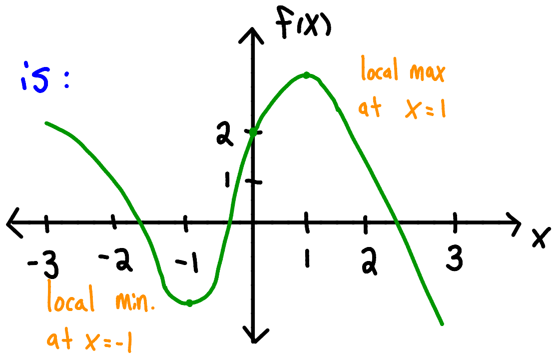
We know $f'(x) \geq 2$ for all $x \geq 5$, and so since c is in $(6, 11)$ we have $f'(c) \geq 2$. Hence, $\dfrac{f(11) - 4}{5} \geq 2 \Rightarrow f(11) - 4 \geq 10 \Rightarrow f(11) \geq 14$. Therefore, the smallest $f(11)$ could be is 14.

Test #2: V33

Written Portion

$\boxed{a}$ We know that f is differentiable for all values of x and $f'(-1) = f'(1) = 0$. Hence $x=-1$ and $x=1$ are critical points of f. Since $f'(x) > 0$ for all x in $(-1,1)$ and $f'(x) < 0$ for all x in $(1,\infty)$, by the First Derivative Test we can conclude that f has a <u>local maximum at $x=1$.</u> Similarly, since $f'(x) < 0$ on $(-1,\infty)$ and $f'(x) > 0$ on $(-1,1)$, by the First Derivative Test we can conclude that f has a <u>local minimum at $x=-1$.</u>

$\boxed{b}$ A possible graph for f on $(-3,3)$ is:



$\boxed{c}$ Since f is differentiable for all values of x, it's also continuous for all values of x. In particular, f is continuous on $[7,11]$ and differentiable on $(7,11)$. Hence, by the Mean Value Theorem, there exists a number c in $(7,11)$ such that $f'(c) = \dfrac{f(11) - f(7)}{11-7} = \dfrac{-10-f(7)}{4}$.

We know $f'(x) \leq -2$ for all $x \geq 5$, and so since c is in $(7,11)$ we have $f'(c) \leq -2$. Hence, $\dfrac{-10-f(7)}{4} \leq -2 \Rightarrow -10-f(7) \leq -8 \Rightarrow f(7) \geq -2$.

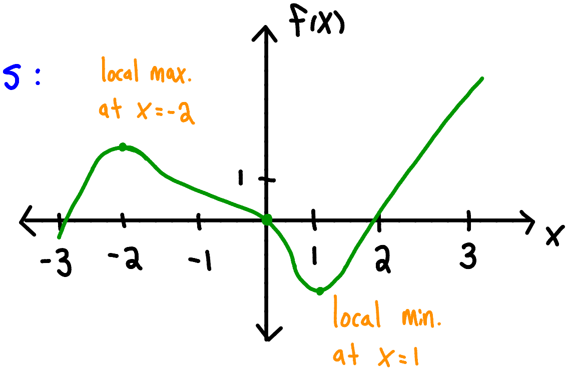
Therefore, the smallest $f(7)$ could be is -2.

Test #2: V55

Written Portion

a We know that f is differentiable for all values of x and $f'(-2) = f'(1) = 0$. Hence $x = -2$ and $x = 1$ are critical points of f. Since $f'(x) > 0$ for all x in $(-\infty, -2)$ and $f'(x) < 0$ for all x in $(-2, 1)$, by the First Derivative Test we can conclude that f has a <u>local maximum at $x = -2$.</u> Similarly, since $f'(x) < 0$ on $(-2, 1)$ and $f'(x) > 0$ on $(1, \infty)$, by the First Derivative Test we can conclude that f has a <u>local minimum at $x = 1$.</u>

b A possible graph for f on $(-3,3)$ is:



c Since f is differentiable for all values of x, it's also continuous for all values of x. In particular, f is continuous on $[7,11]$ and differentiable on $(7,11)$. Hence, by the Mean Value Theorem, there exists a number c in $(7,11)$ such that $f'(c) = \dfrac{f(11)-f(7)}{11-7} = \dfrac{20-f(7)}{4}$.

We know $f'(x) \geq 3$ for all $x \geq 5$, and so since c is in $(7,11)$ we have $f'(c) \geq 3$. Hence, $\dfrac{20-f(7)}{4} \geq 3 \Rightarrow 20-f(7) \geq 12 \Rightarrow f(7) \leq 8$.

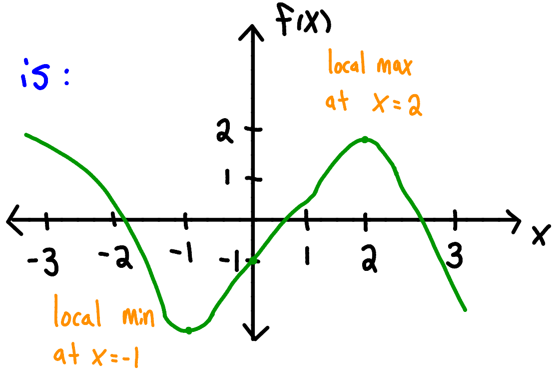
Therefore, the largest $f(7)$ could be is 8.

Test #2: V77

Written Portion

a) We know that f is differentiable for all values of x and $f'(-1) = f'(2) = 0$. Hence $x = -1$ and $x = 2$ are critical points of f. Since $f'(x) > 0$ for all x in $(-1, 2)$ and $f'(x) < 0$ for all x in $(2, \infty)$, by the First Derivative Test we can conclude that f has a local maximum at $x = 2$. Similarly, since $f'(x) < 0$ on $(-\infty, -1)$ and $f'(x) > 0$ on $(-1, 2)$, by the First Derivative Test we can conclude that f has a local minimum at $x = -1$.

$\boxed{b}$ A possible graph for f on $(-3,3)$ is:



$\boxed{c}$ Since f is differentiable for all values of x, it's also continuous for all values of x. In particular, f is continuous on $[6,11]$ and differentiable on $(6,11)$. Hence, by the Mean Value Theorem, there exists a number c in $(6,11)$ such that $f'(c) = \dfrac{f(11)-f(6)}{11-6} = \dfrac{f(11)-(-2)}{5}$.

We know $f'(x) \leq -3$ for all $x \geq 5$, and so since c is in $(6,11)$ we have $f'(c) \leq -3$. Hence, $\dfrac{f(11)-(-2)}{5} \leq -3 \Rightarrow f(11)+2 \leq -15 \Rightarrow f(11) \leq -17$.

Therefore, the largest $f(11)$ could be is -17.