MATH 249 & 265: DINO PROBLEMS
TOPIC 3: LIMITS AT INFINITY, CONTINUITY AND THE INTERMEDIATE VALUE THEOREM

Problem 1. Let b denote the last nonzero digit of your UCID number, that is, if your UCID number is 9876543210 then $b = 1$.

Evaluate the following limits.

(1) $\displaystyle\lim_{x\to-\infty} \frac{bx^3 + 5x}{4x^3 - 2bx^2 + 1}$

(2) $\displaystyle\lim_{x\to\infty} \frac{x^6 - 3x^2 + 4}{bx + 2x^7}$

(3) $\displaystyle\lim_{x\to-\infty} \frac{x^6 - 3x^2 + 4}{bx + 1}$

(4) $\displaystyle\lim_{x\to\infty} \frac{\sin(x^2)}{bx}$

Note: by "Evaluate" we mean: "Calculate the limit if it exists, or show that the limit does not exist. Explain your answers."

Problem 2. Let f be a function such that $\lim_{x\to\infty} f(x) = 8$. Calculate $\lim_{x\to\infty} \left(x - \sqrt{x^2 + xf(x)}\right)$.

Problem 3. Find formulas for two functions, f and g, such that $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to\infty} g(x) = \infty$ and

(1) $\lim_{x\to\infty} (f(x) - g(x)) = \infty$

(2) $\lim_{x\to\infty} (f(x) - g(x)) = -\infty$

(3) $\lim_{x\to\infty} (f(x) - g(x)) = 0$

(4) $\lim_{x\to\infty} (f(x) - g(x)) = 42$

Problem 4. Let $P(x) = a_m x^m + \ldots + a_1 x + a_0$ and $Q(x) = b_n x^n + \ldots + b_1 x + b_0$ be polynomials of degree m and n, respectively. Find $\displaystyle\lim_{x\to\infty} \frac{P(x)}{Q(x)}$ if ...

(1) $m = n$.

(2) $m < n$.

(3) $m > n$.

Problem 5. The population of demogorgons in Hawkins, Indiana is given by the function

$$D(t) = \frac{1600}{1 + 7e^{-0.02t}}$$

where t is time measured in weeks (from time $t = 0$ when Eleven opened The Gate to The Upside Down).

(1) What happens to the population of demogorgons in the long run, i.e., as the time goes to ∞?

(2) Explain what your answer has to do with asymptotes of the population function $D(t)$.

(3) Use an internet search, or your favourite biology textbook, to interpret the results of this problem in the language of biology/ecology. What quantity have we found?

Date: August 31, 2020.

Problem 6. Let C be a positive constant. Calculate the limit

$$\lim_{x\to-\infty} \frac{\sqrt{Cx^2+2x}}{x+1}$$

show all of your work.

Problem 7. Let c and m be unknown constants. Let

$$h(x) = \begin{cases} cx^2 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ -x^3 + mx & \text{if } x > 1 \end{cases}.$$

Are there values of the constants c and m that make $h(x)$ continuous at $x = 1$? Find c and m, or explain why they do not exist.

Problem 8. Recall that a real number is rational if it can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Consider the function f given by

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}.$$

(1) Sketch the graph of $y = f(x)$.

(2) Use the Squeeze Theorem to calculate $\lim_{x\to 0} f(x)$.

(3) Is the function f continuous at $x = 0$?

(4) What does this tell you about what continuous functions "look like"?

Problem 9. Let f be a continuous function on the closed interval $[-4, 6]$. Determine if the following statements are true or false. For each, give a brief (one-sentence) explanation of your answer.

(1) If $f(-1) = 5$ and $f(3) = -5$, then there exists a number c in the open interval $(-1, 3)$ so that $f(c) = 2$.

(2) If $f(-4) = -2$ and $f(6) = 5$, then at some point in the closed interval $[-2, 5]$ the function f must take the value 4.

The following are Multiple-Choice Problems.

Problem 10. Suppose f is a continuous function on the real line and that the following data is known:

$$f(0) = 1,\ f(2) = 4,\ f(3) = 6,\ f(4) = 5,\ \text{and } f(5) = -1.$$

By the Intermediate Value Theorem, which of the following is the **smallest** (i.e., shortest length) interval on which a solution to the equation $f(c) = 0$ is guaranteed?

(a) $[0, 5]$.

(b) $(4, 5)$.

(c) $(0, 4)$.

(d) $(-1, 5)$.

(e) None of the above.

Problem 11. Let $f(x) = \dfrac{1}{x+1}$ and $g(x) = x^2 - 5$. Find all values of x for which $f(g(x))$ is discontinuous.

(a) -1

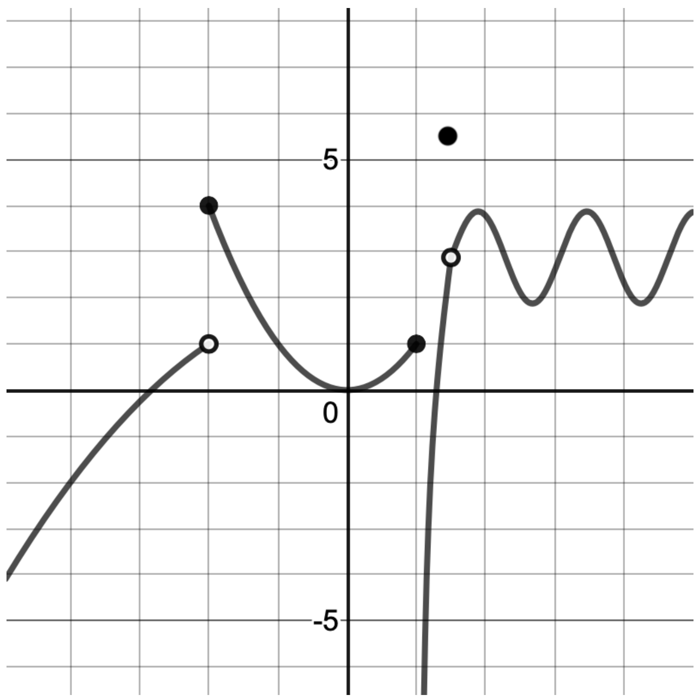
(b) $-2, 2$

(c) $-1, -\sqrt{5}, \sqrt{5}$

(d) $-\sqrt{5}, \sqrt{5}$

(e) None of the above

Problem 12. Examine the graph of the following piecewise function f.



In the portion of the graph of f presented above, which of the following statements is true?

(a) f is continuous at every point in its domain.

(b) f is not continuous at exactly 1 point.

(c) f is not continuous at exactly 2 points.

(d) f is not continuous at exactly 3 points.

(e) f is not continuous at exactly 4 points.

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