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Part B: Multiple Choice

INSTRUCTIONS: Choose the best answer to each of the following questions. Fill in the appropriate circle on the scantron sheet with a pencil AND circle your answer in the booklet. You may keep this booklet when the exam concludes. There are 13 multiple choice problems.

1. Where does the absolute/global maximum value of $f(x) = 2x^3 - 3x^2 - 12x$ on the interval $[0, 3]$ occur?

   (a) $x = -2$

   (b) $x = -1$

   (c) $x = 0$ ***

   (d) $x = 2$

   (e) $x = 3$

2. Suppose that f is a differentiable function and that the degree-four Taylor polynomial of f about $x = 1$ is

$$T_4(x) = -(x-1)^4 + 6(x-1)^3 + 3(x-1)^2 + 7$$

   Which of the following is equal to $f''(1)$?

   (a) 7

   (b) 15

   (c) 6 ***

   (d) 3

   (e) 36

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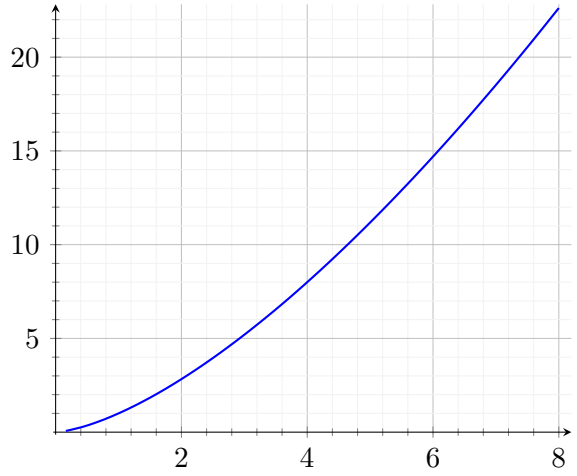
3. Consider the function $f(x) = x^{3/2}$ graphed below. Let $L(x)$ be the linear approximation of f at $x = 4$.



Which of the following statements is true?

(a) $L(x) = 8 + 3(x - 4)$ and $L(4.1)$ is an underestimate of $f(4.1)$. ***

(b) $L(x) = 8 + 3(x - 4)$ and $L(4.1)$ is an overestimate of $f(4.1)$.

(c) $L(x) = 4 + \frac{1}{3}(x - 8)$ and $L(7.9)$ is an underestimate of $f(7.9)$

(d) $L(x) = 4 + \frac{1}{3}(x - 8)$ and $L(7.9)$ is an overestimate of $f(7.9)$

(e) There is not enough information to determine if the linear approximation is an over/under estimate.

4. Suppose that f is a differentiable function and the **second derivative** of f is $f''(x) = \dfrac{(x-1)^2(x-3)}{x^2+4}$.

How many inflection points does the function f have?

(a) f does not have any inflection points

(b) f has exactly one inflection point ***

(c) f has exactly two inflection points

(d) f has exactly three inflection points

(e) f has exactly four inflection points

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5. Evaluate the limit $\lim_{x\to 0^+}(1+2x)^{3/x}$.

   (a) 1

   (b) 3

   (c) $e^{2/3}$

   (d) e^3

   (e) e^6 ***

6. The area of a circle is decreasing at a rate of 3 square-units per second.
   At what rate is the circumference of the circle changing when the circumference is 4π units?

   (a) The circumference is decreasing at $\frac{3}{2}$ units per second. ***

   (b) The circumference is decreasing at $\frac{3}{4\pi}$ units per second.

   (c) The circumference is decreasing at $\frac{2}{3\pi}$ units per second.

   (d) The circumference is decreasing at $\frac{4}{3}$ units per second.

   (e) The circumference is increasing at 12π units per second.

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7. An open box is to be made from one sheet of metal with dimensions $6\,\text{ft} \times 6\,\text{ft}$ by cutting out squares from the corners and bending up the sides. What is the maximum possible volume of such a box?

   (a) $4\,\text{ft}^3$

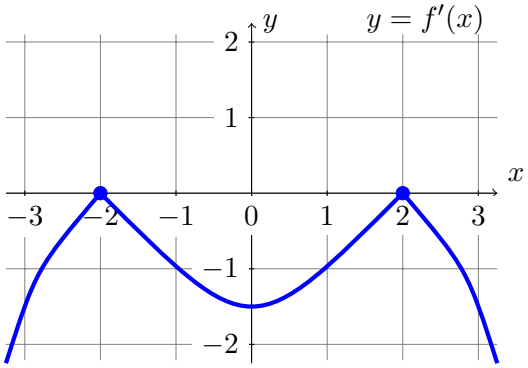
   (b) $9\,\text{ft}^3$

   (c) $16\,\text{ft}^3$ ***

   (d) $25\,\text{ft}^3$

   (e) $36\,\text{ft}^3$

8. Suppose that the function f is continuous on the interval $[-3, 3]$.
   Consider the following graph of the **derivative** of f.



   Which of the following statements is **true** about the function f?

   (a) f is decreasing on the intervals $(-3, -2)$, $(-2, 2)$ and $(2, 3)$ ***

   (b) f is increasing on the intervals $(-3, -2)$ and $(0, 2)$

   (c) f is increasing on the intervals $(-3, -2)$, $(-2, 2)$ and $(2, 3)$

   (d) f is not differentiable at $x = 0$

   (e) f is concave down on the interval $(-2, 2)$

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9. Suppose that f and g are differentiable functions so that

$$\lim_{x\to\infty} f(x) = 1 \qquad \lim_{x\to\infty} g(x) = 0 \qquad \lim_{x\to\infty} f'(x) = 5 \qquad \lim_{x\to\infty} g'(x) = 10$$

Evaluate the limit $\lim_{x\to\infty} \dfrac{\ln(f(x))}{g(x)}$.

(a) 0

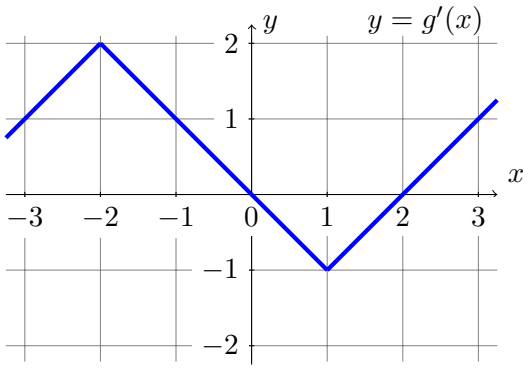
(b) $\dfrac{1}{2}$ ***

(c) $\dfrac{1}{50}$

(d) 2

(e) The limit does not exist.

10. Suppose that the function g is continuous on the interval $(-3, 3)$.
Consider the following graph of the **derivative** of g.



Which of the following statements is **true** about the function g?

(a) g also consists of straight line segments and does not have a notion of concavity

(b) g is concave up on the intervals $(-3, 0)$ and $(2, 3)$

(c) g is concave up on the intervals $(-3, -2)$ and $(1, 3)$ ***

(d) g is concave up on the interval $(-2, 1)$

(e) $g''(x) = 0$ for all x in the interval $(-3, 3)$

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11. Which of the following statements is **always true** about a continuous function f?

    Hint: think about different examples of continuous functions as you consider the different statements.

    (a) If $f(c)$ is a local maximum value of f, then $f'(c) = 0$.

    (b) If $x = c$ is a critical point of f, then $f(c)$ is a local maximum or minimum value of f.

    (c) If $x = c$ is a singular point of f, then $f(c)$ **cannot** be a local maximum nor minimum value of f.

    (d) If $f(c)$ is a local minimum value of f, then $x = c$ is a critical point or a singular point of f. ***

    (e) If $x = c$ is a critical point of f, then $(c, f(c))$ is an inflection point.

12. You are studying the water flow in and out of the Glenmore Reservoir (an artificial lake that stores drinking water in Southwest Calgary). During your many visits to the reservoir, you have measured the depth of the water (in metres) and collected the following data about the depth of the water in the reservoir (one measurement per day).

Date	Nov 2	Nov 3	Nov 4	Nov 5	Nov 6	Nov 7	Nov 8
Water level (m)	23.5	21.5	19	21.5	22.5	19.5	18.5

    What does the **Mean Value Theorem** allow you to conclude over the time period from November 2 to November 7?

    (a) At some moment in time between November 2 and November 7, the water level was decreasing at a rate of exactly $4/5$ metres per day. ***

    (b) During the 5-day period from November 2 to November 7, the water level changes at a constant rate of $4/5$ metres per day.

    (c) During the 5-day period from November 2 to November 7, the rate of change of the water level never exceeds $4/5$ metres per day.

    (d) At some moment in time between November 2 and November 7, the water level was decreasing at a rate of exactly $-1/2$ metres per day.

    (e) At some moment in time between November 2 and November 7, the water level was increasing at a rate of exactly $5/6$ metres per day.

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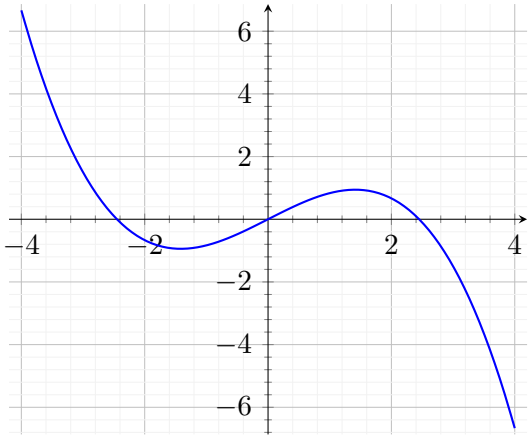
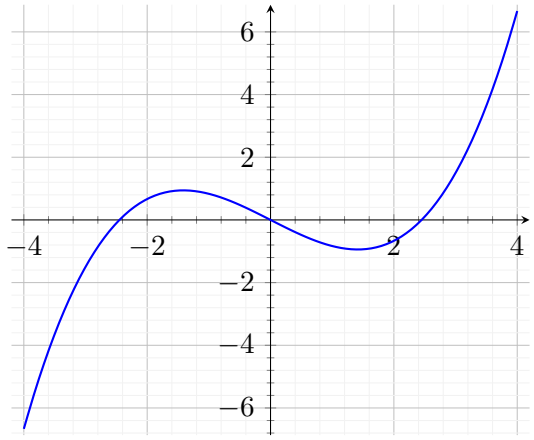
13. Which of the following could be the graph of the degree-three Maclaurin polynomial (i.e., the third degree Taylor polynomial centred at $x = 0$) of the function $f(x) = \sin(x)$?

(a) ***



(b)



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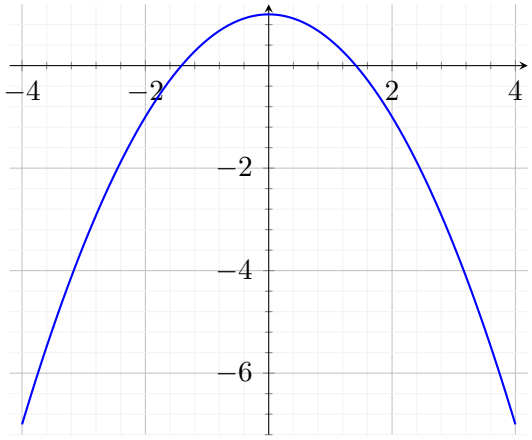
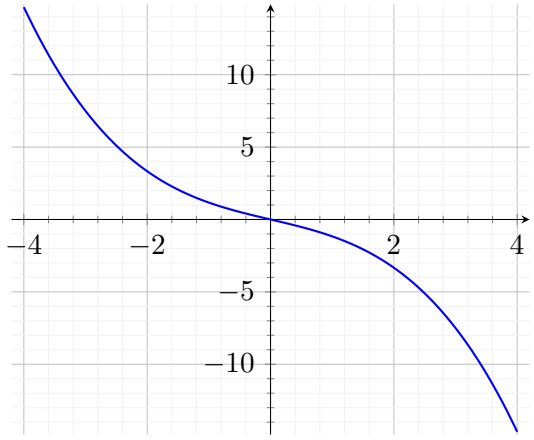
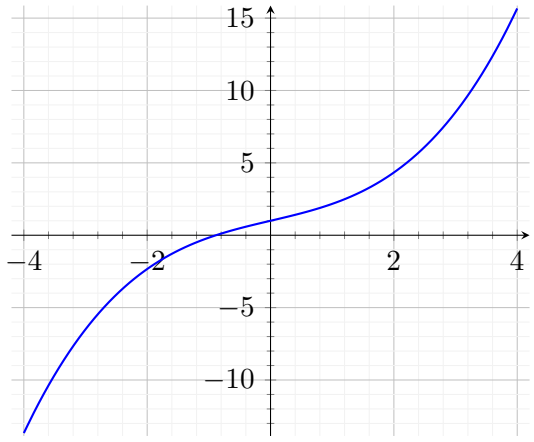


(d)



(e)



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