MATH 249 & 265: DINO PROBLEMS
TOPIC 2: LIMITS, INFINITE LIMITS, AND THE SQUEEZE THEOREM

Problem 1. Your friend says that the limit law $\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$ is always true. After all, it's called a limit *law*. Of course, you have been studying, so you know that something can go wrong. Evaluate the limits below (if possible) to help explain to your friend when the limit law can break down.

(1) $\lim_{x\to 0} x^2$

(2) $\lim_{x\to 0} \dfrac{1}{x^2}$

(3) Is $\lim_{x\to 0} x^2 \left(\dfrac{1}{x^2}\right)$ the product of the two limits you found in the previous parts? If not, **explain** why the limit law does not hold.

Problem 2. Calculate the following limits, if they exist:

(1) $\lim_{x\to -1} \dfrac{x^2+3x+2}{x^3+1}$ (2) $\lim_{x\to 1} \dfrac{\sqrt{x}-1}{x-1}$ (3) $\lim_{x\to 2} \dfrac{x^2-3x+2}{|2-x|}$

Problem 3. Let a be an unknown constant. Consider the function f given by

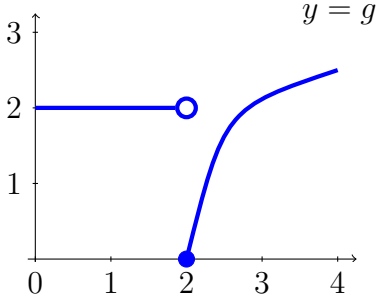
$$f(x) = \frac{x^2+(a-2)x-2a}{x^2-4x-5}.$$

(1) For which values of the constant a does the limit $\lim_{x\to 5} f(x)$ exist? Explain.

(2) For each of the values of a that you found above, calculate the limit $\lim_{x\to 5} f(x)$. Show all of your work, neatly and carefully.

Problem 4. Given the following functions defined graphically and algebraically, compute each of the following limits, or explain why they do not exist.

$y = g(x)$



$$P(x) = \begin{cases} 2 & \text{if } x \geq 2 \\ 0 & \text{if } x < 2 \end{cases}$$

$$j(x) = \frac{x^2-9}{x+3}$$

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(1) $\lim_{x\to 2} g(x)$, $\lim_{x\to 2} P(x)$ and $\lim_{x\to 2} j(x)$

(2) $\lim_{x\to 2}[g(x)+P(x)]$

(3) $\lim_{x\to 2}[g(x)P(x)]$

(4) $\lim_{x\to 2}[P(x)+j(x)]$

Problem 5. Let b denote the last nonzero digit of your UCID number, that is, if your UCID number is 9876543210 then $b = 1$.

Suppose that f is a continuous function such that $2f(x)+bx^2$ is between the two functions $g(x) = x^2-1$ and $h(x) = -(1-x)$, for all x in the closed interval $[-\pi, \pi]$. If possible, calculate the following limits. Explain your answers.

(1) $\lim_{x\to 0} f(x)$

(2) $\lim_{x\to 1} f(x)$

(3) $\lim_{x\to -1} f(x)$

Problem 6.

(1) In general, if $\lim_{x\to a} s(x)$ and $\lim_{x\to a} t(x)$ exist, is it true that the limit $\lim_{x\to a} s(x)+t(x)$ exists? What about $\lim_{x\to a} s(x)t(x)$? Compare your answers here to your answers to Problem 4.

(2) In general, if $\lim_{x\to a} s(x)$ and $\lim_{x\to a} t(x)$ does not exist, is it true that the limit $\lim_{x\to a} s(x)+t(x)$ does not exist? What about $\lim_{x\to a} s(x)t(x)$? Compare your answers here to your answers to Problem 4.

Problem 7. Consider the function

$$f(x) = \frac{|x-6|}{x+6}.$$

Calculate the one-sided limits $\lim_{x\to -6^+} f(x)$ and $\lim_{x\to -6^-} f(x)$ and explain your calculations, or explain why these limits do not exist. Does $\lim_{x\to -6} f(x)$ exist? Make sure that you mention any theorems that you use.

Problem 8. Consider the functions

$$g(x) = \cos\left(\frac{1}{\sqrt{x^2}}\right)$$

and

$$h(x) = x^2\cos\left(\frac{1}{\sqrt{x^2}}\right).$$

Calculate the limits $\lim_{x\to 0} g(x)$ and $\lim_{x\to 0} h(x)$ and explain your calculations, or explain why these limits do not exist. You may, or may not, want to consider one-sided limits. Make sure that you mention any theorems that you use.

Problem 9. Let $F(x) = \dfrac{1}{x^2-1}$ and $G(x) = \dfrac{1}{1-x^2}$.

(1) Evaluate the limits $\lim_{x\to 1} F(x)$ and $\lim_{x\to 1} G(x)$.

(2) Evaluate $\lim_{x\to 1}[F(x)+G(x)]$.

Compare your answers to this problem with your answers in Problems 4 and 6. Explain your observations.

Note: by "Evaluate" we mean: "Calculate the limit if it exists, or show that the limit does not exist. Explain your answers."

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