

Log loss is hard to interpret  
If I say  $\text{log-loss} = 1.0$  ~~But~~  
we cannot interpret it easily

<u>30.6</u>	<u><math>R^2</math> or Coefficient of determination</u>
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Lets look at some methods to determine how good a regression technique is.

We know that  $x_i, y_i, \hat{y}_i$   
                      $\underbrace{\hspace{1cm}}$           $\uparrow$   
                     dataset     model output

$$e_i = \text{error for point } i = y_i - \hat{y}_i = \text{difference between actual \& model output}$$

Sum of squares (SS)

$$SS_{\text{total}} = \sum_{i=1}^n (y_i - \bar{y})^2 = \text{Total sum of squares}$$

$$\bar{y} = \text{Average value of all } y_i$$

$$= \frac{1}{n} \sum_{i=1}^n y_i$$

When we are performing regression, what is simplest model we can build  $\rightarrow$  return mean ( $y_i$ )

Imagine a case where we have to predict a height based on features like weight, skin colour, hair colour, etc.

Given any new person  $x_q$ , in my whole training data, if I know that avg height of a human being is 152 cm.

So, I will predict the avg height of  $x_q \rightarrow 152 \text{ cm}$

That's the simplest model we can build.

$\therefore$  Simple Mean Model

$x_q \rightarrow \text{Mean}(y_i) \text{ as } y_q.$

$\Downarrow$   
 $\bar{y}$

$$\therefore SS_{\text{Total}} = \sum_{i=1}^n (y_i - \bar{y})^2$$

Sum of squared errors  
using a simple Mean  
Model.



$$SS_{\text{Residues}} = \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \sum_{i=1}^n e_i^2$$

$$\text{Residue} = e_i = \underbrace{y_i}_{\text{Actual}} - \underbrace{\hat{y}_i}_{\text{Predicted}}$$

$$R^2 \equiv \left( 1 - \frac{SS_{\text{Residue}}}{SS_{\text{Total}}} \right)$$

Case 1:

$$SS_{\text{res}} = 0 \text{ (i.e. } e_i = 0) \Rightarrow R^2 = 1$$

(Best Value)

It means all of residues or errors are 0.

Case 2:

$$SS_{\text{res}} < SS_{\text{Total}} \quad ; \quad R^2 = 0 \text{ to } 1$$

Case 3:

$$SS_{\text{res}} = SS_{\text{Total}} \quad ; \quad R^2 = 0$$

It means the model that generated residues is same as simple mean model.

It mean

Case 4:

$$SS_{\text{Residue}} > SS_{\text{Total}} ; R^2 = -ve.$$

Model is worse than a simple Mean Model.

So, when someone says.

$R^2 = 0.9 \Rightarrow$  the model is near to perfection

$R^2 \approx 0.1 \Rightarrow$  the model is near to simple Mean model

$R^2 < 0 \Rightarrow$  the model is worse than mean model.

### 30.7 Median Absolute Deviation of Errors

lets say if one  $e_i$  is very large then  $SS_{\text{Residue}}$  would get corrupted.

$\therefore R^2$  is not very Robust to outliers

For every datapoint  $x_i$ , I have  $y_i$  &  $\hat{y}_i$  and a corresponding  $e_i$

In Regression, the measure of how good a model is ~~abs~~ all about  $e_i$

$\therefore$  If  $|e_i| \rightarrow 0 \Rightarrow$  Great

$|e_i| \rightarrow \text{Large} \Rightarrow$  Not so Good



If  $e_i$  = Random Variable  
then

$$\begin{aligned} \text{Median}(e_i) &= \text{Central Value of errors} \\ \text{M.A.D}(e_i) &= \text{Median}(|e_i - \text{Median}(e_i)|) \end{aligned}$$

If I know that  
Median = small } then error is  
and M.A.D = small. } small.

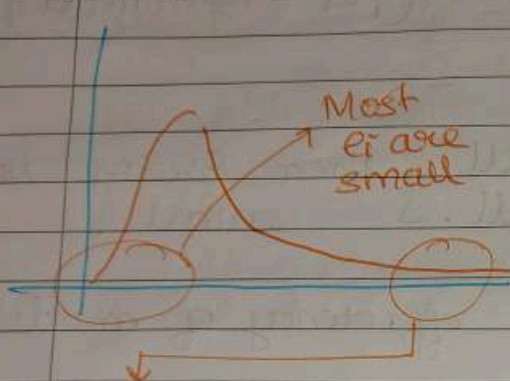
For calculating the efficiency of module,  
we can use.

Mean OR Median of  $e_i$ s.  
Std-Dev OR M.A.D.

↓  
Robust  
to Outliers

### 30.8 Distribution of errors

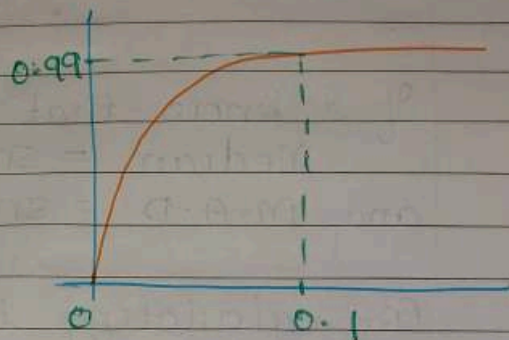
If we plot the PDF and CDF of errors  $e_i$



Very few  $e_i$  are large

Good sign.

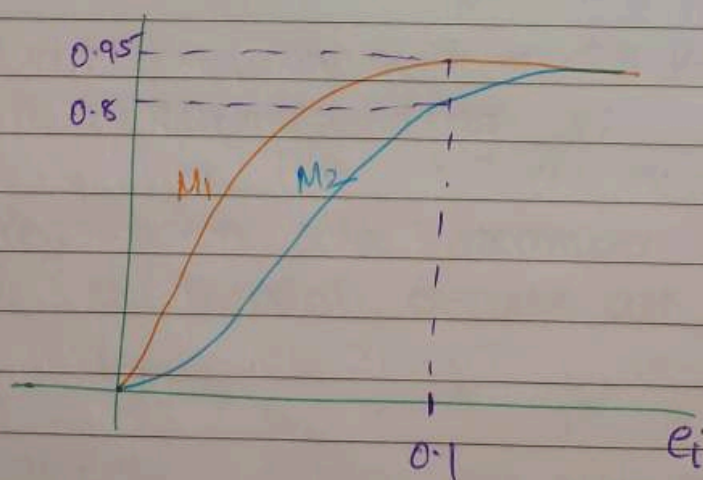
We should have  $e_i$  as close to 0



99% of errors are  $< 0.1$   
only 1% of error are  $> 0.1$

Good sign

Let's say I have CDF of two Model



$M_2$ : cdf is below  $M_1$