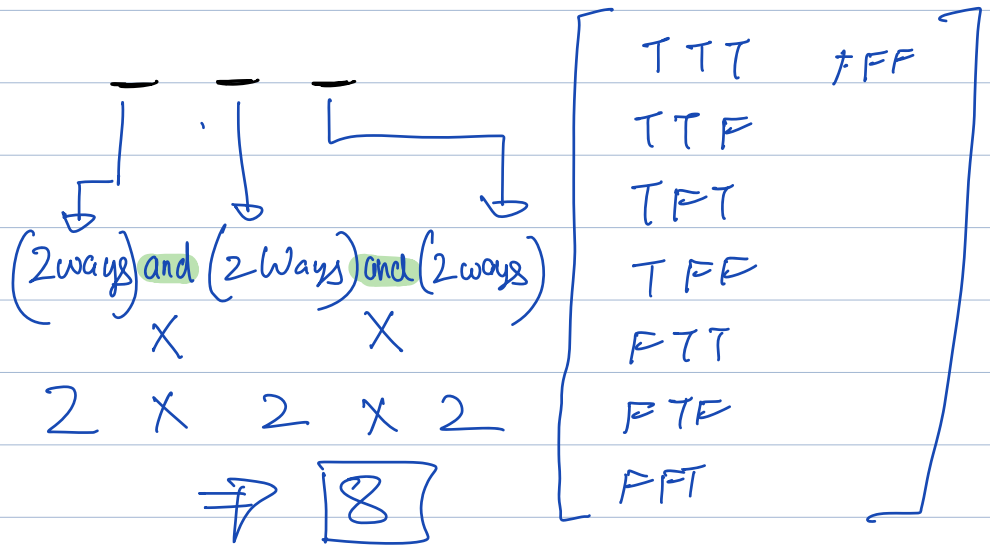
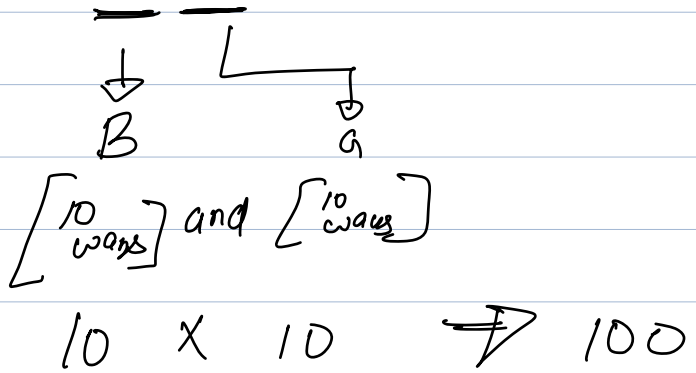
Multiplication & Addition Rule

Q You have 3 Questions. Answer is True or False. How many ways are there to fill the answer.

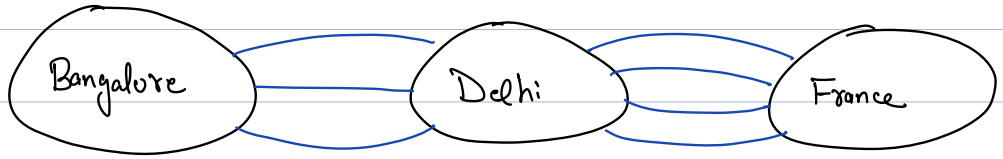
Ans



Q2 Given 10 Girls and 10 Boys, how many different ways are there to pair them.



Q3 How many ways are there to travel from Bangalore to France.



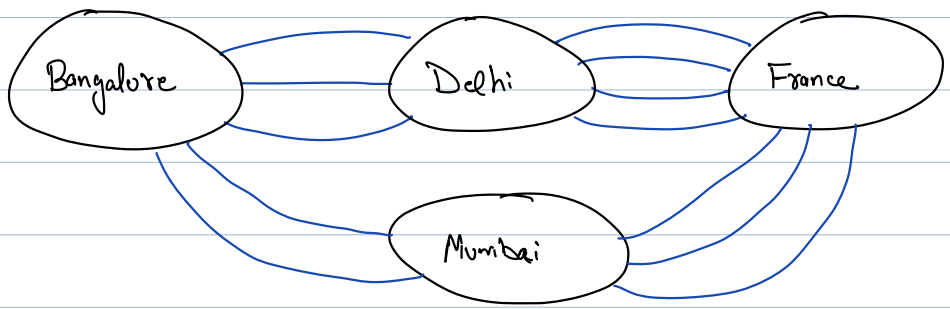
(Bangalor to Delhi) and (Delhi to France)

⇓ ⇓

3 X 4

⇒ 12

Q4 How many ways are there to travel from Bangalore to France.



(Bang - France) ⇒ (Travel via Delhi) OR (Travel via Mumbai)

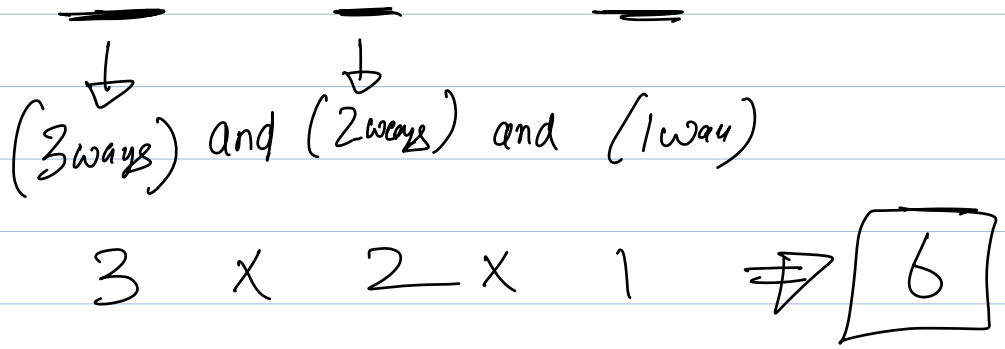
⇓ ⇓

3 X 4 = 12 + 2 X 3 = 6

12 + 6 ⇒ 18

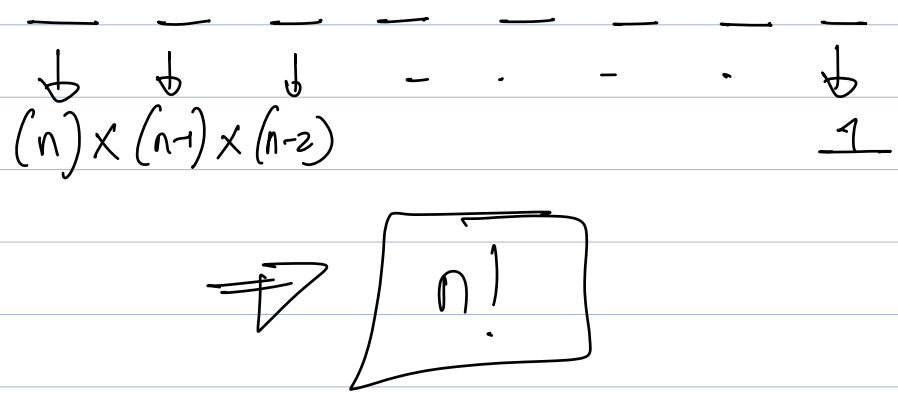
Q5 Given 3 distinct characters. How many different ways are there to arrange them. You can't repeat a character.

Ex1 A B C

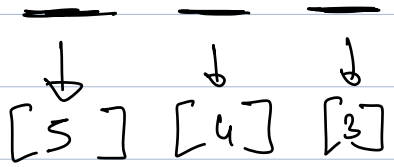


Q6 Given n distinct characters. How many different ways are there to arrange them.

Ex1



Q: 5 distinct characters. Pick 3 and arrange. How many ways to do this.



$\Rightarrow 5 \times 4 \times 3 \Rightarrow 60$

Permutation $\Rightarrow$ Pick and arrange

\# n distinct items
pick r and arrange.

1 2 3 r

$n \quad (n-1) \quad (n-2) \quad \ldots \quad (n-(r-1))$

$= n-r+1$

Totals $\Rightarrow n \times (n-1) \times (n-2) \cdots \times (n-r+1)$

Total $\Rightarrow \dfrac{\text{Total} \times (n-r)!}{(n-r)!} \Rightarrow \dfrac{n!}{(n-r)!}$

Total $\Rightarrow \frac{n!}{(n-r)!} \Rightarrow {}^{n}P_{r}$

Ex1 $n = 5$, $r = 3$

$\Rightarrow \frac{5!}{2!} \Rightarrow \frac{120}{2} \Rightarrow 60$

Permutation: n distinct items
pick r and arrange. $\Rightarrow {}^{n}P_{r}$

Combination

Q Given 3 distinct characters. How many ways are there to choose 2 characters.

A B C ⇒ AB BC AC

AB → 2, BC → 2, AC → 2

Combination = 3

Permutation ⇒ 6

Combination ⇒ Permutation

no of ways of arrange r items

⇒ n!

n items pick r

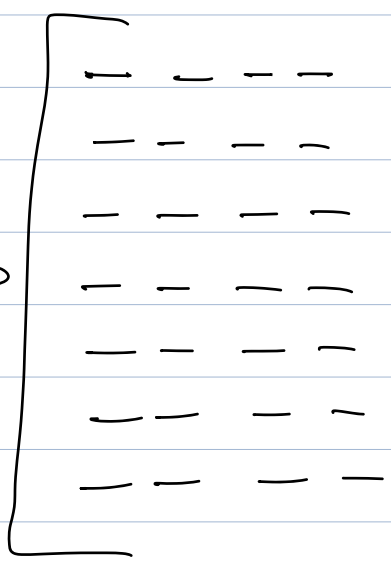
no of Combinations ⇒ comb

no of Permutations ⇒ comb × r!

$\frac{n!}{(n-r)!}$ ⇒ comb × r!

$\frac{n!}{(n-r)!\,r!}$ ⇒ comb ⇒ ^nC_r

comb



Properties $\quad {}^nC_r = \frac{n!}{(n-r)! \times r!}$

1) ${}^nC_0 \Rightarrow \frac{n!}{n! \times 0!} \Rightarrow 1$

2) ${}^nC_n \Rightarrow 1$

3) ${}^nC_1 \Rightarrow n$

4) n distinct items. How many different ways are there to pick something. You can choose to pick nothing.

$$[a_1, a_2, a_3 \ldots a_n] \Rightarrow 2^n$$

$${}^nC_0 + {}^nC_1 + {}^nC_2 \ldots {}^nC_n \Rightarrow 2^n$$

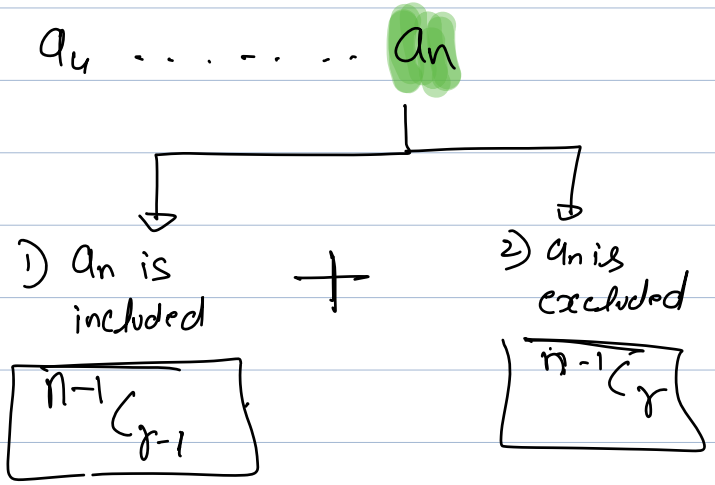
Ex1 $\quad a \; b \; c \Rightarrow 2^3 \Rightarrow (8)$

${}^3C_0 \Rightarrow 1$		[ ]
${}^3C_1 \Rightarrow 3$		[a] [b] [c]
${}^3C_2 \Rightarrow 3$		[ab] [ac] [bc]
${}^3C_3 \Rightarrow 1$		[a b c]
	8	

5) n distinct items ⇒ ${}^nC_r$

pick r

$a_1, a_2, a_3, a_4 \ldots\ldots a_n$



${}^{n-1}C_{r-1} + {}^{n-1}C_r \Rightarrow {}^nC_r$

$\frac{(n-1)!}{\quad}$

$$ {}^{n-1}C_{r-1} + {}^{n-1}C_{r} \Rightarrow {}^{n}C_{r} $$

$$ \frac{(n-1)!}{(n-r)!\,(r-1)!} + \frac{(n-1)!}{(n-r-1)!\,r!} \Rightarrow \frac{n!}{(n-r)!\,r!} $$

$$ (n-1)! \left[\frac{1}{(n-r-1)! \times (n-r) \times (r-1)!} + \frac{1}{(n-r-1)! \times (r-1)! \times r} \right] $$

$$ \frac{(n-1)!}{(n-r-1)! \times (r-1)!} \left[\frac{1}{n-r} + \frac{1}{r} \right] $$

$$ \Rightarrow \frac{r + n - r}{(n-r) \times r} $$

$$ \frac{(n-1)!}{(n-r-1)! \times (r-1)!} \left[\frac{n}{(n-r) \times r} \right] = \frac{n!}{(n-r)! \times r!} $$

$$ \Rightarrow {}^{n}C_{r} $$

6) 5 Boys | Choose 2 | $\Rightarrow {}^5C_2 \Rightarrow 10$

Rejecting 3 $\Rightarrow {}^5C_3 \Rightarrow 10$

B_1 B_2 B_3 B_4 B_5

B_1 B_2 | B_3 B_4 B_5

Choosing r $\Leftrightarrow$ rejecting $n-r$

$${}^nC_r \Rightarrow {}^nC_{n-r}$$

How to code ^nC_r

$n = 50$, $r \Rightarrow 20$

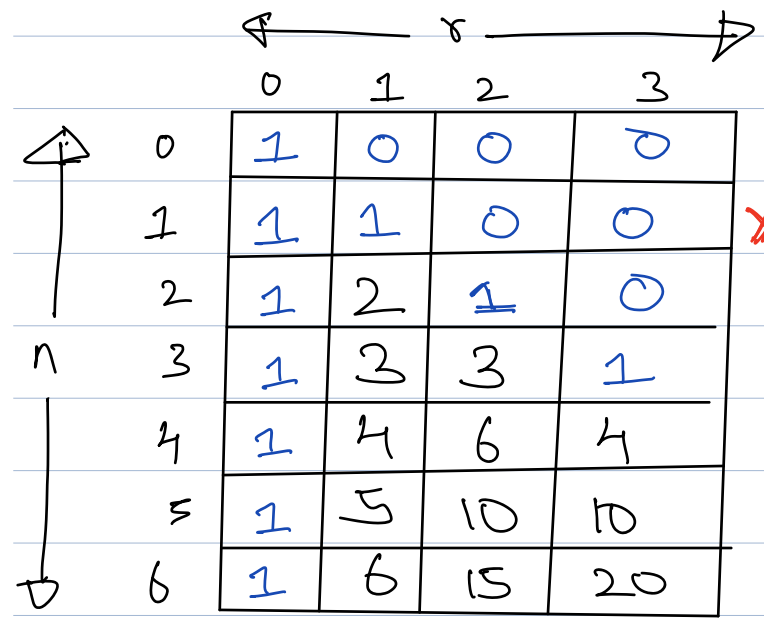
$^{50}C_{20} \Rightarrow \frac{(50)!}{(20)! \times (30)!}$

They will ask $^nC_r \% m$

$^nC_r \Rightarrow {}^{n-1}C_{r-1} + {}^{n-1}C_r$

$^0C_1$ $n = 6$ $r = 3$ $^6C_3$

$^0C_0 \Rightarrow 1$



ans[N+1][r+1]

⇓

ans[i][j] = ^iC_j

return ans[N][r]

$^iC_j \Rightarrow {}^{i-1}C_{j-1} + {}^{i-1}C_j$

ans[i][j] ⇒ ans[i-1][j-1] + ans[i-1][j]

Trivial Case : 1) $^{n}C_{0} = 1 \quad n \geq 0$

2) $^{n}C_{\gamma} \Rightarrow 0 \quad \gamma > n$

3) $^{n}C_{n} \Rightarrow 1$

TC : $O(n \times \gamma)$
SC : $O(n \times \gamma)$ $\xrightarrow{\text{optimised}}$ $O(\gamma)$

$^{1000}C_{50}$

$^{10^6}C_{10^4} \Rightarrow TC \neq 10^{10}$

$p > n$ & $p > \gamma$

Approach 2 [valid if mod value = prime]

↳ 10^9+7, 10^9+9

$$^nC_r \% P \Rightarrow \left[\frac{n!}{(n-r)!\,(r)!}\right] \% P$$

$$\left[n! \times [(n-r)!]^{-1} \times [(r)!]^{-1}\right] \% P$$

$$[(a \times b) \times c] \% P$$

$$[(a \times b) \% P \times c \% P] \% P$$

$$[(a \% P \times b \% P) \% P \times c \% P] \% P$$

$(a \times b) \% m \Rightarrow (a \% m \times b \% m) \% m$

1) $a \% P \Rightarrow n! \% P \Rightarrow O(1)$

2) $b \% P \Rightarrow [(n-r)!]^{-1} \% P \Rightarrow O(\log P)$

3) $c \% P \Rightarrow [(r)!]^{-1} \% P \Rightarrow O(\log P)$

Inverse $(a^{-1}) \% P$ and $\gcd(a, P) \Rightarrow 1$

$\Rightarrow (a^{P-2}) \% P \Rightarrow$

$a^b \% M \Rightarrow$ Fast power $\Rightarrow O(\log b)$

2) $b \% P \Rightarrow [(n-r)!]^{-1} \% P$

$\Downarrow$

$((n-r)!)^{P-2} \% P \Rightarrow \log P$

3) $c \% P \Rightarrow [(r)!]^{-1} \% P$

$\Downarrow$

$[(r)!]^{P-2} \% P \Rightarrow \log P$

0	1	2	3	4	5	6	7	8	9	10
1	1	2	6	24	120	720				

$fact[i] \Rightarrow fact[i-1] \times i$

$\Rightarrow O(n)$

$ans[i] = i! \% m$

$p > n$, $p > r$

$\gcd((n-r)!, p) = 1$

$p > n-r$

$(n-r)! = 1 \times 2 \times 3 \times 4 \ldots (n-r)$.

$\gcd(r!, p) = 1$

T.C

1) Find factorial $\Rightarrow O(n)$

2) Find $^nC_r \Rightarrow O(\log p)$

TC : $O(n + \log p)$ $n \times r$