- GCD Intro
- Properties of GCD
- How to find GCD
- GCD of all elements in array.
- Check if there exist a subsequence.
- Delete 1 element, find max GCD

GCD = GREATEST COMMON DIVISOR

$\gcd(a,b) \Rightarrow x$

x is the largest number for which

$a \% x = 0$ & $b \% x = 0$

Quizzes

1) $\gcd(10, 15)$

factors

10	15
1	1
2	3
5	5
10	15

ans = 5

2) $\gcd(1, 16) = 1$

1	16
1	1
	2
	4
	8
	16

3) $\gcd(10, -25) \Rightarrow \underline{5}$

10	-25
1	1
2	5
5	25
10	

4) $\gcd(-16, -24) = 8$

-16	-24
1	1
2	2
4	3
8	4
16	6
	8
	12
	24

5) $\gcd(0, 8) = 8$ // $\boxed{\gcd(0,a) = a}$

0	8
1	1
2	2
4	4
8	8

6) $\gcd(0, -10) \Rightarrow \underline{10}$

-10
1
2
5
10

Properties of GCD

1) $\gcd(a,b) = \gcd(|a|,|b|)$

2) $\gcd(a,b) = \gcd(b,a)$

3) $\gcd(a,0) = |a|$

$\gcd(0,0) \Rightarrow$ undefined.

4) $\gcd(a,b,c) \Rightarrow \gcd(\gcd(a,b),c)$

$\gcd(\gcd(a,c),b)$

Q1) Given 2 numbers A & B find gcd(A, B).

$A, B > 0$.

Approach 1 :

```
int gcd = min(A,B);
for (int i = gcd; i >= 1; i--) {
    if (a%i == 0 && b%i == 0) {
        return i;
    }
}
```

$TC: O(\min(A,B))$ $SC: O(1)$

Approach 2

1) Find factors of min(A, B).
2) Check for largest factor found above for which max(A, B) % factor $\neq 0$

$TC: O(\sqrt{\min(A,B)})$

Euclid Algorithm

$a \geq b$

$$\gcd(a,b) = \gcd(a-b,b)$$
$$\Downarrow \qquad\qquad \Downarrow$$
$$x \qquad\qquad x$$

$\gcd(a,b) \Rightarrow x$

$$a \% x \Rightarrow 0 \quad \& \quad b \% x \Rightarrow 0$$

$$\Rightarrow \gcd(a-b,b) \Rightarrow x$$

$$(a-b)\% x \Rightarrow 0 \quad \& \quad b \% x \Rightarrow 0$$

PROOF $\Rightarrow$

$$\boxed{\begin{array}{l}(a \% x - b \% x + x) \% x \\ (0 - 0 + x) \% x \Rightarrow 0\end{array}}$$

$a \geq b$

$$\gcd(a,b) = \gcd(a-b, b)$$

$\gcd(20,8)$

$\Rightarrow \gcd(12,8)$

$\Rightarrow \gcd(4,8) \Rightarrow \gcd(8,4)$

$\Rightarrow \gcd(4,4)$

$\Rightarrow \gcd(0,4) \Rightarrow 4$

TC?

$\Rightarrow O(\mathrm{Max}(A,B))$

$\gcd(23,5)$

$\Rightarrow \gcd(18,5) \Rightarrow 23-5(1)$

$\Rightarrow \gcd(13,5) \Rightarrow 23-5(2)$

$\Rightarrow \gcd(8,5) \Rightarrow 23-5(3)$

$\Rightarrow \gcd(3,5) \Rightarrow 23-5(4)$

$\Rightarrow \gcd(5,3)$

$\gcd(a,b)$

$\Rightarrow \gcd(a\%b, b)$

$\Rightarrow \gcd(b, a\%b)$

$gcd(26, 12) \Rightarrow gcd(12, 26\%12)$

$\Rightarrow gcd(12, 2) \Rightarrow gcd(2, 12\%2)$

$\Rightarrow gcd(2, 0) \Rightarrow 2$

a b

$gcd(12, 26) \Rightarrow gcd(26, 12\%26)$

$\Rightarrow gcd(26, 12)$

Pseudo Code

```
int gcd(int a, int b) {
    if (b == 0)
        return a;

    return gcd(b, a%b);
}
```

$TC: O(\log_2 max(A, B))$

Q2) Given N Array elements. Find GCD of all elements.

[2, 4, 6] ⇒ 2

[13, 69, 10, 14] ⇒ 1

Approach 1:

```
int ans ⇒ 0 / A[0]
for (int i=0 ; i<n ; i++) {
    ans ⇒ gcd(ans, arr[i]);
}
```

TC: $O(n \times \log(\max(arr)))$

↓

max of array

Q3) Given N Array Elements. Find GCD of factorials of all given elements ?

Amazon

Ex1: [4, 3, 8, 6]

$arr[i] < n$

⇓

[4!, 3!, 8!, 6!]

$4! \Rightarrow 1 \times 2 \times 3 \times 4$

$3! \Rightarrow 1 \times 2 \times 3$

$8! \Rightarrow 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$

$6! \Rightarrow 1 \times 2 \times 3 \times 4 \times 5 \times 6$

$$gcd(a,b,c,d) \leq min(a,b,c,d)$$

⇒ Find factorial of minimum Number

TC: 1) To find min ⇒ O(n)
2) To find fact ⇒ O(n)
⇒ O(n)

SC: O(1)

SUBSEQUENCE

- A sequence generated by deleting 0 or more elements from array.

Ex1

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \end{array}$$

[4, 2, 1, 4, 5]

⇓

[2, 1, 5] ✓

⇒ [1, 2, 5] ✗

Q4) Given an array return if there exists a subsequence where gcd = 1.

Ex1) [4 6 3 8] ⟹ $\boxed{\text{True}}$

$[\boxed{a_1}, a_2, \boxed{a_3}, a_4, \boxed{a_5}, a_6]$

$gcd(a_1, a_3, a_5) \Rightarrow 1$

$gcd(a_2, a_4, a_6) \Rightarrow X$

$\underline{1}$

Approach 1 : Find GCD of array.

if GCD == 1
return true

$TC : O(n \max(arr))$

Q4) Given an array. Delete exactly 1 element such that the GCD of the remaining array is Maximised.

Ex $A = [12, 15, 18]$

1) Delete 12
   $\gcd(15, 18) \Rightarrow 3$

2) Delete 15
   $\gcd(12, 18) = 6$

3) Delete 18
   $\gcd(12, 15) \Rightarrow 3$

$n^2 \log_2 \max(arr)$

ans $\Rightarrow$ 1 1 4 1 1

4	8	3	12	16

prefix gcd

4	4	1	1	1

Suffix

1	1	1	4	16

TC: $n \times \log \max(A)$

Q5) There are N players, each with strength A[i].
When a player i, attacks player j strength reduces to $|A[i] - A[j]|$ or 0, depending on who attacked.
When a player strength reaches zero, it loses the game, and the game continues in the same manner among other players until only 1 survivor remains.

FB, Tower Research

Can you tell the minimum health last surviving person can have?

#	A	B
	4	6

Case 1: A attacks B

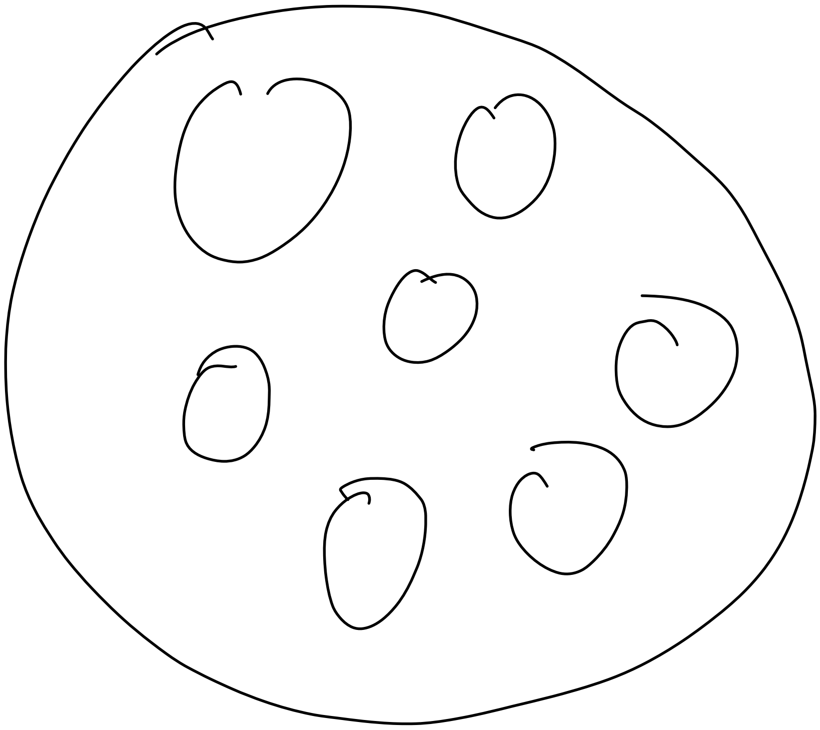
4 6

$\Rightarrow$ 4 2

Case 2: B attack A

6 4

6 0



A		B
15	$\leftarrow$	4
11		4
7		4
3		4

#	A	B
	6	4

$\Rightarrow$ A hits B

$A=6$

$B \Rightarrow 0$

$\downarrow$

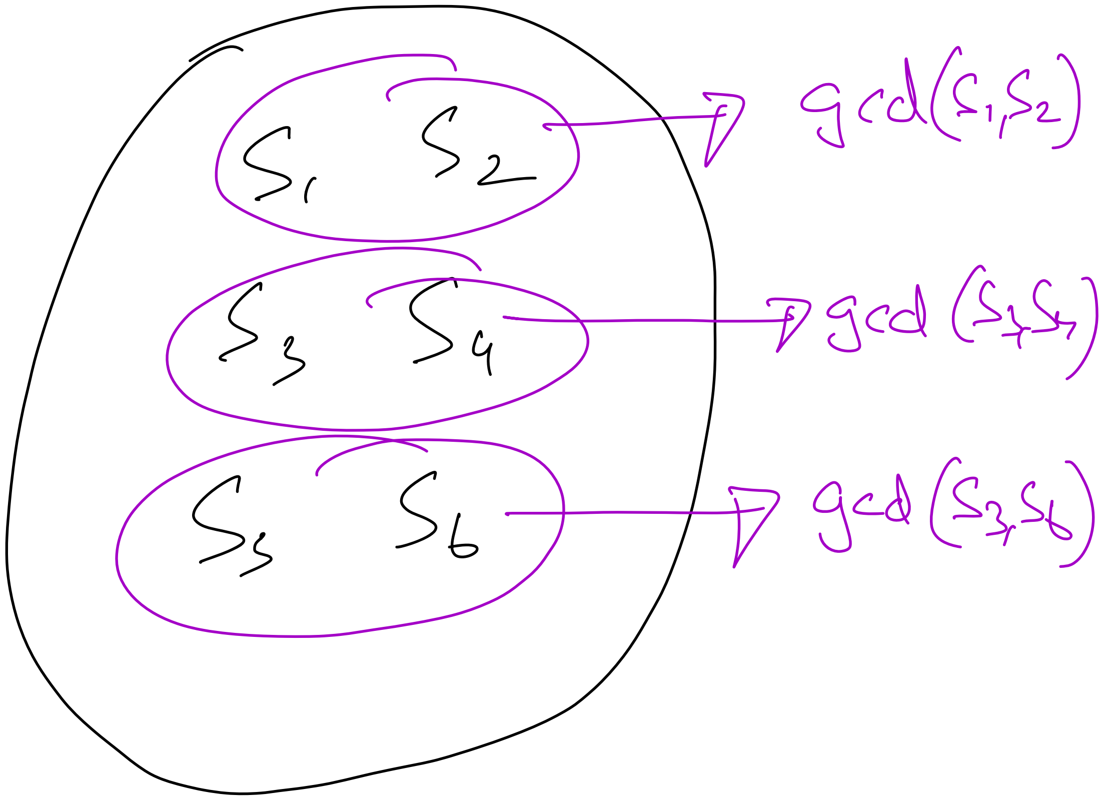
B hits A

A	B
2	4

$\xrightarrow{\text{A hits B}}$

A	B
2	2

$(6,4) \to (4,2) \to (2,2) \to (2,0)$



Inverse modulo !

$$\left(\frac{b}{a}\right)\%m \;\Rightarrow\; \left(\frac{b\,\%m}{a\,\%m}\right)\%m$$

$$\Downarrow$$

$$\left(b \times \frac{1}{a}\right)\%m \;\Rightarrow\; (b \times a^{-1})\%m$$

$$\Downarrow$$

$$(b\%m \times a^{-1}\%m)\%m$$

$\boxed{(a^{-1})\%m}$

$(a^{-1} \% m) \Rightarrow b$

b will exist if $(gcd(a, m) \Rightarrow 1)$

$b \Rightarrow [0, m-1]$

$$(a \times b) \% m \Rightarrow 1$$

1) $(a^{-1}) \% m \qquad a \Rightarrow 10, m \Rightarrow 7$

1) $gcd(10, 7) \Rightarrow 1$

2) $b \Rightarrow$
0
1
2
3
4
5
$(m-1)$ 6

$$\boxed{(a \times b) \% m \Rightarrow 1}$$

$$\boxed{(10 \times 5) \% 7 \Rightarrow 1}$$

Little fermat

$m \Rightarrow$ prime no $\{10^9 + 7\}$

1) $(a^{p-1}) \% p = 1$ $\left[\begin{array}{l} p = \text{prime} \\ a \% p \,!= 0 \end{array} \right]$

2) $(a^{-1}) \% p \Rightarrow (a^{p-2}) \% p$

$\Downarrow$

$TC : O(\log P)$

$\Downarrow$

$\boxed{\text{Fast Power}}$

\# $\left({}^{n}C_{r}\right) \Rightarrow \left[\dfrac{n!}{(n-r)!\,r!}\right] \% m$