What is a % operator

a % b ⇒ remainder when b divides a.

$$\underbrace{20}_{\text{Dividend}} \% \overbrace{3}^{\text{Divisor}} \Rightarrow \underbrace{2}_{\text{remainder}}$$

Dividend ⇒ Quotient x Divisor + remainder.

Quotient x Divisor ⇒ X ⇒ max multiple of Divisor ≤ Dividend

remainder ⇒ a % b

20 ⇒ 6 x 3 + 2

6 x 3 ⇒ 18

Ex1 : 40 % 7

40 ⇒ 5x7 + 5

40 ⇒ 35 + 5

40 % 7 ⇒ 5

Ex2 : −40 % 7

−40 ⇒ 7x(−6) + rem

−40 ⇒ −42 + rem

rem ⇒ 2

$a \% m \xrightarrow{\text{range}} [0, m-1]$

Languages handle modulo differently.

	C++/JAVA	Python
$-40 \% 7$	-5 $+7$	2

$a \% m \Rightarrow$ if $(a < 0)$ {
int ans $\Rightarrow a \% m + m$
}

$40 \% -7$

$\Downarrow$

$[0, 6]$

$40 \Rightarrow -7(-5) + \text{rem}$

$40 \Rightarrow 35 \quad + \text{rem}$

$\text{rem} \Rightarrow 5$

$58 \Rightarrow -7(-8) + \text{rem}$

$|0, |-7| - 1|$

$= |0, 6|$

$|0, |\text{mod}| - 1|$

Modular Airthemetic Properties.

1) $(a+b)\%m \Rightarrow (a\%m + b\%m)\%m$

2) $(a\times b)\%m \Rightarrow (a\%m \times b\%m)\%m$

3) $(a)\%m \Rightarrow (a+m)\%m$

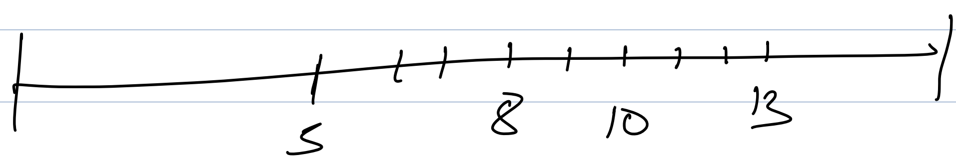
4) $(a-b)\%m \Rightarrow (a\%m - b\%m + m)\%m$

$a \Rightarrow 8,\ b \Rightarrow 4,\ m \Rightarrow 5$

$(8-4)\%5 \Rightarrow 4 \qquad (8\%5 - 4\%5 + 5)\%5$

$4\%5 \Rightarrow 4.$

$8\%5 \Rightarrow 3 \qquad 13\%5 \Rightarrow 3 \qquad 18\%5 \Rightarrow 3$



Q1 Given A & B, $A > B$, find no of M such that $A\%M = B\%M$ & $M > 1$.

Ex $A = 16$, $B = 4$, $M \Rightarrow$ 2, 3, 4, 6, 12

$16\%M = 4\%M$

$A\%M \Rightarrow B\%M$

$A\%M - B\%M \Rightarrow 0$

$(a-b)\%m \Rightarrow (a\%m - b\%m + m)\%m$

Add m on both sides.

$A\%M - B\%M + M \Rightarrow M.$

Do modulo on both sides.

$(A\%M - B\%M + M)\%M \Rightarrow M\%M \Rightarrow 0$

$$\boxed{(A-B)\%M \Rightarrow 0}$$

no of

Q2 Given n elements. Find a pair (i, j) such that $(a[i] + a[j]) \% M \Rightarrow 0$.
M is given

1) i != j

2) (i, j) is same as (j, i)

Ex1 arr =

0	1	2	3	4	5
4	7	6	5	5	3

$M \Rightarrow 3$.

Pairs

i	j	arr[i] + arr[j]
0	3	4 + 5
0	4	4 + 5
1	3	7 + 5
1	4	7 + 5
2	5	6 + 3

$\Rightarrow 5$

$(a+b)\%m \Rightarrow 0$ $\qquad (4+5)\%3 \Rightarrow 0$

$(a\%m + b\%m)\%m \Rightarrow 0 \qquad (1+2)\%3 \Rightarrow 0$

$\Downarrow \qquad \Downarrow$

X $\qquad$ Y

X	Y
1	m-1
2	m-2

mod freq

0	1	2	3	4	5
4	7	6	5	5	3

arr =

$\downarrow \%3$

1	1	0	2	2	0

$0 \Rightarrow 2$
$1 \Rightarrow 2$
$2 \Rightarrow 2$

1) $i=0$ $\quad arr[0] = 1 \quad arr[j] \Rightarrow 2 \quad \{m-1\}$

2

2) $i=1$ $\quad arr[1] = 1 \quad arr[j] \Rightarrow 2 \quad \{m-1\}$

2

arr[14] $\Rightarrow$ {24, 42, 16, 8, 7, 36, 17, 23, 35, 20, 26
28, 34, 21}.

$\boxed{M \Rightarrow 6}$

$x+y \Rightarrow 0$
$\Rightarrow m$

$0 \Rightarrow 3$
$1 \Rightarrow 1$
$2 \Rightarrow 3$
$3 \Rightarrow 1$
$4 \Rightarrow 3$
$5 \Rightarrow 3$

$x \Rightarrow$, $y \Rightarrow 0$

$^3C_2 \Rightarrow 3$

$3 \times 1 \Rightarrow 3$
$3 \times 3 \Rightarrow 9$

$^1C_2 \Rightarrow 0$

x	y
1	5
2	4
3	3
4	2
5	1

Pseudo Code

```
map <int,int> hm;
for (int i=0 ; i<n ; i++) {
        hm[arr[i]%m]++;                                0 -> 0
}
ans => 0
ans => hm[0] C 2        // Edge case
i = 1 , j => m-1
while ( i < j ) {
   ans = ans + hm[i] * hm[j];
   i++ , j--;
}
if (m%2 == 0) {
   ans = ans + hm[m/2] C 2 ;
}
   return ans;
```

TC: O(n+m)

SC:

Space Complexity

1) $n >> m$

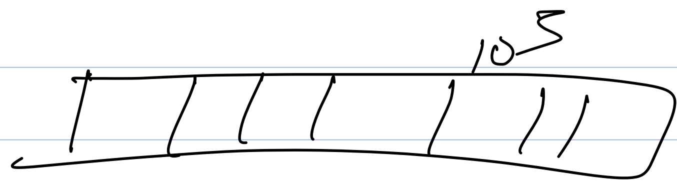
$SC: O(M)$

2) $m >> n$

$SC: O(N)$

$O(\min(m, n))$

$n =$



$m \Rightarrow 10^3$

Q3 Given an array of all distinct integers where $(0 \le arr[i] \le n-1 \}$, n is the size of array. $\boxed{0 \le arr[i] < n}$

Replace arr[i] with arr[arr[i]].

Ex1

arr =

0	1	2	3	4
3	2	4	1	0

# arr[0] = arr[arr[0]]
arr[0] = arr[3]
arr[0] => 1

arr[2] = arr[arr[2]]
= arr[4]
=> 0

[1, 4, 0, 2, 3]

Ex1

0	1	2	3
1	2	3	0

arr[3] = arr[arr[3]]
= arr[0]

OLD Value
NEW Value

Big Bang. $\Rightarrow x/24$

$\Downarrow$ Day. hour $\Rightarrow x\%24$

	Day	hour
22 hours	0	22
50 hours	2	2

$arr[i] \times n + m$ $\Rightarrow$ old $\Rightarrow arr(i)/n$

$\Rightarrow$ new $\Rightarrow arr[i]\%n$

$\Downarrow$

$[0, n-1]$

	0	1	2	3	4
arr[5] $\Rightarrow$	3	2	4	1	0
multiply bus $\Rightarrow$	15	10	20	5	0
updation $\Rightarrow$	index $\Rightarrow$ 3 value $\Rightarrow$ arr[3]/5 $\Rightarrow$ 1. 15 + 1 = 16	index $\Rightarrow$ 10/5 $\Rightarrow$ 2 value $\Rightarrow$ arr[2]/5 10 + 4 = 14		index $\Rightarrow$ 1 value $\Rightarrow$ 2	

16	14	20	7	3

$\Downarrow \% n$

1	4	0	2	3

$2 \times 5 \Rightarrow 10$

$n = 5$

$10 + 4$

4

m

Pseudo Code

```
for (int i=0 ; i<n ; i++)
    arr[i] *= n;

for (int i=0 ; i<n ; i++) {
    int index => arr[i]/n;
    int value => arr[index]/n;
    arr[i] += value;
}

for (int i=0 ; i<n ; i++) {
    arr[i] %= n;
}
```

TC : O(n)
SC : O(1)