Optimal Control WS20/21: Homework 2

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Problem 1

a) Formulating the problem as an discrete-time, infinite-horizon o. c. problem yields

$$\begin{aligned} \min_{u_0,u_1,\dots} \quad & \sum_{k=0}^{\infty} f_0(x_k,u_k) = 0.9^k f_0(x_k,u_k) \\ \text{subject to} \quad & x_{k+1} = f(x_k,u_k), \\ & x_k \in \mathcal{X} = \{\xi_1,\dots,\xi_8\}, \\ & u_k \in \mathcal{U} = \{0,1,2\}, \\ & x_0 = \xi_1, \end{aligned} \tag{1}$$

where the dynamics $f : \mathcal{X} \times \mathcal{U} \longrightarrow \mathcal{X}$ are defined by the arrows in the graph.

The Value function iteration is defined by the Bellman operator

$$V_{k+1}(x) = TV_k(x) = \min_{u\in\mathcal{U}}\{f_0(x,u) + \alpha V_k(x)\} \quad \text{with} \quad \alpha = 0.9. \tag{2}$$

Evaluated for this particular problem, this yields

$$\begin{aligned} V_{k+1}(\xi_1) &= \min_{u\in\mathcal{U}}\{1 + 0.9V_k(\xi_1), 1 + 0.9V_k(\xi_1))\} = 1 + 0.9V_k(\xi_1) \\ V_{k+1}(\xi_2) &= \min_{u\in\mathcal{U}}\{3 + 0.9V_k(\xi_1), 6 + 0.9V_k(\xi_1)), 3 + 0.9V_k(\xi_1))\} = 2 + 0.9V_k(\xi_2), \\ V_{k+1}(\xi_3) &= \min_{u\in\mathcal{U}}\{1 + 0.9V_k(\xi_3))\} \\ V_{k+1}(\xi_4) &= \min_{u\in\mathcal{U}}\{2 + 0.9V_k(\xi_4))\} \\ V_{k+1}(\xi_5) &= \min_{u\in\mathcal{U}}\{0 + 0.9V_k(\xi_5))\} \\ V_{k+1}(\xi_6) &= \min_{u\in\mathcal{U}}\{1 + 0.9V_k(\xi_6))\} \\ V_{k+1}(\xi_7) &= \min_{u\in\mathcal{U}}\{2 + 0.9V_k(\xi_7))\} \\ V_{k+1}(\xi_8) &= \min_{u\in\mathcal{U}}\{0 + 0.9V_k(\xi_8))\}, \end{aligned} \tag{3}$$

starting with an arbitrary inital value function $V_0 : \mathcal{X} \longrightarrow \mathbb{R}$.

b)

Problem 2

a)

b)