# Exercise 1

The main goals of this exercise are:

- Learn how to implement an MPC algorithm.
- Explore the influence of the prediction horizon and the constraints on the closed-loop behavior.

Consider the following continuous-time constrained optimal control problem

$$\begin{aligned}
\min_{\bar{u}(\cdot;t)} \int_{\tau=t}^{t+T} & \|\bar{x}(\tau;t)\|_Q^2 + \|\bar{u}(\tau;t)\|_R^2 \, \mathrm{d}\tau \\
\text{s.t. } & \dot{\bar{x}}(\tau;t) = f(\bar{x}(\tau;t), \bar{u}(\tau;t)) \ , \quad \bar{x}(t;t) = x(t) \\
& \bar{x}(\tau;t) \in \mathcal{X} \ , \quad \forall \tau \in [t, t+T] \\
& \bar{u}(\tau;t) \in \mathcal{U} \ , \quad \forall \tau \in [t, t+T] \\
& \bar{x}(t+T;t) = 0
\end{aligned}$$

with the linear system dynamics

$$f(x,u) = \begin{pmatrix} -1 & 1 \\ 0 & 0.5 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

and the constraint sets

$$\mathcal{X} := \{x \in \mathbb{R}^2 : \|x\|_\infty \le 1\} \quad \text{and} \quad \mathcal{U} := \{u \in \mathbb{R} : |u| \le 1\} \ .$$

Moreover, the weighting matrices are given by

$$Q = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \quad \text{and} \quad R = 1.0 \ .$$

**Problem 1** In order to solve the above optimal control problem, a suitable discretization is needed. To this end, a sampling time of $\delta = T/N$ is chosen, i.e., there are $N$ sampling intervals over the prediction horizon $T$. During each sampling interval, the open-loop input $\bar{u}$ is chosen to be piecewise constant (zero-order hold), that is,

$$\bar{u}(t + k \cdot \delta + \tau; t) = \bar{u}_k \ , \quad \forall \tau \in [0, \delta) \ ,$$

where $k = 0, \ldots, N-1$. Reformulate the above optimal control problem using exact discretization with zero-order hold.

**Problem 2** Implement an MPC algorithm by means of the file `MPC_Exercise1.m` using the Matlab function `quadprog.m` as presented in the comments of the code. The sampling time $\delta$ is chosen to be 0.1 time-units and the horizon $T$ is chosen to be 4 time-units. The initial state is given by $x_0 = [0.6, 0.8]^\top$.

**Problem 3** Now use different initial states, different prediction horizon lengths, different weights $Q$ and $R$, and different constraint sets $\mathcal{X}$ and $\mathcal{U}$.

What do you expect for each change? Can you observe what you expected?