Lecture 7: Tree Recursion

Brian Hou
June 29, 2016

Announcements

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- Alternate Exam Request: goo.gl/forms/FDQix4I5dNXPQDgw2

Hog Contest Rules

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- Up to two people submit one entry;
  max one entry per person

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Ready? cs61a.org/proj/hog_contest

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Environments

- We talk about environments a lot in this class…

Roadmap

Introduction

Functions

Data

Mutability

Objects

Interpretation

Paradigms

Applications

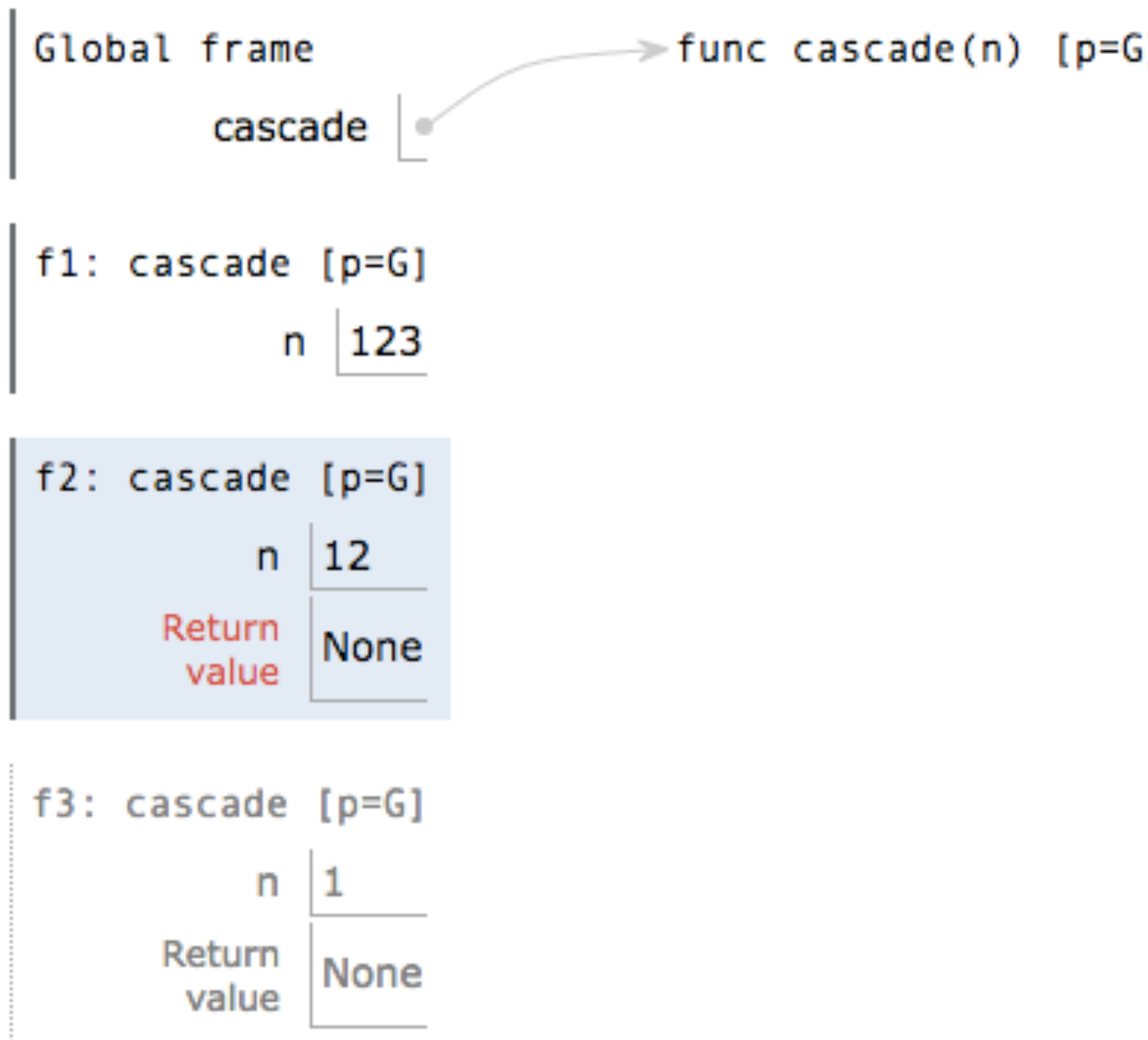
- This week (Functions), the goals are:
  - To understand the idea of *functional abstraction*
  - To study this idea through:
    - higher-order functions
    - recursion
    - orders of growth

Recursion

The Cascade Function

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



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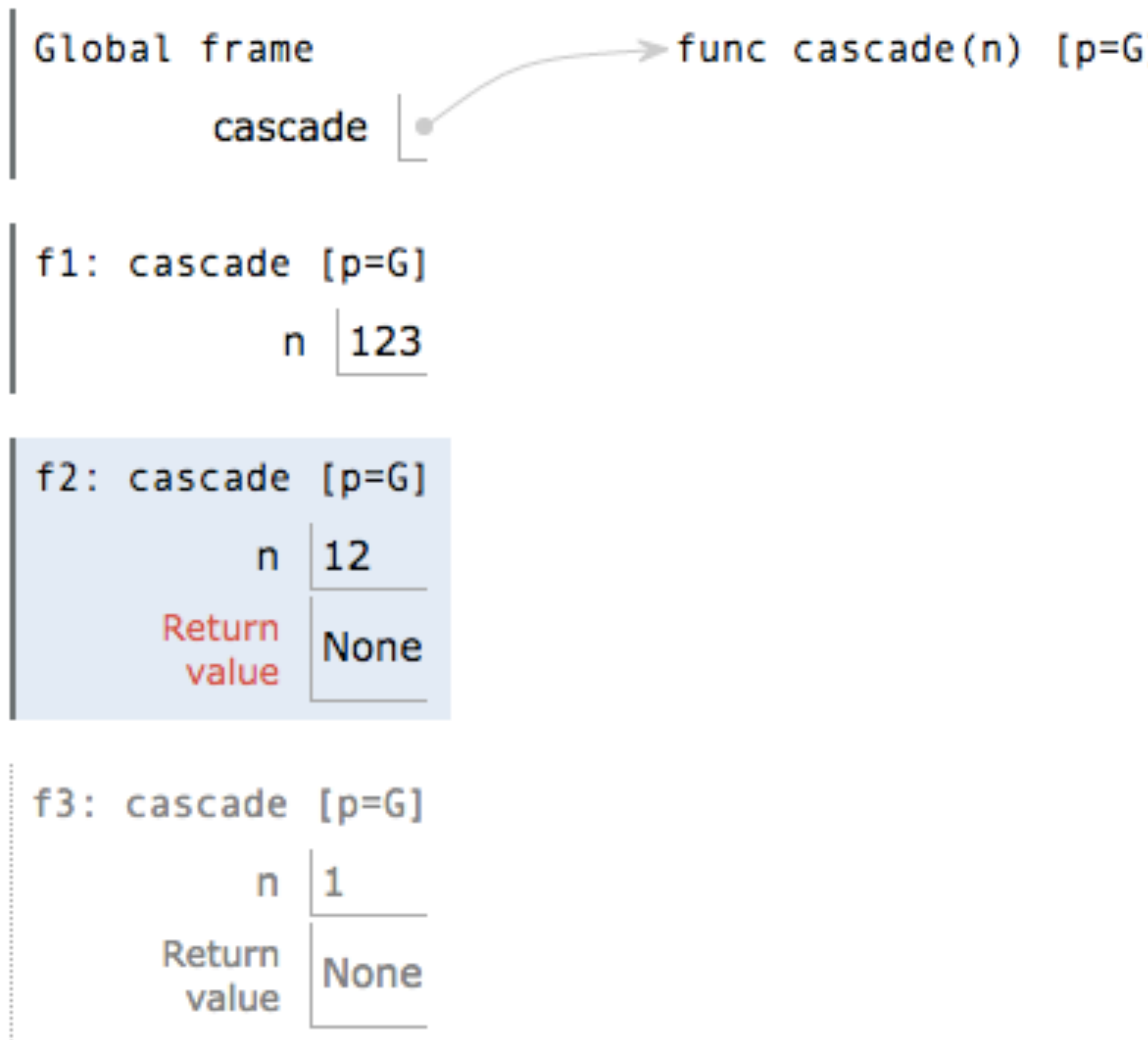

Output

123
12
1
12

The Cascade Function

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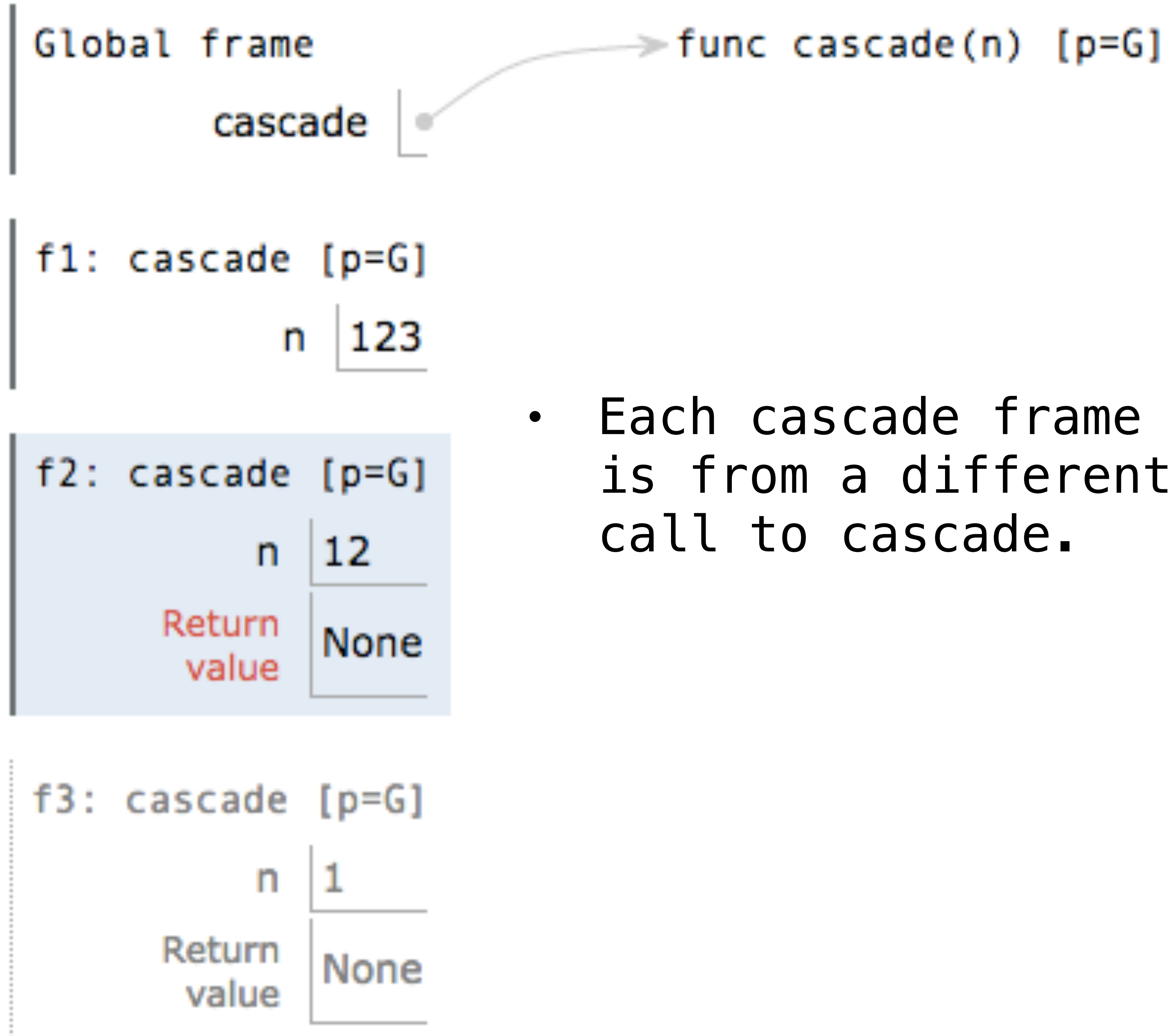

- Each cascade frame is from a different call to cascade.

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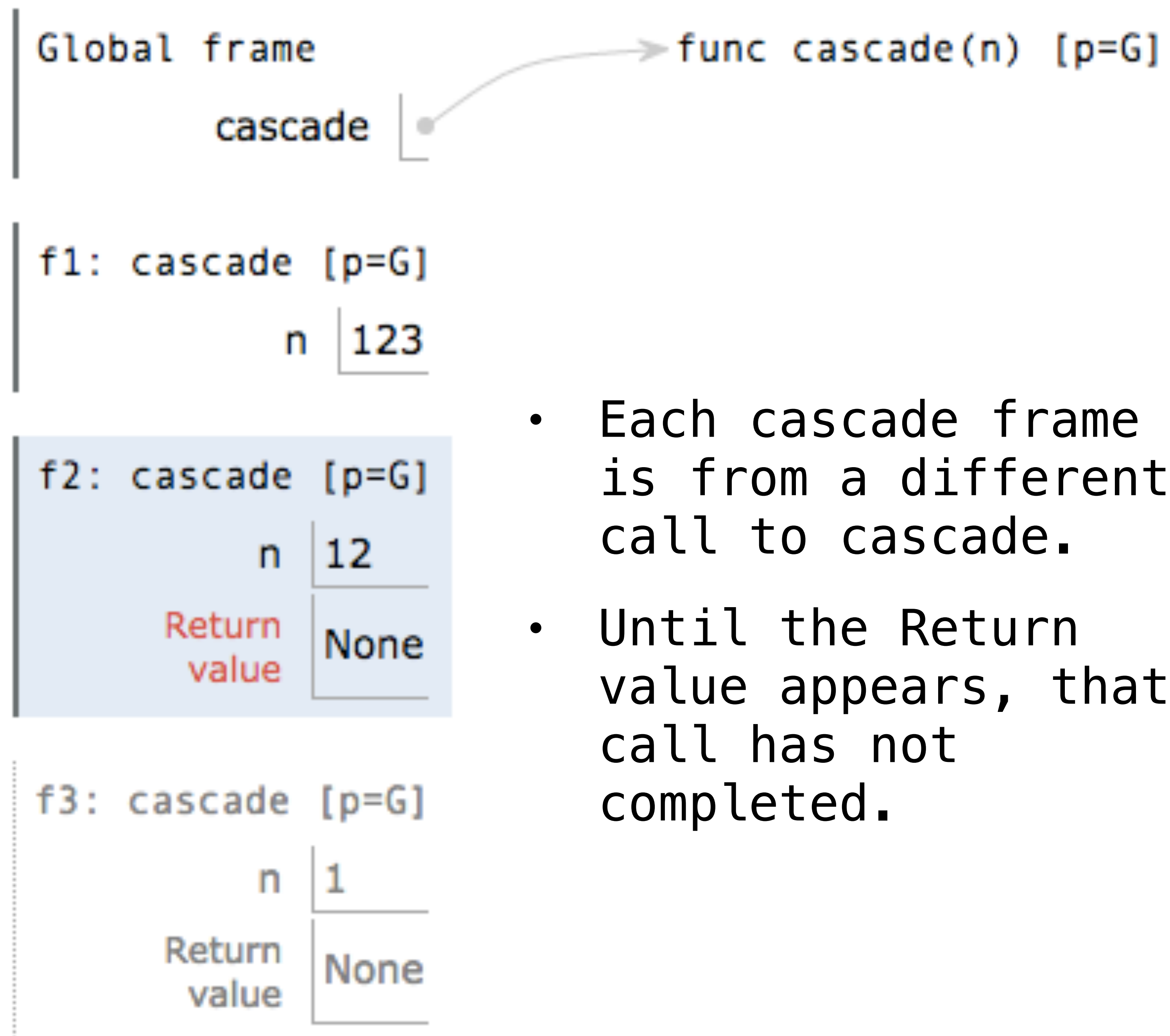

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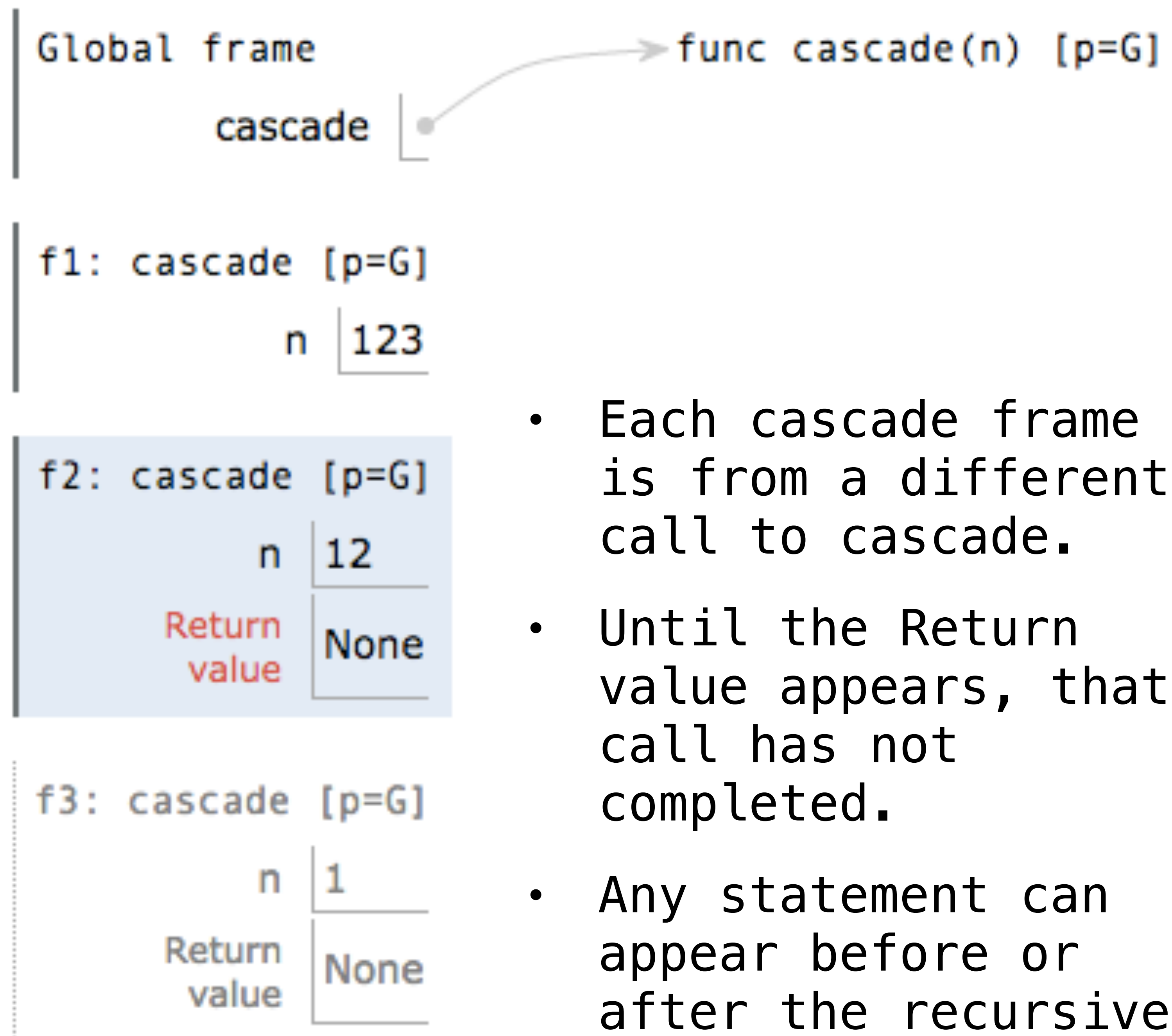

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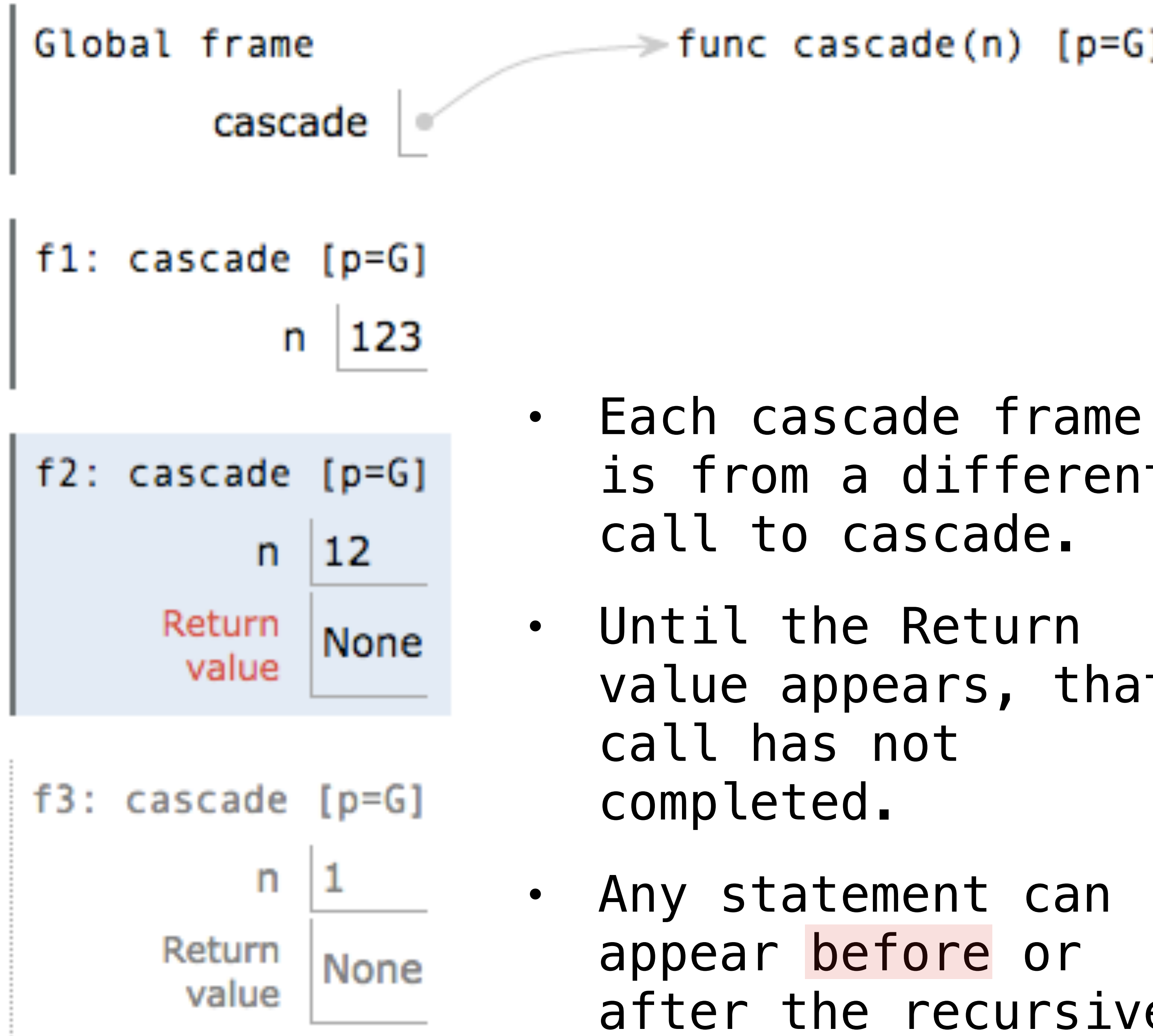

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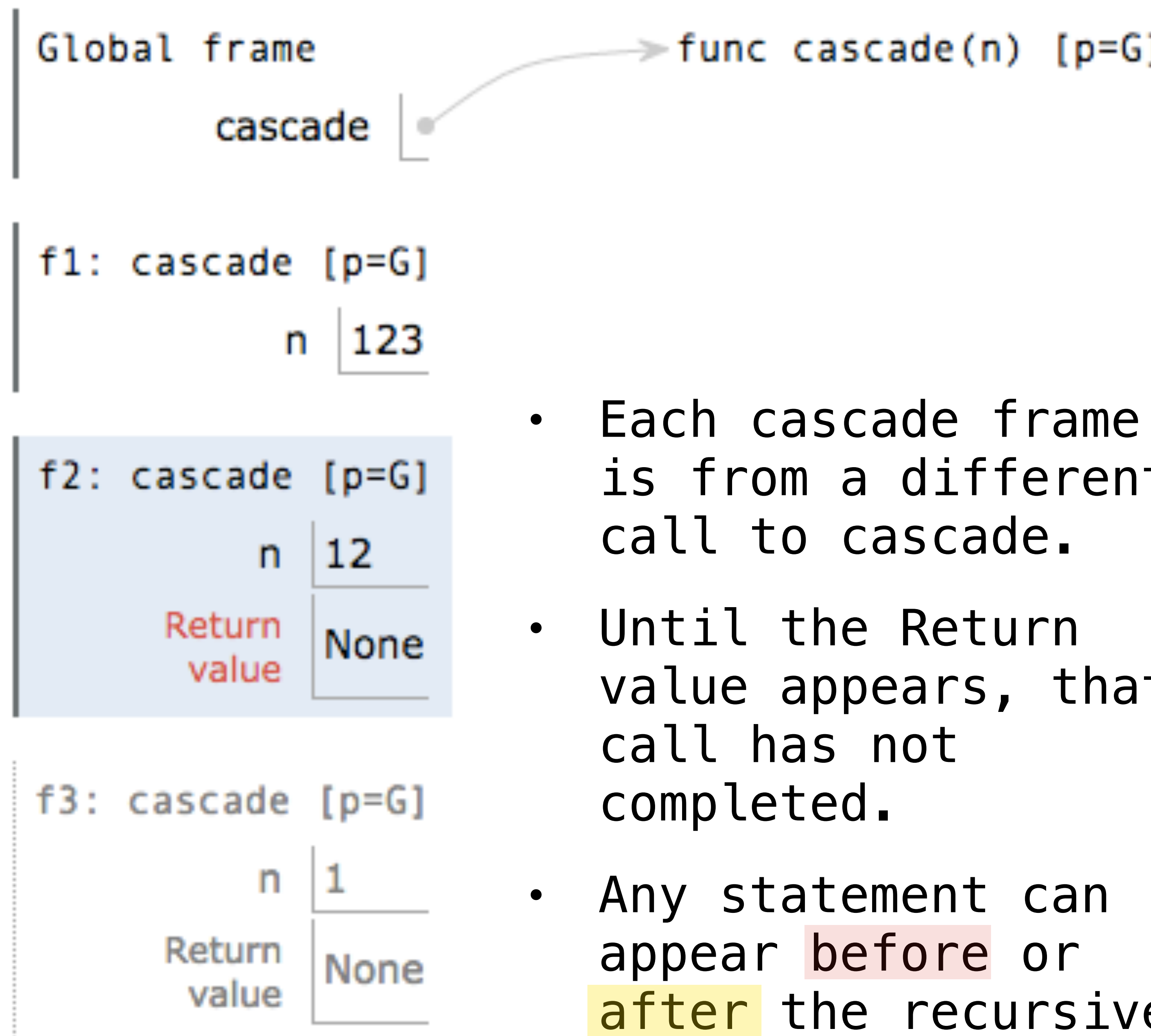

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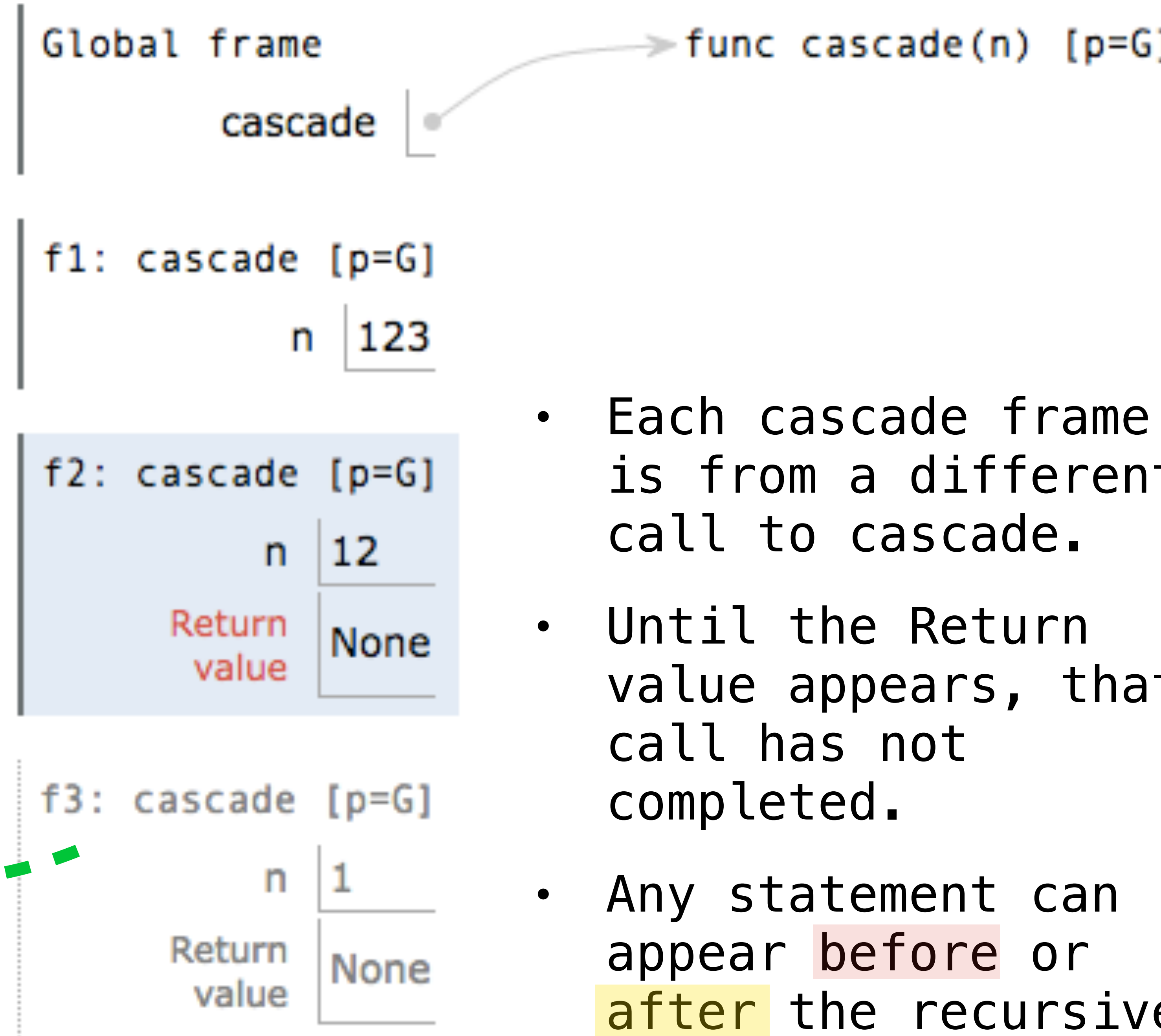

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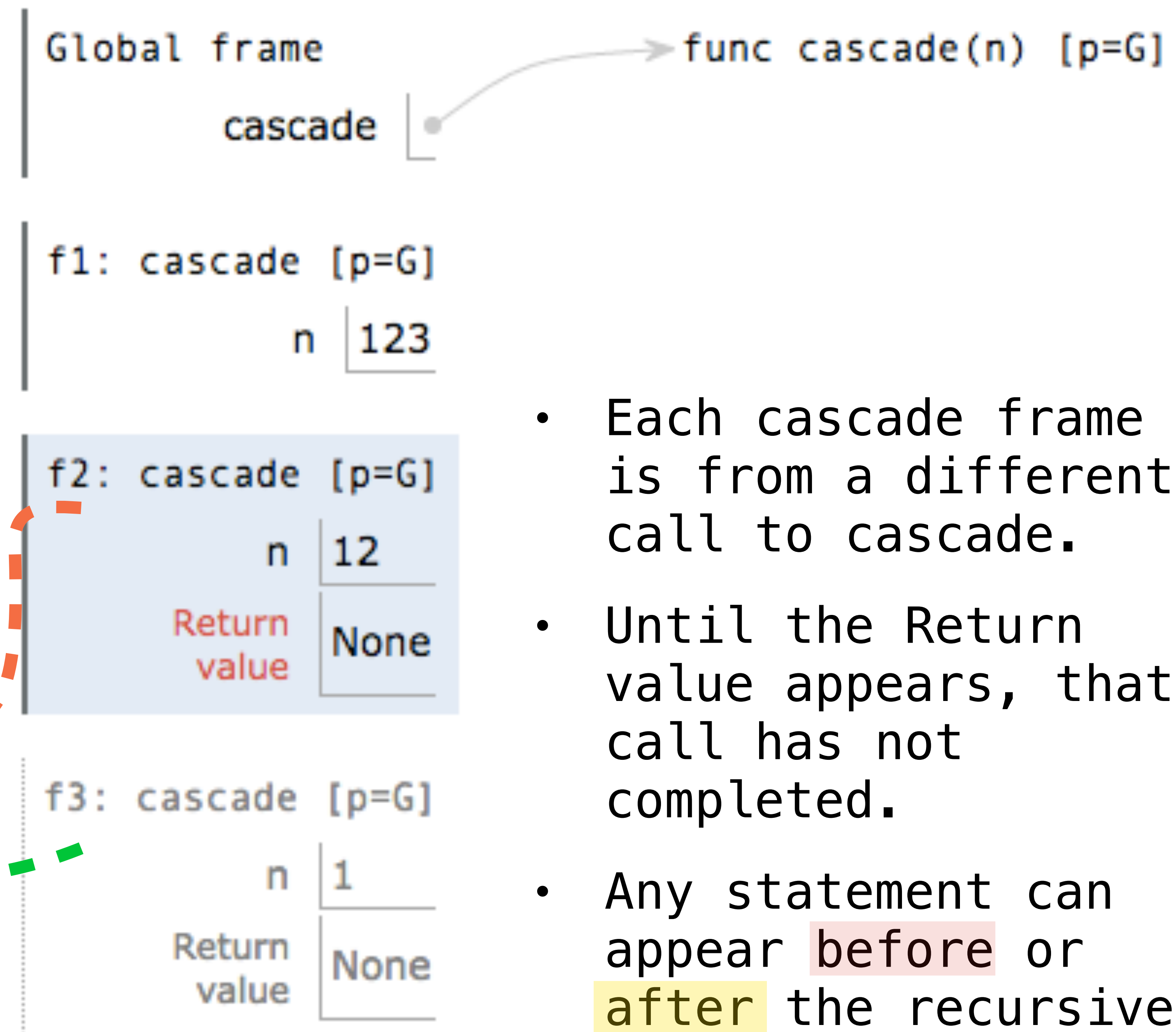

Output

123
12
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- Each cascade frame is from a different call to cascade.
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Two Definitions of Cascade

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n // 10)
        print(n)
```

```python
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n // 10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (to me)
- When learning to write recursive functions, put base cases first

Two Definitions of Cascade (demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n // 10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n // 10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (to me)
- When learning to write recursive functions, put base cases first

Inverse Cascade

Inverse Cascade

Output

```
1
12
123
1234
123
12
1
```

```python
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

Inverse Cascade

Output

```
1
12
123
1234
123
12
1
```

```python
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

```python
def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

Inverse Cascade

Output

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

```
def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

```
grow   = lambda n: f_then_g(                    )

shrink = lambda n: f_then_g(                    )
```

Inverse Cascade

Output

```
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12
123
1234
123
12
1
```

```python
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

```python
def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

```python
grow   = lambda n: f_then_g(grow,  print,  n // 10)
shrink = lambda n: f_then_g(print, shrink, n // 10)
```

Fibonacci

The Fibonacci Sequence

The Fibonacci Sequence

n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

The Fibonacci Sequence

n:	0, 1, 2, 3, 4, 5, 6, 7, 8,
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21,

The Fibonacci Sequence

n:	0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35
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```

```
def fib(n):
    pred, curr = 0, 1
```

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n:	0, 1, 2, 3, 4, 5, 6, 7, 8,
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```
def fib(n):
    pred, curr = 0, 1
    k = 1
```

The Fibonacci Sequence

n:	0, 1, 2, 3, 4, 5, 6, 7, 8,
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21,

```
def fib(n):
    pred, curr = 0, 1
    k = 1
    while k < n:
```

The Fibonacci Sequence

```
      n: 0, 1, 2, 3, 4, 5, 6,  7,  8,
fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21,
```

```
def fib(n):
    pred, curr = 0, 1
    k = 1
    while k < n:
        pred, curr = curr, pred + curr
```

The Fibonacci Sequence

n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

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```
def fib(n):
    pred, curr = 0, 1
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```

The next Fibonacci number is the sum of the two previous Fibonacci numbers

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    k = 1
    while k < n:
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        k += 1
    return curr
```

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```
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```

The Fibonacci Sequence

n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

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```
def fib(n):
    if n == 0:
```

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```

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```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

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Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

```
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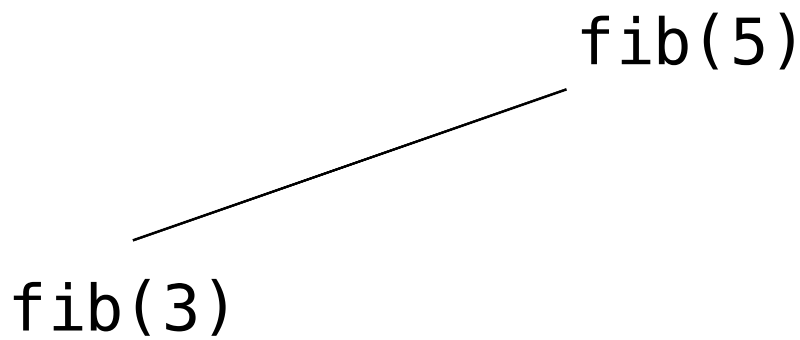
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

A Tree-Recursive Process

A Tree-Recursive Process

fib(5)

A Tree-Recursive Process



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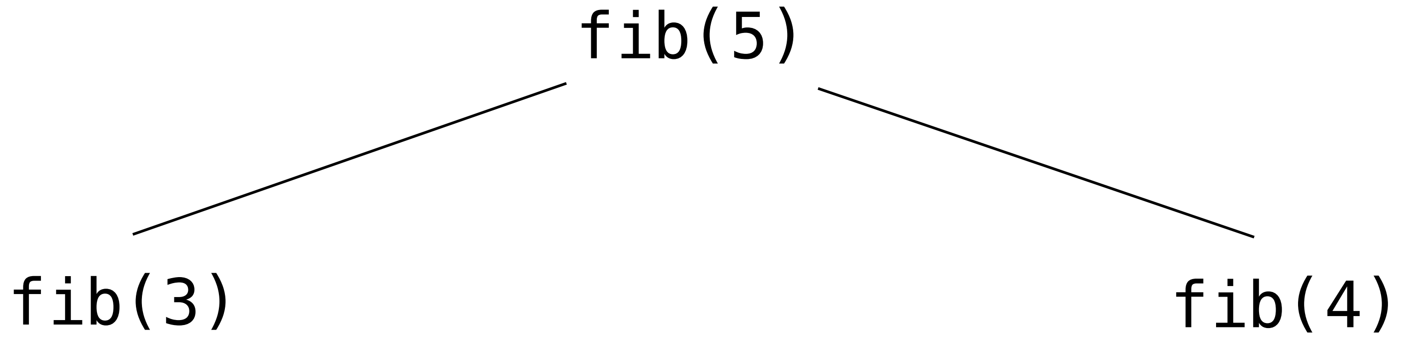

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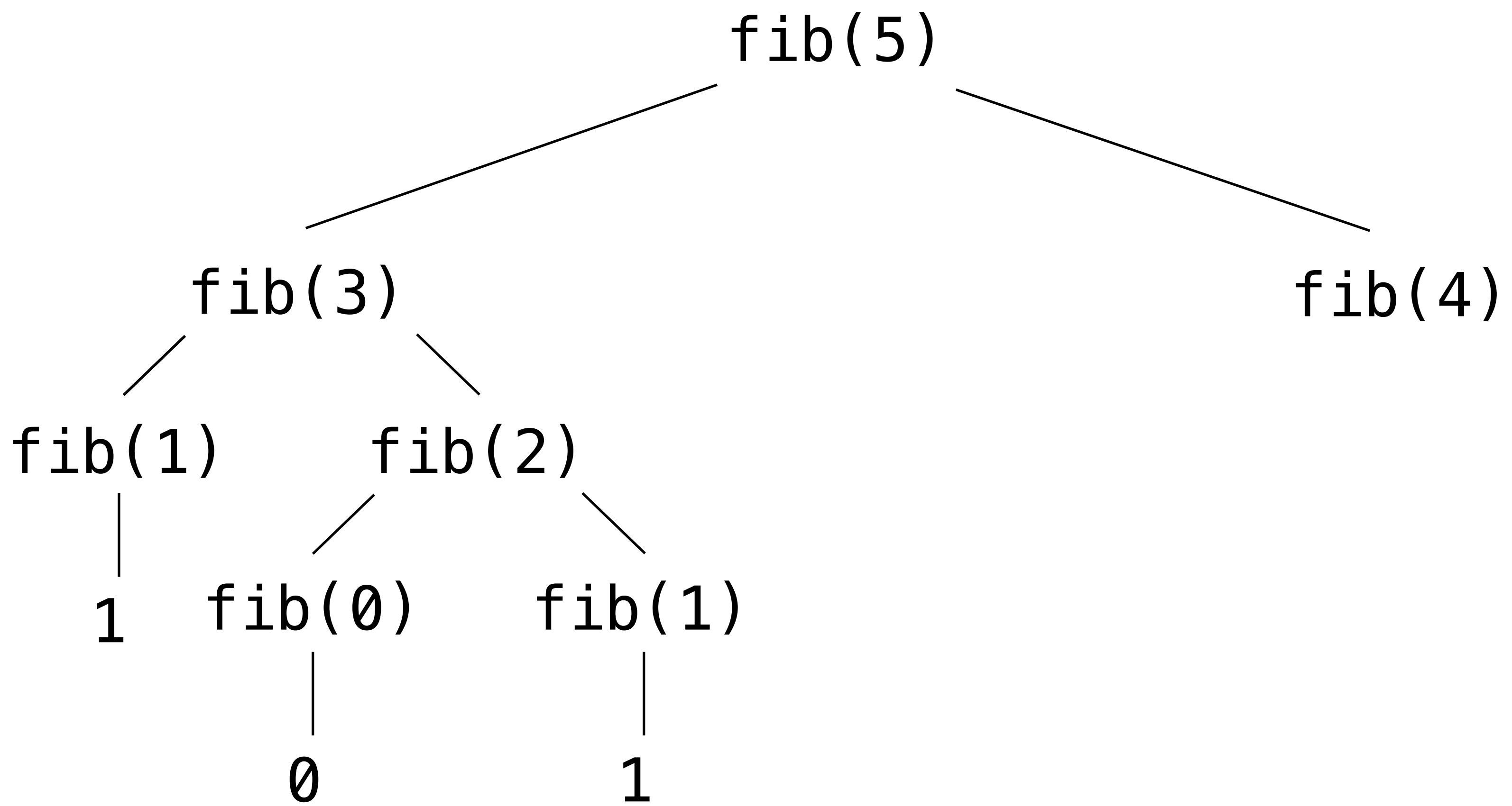

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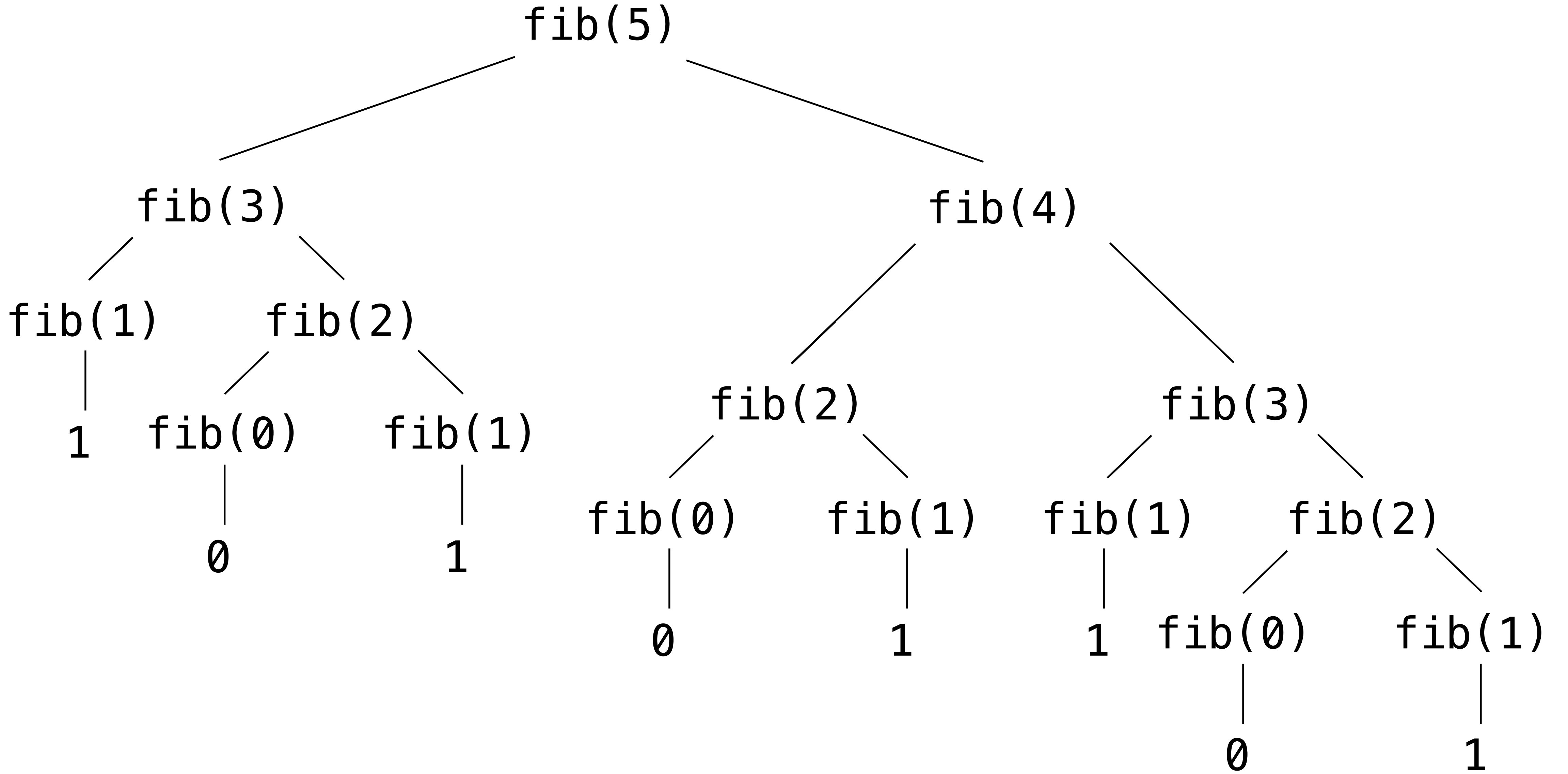

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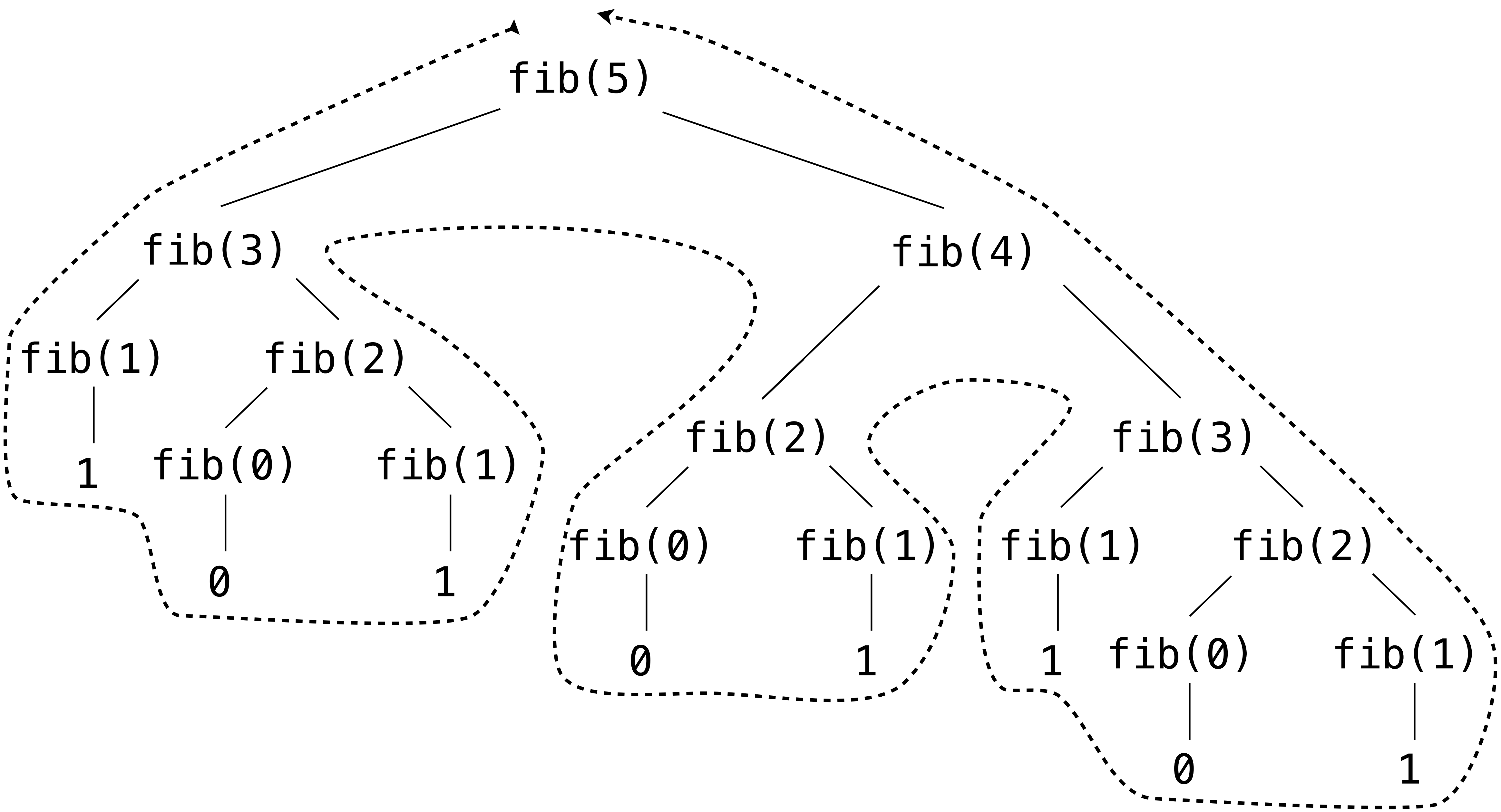

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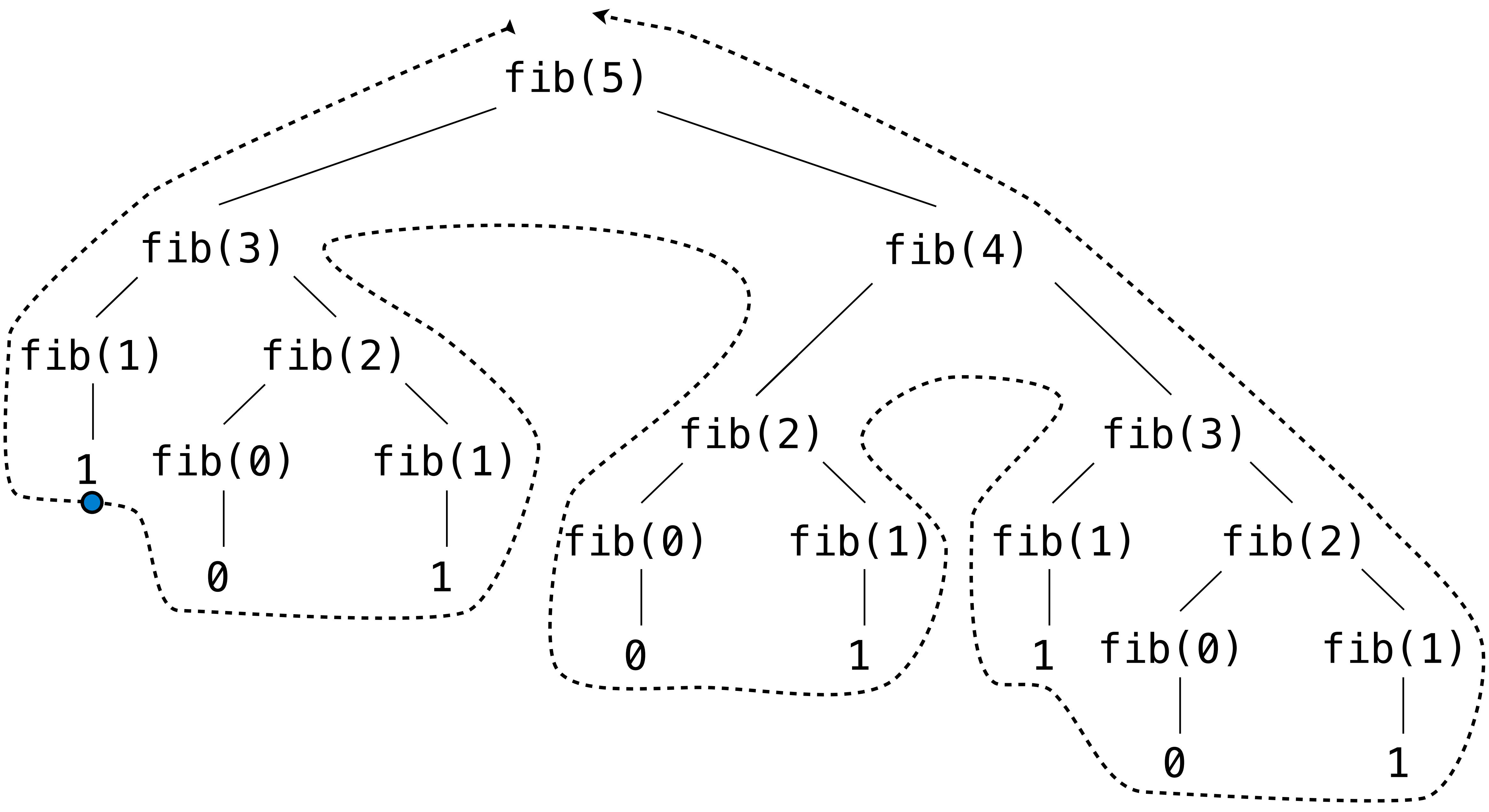

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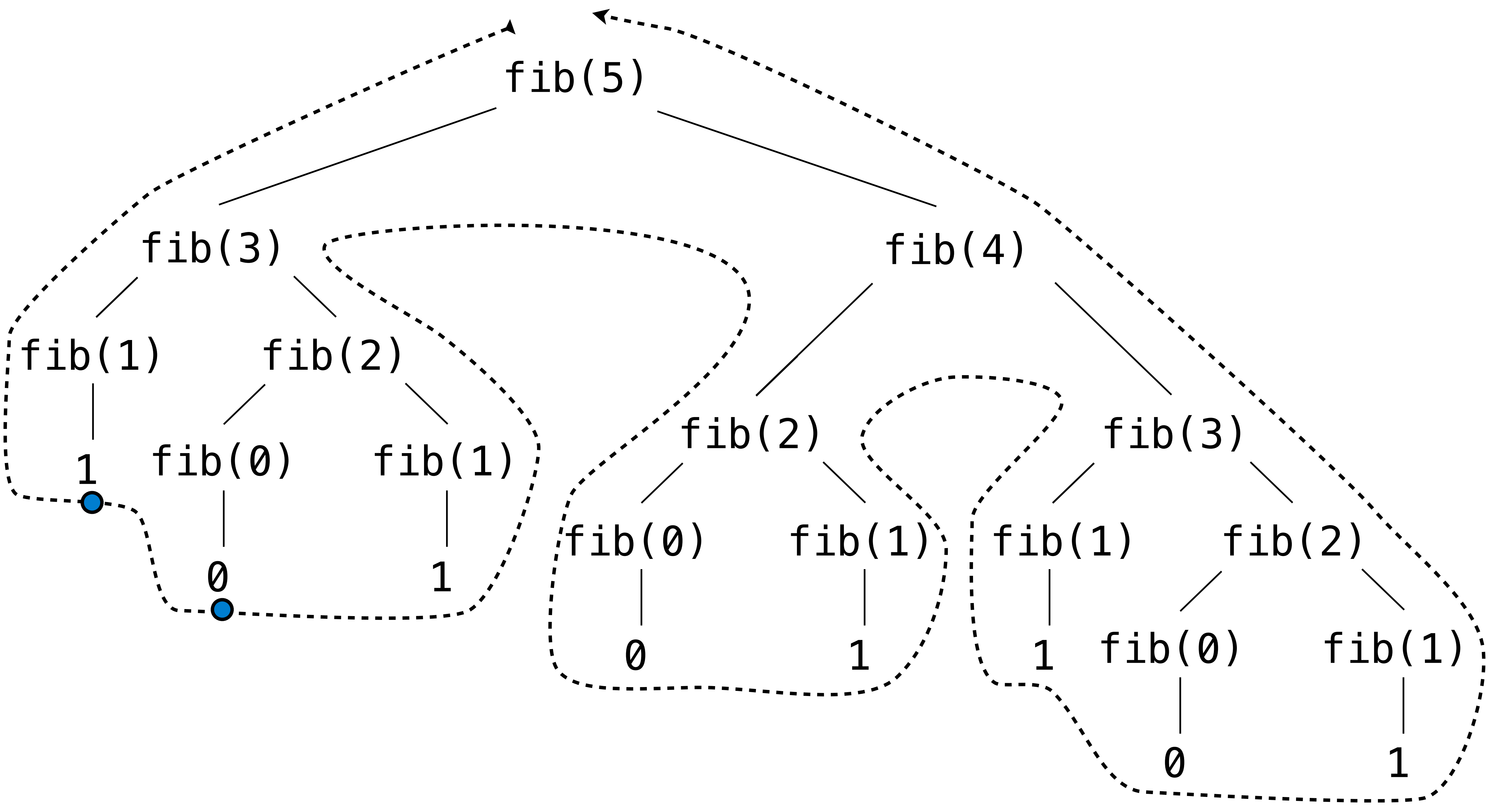

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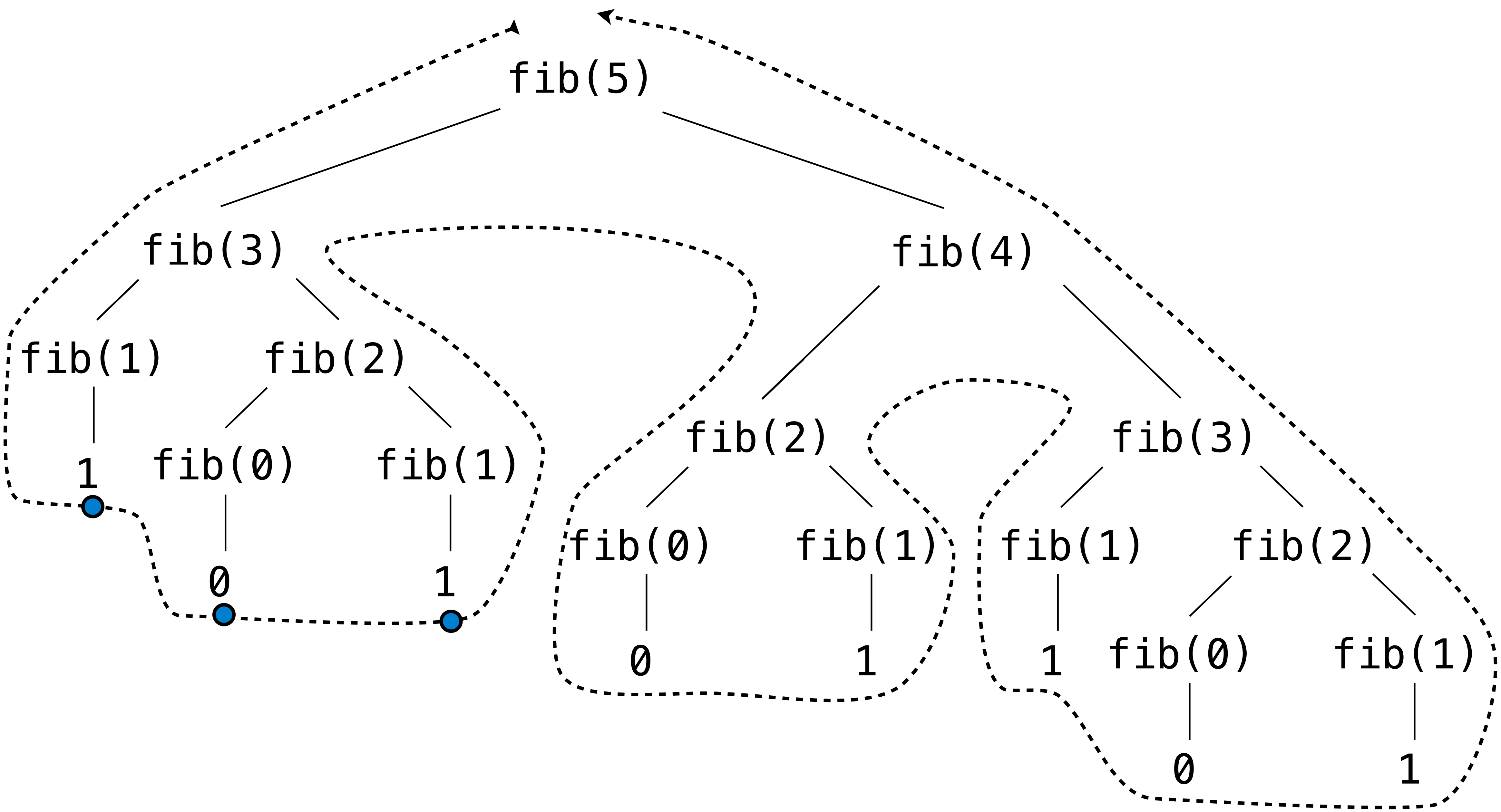

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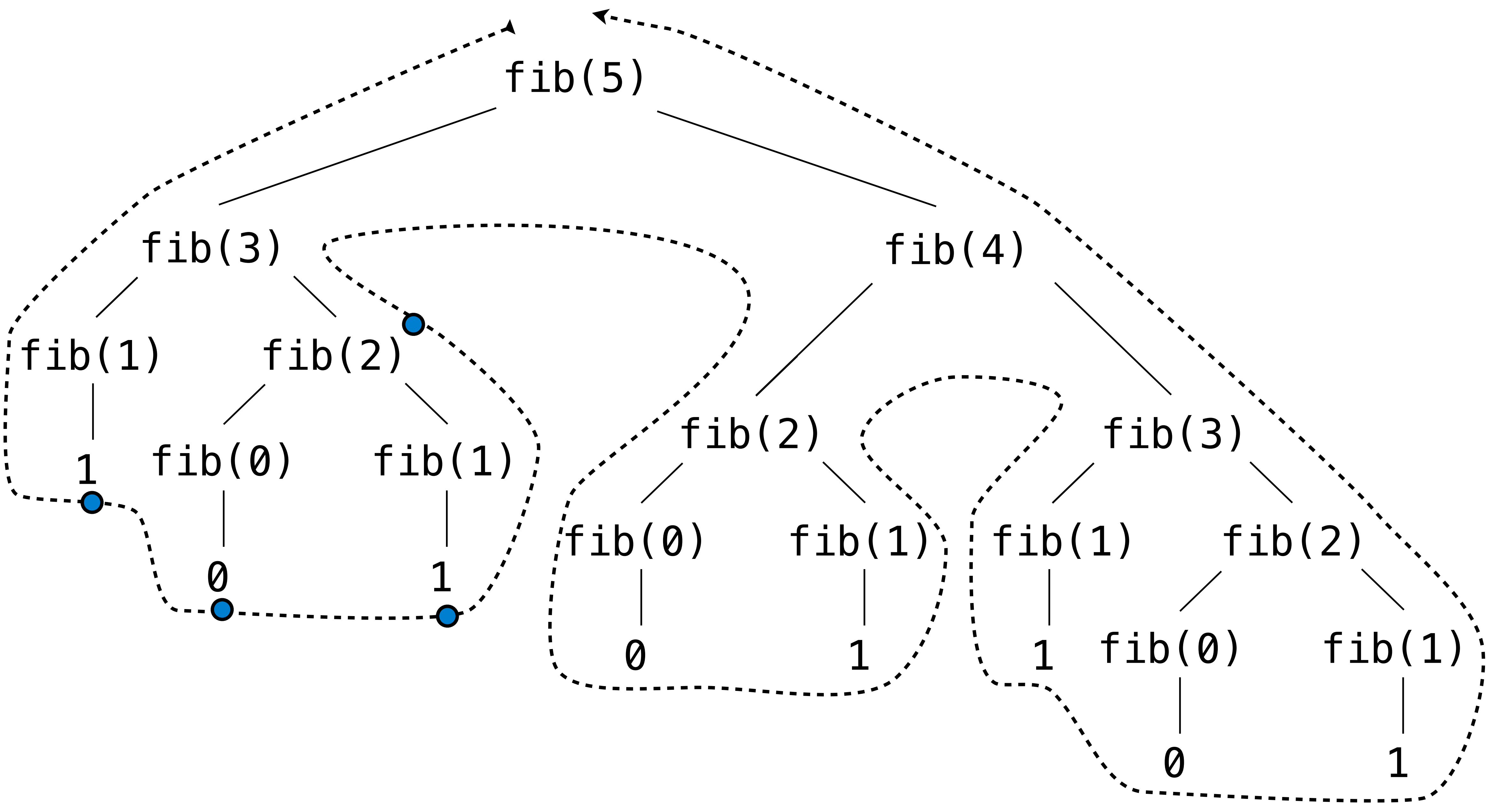

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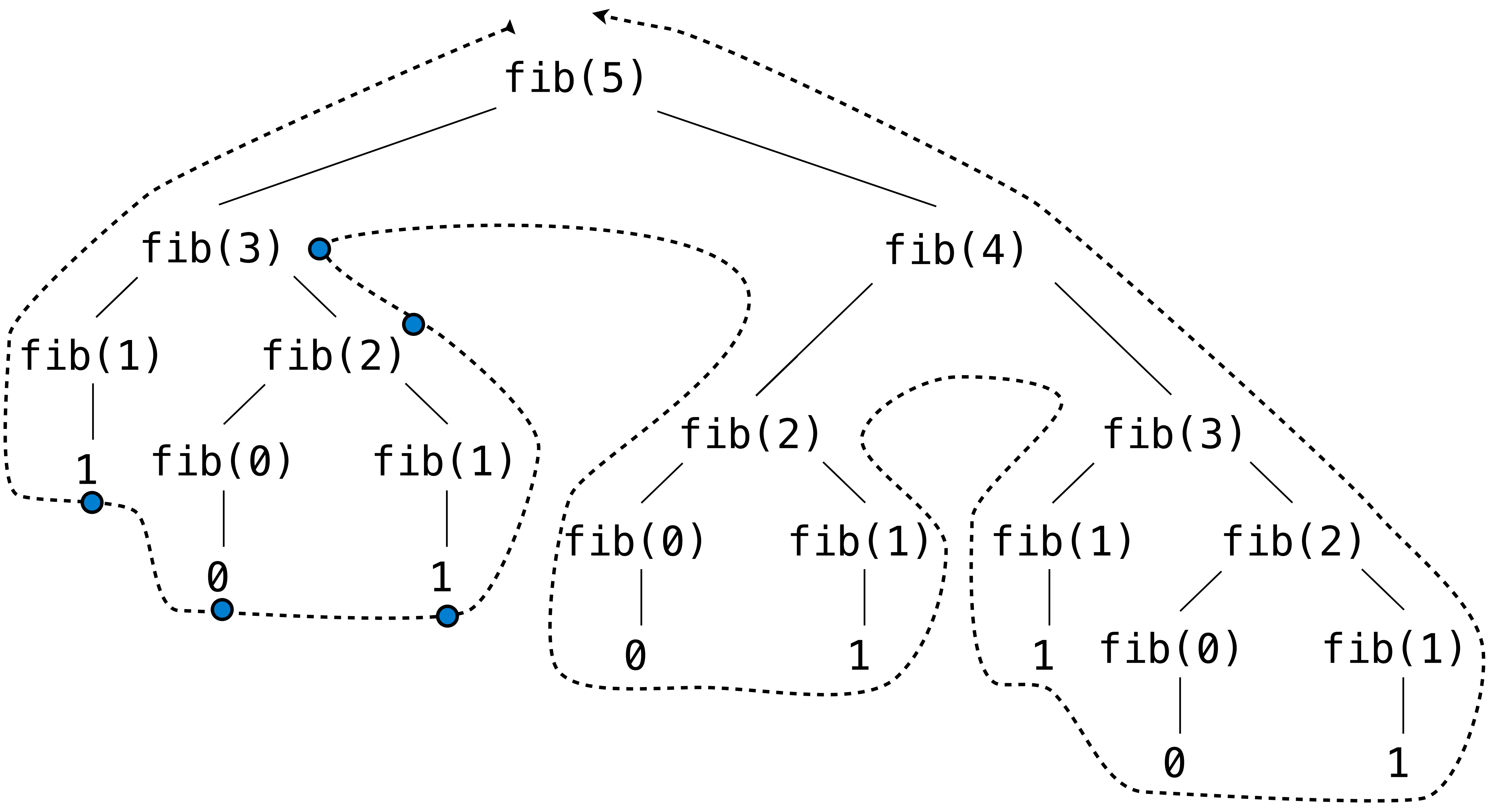

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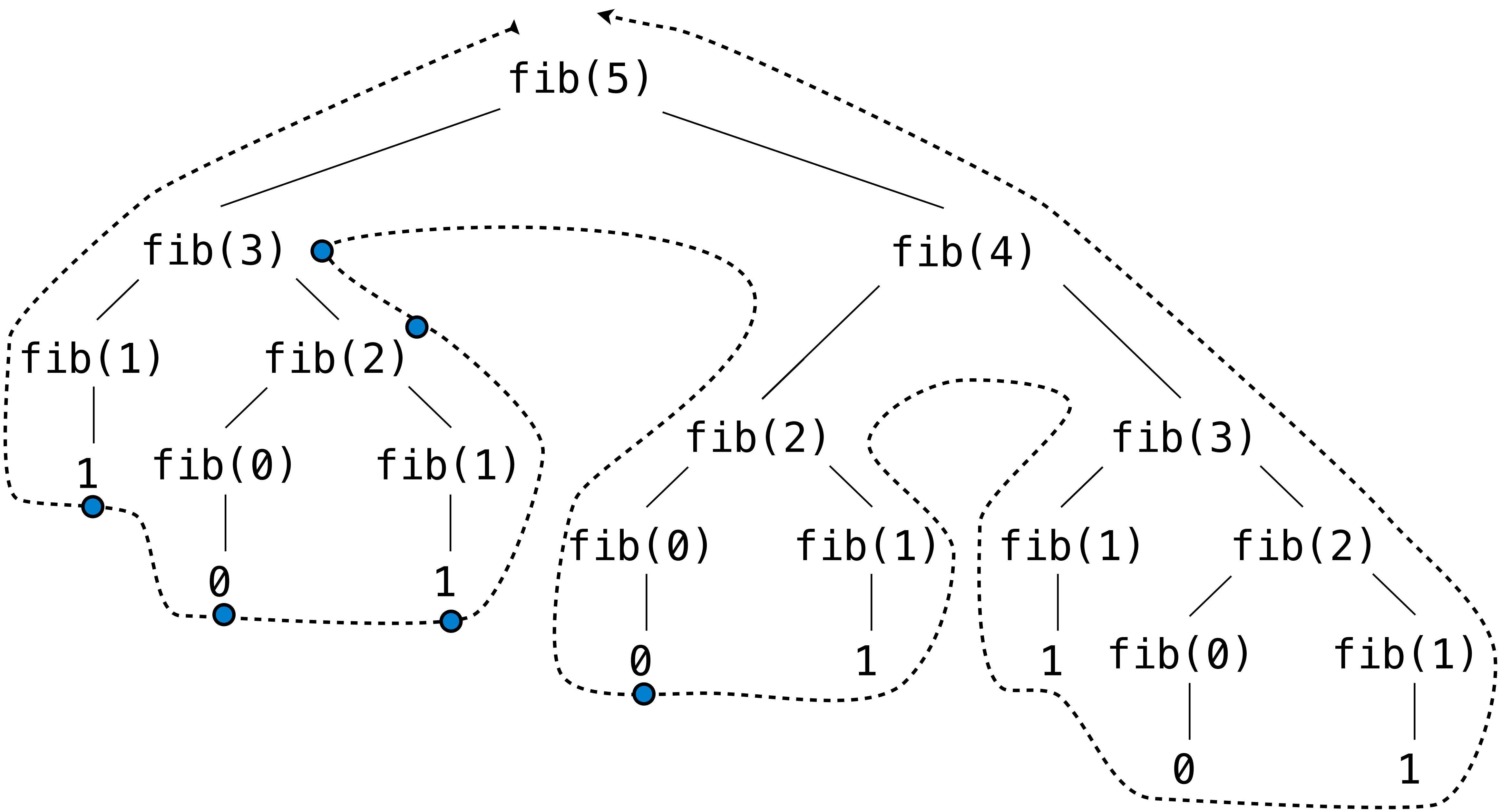

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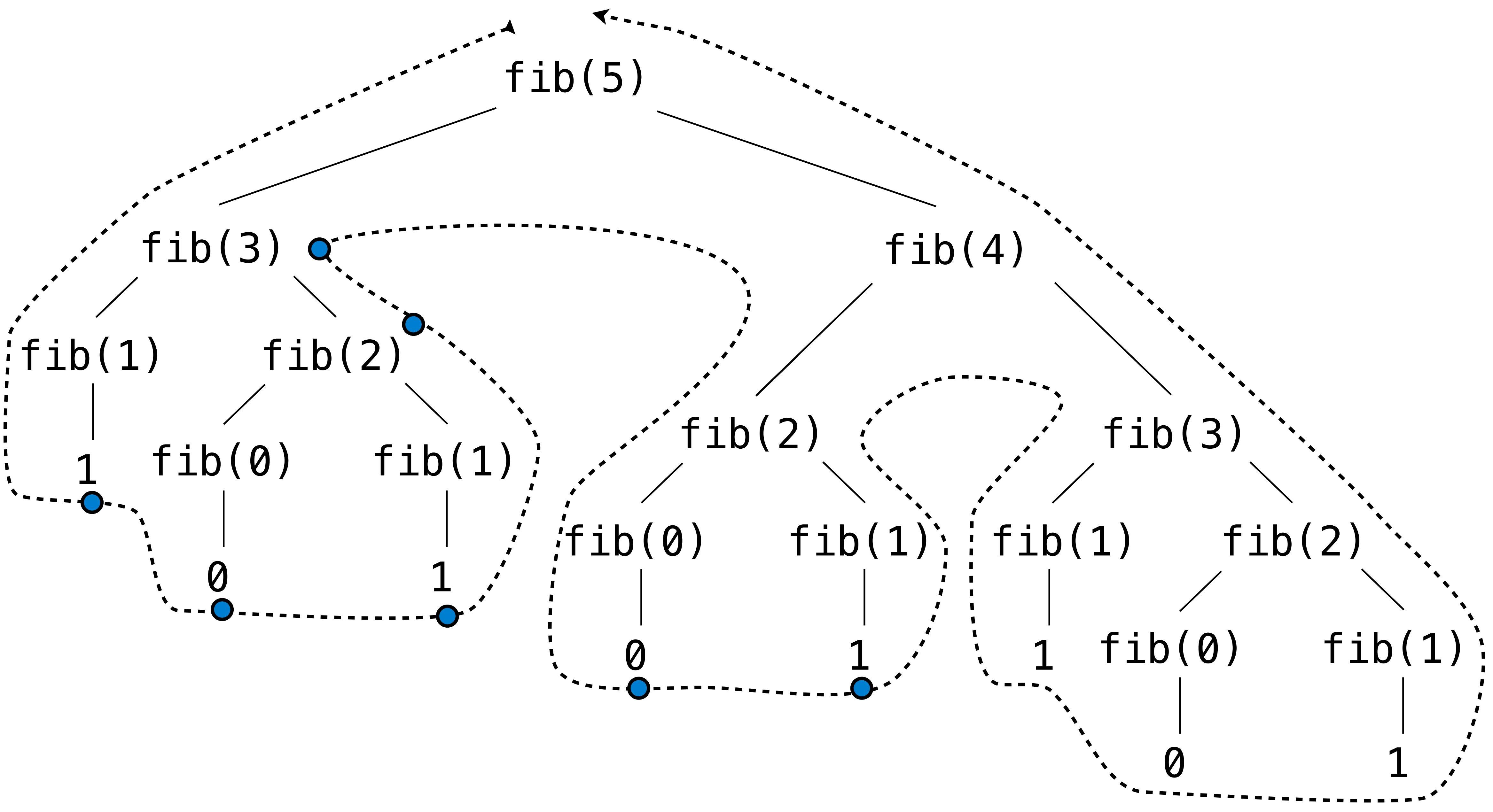
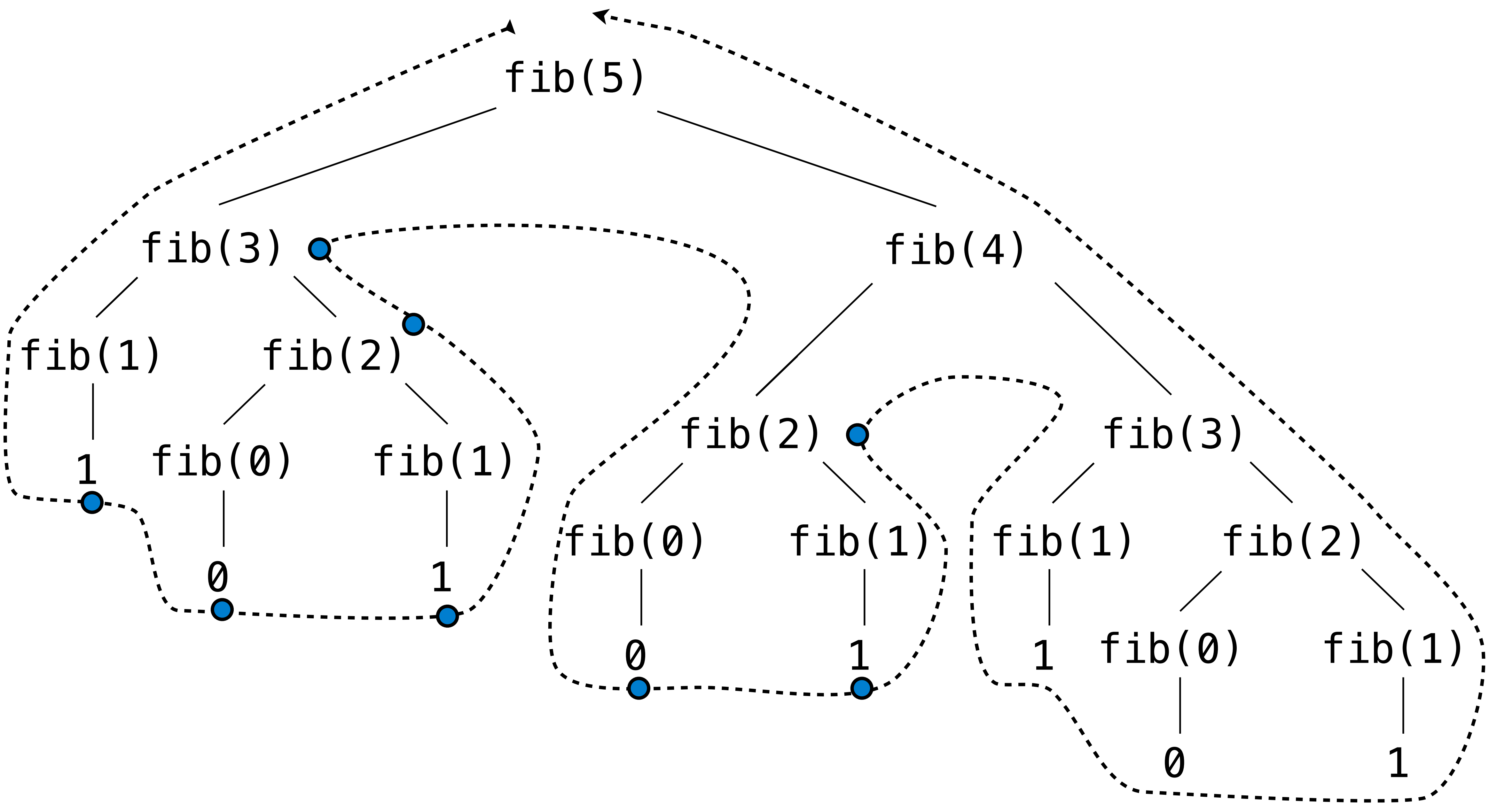


A Tree-Recursive Process



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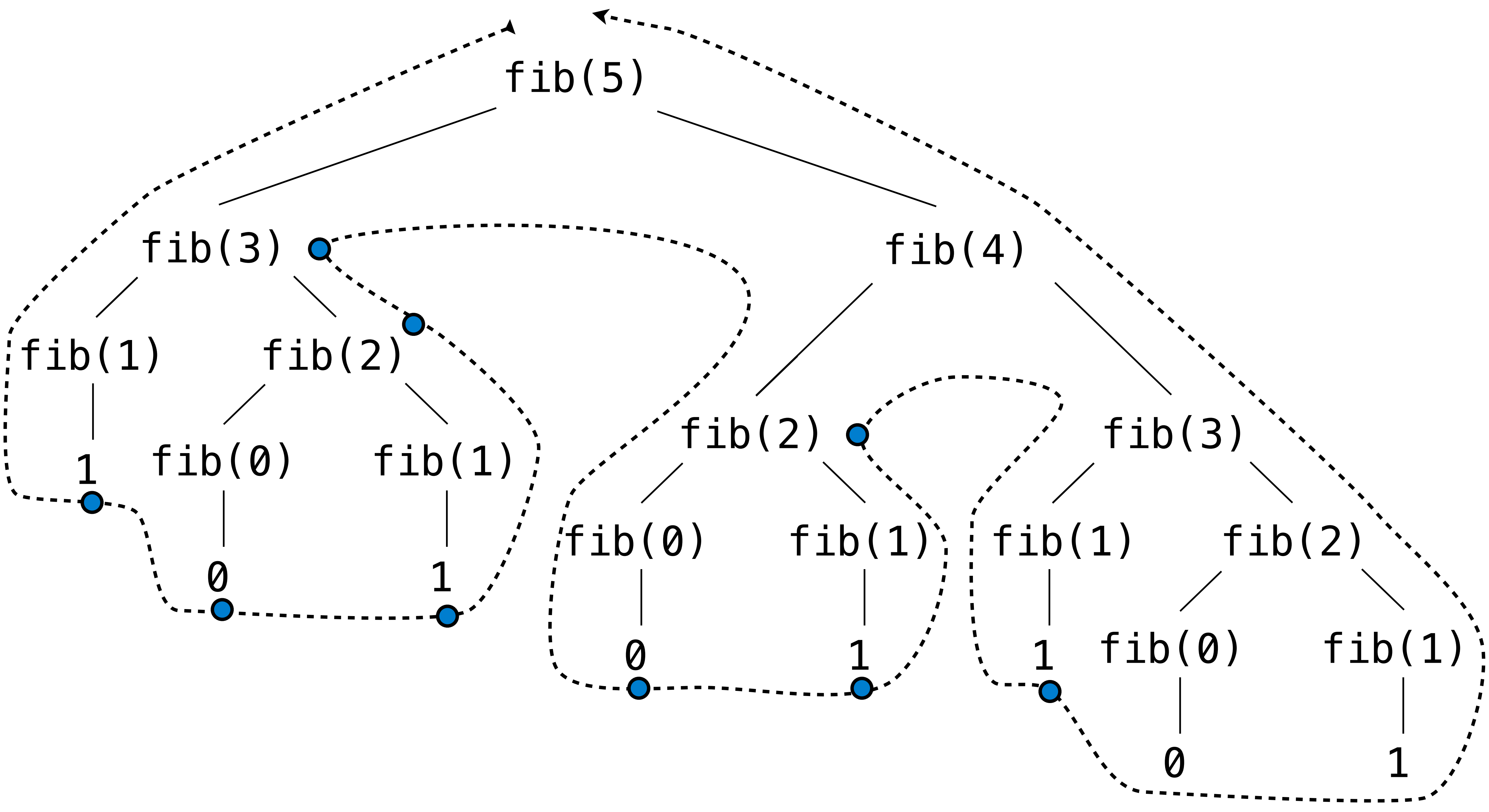

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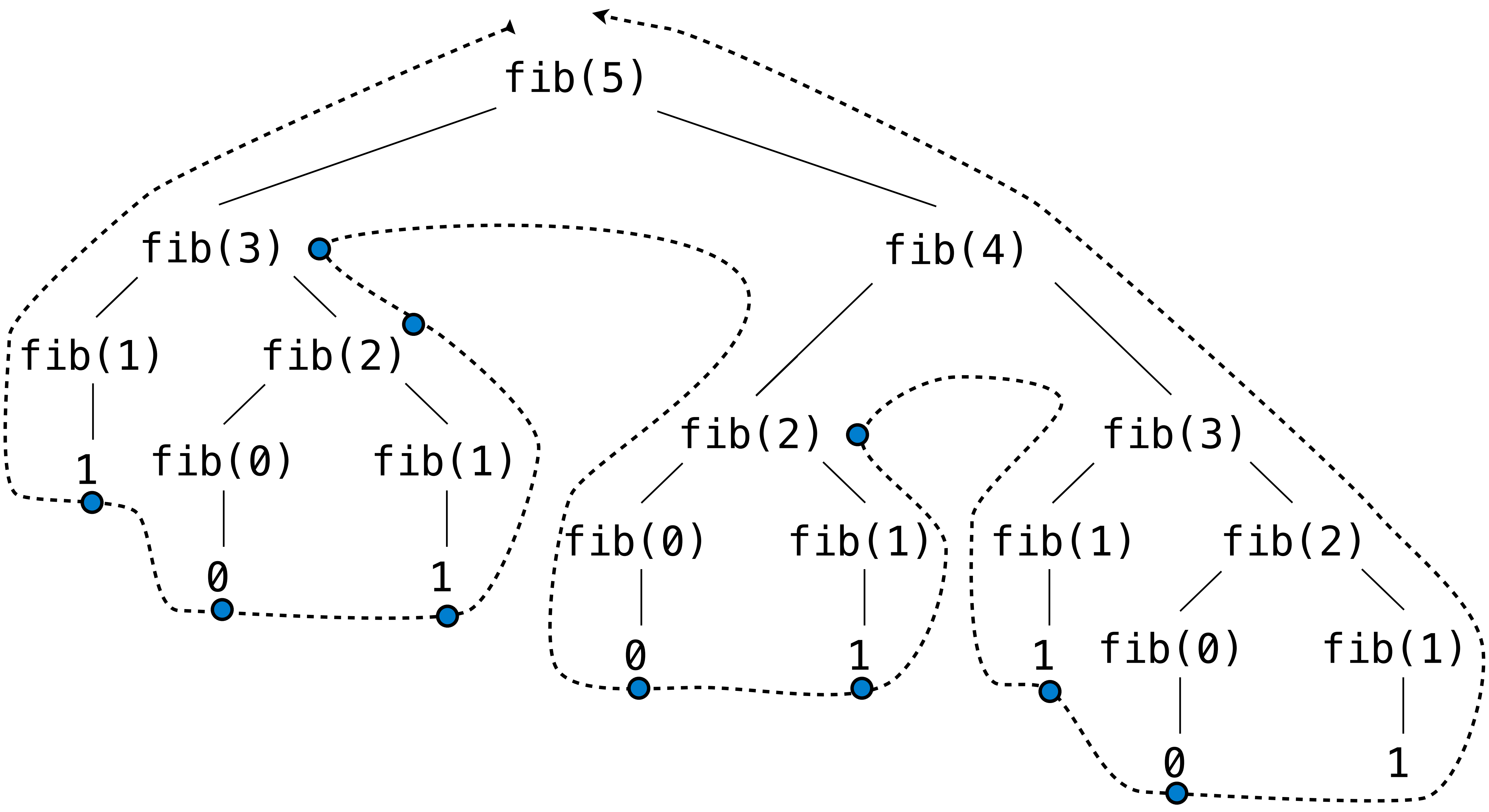

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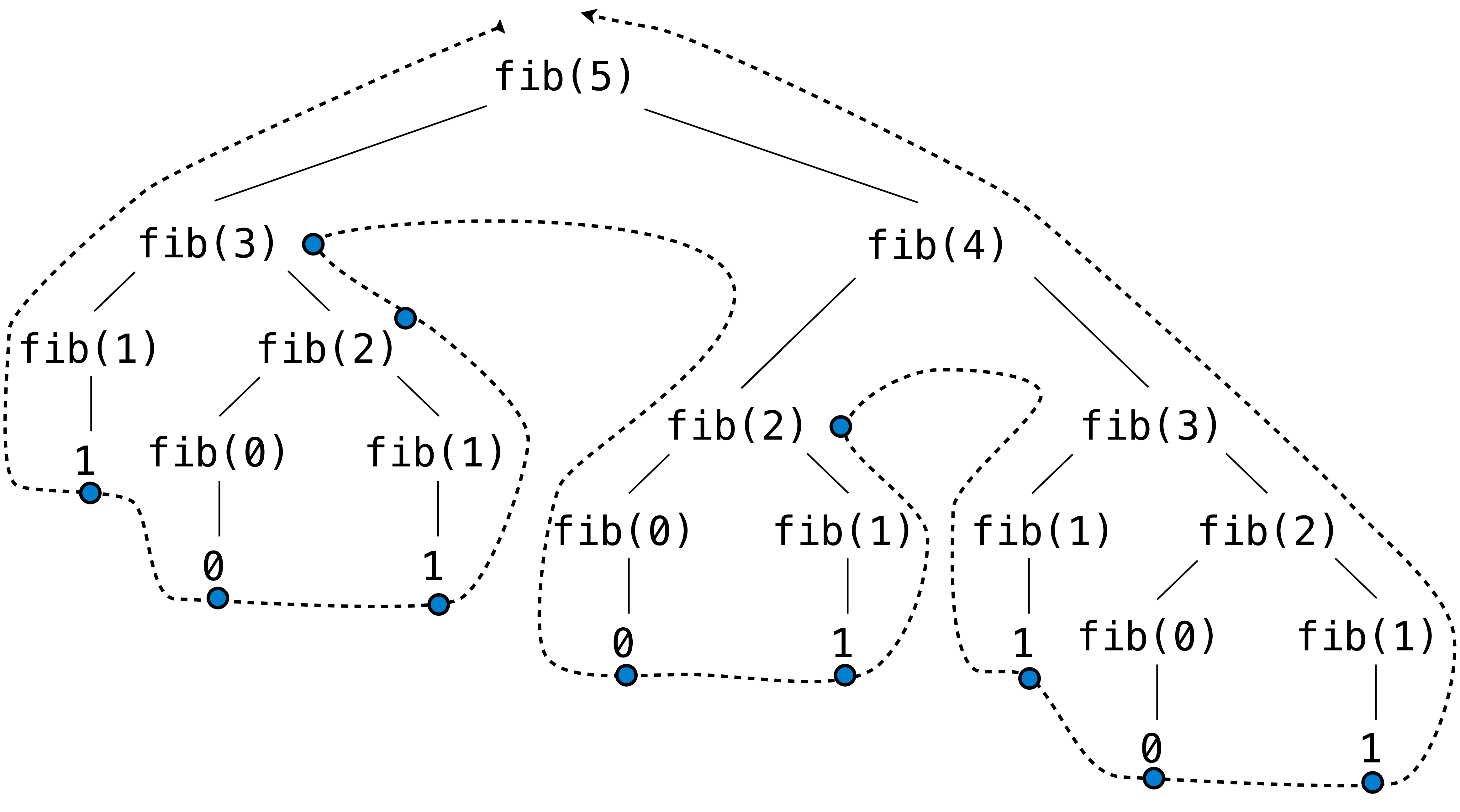

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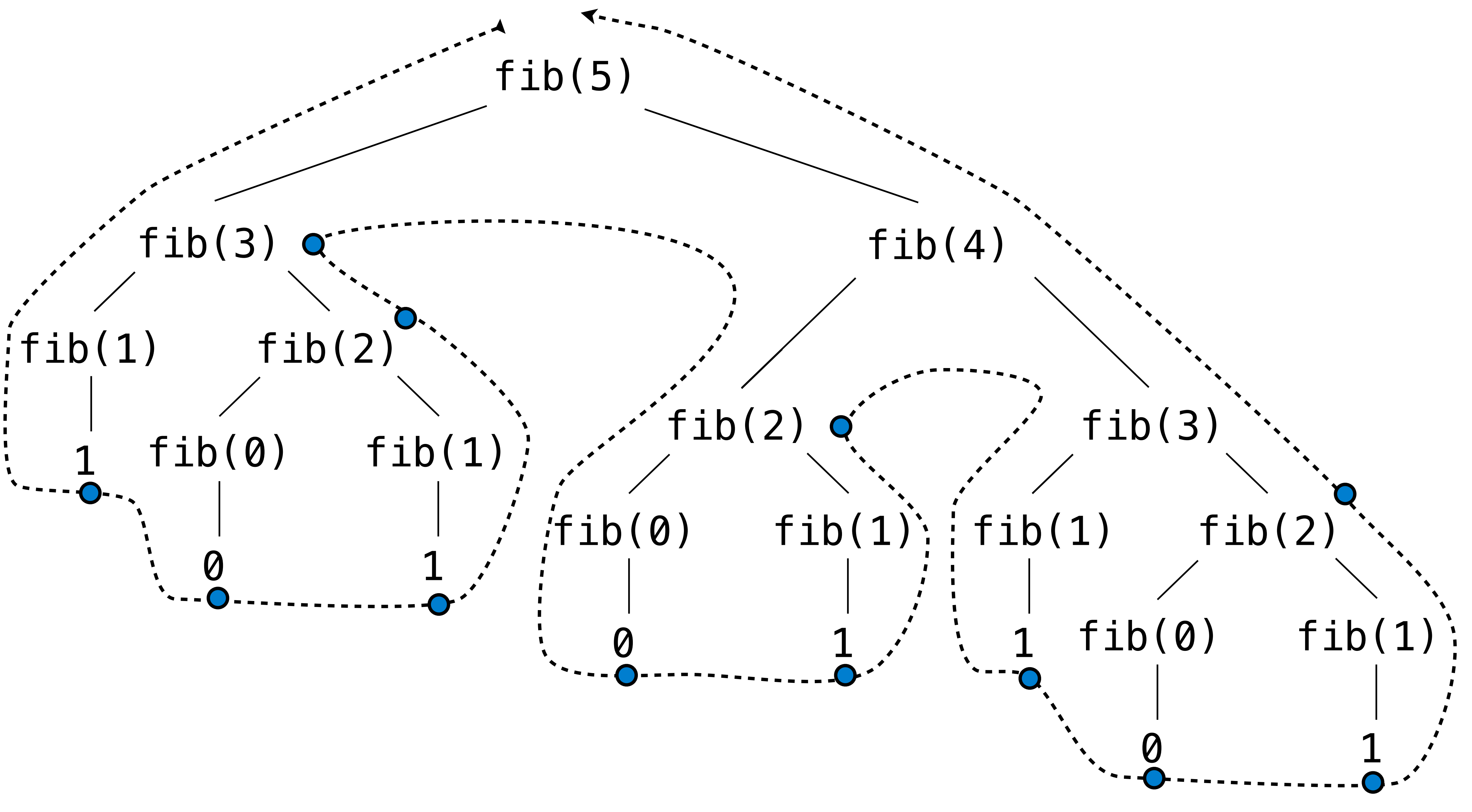

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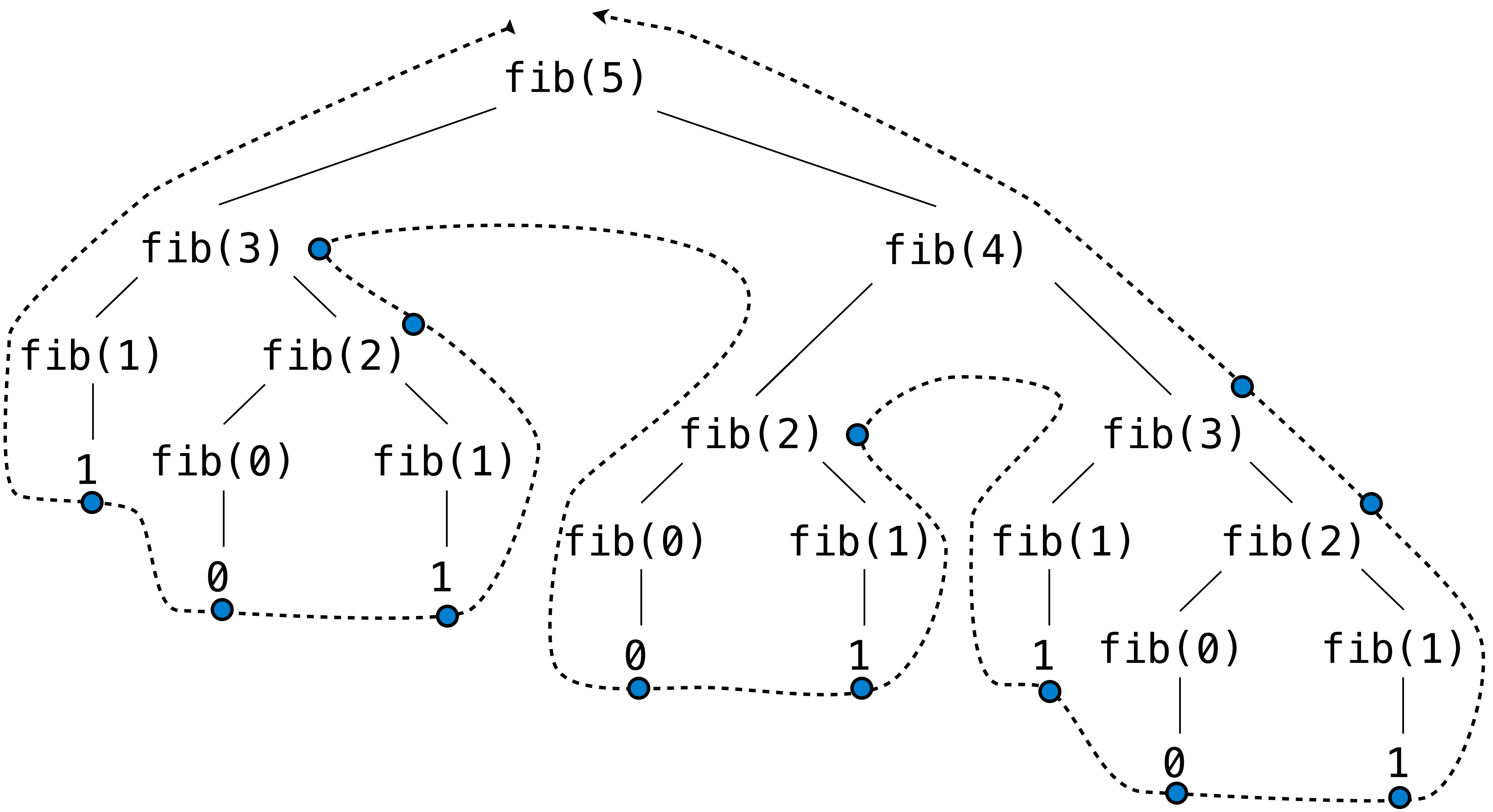

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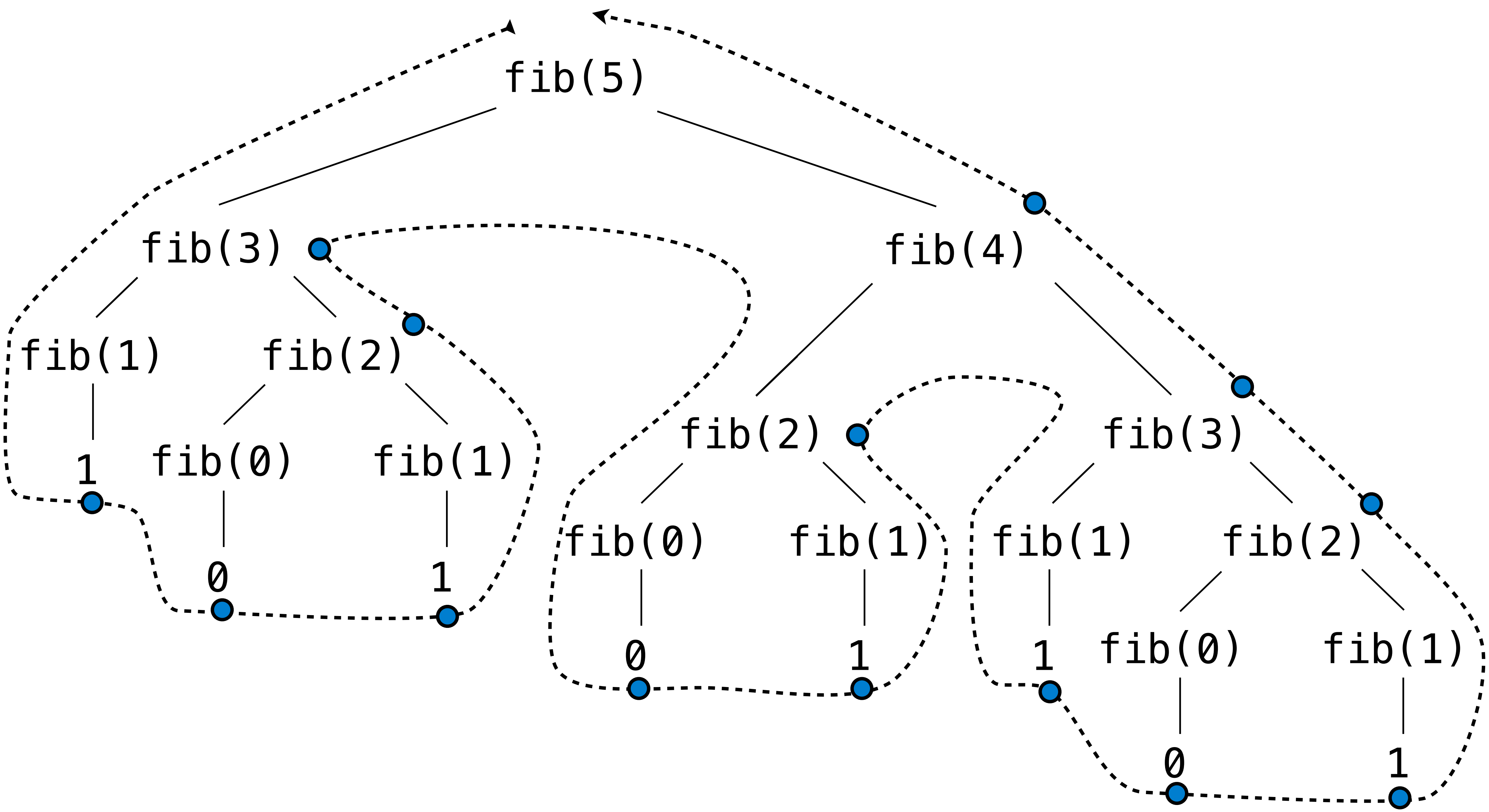

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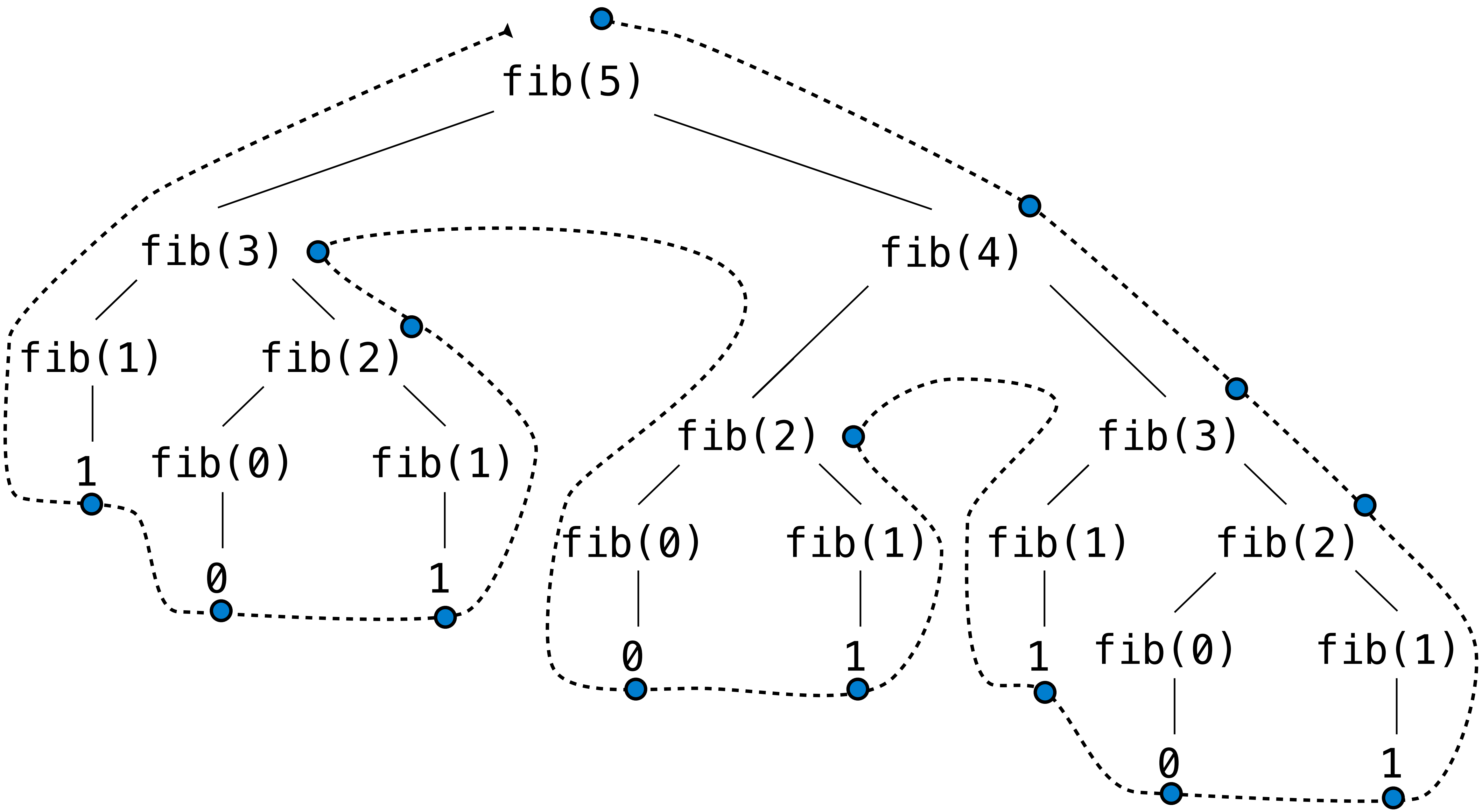
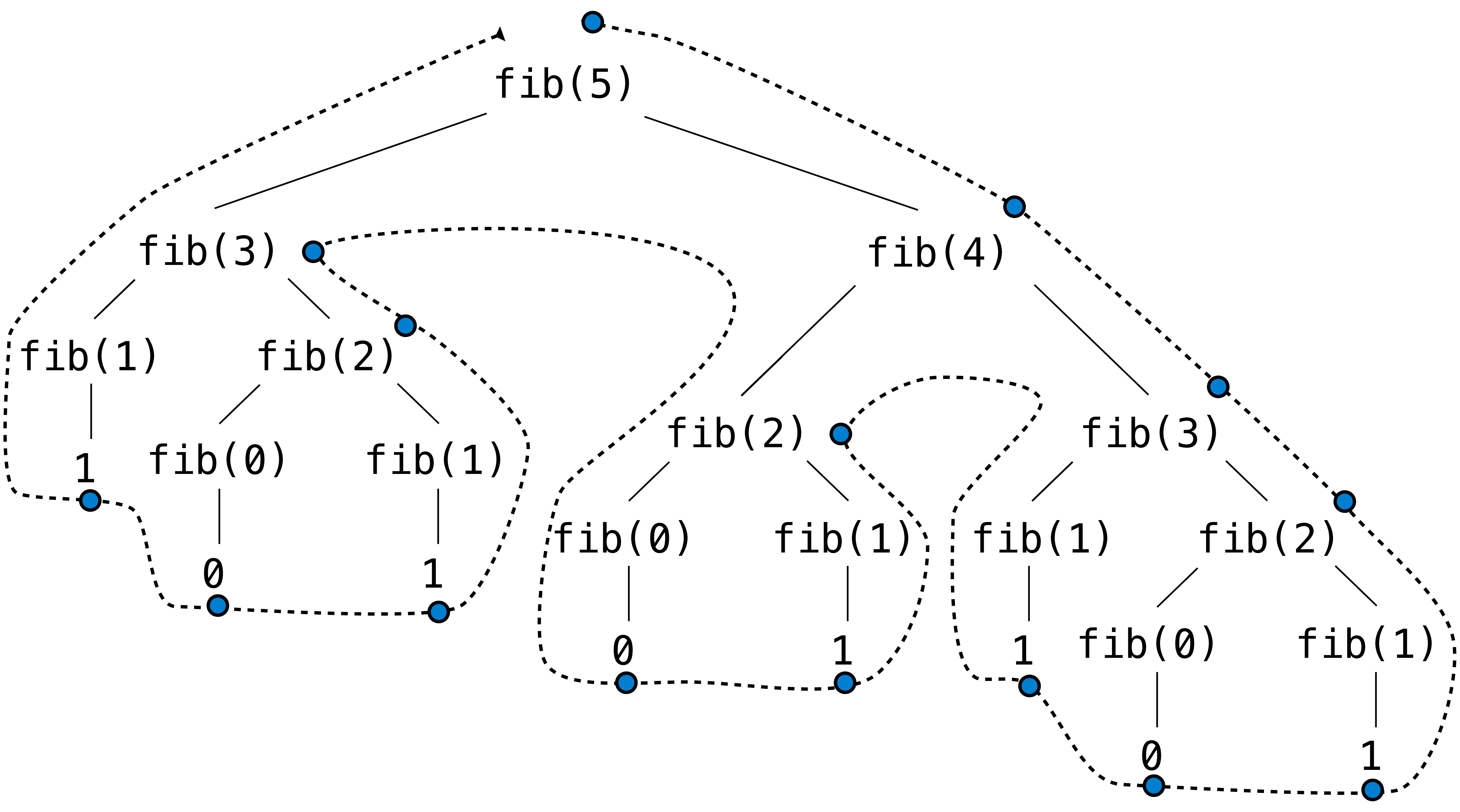


A Tree-Recursive Process (demo)



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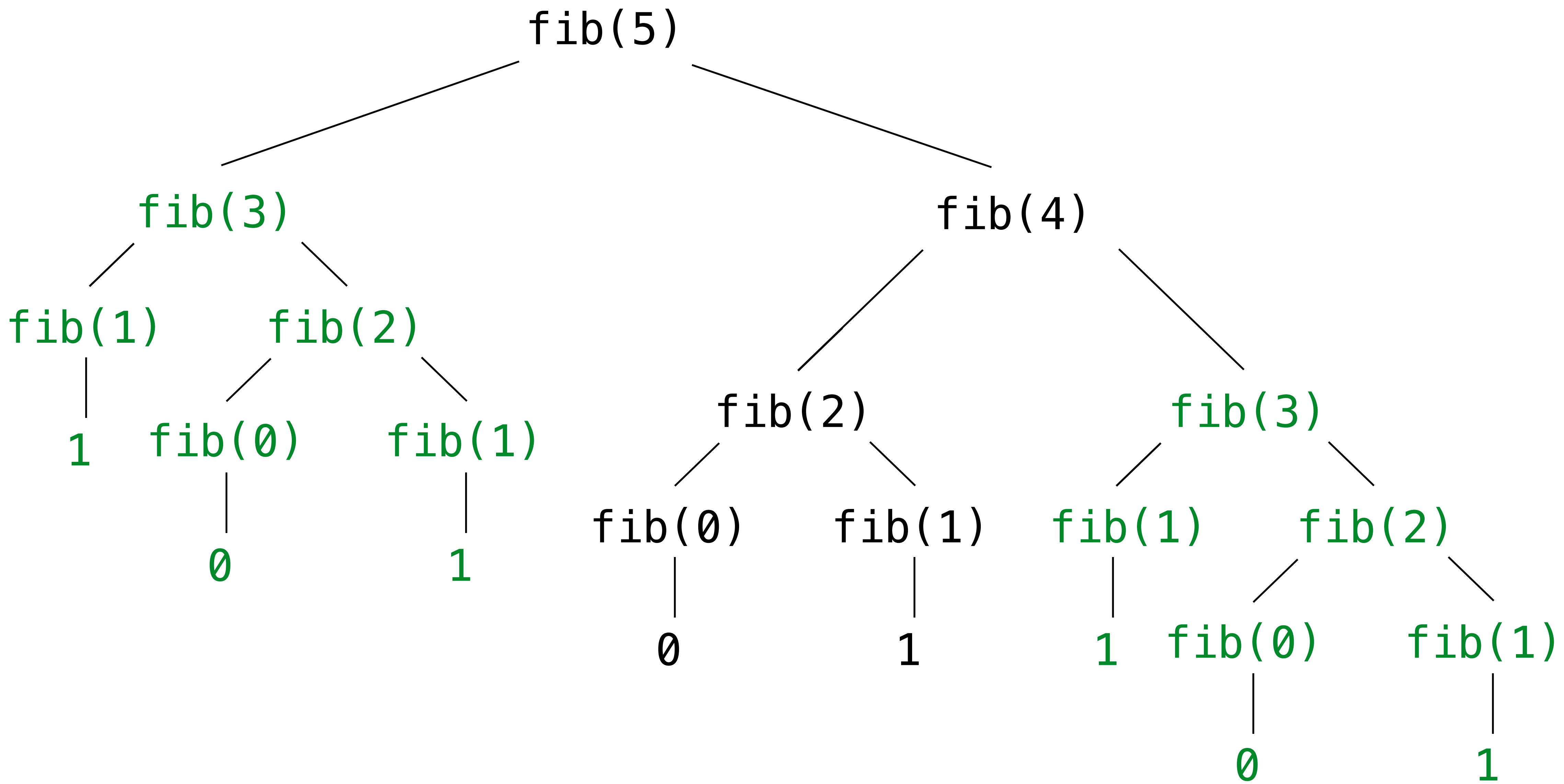

Break!

Counting Partitions

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count_partitions(6, 4)

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How many different ways can I give out
6 pieces of chocolate if nobody can
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1 + 1 + 4 = 6

3 + 3 = 6

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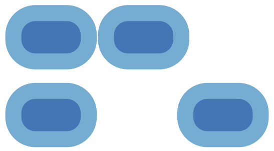

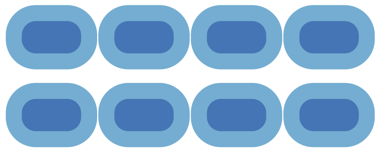

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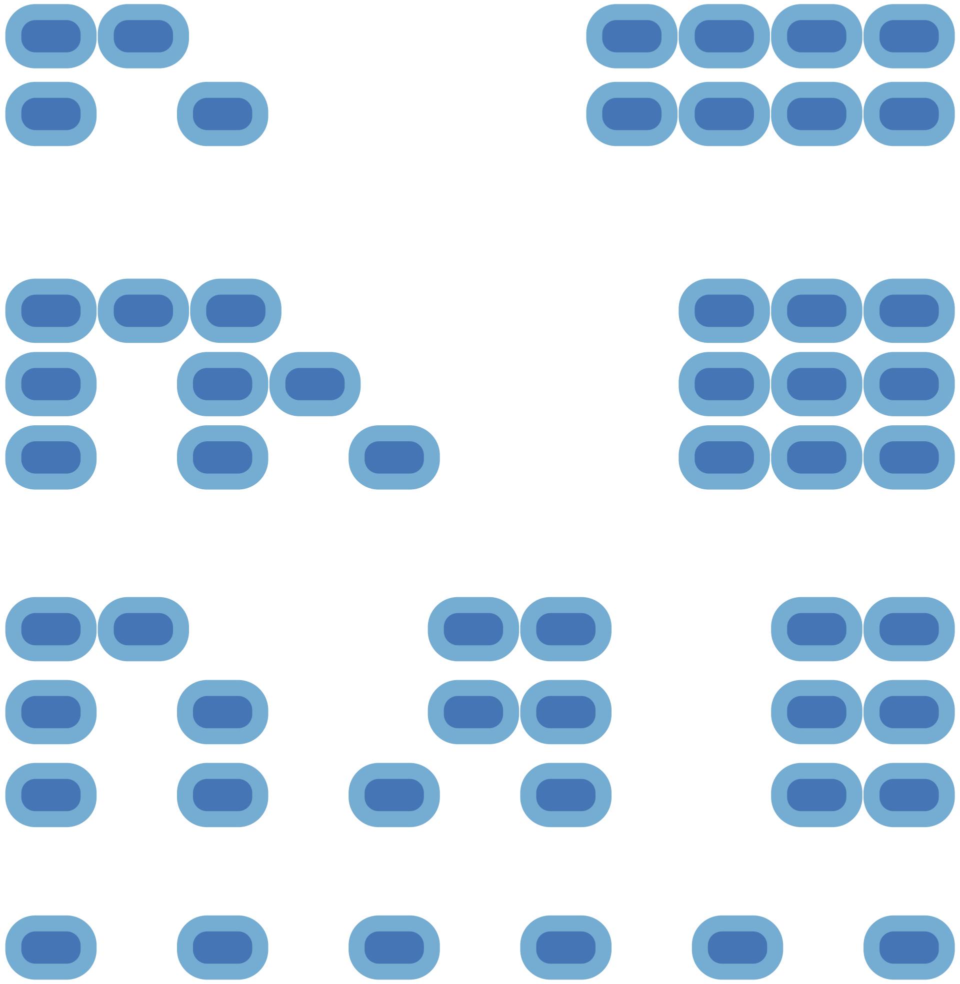

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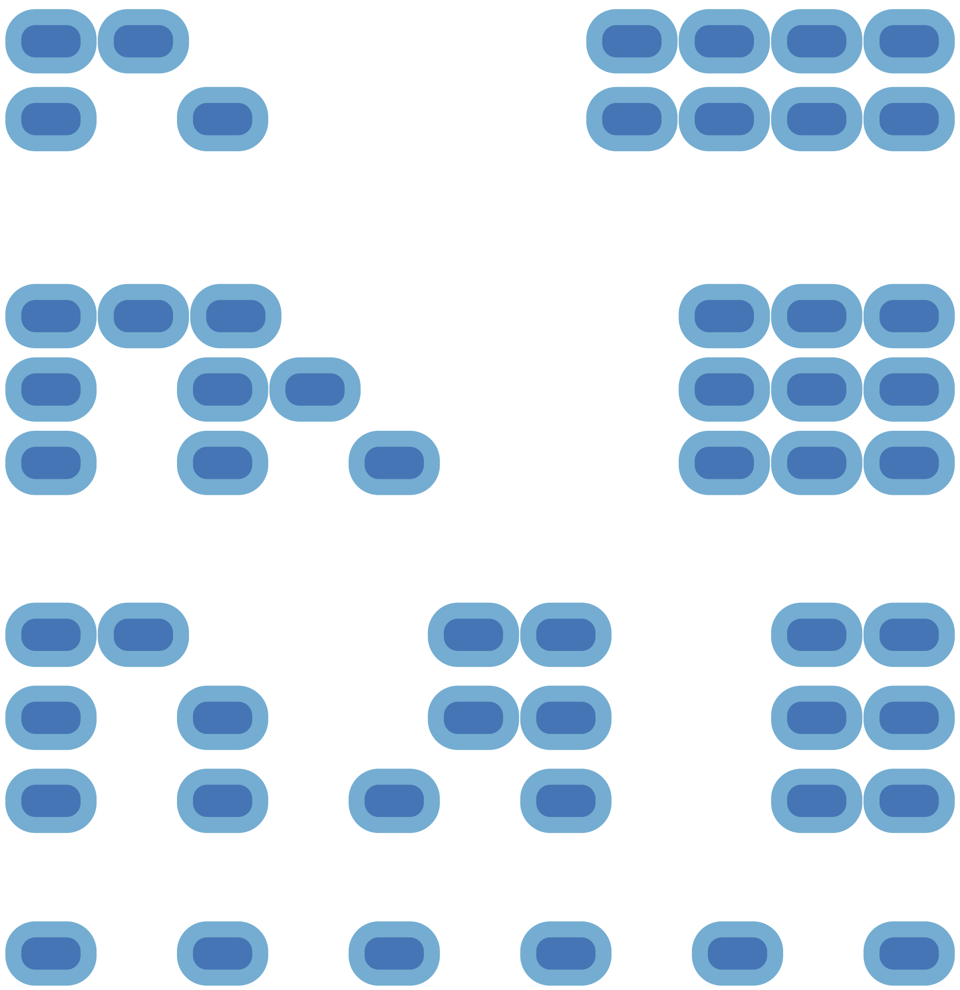

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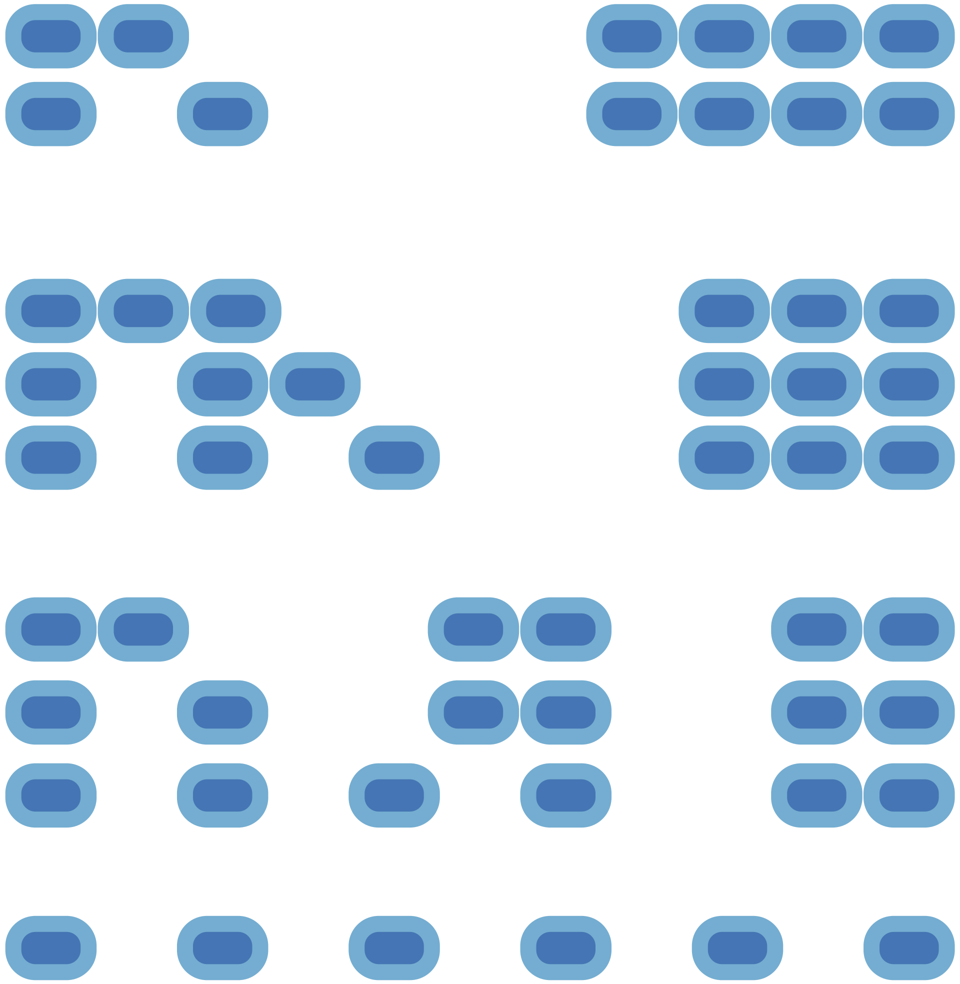

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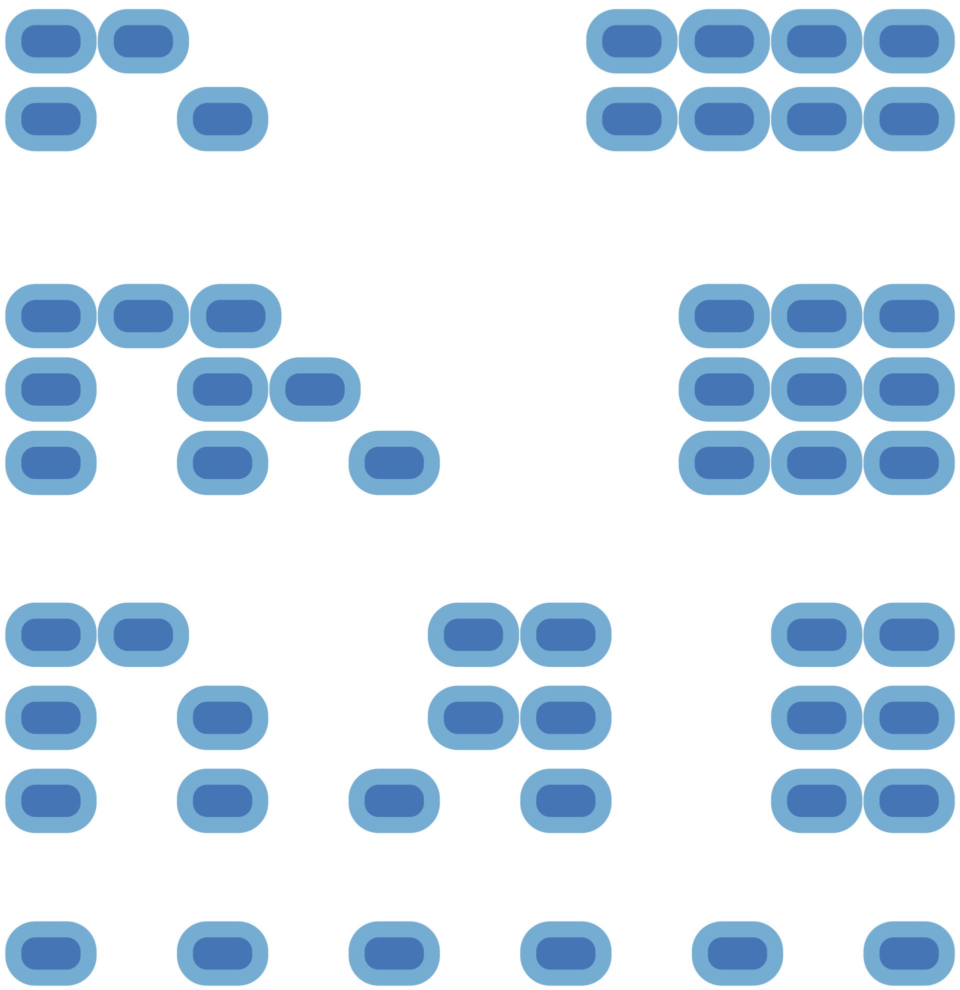

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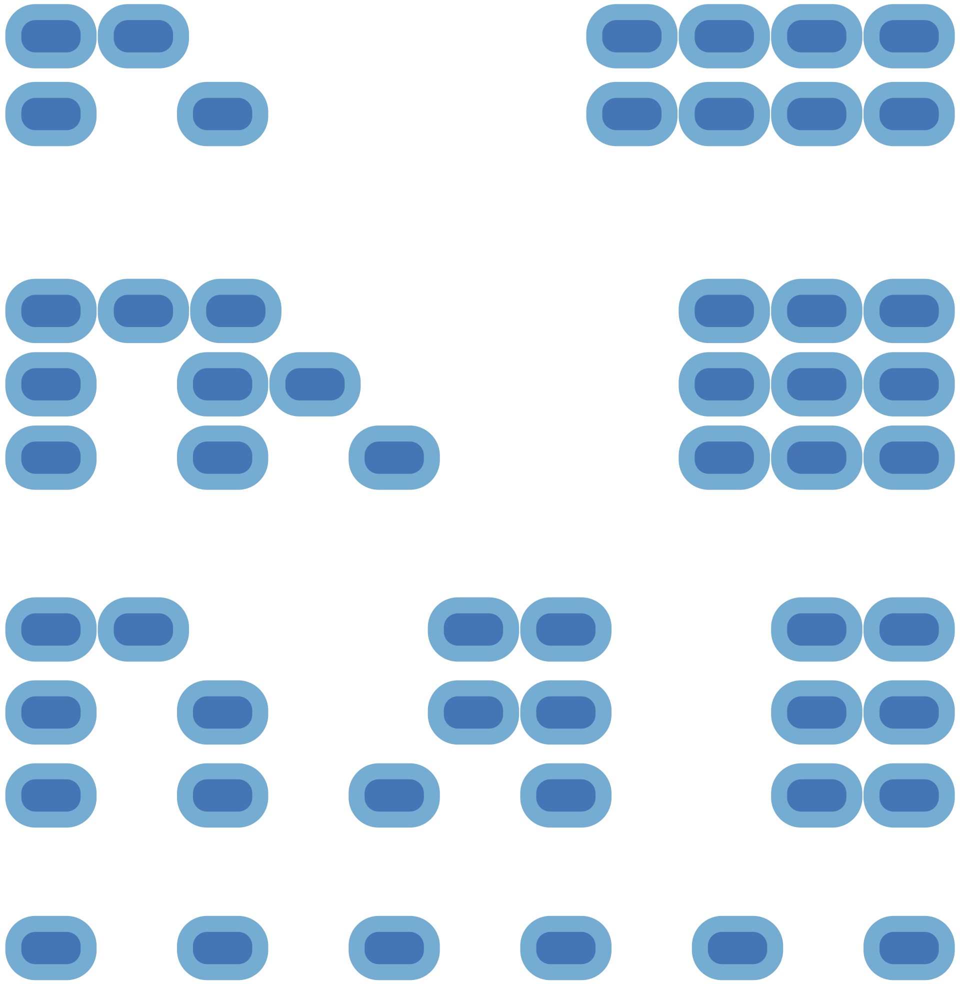

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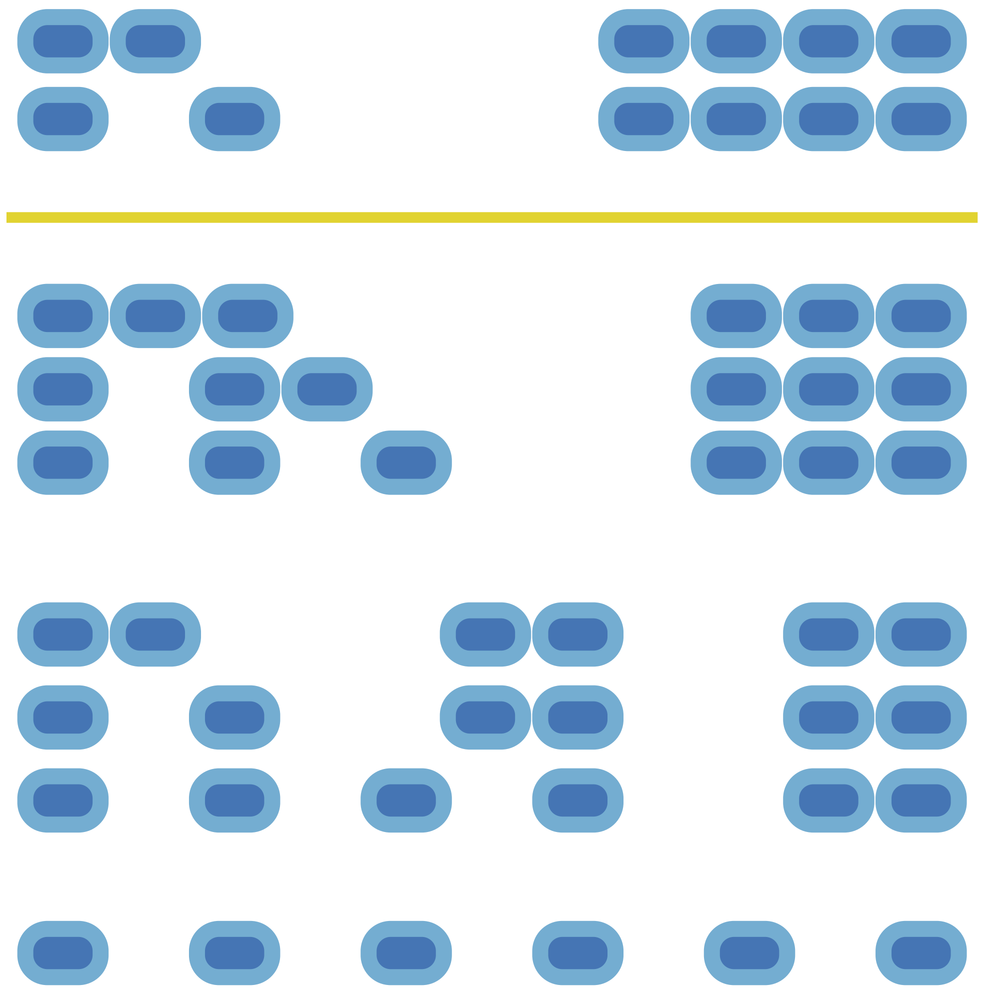

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- Explore two possibilities:
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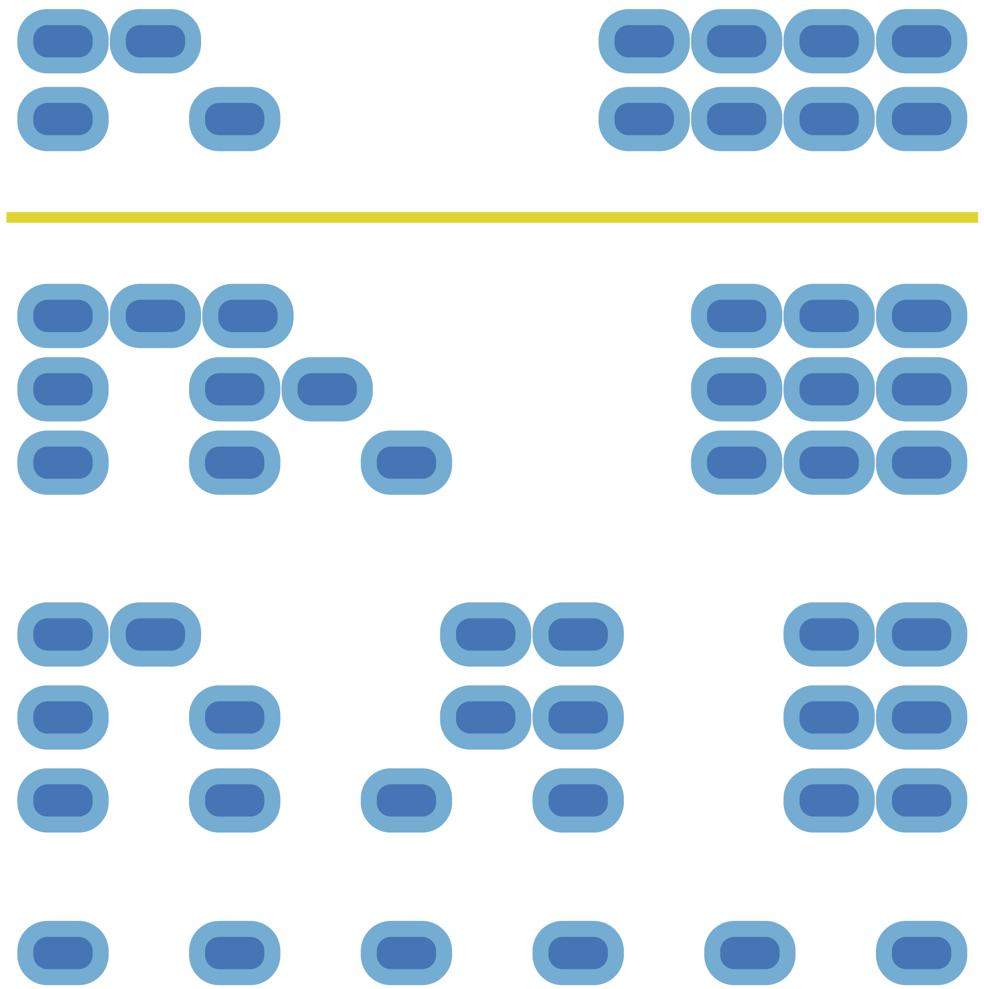

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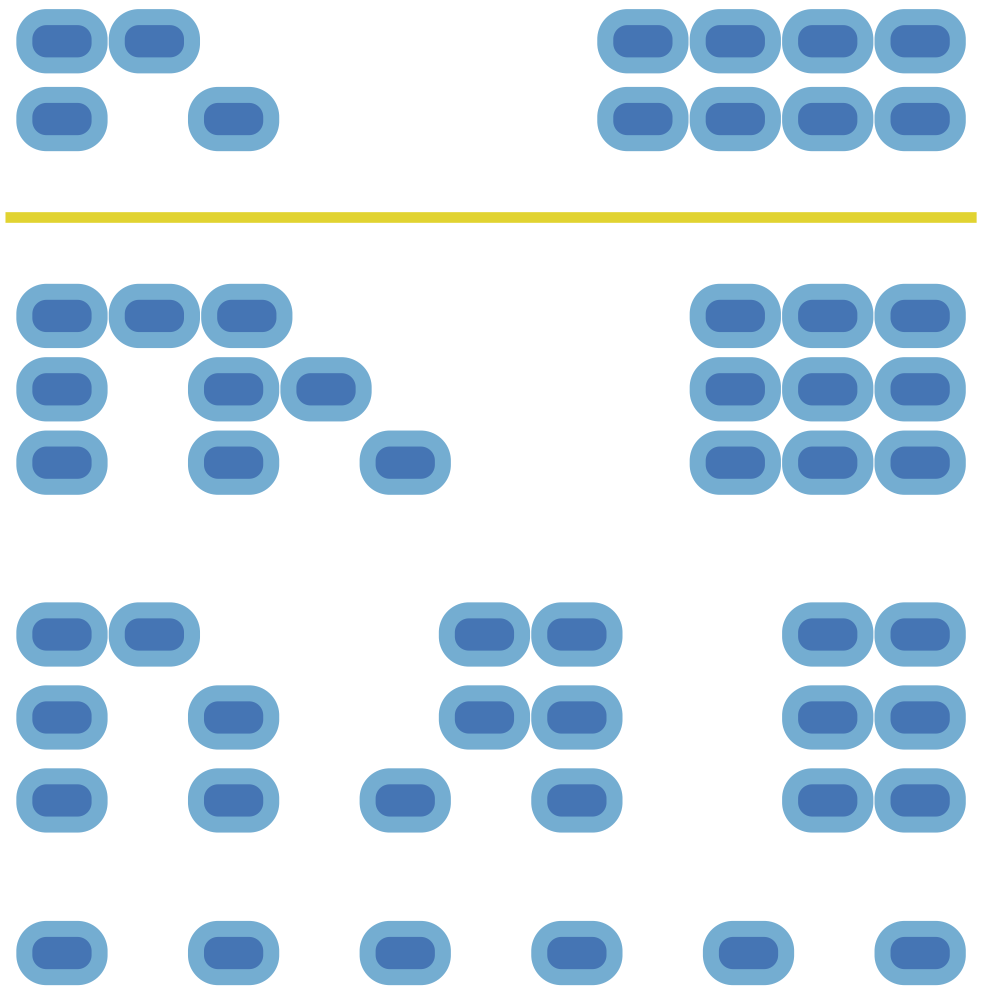

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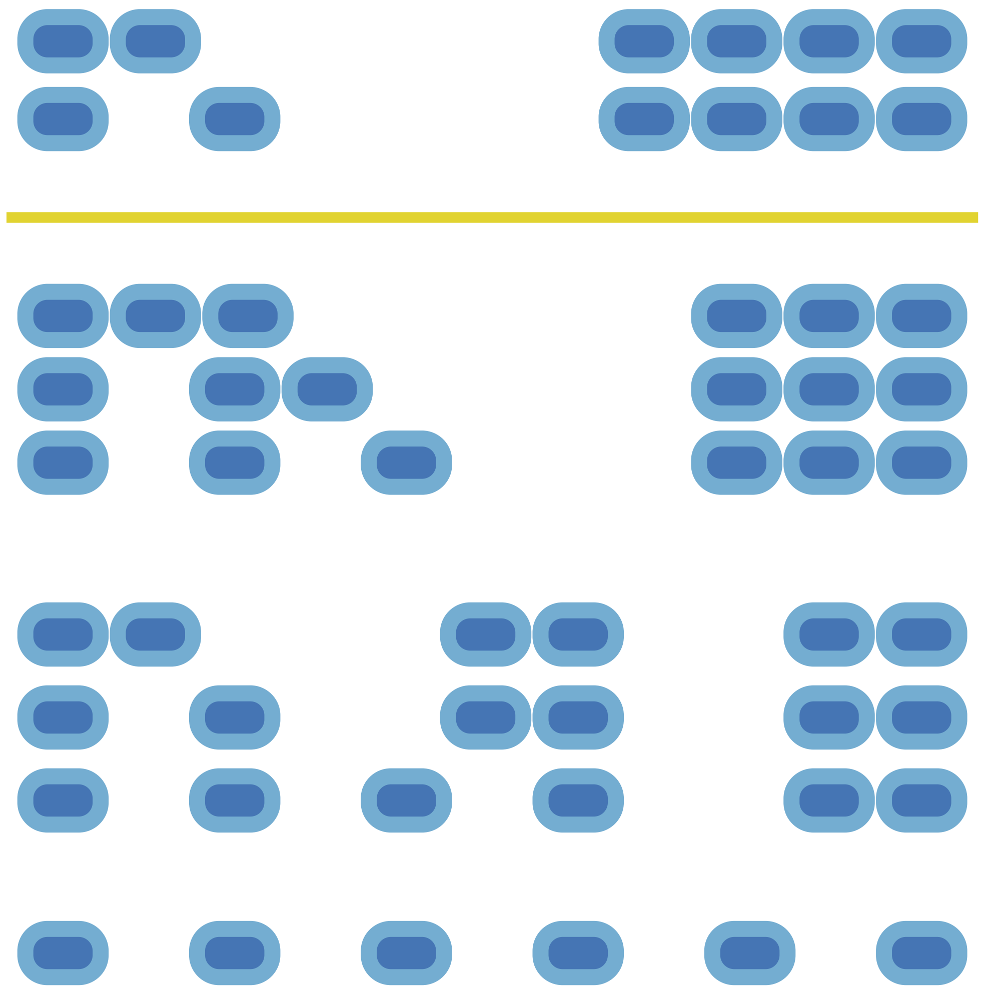

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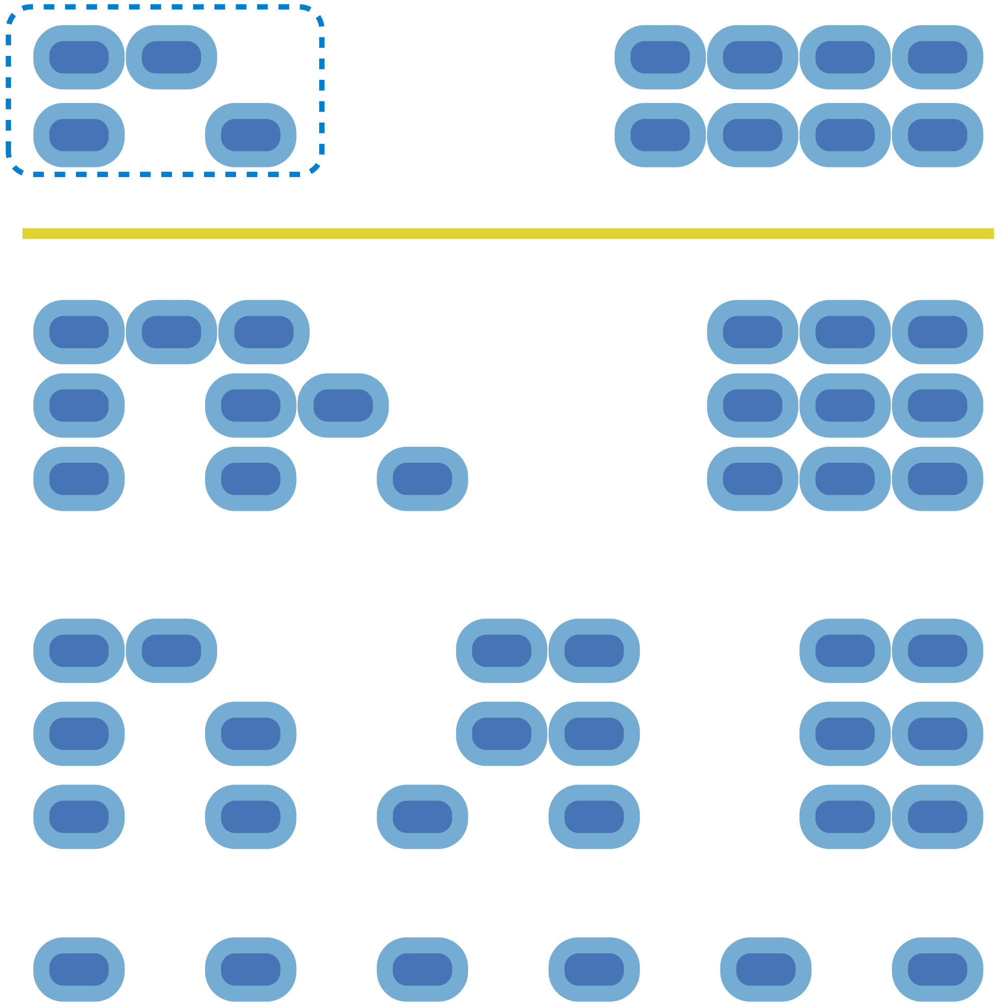

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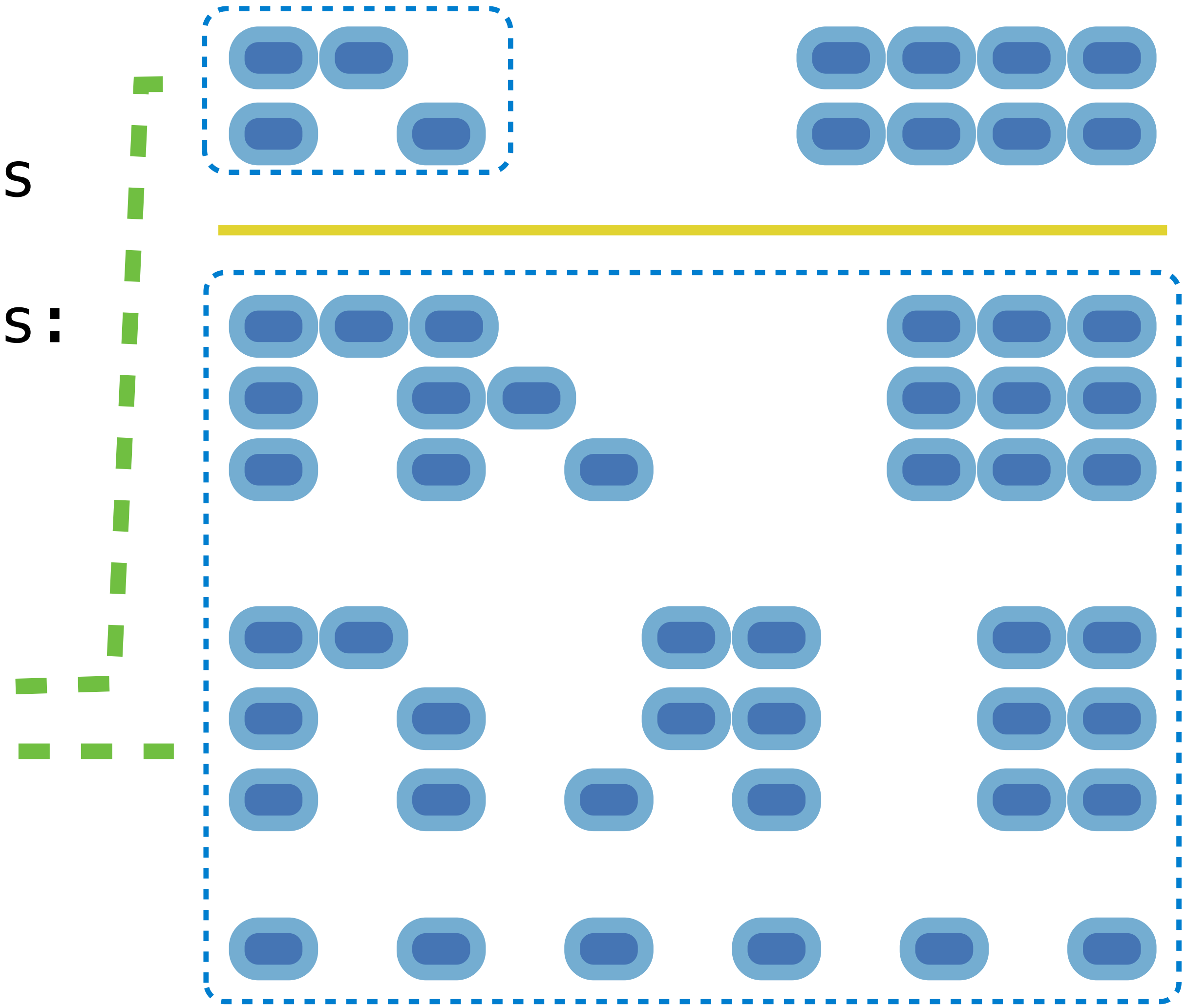

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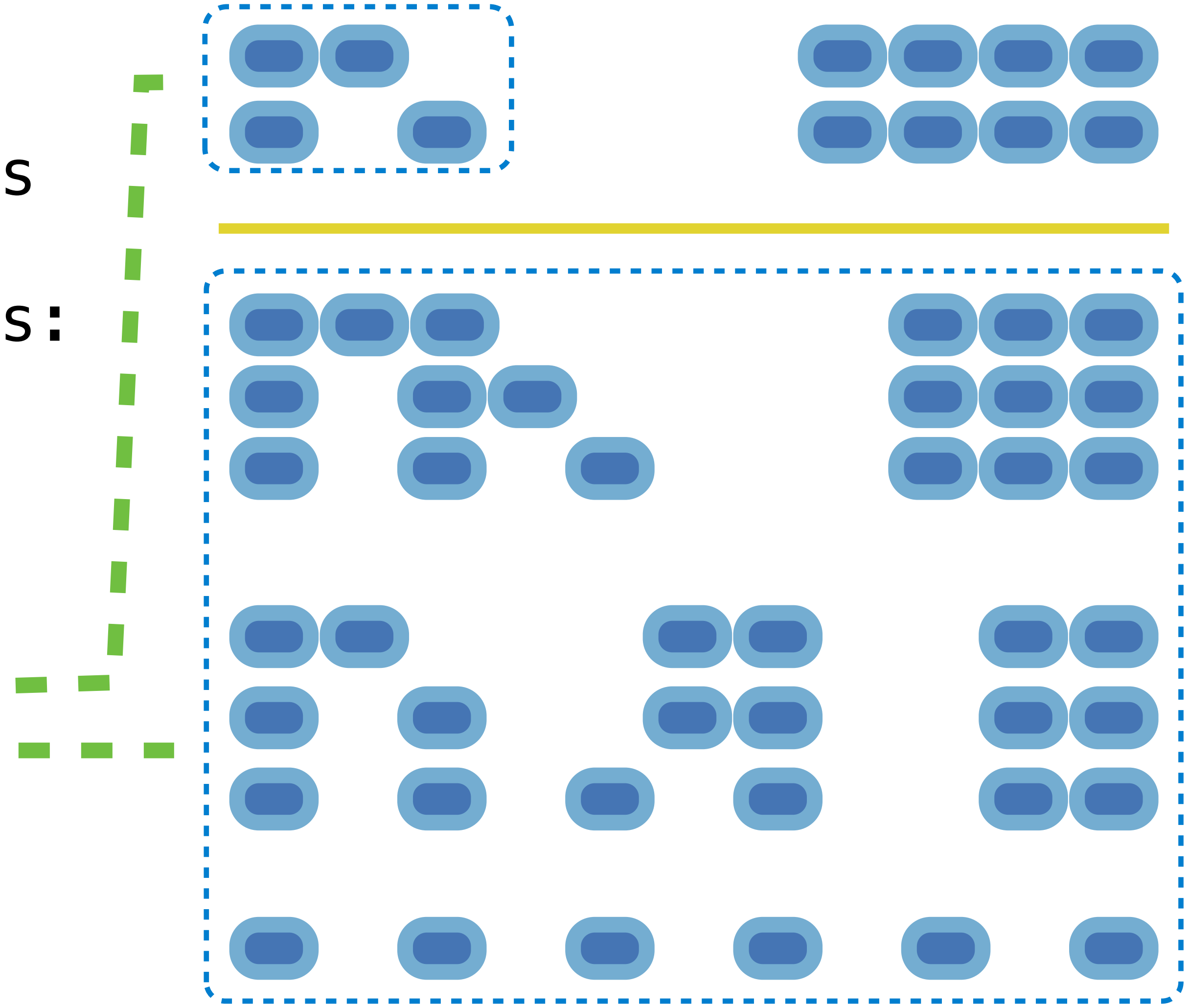

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    with_m = count_partitions(n-m, m)
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```
def count_partitions(n, m):
    if n == 0:

        with_m = count_partitions(n-m, m)
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```

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def count_partitions(n, m):
    if n == 0:
        return 1

    with_m = count_partitions(n-m, m)
    without_m = count_partitions(n, m-1)
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