

# Lecture 10: Linked Lists

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Brian Hou  
July 6, 2016

# Announcements

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  - Bring food and board games!

# Hog Contest

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# Roadmap

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Introduction

Functions

Data

Mutability

Objects

Interpretation

Paradigms

Applications

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- This week (Data), the goals are:

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- This week (Data), the goals are:
  - To continue our journey through abstraction with *data abstraction*



# Roadmap

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Introduction

Functions

Data

Mutability

Objects

Interpretation

Paradigms

Applications

- This week (Data), the goals are:
  - To continue our journey through abstraction with *data abstraction*
  - To study useful data types we can construct with data abstraction

# Data Abstraction

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- Great programmers use data abstraction to separate:

# Data Abstraction

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- Great programmers use data abstraction to separate:
  - How compound values are *used* (the unit)

# Data Abstraction

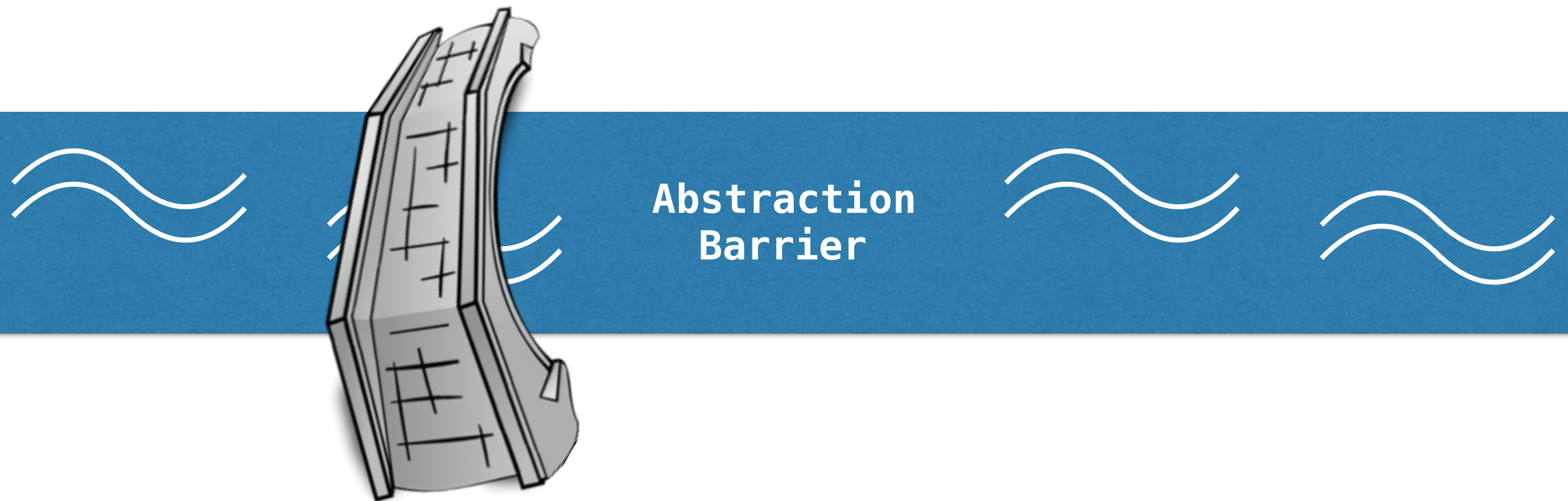
---

- Great programmers use data abstraction to separate:
  - How compound values are *used* (the unit)
  - How compound values are *represented* (the parts)

# Data Abstraction

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- Great programmers use data abstraction to separate:
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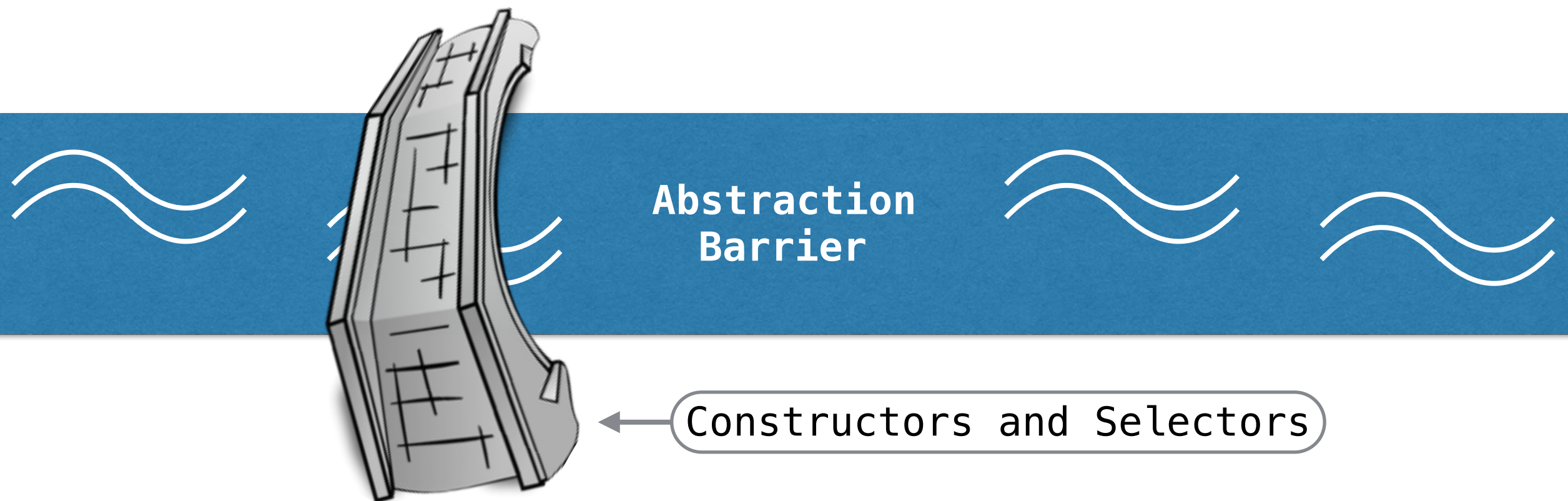


- How compound values are *represented* (the parts)

# Data Abstraction

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- Great programmers use data abstraction to separate:
  - How compound values are *used* (the unit)



- How compound values are *represented* (the parts)



# Abstraction Barrier Violations

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# Abstraction Barrier Violations

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- Constructors and selectors provide us with abstraction, allowing us to use the data type without having to know its implementation
- An abstraction barrier violation is when we assume knowledge about the data type implementation, rather than using constructors and selectors

**Never violate the abstraction barrier!**

# Sequences

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# The Sequence Abstraction

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**Length.** A sequence has a finite length.

**Element selection.** A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0.



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Lists and strings are both examples of sequences.

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**Length.** A sequence has a finite length.

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Lists and strings are both examples of sequences.

We can use built-in syntax associated with this behavior.  
We can also use functions.

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(demo)

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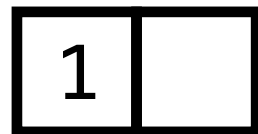
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- Another way to implement the sequence abstraction
- Links have two parts
  - first: the element in the link
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- This is a recursive definition: the rest of a linked list is another linked list

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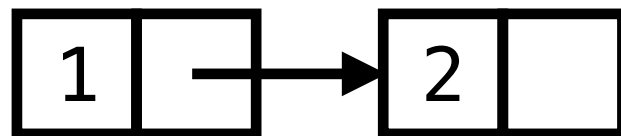
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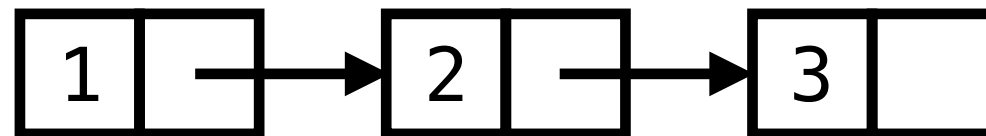
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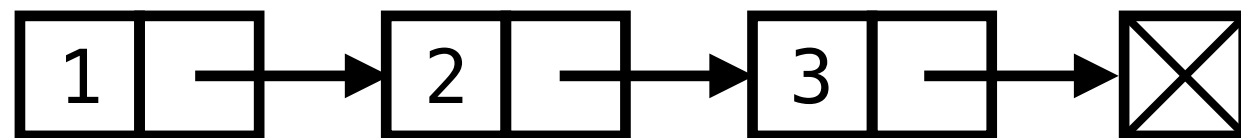
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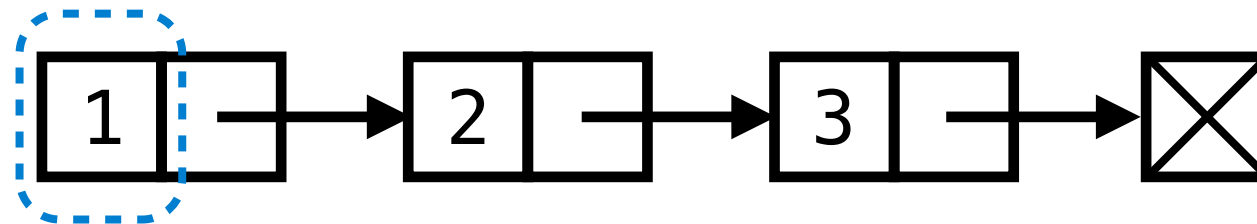
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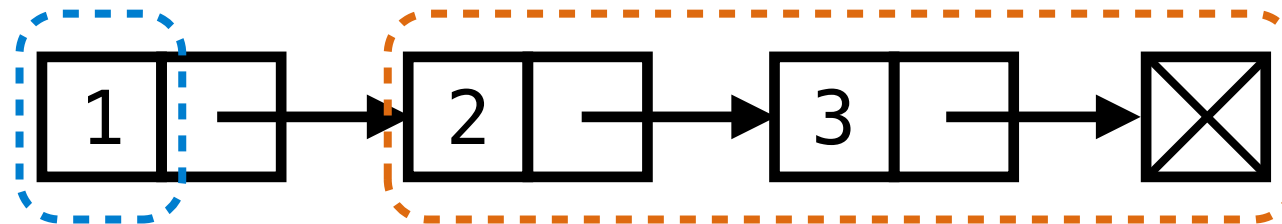
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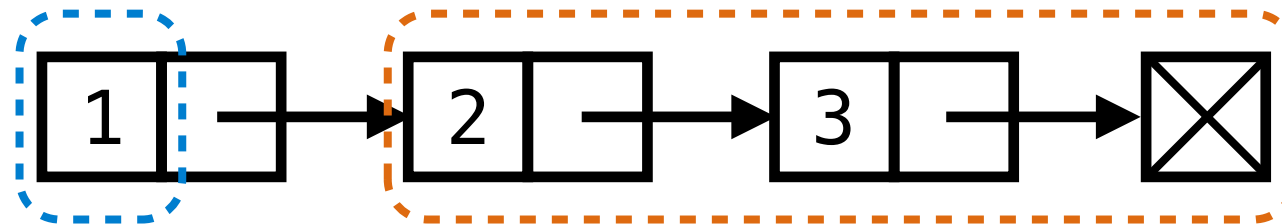




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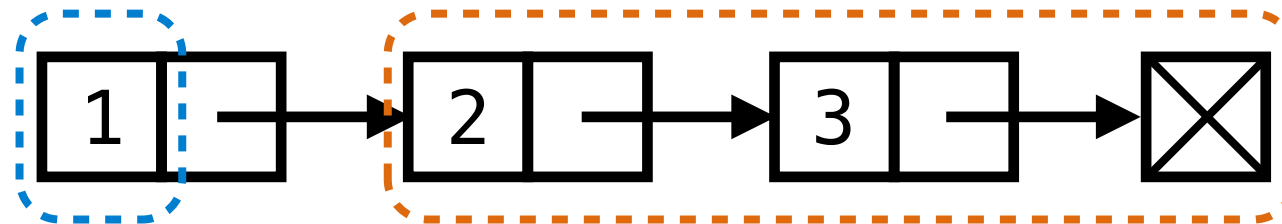


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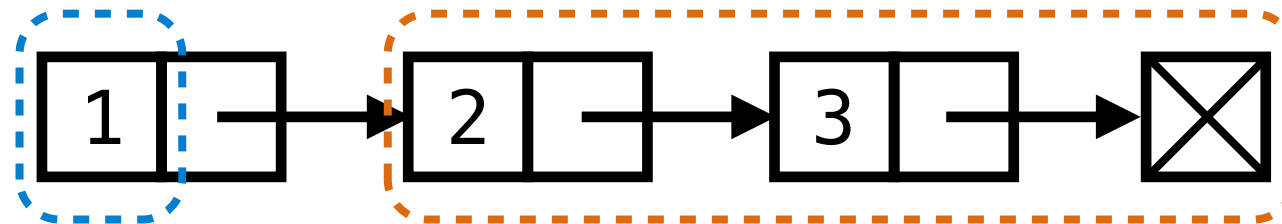


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  - List (Lisp)
  - Forward list (C++)

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def link(first, rest):  
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    return [first, rest]
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def first(s):  
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# Linked Lists are Sequences

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(demo)

```
def len_link(s):  
    """Return the length of the linked list."""  
    length = 0  
    while s != empty:  
        s, length = rest(s), length + 1  
    return length
```

# Linked Lists are Sequences

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(demo)

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def len_link(s):  
    """Return the length of the linked list."""  
    length = 0  
    while s != empty:  
        s, length = rest(s), length + 1  
    return length  
  
def getitem_link(s, i):  
    """Return the element at index i."""  
    while i > 0:  
        s, i = rest(s), i - 1  
    return first(s)
```

# Linked Lists are Sequences

(demo)

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**Never violate the abstraction barrier!**

# Linked Lists are Recursive

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# Linked Lists are Recursive (demo)

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# Linked Lists are Recursive

(demo)

---

```
def len_link(s):  
    """Return the length of the linked list."""  
    if s == empty:  
        return 0  
    else:  
        return 1 + len_link(rest(s))
```



# Linked Lists are Recursive

(demo)

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def len_link(s):  
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def getitem_link(s, i):  
    """Return the element at index i."""  
    if i == 0:  
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    else:  
        return getitem_link(rest(s), i - 1)
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# Break!

---

# Linked List Processing

---

# Sequences as Containers

---

# Sequences as Containers

---

(demo)

# Sequences as Containers

---

(demo)

```
def contains(s, elem):  
    """Return whether ELEM is in the sequence S.  
    >>> contains([1, 2, 3], 1)  
    True  
    >>> contains([1, 2, 3], 4)  
    False  
    """  
  
    for x in s:  
        if x == elem:  
            return True  
    return False
```

# Linked Lists as Containers (demo)

---



# Linked Lists as Containers

(demo)

---

```
def contains_link(s, elem):
    """Return whether ELEM is in the sequence S.
    >>> contains_link(link(1, link(2, link(3, empty))), 1)
    True
    >>> contains_link(link(1, link(2, link(3, empty))), 4)
    False
    """
    if s == empty:
        return False
    if first(s) == elem:
        return True
    else:
        return contains(rest(s), elem)
```

# Linked List Examples

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# Counting Partitions

---

```
def count_partitions(n, m):  
    if n == 0:  
        return 1  
    elif n < 0:  
        return 0  
    elif m == 0:  
        return 0  
    else:  
        with_m = count_partitions(n-m, m)  
        without_m = count_partitions(n, m-1)  
        return with_m + without_m
```

# Enumerating Partitions

---

# Enumerating Partitions

(demo)



# Enumerating Partitions

---

(demo)

```
def partitions(n, m):
```

# Enumerating Partitions

---

(demo)

```
def partitions(n, m):  
    if n == 0:
```

# Enumerating Partitions

---

(demo)

```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)
```



# Enumerating Partitions

---

(demo)

```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)  
    elif n < 0 or m == 0:
```

# Enumerating Partitions

---

(demo)

```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)  
    elif n < 0 or m == 0:  
        return empty
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# Enumerating Partitions

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(demo)

```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)  
    elif n < 0 or m == 0:  
        return empty  
    else:
```

# Enumerating Partitions

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(demo)

```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)  
    elif n < 0 or m == 0:  
        return empty  
    else:  
        with_m = partitions(n-m, m)
```

# Enumerating Partitions

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(demo)

```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)  
    elif n < 0 or m == 0:  
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# Enumerating Partitions

(demo)

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```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)  
    elif n < 0 or m == 0:  
        return empty  
    else:  
        with_m = partitions(n-m, m)  
        without_m = partitions(n, m-1)  
        add_m = lambda s: link(m, s)
```

# Enumerating Partitions

(demo)

---

```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)  
    elif n < 0 or m == 0:  
        return empty  
    else:  
        with_m = partitions(n-m, m)  
        without_m = partitions(n, m-1)  
        add_m = lambda s: link(m, s)  
        with_m = map_link(add_m, with_m)
```

# Enumerating Partitions

(demo)

---

```
def partitions(n, m):  
    if n == 0:  
        return link(empty, empty)  
    elif n < 0 or m == 0:  
        return empty  
    else:  
        with_m = partitions(n-m, m)  
        without_m = partitions(n, m-1)  
        add_m = lambda s: link(m, s)  
        with_m = map_link(add_m, with_m)  
        return extend(with_m, without_m)
```



# Break?

---

# Other Linked List Implementations

---

# Implementing Linked Lists (v1)

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```
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# Implementing Linked Lists (v2)

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```
def link(first, rest):
```

# Implementing Linked Lists (v2)

---

```
def link(first, rest):
```

```
def first(s):
```

```
def rest(s):
```

# Implementing Linked Lists (v2)

---

```
def link(first, rest):  
    def dispatch(msg):
```

```
def first(s):
```

```
def rest(s):
```

# Implementing Linked Lists (v2)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':
```

```
def first(s):
```

```
def rest(s):
```



# Implementing Linked Lists (v2)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':  
            return first
```

```
def first(s):
```

```
def rest(s):
```

# Implementing Linked Lists (v2)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':  
            return first  
        elif msg == 'rest':
```

```
def first(s):
```

```
def rest(s):
```

# Implementing Linked Lists (v2)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':  
            return first  
        elif msg == 'rest':  
            return rest
```

```
def first(s):
```

```
def rest(s):
```

# Implementing Linked Lists (v2)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':  
            return first  
        elif msg == 'rest':  
            return rest  
    return dispatch
```

```
def first(s):
```

```
def rest(s):
```

# Implementing Linked Lists (v2)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':  
            return first  
        elif msg == 'rest':  
            return rest  
    return dispatch
```

```
def first(s):  
    return s('first')  
  
def rest(s):
```

# Implementing Linked Lists (v2)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':  
            return first  
        elif msg == 'rest':  
            return rest  
    return dispatch
```

```
def first(s):  
    return s('first')  
  
def rest(s):  
    return s('rest')
```

# Implementing Linked Lists (v2) (demo)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':  
            return first  
        elif msg == 'rest':  
            return rest  
    return dispatch
```

```
def first(s):  
    return s('first')  
  
def rest(s):  
    return s('rest')
```

# Implementing Linked Lists (v2) (demo)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'first':  
            return first  
        elif msg == 'rest':  
            return rest  
    return dispatch
```

```
def first(s):  
    return s('first')  
  
def rest(s):  
    return s('rest')
```



# Implementing Linked Lists (v3)

---

```
def link(first, rest):  
    def dispatch(msg):  
        if msg == 'brian':  
            return first  
        elif msg == 'marvin':  
            return rest  
    return dispatch
```

```
def first(s):  
    return s('brian')  
  
def rest(s):  
    return s('marvin')
```

# Summary

---

- Linked lists are one implementation of the sequence abstraction
- Linked lists are composed of two parts:
  - first: the element in the link
  - rest: the next link in the list (may be empty)
- Data abstraction means that the implementation details of the first and rest selectors are unnecessary
- We can use functions to implement linked lists
  - We can use lists to implement dictionaries
  - Therefore, we can use functions to implement dictionaries