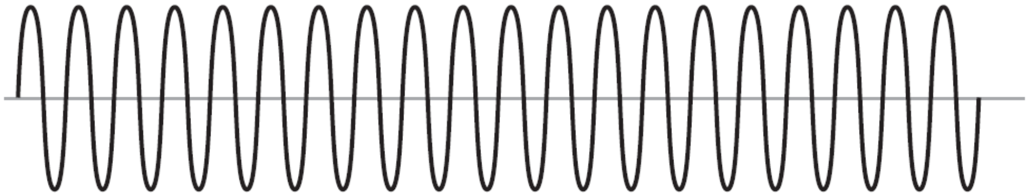
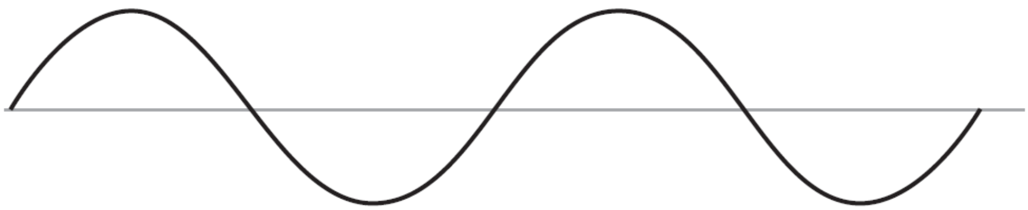
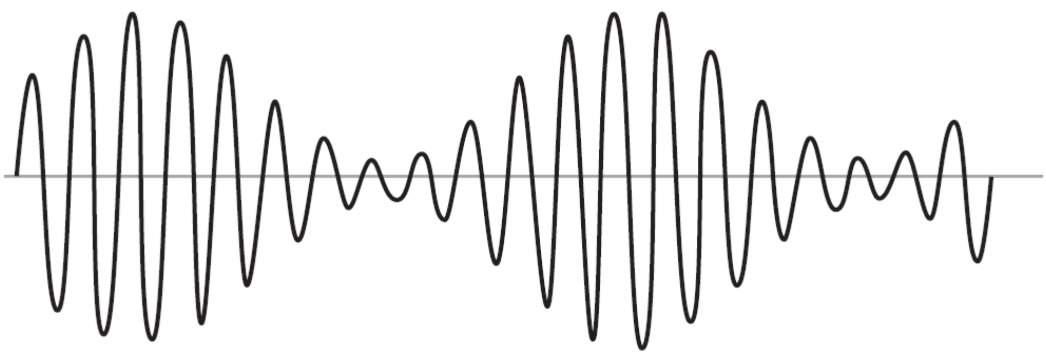


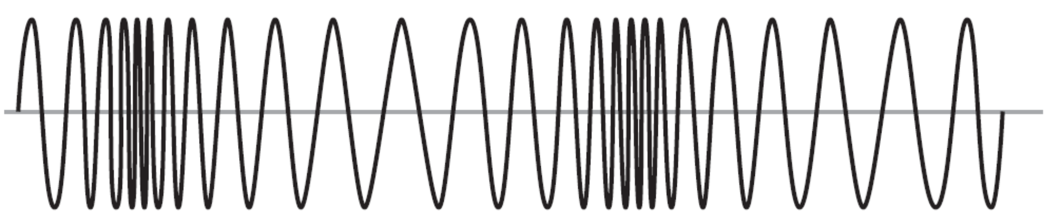
(a)



(b)



(c)



(e) time

Mensaje simplificado $$m(t) = \boxed{A_m \cos(2\pi f_m t)}$$

Frecuencia instantánea de la portadora

$$\begin{aligned} f_i(t) &= f_c + k_f \boxed{A_m \cos(2\pi f_m t)} \\ &= f_c + \Delta f \cos(2\pi f_m t) \end{aligned}$$

Ángulo instantáneo (la integral de la fi)

$$\boxed{\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)}$$

Con $\beta = \dfrac{\Delta f}{f_m}$ entonces

$$s(t) = A_c \cos[\boxed{2\pi f_c t + \beta \sin(2\pi f_m t)}]$$

$$\boxed{s(t) = \mathrm{Re}[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))]}$$

$$= \mathrm{Re}[\tilde{s}(t) \exp(j2\pi f_c t)]$$

$$\tilde{s}(t) = A_c \exp[j\beta \sin(2\pi f_m t)]$$

Periódica

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t)$$

.. con coeficientes ...

$$c_n = f_m \int_{-1/(2f_m)}^{1/(2f_m)} \tilde{s}(t) \exp(-j2\pi n f_m t)\, dt$$

$$= f_m A_c \int_{-1/(2f_m)}^{1/(2f_m)} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]\, dt$$

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx \quad \text{con} \quad x = 2\pi f_m t$$

Los coeficientes tienen la forma de la función de Bessel

$$c_n = A_c J_n(\beta)$$

... es decir que

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t)$$

Luego

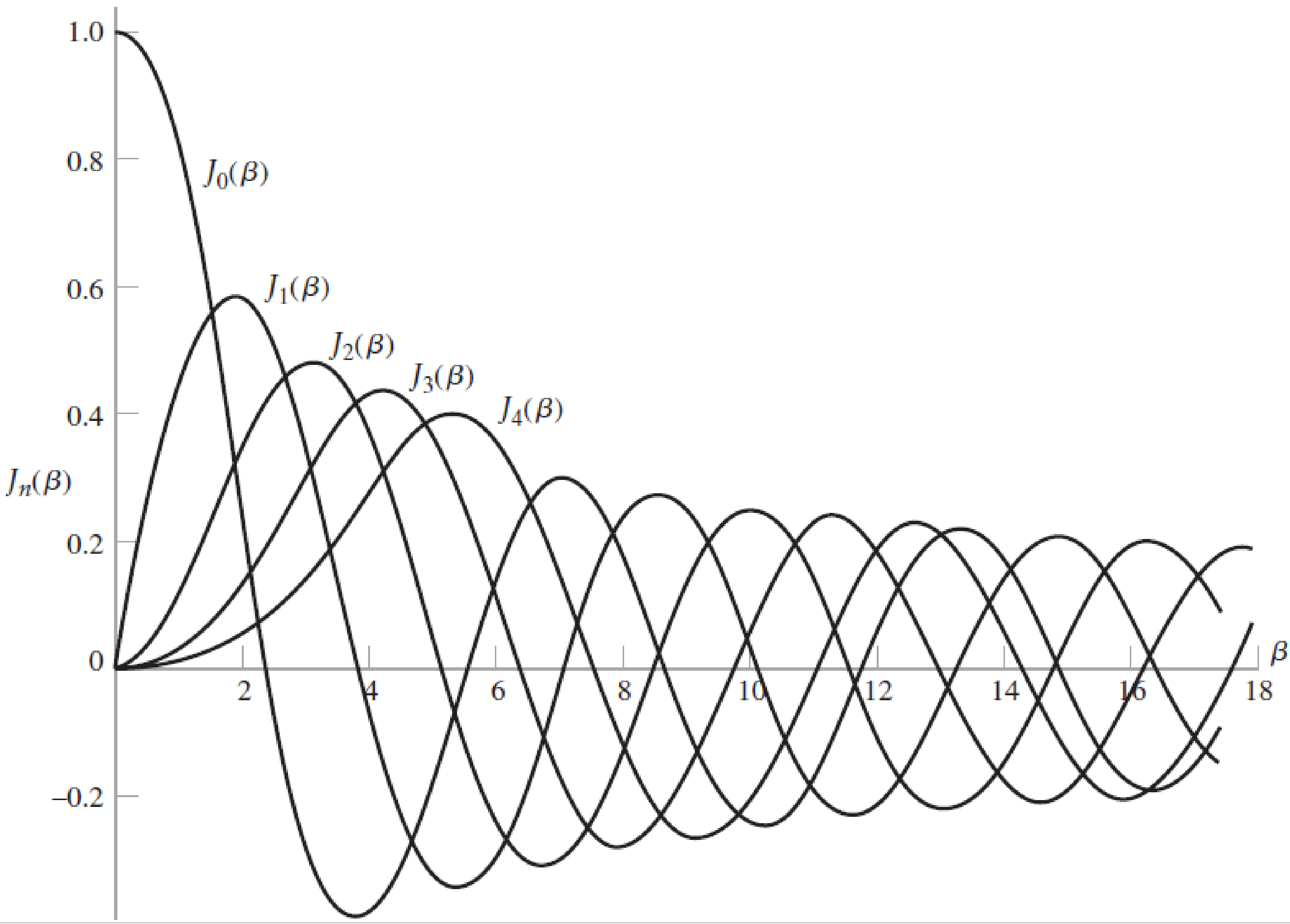
$$\boxed{s(t) = \mathbf{Re}\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t]\right]}$$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]$$

¿Cómo es este espectro?

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$

¿cómo depende de beta? Recuerde que $\beta = \dfrac{\Delta f}{f_m}$



Propiedades de las funciones de Bessel

1. For different integer (positive and negative) values of n, we have

$$J_n(\beta) = J_{-n}(\beta), \qquad \text{for } n \text{ even}$$

and

$$J_n(\beta) = -J_{-n}(\beta), \qquad \text{for } n \text{ odd}$$

2. For small values of the modulation index β, we have

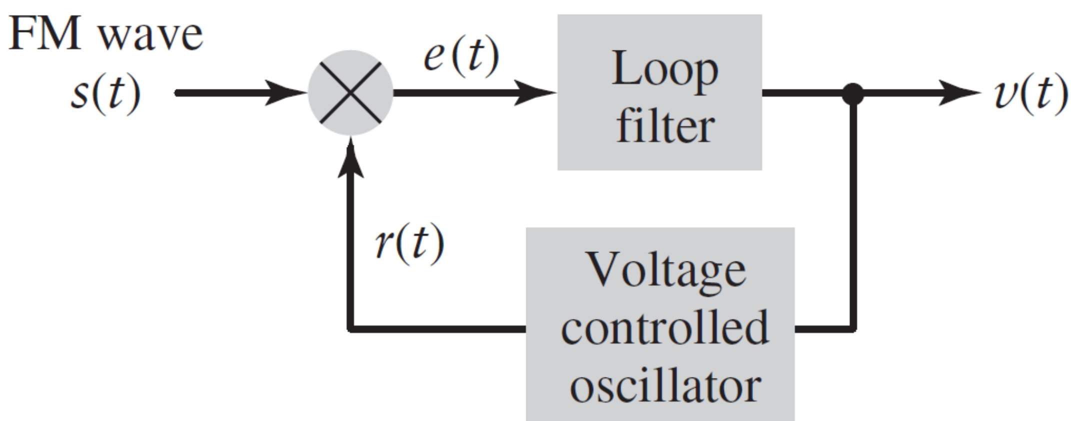
$$\left.\begin{aligned} J_0(\beta) &\approx 1, \\ J_1(\beta) &\approx \frac{\beta}{2}, \\ J_n(\beta) &\approx 0, \qquad n > 2 \end{aligned}\right\}$$

3. The equality

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

holds exactly for arbitrary β.

Circuito PLL (Phase-Locked Loop) – como receptor de FM



s(t) entra al circuito

$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)]$$

$$\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

… y el VCO está calibrado para …

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)]$$

$$\phi_2(t) = 2\pi k_v \int_0^t v(\tau)\, d\tau$$

Calcule e(t) y aplique el efecto del filtro, usted ya sabe cómo es eso… $\sin \boldsymbol{\alpha} \cos \boldsymbol{\beta} = 0.5[\sin(\alpha + \beta) + \sin(\boldsymbol{\alpha} - \boldsymbol{\beta})]$

$$e(t) = k_m A_c A_v \sin[\phi_e(t)]$$

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

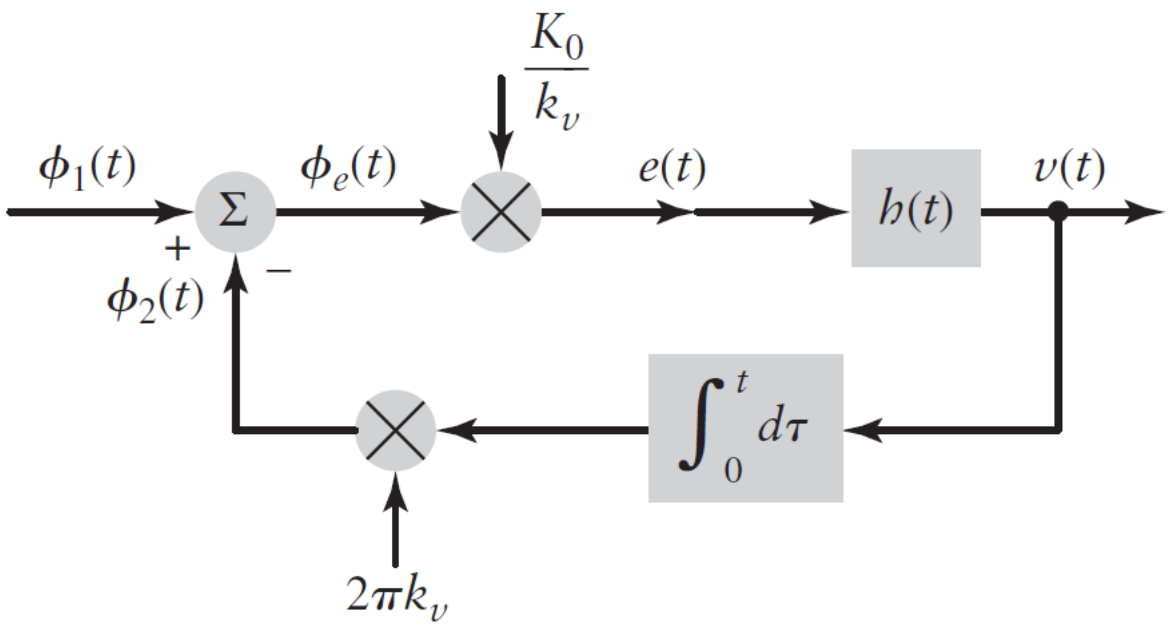
¡Que tal esto! $$\sin[\phi_e(t)] \approx \phi_e(t)$$

Entonces…

$$\begin{aligned} e(t) &\approx k_m A_c A_v \phi_e(t) \\ &= \frac{K_0}{k_v}\phi_e(t) \end{aligned}$$

$$K_0 = k_m k_v A_c A_v$$

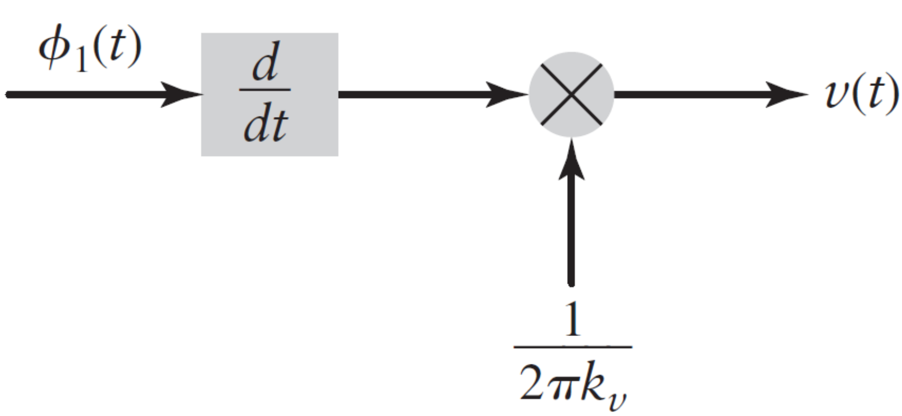
Considere esta equivalencia …



Para un lazo cerrado de retroalimentación (con operadores lineales)

$$A = \frac{\mu}{1 + \mu\beta} \qquad\qquad A \approx \frac{1}{\beta}$$

Entonces, con $K_0 \gg 1$…



En conclusión:

$$v(t) \approx \frac{1}{2\pi k_v}\left(\frac{d\phi_1(t)}{dt}\right)$$

Luego $\phi_1(t) \approx \phi_2(t)$

¿Cuánto vale $v(t)$?