



Problem Formulation

Building CP and MIP model

Denotation

- $R = \{1, \dots, n\}$ is the set of given rectangles

Item i has weight w_i and height h_i

- $B = \{1, \dots, m\}$ is the set of given bins

Bin k has width W_k , height H_k and cost c_k

Variables

- $o_i \in \{0, 1\}$ represents the orientation of item i
- l_i, r_i, t_i, b_i are left, right, top and bottom coordinates of item i
- Binary variable u_k is 1 iff bin k is used
- Binary variable p_{ik} is 1 iff item i is placed in bin k
- $y_i = k \in \{1, \dots, m\}$, i.e. item i is placed in bin k

CP model - Constraints

$$\bullet \quad o_i = 0 \Rightarrow r_i = l_i + w_i \wedge t_i = b_i + h_i \quad \forall i \in R \quad (1)$$

$$\bullet \quad o_i = 1 \Rightarrow r_i = l_i + h_i \wedge t_i = b_i + w_i \quad \forall i \in R \quad (2)$$

$$\bullet \quad y_i = y_j \Rightarrow r_i \leq l_j \vee r_j \leq l_i \vee t_i \leq b_j \vee t_j \leq b_i \quad \forall i, j \in R, i < j \quad (3)$$

$$\bullet \quad y_i = k \Rightarrow r_i \leq W_i \wedge t_i \leq H_i \quad \forall i, j \in R, k \in B \quad (4)$$

$$\bullet \quad p_{ik} = 1 \Rightarrow y_i = k \quad \forall i \in R, k \in B \quad (5)$$

$$\bullet \quad \sum_{i=1}^n p_{ik} \geq 1 \Rightarrow u_k = 1 \quad \forall k \in B \quad (6)$$

Constraints

- (1) If an item doesn't rotate, its $\text{right} = \text{left} + \text{width}$ and its $\text{top} = \text{bottom} + \text{height}$
- (2) If the item rotates then its $\text{right} = \text{left} + \text{height}$ and its $\text{top} = \text{bottom} + \text{width}$
- (3) If two items are placed in the same bin, they can't overlap each other
- (4) If one item is placed in a bin then its right and top coordinates can't exceed the bin
- (5) Item i is placed in bin k
- (6) Bin k is used when at least one item is placed in it