Thứ ngày

BT ập giới hạn bổ sung:

a, $f(x,y) = \dfrac{x^2}{x^2+y^2}$

xét $M_1\left(\frac{1}{k},\frac{1}{k}\right)$; $M_2\left(\frac{1}{k},\frac{2}{k}\right)$ $[k\to\infty]$

$\Rightarrow \lim\limits_{k\to\infty} f(M_1) = \dfrac{\left(\frac{1}{k}\right)^2}{\left(\frac{1}{k}\right)^2+\left(\frac{1}{k}\right)^2} = \dfrac{1}{2}$ $\neq$

$\lim\limits_{k\to\infty} f(M_2) = \dfrac{\left(\frac{1}{k}\right)^2}{\left(\frac{1}{k}\right)^2+\left(\frac{2}{k}\right)^2} = \dfrac{1}{5}$

$\Rightarrow$ ko tồn tại giới hạn.

b, $f(x,y) = \dfrac{(x+y)^2}{x^2+y^2}$

xét $M_1\left(\frac{1}{k},\frac{1}{k}\right)$; $M_2\left(\frac{1}{k},-\frac{1}{k}\right)$ $[k\to\infty]$

$\Rightarrow \lim\limits_{k\to\infty} f(M_1) = \dfrac{\left(\frac{1}{k}+\frac{1}{k}\right)^2}{\left(\frac{1}{k}\right)^2+\left(\frac{1}{k}\right)^2} = 2$ $\neq$

$\lim\limits_{k\to\infty} f(M_2) = \dfrac{\left(\frac{1}{k}-\frac{1}{k}\right)^2}{\left(\frac{1}{k}\right)^2+\left(\frac{1}{k}\right)^2} = 0$

$\Rightarrow$ ko tồn tại gh.

c, $f(x,y) = \dfrac{8x^2y^2}{x^4+y^4}$

xét $M_1\left(\frac{1}{k},\frac{1}{k}\right)$ $k\to(\infty)$

$M_2\left(\frac{1}{k},\frac{2}{k}\right)$

$$\lim_{k\to\infty} M_n = \frac{8.\frac{1}{k^4}}{\frac{2}{k^4}} = 4.$$

$$\lim_{k\to\infty} M_n = \frac{\frac{32}{k^4}}{\frac{17}{k^4}} = \frac{32}{17} \neq$$

$\Rightarrow$ M_n tồn tại gh.

d, $\lim_{(x,y)\to(0,0)} f(x,y) = \lim \frac{x^3+xy^2}{x^2+y^2} = \lim \frac{x(x^2+y^2)}{x^2+y^2} = \lim_{(x,y)\to(0,0)} x = 0$

e, $f_{(x,y)\to(0,0)}(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$

có: $x^2+y^2 \geq 2\sqrt{x^2y^2} = |2xy|$

$\Rightarrow \left|\frac{xy}{\sqrt{x^2+y^2}}\right| \leq \frac{1}{2}\sqrt{x^2+y^2}$ Mà $\lim_{(x,y)\to(0,0)}\sqrt{x^2+y^2} = 0$

$\Rightarrow \lim_{(x,y)\to(0,0)} f(x,y) = 0$

f, $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy+1}{x^2+y^2+1} = \frac{0+1}{0+0+1} = 1$

g, $f_{(x,y)\to(0,0)}(x,y) = \frac{2x^2y}{x^4+y^2}$ Đặt $y = kx$

$\Rightarrow \lim_{x\to 0} f(x,kx) = \lim_{x\to 0} \frac{2kx^3}{x^4+k^2x^2} = \frac{2kx^3}{x^2(k^2+x^2)} = \frac{2kx}{k^2+x^2}$

$= \frac{2k.0}{0+k^2} = \frac{0}{k^2} = 0$

h, $f_{(x,y)\to(0,0)}(x,y) = \frac{x^3y^2}{x^2+y^2}$

~~Lim f(x,y~~ xét $x^2+y^2 \geq 2|xy|$

$$\Rightarrow \left|\frac{xy}{x^2+y^2}\right| \leq \frac{1}{2}.$$

Mà $\lim\limits_{(x,y)\to(0,0)} x^2y = 0 \Rightarrow \lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$ (ĐL kẹp)

j, $f(x,y) = \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = \dfrac{(x^2+y^2)\left[\sqrt{x^2+y^2+1}+1\right]}{x^2+y^2}$

$= \sqrt{x^2+y^2+1}+1$

$\Rightarrow \lim\limits_{(x,y)\to(0,0)} (x,y) = \lim\limits_{(x,y)\to(0,0)} \sqrt{x^2+y^2+1}+1 = 2.$

k, $\lim\limits_{(x,y,z)\to(2,0,1)} f(x,y,z) = e^{-xy}\sin\dfrac{\pi z}{2} = e^{-2.0}\cdot\sin\dfrac{\pi}{2} = 1$

l, $f(x,y,z) = \dfrac{x^2+2y^2+3z^2}{x^2+y^2+z^2}$

Đặt $x=n$, $y=an$, $z=bn$

$\Rightarrow \lim\limits_{n\to 0} f(n,an,bn) = \dfrac{n^2+2a^2n^2+3b^2n^2}{n^2+a^2n^2+b^2n^2} = \dfrac{1+2a^2+3b^2}{1+a^2+b^2}$

2,

a, $f(x,y) = x^4+y^4-4x^2y^2$ liên tục trên R.

b, $\ln(x^2+y^2) \Rightarrow$ Để hs liên tục trên R thì $\left[\begin{array}{l} x\neq 0 \\ y\neq 0\end{array}\right.$

c, $f(x,y) = \frac{1}{y}\cos^2 x$

ĐK̄ xđ: $x \in R$ và $y \neq 0$

d, $f(x,y) = \tan\frac{x^2}{y}$

$\Rightarrow \begin{cases} x \in R \\ y \neq 0 \end{cases}$

e, $f(x,y) = \arctan\frac{y}{x} \Rightarrow \begin{cases} y \in R \\ x \neq 0 \end{cases}$

f: $f(x,y) = \arcsin\frac{x}{x^2+y^2} \Rightarrow \left[\, x \neq 0 \text{ hoặc } y \neq 0 \right.$

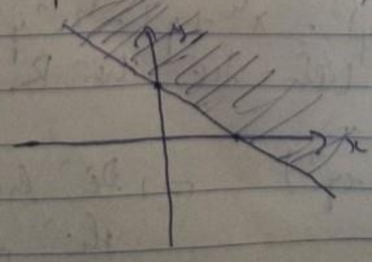
g, $\arctan\left(\frac{x+y}{1-xy}\right) \Rightarrow 1 - xy \neq 0 \Rightarrow xy \neq 1 \Rightarrow x \neq \frac{1}{y}$

h, $f(x,y) = \arctan\frac{x}{x^2+y^2} \Rightarrow x \neq 0$ hoặc $y \neq 0$

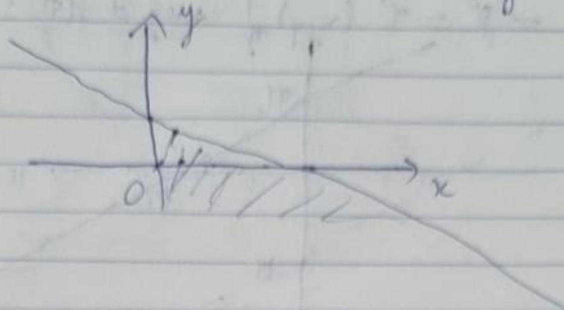
Exc 13-22:

13: $f(x,y) = \sqrt{x-2} + \sqrt{y-1}$

$D = \{(x,y) \in R^2 \mid x \geq 2, y \geq 1\}$

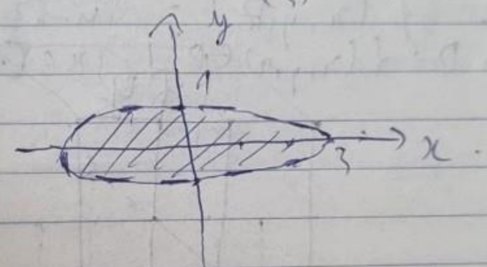


14. $f(x,y) = \sqrt[4]{x-3y} \Rightarrow x > 3y$

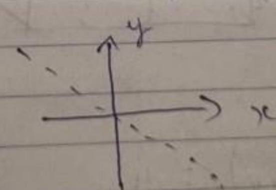


15. $\ln(9 - x^2 - 9y^2) \Rightarrow 9 - x^2 - 9y^2 > 0$

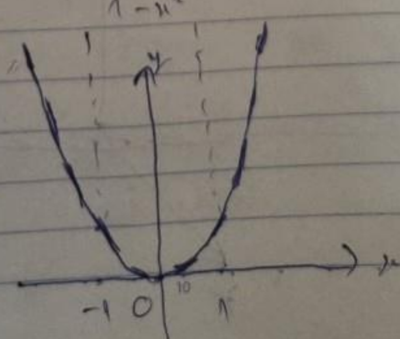
$\Rightarrow D = \{ (x,y) \in \mathbb{R}^2 \mid (\frac{x}{3})^2 + y^2 < 1 \}$



17. $g(x,y) = \frac{x-y}{x+y} \Rightarrow D = \{ (x,y) \mid x \neq -y \}$

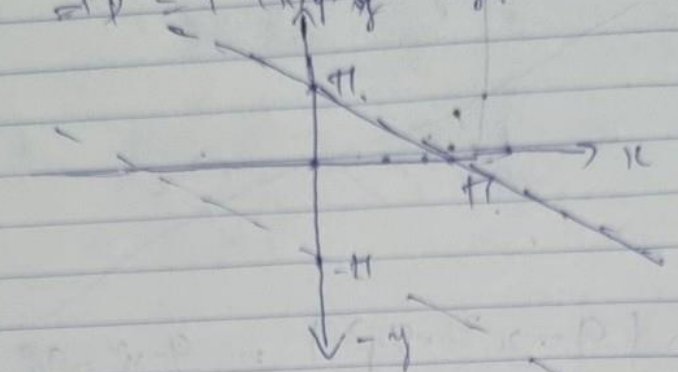


19. $f(x,y) = \frac{\sqrt{y-x^2}}{1-x^2} \Rightarrow D = \{ (x,y) \mid x \neq \pm 1 \quad y > x^2$



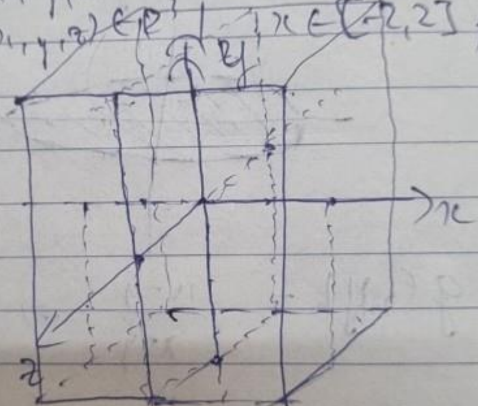
20. $f(x,y) = \sin^{-1}(x+y)$

$\Rightarrow D = \{(x,y) \mid x+y \neq k\pi\}$

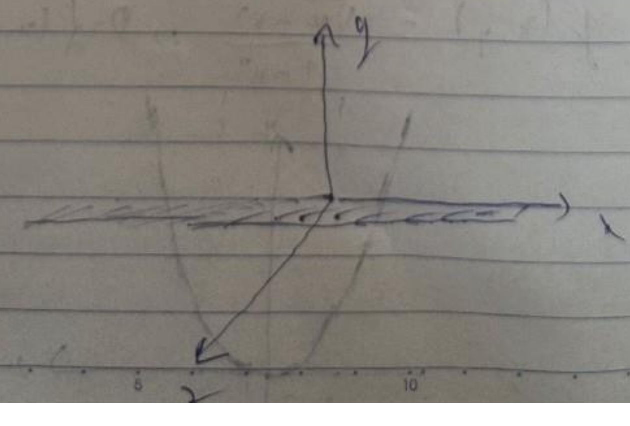


21: $f(x,y,z) = \sqrt{4-x^2} + \sqrt{9-y^2} + \sqrt{1-z^2}$

$\Rightarrow D = \{(x,y,z) \in \mathbb{R}^3 \mid x \in [-2,2], y \in [-3,3], z \in [-1$



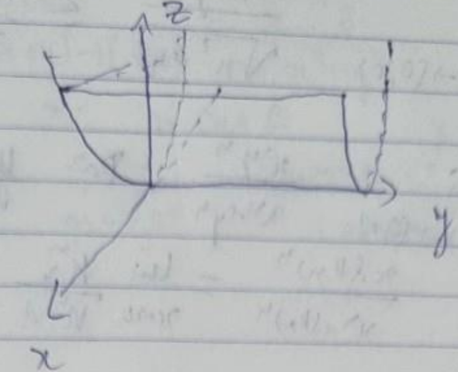
23: $f(x,y) = y$. Đặt $f(x,y) = z$.



Thứ ngày

$f(x,y) = x^2$

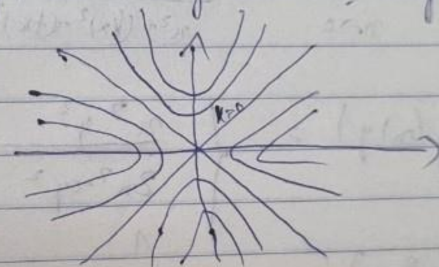
24:



45: (kg (2D)):

$f(x,y) = x^2 - y^2$ đặt $f(x,y) = K$

$\Rightarrow x^2 = y^2 + K$



5, $\lim\limits_{(x,y)\to(3,2)} (x^2y^3 - 4y^2) = 56$

9, $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 4y^2}{x^2 + 2y^2}$ Đặt $y = kx$

$= \lim\limits_{(x\to 0)} \dfrac{x^4 - 4k^2x^2}{x^2 + 2k^2x^2} = \dfrac{x^2 - 4k^2}{1 + 2k^2}$

$= \dfrac{-4k^2}{1+2k^2}.$

10, $\lim\limits_{(x,y)\to(0,0)} \dfrac{5y^4\cos^2 x}{x^4 + y^4}$ Đặt $y = kx$

$= \lim\limits_{x\to 0} \dfrac{5k^4x^4\cos^2 x}{x^4 + k^4x^4} = \dfrac{5k^4x^2}{(k^4+1)} = 0$

17: $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}} \le \dfrac{1}{2}\sqrt{x^2+y^2} = 0$

18: $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^4}{x^4+y^4}$ Đặt $y = kx$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{x.(kx)^4}{x^4+(kx)^4} = \lim\limits_{x\to 0} \dfrac{k^4x}{k^4+1} = 0$

20: $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{x^2y^2z^2}{x^2+y^2+z^2}$ Đặt $y = kx$, $z = tx$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{x^2.(kx)^2.(tx)^2}{x^2+(kx)^2+(tx)^2} = \dfrac{k^2t^2x^6}{(k^2+t^2+1)x^2} = \dfrac{k^2t^2x^4}{k^2+t^2+1}$

37: $f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2} & \text{if } (x,y) \ne (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$

$\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^3}{2x^2+y^2} = 0 \Rightarrow$ liên tục

38: $f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & \text{if } (x,y) \ne (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

$\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+xy+y^2}$ Đặt $y = kx \Rightarrow \dfrac{k}{(k^2+k+1)} \Rightarrow$ ko lt

39: $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{x^2+y^2}$.

$x = r\cos\theta$, $y = r\sin\theta$.

$\Rightarrow \lim\limits_{r\to 0^+} \dfrac{(r\cos\theta)^3+(r\sin\theta)^3}{(r\cos\theta)^2+(r\sin\theta)^2} = 0.$

40: $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$

Đặt $t = x^2+y^2 \Rightarrow t\to 0$

$\Rightarrow \lim\limits_{t\to 0} t\ln t \Leftrightarrow \lim\limits_{t\to 0} \dfrac{\ln t}{\frac{1}{t}} =$

$\xrightarrow{L} \lim\limits_{t\to 0} \dfrac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim\limits_{t\to 0} -t = 0$

41: $\lim\limits_{(x,y)\to(0,0)} \dfrac{e^{-x^2-y^2}-1}{x^2+y^2}$

Đặt $x^2+y^2 = t \Rightarrow \lim\limits_{t\to 0} \dfrac{e^{-t}-1}{t}$

$\Rightarrow \lim\limits_{t\to 0} \dfrac{-t}{t} = -1.$

43: $f(x,y) \begin{cases} \dfrac{\sin xy}{xy} & xy \neq 0 \\ 1 & xy = 0 \end{cases}$

xét $\lim\limits_{xy\to 0} \dfrac{\sin xy}{xy} = 1 \Rightarrow$ liên tục.