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BT:

I, Calculus with Parametric Curves

3, $\begin{cases} x = t^3+1 = 0 \\ y = t^4+t = 0 \\ t = -1 \end{cases} \Rightarrow \begin{cases} x' = 3t^2 \\ y' = 4t^3 \\ t = -1 \end{cases} \Rightarrow \begin{cases} x'_{(1)} = 3 \\ y'_{(1)} = -4 \end{cases}$

$\Rightarrow$ tangent equation: $\dfrac{x-0}{3} = \dfrac{y-0}{-4} \Leftrightarrow \dfrac{x}{3} = \dfrac{y}{-4}$

4, $\begin{cases} x = \sqrt{t} \\ y = t^2-2t \\ t = 4 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ y = 8 \end{cases} \Rightarrow \begin{cases} x'_{(4)} = \dfrac{1}{2\sqrt{t}} = \dfrac{1}{4} \\ y'_{(4)} = 2t-2 = 6 \end{cases}$

$\Rightarrow$ tangent equation: $\dfrac{x-2}{\frac{1}{4}} = \dfrac{y-8}{6} \Leftrightarrow 4(x-2) = \dfrac{y-8}{6}$

5, $\begin{cases} x = t\cos t = -\pi \\ y = t\sin t = 0 \\ t = \pi \end{cases} \Rightarrow \begin{cases} x' = \cos t - t\sin t = -1 \\ y' = \sin t + t\cos t = -\pi \\ t = \pi \end{cases}$

$\Rightarrow$ tangent equation: $\dfrac{x+\pi}{-1} = \dfrac{y}{-\pi} \Rightarrow x+\pi = \dfrac{y}{\pi}$

7, C_1: $\begin{cases} x = 1+\ln t \\ y = t^2+2 \end{cases} \Rightarrow \begin{cases} x' = \dfrac{1}{t} = 1 \\ y' = 2t = 2 \end{cases}$

$\Rightarrow$ tangent point: $\dfrac{x-1}{\frac{1}{t}} = \dfrac{y-3}{2t}$.

C_2: $\begin{cases} x = 1+\ln t = 1 \\ y = t^2+2 = 3 \end{cases} \Rightarrow \begin{cases} \ln t = 0 \\ t^2 = 1 \end{cases} \Rightarrow t = 1 \Rightarrow$ tangent point: $\dfrac{x-1}{1} = \dfrac{y-3}{2}$

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17: $\begin{cases} x = t^3 - 3t \\ y = t^2 - 3 \end{cases}$ xét $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2t}{3t^2-3}$

$\Rightarrow$ ptr có tiếp tuyến ngang khi $t = 0$ $\left(\dfrac{dy}{dx} = 0\right)$

dọc khi $t = \pm 1$ $\left(\dfrac{dy}{dx} = \infty\right)$

18: $\begin{cases} x = t^3 - 3t \\ y = t^3 - 3t^2 \end{cases}$ xét $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{3t^2-3}{3t^2-6t}$

(Quy ước tắt)

$\Rightarrow$ ptr có tiếp tuyến ngang khi $\dfrac{dy}{dx} = 0 \Rightarrow t = \pm 1$

ptr có tiếp tuyến dọc khi $\dfrac{dy}{dx} = \infty \Rightarrow \left[\begin{array}{l} t = 2 \\ t = 0 \end{array}\right.$

19: $\begin{cases} x = \cos\theta \\ y = \cos 3\theta \end{cases}$ xét $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{-3\sin 3\theta}{-\sin\theta} = \dfrac{3\sin 3\theta}{\sin\theta}$.

$\Rightarrow$ ptr có tt ngang khi $\sin 3\theta = 0 \Rightarrow 3\theta = k\pi \Rightarrow \theta = \dfrac{k\pi}{3}$

dọc khi $\sin\theta = 0 \Rightarrow \theta = k\pi$

~~25: $\begin{cases} x = \cos t \\ y = \sin t \cos t \end{cases}$ $\Rightarrow y'_x = \dfrac{y'_t}{x'_t} = \dfrac{\cos 2t}{-\sin t}$~~

~~(*) Có tt ngang khi $t = \pi + k2\pi$ $\Rightarrow$ xét $t = \pi$ (đúng)~~

~~dọc khi $t = k\pi$~~

~~$\Rightarrow \begin{cases} x = 0 \\ y = \end{cases}$~~

25: $\begin{cases} x = \cos t \\ y = \sin t \cos t \end{cases}$ $\Rightarrow$ tại $M(0,0) \Rightarrow \begin{cases} \cos t = 0 \\ \sin t \cos t = 0 \end{cases} \Leftrightarrow \begin{cases} t = \frac{\pi}{2} + k\pi \\ \left[\begin{array}{l} t = \frac{\pi}{2} + k\pi \\ t = k\pi \end{array}\right. \end{cases}$

$\Rightarrow t = \dfrac{\pi}{2} + k\pi \Rightarrow$ lấy 2 điểm $\begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases}$ (các đ² t cho ra kq tương tự)

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8, $C_1: \begin{cases} x = \sqrt{t^2+1} \\ y = e^{t^2} \end{cases} \Rightarrow \begin{cases} x' = \frac{1}{2\sqrt{t}} \\ y' = 2t\, e^{t^2} \end{cases}$ $(2, e)$

$\Rightarrow$ tangent equation : $\frac{x-2}{\frac{1}{2}} = \frac{y-e}{2e}$

$C_2: \begin{cases} x = \sqrt{t}+1 \\ y = e^{t^2} \end{cases} \Rightarrow \begin{cases} t = (x-1)^2 \\ \ln y = t^2 \end{cases} \Rightarrow \underbrace{(x-1)^4 - \ln y}_{f(x,y)} = 0$

$\Rightarrow \begin{cases} f'_x = 4(x-1)^3 = 4 \\ f'_y = -\frac{1}{y} = -\frac{1}{e}. \end{cases}$ $\Rightarrow$ tangent equation $= 4(x-2) - \frac{1}{e}(y-e)$

9, $C_1: \begin{cases} x = t^2 - t = 0 \\ y = t^2+t+1 = 3 \\ \Rightarrow t = 1 \end{cases} \Rightarrow \begin{cases} x' = 2t-1 = 1 \\ y' = 2t+1 = 3 \end{cases}$

$\Rightarrow$ tangent equation : $\frac{x-0}{1} = \frac{y-3}{3}$

~~$C_2: \begin{cases} x = t^2 - t \\ y = t^2+t+1 \end{cases}$~~

10, $\begin{cases} x = \sin \pi t = 0 \\ y = t^2+t = 2 \\ \Rightarrow t = 1 \end{cases} \Rightarrow \begin{cases} x' = \pi \cos \pi t = -\pi \\ y' = 2t+1 = 3 \end{cases}$

$\Rightarrow$ tangent equation : $\frac{x}{-\pi} = \frac{y-2}{3}$

11, $\begin{cases} x = t^2+1 \\ y = t^2+t \end{cases} \Rightarrow \begin{cases} t^2 = x-1 \\ y = t^2+t \end{cases} \Rightarrow \begin{cases} t^2 = x-1 \\ y = x-1+\sqrt{x-1} \end{cases}$ $(x>1)$

$\Rightarrow y' = 1 + \frac{1}{2\sqrt{x-1}}$ $y'' = -\frac{\frac{2}{2\sqrt{x-1}}}{(2\sqrt{x-1})^2} = -\frac{1}{4(\sqrt{x-1})^{\frac{3}{2}}}$

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13. $\begin{cases} x = e^t \\ y = t\,e^{-t} \end{cases} \Rightarrow \begin{cases} \ln x = t \\ y = \dfrac{x}{\ln x} \end{cases} \Rightarrow y' = \dfrac{\ln x - 1}{(\ln x)^2} \quad y'' = -\dfrac{1}{x}$

14. $\begin{cases} x = t^2 + 1 \\ y = e^t - 1 \end{cases} \Rightarrow \begin{cases} t = \sqrt{x-1} \\ y = e^{\sqrt{x-1}} - 1 \end{cases} \Rightarrow y' = \dfrac{1}{2\sqrt{x-1}} e^{\sqrt{x-1}}$

$\Rightarrow y'' = -\dfrac{1}{8(\sqrt{x-1})^{\frac{3}{2}}} \cdot e^{\sqrt{x-1}} + \dfrac{1}{4(x-1)} e^{\sqrt{x-1}}$

15. $\begin{cases} x = t - \ln t \\ y = t + \ln t \end{cases} \Rightarrow \begin{cases} x + y = 2t \\ y = t + \ln t \end{cases} \Rightarrow \begin{cases} t = \dfrac{x+y}{2} \\ y = t + \ln t \end{cases}$

$\Rightarrow y = \dfrac{x+y}{2} + \ln \dfrac{x+y}{2} \Rightarrow \dfrac{y}{2} - \dfrac{x}{2} + \ln (x+y) - \ln 2 = c = F(x,y)$

$y'_x = -\dfrac{F'_x}{F'_y} = -\dfrac{-\frac{1}{2} + \frac{1}{x+y}}{\frac{1}{2} + \frac{1}{x+y}} = -\dfrac{2-x-y}{x+y+2} = \dfrac{x+y-2}{x+y+2}$

$\Rightarrow y''_{xx} = -\dfrac{F''_{xx}(F'_y)^2 - 2F''_{xy} \cdot F_x F_y - F''_{yy}(F'_x)^2}{(F_y)^3}$

$= -\dfrac{-\frac{1}{(x+y)^2} \cdot \left(\frac{1}{2} + \frac{1}{x+y}\right)^2 - 2 \cdot \left(-\frac{1}{(x+y)^2}\right) \cdot \left(\frac{1}{x+y} - \frac{1}{2}\right)\left(\frac{1}{x+y} + \frac{1}{2}\right) - \left(\frac{1}{(x+y)^2}\right)\left(\frac{1}{x+y} - \frac{1}{2}\right)^2}{\left(\frac{1}{2} + \frac{1}{x+y}\right)^3}$

~~17. $\begin{cases} x = t^3 - 3t \\ y = t^2 - 3 \end{cases} \Leftrightarrow \begin{cases} x' = 3t^2 - 3 = 0 \\ y' = 2t = 0 \end{cases} \Rightarrow \begin{cases} t = \pm 1 \\ t = 0 \end{cases} \Rightarrow$ Ko có~~

~~18. $\begin{cases} x = t^3 - 3t \\ y = t^3 - 3t^2 \end{cases} \Rightarrow \begin{cases} x' = 3t^2 - 3 \\ y' = 3t^2 - 6t \end{cases} \Rightarrow \begin{cases} \left[\begin{array}{l} t = -1 \\ t = 1 \end{array}\right. \\ \left[\begin{array}{l} t = 0 \\ t = 2 \end{array}\right. \end{cases}$~~

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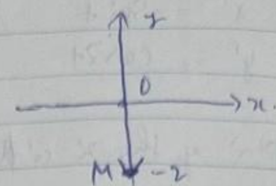
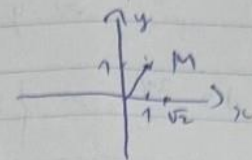
Polar coordinates

3, a: $(2, \frac{3\pi}{2})$



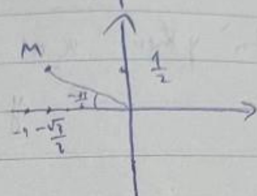
$(x=0, y=-2)$

b, $(\sqrt{2}, \frac{\pi}{4})$



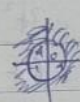
$(x = y = 1)$

c, $(-1, -\frac{\pi}{6})$

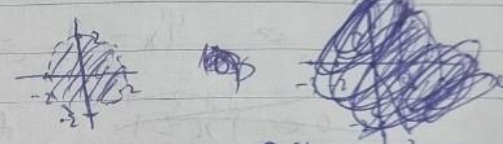


$\begin{cases} x = -\frac{\sqrt{3}}{2} \\ y = \frac{1}{2} \end{cases}$

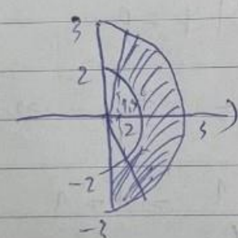
7,



8,



11,



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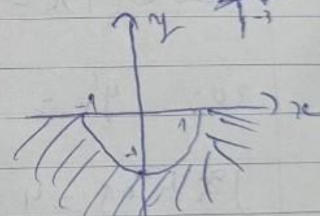

15, $r^2 = 5 \Rightarrow r = \pm\sqrt{5} \Rightarrow \left[\begin{array}{l} \begin{cases} x = \sqrt{5}\cos\theta \\ y = \sqrt{5}\sin\theta \end{cases} \\ \begin{cases} x = -\sqrt{5}\cos\theta \\ y = -\sqrt{5}\sin\theta \end{cases} \end{array} \right.$

16, $r = 4\sec\theta \Rightarrow \begin{cases} x = 4 \\ y = 4\sec\theta\sin\theta = 4\tan\theta \end{cases}$

19: $r^2 = \cos 2\theta^{(-1)} \Rightarrow r = \pm\sqrt{\frac{1}{\cos 2\theta}}$

$\Rightarrow \begin{cases} x = \frac{1}{\sqrt{\cos 2\theta}}\cos\theta \\ y = \frac{1}{\sqrt{\cos 2\theta}}\sin\theta \end{cases}$ hoặc $\begin{cases} x = -\frac{1}{\sqrt{\cos 2\theta}}\cos\theta \\ y = \frac{1}{\sqrt{\cos 2\theta}}\sin\theta \end{cases}$

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21, $\begin{cases} \theta_1 \le \theta \le \theta_2 \\ r = \dfrac{R}{\sin\theta} \end{cases}$

22, $\begin{cases} \left[\begin{array}{l} \theta = 45^\circ \\ \theta = -135^\circ \end{array}\right. \\ r = \sqrt{2}\, x \end{cases}$

$\begin{cases} r^2 = x^2 + y^2 \\ \theta = \arctan \dfrac{y}{x} \end{cases}$

23. $y = 1 + 3x \Rightarrow \begin{cases} r^2 = x^2 + (1+3x)^2 \\ \theta = \arctan \dfrac{1+3x}{x} \end{cases}$

$\Rightarrow \begin{cases} r = \sqrt{10x^2 + 6x + 1} \\ \theta = \arctan\left(\frac{1}{x} + 3\right) \end{cases}$

24. $4y^2 = x \Rightarrow \begin{cases} r^2 = x^2 + \frac{x}{4} \\ \theta = \arctan\left(\dfrac{\sqrt{\frac{x}{4}}}{x}\right) \end{cases} \Rightarrow \begin{cases} r = \sqrt{x^2 + \frac{x}{4}} \\ \theta = \arctan\left(\dfrac{\sqrt{\frac{x}{4}}}{x}\right) \end{cases}$

25, $x^2 + y^2 = 2cx \Rightarrow \begin{cases} r = \sqrt{2cx} \\ \theta = \arctan\left(\dfrac{\sqrt{x^2 - 2cx}}{x}\right) \end{cases}$

26, $x^2 - y^2 = 4 \Rightarrow \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \arctan\left(\dfrac{y}{x}\right) \end{cases} \Rightarrow \begin{cases} r = \sqrt{2x^2 - 4} \\ \theta = \arctan\left(\dfrac{\sqrt{x^2-4}}{x}\right) \end{cases}$

51: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Rightarrow \begin{array}{l} x = \sin^2\theta \\ y = \sin^2\theta \tan\theta. \end{array}$ $0 \le x < 1$ (has $x=1$ is vertical)

$(\cos\theta \ne 0) \Rightarrow \sin\theta \ne \pm 1 \Rightarrow 0 \le \sin^2\theta < 1$

61, $r = 3\cos\theta \Rightarrow \begin{cases} x = 3\cos^2\theta \\ y = \frac{3}{2}\sin 2\theta \end{cases} \Rightarrow \begin{cases} x'_\theta = -3\sin 2\theta \\ y' = 3\cos 2\theta \end{cases}$

$\Rightarrow k = \dfrac{y'_\theta}{x'_\theta} = y'_x = \cot 2\theta \Rightarrow \begin{cases} \sin 2\theta = 0 \Rightarrow \theta = \frac{\pi}{2} + \frac{k\pi}{2} \\ \cos 2\theta = 0 \Rightarrow \theta = \frac{\pi}{4} + \frac{k\pi}{2} \end{cases}$

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25, $\begin{cases} x = e^{-t}\cos t \\ y = e^{-t}\sin t \\ z = e^{-t} \end{cases} \Rightarrow \begin{cases} x' = -e^{-t}\cos t - \sin t\, e^{-t} = -\frac{\cos(1)}{e} - \frac{\sin(1)}{e} \\ y' = -e^{-t}\sin t + \cos t\, e^{-t} = -\frac{\sin(1)}{e} + \frac{\cos(1)}{e} \\ z' = -e^{-t} = -\frac{1}{e}. \end{cases}$ M (1, 0, 1)

$\Rightarrow$ tangent equation : $\dfrac{x-1}{-\frac{(\sin+\cos)}{e}} = \dfrac{y}{\frac{\cos-\sin}{e}} = \dfrac{z-1}{-\frac{1}{e}}$

$\Rightarrow \begin{cases} x = (\sin+\cos)t + 1 \\ y = (\sin-\cos)t \\ z = t+1 \end{cases}$

ARC length.

27: $y = x^4 \Rightarrow y' = 4x^3 \quad y'' = 12x^2$

$\Rightarrow C(M) = \dfrac{|y''|}{(1+(y')^2)^{\frac{3}{2}}} = \dfrac{|12x^2|}{(1+16x^6)^{\frac{3}{2}}} = \dfrac{12x^2}{(1+16x^6)^{\frac{3}{2}}}$

28, $y = \tan x \Rightarrow y' = \dfrac{1}{\cos^2 x} = \tan^2 x + 1$, $y'' = 2(\tan^2 x + 1)\tan x$
$= 2\tan^3 x + 2\tan x$

$\Rightarrow C(M) = \dfrac{|y''|}{(1+y'^2)^{\frac{3}{2}}} = \dfrac{2\tan^3 x + 2\tan x}{[(\tan^2 x + 1)^2 + 1]^{\frac{3}{2}}}$

30, $y = \ln x$, $y' = \dfrac{1}{x}$, $y'' = -\dfrac{1}{x^2}$.

$\Rightarrow C(M) = \dfrac{|\frac{1}{x^2}|}{[1+(\frac{1}{x})^2]^{\frac{3}{2}}} = \dfrac{1}{x^2(1+\frac{1}{x^2})^{\frac{3}{2}}}$

$= \dfrac{1}{(x^{\frac{4}{3}} + x^{-\frac{2}{3}})^{\frac{3}{2}}} \Rightarrow C'(M) = \dfrac{\frac{3}{2}(x^{\frac{4}{3}} + x^{-\frac{2}{3}})^{\frac{1}{2}}.(\frac{4}{3}x^{\frac{1}{3}} - \frac{2}{3}x^{-\frac{5}{3}})}{(x^{\frac{4}{3}} + x^{-\frac{2}{3}})^{3}} = 0$

$\Rightarrow \left[\begin{array}{l} x^{\frac{4}{3}} = -x^{-\frac{2}{3}} \\ \frac{4}{3}x^{\frac{1}{3}} = \frac{2}{3}x^{-\frac{5}{3}} \end{array}\right. \Rightarrow \begin{array}{l} x^2 = -1 \text{ (loại)} \\ x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}} \end{array}$

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$\Rightarrow$ Max khi

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68, $r = (1 - \sin\theta)$ $\Rightarrow \begin{cases} x = (1-\sin\theta)\cos\theta \\ y = (1-\sin\theta)\sin\theta \end{cases} \Rightarrow \begin{cases} x'_\theta = -\sin\theta + \cos 2\theta \\ y'_\theta = \cos\theta + \cos 2\theta \end{cases}$

$\Rightarrow k = \frac{y'_\theta}{x'_\theta} = y'_x = \frac{\cos\theta + \cos 2\theta}{\cos 2\theta - \sin\theta} \Rightarrow \begin{cases} \text{vertical} \Rightarrow y'_\theta = 0 \\ \text{horizontal} \Rightarrow x'_\theta = 0 \end{cases}$

64, $r = e^\theta \Rightarrow \begin{cases} x = e^\theta \cos\theta \\ y = e^\theta \sin\theta \end{cases} \Rightarrow \begin{cases} x'_\theta = e^\theta\cos\theta - e^\theta \sin\theta \\ y'_\theta = e^\theta \sin\theta + e^\theta \cos\theta \end{cases}$

$\begin{cases} \text{horizontal} \Rightarrow x'_\theta = 0 \\ \text{vertical} \Rightarrow y'_\theta = 0 \end{cases} \Rightarrow \begin{cases} \cos\theta - \sin\theta = 0 \\ \sin\theta + \cos\theta = 0 \end{cases} \Rightarrow \begin{cases} \tan\theta = 1 \\ \tan\theta = -1 \end{cases}$

57, $r = \frac{1}{\theta}$; $\theta = \pi \Rightarrow r = \frac{1}{\pi} \Rightarrow \begin{cases} x = \frac{\cos\pi}{\pi} = \frac{-1}{\pi} \\ y = \frac{\sin\pi}{\pi} = 0 \end{cases}$

$\begin{cases} x'_\theta = \left(\frac{\cos\theta}{\theta}\right)' = \frac{-\sin\theta . \theta - \cos\theta}{\theta^2} \\ y'_\theta = \left(\frac{\sin\theta}{\theta}\right)' = \frac{\cos\theta . \theta - \sin\theta}{\theta^2} \end{cases}$ $\Rightarrow k = y'_x = \frac{\theta . \cos\theta - \sin\theta}{\theta(-\sin\theta) - \cos\theta}$

$= \frac{-\pi}{\pi} = -1$

Derivatives and Integrals of vector Funtions (1)

23, $\begin{cases} x = t^2 + 1 \\ y = 4\sqrt{t} \\ z = e^{t^2 - t} \end{cases} \Rightarrow \begin{cases} x'_t = 2t = 4 \\ y'_t = \frac{2}{\sqrt{t}} = 1 \\ z'_t = (2t-1)\, e^{t^2-t} = 1 \end{cases}$ $M(2, 4, 1)$

$\Rightarrow$ tangent equation : $\frac{x-2}{4} = \frac{y-4}{1} = \frac{z-1}{1}$

$\Rightarrow \begin{cases} x = 4t + 2 \\ y = t + 4 \\ z = t + 1 \end{cases}$

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xét $\begin{cases} x' = -\sin t \\ y' = \cos 2t \end{cases}$ tại $t = \frac{\pi}{2} \Rightarrow \begin{cases} x' = -1 \\ y' = -1 \end{cases} \Rightarrow y'_x = 1$

$\Rightarrow$ pt y theo x có ~~pháp~~ tiếp tuyến với hệ số $K = 1$ (1)

$\Rightarrow y = k(x - x_0) + y_0 \Rightarrow y = x$

xét tại $t = \frac{3\pi}{2} \Rightarrow \begin{cases} x' = 1 \\ y' = -1 \end{cases} \Rightarrow y'_x = -1$

$\Rightarrow$ pt y theo x có tiếp tuyến với hệ số $K = -1$ (2)

$\Rightarrow y = K(x - x_0) \Leftrightarrow y = -x$

29. $\Rightarrow y'_x = \frac{1}{2} \Rightarrow \frac{y'_t}{x'_t} = \frac{1}{2} \Rightarrow \frac{3t^2}{6t} = \frac{1}{2} \Rightarrow 6t^2 = 6t \Rightarrow \left[\begin{array}{l} \cancel{t = 0} \\ t = 1 \end{array} \right.$

~~$t = 0 \Rightarrow \begin{cases} y = -1 \\ x = 1 \end{cases}$~~ $t = 1 \Rightarrow \begin{cases} x = 4 \\ y = 0 \end{cases}$

30: $y'_x = \frac{y'_t}{x'_t} = \frac{6t^2}{6t} = t \Rightarrow K = y'_x = 1$

$\begin{cases} 3t^2 + 1 = 4 \\ 2t^3 + 1 = 3 \end{cases} \Rightarrow \begin{cases} t = \pm 1 \\ t = 1 \end{cases} \Rightarrow t = 1$

$\Rightarrow y_{tt} = K(x - x_0) + y_0 \Rightarrow y_{tt} = x_{tt} - 1$

70: $y'' = 2$ $y = 2x \Rightarrow$ curvature $= \left| \frac{2}{(1 + 4x^2)^{\frac{3}{2}}} \right|$

$= \frac{2}{5\sqrt{5}}$

71: $\begin{cases} x = \theta - \sin\theta \\ y = 1 - \cos\theta \end{cases} \Rightarrow \begin{cases} x'_\theta = 1 - \cos\theta \\ y'_\theta = \sin\theta \end{cases} \Rightarrow \begin{cases} x''_{\theta\theta} = \sin\theta \\ y''_{\theta\theta} = \cos\theta \end{cases}$

$(C_M) = \frac{|x' \cdot y'' - x'' y'|}{(\ldots)^{\frac{3}{2}}} = \frac{|(1 - \cos\theta) \cdot (\cos\theta) - \sin^2\theta|}{[(1 - \cos\theta)^2 + \sin^2\theta]^{\frac{3}{2}}}$

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11, $y = e^x \Rightarrow y' = e^x \quad y'' = e^x$

$\Rightarrow C(M) = \dfrac{e^x}{(1+e^{2x})^{\frac{3}{2}}} \Rightarrow C'(M) = \dfrac{e^x(1+e^{2x})^{\frac{3}{2}} - e^x \cdot \frac{3}{2}(1+e^{2x})^{\frac{1}{2}} \cdot (1+e^{2x})'}{(1+e^{2x})^3}$

$= \dfrac{e^x(1+e^{2x})^{\frac{3}{2}} - 3e^{3x}(1+e^{2x})^{\frac{1}{2}}}{(1+e^{2x})^3} = 0$

$(1+e^{2x})^{\frac{3}{2}} = 3e^{2x}(1+e^{2x})^{\frac{1}{2}}$

$\Rightarrow (1+e^{2x})^2 = 3e^{2x} \Rightarrow e^{4x} + 2e^{2x} + 1 = 3e^{2x}$

Đặt $e^{2x} = t \Rightarrow t^2 - t + 1 = 0 \Leftrightarrow$ VN $(C'(M) > 0 \ \forall x)$

$\Rightarrow x \to \infty \Rightarrow C(M)$ tăng.

32: $y = ax^2 + bx + c \qquad (0, 0) \quad k = 4$

$\Rightarrow y' = 2ax + b \qquad \Rightarrow \dfrac{|2a|}{((2ax+b)^2+1)^{\frac{3}{2}}} = 4$

$y'' = 2a$

$\Rightarrow \dfrac{|2a|}{(b^2+1)^{\frac{3}{2}}} = 4$

$\Rightarrow |a| = 2(b^2+1)^{\frac{3}{2}} \quad ; c = \text{any}$

43. $\begin{cases} x' = 2t \\ y' = 3t^2 \end{cases} \begin{cases} x'' = 2 \\ y'' = 6t \end{cases} \Rightarrow C(M) = \dfrac{|x'y'' - y'x''|}{[(x')^2 + (y')^2]^{\frac{3}{2}}}$

$= \dfrac{|12t^2 - 6t^2|}{(4t^2 + 9t^4)^{\frac{3}{2}}} = \dfrac{6t^2}{(4t^2+9t^4)^{\frac{3}{2}}}$

45, $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases} \Rightarrow \begin{cases} x' = e^t \cos t - e^t \sin t \\ y' = e^t \sin t + e^t \cos t \end{cases}$

$\begin{cases} x'' = -e^t \sin t + e^t \cos t - e^t \sin t - e^t \cos t = -2e^t \sin t \\ y'' = e^t \sin t + e^t \cos t + e^t \cos t - e^t \sin t = 2e^t \cos t \end{cases}$

$\Rightarrow C(M) = \dfrac{|x'y'' + y'x''|}{(x'^2 + y'^2)^{\frac{3}{2}}} = \dfrac{|2(e^t\cos t - e^t \sin t)e^t \cos t - 2e^t \sin t(e^t \sin t + e^t \cos t)|}{[(e^t\cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2]^{\frac{3}{2}}}$

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5, $\begin{cases} x = 2t-1 \\ y = \frac{1}{2}t+1 \end{cases} \Rightarrow \begin{cases} t = \frac{x-1}{2} \\ y = \frac{x-1}{4}+1 \end{cases} \Rightarrow x - 4y + 3 = 0$

7, $\begin{cases} x = t^2-3 \\ y = 2+t \end{cases} \Rightarrow \begin{cases} y-2 = t \\ x = (y-2)^2-3 \end{cases} \Rightarrow y^2 - 4y - x + 1 = 0$

8, $\begin{cases} x = \sin t \\ y = 1-\cos t \end{cases} \Rightarrow \begin{cases} t = \arcsin x \\ y = 1-\cos(\arcsin x) \end{cases} \Rightarrow y + \cos(\arcsin x) - 1 = 0$

11, $\begin{cases} x = \sin\frac{1}{2}\theta \\ y = \cos\frac{1}{2}\theta \end{cases} \Rightarrow x^2 + y^2 - 1 = 0 \quad (-\pi \leq \theta \leq \pi)$
$\Rightarrow (-1 \leq x \leq 1 ; 0 \leq y \leq 1)$

12: $\begin{cases} x = \frac{1}{2}\cos\theta \\ y = 2\sin\theta \end{cases} \Rightarrow 16x^2 + y^2 - 4 = 0 \quad (-1 \leq x \leq 1 ; 0 \leq y \leq 1)$

13, $\begin{cases} x = \sin t \\ y = \csc t \end{cases} \Rightarrow \begin{cases} x = \sin t \\ y = \frac{1}{\sin t} \end{cases} \Rightarrow xy - 1 = 0$
$(0 < x \leq \frac{\sqrt{2}}{2}), (y \geq \sqrt{2})$

15, $\begin{cases} x = t^2 \\ y = \ln t \end{cases} \Rightarrow \begin{cases} \sqrt{x} = t \\ y = \frac{1}{2}\ln x \end{cases} \Rightarrow y - \frac{1}{2}\ln x = 0$

16, $\begin{cases} x = \sqrt{t+1} \\ y = \sqrt{t-1} \end{cases} \Rightarrow \begin{cases} x^2 = t+1 \\ y^2 = t-1 \end{cases} \Rightarrow y^2 - x^2 + 2 = 0$

18, $\begin{cases} x = \tan^2\theta \\ y = \sec\theta = \frac{1}{\cos\theta} \end{cases} \Rightarrow \begin{cases} \theta = \arccos\left(\frac{1}{y}\right) \\ x = \tan^2 \arccos\left(\frac{1}{y}\right) \end{cases} \Rightarrow x - \tan^2 \arccos\left(\frac{1}{y}\right)$

$\tan\theta = \frac{1}{\cos^2\theta} - 1 \Rightarrow x = (y^2-1)^2 \Rightarrow y^4 - 2y^2 + 1 - x = 0$

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