Thứ ngày

29: $F(x,y) = \int_y^x \cos(e^t)\,dt = \frac{\sin(e^t)}{e^t}\Big|_y^x = \frac{\sin e^x}{e^x} - \frac{\sin e^y}{e^y}$

$\Rightarrow F'_x(x,y) = \frac{e^{2x}\cos e^x - e^x \sin e^x}{e^{2x}} = \frac{e^x\cos e^x - \sin e^x}{e^x}$

$F'_y(x,y) = -\frac{e^{2y}\cos e^y - e^y \sin e^y}{e^{2y}} = \frac{\sin e^y - e^y\cos e^y}{e^y}$

30. $F(\alpha,\beta) = \int_\alpha^\beta \sqrt{t^2+1}\,dt =$

33: $w = \ln(x+2y+3z)$

$\Rightarrow w'_x = \frac{1}{x+2y+3z}$ $w'_y = \frac{2}{x+2y+3z}$ $w'_z = \frac{3}{x+2y+3z}$

34 $w = y\tan(x+2z)$

$\Rightarrow w'_x = y \cdot \frac{1}{\cos^2(x+2z)}$ $w'_z = y \cdot \frac{2}{\cos^2(x+2z)}$

$w'_y = \tan(x+2z)$

35: $p = \sqrt{t^4 + u^2\cos v}$

$\Rightarrow P'_t = \frac{1}{2}\frac{4t^3}{\sqrt{t^4+u^2\cos v}}$ $P'_u = \frac{1}{2}\frac{2(\cos v)u}{\sqrt{t^4+u^2\cos v}}$

$P'_v = \frac{1}{2}\frac{-u^2\sin v}{\sqrt{t^4+u^2\cos v}}$

④ 36: $u = x^{y/z}$ $\Rightarrow u'_x = \frac{y}{z}\cdot x^{(\frac{y}{z}-1)}$

$u'_y = \frac{1}{z}\cdot x^{\frac{y}{z}}\ln(x)$

$u'_z = -\frac{y}{z^2}\cdot x^{\frac{y}{z}}\ln(x)$

BT chương

Thứ ngày

10, $f_x(2,1) = 2 \qquad f_y(2,1) = 6$

11, $f(x,y) = 16 - 4x^2 - y^2$

$\Rightarrow f_x(1,2) = -8x = -8$

$f_y(1,2) = -2y = -4$

13, a, $f(x,y) = x^2y^3 \Rightarrow \begin{cases} f_y = 3x^2y^2 \\ f_x = 2y^3x \end{cases}$

b, $f(x,y) = \dfrac{y}{1+x^2y^2} \Rightarrow \begin{cases} f_x = \dfrac{-2y^3 \cdot x}{(1+x^2y^2)^2} \\ f_y = \dfrac{(1+x^2y^2) - y(2x^2y)}{(1+x^2y^2)^2} \end{cases}$

$= \dfrac{1 - x^2y^2}{(1+x^2y^2)^2}$

15: a, $f(x,y) = x^4 + 5xy^3 \Rightarrow f'_x(x,y) = 4x^3 + 5y^3$

$f'_y(x,y) = 15xy^2$

16 $f(x,y) = x^2y - 3y^4 \Rightarrow f'_x(x,y) = 2xy - 0$

$f'_y(x,y) = x^2 - 12y^3$

HONG HA

Thứ ngày

18 $f(x,t) = \sqrt{3x+4t} \Rightarrow f_x(x,t) = \frac{3}{2}\frac{1}{\sqrt{3x+t}}$

$f_t(x,t) = \frac{2}{\sqrt{3x+4t}}$

20 $f(z) = x\sin(xy) \Rightarrow f'_x = \sin(xy) + xy\cos(xy)$

$f'_y = x^2\cos(xy)$

21: $f(x,y) = \frac{x}{y} \Rightarrow f'_x = \frac{1}{y}$

$f'_y = -\frac{x}{y^2}$

22: $f(x,y) = \frac{x}{(x+y)^2} \Rightarrow f'_x = \frac{(x+y)^2 - 2x(x+y)}{(x+y)^2}$

$= \frac{y^2 - x^2}{(x+y)^2}$

$f'_y = \frac{-2x(x+y)}{(x+y)^4}$

25, $g(u,v) = (u^2v - v^3)^5 \Rightarrow g'_u(u,v) = 5(2uv).(u^2v - v^3)^4$

$= 10uv(u^2v - v^3)^4$

$g'_v(u,v) = 5(u^2 - 3v^2)(u^2v - v^3)^4$

26, $u(r,\theta) = \sin(r\cos\theta) \Rightarrow u_r(r,\theta) = \cos\theta\cos(r\cos\theta)$

$u_\theta(r,\theta) = -r\sin\theta\cos(r\cos\theta)$

27, $R(p,q) = \tan^{-1}(pq^2) \Rightarrow R_p(p,q) = \frac{-q^2}{\sin^2(pq^2)}$

$R_q(p,q) = \frac{-2pq}{\sin^2(pq^2)}$

28: $f(x,y) = x^y \Rightarrow f'_x(x,y) = y.x^{(y-1)}$

$f'_y(x,y) = x^y\ln(x)$

U10: $u = \sin(x_1 + 2x_2 + \dots + nx_n)$

$u'_{x_1} = \cos(x_1 + 2x_2 + \dots + nx_n)$

$u'_{x_2} = 2\cos(x_1 + 2x_2 + \dots + nx_n)$

$\Rightarrow u'_{x_n} = n\cos(x_1 + 2x_2 + \dots + nx_n)$

U1: $R(s,t) = te^{s/t} \Rightarrow R_t(s,t) = e^{s/t} + \left(-\frac{s}{t}\right) \cdot e^{s/t}$

$\Rightarrow R_t(0,1) = 1$

U2: $f(x,y) = y\sin^{-1}(xy)$

$\Rightarrow f'_y(x,y) = \dfrac{\sin(xy) - xy\cos(xy)}{[\sin(xy)]^2}$

$\Rightarrow f'_y(1, \frac{1}{2}) = \dfrac{\sin(\frac{1}{2}) - \frac{1}{2}\cos\frac{1}{2}}{\sin^2(\frac{1}{2})}$

U3, $f(x,y,z) = \ln\dfrac{1 - \sqrt{x^2+y^2+z^2}}{1 + \sqrt{x^2+y^2+z^2}} = \ln(1-\sqrt{x^2+y^2+z^2}) - \ln(1+\sqrt{x^2+y^2+z^2})$

$\Leftrightarrow f'_y(x,y,z) = \dfrac{-\frac{2y}{\sqrt{x^2+y^2+z^2}}}{1-\sqrt{x^2+y^2+z^2}} - \dfrac{\frac{2y}{\sqrt{x^2+y^2+z^2}}}{1+\sqrt{x^2+y^2+z^2}}$

$\Rightarrow f'_y(1,2,2) = \dfrac{-\frac{4}{3}}{1-3} - \dfrac{\frac{4}{3}}{1+3}$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

U5. $f(x,y) = xy^2 - x^3y$

$\Rightarrow f'_x(x,y) = \lim\limits_{x \to x_0} \dfrac{f(x,y_0) - f(x_0,y_0)}{x - x_0}$

$f'_y(x,y) = \lim\limits_{y \to y_0} \dfrac{f(x_0,y) - f(x_0,y_0)}{y - y_0}$

Thứ ngày

57: $f(x,y) = x^4 y - 2x^3 y^2$

$\Rightarrow f_x = 4x^3 y - 6x^2 y^2$

$f_y = x^4 - 4x^3 y$

$f_{xx} = 12x^2 y - 12x y^2$

$f_{yy} = -4x^3$

$f_{xy} = 4x^3 - 12x y^2$

$f_{yx} = 4x^3 - 12x^2 y$

56: $T = e^{-2r} \cos\theta$.

$\Rightarrow T_r = -2e^{-2r}\cos\theta$

$T_\theta = -e^{-2r}\sin\theta$

$T_{rr} = 4e^{-2r}\cos\theta$

$T_{\theta\theta} = -e^{-2r}\cos\theta$.

$T_{r\theta} = 2e^{-2r}\sin\theta$.

$T_{\theta r} = 2e^{-2r}\sin\theta$.

58, $w = \sqrt{1+uv^2}$

$\Rightarrow w_u = \dfrac{v^2}{2\sqrt{1+uv^2}}$ $\qquad w_v = \dfrac{2uv}{\sqrt{1+uv^2}}$

$w_{uu} = -\dfrac{v^2 \cdot \frac{v^2}{2\sqrt{1+uv^2}}}{2(1+uv^2)}$. $\qquad w_{vv} = \dfrac{u\sqrt{1+uv^2} - uv\frac{uv}{\sqrt{1+uv^2}}}{(1+uv^2)^2}$

$= -\dfrac{v^4}{4(\sqrt{1+uv^2})^3}$ $\qquad = \dfrac{u}{(\sqrt{1+uv^2})^3}$

Thứ ngày

59: $u = x^4y^3 - y^4$

$u_x = 4x^3y^3 - 0 \Rightarrow u_{xy} = 12x^3y^2$ = correct.

$u_y = 3x^4y^2 - 4y^3 \Rightarrow u_{yx} = 12x^3y^2$

61, $u = \cos(x^2y)$

$u_x = -2xy\sin(x^2y) \Rightarrow u_{xy} = -2x\sin(x^2y) + (-2xy).x^2\cos(x^2y)$

$u_y = -x^2\sin(x^2y)$ $= -2x\sin(x^2y) - 2x^3y\cos(x^2y)$

$\Rightarrow u_{yx} = -2x\sin(x^2y) - 2x^3y\cos(x^2y)$

63: $\sin(2x+5y)$

$f_y = 5\cos(2x+5y)$ $f_{yx} = -10\sin(2x+5y)$

$f_{yxy} = -50\cos(2x+5y)$

65, $f(x,y,z) = e^{xyz^2}$

$\Rightarrow f_x = yz^2e^{xyz^2}$ $f_{xy} = xyz^4e^{xyz^2}$

$f_{xyz} = xy(4z^3e^{xyz^2} + z^4 2xyz e^{xyz^2})$

$= (4xyz^3 + 2z^5x^2y^2)e^{xyz^2}$

67 $W = \sqrt{u+v^2}$

$\Rightarrow \frac{\partial W}{\partial v} = \frac{v}{\sqrt{u+v^2}}$ $\frac{\partial W^2}{\partial u \partial v} = -\frac{v}{2(\sqrt{u+v^2})^3}$

$\Rightarrow \frac{\partial W^3}{\partial^2 u . \partial v} = \left[-\frac{v}{2(\sqrt{u+v^2})^3}\right]'_u$

68: $V = \ln(r+s^2+t^3)$

$\frac{\partial^3 V}{\partial r \partial s \partial t} = V'''_{tsr} \Rightarrow \frac{3t^2}{r+s^2+t^3} \rightarrow -\frac{3t^2.2s}{(r+s^2+t^3)^2}$

$= \frac{12t^2s(r+s^2+t^3)}{(r+s^2+t^3)^4} = \frac{12t^2s}{(r+s^2+t^3)^3}$

HONGHA

70, $u = x^a y^b z^c$

$\Rightarrow \dfrac{\partial^6 u}{\partial x \, \partial y^2 \, \partial z^3} = u_{zzzyyx}$

$u \Rightarrow u_z = c x^a y^b . z^{(c-1)} \rightarrow u_{zzz} = c(c-1)(c-2). x^a y^b . z^{(c-3)}$

$\rightarrow u_{zzzyyx} = a c b(b-1).c(c-1)(c-2) . x^{(a-1)} y^{(b-2)} z^{(c-3)}$

Chain Rule

1, $z = xy^3 - x^2 y$; $x = t^2+1$, $y = t^2-1$

$\dfrac{\partial z}{\partial t} = \dfrac{\partial x}{\partial t} . \dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} . \dfrac{\partial y}{\partial t}$

$= 2t . (y^3 - 2xy) + (3xy^2 - x^2). 2t.$

$= 2t . [(t^2-1)^3 - 2(t^2+1)(t^2-1)] + (3.(t^2+1)(t^2-1)^2 - (t^2+1)^2)$

2, $z = \dfrac{x-y}{x+2y}$; $x = e^{\pi t}$ $y = e^{-\pi t}$

$\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x} . \dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y} . \dfrac{\partial y}{\partial t}$

$= \dfrac{(x+2y)-(x-y)}{(x+2y)^2} . \pi e^{\pi t} + \dfrac{-(x+2y)+2(x-y)}{(x+2y)^2} . -\pi e^{-\pi t}$

$= \pi e^{\pi t} \dfrac{3y}{(x+2y)^2} + \pi e^{-\pi t} . \dfrac{x - 4y}{(x+2y)^2}$

$= \pi e^{\pi t} . \dfrac{3e^{-\pi t}}{(e^{\pi t} + 2e^{-\pi t})^2} - \pi e^{-\pi t} . \dfrac{e^{\pi t} - 4.e^{-\pi t}}{(e^{\pi t} + 2e^{-\pi t})^2}$

$= \dfrac{3 - \pi + 4\pi e^{-2\pi t}}{(e^{\pi t} + 2e^{-\pi t})^2}$

HONG HA

Thứ ngày

4, $z = \sqrt{1+xy}$ $x = \tan t$ $y = \arctan t$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= \frac{y}{2\sqrt{1+xy}}\cdot\frac{1}{\cos^2 t} + \frac{x}{2\sqrt{1+xy}}\cdot\frac{1}{1+t^2}$$

$$= \frac{\arctan t}{2\sqrt{1+\tan t.\arctan t}}\cdot\frac{1}{\cos^2 t} + \frac{\tan t}{2\sqrt{1+\tan t.\arctan t}}\cdot\frac{1}{1+t^2}$$

5, $w = x e^{y/z}$; $x = t^2$; $y = 1-t$; $z = 1+2t$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial t}$$

$$= e^{y/z}.2t + \frac{x}{z}e^{\frac{y}{z}}.(-1) + \left(-\frac{xy}{z^2}\right).e^{\frac{y}{z}}.2$$

$$= \left(2t - \frac{x}{z}e^{\frac{y}{z}} - \frac{2xy}{z^2}\right).e^{\frac{y}{z}}$$

7, $z = (x-y)^5$; $x = s^2 t$; $y = st^2$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= 5(x-y)^4.s^2 - 5(x-y)^4.\cancel{2}st$$

$$\frac{\partial z}{\partial s} = 5(x-y)^4.2st - 5(x-y)^4.t^2$$

9, $z = \ln(3x+2y)$; $x = s\sin t$; $y = t\cos s$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= \left(\frac{3}{3x+2y}\right).(s\cos t) + \left(\frac{2}{3x+2y}\right).\cos s$$

$$\frac{\partial z}{\partial s} = \left(\frac{3}{3x+2y}\right).\sin t + \left(\frac{2}{3x+2y}\right).(-t\sin s)$$

Thứ ngày

11, $z = e^r \cos\theta$; $r = st$; $\theta = \sqrt{s^2+t^2}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r}\cdot\frac{\partial r}{\partial t} + \frac{\partial z}{\partial \theta}\cdot\frac{\partial \theta}{\partial t}$$

$$= e^r\cos\theta \cdot s - e^r\sin\theta \cdot \frac{t}{\sqrt{s^2+t^2}}$$

$$\frac{\partial z}{\partial s} = e^r\cos\theta\, t - e^r\sin\theta \cdot \frac{s}{\sqrt{s^2+t^2}}$$

12, $z = \tan\left(\frac{u}{v}\right)$; $u = 2s+3t$; $v = 3s-2t$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial t} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial t}$$

$$= \frac{3}{v}\cdot\frac{1}{\cos^2\left(\frac{u}{v}\right)} + \frac{2u}{v^2}\cdot\frac{1}{\cos^2\left(\frac{u}{v}\right)}$$

$$\frac{dz}{ds} = \frac{2}{v\cdot\cos^2\left(\frac{u}{v}\right)} - \frac{3u}{v^2\cdot\cos^2\left(\frac{u}{v}\right)}$$

27,

a, $y\cos x = x^2+y^2 \Rightarrow x^2+y^2 - y\cos x = 0$

có: $f(x,y) = x^2+y^2 - y\cos x = 0$

$$\Rightarrow \frac{dy}{dx} = f'_x(x,y) = -\frac{f'_x(x,y)}{f'_y(x,y)} = -\frac{2x+y\sin x}{2y-\cos x}$$

28, $f(x,y) = \cos(xy) - 1 - \sin y = 0$

$$\Rightarrow \frac{dy}{dx} = f'_x(y) = -\frac{f'_x(x,y)}{f'_y(x,y)} = -\frac{-y\sin(xy)}{-x\sin(xy) - \cos y}$$

29. $f(x,y) = xy^2+x - \cot(x^2y)$

$$\frac{dy}{dx} = f'(y) = -\frac{f'_x(x,y)}{f'_y(x,y)} = -\frac{y^2+1+\frac{2xy}{\sin^2(x^2y)}}{2xy+\frac{x^2}{\sin^2(x^2y)}}$$

HONG HA

Thứ ngày

30. $f(x,y) = x + xy - e^y \sin x$

$\frac{dy}{dx} = y' = -\frac{f'_x(x,y)}{f'_y(x,y)} = -\frac{y+1-e^y\cos x}{x - e^y \sin x}$

31, $f(x,y,z) = x^2 + 2y^2 + 3z^2 - 1 = 0$

$\Rightarrow \frac{\partial z}{\partial x} = z'_x = \frac{f_x(x,y,z)}{f_z(x,y,z)} = \frac{2x}{6z} = -\frac{x}{3z}$.

$\frac{\partial z}{\partial y} = z'_y = \frac{f'_y(x,y,z)}{f_z(x,y,z)} = -\frac{2y}{3z}$.

32. $f(x,y,z) = x^2 - y^2 + z^2 - 2z - 4 = 0$

$\Rightarrow \frac{\partial z}{\partial x} = \frac{f'_x(x,y,z)}{f'_z(x,y,z)} = -\frac{2x}{2z-2}$.

$\frac{\partial z}{\partial y} = -\frac{-2y}{2z-2}$.

33. $f(x,y,z) = e^z - xyz$.

$\frac{\partial z}{\partial x} = -\frac{-yz}{e^z - xy}$. $\quad \frac{\partial z}{\partial y} = -\frac{-xz}{e^z - xy}$.

34. $f(x,y,z) = yz + x\ln y - z^2 = 0$

$\frac{\partial z}{\partial x} = -\frac{\ln y}{y - 2z}$. $\quad \frac{\partial z}{\partial y} = -\frac{z + \frac{x}{y}}{y - 2z}$.

52, a: $z = f(x)g(y)$

$\Rightarrow \frac{\partial z}{\partial x} = f'(x)g(y) \quad \frac{\partial z}{\partial y} = f(x)g'(y)$

b, $z = f(xy) \Rightarrow \frac{\partial z}{\partial x} = y.f'(xy)$

$\frac{\partial z}{\partial y} = xf'(xy)$

HONGHA