34: $f(x,y,z) = x\ln y + yz - z^2 = 0$

$\Rightarrow \dfrac{\partial z}{\partial x} = -\dfrac{f'_x}{f'_z} = -\dfrac{\ln y}{y-2z}$

$\dfrac{\partial z}{\partial y} = -\dfrac{f'_y}{f'_z} = -\dfrac{\frac{x}{y}+z}{y-2z}$

Maximum / Minimum Values

1, a: $f_{xx}(1,1) = 4$; $f_{xy}(1,1) = 1$; $f_{yy}(1,1) = 2$.

$\Rightarrow \begin{cases} a_{11} \cdot a_{22} - a_{12}^2 = 7 > 0 \\ a_{11} = 4 > 0 \end{cases} \Rightarrow f(1,1)$ is Minimum

b. $a_{11} = 4 \quad a_{12} = a_{21} = 3 \quad a_{22} = 2$

$\Rightarrow \begin{cases} a_{11} \cdot a_{22} - a_{12}^2 = -1 < 0 \\ a_{11} = 4 > 0 \end{cases} \Rightarrow f(1,1)$ is saddles point

2, a: $a_{11} = -1 \quad a_{12} = a_{21} = 6$; $a_{22} = 1$

Có: $a_{11} \cdot a_{22} - a_{12}^2 = -37 < 0$

$\Rightarrow g(0,2)$ is saddles point

b, $a_{11} = -1 \quad a_{12} = a_{21} = 2 \quad a_{22} = -8$

Có: $\begin{cases} a_{11} \cdot a_{22} - a_{12}^2 = 4 > 0 \\ a_{11} = -1 < 0 \end{cases} \Rightarrow g(0,2)$ is maximum point

3, $f(x,y) = 4 + x^3 + y^3 - 3xy$

$f'_x = 3x^2 - 3y = 0$

$f'_y = 3y^2 - 3x = 0$

$\Rightarrow \begin{cases} x^2 = y \\ y^2 = x \end{cases} \Rightarrow \left[\begin{array}{l} x = 0, y = 0 \\ x = 1, y = 1 \end{array} \right.$

33: $f(x,y) = x^2 + y^2 + 2x^2y + 4$ $\quad D = \{(x,y) \mid |x| \le 1, |y| \le 1\}$

$\Rightarrow f'_x = 2x + 2xy = 0 \Rightarrow \begin{cases} x = 0, y = 0 \\ y = -1, x = \pm\sqrt{2}. \end{cases} \Rightarrow M(0,0)$

$f'_y = 2y + x^2 = 0$

$a_{11} = 2 + 2y \quad a_{12} = a_{21} = 2x \quad a_{22} = 2$

$\Rightarrow D = 2.(2+2y) - 4x^2 = 4 + 4y - 4x^2 \Rightarrow \begin{cases} 4 > 0 \Rightarrow \text{local} \\ a_{11} = 2 > 0 \text{ min.} \end{cases}$

34: $f(x,y) = x^2 + xy + y^2 - 6y.$ $\quad D = \{(x,y) \mid -3 \le x \le 3, 0 \le y \le 5\}$

$f'_x = 2x + y = 0 \Rightarrow \begin{cases} x = -2 \\ y = 4 \end{cases} \Rightarrow M(-2, 4)$

$f'_y = x + 2y - 6 = 0$

$a_{11} = 2 \quad a_{12} = a_{21} = 1 \quad a_{22} = 2$

$\Rightarrow \begin{cases} D = 3 > 0 \\ a_{11} = 2 > 0 \end{cases} \Rightarrow$ M is maximum point

36, $f(x,y) = xy^2$ $\quad D = \{(x,y) \mid x \ge 0, y \ge 0, x^2 + y^2 \le 3\}$

$f'_x = y^2 = 0 \Rightarrow$ $\quad y = 0, x \in R \Rightarrow M(R, 0)$

$f'_y = 2xy = 0$

$a_{11} = 0 \quad a_{12} = a_{21} = 2y \quad a_{22} = 2x.$

$\Rightarrow D = -4y^2 = 0 \Rightarrow$ chưa thể xác định.

Xét $f(x_0 + \delta, y_0 + \delta)$ $(|\delta| < 1) = f(R, \delta)$

$x\delta^2$ mà $x \ge 0 \Rightarrow x\delta^2 \ge 0$

$\Rightarrow$ minimum point

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$f''_{xx} = 6x \quad f''_{xy} = f''_{yx} = -3 \quad f''_{yy} = 6y^2$

$\Rightarrow D = 6x.6y - (-3)^2 = 36xy - 9$

xét $M(0,0) \Rightarrow D = -9 < 0 \Rightarrow M(0,0)$ is saddles point

$M(1,1) \Rightarrow \begin{cases} D = 27 > 0 \\ f''_{xx} = 6 > 0 \end{cases} \Rightarrow M(1,1)$ is minimum point.

5, $f(x,y) = x^2 + xy + y^2 + y$

$\Rightarrow f'_x = 2x + y = 0 \Rightarrow \begin{cases} x = \frac{1}{3} \\ y = -\frac{2}{3} \end{cases}$

$f'_y = x + 2y + 1 = 0$

có: $a_{11} = 2 \quad a_{12} = a_{21} = 1 \quad a_{22} = 2$

$\Rightarrow \begin{cases} D = 3 > 0 \\ a_{11} = 2 > 0 \end{cases} \Rightarrow M\left(\frac{1}{3}; \frac{-2}{3}\right)$ is minimum point.

8, $f(x,y) = ye^x - y$

$\Rightarrow f'_x = ye^x = 0 \Rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases} \Rightarrow M(0,0)$ is critical point

$f'_y = e^x - 1. = 0$

$a_{11} = ye^x \quad a_{12} = a_{21} = e^x \quad a_{22} = 0$

$\Rightarrow D = -e^{2x} = -1 \Rightarrow M(0,0)$ is saddles point

9, $f(x,y) = x^2 + y^4 + 2xy$

$\Rightarrow f'_x = 2x + 2y = 0 \Leftrightarrow \begin{cases} y = 0, x = 0 \\ y = \frac{\sqrt{2}}{2}, x = \frac{\sqrt{2}}{2} \\ y = -\frac{\sqrt{2}}{2}, x = \frac{\sqrt{2}}{2} \end{cases}$

$f'_y = 4y^3 + 2x = 0$

$a_{11} = 2 \quad a_{12} = a_{21} = 2 \quad a_{22} = 12y^2$

$\Rightarrow D = 24y^2 - 4.$

xét $M_1(0,0) \Rightarrow D = -4 \Rightarrow$ is saddles point

$\begin{cases} M_2\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \\ M_3\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \end{cases} \Rightarrow \begin{cases} D = 8 > 0 \\ a_{11} = 2 > 0 \end{cases} \Rightarrow M_2, M_3$ is minimum

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20: $f(x,y) = \sin x \sin y \Rightarrow f'_x = \sin y \cos x = 0$

$f'_y = \sin x \cos y = 0$

$\Rightarrow \begin{cases} \left[\begin{array}{l} \sin y = 0 \Rightarrow y = 0 \\ \cos x = 0 \Rightarrow x = \pm\frac{\pi}{2} \end{array}\right. \\ \left[\begin{array}{l} \sin x = 0 \Rightarrow x = 0 \\ \cos y = 0 \Rightarrow y = \pm\frac{\pi}{2} \end{array}\right. \end{cases} \Rightarrow M_1(0,0)\ M_2(\frac{\pi}{2}, \frac{\pi}{2})\ M_3(\frac{\pi}{2}, -\frac{\pi}{2})$

$M_4(-\frac{\pi}{2}, \frac{\pi}{2})\ M_5(-\frac{\pi}{2}, \frac{\pi}{2})$

$a_{11} = -\sin x \sin y \quad a_{12} = a_{21} = \cos x \cos y \quad a_{22} = -\sin x \sin y$

$\Rightarrow D = \sin^2 x \sin^2 y - \cos^2 x \cos^2 y$

$\Rightarrow$ tự thay tiếp vào rồi xét

21: $f(x,y) = x^2 + 4y^2 - 4xy + 2$.

$f'_x = 2x - 4y = 0 \Leftrightarrow$ ptr vô số nghiệm $\Rightarrow$ có vô số

$f'_y = 8y - 4x = 0$ critical point

$a_{11} = 2 \quad a_{12} = a_{21} = -4 \quad a_{22} = 8$

$\Rightarrow D = a_{11}a_{22} - (a_{12})^2 = 0$

xét $0 < \delta < 1$

$f(x,y) > f(x_0, y_0)$

Có: $f(x_0, y_0) = x_0^2 + 4y_0^2 - 4x_0y_0 + 2$

$f(x,y) = f(x_0+\delta, y_0+\delta) = (x_0+\delta)^2 + 4(y_0+\delta)^2 - 4(x_0+\delta)(y_0+\delta) + 2$

$= x_0^2 + 4y_0^2 - 4x_0y_0 + 2 + 2x_0\delta + 8y_0\delta + \delta^2 - 4(x_0y_0\delta) - 4\delta^2$

$= f(x_0, y_0) + 2x_0\delta + 8y_0\delta + \delta^2 - 4x_0\delta - 4y_0\delta$

$= f(x_0, y_0) \underbrace{+ 4y_0\delta - 2x_0\delta + \delta^2}_{K}$

$\left[\begin{array}{l} K \text{ can } > 0 \text{ because } \delta > 0 \text{ and } y_0, x_0 \in \mathbb{R} \\ K \text{ can } < 0 \text{ because } \delta > 0 \text{ and } y_0, x_0 \in \mathbb{R} \end{array}\right.$

$\Rightarrow$ f has both of maximum and minimum at each critical point

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14, $f(x,y) = y\cos x \Rightarrow f'_x = -y\sin x = 0 \Rightarrow \begin{cases} y = 0 \\ x = \frac{\pi}{2} + k\pi \end{cases}$, $f'_y = \cos x = 0$

$a_{11} = -y\cos x \quad a_{12} = a_{21} = -\sin x \quad a_{22} = -\sin x$

$\Rightarrow D = y\sin x\cos x - \sin^2 x$

$\Rightarrow M = -\sin^2(\frac{\pi}{2} + k\pi) = -1 < 0$

$\Rightarrow$ M is saddles point

15, $f(x,y) = e^x\cos y \Rightarrow f'_x = e^x\cos y$; $f'_y = -e^x\sin y \Rightarrow$

$\Rightarrow \begin{cases} e^x = \cos y \\ \left[\begin{array}{l} \sin y = 0 \\ \cos y = 0 \end{array}\right. \end{cases} \Rightarrow \begin{cases} x \in R \\ y = k\frac{\pi}{2} \end{cases}$

$a_{11} = e^x\cos y \quad a_{12} = a_{21} = -e^x\sin y \quad a_{22} = -e^x\cos y$

$\Rightarrow D = -e^{2x}\cos^2 y - e^{2x}\sin^2 y = -e^{2x}$

$\Rightarrow$ with $x \in R$, M is saddles point

~~16, $f(x,y) = xy\,e^{-\frac{(x^2+y^2)}{2}}$~~

~~$\Rightarrow f'_x = y\left(e^{-\frac{(x^2+y^2)}{2}} - x^2 e^{-\frac{(x^2+y^2)}{2}}\right)$~~

~~$f'_y =$~~

17, $f(x,y) = xy + e^{-xy}$

$\Rightarrow f'_x = y - ye^{-xy} = 0 \Rightarrow \begin{cases} \left[\begin{array}{l} y = 0 \\ e^{-xy} = 1 \end{array}\right. \\ \left[\begin{array}{l} x = 0 \\ e^{-xy} = 1 \end{array}\right. \end{cases}$

$f'_y = x - xe^{-xy} = 0$

$\Rightarrow M_1 (0,0) \quad M_2 (R,0) \quad M_3 (0,R)$

$a_{11} = y^2 e^{-xy} \quad a_{12} = a_{21} = 1 - (e^{-xy} - y^2 e^{-xy}) \quad a_{22} = x^2 e^{-xy}$

$\Rightarrow D = a_{11}a_{22} - (a_{12})^2 = x^2y^2 . e^{-2xy} - (1 - e^{-xy} + y^2 e^{-xy})^2$

$y = 0 \Rightarrow D = 0 \qquad \Rightarrow M_1, M_2$ ko xác định

$x = 0$ với $y > 0 \Rightarrow D < 0 \qquad M_3$ is saddles point

37: $f(x,y) = 2x^3 + y^4$, $D = \{(x,y) \mid x^2+y^2 \le 1\}$

$f'_x = 6x^2 = 0 \Rightarrow x=0, y=0 \Rightarrow M(0,0)$

$f'_y = 4y^3 = 0$

$D = a_{11} \cdot a_{22} - a_{12}^2 = 12^2 x y^2 = 0$

xét $f(x+\delta, y+\delta)$ với $0<\delta<1$.

~~$2(x+\delta)^3 + (y+\delta)^4$~~ xét $f(\delta, \delta) = 2\delta^3 + \delta^4 > 0$

$f(-\delta, \delta) = -2\delta^3 + \delta^4 < 0$ do $\delta \in (0,1)$

$\Rightarrow$ Ko phải cực trị.

Lagrange Multipliers

3, $f(x,y) = x^2 - y^2$ $\quad x^2+y^2 = 1$

$\Rightarrow L = x^2 - y^2 + \lambda(1 - x^2 - y^2)$

$\Rightarrow L'_x = 2x - 2\lambda x = 0$

$L'_y = -2y - 2\lambda y = 0$

$L'_\lambda = 1 - x^2 - y^2 = 0$

$\Rightarrow \begin{cases} \left[\begin{array}{l} x=0 \\ \lambda = 1 \end{array}\right. \\ \left[\begin{array}{l} y=0 \\ \lambda = -1 \end{array}\right. \\ 1 - x^2 - y^2 = 0 \end{cases} \Rightarrow$ Vô lý.

$\Rightarrow$ Ko có cực trị.

4, $f(x,y) = 3x + y$, $\quad x^2+y^2 = 10$

$\Rightarrow L = 3x + y + \lambda(10 - x^2 - y^2)$

$L'_x = 3 - 2\lambda x = 0$

$L'_y = 1 - 2\lambda y = 0$

$L'_\lambda = 10 - x^2 - y^2 = 0$

$\Rightarrow \begin{cases} \lambda = \dfrac{3}{2x} = \dfrac{1}{2y} \\ 10 - x^2 - y^2 = 0 \\ x = 3y \end{cases} \Rightarrow \left[\begin{array}{l} y=1, x=3, \lambda= \\ y=-1, x=-3, \lambda \end{array}\right.$

$$|D| = \begin{vmatrix} 0 & g'_x & g'_y \\ g'_x & L''_{xx} & L''_{xy} \\ g'_y & L''_{yx} & L''_{yy} \end{vmatrix} = \begin{vmatrix} 0 & 2x & 2y \\ 2x & -2\lambda & 0 \\ 2y & 0 & -2\lambda \end{vmatrix} = -(-8y^2\lambda - 8x^2\lambda) = 8y^2\lambda + 8x^2\lambda.$$

Với $M_1 \Rightarrow D = 40 > 0 \Rightarrow$ cực đại

$M_2 \Rightarrow D = -40 < 0 \Rightarrow$ cực tiểu

9, $f(x,y,z) = xy^2z \qquad x^2+y^2+z^2=4$

$L = xy^2z + \lambda(4-x^2-y^2-z^2)$

$L'_x = y^2z - 2\lambda x = 0$

$L'_y = 2xyz - 2\lambda y = 0$

$L'_z = x y^2 - 2\lambda z = 0$

$L'_\lambda = 4-x^2-y^2-z^2 = 0$

$\Rightarrow \begin{cases} \lambda = \frac{y^2z}{2x} \\ \lambda = xz \\ \lambda = \frac{xy^2}{2z} \\ 4-x^2-y^2-z^2=0 \end{cases}$

xét $\frac{y^2z}{2x} = xz \Leftrightarrow y^2z = 2x^2z \Rightarrow \left[\begin{array}{l} z=0 \\ x = \pm y \end{array}\right.$

xét $xz = \frac{xy^2}{2z} \Rightarrow 2xz^2 = xy^2 \Leftrightarrow \left[\begin{array}{l} x=0 \\ y = \pm\sqrt{2}z \end{array}\right.$

xét $\frac{y^2z}{2x} = \frac{xy^2}{2z} \Rightarrow \left[\begin{array}{l} y=0 \\ x = \pm z \end{array}\right.$ ~~(0,0,0)~~

và $4-x^2-y^2-z^2=0$

11, $f(x,y,z) = x^2+y^2+z^2 \quad , \quad x^4+y^4+z^4=1$

$\Rightarrow L = x^2+y^2+z^2 + \lambda(1-x^4-y^4-z^4)$

$\Rightarrow L'_x = 2x - 4\lambda x^3$

$L'_y = 2y - 4\lambda y^3$

$L'_z = 2z - 4\lambda z^3$

$L'_\lambda = 1-x^4-y^4-z^4$

$\Rightarrow \begin{cases} \lambda = \frac{1}{2x^2} = \frac{1}{2y^2} = \frac{1}{2z^2} \\ 1-x^4-y^4-z^4=0 \end{cases}$

$\Leftrightarrow x^2=y^2=z^2 \Rightarrow 1-3x^4=0 \Leftrightarrow x^4 = \frac{1}{3}$

$\Rightarrow \left[\begin{array}{l} x = \sqrt[4]{\frac{1}{3}} \Rightarrow \left[\begin{array}{l} y=\sqrt[4]{\frac{1}{3}} \Rightarrow \left[\begin{array}{l} z=\sqrt[4]{\frac{1}{3}} \\ z=-\sqrt[4]{\frac{1}{3}} \end{array}\right. \\ y=-\sqrt[4]{\frac{1}{3}} \Rightarrow \left[\begin{array}{l} z=\sqrt[4]{\frac{1}{3}} \\ z=-\sqrt[4]{\frac{1}{3}} \end{array}\right. \end{array}\right. \\ x = -\sqrt[4]{\frac{1}{3}} \Rightarrow \left[\begin{array}{l} y=\sqrt[4]{\frac{1}{3}} \Rightarrow \left[\begin{array}{l} z=\sqrt[4]{\frac{1}{3}} \\ z=-\sqrt[4]{\frac{1}{3}} \end{array}\right. \\ y=\sqrt[4]{\frac{1}{3}} \Rightarrow \left[\begin{array}{l} z=\sqrt[4]{\frac{1}{3}} \\ z=-\sqrt[4]{\frac{1}{3}} \end{array}\right. \end{array}\right. \end{array}\right.$

$$|\bar{H}| = \begin{vmatrix} 0 & 4x^3 & 4y^3 & 4z^3 \\ 4x^3 & (2-12xz^2) & 0 & 0 \\ 4y^3 & 0 & (2-12xy^2) & 0 \\ 4z^3 & 0 & 0 & (2-12xz^2) \end{vmatrix}$$

xét $|H_2| = \begin{vmatrix} 0 & 4x^3 & 4y^3 \\ 4x^3 & (2-12xz^2) & 0 \\ 4y^3 & 0 & (2-12xy^2) \end{vmatrix} = -\left[4y^3 \cdot 4y^3 (2-12xz^2) + 4x^3 \cdot 4x^3 \cdot (2-12xy^2)\right]$

xét $|H_3| = 4x^3 \cdot (-1)^{(2+1)} \begin{vmatrix} 4x^3 & 0 & 0 \\ 4y^3 & (2-12xy^2) & 0 \\ 4z^3 & 0 & (2-12xz^2) \end{vmatrix} + (2-12xz^2)(-1)^{(2+2)} \begin{vmatrix} 4x^3 & 0 & 0 \\ 4y^3 & (2-12xy^2) & 0 \\ 4z^3 & 0 & (2-12xz^2) \end{vmatrix}$

$= -4x^3 \cdot 4x^3 \cdot (2-12xy^2) \cdot (2-12xz^2) + (2-12xz^2) \cdot 4x^3 \cdot (2-12xy^2) \cdot (2-12xz^2)$

Khoảng tùy điều với H_1, H_2

nếu $H_1 > 0$, $H_2 > 0 \Rightarrow$ cực đại

$H_1 < 0$, $H_2 > 0 \Rightarrow$ cực tiểu

23: $f(x,y) = e^{-xy}$, $x^2 + 4y^2 \le 1$

$f'_x = -ye^{-xy} = 0$
$f'_y = -xe^{-xy} = 0$
$\Rightarrow \begin{cases} \left[\begin{array}{l} e^{-xy} = 0 \\ y = 0 \end{array}\right. \\ \left[\begin{array}{l} e^{-xy} = 0 \\ x = 0 \end{array}\right. \end{cases} \Rightarrow x = 0, y = 0 \Rightarrow M(0,0)$ (TM)

$a_{11} = y^2 e^{-xy}$ $\quad a_{12} = a_{21} = xye^{-xy}$ $\quad a_{22} = x^2 e^{-xy}$

$\Rightarrow D = a_{11}a_{22} - a_{12}^2 = x^2y^2e^{-2xy} - x^2y^2e^{-2xy}$
$= 0 \Rightarrow$ ko xác định được

xét $0 < \delta < 1$

$\Rightarrow f(\delta,\delta) = e^{-\delta^2} < 1$
$f(-\delta,\delta) = e^{\delta^2} > 1$ $\Rightarrow$ ko phải cực trị.

R M a M

1, $f(x,y) = (y-2)x^2y^2$

$\Rightarrow f'_x = 2y^2(y-2)x = 0 \Rightarrow \left[\begin{array}{l} y=0 \\ x=0 \\ y=2 \end{array}\right.$

$f'_y = 3x^2y^2 - 4x^2y = 0 \Rightarrow \left[\begin{array}{l} x=0 \\ y=0 \\ y=\frac{4}{3} \end{array}\right.$

$\Rightarrow M_1(0,0);\ M_2(0,\frac{4}{3})\ M_3(0,2)$

4, $f(x,y) = 3y^3 - x^2y^2 + 8y^2 + 4x^2 - 20y$

$\Rightarrow f'_x = -2xy^2 + 8x = 0$

$f'_y = 9y^2 - 2x^2y + 16y - 20 = 0$ $\Rightarrow \left\{ \left[\begin{array}{l} y=\pm 2 \\ x=0 \end{array}\right. \right.$

Với $x=0 \Rightarrow \left[\begin{array}{l} y = \frac{-8+2\sqrt{61}}{9} \\ y = \frac{-8-2\sqrt{61}}{9} \end{array}\right.$ $\Rightarrow M_1\left(0, \frac{-8+2\sqrt{61}}{9}\right)$

$y=2 \Rightarrow x = \emptyset.$ $M_2\left(0, \frac{-8-2\sqrt{61}}{9}\right)$

$y=-2 \Rightarrow x = \emptyset.$

1, $f(x,y) = 81x^2 + y^2$ $4x^2+y^2=9$

$L = 81x^2 + y^2 + \lambda(9-4x^2-y^2)$

$\Rightarrow L'_x = 162x - 8\lambda x \Rightarrow \left\{ \begin{array}{l} \left[\begin{array}{l} x=0 \\ \lambda=\frac{81}{4} \end{array}\right. \\ \left[\begin{array}{l} y=0 \\ \lambda=1 \end{array}\right. \\ 9-4x^2-y^2=0 \end{array}\right.$ $\Rightarrow$ VS nghiệm

$L'_y = 2y - 2\lambda y$

$L'_\lambda = 9-4x^2-y^2$

2, $f(x,y) = 8x^2 - 2y$ $x^2+y^2=1,$

$L = 8x^2 - 2y + \lambda(1-x^2-y^2)$

$\Rightarrow L'_x = 16x - 2\lambda x \Rightarrow \left\{ \begin{array}{l} \left[\begin{array}{l} x=0 \\ \lambda=8 \end{array}\right. \\ \lambda = -\frac{1}{y} \\ 1-x \end{array}\right.$

$L'_y = -2 - 2\lambda y.$

$L'_\lambda = 1-x^2-y^2 = 0.$

$x=0,\ y=\pm1 \Rightarrow \lambda = \mp 1.$

6, $xe^y = f(x,y)$ $\quad x^2+y^2=2$.

$\Rightarrow L = xe^y + \lambda(2-x^2-y^2)$

$$\Rightarrow \begin{cases} L'_x = e^y - 2\lambda x = 0 \\ L'_y = xe^y - 2\lambda y = 0 \\ L'_\lambda = 2-x^2-y^2 = 0 \end{cases} \Rightarrow \begin{cases} \lambda = \dfrac{e^y}{2x} \\ \lambda = \dfrac{xe^y}{2y} \\ 2-x^2-y^2=0 \end{cases} \Rightarrow y = x^2$$

$$\Rightarrow \left[\begin{array}{l} y = 1 \Rightarrow x = \pm 1 \\ y = -2 \text{ VN.} \end{array}\right. \Rightarrow \begin{array}{l} M_1\left(1, 1, \frac{e}{2}\right) \\ M_2\left(-1, 1, -\frac{e}{2}\right) \end{array}$$

$f(1,1) = e \qquad f(-1,1) = -e$

$\Rightarrow$ cực đại tại $(1,1)$ cực tiểu tại $(-1,1)$

7, $f(x,y,z) = e^{xyz} \qquad 2x^2+y^2+z^2 = 24$.

$\Rightarrow L = e^{xyz} + \lambda(24-2x^2-y^2-z^2)$

$$\Rightarrow \begin{cases} L'_x = yze^{xyz} - 4\lambda x = 0 \\ L'_y = xze^{xyz} - 2\lambda y = 0 \\ L'_z = xye^{xyz} - 2\lambda z = 0 \\ L'_\lambda = 24-2x^2-y^2-z^2 = 0 \end{cases} \Rightarrow \begin{cases} \lambda = \dfrac{yze^{xyz}}{4x} \\ \lambda = \dfrac{xze^{xyz}}{2y} \\ \lambda = \dfrac{xye^{xyz}}{2z} \\ 24-2x^2-y^2-z^2=0 \end{cases}$$

$$\Rightarrow \begin{cases} y = \pm\sqrt{2}x \\ z = \pm y \\ z = \pm\sqrt{2}x \end{cases} \Rightarrow \left[\begin{array}{l} x = 2 \Rightarrow \left[\begin{array}{l} y = 2\sqrt{2} \Rightarrow \left[\begin{array}{l} z = 2\sqrt{2} \\ z = -2\sqrt{2} \end{array}\right. \\ y = -2\sqrt{2} \Rightarrow \left[\begin{array}{l} z = -2\sqrt{2} \\ z = 2\sqrt{2} \end{array}\right. \end{array}\right. \\ x = -2 \Rightarrow \left[\begin{array}{l} y = -2\sqrt{2} \Rightarrow \left[\begin{array}{l} z = -2\sqrt{2} \\ z = 2\sqrt{2} \end{array}\right. \\ y = 2\sqrt{2} \Rightarrow \left[\begin{array}{l} z = 2\sqrt{2} \\ z = -2\sqrt{2} \end{array}\right. \end{array}\right. \end{array}\right.$$

$$|\bar{H}| = \begin{vmatrix} 0 & g'_x & g'_y & g'_z \\ g'_x & L''_{xx} & L''_{xy} & L''_{xz} \\ g'_y & L''_{yx} & L''_{yy} & L''_{yz} \\ g'_z & L''_{zx} & L''_{zy} & L''_{zz} \end{vmatrix} = \begin{vmatrix} 0 & 4x & 2y & 2z \\ 4x & (y^2z^2e^{xyz} - 4\lambda) & & \\ 2y & L''_{yx} & L''_{yy} & L''_{yz} \\ 2z & L''_{zx} & L''_{zy} & L''_{zz} \end{vmatrix} \Rightarrow \text{đạo giản tiếp}$$

$|\bar{H}_1| > 0$, $|\bar{H}_2| < 0 \Rightarrow$ cực đại

$|\bar{H}_1| < 0$, $|\bar{H}_2| < 0 \Rightarrow$ cực tiểu

$$|H| = \begin{vmatrix} 0 & 8x & 2y \\ 8x & 162-8\lambda & 0 \\ 2y & 0 & 2-2\lambda \end{vmatrix} \Rightarrow |H| = -(4y^2)(162-8\lambda) - 64x^2(2-2\lambda)$$

Nos' $|H| > 0 \Rightarrow$ cực đại $\Rightarrow M_1, M_2$ (là cực đại)

$|H| < 0 \Rightarrow$ cực tiểu

59.

$$F(x,y) = 0 \Rightarrow \frac{dy}{dx} = -\frac{F_x}{F_y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(-\frac{F_x}{F_y}\right) = -\frac{(F_x)'F_y - F_x(F_y)'}{F_y^2}$$

$$(F_x)' = F_{xx} + F_{xy}y' = F_{xx} + F_{xy}\left(-\frac{F_x}{F_y}\right)$$

$$(F_y)' = F_{yx} + F_{yy}y' = F_{yx} + F_{yy}\left(-\frac{F_x}{F_y}\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{F_{xx}F_y^2 - 2F_{xy}F_xF_y - F_{yy}F_x^2}{F_x^3}$$

Vì $(F_x)' = \frac{\partial(F_x)'}{\partial x}\cdot\frac{\partial x}{\partial x} + \frac{\partial(F_x)'}{\partial y}\cdot\frac{\partial y}{\partial x}$

1': gọi $M(x,y,z) \in x+y+z=1$

$$d((2,0,-3),M) = \sqrt{(x-2)^2+y^2+(z+3)^2}$$

Z: $d^2 = (x-2)^2 + y^2 + (z+3)^2$

$\Rightarrow$ [Điểm cực trị, đặt $d^2 = f(x,y,z)$
Đặt z theo x, y rồi tìm đạo hàm cực trị.

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Chương 1-3: Hàm ẩn, cực trị.

27: $y\cos x = x^2+y^2 \Rightarrow f(x,y) = x^2+y^2 - y\cos x = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{f'_x(x,y)}{f'_y(x,y)} = -\frac{2x+y\sin x}{2y-\cos x}.$$

28: $f(x,y) = \cos(xy) - \sin y - 1 = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{f'_x(x,y)}{f'_y(x,y)} = -\frac{-y\sin(xy)}{-x\sin(xy)-\cos y} = \frac{-y\sin(xy)}{x\sin(xy)+\cos y}$$

29: $f(x,y) = \cot(x^2y) - x - xy^2 = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{f'_x(x,y)}{f'_y(x,y)} = -\frac{\frac{-2xy}{\sin^2(x^2y)} - 1 - y^2}{\frac{-x^2}{\sin^2(x^2y)} - 2xy}.$$

30: $f(x,y) = e^y\sin x - x - xy = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{f'_x(x,y)}{f'_y(x,y)} = -\frac{e^y\cos x - 1 - y}{e^y\sin x - x}$$

31: $f(x,y,z) = x^2+2y^2+3z^2 - 1 = 0$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{f'_x}{f'_z} = -\frac{2x}{6z} = -\frac{x}{3z}.$$

$$\frac{\partial z}{\partial y} = -\frac{f'_y}{f'_z} = -\frac{4y}{6z} = -\frac{2y}{3z}.$$

32: $f(x,y,z) = e^z - xyz = 0$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{f'_x}{f'_z} = -\frac{-yz}{e^z-xy} = \frac{yz}{e^z-xy}$$

$$\frac{\partial z}{\partial y} = -\frac{f'_y}{f'_z} = \frac{xz}{e^z-xz}.$$

KOKUYO