Bayesian inference of Hubble constant from simulated time delays

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Time delay

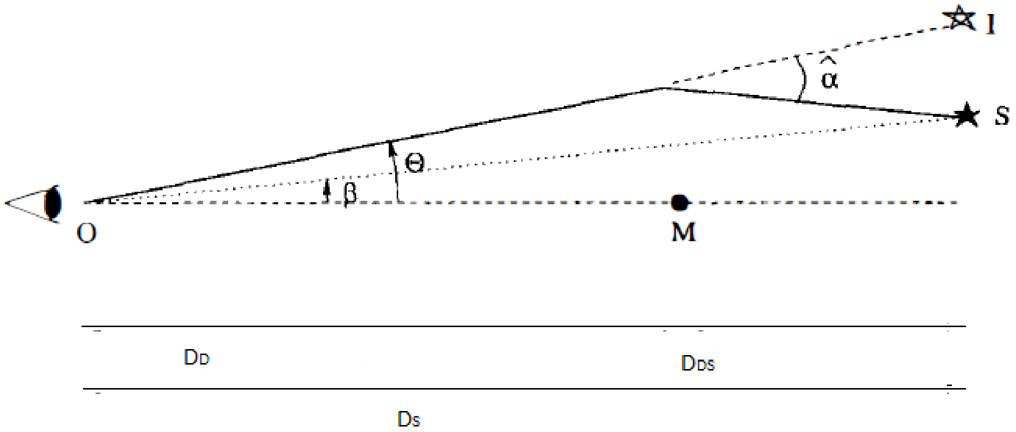
- time delay é a diferença de tempo de chagada entre as imagens formadas;
- O tempo de viagem da luz através de uma única lente isolada e fina é dado por:

$$\tau(\vec{\theta}) = \frac{D_{\Delta t}}{c}\phi(\vec{\theta}, \vec{\beta})$$

onde

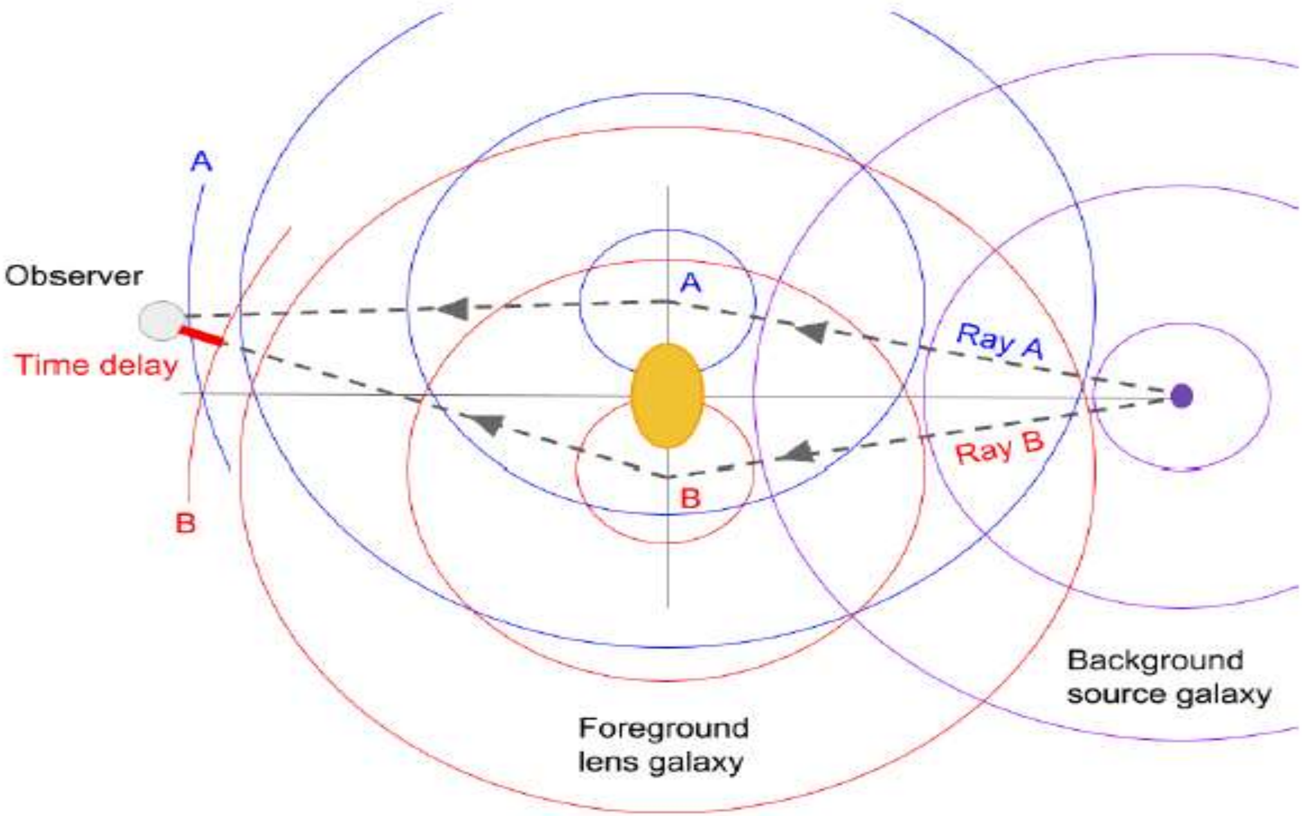
$$\phi(\vec{\theta}) = \frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta})$$

$\phi(\vec{\theta})$: potencial de Fermat;
$\vec{\theta}$: posição da imagem;
$\vec{\beta}$: posição da fonte;
$\psi(\vec{\theta})$: potencial gravitacional da lente;



Time delay

- arXiv:1605.05333v1



- Time delay entre duas imagens:

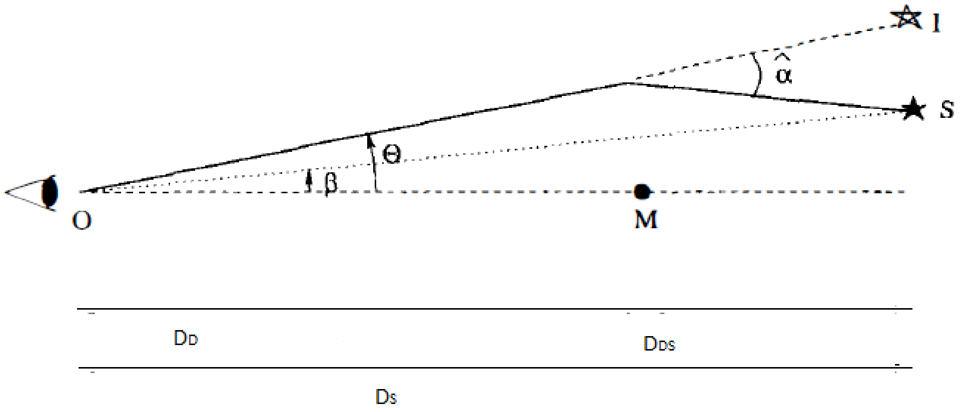
$$\Delta\tau_{\mathrm{AB}} = \frac{D_{\Delta t}}{c}\Delta\Phi_{\mathrm{AB}}$$

Time delay

- Time delay distance:

$$D_{\Delta t} = (1 + z_D)\frac{D_D D_S}{D_{DS}} = c\,\frac{\Delta\tau_{AB}}{\Delta\phi_{AB}}$$

- Angular diameter distances:



D_D: distância observador-lente;

D_S: distância observador-fonte;

D_{DS}: distância lente-fonte;

z_D: redshift da lente;

- A constante de Hubble é sensível ao time delay distance:

$$H_0 \propto D_{\Delta t}^{-1}$$

Escolhas de simulação

	Nome	zl	zs	Sigma ap	dSigma ap	thetaE	Survey	theta ap	Theta eff	sigma0	dsigma0	sigma_atm_(arcsec)
0	J0830+5116	0.5300	1.3320	268.00	36.00	1.14	BELLS	1.0	1.100	274.00	37.00	1.8
1	J1215+0047	0.6420	1.2970	262.00	45.00	1.37	BELLS	1.0	1.420	266.00	46.00	1.8
2	J1337+3620	0.5640	1.1820	225.00	35.00	1.39	BELLS	1.0	1.600	227.00	35.00	1.8
3	J1542+1629	0.3520	1.0230	210.00	16.00	1.04	BELLS	1.0	1.450	213.00	16.00	1.8
4	J1545+2748	0.5220	1.2890	250.00	37.00	1.21	BELLS	1.0	2.650	247.00	37.00	1.8
...	...	...	...	...	...	...	...	...	...	...	...	...
100	J2309-0039	0.2905	1.0048	184.00	13.00	1.14	SLACS2017	1.5	2.080	187.00	13.00	1.4
101	J090311.6+003906	0.2997	3.0420	238.58	16.49	1.52	BELLS	1.0	1.815	239.51	16.56	1.8
102	J134429.4+303036	0.6720	2.3010	422.43	96.79	0.92	BELLS	1.0	0.695	440.67	100.97	1.8
103	J110016.3+571736	0.7810	1.6510	265.99	63.34	1.14	BELLS	1.0	5.650	255.17	60.76	1.8
104	J141351.9-000026	0.5480	2.4782	322.68	38.01	1.13	BELLS	1.0	1.980	322.81	38.02	1.8

105 rows × 12 columns

. $\theta_E \in [0.9, 1.6] \longrightarrow$ 105 sistemas de lentes

Escolhas de simulação

```
# define lens configuration and cosmology
z_lens =  tabela['zl']
z_source = tabela['zs']
theta_E = tabela['thetaE']

from astropy.cosmology import FlatLambdaCDM
cosmo = FlatLambdaCDM(H0=70, Om0=0.3, Ob0=0.)

# data specifics
sigma_bkg = .05  #  background noise per pixel (Gaussian)
exp_time = 100.  #  exposure time (arbitrary units, flux per pixel is in units #photons/exp_time unit)
numPix = 100  #  cutout pixel size
deltaPix = 0.05  #  pixel size in arcsec (area per pixel = deltaPix**2)
fwhm = 0.1  # full width half max of PSF (only valid when psf_type='gaussian')
psf_type = 'GAUSSIAN'  # 'GAUSSIAN', 'PIXEL', 'NONE'
kernel_size = 91

# initial input simulation

# generate the coordinate grid and image properties
kwargs_data = sim_util.data_configure_simple(numPix, deltaPix, exp_time, sigma_bkg)
data_class = ImageData(**kwargs_data)

# generate the psf variables
kwargs_psf = {'psf_type': psf_type, 'pixel_size': deltaPix, 'fwhm': fwhm}
psf_class = PSF(**kwargs_psf)
```

- https://github.com/sibirrer/lenstronomy
- lenstronomy_extensions/time-delay cosmography.ipynb at main · sibirrer/lenstronomy_extensions (github.com)

Escolhas de simulação

```python
# lensing quantities
gamma1, gamma2 = param_util.shear_polar2cartesian(phi=-0.5, gamma=0.06)
kwargs_shear = {'gamma1': gamma1, 'gamma2': gamma2}  # shear values

lista_de_dicionarios = []

for index in tabela['thetaE']:

    kwargs_spemd={}
    kwargs_spemd['theta_E'] = index
    kwargs_spemd['gamma'] = 1.98
    kwargs_spemd['center_x'] = 0.0
    kwargs_spemd['center_y'] = 0.0
    kwargs_spemd['e1'] = 0.05
    kwargs_spemd['e2'] = 0.05
    lista_de_dicionarios.append(kwargs_spemd)

# the lens model is a supperposition of an elliptical lens model with external shear
lens_model_list = ['SPEP', 'SHEAR']
kwargs_lens = [lista_de_dicionarios, kwargs_shear]
```

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Escolhas de simulação

```
# the lens model is a supperposition of an elliptical lens model with external shear
lens_model_list = ['SPEP', 'SHEAR']
kwargs_lens = [lista_de_dicionarios, kwargs_shear]


lens_model_class_lista= []

for a in range(0,104):

    lens_model_class = LensModel(lens_model_list=lens_model_list, z_lens=z_lens[a], z_source=z_source[a], cosmo=cosmo)
    lens_model_class_lista.append(lens_model_class)



# choice of source type
source_type = 'SERSIC'  # 'SERSIC' or 'SHAPELETS'
source_x = 0.
source_y = 0.1

# Sersic parameters in the initial simulation
phi_G, q = 0.5, 0.8
e1, e2 = param_util.phi_q2_ellipticity(phi_G, q)
kwargs_sersic_source = {'amp': 4000, 'R_sersic': 0.2, 'n_sersic': 1, 'e1': e1, 'e2': e2, 'center_x': source_x, 'center_y': source_y}
source_model_list = ['SERSIC_ELLIPSE']

kwargs_source = [kwargs_sersic_source]

source_model_class = LightModel(light_model_list=source_model_list)
```

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- lenstronomy_extensions/time-delay cosmography.ipynb at main · sibirrer/lenstronomy_extensions (github.com)

Escolhas de simulação

```python
# lens light model
phi_G, q = 0.9, 0.9
e1, e2 = param_util.phi_q2_ellipticity(phi_G, q)
kwargs_sersic_lens = {'amp': 8000, 'R_sersic': 0.4, 'n_sersic': 2., 'e1': e1, 'e2': e2, 'center_x': 0.0, 'center_y': 0}
lens_light_model_list = ['SERSIC_ELLIPSE']
kwargs_lens_light = [kwargs_sersic_lens]
lens_light_model_class = LightModel(light_model_list=lens_light_model_list)

lensEquationSolver = LensEquationSolver(lens_model_class)

x_image_lista = []
y_image_lista = []

for a in range(0, 104):

    x_image, y_image = lensEquationSolver.findBrightImage(source_x, source_y, [lista_de_dicionarios[a], kwargs_shear], numImages=4,
                                                          min_distance=deltaPix, search_window=numPix * deltaPix)
    x_image_lista.append(x_image)
    y_image_lista.append(y_image)


mag_lista = []

for a in range(0,104):
    mag = lens_model_class.magnification(x_image_lista[a], y_image_lista[a], kwargs=[lista_de_dicionarios[a], kwargs_shear])
    mag_lista.append(mag)
```

- https://github.com/sibirrer/lenstronomy
- lenstronomy_extensions/time-delay cosmography.ipynb at main · sibirrer/lenstronomy_extensions (github.com)

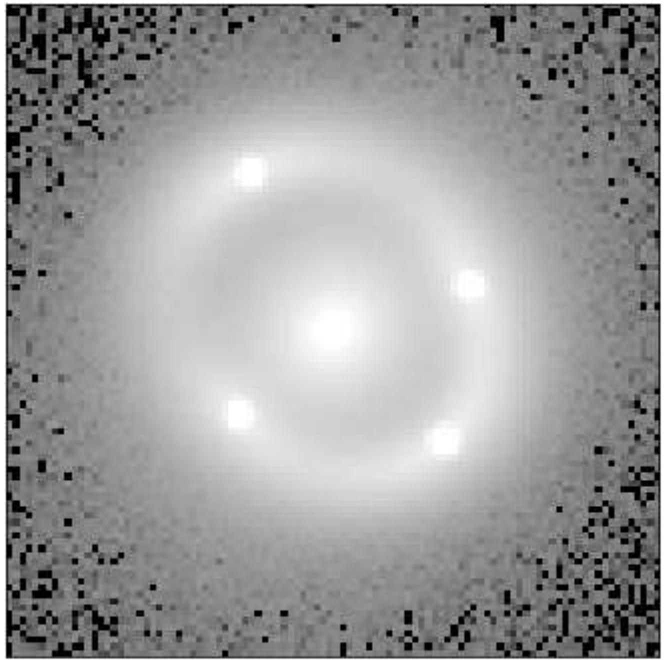
Escolhas de simulação

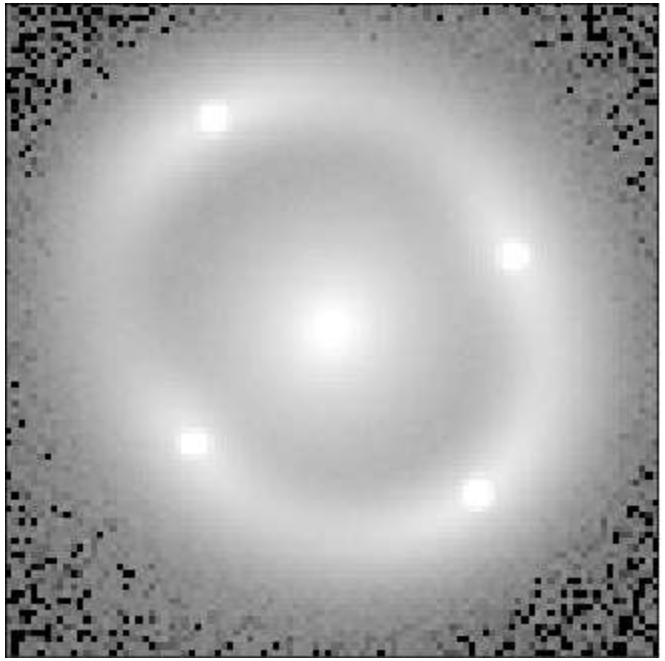
```python
kwargs_ps_lista = []

for a in range(0, 104):
    kwargs_ps = [{'ra_image': x_image_lista[a], 'dec_image': y_image_lista[a],
                  'point_amp': np.abs(mag_lista[a])*1000}]  # quasar point source position in the source plane and intrinsic brightness
    kwargs_ps_lista.append(kwargs_ps)



point_source_list = ['LENSED_POSITION']
point_source_class = PointSource(point_source_type_list=point_source_list, fixed_magnification_list=[False])

kwargs_numerics = {'supersampling_factor': 1}

imageModel_lista = []

for a in range(0, 104):
    imageModel = ImageModel(data_class, psf_class, lens_model_class_lista[a], source_model_class,
                            lens_light_model_class, point_source_class, kwargs_numerics=kwargs_numerics)
    imageModel_lista.append(imageModel)
```

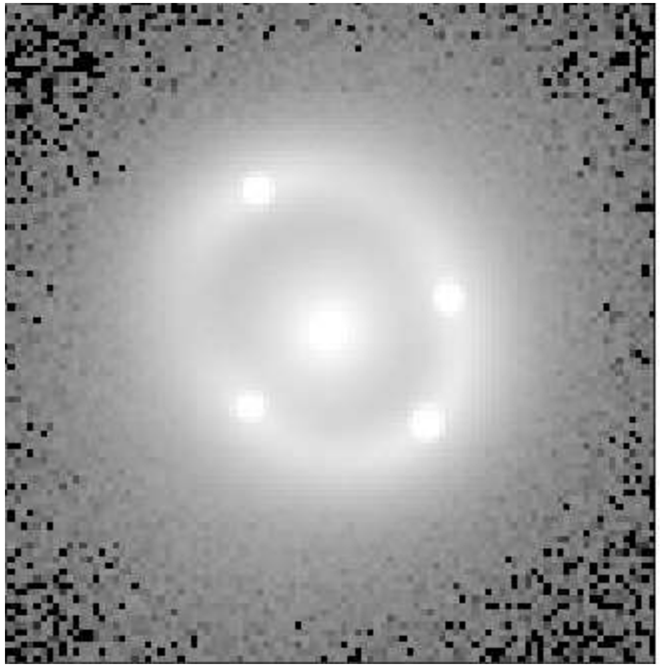
- https://github.com/sibirrer/lenstronomy
- lenstronomy_extensions/time-delay cosmography.ipynb at main · sibirrer/lenstronomy_extensions (github.com)

Escolhas de simulação

- Algumas lentes simuladas







- https://github.com/sibirrer/lenstronomy
- lenstronomy_extensions/time-delay cosmography.ipynb at main · sibirrer/lenstronomy_extensions (github.com)

Determinando os time delays

```
from lenstronomy.Analysis.td_cosmography import TDCosmography

td_cosmo_lista = []
t_days_lista = []
printar_0_lista = []
dt_days_lista = []
dt_measured_lista = []
printar_1_lista = []

for k in range(0,104):
    td_cosmo = TDCosmography(z_lens[k], z_source[k], kwargs_model, cosmo_fiducial=cosmo)
    td_cosmo_lista.append(td_cosmo)

# time delays, the unit [days] is matched when the lensing angles are in arcsec
    t_days = td_cosmo_lista[k].time_delays([lista_de_dicionarios[k], kwargs_shear], kwargs_ps_lista[k], kappa_ext=0)
    t_days_lista.append(t_days)
    printar_0 = print("the time delays for the images at position ", kwargs_ps_lista[k][0]['ra_image'], kwargs_ps_lista[k][0]['dec_image'], "are: ",
                     t_days_lista[k])
    printar_0_lista.append(printar_0)

# relative delays (observable). The convention is relative to the first image
    dt_days =  t_days_lista[k][1:] - t_days_lista[k][0]
    dt_days_lista.append(dt_days)

# and errors can be assigned to the measured relative delays (full covariance matrix not yet implemented)
    dt_sigma = [3, 5, 10]
    np.random.seed(2)
# and here a realisation of the measurement with the quoted error bars
    dt_measured = np.random.normal(dt_days_lista[k], dt_sigma)
    dt_measured_lista.append(dt_measured)
```

```
the time delays for the images at position  [-0.63063198  0.84718868  1.04061186 -0.71358134] [ 1.19566337 -0.8455816   0.32896767 -0.65281221] are:  [-93.4821403
2 -70.74911216 -62.26549017 -50.79170023]
the measured relative delays are:  [21.48275462 30.93531602 21.32847914]
```

Determinando distâncias

Fermat Potential

```
from lenstronomy.Cosmo.lens_cosmo import LensCosmo

lens_cosmo_lista = []
d_fermat_model_lista = []

for k in range(0,104):

    lens_cosmo = LensCosmo(z_lens[k], z_source[k], cosmo)
    lens_cosmo_lista.append(lens_cosmo)

    d_fermat_model = lens_cosmo_lista[k].time_delay2fermat_pot(dt=dt_days_lista[k])
    d_fermat_model_lista.append(d_fermat_model)
```

Time delay distance

```
td_distance_lista = []

for k in range(0,104):

    td_distance = td_cosmo_lista[k].ddt_from_time_delay(d_fermat_model_lista[k], dt_measured_lista[k]
                                                        , kappa_s=0, kappa_ds=0, kappa_d=0)
    td_distance_lista.append(td_distance)
```

$$D_{\Delta t} = c\,\frac{\Delta\tau_{AB}}{\Delta\phi_{AB}}$$

Determinando distâncias

Dispersão de velocidades estelar projetada na linha de visão da galáxia defletora:

$$(\sigma^{\mathrm{P}})^2 = \frac{D_{\mathrm{s}}}{D_{\mathrm{ds}}} c^2 J(\xi_{\mathrm{lens}}, \xi_{\mathrm{light}}, \beta_{\mathrm{ani}}),$$

- arXiv:1910.06306v3

```
R_slit = 1. # slit length in arcsec
dR_slit = 1.  # slit width in arcsec
psf_fwhm = 0.7  # Full width at half maximum of the PSF
kwargs_aperture = {'aperture_type': 'slit', 'length': R_slit, 'width': dR_slit, 'center_ra': 0.05, 'center_dec': 0, 'angle': 0}
anisotropy_model = 'OM'
aperture_type = 'slit'

kwargs_numerics_galkin = {'interpol_grid_num': 1000,  # numerical interpolation, should converge -> infinity
                          'log_integration': True,  # log or linear interpolation of surface brightness and mass models
                           'max_integrate': 100, 'min_integrate': 0.001}  # lower/upper bound of numerical integrals
r_ani = 1.
r_eff = 0.2
kwargs_anisotropy = {'r_ani': r_ani}
kwargs_seeing = {'psf_type': 'GAUSSIAN', 'fwhm': psf_fwhm}

from lenstronomy.Analysis.kinematics_api import KinematicsAPI
kin_api_lista = []
vel_disp_lista = []

for l in range(0,104):
    kin_api = KinematicsAPI(z_lens[l], z_source[l], kwargs_model, cosmo=cosmo,
                        lens_model_kinematics_bool=[True, False], light_model_kinematics_bool=[True],
                        kwargs_aperture=kwargs_aperture, kwargs_seeing=kwargs_seeing,
                        anisotropy_model=anisotropy_model, kwargs_numerics_galkin=kwargs_numerics_galkin,
                        sampling_number=10000,  # numerical ray-shooting, should converge -> infinity
                        Hernquist_approx=True)
    kin_api_lista.append(kin_api)

    vel_disp = kin_api_lista[l].velocity_dispersion([lista_de_dicionarios[l], kwargs_shear], kwargs_lens_light,
                                                    kwargs_anisotropy, r_eff=r_eff, theta_E=None, kappa_ext=0)
    vel_disp_lista.append(vel_disp)
```

Determinando distâncias

D_s / D_{ds}

```
ds_dds_lista = []

for k in range(0,104):

    ds_dds = td_cosmo_lista[k].ds_dds_from_kinematics(vel_disp_lista[k], J_lista[k], kappa_s=0, kappa_ds=0)
    ds_dds_lista.append(ds_dds)
```

Angular diameter distance to the lens (D_d)

```
dd_lista = []
#D_d_sample = []

for k in range(0,104):

    dd =  td_distance_lista[k] / (ds_dds_lista[k] * (1 + z_lens[k]))
    dd_lista.append(dd)
```

Determinando a posterior

A likelihood para os parâmetros cosmológicos relevantes, η, está totalmente contida nas "distâncias de diâmetro angular" inferidas dos dados $\{D_D, D_S, D_{DS}\} \equiv D_{D,S,DS}$. Então,

$$\begin{aligned} P(\eta|d) &\propto P(d|\eta)P(\eta) \\ &= P(d|D_{D,S,DS}(\eta))P(\eta) \end{aligned}$$

- arXiv:1809.01274v3

Considerando que as medidas realizadas em lentes diferentes são independentes umas das outras, a likelihood será:

$$P(d|D_{D,S,DS}) = \prod_i P(d_i|D_{D,S,DS})$$

- arXiv:2109.00009v1

```
from lenstronomy.Cosmo.kde_likelihood import KDELikelihood

kde_lista = []

for k in range(0,104):

    kde = KDELikelihood(D_d_sample = dd_lista[k], D_delta_t_sample = td_distance_lista[k], kde_type='gaussian', bandwidth=20)
    kde_lista.append(kde)
```

Determinando a posterior

```
def log_likelihood_100(params):
    h0, om_m = params
    cosmo = FlatLambdaCDM(H0=h0, Om0=om_m)

    log_like_lista = []
    D_d_r_lista = []
    D_dt_r_lista = []
    D_d_lista = []
    D_s_lista = []
    D_ds_lista = []
    z_lens_lista = []

    for k in range(0,104):
        D_d = cosmo.angular_diameter_distance(z_lens[k]).value
        D_s = cosmo.angular_diameter_distance(z_source[k]).value
        D_ds = cosmo.angular_diameter_distance_z1z2(z_lens[k], z_source[k]).value

        D_d_lista.append(D_d)
        D_s_lista.append(D_s)
        D_ds_lista.append(D_ds)
        z_lens_lista.append(z_lens[k])


        D_dt_r = (1+z_lens[k])*(D_d_lista[k] * D_s_lista[k] / D_ds_lista[k])
        D_d_r_lista.append(D_d_lista[k])
        D_dt_r_lista.append(D_dt_r)

        log_like = kde_lista[k].logLikelihood(D_d_r_lista[k], D_dt_r_lista[k])
        log_like_lista.append(log_like)

    return np.sum(log_like_lista)
```

```
def log_prior_100(params):
    h0, om_m = params

    if not 0. < h0 < 150.:
        return -np.inf

    if not 0.05 < om_m < 0.5:
        return -np.inf

    return 0.


def log_probability_100(params):
    prior = log_prior_100(params)

    if not np.isinf(prior):
        return log_likelihood_100(params) + prior
    else:
        return prior
```

- https://github.com/ajshajib/DESJ0408_time_delay_cosmography

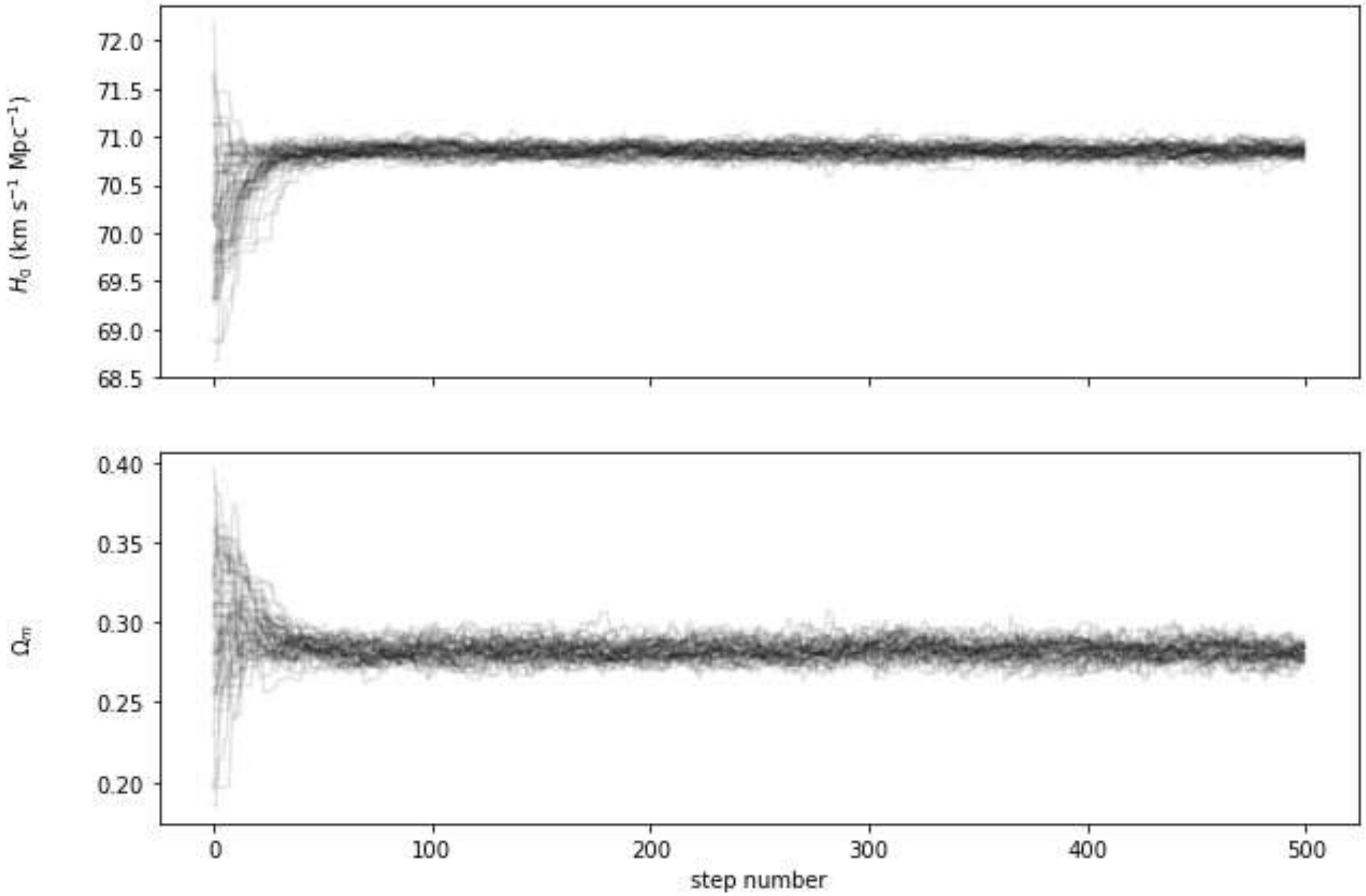
Processo de MCMC e resultados

```python
from multiprocessing import Pool

n_walkers, n_dim = 50, 2

with Pool(processes = 18) as pool:

    np.random.seed(1)
    pos = np.random.normal(loc=[70., 0.3], size=[n_walkers, n_dim], scale=[1, 5e-2])

    sampler = emcee.EnsembleSampler(n_walkers, n_dim, log_probability_100, pool = pool)

    state = sampler.run_mcmc(pos, 500, progress = True)
```

```python
import copy

fig, axes = plt.subplots(n_dim, figsize=(10, 7), sharex=True)
samples = copy.deepcopy(sampler.chain)

#samples[:, :, 0] -= np.mean(samples[:, :, 0])
#samples[:, :, 1] -= np.median(samples[:, :, 1])

labels = [r"$H_0$ (km s$^{-1}$ Mpc$^{-1}$)", r"$\Omega_m$"]

for i in range(n_dim):
    ax = axes[i]
    ax.plot(samples[:, :, i].T, "k", alpha=0.1)

    ax.set_ylabel(labels[i])
    ax.yaxis.set_label_coords(-0.1, 0.5)

axes[-1].set_xlabel("step number");
```

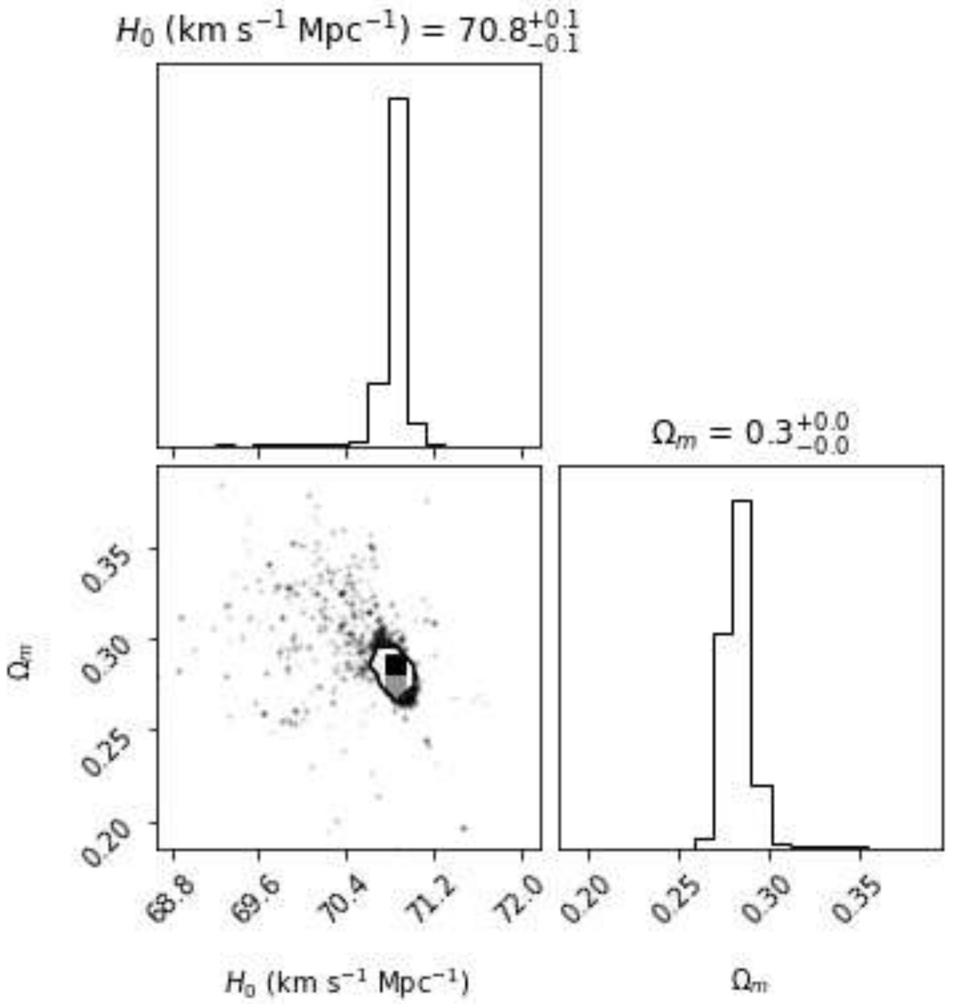
Processo de MCMC e resultados

- Para o sistema de lentes:



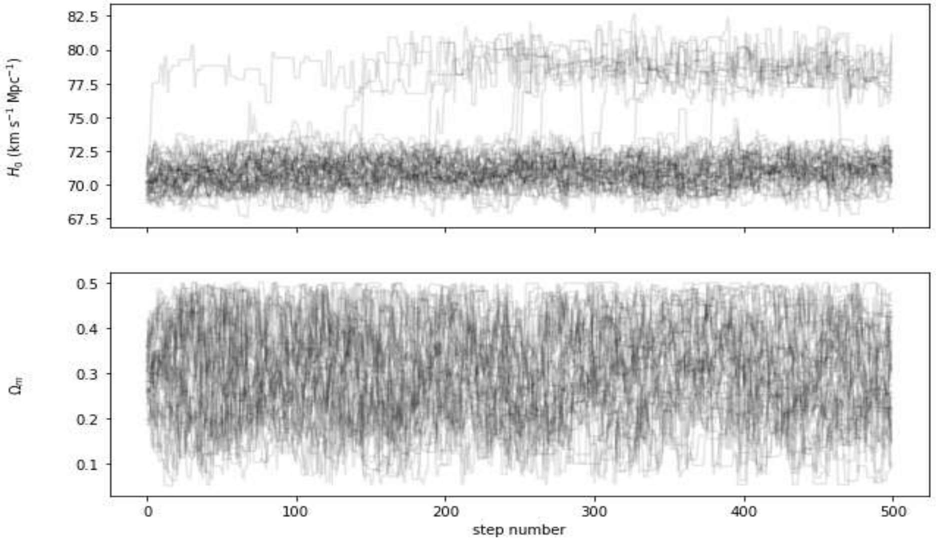
Processo de MCMC e resultados

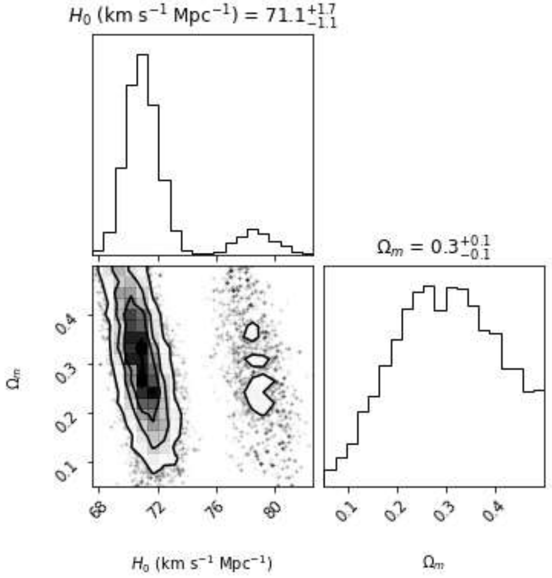
```
corner.corner(samples.reshape(-1, 2), show_titles=True, labels=labels,
              title_fmt='.1f'
             );
```

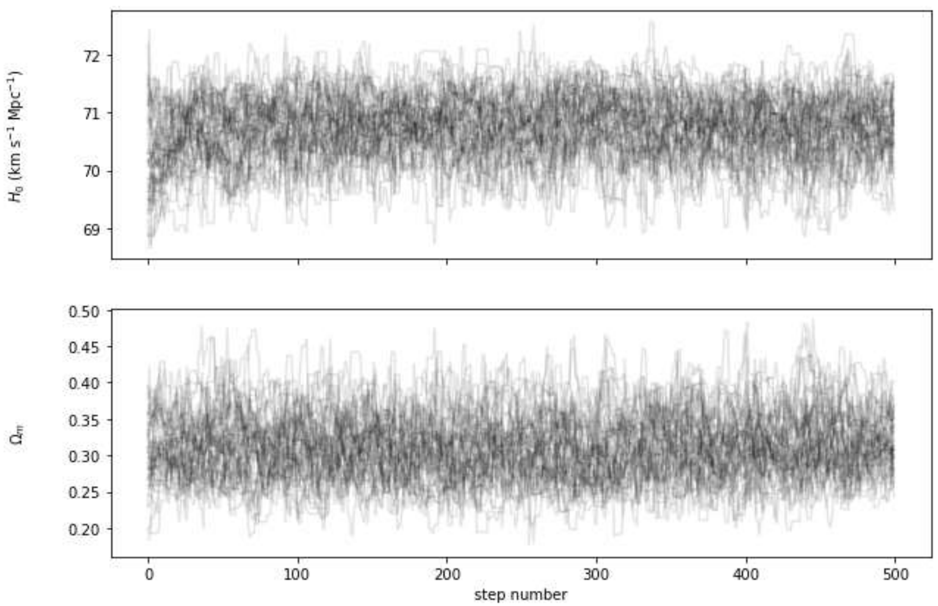


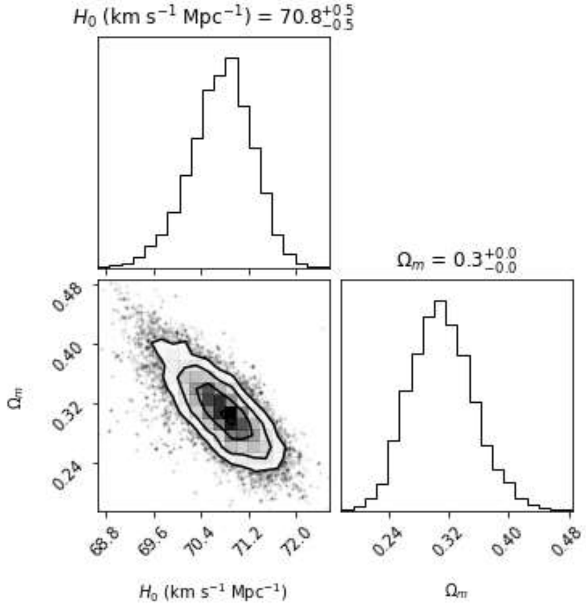
Processo de MCMC e resultados

- Para uma lente:









Obrigado!