№1. $f(x)=e^{e^x}+tg\,(e^{x^2}-\cos x)-\sqrt{\cos x^3}$

$$f'(x)=e^{e^x}\cdot e^x+\frac{e^{x^2}\cdot 2x+\sin x}{\cos^2(e^{x^2}-\cos x)}+\frac{3x^2\sin x^3}{2\sqrt{\cos x^3}}$$

$$f'(0)=e+\frac{2e\cdot 0+\sin 0}{\cos^2(e-\cos 0)}+\frac{\sin 0\cdot 3\cdot 0}{2\sqrt{\cos 0}}=e.$$

Відповідь: $f'(0)=e$.

№2. $e^x<(1+x)^{1+x}$, $x>0$

$$e^x<e^{(1+x)\ln(1+x)},\quad x>0$$

$$e>1,\quad \underbrace{x}_{\varphi(x)}<\underbrace{(1+x)\ln(1+x)}_{\psi(x)},\quad x>0$$

$$\left.\begin{array}{l}\varphi(0)=0\\ \psi(0)=0\end{array}\right. \Rightarrow \varphi(0)=\psi(0);$$

$$\left.\begin{array}{l}\varphi'(x)=1\\ \psi'(x)=\ln(1+x)+(1+x)\cdot\frac{1}{1+x}\end{array}\right. \Rightarrow \begin{array}{l}\varphi'(0)=1\\ \psi'(0)=1\end{array} \Rightarrow \varphi'(0)=\psi'(0);$$

$$\left.\begin{array}{l}\varphi''(x)=0\\ \psi''(x)=\frac{1}{1+x}+0\end{array}\right. \Rightarrow \begin{array}{l}\varphi''(0)=0\\ \psi''(0)=1\end{array} \Rightarrow \psi''(x)>\varphi''(x)$$

Отже, нерівність доведено.

№ 4. $y(x) = \frac{1}{x^2 - x - 2} = (x^2 - x - 2)^{-1} = \frac{1}{(x+1)(x-2)} \ominus$

$$(x^{\alpha})^{(n)} = \alpha(\alpha-1)(\alpha-2)\cdot\ldots\cdot(\alpha-n+1)\,x^{\alpha-n}$$

$$\ominus \frac{\alpha}{x+1} + \frac{\beta}{x-2} \Rightarrow 1 = (x-2)\alpha + (x+1)\beta$$

$$\begin{cases} \alpha + \beta = 0 \\ -2\alpha + \beta = 1 \end{cases} \Rightarrow \begin{cases} \alpha = -\frac{1}{3} \\ \beta = \frac{1}{3} \end{cases}$$

$$y(x) = \frac{1}{3(x-2)} - \frac{1}{3(x+1)} = \frac{1}{3}\left(\frac{1}{x-2} - \frac{1}{x+1}\right);$$

$$y^{(n)}(x) = \frac{1}{3}\cdot(-1)^n\cdot n!\left(\frac{1}{(x-2)^{n+1}} - \frac{1}{(x+1)^{n+1}}\right).$$

№ 5. $y = x^n e^{-x}$, $n \in \mathbb{N}$;

$$f'(x) = n x^{n-1}\cdot e^{-x} - x^n \cdot e^{-x}$$

$$f'(0) = 0$$

$$f''(x) = -n\cdot x^{n-2}\cdot e^{-x} + n^2 x^{n-2} e^{-x} - 2n x^{n-1}\cdot e^{-x} + x^n \cdot e^{-x}$$

$$f''(0) = 0$$

$$f^{(k)}(0) = 0, \quad k = \overline{1, n-2}$$

$$f^{(n)}(x) = n!\cdot e^{-x}$$

| $\exists f^{(k)}(x_0) = 0$, $k = \overline{1, n-1}$
| $\exists f^{(n)}(x_0) \neq 0$, тоді при $2n$ є екстр., а при $2n+1$ - немає

$f^{(n)}(0) = n! > 0$, в т. $x_0 = 0$ при $2n$ точка min, а при $2n+1$ точки екстремуму немає.

№ 8. $f(x) = (1+\cos x)^{\cos x} = e^{\cos x \ln(1+\cos x)}$

$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + o(x^5)$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + o(x^5)$

1) $\ln(1+\cos x) = \ln\left(2 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right) =$
$= \ln\left(2\left(1 - \frac{x^2}{4} + \frac{x^4}{48} + o(x^5)\right)\right) = \ln 2 + \ln\left(1 - \frac{x^2}{4} + \frac{x^4}{48} + o(x^5)\right) = \ln 2 + \left(-\frac{x^2}{4} + \frac{x^4}{48} - \frac{x^4}{32}\right) = \ln 2 - \frac{x^2}{4} - \frac{x^4}{96} + o(x^5);$

2) $\cos x \ln(1+\cos x) = \left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)\left(\ln 2 - \frac{x^2}{4} - \frac{x^4}{96}\right) + o(x^5) =$
$= \ln 2 - \frac{x^2}{4} - \frac{x^4}{96} - \ln 2 \cdot \frac{x^2}{2} + \frac{x^4}{8} + \ln 2 \cdot \frac{x^4}{24} + o(x^5) =$
$= \ln 2 + x^2\left(-\frac{\ln 2}{2} - \frac{1}{4}\right) + x^4\left(\frac{\ln 2}{24} + \frac{11}{96}\right) + o(x^5);$

3) $e^{\cos x \ln(1+\cos x)} = e^{\ln 2 \left(-\frac{\ln(2) x^2}{2} + \frac{1}{4}\right) - \frac{x^2}{4} + \frac{\ln 2}{24}x^4 + \frac{11}{96}x^4 + o(x^5)} =$

$= e^{\ln 2} \cdot e^{-x^2 \cdot \left(\frac{\ln 2}{2} + \frac{1}{4}\right) + x^4\left(\frac{\ln 2}{24} + \frac{11}{96}\right) + o(x^5)} =$

$= 2 \cdot \left(1 - x^2\left(\frac{\ln 2}{2} + \frac{1}{4}\right) + x^4\left(\frac{\ln 2}{24} + \frac{11}{96}\right) + \frac{\left(-x^2\left(\frac{\ln 2}{2} + \frac{1}{4}\right)\right)^2}{2} + o(x^5)\right)$

$= 2\left(1 - x^2\left(\frac{\ln 2}{2} + \frac{1}{4}\right) + x^4\left(\frac{\ln 2}{48} + \frac{11}{192} + \frac{(\ln 2)^2}{8} + \frac{2\ln 2}{16} + \frac{1}{32}\right) + o(x^5)\right) =$

$= 2\left(1 - x^2\left(\frac{2\ln 2 + 1}{4}\right) + x^4\left(\frac{\ln^2(2)}{8} + \frac{\ln 2}{6} + \frac{7}{48}\right) + o(x^5)\right) =$

$= 2 - \left(\frac{\ln 2}{2} + \frac{1}{2}\right)x^2 + \left(\frac{\ln^2(2)}{4} + \frac{\ln 2}{3} + \frac{7}{24}\right)x^4 + o(x^5),$