№ 84. р) $\int_0^1 \ln(1+\sqrt{1+x})\,dx = \left| \begin{matrix} 1+x=t, & dx=dt \\ x=0, & t=1 \\ x=1, & t=2 \end{matrix} \right| =$ 12.04.

$= \int_1^2 \ln(1+\sqrt{t})\,dt = \left| \begin{matrix} 1+\sqrt{t}=a, \ \sqrt{t}=a-1 \\ da=\frac{1}{2\sqrt{t}}dt, \ dt=2\sqrt{t}\,da= \\ t=2, \ a=1+\sqrt{2}. \quad =2\sqrt{a-1}\,da \\ t=1, \ a=2 \end{matrix} \right| =$

$= \int_2^{1+\sqrt{2}} \ln a \cdot 2(a-1)\,da = 2\int_2^{1+\sqrt{2}} (a\ln a - \ln a)\,da =$

$= 2\left(\frac{\ln a \cdot a^2}{2} - \frac{a^2}{4} - \ln a \cdot a + a\right)\Big|_2^{1+\sqrt{2}} =$

$= 2\left(\frac{\ln(1+\sqrt{2})(1+\sqrt{2})^2}{2} - \frac{(1+\sqrt{2})^2}{4} - \ln(1+\sqrt{2})\cdot(1+\sqrt{2}) +\right.$

$\left. + (1+\sqrt{2}) - \left(\frac{\ln 2\cdot 4}{2} - 1 - \ln 2\cdot 2 + 2\right)\right) =$

$= \ln(1+\sqrt{2})\cdot(1+\sqrt{2})^2 - \frac{(1+\sqrt{2})^2}{2} - 2\ln(1+\sqrt{2})\cdot(1+\sqrt{2}) + 2(1+\sqrt{2}) -$

$- \cancel{4\ln 2} + 2 + \cancel{4\ln 2} + 4 = \ln(1+\sqrt{2}) - \frac{3-2\sqrt{2}}{2}.$

№ 89. е) $\lim\limits_{x\to+\infty} \dfrac{\int_0^x \frac{dt}{\sqrt[4]{1+t^4}}}{\ln x}$

$$\int_0^x \frac{dt}{\sqrt[4]{1+t^4}} = \frac{1}{\sqrt[4]{1+\xi^4}}\cdot x \xrightarrow[x\to+\infty]{} \infty;$$

$$\lim_{x\to+\infty} \frac{\frac{1}{\sqrt[4]{1+x^4}}}{\frac{1}{x}} = \lim_{x\to+\infty}\frac{x}{\sqrt[4]{1+x^4}} = \lim_{x\to+\infty}\frac{\cancel{x}}{\cancel{x}\sqrt[4]{\frac{1}{x^4}+1}} =$$

$$= \lim_{x\to+\infty}\frac{1}{\sqrt[4]{\frac{1}{x^4}+1}} = 1.$$

№ 104. и) $\displaystyle\int_{50}^{150}\frac{x\cos x\,dx}{2^x} = \xi\cos\xi\int_{50}^{150}\frac{1}{2^x}dx =$

$$= \xi\cos\xi\cdot\left(\frac{-2^{-x}}{\ln 2}\right)\Big|_{50}^{150} = \xi\cos\xi\cdot\frac{2^{-50}-2^{-150}}{\ln 2};$$

$$-150\cdot 1\cdot\frac{2^{-50}(1-2^{-100})}{\ln 2} \le \int \le 150\cdot 1\cdot \frac{2^{-50}(1-2^{-100})}{\ln 2}.$$

№ 74. б) $S_n = \displaystyle\sum_{k=1}^{n-1}\left(1+\frac{k}{n}\right)\sin\frac{\pi k}{n^2}$

$$\sum_{k=1}^{n-1}\left(\left(1-\frac{k}{n}\right)\sin\frac{\pi k}{n^2}\right) = \sum_{k=1}^{n-1}\left(\left(1-\frac{k}{n}\right)\left(\frac{\pi k}{n^2}+o\left(\frac{1}{n^2}\right)\right)\right) =$$

$$=\sum_{k=1}^{n-1}\left(\frac{\pi k}{n^2}-\frac{\pi k^2}{n^3}+o\left(\frac{k}{n^3}\right)\right) = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{\pi k}{n}-\frac{\pi k^2}{n^2}+o\left(\frac{k}{n^3}\right)\right)-$$

$$-\frac{1}{n}\left(\frac{\pi n}{n}-\frac{\pi n^2}{n}+o\left(\frac{n}{n^2}\right)\right)\xrightarrow[n\to\infty]{}\int_0^1(\pi x-\pi x^2)\,dx =$$

$$=\pi\int_0^1(x-x^2)\,dx = \left(\pi\frac{x^2}{2}-\pi\frac{x^3}{3}\right)\Big|_0^1 = \frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}.$$

№ 162. и) $x = 3t^2$

$y = 3t - t^3$

$$\boxed{S = \int_{t_1}^{t_2} x(t)\, y'(t)\, dt}$$

$$\begin{cases} 3t_1^2 = 3t_2^2 \Rightarrow t_1 = -t_2 \Rightarrow \cancel{t_1 = t_2 = 0} \\ 3t_1 - t_1^3 = 3t_2 - t_2^3 \Rightarrow \left[\begin{array}{l} t_1 = \sqrt{3} \\ t_2 = \sqrt{3} \end{array}\right. \\ t_1 \neq t_2 \end{cases}$$

$$S = \int_{-\sqrt{3}}^{\sqrt{3}} 3t^2 (t - 3t^2)\, dt = 3\int_{-\sqrt{3}}^{\sqrt{3}} (t^3 - 3t^4)\, dt =$$

$$= 3\left(\frac{t^4}{4} - 3\frac{t^5}{5}\right)\Bigg|_{-\sqrt{3}}^{\sqrt{3}} = 3\left(\cancel{\frac{9}{4}} - \frac{3\cdot 9\sqrt{3}}{5} - \cancel{\frac{9}{4}} - \frac{3\cdot 9\sqrt{3}}{5}\right) =$$

$$= -3\cdot\frac{54\sqrt{3}}{5} = -\frac{162\sqrt{3}}{5}.$$