23.11. Сенечко Д., ТВС-11

№1. $\lim\limits_{x\to\frac{\pi}{2}} (\sin x)^{\operatorname{tg} x} = \lim\limits_{x\to\frac{\pi}{2}} \left((\sin x - 1 + 1)^{\frac{1}{\sin x - 1}}\right)^{\operatorname{tg} x(\sin x - 1)} =$

$= \lim\limits_{x\to\frac{\pi}{2}} e^{\operatorname{tg} x(\sin x - 1)} = e^0 = 1$

$\lim\limits_{x\to\frac{\pi}{2}} \operatorname{tg} x(\sin x - 1) = \lim\limits_{x\to\frac{\pi}{2}} \sin x \cdot \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x - 1}{\cos x} \Rightarrow$

1) $\lim\limits_{x\to\frac{\pi}{2}} \sin x = \sin\frac{\pi}{2} = 1;$

2) $\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x - 1}{\cos x} = \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x - 1}{\cos x} \cdot \frac{\sin x + 1}{\sin x + 1} =$

$= \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin^2 x - 1}{\cos x(\sin x + 1)} = \lim\limits_{x\to\frac{\pi}{2}} \frac{-\cos^2 x}{\cancel{\cos x}(\sin x + 1)} = \lim\limits_{x\to\frac{\pi}{2}} \frac{-\cos x}{\sin x + 1} = 0$

3) $\lim\limits_{x\to\frac{\pi}{2}} e^{\operatorname{tg} x(\sin x - 1)} = e^0 = 1.$

№2. а) $f(x) = \frac{x^2}{\cos x - 1}$, $X = (0, 1)$

$\lim\limits_{x_0\to 2\pi+0} \frac{x^2}{\cos x - 1} = \frac{4\pi^2}{1-1} = \frac{4\pi^2}{0} = -\infty;$ $x \neq 2\pi n, n \in \mathbb{Z}$

$x_0 = 2\pi n, n \in \mathbb{Z}.$

$\lim\limits_{x_0\to 2\pi-0} \frac{x^2}{\cos x - 1} = \frac{4\pi^2}{1-1} = -\infty;$

$\lim\limits_{x_0\to 2\pi+0} f(x) = \lim\limits_{x_0\to 2\pi+0} f(x)$, $x_0 = 2\pi n, n \in \mathbb{Z}$ – точки розриву 2-го роду.

№ 2. б) $f(x) = \frac{1}{2}(x - [x+\frac{1}{2}]) \cdot |x - [x+\frac{1}{2}]| + \frac{1}{4}[x+\frac{1}{2}]$

$x = n + \frac{1}{2}$

1) $f(n+\frac{1}{2}+0) = \frac{1}{2}(n+\frac{1}{2}-(n+1)) \cdot |n+\frac{1}{2}-(n+1)| +$

$+ \frac{1}{4}(n+1) = -\frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}n + \frac{1}{4} = -\frac{1}{8} + \frac{1}{4}n + \frac{1}{4} =$

$= \frac{2}{8} - \frac{1}{8} + \frac{1}{4}n = \frac{1}{8} + \frac{1}{4}n;$

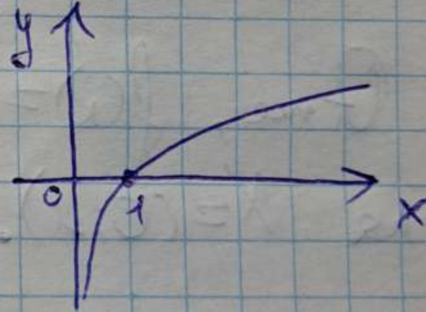
2) $f(n+\frac{1}{2}-0) = \frac{1}{2}(n+\frac{1}{2}-n)|n+\frac{1}{2}-n| + \frac{1}{4}n =$

$= \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}n = \frac{1}{8} + \frac{1}{4}n;$

$f(x_0+0) = f(x_0-0) = f(x_0)$

$f(x) \in C(\mathbb{R})$ – всюди неперервна.

№ 3. б) $f(x) = \ln x$, $X = (0,1)$



$x_n' = \frac{1}{e^n}$

$x_n'' = \frac{2}{e^n}$ $\quad x_n' - x_n'' \longrightarrow 0$

$f(x_n') = \ln e^{-n} - \ln\frac{2}{e^n} = -n - \ln 2 + \ln e^n = \cancel{-n} - \ln 2 +$

$+ n = -\ln 2 \not\xrightarrow[n\to\infty]{} 0;$

Отже, $f(x) = \ln x$ не є рівномірно неперервною на $X = (0,1)$.

№3.а) $f(x)=x\sin\frac{1}{x^2}$, $X=(0,1)$

$x_n'=\frac{1}{n}$

$x_n''=\frac{2}{n}$ $\quad x_n'-x_n''\longrightarrow 0$

$$f(x_n')-f(x_n'')=\frac{1}{n}\sin n^2-\frac{2}{n}\sin\frac{n^2}{4}=$$

$$=\frac{\sin n^2-2\sin\frac{n^2}{4}}{n}\xrightarrow[n\to\infty]{}0.$$

Отже, $f(x)=x\sin\frac{1}{x^2}$ є рівномірно неперервною на $X=(0,1)$.