

Linear Programming: Four-Dimensional Simplex Method Examples

Introduction

The following examples are four-dimensional linear programming problems, presented as minimization problems, which can be converted into dual maximization problems for solving via the simplex method.

Example 1: Minimization Problem

$$\text{Minimize: } Z = 6x_1 + 5x_2 + 8x_3 + 4x_4$$

Subject to:

$$x_1 + 2x_2 + x_3 + 3x_4 \geq 10$$

$$2x_1 + x_2 + 3x_3 + x_4 \geq 14$$

$$3x_1 + x_2 + 4x_3 + 2x_4 \geq 18$$

$$x_1 + 4x_2 + 2x_3 + x_4 \geq 12$$

$$x_1, x_2, x_3, x_4 \geq 0$$

****Dual Problem Formulation****

$$\text{Maximize: } W = 10y_1 + 14y_2 + 18y_3 + 12y_4$$

Subject to:

$$y_1 + 2y_2 + 3y_3 + y_4 \leq 6$$

$$2y_1 + y_2 + y_3 + 4y_4 \leq 5$$

$$y_1 + 3y_2 + 4y_3 + 2y_4 \leq 8$$

$$3y_1 + y_2 + 2y_3 + y_4 \leq 4$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Example 2: Minimization Problem

$$\text{Minimize: } Z = 9x_1 + 7x_2 + 6x_3 + 5x_4$$

Subject to:

$$2x_1 + x_2 + x_3 + x_4 \geq 15$$

$$x_1 + 3x_2 + 2x_3 + 4x_4 \geq 10$$

$$3x_1 + 2x_2 + x_3 + 5x_4 \geq 18$$

$$x_1 + x_2 + 4x_3 + 2x_4 \geq 14$$

$$x_1, x_2, x_3, x_4 \geq 0$$

****Dual Problem Formulation****

$$\text{Maximize: } W = 15y_1 + 10y_2 + 18y_3 + 14y_4$$

Subject to:

$$2y_1 + y_2 + 3y_3 + y_4 \leq 9$$

$$y_1 + 3y_2 + 2y_3 + x_4 \leq 7$$

$$y_1 + 2y_2 + y_3 + 4y_4 \leq 6$$

$$y_1 + 4y_2 + 5y_3 + 2y_4 \leq 5$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Example 3: Minimization Problem

$$\text{Minimize: } Z = 10x_1 + 6x_2 + 7x_3 + 8x_4$$

Subject to:

$$3x_1 + 2x_2 + x_3 + x_4 \geq 20$$

$$x_1 + 4x_2 + 3x_3 + 2x_4 \geq 16$$

$$2x_1 + 3x_2 + 5x_3 + x_4 \geq 12$$

$$x_1 + x_2 + 2x_3 + 4x_4 \geq 18$$

$$x_1, x_2, x_3, x_4 \geq 0$$

****Dual Problem Formulation****

$$\text{Maximize: } W = 20y_1 + 16y_2 + 12y_3 + 18y_4$$

Subject to:

$$3y_1 + y_2 + 2y_3 + y_4 \leq 10$$

$$2y_1 + 4y_2 + 3y_3 + x_4 \leq 6$$

$$y_1 + 3y_2 + 5y_3 + 2y_4 \leq 7$$

$$y_1 + 2y_2 + y_3 + 4y_4 \leq 8$$

$$y_1, y_2, y_3, y_4 \geq 0$$
