RC-CIR

RC Circuits

revised August, 2021

Learning Objectives:

During this lab, you will

1. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
2. fit an exponential model to experimental data.
3. learn how to use a residual plot to determine how well a model fits experimental data.
4. compare quantities measured by different experiments.
5. study the behavior of circuits that perform analog calculations.

A. Introduction

A capacitor can be constructed from two parallel conducting plates separated by a small gap. This gap is usually filled with an insulating material that improves various properties of the capacitor. Capacitors are used in circuits to store energy and to introduce a time or frequency dependence to signals.

You will investigate properties of capacitors and measure the time constants for charging and discharging them through a resistor. Not only should this lab help you understand simple RC (*resistor-capacitor*) circuits and transient signals in electronic systems, it should also help you appreciate a wide variety of mechanical systems which are analogous to RC circuits.

This laboratory experiment requires that you **fill in the worksheet in the appendix and submit it, along with your graphs and a cover sheet to Canvas.**

B. Theory

B.1. Capacitors

A capacitor stores charge; positive charge on one plate and an equal amount of negative charge on a second, parallel plate. Since the net stored charge is zero, you might wonder, why bother? The answer is that it takes work to separate equal charges $+Q$ and $-Q$ in space. When such charges are held separated, electrical energy is stored. A convenient analogy is the mechanical potential energy stored in a stretched spring. This stored energy can be liberated and put to work.

The defining relationship for capacitance, C, is

$$C = Q/V \qquad (1)$$

where Q is the magnitude of the charge stored on either plate for a given voltage difference V between the plates. The value of the capacitance is a function of the size and shape of the capacitor and of the material between the plates. For an ideal capacitor, the ratio Q/V is a constant, independent of Q or V.

In the SI system, capacitance is measured in farads (F) where 1 F = 1 Coulomb/Volt. Common sizes for capacitors used in electronics are in the ranges of pF (*pico-farads or* 10^{-12} *F*), nF (*nano-farads or* 10^{-9} *F*), μF (*micro-farads or* 10^{-6} *F*) and mF (*milli-farads or* 10^{-3} *F*). Penny-size capacitors of 1 F (*a lot of capacitance*) have only become available in the last few years. (*Four decades ago, a 1 F capacitor might have been the size of a file cabinet. The technology for producing small, high value capacitors was developed in Cleveland, Ohio in the 1980s.*)

Capacitors come in many shapes and sizes. In this laboratory, we generally use ceramic disk capacitors. These tend to be marked in nF or μF, although the notation printed on the capacitor is not always obvious. Some capacitors, such as electrolytics, use special insulators between the parallel plates and have polarity markings. The positive (+) terminal of these capacitors must always be at a higher potential than the negative (-) terminal or the capacitor may be damaged.

B.2. RC Time Constants

B.2.1. Charging a Capacitor

Figure 1 shows a schematic diagram of a simple circuit with a capacitor connected in series with a resistor R, a switch S and a voltage source or battery V_0. Note the symbols used to represent the components. (*The symbol for a capacitor sometimes has one plate slightly curved to more carefully distinguish it from a battery.*)

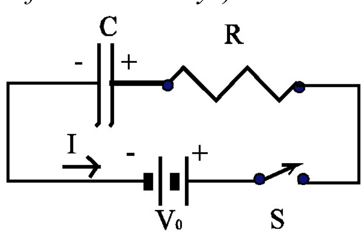

Figure 1: Charging RC Circuit

Assume that the circuit has been sitting with the switch open for a long time and there is no charge on the capacitor. Closing the switch will allow charge to flow from the battery through the resistor into the capacitor. Initially, charge will flow freely into an uncharged capacitor, limited only by the resistance in the circuit. (*A capacitor behaves like a piece of conducting wire,* (i.e., *a 'short-circuit'*) *for quickly changing signals.*) As the capacitor becomes charged, the flow will decrease as charges collecting on the plates repel incoming charges more strongly. Eventually, when the voltage across the capacitor equals the voltage across the battery, the capacitor will be filled ($Q = CV_0$) and the flow of charge will cease ($I = 0$). (*A capacitor behaves like a break in the wire* (i.e., *an 'open-circuit'*) *for constant signals.*)

The current in the circuit while the capacitor is charging is described as an exponential decay function

$$I(t) = \frac{V_0}{R} e^{-t/RC} \qquad (2)$$

where the quantity RC (*resistance* × *capacitance*) is called the *time constant* of the circuit, usually represented by the Greek letter tau, τ defined by

$$\tau = RC. \qquad (3)$$

The time constant's units of *ohm-farads* in the MKS or SI system must be equal to units of time, *seconds*.

It is useful to think for a moment about the behavior predicted by Eq. 2. Consider the current at $t = 0$. Eq. 2 says that this should be $I = V_0/R$, the same value it would have if the capacitor was replaced with a piece of wire. This reflects the general principle mentioned earlier that capacitors behave like a short circuit for quickly changing signals. Now consider what happens if you try a very large t in Eq. 2. The current falls to zero as the capacitor acts more like an open circuit.

An example of the exponential decay of the current, with $\tau = 1$ second, is shown in Fig. 2. It should be pointed out that the initial value of the current, at $t = 0$, could be positive or negative but in either case the current decays towards 0 as time increases.

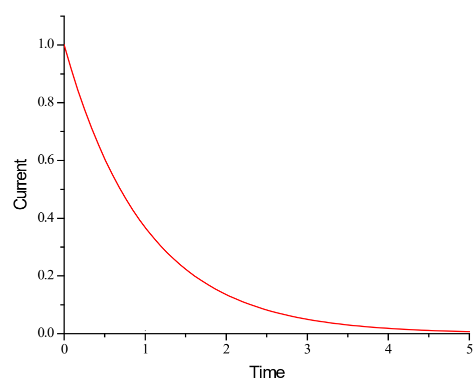

Figure 2: Exponential decay of current in a charging RC circuit.

During charging, the time dependence of the charge on the capacitor is described by an *inverse* or *reverse* exponential function

$$Q_C(t) = CV_0\left(1 - e^{-t/RC}\right) \qquad (4)$$

Note the form of this function; at $t = 0$, $Q_C = 0$, with Q_C increasing to its maximum value $Q_C = CV_0$ at large times. Technically, it takes an infinitely long time to charge a capacitor completely, but practically, the capacitor can be considered to be fully charged after a few time constants; if $\tau = 1$ second, the capacitor is 95% charged after 3 seconds. An example of an inverse or reverse exponential growth of current, with a 1 second time constant, is shown in Figure 3. The value starts at zero and approaches some finite limit, which could be positive, as shown, or negative.

Combining Eqs. 1 and 4, the voltage across the capacitor while the capacitor is charging is

$$V_C(t) = V_0\left(1 - e^{-t/RC}\right) \qquad (5)$$

Kirchoff's Loop Rule, in this case $V_R + V_C = V_0$, gives us the voltage across the resistor

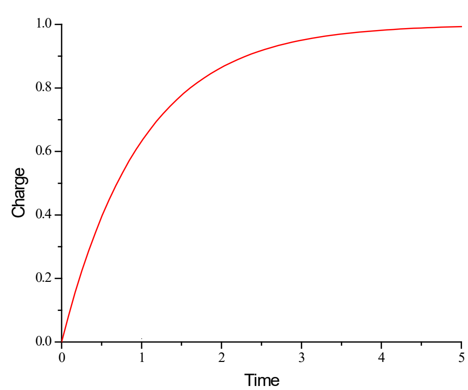

Figure 3: Inverse or reverse exponential of charge on the capacitor of a charging RC circuit.

$$V_R(t) = V_0 e^{-t/RC} \qquad (6)$$

(Using Ohm's law ($V = IR$) and Eq. 2 also gives us Eq. 6.)

B.2.2. Discharging a Capacitor

An ideal capacitor that has been charged may hold its charge for many hours even if it is removed from the circuit. (*People who work with electronics, including the author, usually learn this the hard way, after which they carefully discharge any high voltage capacitor before touching it.*) A capacitor can be discharged through a resistor by removing the battery and closing the switch as illustrated in Fig. 4. Electrons will flow from the negative to the positive plate of the capacitor so that the charge on the capacitor decreases with time as

$$Q_D(t) = CV_0 e^{-t/RC} \qquad (7)$$

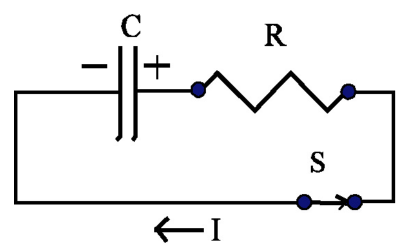

Figure 2: Discharging RC circuit

The voltage difference between the plates during discharge is

$$V_C(t) = Q_C(t)/C = V_0 e^{-t/RC} \qquad (8)$$

The current through the capacitor is the same as that through the resistor and is given by

$$I(t) = V_C(t)/R = Q_C(t)/RC \qquad (9)$$

where the direction of the current is opposite to the charging current and is shown in Fig. 4, so that the voltage across the resistor during discharge is

$$V_R(t) = I(t)R = -V_0 e^{-t/RC} \qquad (10)$$

Note that $V_R(t) + V_C(t) = 0$ at all times. This is just Kirchoff's Loop Rule again. It should make sense since during discharge the resistor and capacitor are directly connected to each other.

(*It might seem that you can discharge the capacitor more quickly by letting R be zero, just shorting the ends of the capacitor to each other. Sometimes this will work, other times it will destroy the capacitor because the initial current is too high.* ***So don't do it!***)

Note that charge flows *through* a resistor but *into* or *out of* a capacitor. There is no path for charge to pass from one plate to the other inside the capacitor.

B.3. Capacitors in Series and Parallel

In the DC Circuits Lab you investigated resistors in series and parallel combinations. The equations that describe such combinations are:

Series: $R_{eq} = R_1 + R_2 + R_3 + \ldots. \qquad (11)$

Parallel: $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots \qquad (12)$

The corresponding equations for capacitors connected in series or parallel are derived in your textbook. They are similar to the equations for resistors except that the forms for series and parallel are reversed.

Series: $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots. \qquad (13)$

Parallel: $C_{eq} = C_1 + C_2 + C_3 + \ldots. \qquad (14)$

C. Apparatus

For the first part of this lab your TA will use a small light bulb mounted on a board with a 1 F, 5.5 V capacitor, charging the capacitor from a 5 V power supply. The bulb is a #48, rated at 2 V, 0.06 A, 1/8 watt. For the second part of this lab you will use a Pasco board, 1 kΩ and 10 kΩ resistors, a 47 μF capacitor, DMM, scope, function generator and a computer with data acquisition capabilities. There will be two LC meters to measure capacitances. The manufacturer of the LC meters claims they have an accuracy of ± 2% ± 1 digit. (The regular DMMs can only measure capacitances less than 20 μF.) The hardware and software used with the computers are called the LabPro interface with *LoggerPro 3* software.

D. Qualitative Measurements

(Your TA will show you this part.) The apparatus for this section is illustrated in Figure 5 (next page). This quick experiment is designed to demonstrate charging and discharging of a capacitor with a system that lets you observe the flow of charge by watching the glow of a small incandescent bulb as current flows through the circuit.

You might remember from an earlier lab that the resistance of a light bulb filament is typically a few ohms. When connected to a 1 F capacitor, as in this circuit, the time constant is a few seconds. Assuming that the capacitor is initially discharged, consider what should happen when you first connect a 5 V power supply to this circuit. The capacitor acts as a short circuit, since initially there are no charges on the plates to repel other charges, and the current immediately increases to $I = (5\text{ V})/R_{\text{bulb}}$. As the capacitor charges the current decreases, in the form of an exponential decay, to zero as described by Eq. 2 and shown in Fig. 2. What you should observe is that the light bulb glows when you connect the power supply and a large current begins to flow. The bulb then dims over a few seconds as the current diminishes. Even if the power supply remains connected indefinitely, the capacitor soon acts as an open circuit, the current falls to zero and the bulb gets dimmer until it stops glowing.

If you now disconnect the power supply (*disconnecting itself will cause no change*) and short out the connectors so that the capacitor can discharge through the bulb, the bulb will again glow but will also again dim over a few seconds. The current undergoes an exponential decay as it did during the charging of the capacitor; the only difference is that the current is now flowing in the opposite direction, something that you can't tell just by looking at the bulb. In the more careful measurements that you make later, you will be able to see the change in sign of the current.

You may wonder where the inverse exponential comes into the picture. If you could see the voltage across the capacitor during the original charging process in addition to the current through the bulb you would see that this voltage does behave as an inverse exponential, starting at zero and increasing until it matches the output of the power supply.

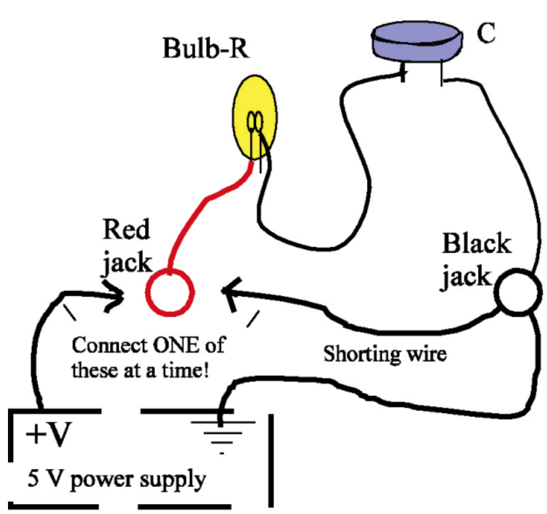

Figure 5: One farad capacitor and bulb.

E. Quantitative Measurements

E.1. Measuring *Q* and *I*

While we can't measure the charge on a capacitor directly with our DMMs, we can measure the voltage across the capacitor and calculate the charge from Eq. 1. Similarly, we can determine the current passing through a resistor by measuring the voltage across the resistor and applying Ohm's Law. In the following sections, we refer to these operations as "measuring" the charge and current.

E.2. RC Circuits

The circuit diagram for this experiment is illustrated in Figure 6 (see next page). When the charging pushbutton, CP, is held down, the RC circuit is connected to the DC voltage source V and the capacitor is charged through the resistor. The capacitor should remain charged when this pushbutton is released. *A real capacitor will slowly discharge through its own internal resistance. This can take seconds or weeks, depending on the type of capacitor*. When the discharging pushbutton, DP, is subsequently held down, the capacitor will discharge through the resistor.

The circuit *layout* is shown in Figure 7. Study the diagram carefully until you understand the current paths through the circuit. Note the function of the two pushbutton switches. These only make contact while you depress the button. You must **construct this circuit on the Pasco board using the 3 V battery pair, a 10 kΩ resistor and a 47 μF capacitor.** (*Note that some of the 47 μF capacitors have a preferred polarity; if one lead on your capacitor is longer than the other, make sure it is on the high-voltage end of the circuit.*) **Measure the values of the resistor and capacitor with a DMM**: you will have to wait your turn for an LC meter; **measure and record the capacitance of *two* of your capacitors**; the second for use in Part H.) and use these measured values rather than

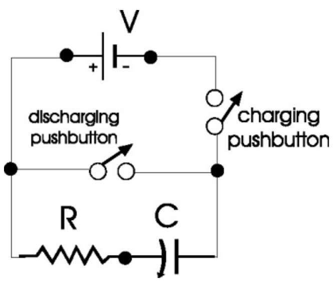

Figure 6: RC Circuit Diagram

the *nominal (labeled)* values in any calculations you are asked to perform. (*To measure capacitance with your DMM, use the F, for farads, setting of the knob and insert the capacitor into the special slot labeled C_x which is located above the banana jacks for current inputs.*)

E.3. *LoggerPro* Program

Bring up the *LoggerPro* program. It uses a modern Windows™ interface. **Open the file P:/LoggerPro 3/_E and M Labs/RCCIR.** (If the default folder hasn't been changed, you should be in the Experiments folder when you go to the Open command.)

E.4. General Measuring Procedures

Note carefully which operations you are asked to record and to fit. Try to fit each curve directly in the *LoggerPro* program as soon as you acquire it. Be certain to save each set of data before taking the next set of measurements so that you can analyze or re-analyze the data later.

Each measurement involves recording the voltage across the resistor or the capacitor

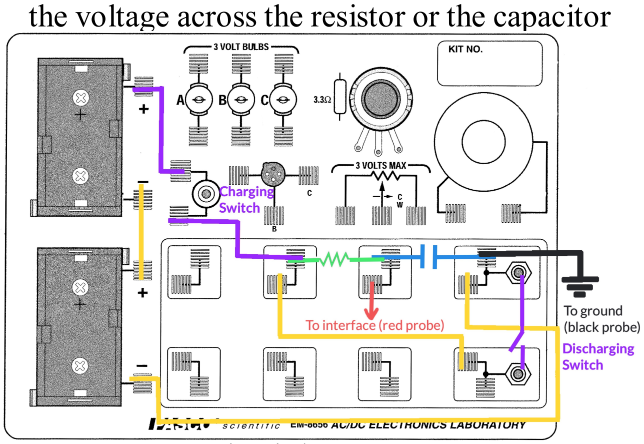

Figure 7: RC circuit layout

as time passes. By fitting the appropriate equation to these data, you are effectively measuring the time constant of the circuit.

E.5. Charging and Discharging

There are two main operations, *Charge and Discharge*. The next two paragraphs describe these processes in general; ***the specific instructions for taking careful data to be included in your worksheet are given in Section E.7.***

E.6.1. Cycling Test

Before you move on to careful experiments, **start a sweep in *LoggerPro* by clicking on the "collect" button. Try pressing and holding first the charge and then the discharge buttons, cycling between them every few seconds, while watching *LoggerPro* plot the data.** You should be able to create *sawtooth* patterns on the plot, ramping the voltage up and down within the limits of zero and +3 V. If you don't see this behavior, check your circuit. If you can't spot the problem, ask a TA for help. If your curve goes up to less than 2 V, you may need to turn the capacitor around.

E.6.2. Charge

Pushing and holding the charging pushbutton (CP) on the left side of the Pasco board should charge the capacitor to about 3V. Before taking any careful measurements of the charging process, you should have first thoroughly discharged the capacitor using the discharging pushbutton (DP) as described below.

After charging the capacitor, release the pushbutton to leave the capacitor in the charged state.

E.6.2.a. Charge with LoggerPro

To record charging data, we should first **set the triggering so that hitting CP will start *LoggerPro* collecting data. Go to EXPERIMENT/ DATA COLLECTION. Choose the "Triggering" tab. Click the "Trigger on Sensor Value" box, make sure**

the "Increasing" radio button is selected, and set the threshold (marked "Across") to 0.05 V. Press the DP until the "Potential" reading on the screen drops to 0.03 V or so. Click the "Collect" button on the screen and then press CP. You should see a graph of potential versus time. If *LoggerPro* improperly triggered (or did not trigger at all), you may have to stop data collection, discharge the capacitor, and click the "Collect" button and press CP again.

E.6.3. Discharge

You will normally do this only after charging it as described above, so a normal sequence to record a discharge will be to hold the left pushbutton down for 30 seconds or so to charge the capacitor completely and then to hold the right pushbutton down to record the discharge.

E.6.3.a. Discharge with LoggerPro

To record discharging data, we should first **set the triggering so that hitting DP will start *LoggerPro* collecting data. Go to EXPERIMENT / DATA COLLECTION**. **Choose the "Trigger" tab. Make sure the "Enable Triggering" box is checked, make sure the "Decreasing" radio button is selected, and set the threshold to about 0.1 V less than the potential of the batteries** (2.9 V for a fresh pair.) **Press the CP until the "Potential" reading on the screen increases to 0.05 V or so above the threshold. Click the "Collect" button on the screen and then press DP.** You should see a graph of potential versus time. If *LoggerPro* improperly triggered (or did not trigger at all), you may have to stop data collection, discharge the capacitor, and click the "Collect" button and press DP again.

E.7. Data for Analysis

E.7.1. Charge on Capacitor

Now we will measure the charge on the capacitor vs. time as we charge and discharge the capacitor. Note that *LoggerPro* only measures voltages but the charge on the capacitor is just $Q = CV$ so the data you take will just need to be multiplied by a constant to convert from volts to charge.

1. **Connect *LoggerPro* across *C*** as in Fig. 7.
2. ***Discharge* fully**
3. ***Charge*** (Record and fit.) Read Section F for guidance on fitting this data; you should make this fit before taking the discharge data below. You should also **save your work before going on**.
4. ***Discharge* (Record, fit and save.)**

E.7.2. Current through Resistor

Measure the variation with time of the current through the resistor during charging and discharging. *LoggerPro* only measures voltages but the current through the resistor is just $I = V/R$.

1. **Connect *LoggerPro* across *R*, with the black connector between *R* and *C* and the red connector on the other side of *R***. This lets the interface monitor the voltage across this resistor. Those using *LoggerPro* should think carefully about the proper settings for the triggering.
2. ***Charge*** (record, fit and save the data.)
3. ***Discharge*** (record, fit and save.)

F. Analysis

F.1. Fitting Functions for the data

You will need to determine for each of your fits whether to use the *Exponential* function or the *Reverse Exponential.* An easy way to think about this is to recognize that the term $e^{-t/RC}$ goes to 1 as $t \to 0$ and goes to 0 as $t \to \infty$. Therefore the standard exponential decay function, $A*e^{-t/RC}$ describes a curve that at time $t = 0$ is at some value, A, which may be positive or negative, and decays to 0 as time passes. By contrast, the reverse exponential function, $A*(1-e^{-t/RC})$ describes a curve that starts at 0 and increases towards the constant A, which again may be positive or negative. You should see that the *Reverse*

Exponential function is the correct equation for analyzing the voltage on the capacitor during charging, since this voltage starts at (*or near*) zero and ends up at about 3 V.

It is possible to choose *LoggerPro* function from the list provided by the software package and then edit it to the form you need. However, it is probably easier to type in your own function from scratch. The basic exponential function should be entered as

A*exp(-t/K)

where K is the time constant that you wish to find. The inverse or reverse exponential usually has the form

A*(1-exp(-t/K))

However, you would probably find that these equations fit your data poorly. The reason is that *LoggerPro* will trigger *after* you press a pushbutton, so your plots don't start at either 0 or 3 volts. The fix for this is to insert another parameter, an offset, into your fitting equations. So, assuming you set the trigger properly with *LoggerPro* you should type one of the following into *LoggerPro*,

Exponential Decay

A*exp(-t/K) + B

or

Reverse Exponential

A*(1-exp(-t/K)) + B

Since B is an arbitrary fitting parameter, this last equation can be simplified to just

B - A*exp(-t/K)

(*Those who are paying attention may notice that the last equation can actually be used for exponential as well as reverse exponential fits. You may do this if you like, even though it obscures the fundamental difference between these two functions.*)

If you pressed the ENTER key before starting the charge or discharge cycle, you have a bit more work to do for a proper fit and will have to work out the details on your own. It may be easier just to retake your data.

The ***LoggerPro* program should be used to perform a least-squares fit and calculate the time constants for each of the four sets of data you collected in Section E.** (*The detailed instructions for doing these fits are included in Appendix II at the end of this write-up.*) **Include printouts from these fits with your worksheet**.

F.2. Presentation Quality fits with *Origin*

LoggerPro does not make presentation-quality graphs; **to make your graphs "pretty," you will need to transfer them to *Origin*. You only need to do this for the <u>charging data for the capacitor graph</u>. Include a printout from this fit with your worksheet.**

You will have to **transfer your data from *LoggerPro* to *Origin*.** This can be accomplished by **highlighting both data columns in *LoggerPro*, then just EDIT and COPY from the menu. Switch to *Origin*, highlight columns A and B and select EDIT/PASTE. You'll have to delete the first few rows of the data copied form *LoggerPro***. You can now **make a scatter plot of this data.** Double-click on one of the data points. A menu should pop up that lets you **change the size of the data points to 3.** This is useful because the default size is too large to let you judge the quality of the fits.

Now **go to ANALYSIS → FITTING → NON-LINEAR CURVE FIT → OPEN DIALOG, enter the proper fitting function within *Origin*, and complete the fit**. (*If you don't remember how to do this, refer back to Lab #2, EPF Section D3.*) You should note how the fitted parameters for this set of data compare between *LoggerPro* and *Origin*.

Important: You are only going to plot *one* of the data sets in *Origin*. However, in order to do the analysis in the best possible way you want to also **plot the *y*-error-bars** (here assume that the *x*-error bars can be ignored). So you need to **make an *estimate* as to the value of delta-y for each point.**

Warning: Do not use the "exponential fit" function in *Origin*. This might plausibly work, but it does not. Instead use the "non-linear fit" and make a user-defined function that looks just like the function from *LoggerPro*:

y = A*(1-exp(-x/K)) + B

Here, A, K, and B will be your parameters that you will allow to vary in the non-linear fit. You can **start by using the numbers that are given by the *LoggerPro* fit, then run several iterations to get the best fit in *Origin*. Be sure to write down the best value for K and the uncertainty on this from *Origin*.**

G. Uncertainties

You now have five independent measurements of the time constant $\tau = RC$:

1. Measured using DMM values of R and C in Eq. 3.
2. Measured voltage across capacitor while **charging** using *LoggerPro*.
3. Measured voltage across capacitor while **discharging** using *LoggerPro*.
4. Measured voltage across resistor while **charging** using *LoggerPro*.
5. Measured voltage across resistor while **discharging** using *LoggerPro*.

For one of these (#2) you also copied the data over to *Origin* and did a non-linear fit. <u>Compare these five values of the time constant to each other and discuss whether or not they are in agreement, within estimated errors.</u>

Our model of an RC circuit says that the time constant from our fits should be the same as the directly measured value (from the DMM measurements). To get one value from the fits to compare with the directly measured values, **calculate the mean and standard deviation of your four values of the time constant from fitting**. (Four numbers is not generally considered enough to justify a statistical analysis but do so anyway.) **<u>Report the mean value as your best estimate of the time constant and the standard deviation as your best estimate of its uncertainty and compare this estimate to the directly measured value.</u>**

Finally, since the uncertainty that you got from the *Origin* plot is only as good as the fit that was done, you should examine the non-

(a) (b)

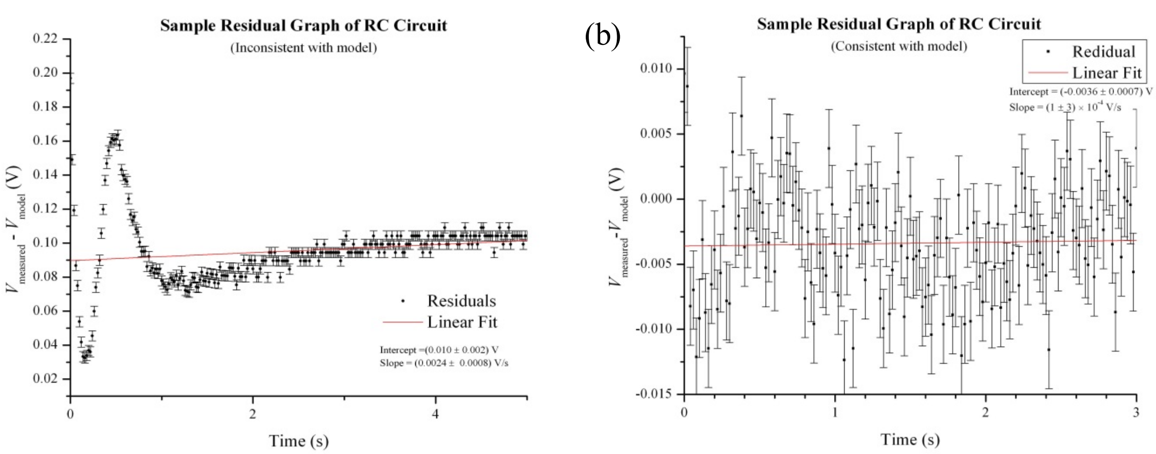

Figure 8: Sample residual graphs. a) Residuals show the model does not fit this data well. b) Residuals show that the model fits this data well.

linear fit that was done in *Origin* and ask yourself this: Does the data that I took really support the (exponential decay) model that I used in the fit? In other words, do the data fall on the model to within the error bars? For this experiment it is not easy to tell, because the error bars are so tiny (about 0.000001 - 0.0001, if you determined them correctly, right?). In cases like this, physicists often make a second plot which is called a plot of **the residuals.** The residuals are defined as the difference between the model (the fitted curve) and the y-value of your data points (the physically measured data) for each x-value. To **make a plot of the residuals** you want to **make a new column in Origin that is defined to be this quantity.**

For example, suppose your non-linear fit gives values for the offset, normalization, and time constants (A, B, and K) of 2.55, 4.33 and 4.88 respectively. If your data are in three columns labeled, e.g., TIME, VOLTAGE, and DVOLTAGE then you can **create a new column of residuals defined in Origin by using the "SET COLUMN" option** as follows:

Col(V)-(2.55*(1-exp(-1.0*Col(T)/4.88))+4.33)

Then make a new plot, using this new column **as the "Y" column and using your old TIME and DVOLTAGE columns for X and error-bar-*y* values respectively**. You will get a plot that should be roughly horizontal with each point randomly scattered above or below the line. If your fit was a **good** fit then two-thirds of the points will have a y-value of zero within one error bar (roughly) and there will be no obvious trend (*e.g.*, first part of residual plot has points all greater than zero, last part all less than zero). Does the residual plot indicate you had a good fit or a bad fit? If the fit wasn't good, then the model isn't accurately describing the data.

Figure 8a shows a sample plot of the residuals where the first 1.5 s or so of data is not well described by the model. Note how the data starts above the best fit line, goes well below it, then well above, then below, then after about 2 s curves up to the best fit linear line. Also note that the slope of the best fit line is non-zero. Fig. 8b was produced from a different data set. Note how many of the error bars overlap zero and the best fit straight line has a slope consistent with zero. The residuals in Fig. 8b. Show that the second data set is well fitted by the model. See Appendix III for a note on interpreting residuals.

H. Capacitors in Parallel and Series

After measuring its value with your DMM, **add a second 47 μF capacitor in parallel with the first one and re-take the charging curve while measuring the voltage across this pair of capacitors. Fit this curve with *LoggerPro*.** Compare the time constant that you find now with the value expected from direct measurement of the

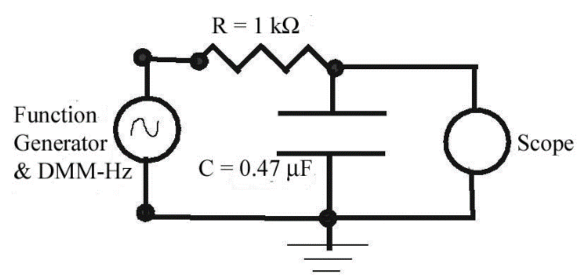

Figure 9: Low Pass Filter Diagram.

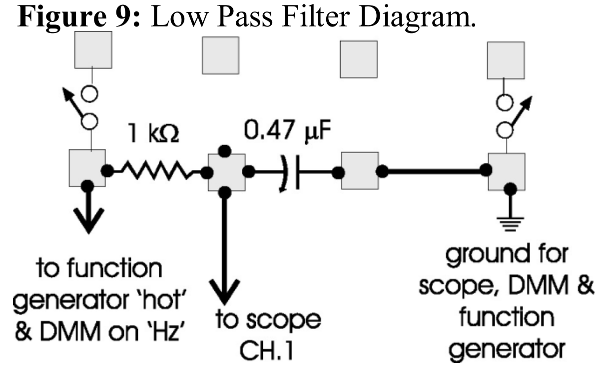

Figure 10: Low Pass Filter Layout

resistor and two capacitors. Explain the change in the time constant from Part G using Equations 13 and/or 14.

Remove the new, parallel capacitor and place it in series with the original capacitor. Once again, re-take the charging curve while measuring the voltage across the pair of capacitors. Fit this curve with *LoggerPro*. Compare the time constant that you find now with the value expected from direct measurement of the resistor and two capacitors. Explain the change in the time constant from Part G using equations 13 and/or 14.

I. Filters and Analog Calculus

If you were to replace the 10 kΩ resistor with a 1 kΩ resistor and the 47 μF capacitor with a 0.47 μF capacitor, what would the theoretical time constant be now?

Disconnect the batteries and probes and, construct the circuit diagrammed in Fig. 9 (and laid out in Fig. 10) **using your function generator and scope. Be certain the function generator is set to deliver sine waves**; previous groups may have left it on another setting. Connect the BNC ends of the two cables to CH1 Output of the function generator and CH1 of the Oscilloscope.

The scope is used to monitor the signal amplitude (Voltage). **Temporarily move the positive end of the scope probe to the function generator "hot" output on the Pasco board** (*"hot" means the positive, in this case the red, connection*) and **set the function generator to 5 V_{pp} at 500 Hz.** (Look on the lab website for images, if you cannot remember how to operate the function generator.) Check to see what the OSC reads for voltage and frequency, and record the values in your notebook.

Put the scope probe back to its operating position as shown in Figs. 9 and 10. Record the peak-to-peak voltage of the signal (*measured with the scope*) **as a function of the frequency** (*measured with the DMM*). **Take data at frequencies near 10, 20, 50, 100, 200, 500, 1000, 2000, 5000, and 10000 Hz.**

When something varies over two or more orders of magnitude, as the frequency does in this plot, one often uses a log scale to plot it. One should vary the spacing of the data points appropriately, so that they are spaced out evenly on the log scale; this is the reason for the frequency values given above. The amplitude you measure will also vary over a wide range; so after you **plot your data in *Origin* you should convert both axes to log scales (*log base 10*) to create a so-called log-log plot**. Origin lets you convert a 'normal' plot to a log scale by **double-clicking on the plot axis and, on the *SCALE* tab, changing "type" from "*linear*" to a "*log10*" setting.**

What is this circuit doing to the signal from the function generator? The log-log plot should make this clear. You should find that the signal is roughly constant at low frequencies but decreases linearly (*on a log-log plot*) at high frequencies, where *low* and *high* are relative to a frequency $f = 1/(2\pi RC)$. (*f should be 200 - 300 Hz for your circuit*). (*This does NOT mean you should attempt a linear fit to your data; it obviously is NOT linear over the entire range of frequencies.*) The falloff is properly described as 6 decibels per octave; this means the signal falls off by a factor of two for each doubling of frequency. You have built a low pass filter; low frequency signals pass through unaffected while high frequency signals are attenuated. You might know this device as a tone control; this circuit cuts the treble.

If you were to reverse the resistor and capacitor in the circuit, you would have a high pass filter; high frequency signals pass easily but low frequency signals are attenuated, *i.e.* a bass control. Both circuits may be understood by thinking of a capacitor as a frequency-dependent resistor; a capacitor has a low resistance for high frequency signals and a high resistance for low frequency signals.

Another aspect of the RC circuit may be understood by switching to the square wave output of your function generator. **Feed in a 5 kHz square wave while watching the output on the scope**. You should see a triangle wave. How does one get a triangle wave from a square wave? By taking the *integral* of the square wave. This should make sense since the voltage across the capacitor depends on the charge it is storing, which equals the time integral of the current flowing into it. *Why didn't you notice that the sine waves you used earlier were also being integrated?*

One can also make an analog differentiator from an RC circuit. **Swap the capacitor and resistor for each other in your circuit and set the frequency to about 70 Hz.** Look first at a sine wave; its derivative should just look like another sine wave. Then **try a triangle wave and a square wave input. <u>Sketch and explain the waveforms you see for these last two inputs.</u>**

Appendix I: *LoggerPro* Fitting

To perform a non-linear fit in *LoggerPro*, you must first go to ANALYZE/ CURVE FIT. Although some of the predefined functions may be usable with some editing, it is usually easier to just click the "Define Function" button. In the "User Defined Function" box, type in the function you want (either A*exp(-t/K) + B or B - A*exp(-t/K); *LoggerPro* will remember a user-defined function until you exit the program.) You should then type in your initial guesses for the coefficients. If the fit type is "Manual," *LoggerPro* will display the function with the parameters you have guessed. This feature is useful to check to make sure your function was typed in correctly and that your initial guesses are reasonable. To have *LoggerPro* do a least-squares fit, click the fit-type radio button to "Automatic" and click the "Try Fit" button. When *LoggerPro* is done fitting, click the "OK" button and the fit and fitting parameters will be displayed on the screen.

You may have to increase the number of significant figures to get enough information to properly write the fitting parameters as measurement intervals. To do this, right-click on the "Auto fit" box and select "Curve-fit helper options." The "Displayed precision" is the option that controls the number of significant figures. Note that *LoggerPro* doesn't know our rounding rules any better than *Origin* does.

Appendix II: A Note on Interpreting Residuals

Analysis of residuals is often important in science to develop more sophisticated models and a better understanding of the Universe. One example of looking at residuals is the analysis of the cosmic microwave background (CMB) data from the Cosmic Background Explorer (COBE).

The top of Fig. 11 shows a map of the nearly uniform temperature of the CMB. According to NASA (http://lambda.gsfc.nasa.gov/product/cobe/), "the cosmic microwave background (CMB) spectrum is that of a nearly perfect blackbody with a temperature of 2.725 ± 0.002 K. This observation matches the predictions of the hot Big Bang theory extraordinarily well, and indicates that nearly all of the radiant energy of the Universe was released within the first year after the Big Bang."

Looking only at the top map of Fig. 11, one might conclude that this model of a nearly uniform CMB temperature adequately described the data. If one subtracts off 2.725

K from every point on the top map, one is left with the middle residual map with variations of 0.003353 K on either side of the average 2.725 K. Furthermore, these variations are not random fluctuations, but organized in a "dipole anisotropy." This anisotropy is explained by our motion relative to the CMB; radiation from the direction we are traveling towards is hotter and radiation from the direction we are traveling away from is cooler.

If we revise our model of the CMB (uniform + our motion) and subtract it from the COBE data, we get the bottom residual map with residuals ranging to 1.8×10^{-5} K from our model. The band at the center of this map is due to microwave emissions from gasses in the Milky Way. Cosmologists are using this map (and similar but more precise

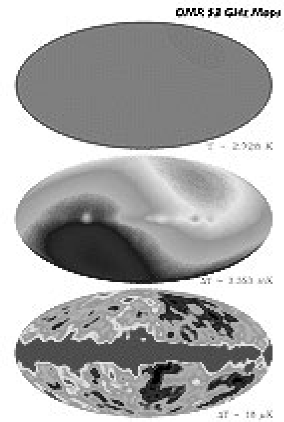

Figure 11: COBE data and residuals. Courtesy NASA/ LAMBDA (http://lambda.gsfc.nasa.gov).

data from the Wilkinson Microwave Anisotropy Probe) to test models of the early Universe.

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