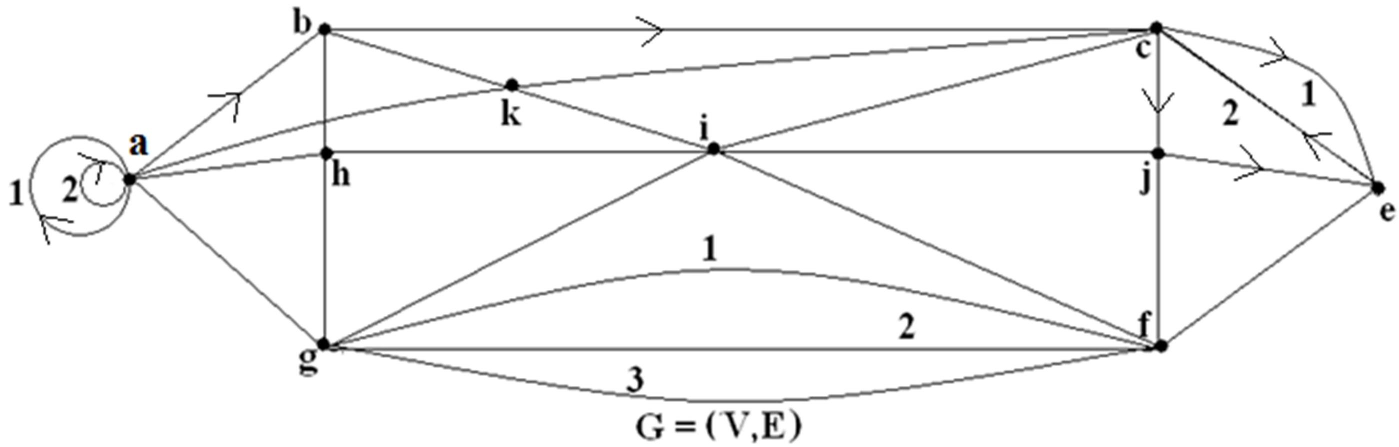
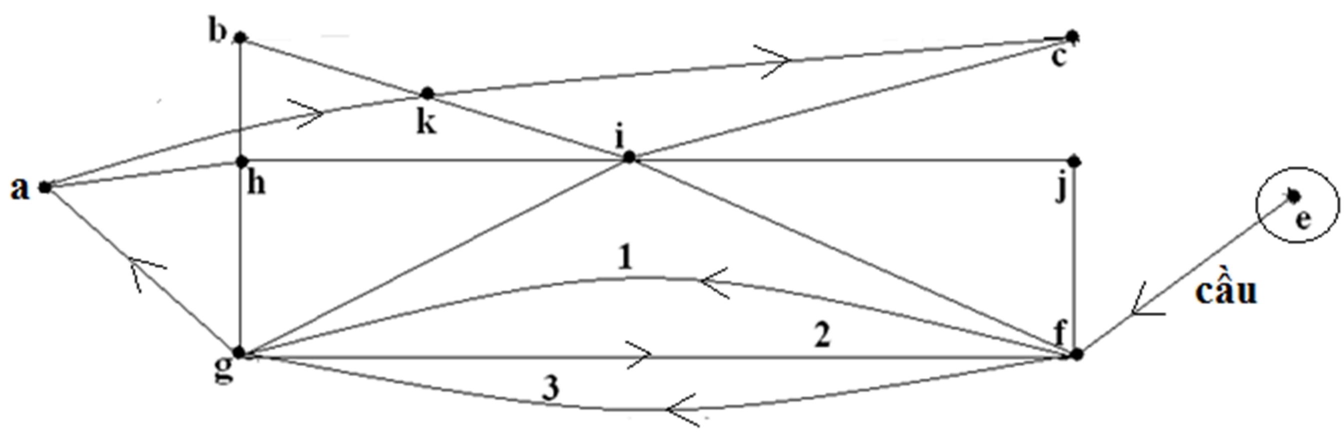
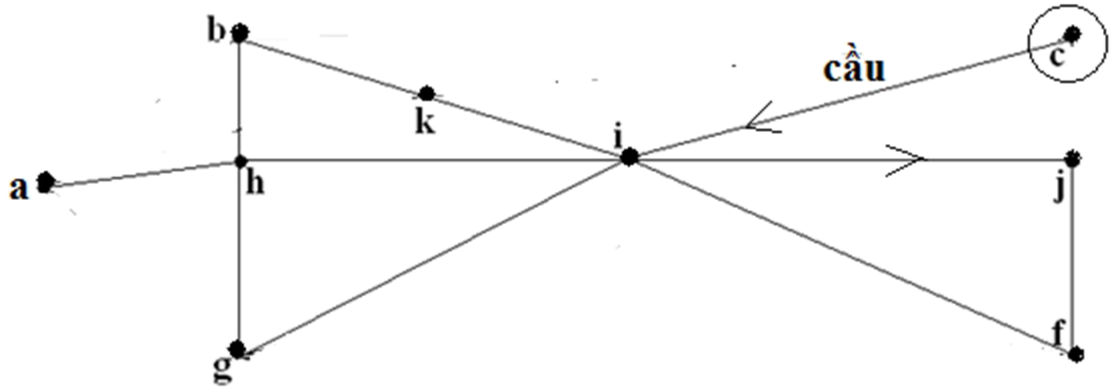
MINH HỌA CÁC BÀI TOÁN VỀ ĐƯỜNG VÀ CHU TRÌNH



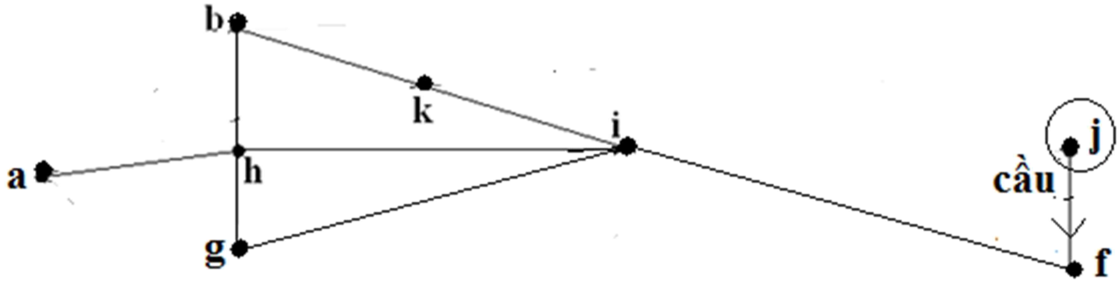
Đường $\overline{a^3bcecje}$



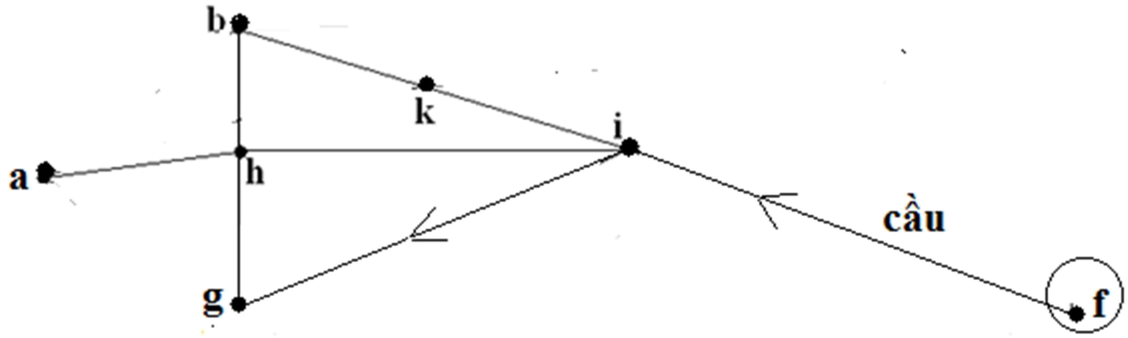
Đường $\overline{efgfgakc}$ (xóa e)



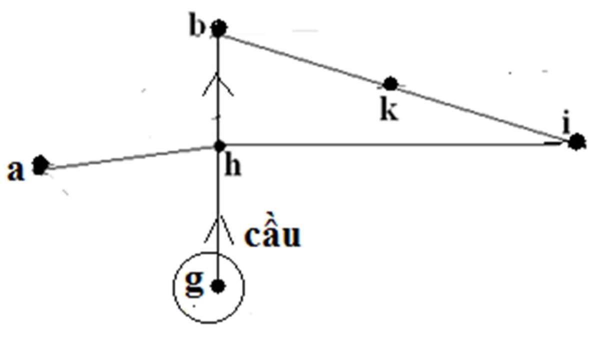
Đường $\overline{cij}$ (xóa c)



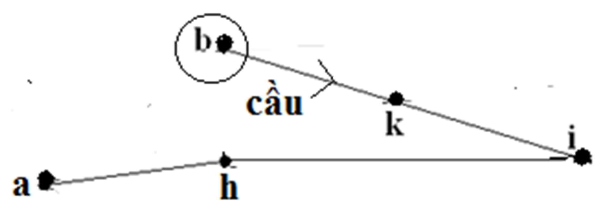
Cầu $\overline{jf}$ (xóa j)



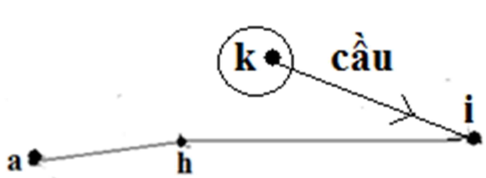
Đường $\overline{fig}$ (xóa f)



Đường $\overline{ghb}$ (xóa g)

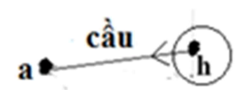


Cầu $\overline{bk}$ (xóa b)



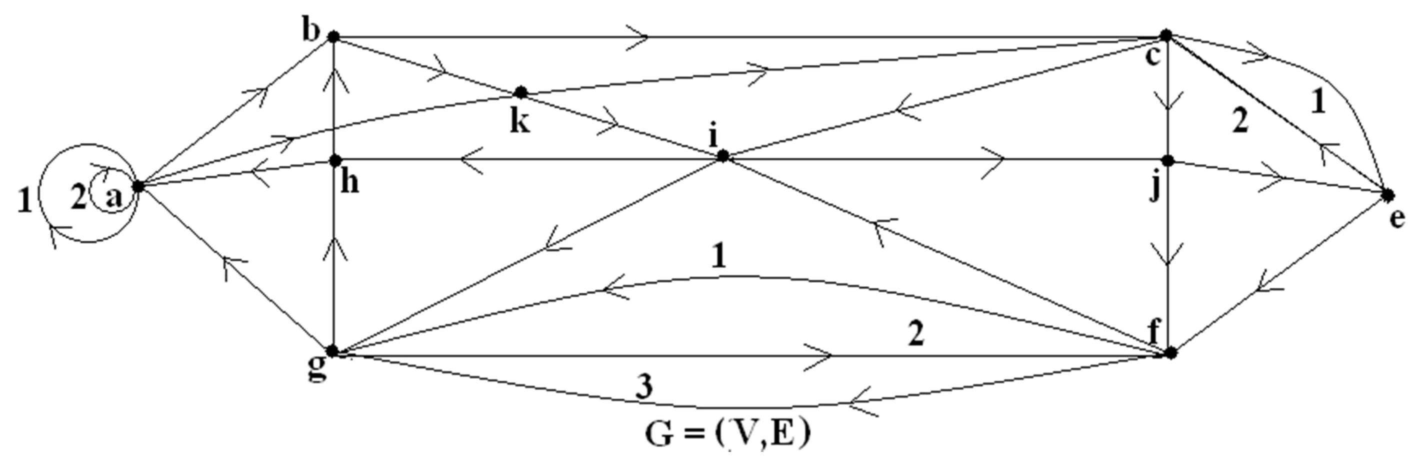
Cầu $\overline{ik}$ (xóa k)

Cầu $\overline{ih}$ (xóa i)

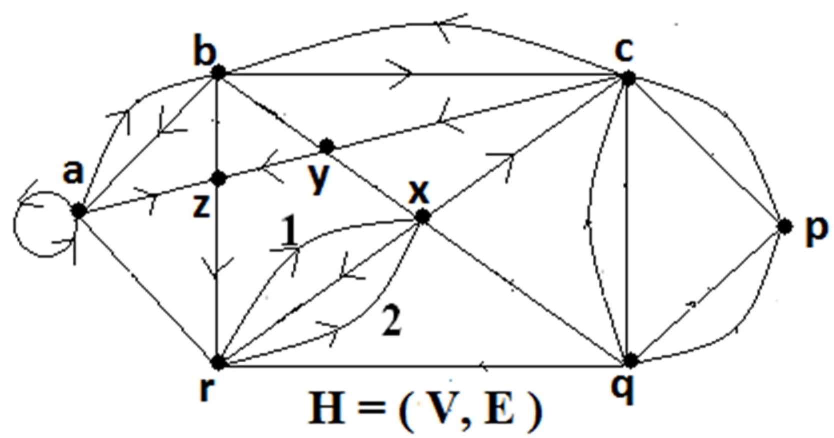


Cầu $\overline{ha}$ (xóa h)

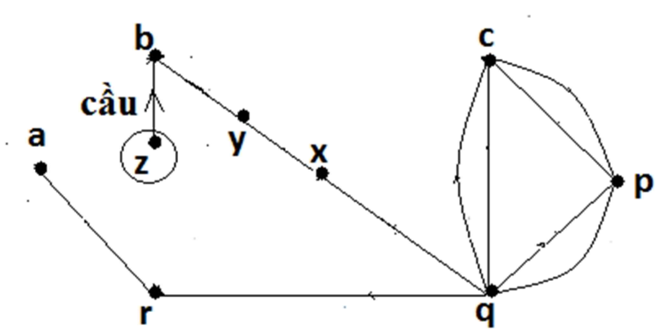
xóa a



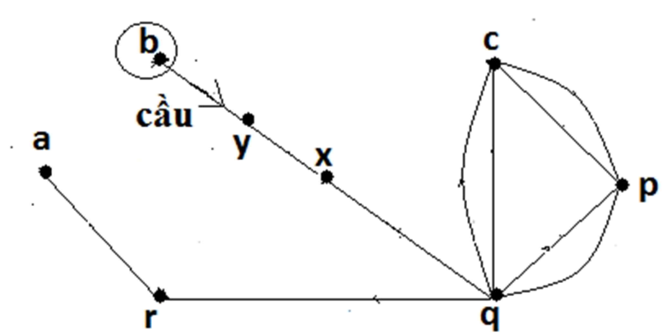
G có chu trình Euler (C) : $\overline{a^3bcecjefgfgakcijfighbkiha}$



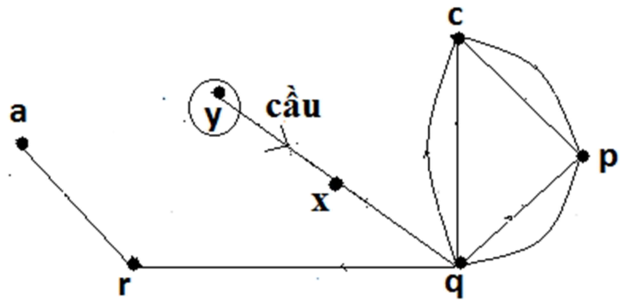
Đường $\overline{a^2bazrxrxcbcyz}$



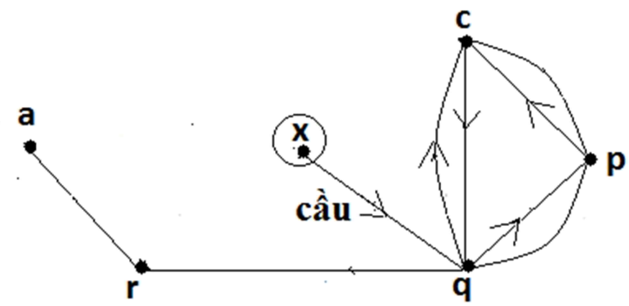
Cầu $\overline{zb}$ (xóa z)



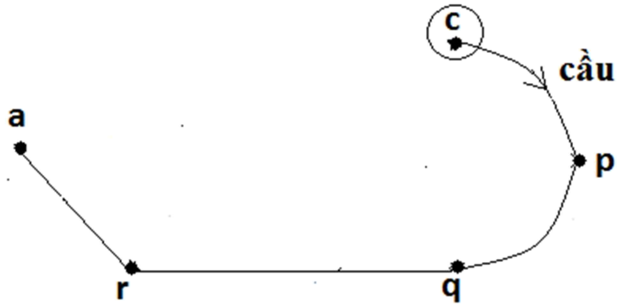
Cầu $\overline{by}$ (xóa b)



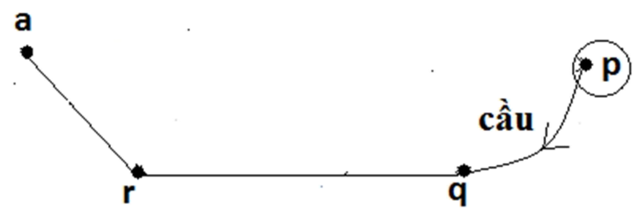
Cầu $\overline{yx}$ (xóa y)



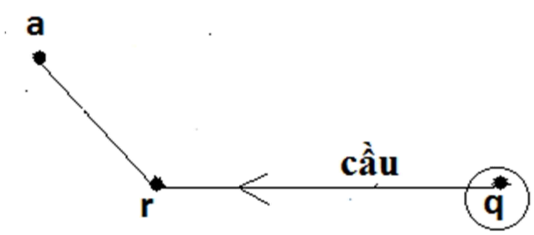
Đường $\overline{xqcqpc}$ (xóa x)



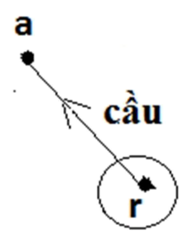
Cầu $\overline{cp}$ (xóa c)



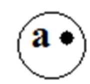
Cầu $\overline{pq}$ (xóa p)



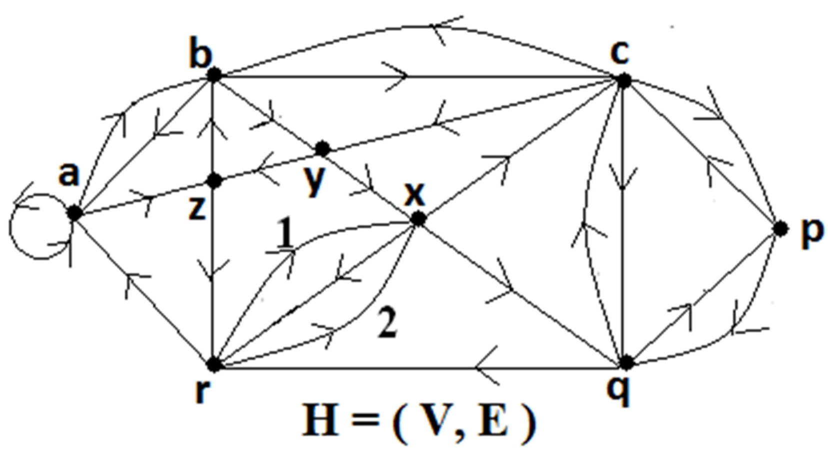
Cầu $\overline{qr}$ (xóa q)



Cầu $\overline{ra}$ (xóa r)

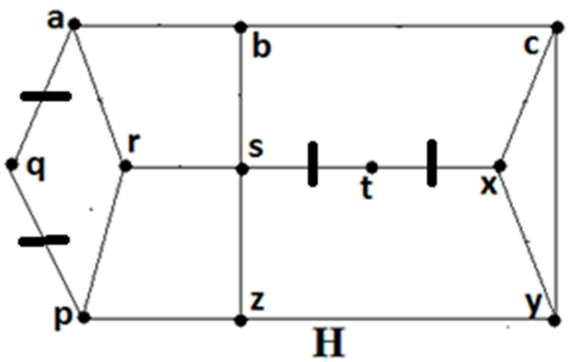


xóa a

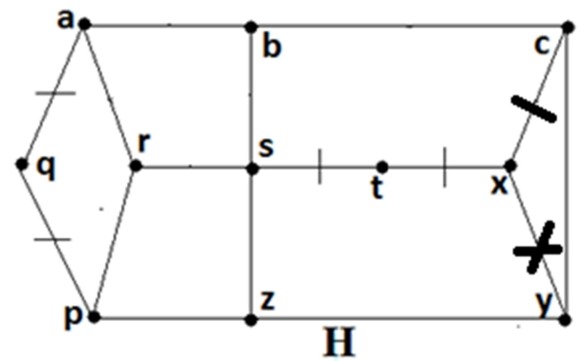


H có chu trình Euler (C): $\overline{a^2bazrxrxcbcyzbyxqcqpcpqra}$.

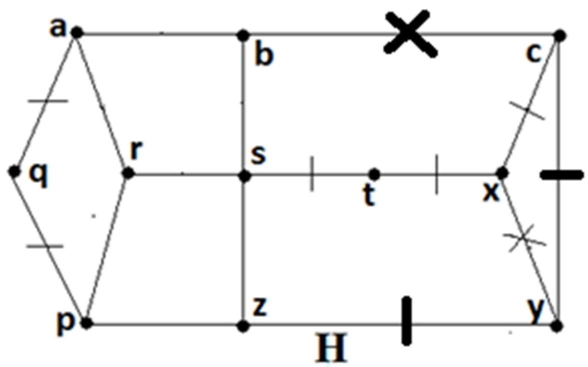
Giả sử H có chu trình Hamilton là (L).



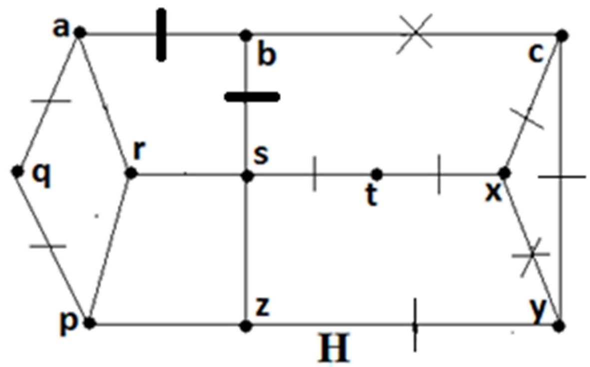
d(q) = d(t) = 2 nên $\overline{aq}$, $\overline{pq}$, $\overline{st}$, $\overline{tx} \in$ (L)



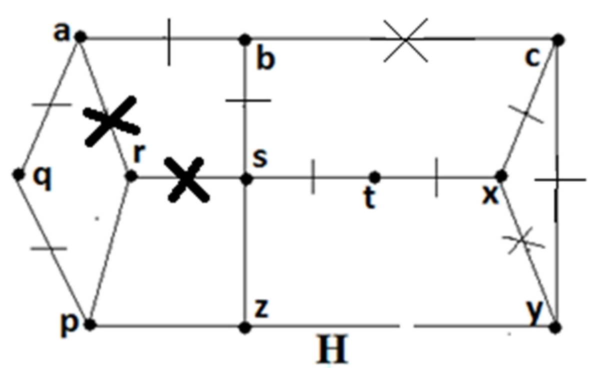
Xem như $\overline{xy} \notin$ (L). Suy ra $\overline{cx} \in$ (L)



$\overline{xy} \notin$ (L) nên $\overline{cy}$, $\overline{yz} \in$ (L) và $\overline{bc} \notin$ (L)

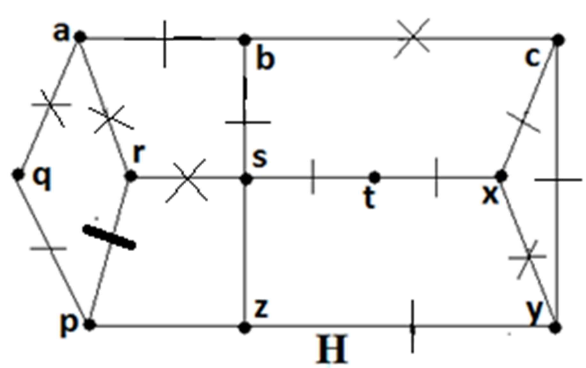


$\overline{bc} \notin$ (L) nên $\overline{ab}$, $\overline{bs} \in$ (L)



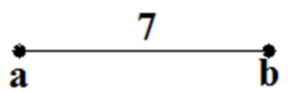
$\overline{aq}$, $\overline{ab} \in$ (L) $\Rightarrow$ $\overline{ar} \notin$ (L).

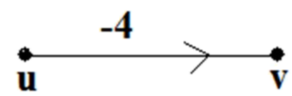
$\overline{st}$, $\overline{bs} \in$ (L) $\Rightarrow$ $\overline{rs} \notin$ (L).

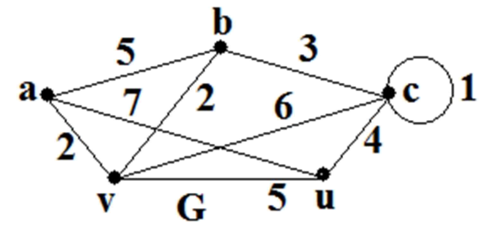


$\overline{ar} \notin$ (L), $\overline{rs} \notin$ (L) $\Rightarrow$ $\overline{pr} \in$ (L)

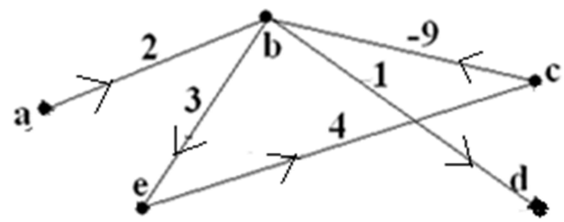
(L) chỉ đi qua một cạnh $\overline{pr}$ tại r !

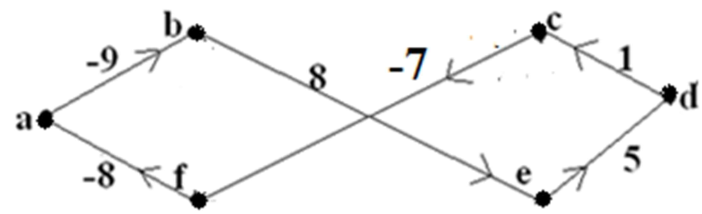






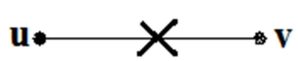
Ta có $w(\overline{ab}) = 7$, $w(\overline{cd}) = -4$ và $w(G) = 1 + 2 + 2 + 3 + 4 + 5 + 5 + 6 + 7 = 35$.



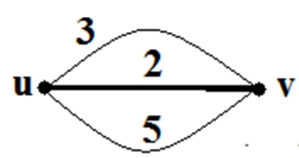


Đường đơn (P): $\overline{abecbd}$ với $w(P) = 2 + 3 + 4 - 9 + 1 = 1$.

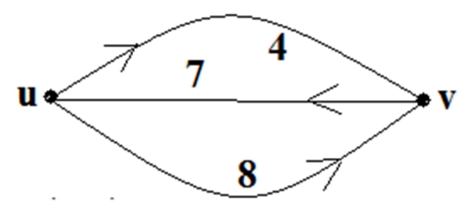
Mạch sơ cấp âm (C): $\overline{abedcfa}$ với $w(C) = -9 + 8 + 5 + 1 - 7 - 8 = -10 < 0$.



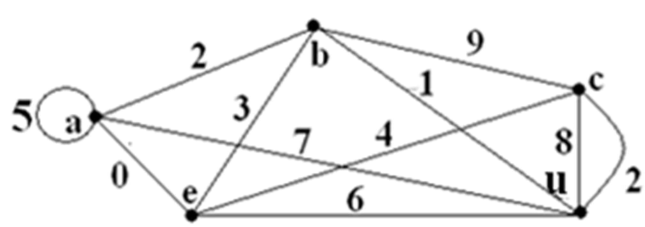
$d(u,v) = +\infty$



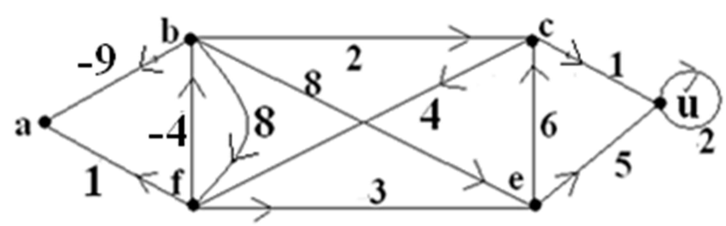
$d(u,v) = 2$



$d(u,v) = 4 \neq 7 = d(v,u)$

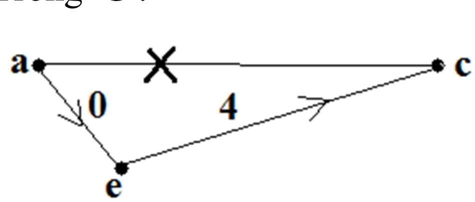


G

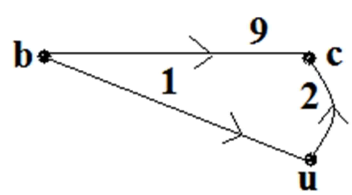


H

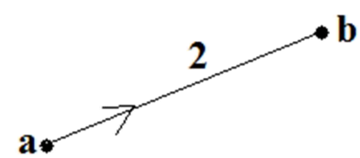
Trong G :



$d^*(a,c) = 4 < d(a,c) = +\infty$

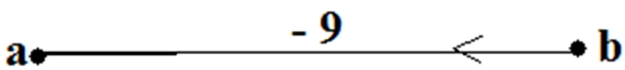


$d^*(b,c) = 3 < d(b,c) = 9$

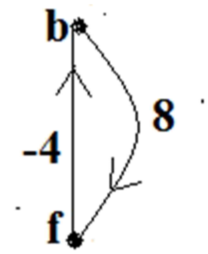


$d^*(a,b) = d(a,b) = 2$

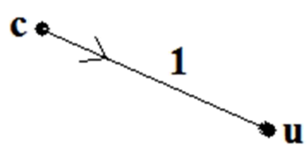
Trong H:



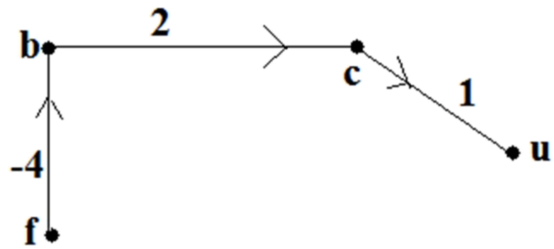
$d(b,a) = -9 \neq d(a,b) = +\infty$



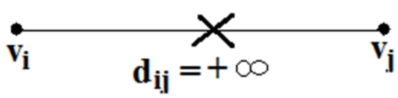
$d(b,f) = 8 \neq d(f,b) = -4$

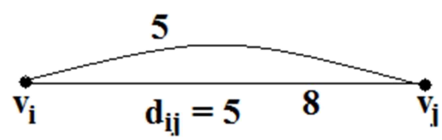


$d^*(u,c) = +\infty = d(u,c)$, $d^*(c,u) = d(c,u) = 1$



$d^*(f,u) = -1 < d(f,u) = +\infty$

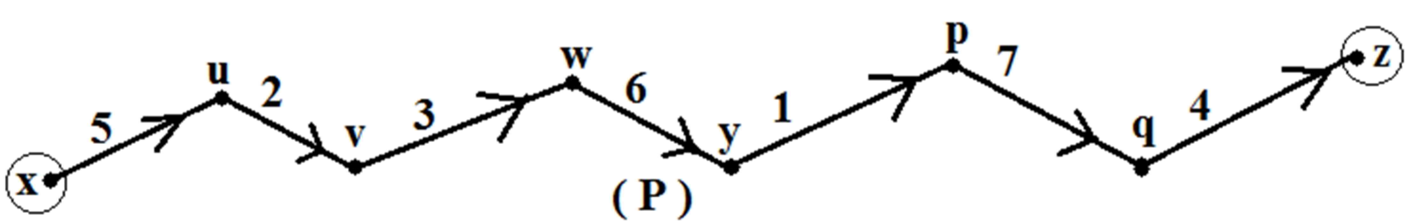


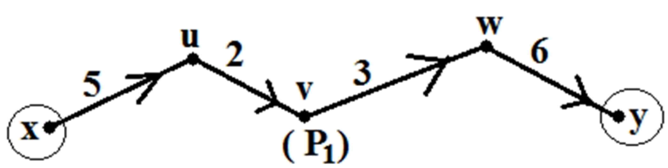


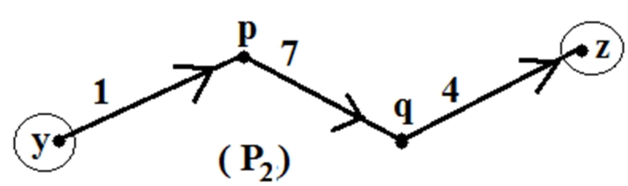
v_i • $(d_{ii} = 0)$

Cho đồ thị liên thông có trọng số G.

* Xét (P) là *đường đi ngắn nhất* (*tuyệt đối*) từ đỉnh x đến đỉnh z trong G. Giả sử đỉnh y (vô tình) thuộc (P). Đặt (P_1) và (P_2) lần lượt là phần đường (P) nối (x đến y) và nối (y đến z). Khi đó (P_1) và (P_2) lần lượt cũng là *đường đi ngắn nhất* nối (x đến y) và nối (y đến z). Hơn nữa $w(P) = w(P_1) + w(P_2)$.

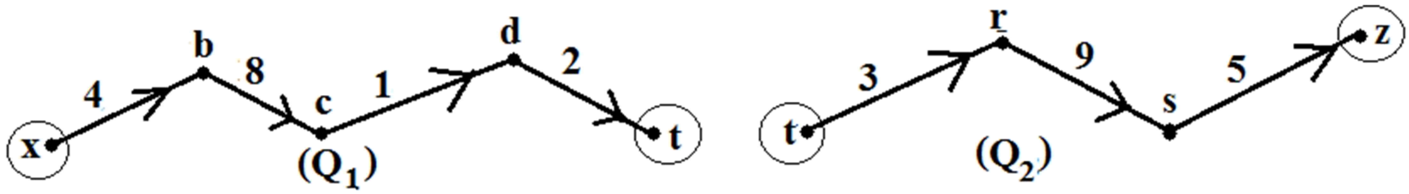


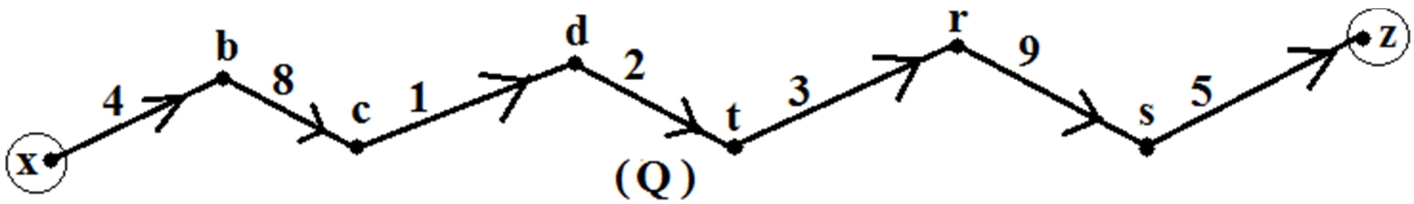




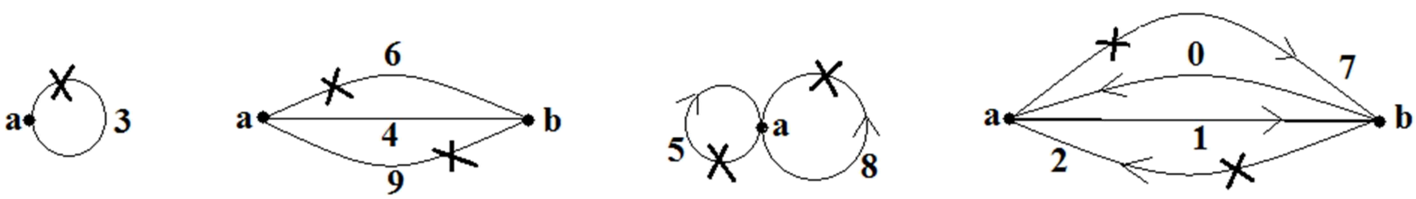
$w(P) = w(P_1) + w(P_2) = (5 + 2 + 3 + 6) + (1 + 7 + 4) = 16 + 12 = 28.$

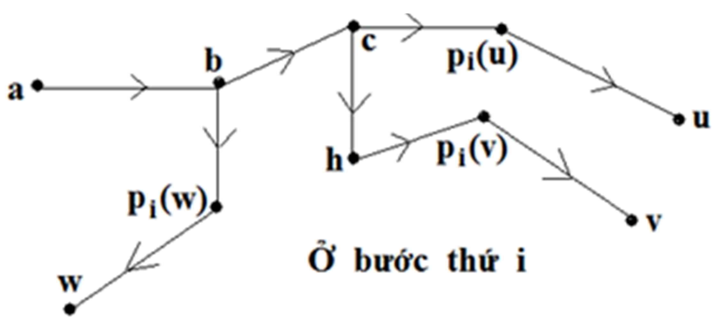
* Xét đỉnh t của G và **t** $\notin$ (P). Gọi (Q) là *đường đi ngắn nhất* từ đỉnh x đến đỉnh z trong G sao cho (Q) *phải đi qua* **t** (*có điều kiện*). Đặt (Q_1) và (Q_2) lần lượt là *đường đi ngắn nhất* nối (x đến **t**) và nối (**t** đến z). Khi đó $w(Q) = w(Q_1) + w(Q_2) \geq w(P)$.





$$w(Q) = w(Q_1) + w(Q_2) = (4 + 8 + 1 + 2) + (3 + 9 + 5) = 15 + 17 = 32 > w(P) = 28.$$



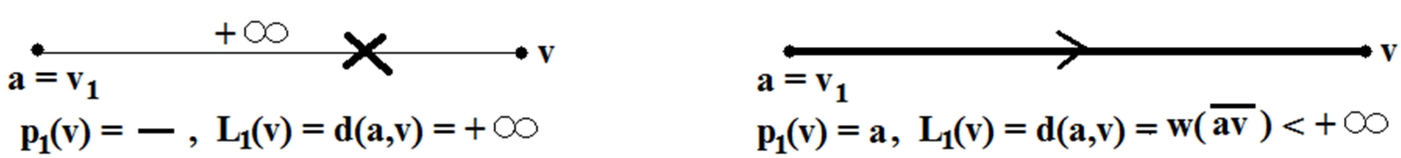


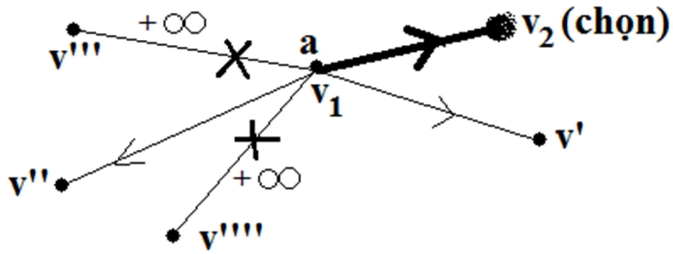
$L_i(u)$ = độ dài đường tạm thời từ a đến u

$L_i(v)$ = độ dài đường tạm thời từ a đến v

$L_i(w)$ = độ dài đường tạm thời từ a đến w

BƯỚC 1:

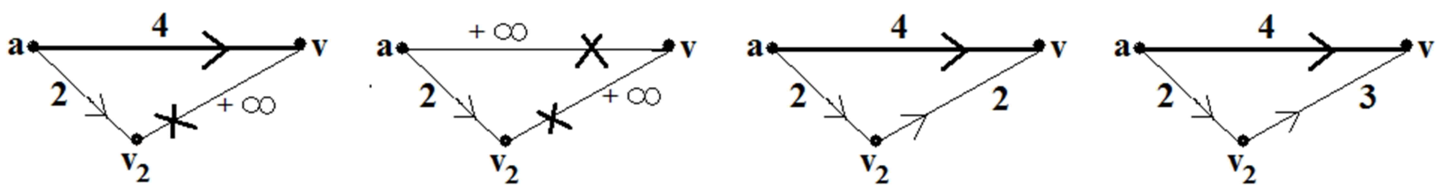




$$L_1(v_2) = \min \{ L_1(v) \mid v \in V \setminus V_1 \}$$

BƯỚC 2:

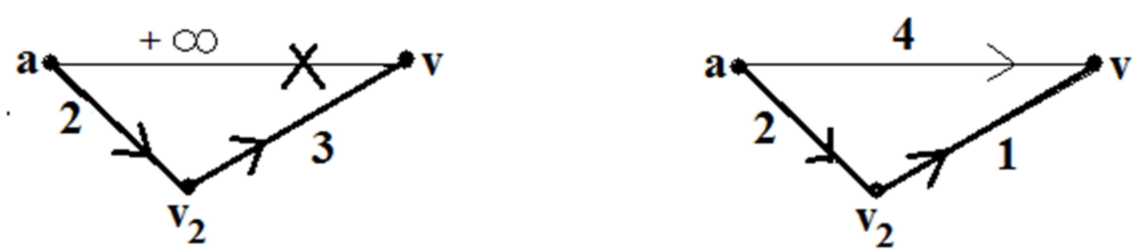
Các trường hợp không cần chỉnh sửa (nếu chỉnh sửa sẽ *không tốt hơn*):



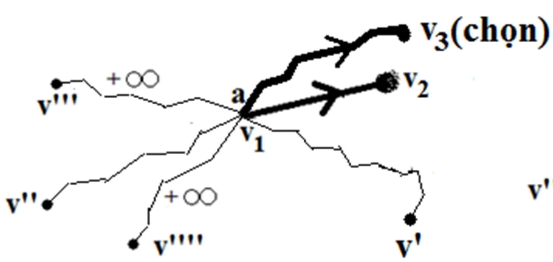
$d(v_2, v) = +\infty$ hoặc $[\, d(v_2, v) < +\infty$ và $L_1(v) \le L_1(v_2) + d(v_2, v) \,]$: giữ nguyên

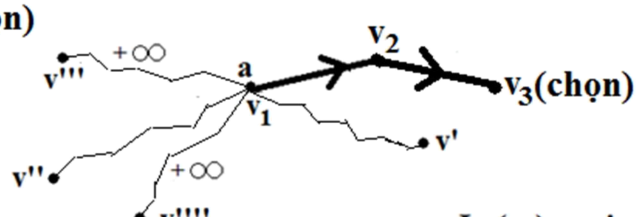
$L_2(v) = L_1(v) = \mathbf{4}$ và $p_2(v) = p_1(v) = a = v_1$.

Các trường hợp cần phải chỉnh sửa cho được *tốt hơn*:



$d(v_2, v) < +\infty$ và $L_1(v) > L_1(v_2) + d(v_2, v)$: chỉnh sửa $L_2(v) = L_1(v_2) + d(v_2, v)$ và $p_2(v) = v_2$.

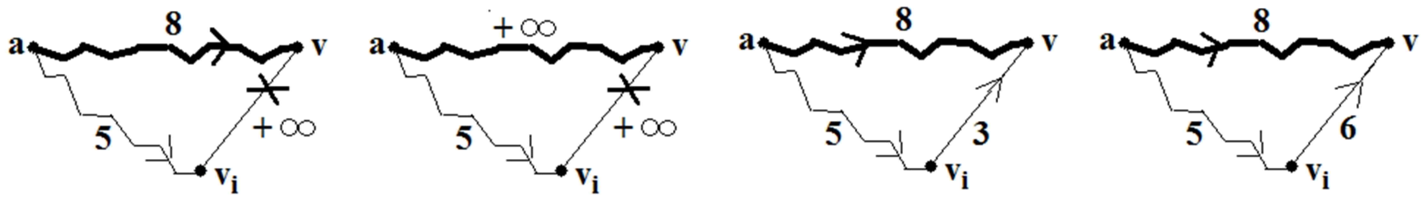




$$L_2(v_3) = \min \{ L_2(v) \mid v \in V \setminus V_2 \}$$

BƯỚC i với $i \geq 3$:

Các trường hợp không cần chỉnh sửa (nếu chỉnh sửa sẽ *không tốt hơn*):



$d(v_i, v) = +\infty$ hoặc $[\, d(v_i, v) < +\infty$ và $L_{i-1}(v) \leq L_{i-1}(v_i) + d(v_i, v)\,]$: giữ nguyên

$L_i(v) = L_{i-1}(v) = \mathbf{8}$ và $p_i(v) = p_{i-1}(v)$.

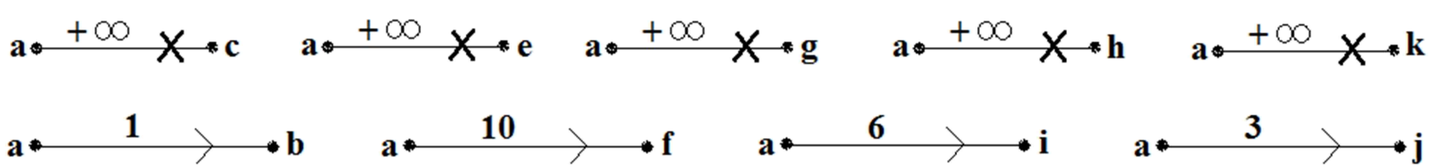
Các trường hợp cần phải chỉnh sửa cho được *tốt hơn* :

$d(v_i, v) < +\infty$ và $L_{i-1}(v) > L_{i-1}(v_i) + d(v_i, v)$: chỉnh sửa

$L_i(v) = L_{i-1}(v_i) + d(v_i, v)$ và $p_i(v) = v_i$.

Bước 1: $T_1 = (\, V_1 = \{\, v_1 = a \,\}, E_1 = \varnothing \,)$.

V	b	c	e	f	g	h	i	j	k	T
a	**(1,a)**	(∞,–)	(∞,–)	(10,a)	(∞,–)	(∞,–)	(6,a)	(3,a)	(∞,–)	

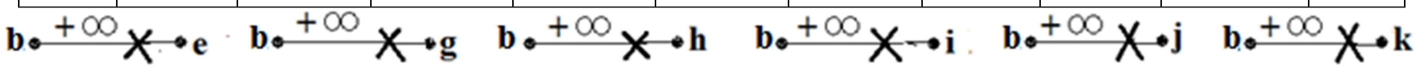


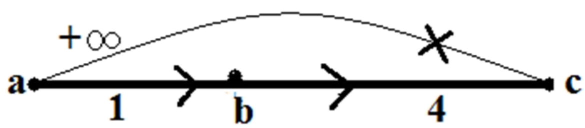
$d(a, c) = d(a, e) = d(a, g) = d(a, h) = d(a, k) = +\infty$ nên $(L_1(c), p_1(c))$, $(L_1(e), p_1(e))$, $(L_1(g), p_1(g))$, $(L_1(h), p_1(h))$ và $(L_1(k), p_1(k))$ đều là $(\infty, -)$.

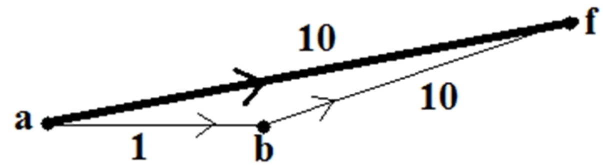
$d(a,b) = 1$, $d(a,f) = 10$, $d(a,i) = 6$, $d(a,j) = 3$ nên $(L_1(b), p_1(b)) = (\mathbf{1}, \mathbf{a})$, $(L_1(f), p_1(f)) = (\mathbf{10},\mathbf{a})$, $(L_1(i), p_1(i)) = (\mathbf{6},\mathbf{a})$ và $(L_1(j), p_1(j)) = (\mathbf{3}, \mathbf{a})$. $L_1(\mathbf{b}) = \mathbf{1} = \min\{\, L_1(v) \mid v \in V \setminus V_1 \,\}$ ($\mathbf{b} = v_2$).

Bước 2: $T_2 = (\, V_2 = V_1 \cup \{\, \mathbf{b} \,\},\ E_2 = E_1 \cup \{\, \overline{p_1(b)b} = \overline{ab} \,\}\,)$.

V	b	c	e	f	g	h	i	j	k	T
a	**(1,a)**	(∞,–)	(∞,–)	(10,a)	(∞,–)	(∞,–)	(6,a)	(3,a)	(∞,–)	
b	–	(5,b)	(∞,–)	(10,a)	(∞,–)	(∞,–)	(6,a)	**(3,a)**	(∞,–)	$\overline{ab}$







$d(b, e) = d(b, g) = d(b, h) = d(b, i) = d(b, j) = d(b, k) = +\infty$ *nên* $(\, L_2(e), p_2(e) \,)$, $(\, L_2(g), p_2(g) \,)$, $(\, L_2(h), p_2(h) \,)$, $(\, L_2(i), p_2(i) \,)$, $(\, L_2(j), p_2(j) \,)$ và $(\, L_2(k), p_2(k) \,)$ *y hệt dòng* 1.

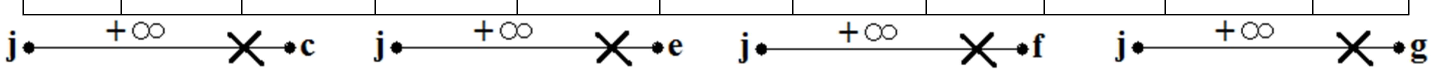
$L_1(c) = \infty > L_1(b) + d(b, c) = 1 + 4 = 5$ *nên có* $(\, L_2(c), p_2(c) \,) = (\mathbf{5}, \mathbf{b})$.

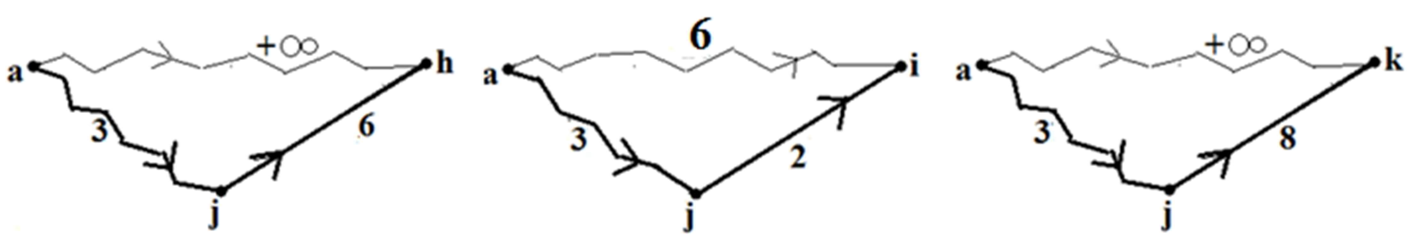
$L_1(f) = 10 \le L_1(b) + d(b, f\,) = 1 + 10 = 11$ *nên* $(\, L_2(f\,), p_2(f\,) \,)$ *y hệt dòng* 1.

Ta có $L_2(\mathbf{j}) = \mathbf{3} = \min\{\, L_2(v) \mid v \in V \setminus V_2 \,\}$ $(\, \mathbf{j} = v_3 \,)$.

Bước 3: $T_3 = (V_3 = V_2 \cup \{\, \mathbf{j} \,\},\ E_3 = E_2 \cup \{\, \overline{p_2(j)j} = \overline{aj} \,\})$.

V	b	c	e	f	g	h	i	j	k	T
b	–	(5,b)	(∞,–)	(10,a)	(∞,–)	(∞,–)	(6,a)	**(3,a)**	(∞,–)	$\overline{ab}$
j	–	**(5,b)**	(∞,–)	(10,a)	(∞,–)	(9, j)	(5,j)	–	(11,j)	$\overline{aj}$





$d(j,c) = d(j,e) = d(j,f) = d(j,g) = +\infty$ *nên* $(L_3(c), p_3(c))$, $(L_3(e), p_3(e))$, $(L_3(f), p_3(f))$ và $(L_3(g), p_3(g))$ *y hệt dòng* 2.

$L_2(h) = \infty > L_2(j) + d(j, h) = 3 + 6 = 9$ *nên có* $(L_3(h), p_3(h)) = (\mathbf{9}, \mathbf{j})$.

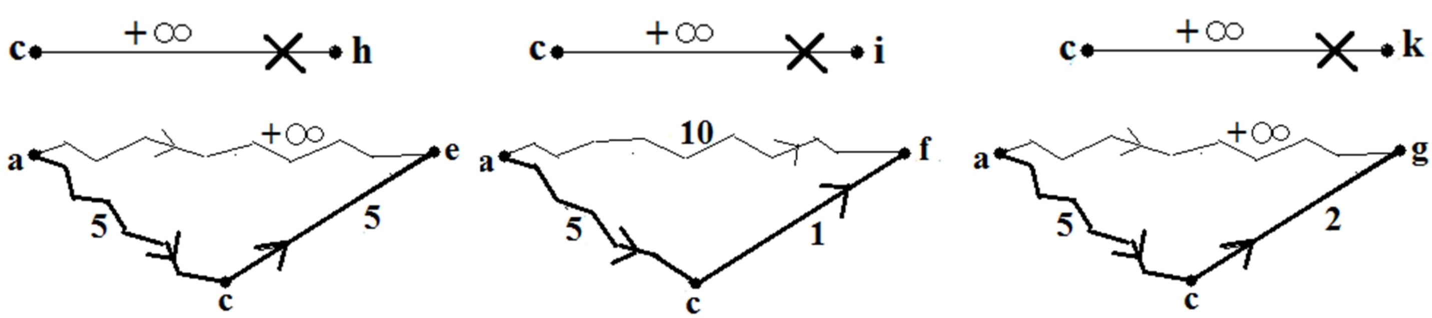
$L_2(i) = 6 > L_2(j) + d(j, i) = 3 + 2 = 5$ *nên có* $(L_3(i), p_3(i)) = (\mathbf{5}, \mathbf{j})$.

$L_2(k) = \infty > L_2(j) + d(j, k) = 3 + 8 = 11$ *nên có* $(L_3(k), p_3(k)) = (\mathbf{11}, \mathbf{j})$.

Để ý $L_3(\mathbf{c}) = \mathbf{5} = \min \{ L_3(v) \mid v \in V \setminus V_3 \}$ ($\mathbf{c} = v_4$).

Bước 4: $T_4 = (V_4 = V_3 \cup \{ \mathbf{c} \}, E_4 = E_3 \cup \{ \overline{p_3(c)c} = \overline{bc} \})$.

V	b	c	e	f	g	h	i	j	k	T
j	–	**(5,b)**	(∞,–)	(10,a)	(∞,–)	(9, j)	(5,j)	–	(11,j)	$\overline{aj}$
c	–	–	(10,c)	(6,c)	(7,c)	(9, j)	**(5,j)**	–	(11,j)	$\overline{bc}$



$d(c, h) = d(c, i) = d(c, k) = +\infty$ *nên* $(L_4(h), p_4(h))$, $(L_4(i), p_4(i))$ và $(L_4(k), p_4(k))$ *y hệt dòng* 3.

$L_3(e) = \infty > L_3(c) + d(c, e) = 5 + 5 = 10$ *nên có* $(L_4(e), p_4(e)) = (\mathbf{10}, \mathbf{c})$.

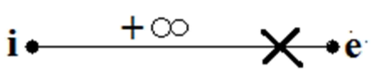
$L_3(f) = 10 > L_3(c) + d(c, f) = 5 + 1 = 6$ *nên có* $(L_4(f), p_4(f)) = (\mathbf{6}, \mathbf{c})$.

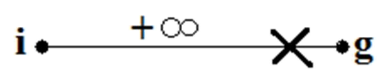
$L_3(g) = \infty > L_3(c) + d(c, g) = 5 + 2 = 7$ *nên có* $(L_4(g), p_4(g)) = (\mathbf{7}, \mathbf{c})$.

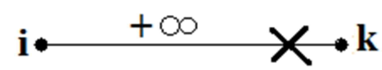
Để ý $L_4(\mathbf{i}) = \mathbf{5} = \min\{ L_4(v) \mid v \in V \setminus V_4 \}$ ($\mathbf{i} = v_5$).

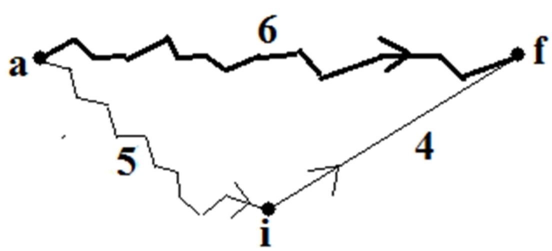
Bước 5: $T_5 = (\ V_5 = V_4 \cup \{\ \mathbf{i}\ \},\ E_5 = E_4 \cup \{\ \overline{p_4(i)i} = \overline{ji}\ \}\ )$.

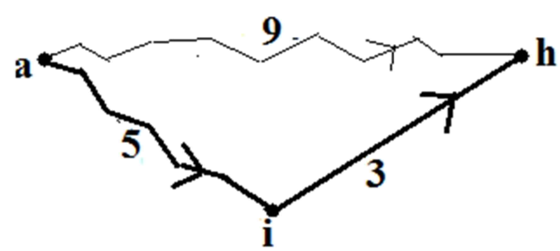
V	b	c	e	f	g	h	i	j	k	T
c	–	–	(10,c)	(6,c)	(7,c)	(9, j)	**(5,j)**	–	(11,j)	$\overline{bc}$
i	–	–	(10,c)	**(6,c)**	(7,c)	(8,i)	–	–	(11,j)	$\overline{ji}$











d(i, e) = d(i, g) = d(i, k) = + ∞ *nên* (L_5(e), p_5(e)), (L_5(g), p_5(g)) và (L_5(k), p_5(k)) *y hệt dòng* 4.

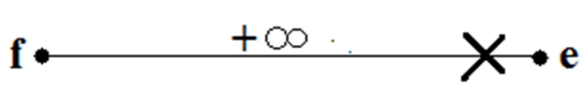
L_4(f) = 6 ≤ L_4(i) + d(i, f) = 5 + 4 = 9 *nên* (L_5(f), p_5(f)) *y hệt dòng* 4.

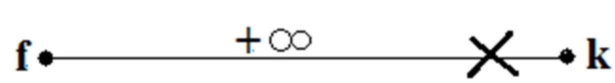
L_4(h) = 9 > L_4(i) + d(i, h) = 5 + 3 = 8 *nên có* (L_5(h), p_5(h)) = (**8**, **i**).

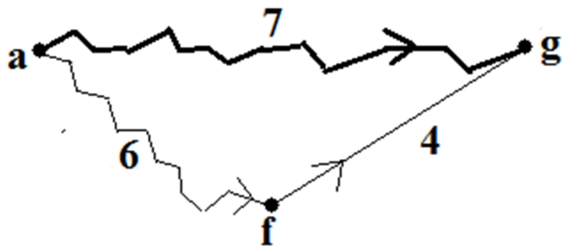
Để ý L_5(**f**) = **6** = min{ L_5(v) | v ∈ V \ V_5 } (**f** = v_6).

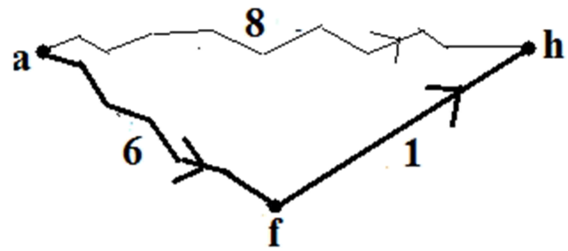
Bước 6: $T_6 = (\ V_6 = V_5 \cup \{\ \mathbf{f}\ \},\ E_6 = E_5 \cup \{\ \overline{p_5(f)f} = \overline{cf}\ \}\ )$.

V	b	c	e	f	g	h	i	j	k	T
i	–	–	(10,c)	**(6,c)**	(7,c)	(8,i)	–	–	(11,j)	$\overline{ji}$
f	–	–	(10,c)	–	**(7,c)**	(7,f)	–	–	(11,j)	$\overline{cf}$









d (f, e) = d(f, k) = + ∞ *nên* (L_6(e), p_6 (e)) và (L_6(k), p_6(k)) *y hệt dòng* 5.

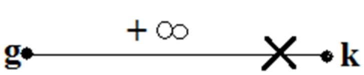
L_5(g) = 7 ≤ L_5(f) + d(f, g) = 6 + 4 = 10 *nên* (L_6(g), p_6(g)) *y hệt dòng* 5.

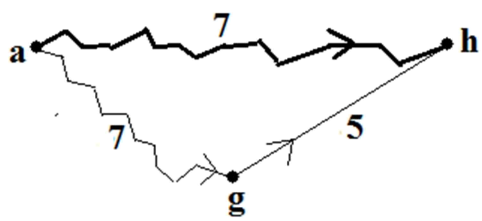
L_5(h) = 8 > L_5(f) + d(f, h) = 6 + 1 = 7 *nên có* (L_6(h), p_6(h)) = (**7**, **f**).

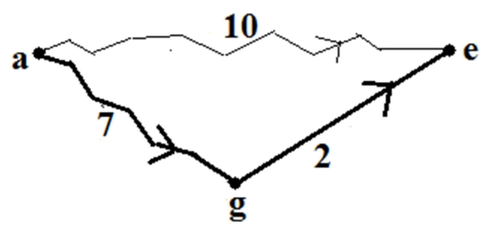
Để ý L_6(**g**) = **7** = min{ L_6(v) | v ∈ V \ V_6 } (**g** = v_7).

Bước 7: T_7 = (V_7 = V_6 ∪ { **g** }, E_7 = E_6 ∪ { $\overline{p_6(g)g} = \overline{cg}$ }).

V	b	c	e	f	g	h	i	j	k	T
f	–	–	(10,c)	–	**(7,c)**	(7,f)	–	–	(11,j)	$\overline{cf}$
g	–	–	(9,g)	–	–	**(7,f)**	–	–	(11,j)	$\overline{cg}$







d(g, k) = + ∞ *nên* (L_7(k), p_7(k)) *y hệt dòng* 6.

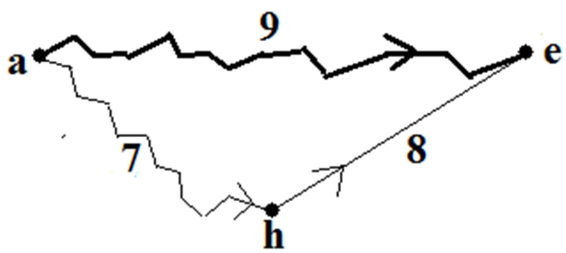
L_6(h) = 7 ≤ L_6(g) + d(g, h) = 7 + 5 = 12 *nên* (L_7(h), p_7(h)) *y hệt dòng* 6.

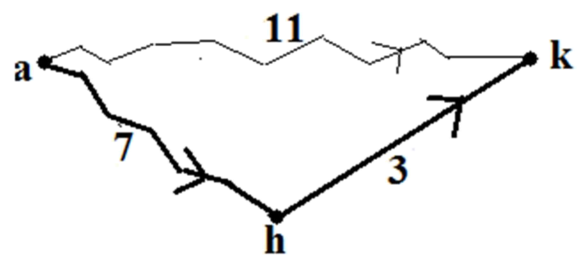
L_6(e) = 10 > L_6(g) + d(g, h) = 7 + 2 = 9 *nên có* (L_7(e), p_7(e)) = (**9**, **g**).

Để ý L_7(**h**) = **7** = min{ L_7(v) | v ∈ V \ V_7 } (**h** = v_8).

Bước 8: $T_8 = (V_8 = V_7 \cup \{ \mathbf{h} \}, E_8 = E_7 \cup \{ \overline{p_7(h)h} = \overline{fh} \})$.

V	b	c	e	f	g	h	i	j	k	T
g	–	–	(9,g)	–	–	**(7,f)**	–	–	(11,j)	$\overline{cg}$
h	–	–	**(9,g)**	–	–	–	–	–	(10,h)	$\overline{fh}$





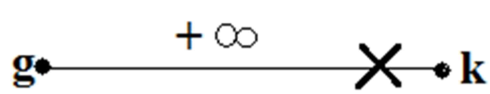
$L_7(e) = 9 \le L_7(h) + d(h, e) = 7 + 8 = 15$ *nên* $(L_8(e), p_8(e))$ *y hệt dòng* 7.

$L_7(k) = 11 > L_7(h) + d(h, k) = 7 + 3 = 10$ *nên có* $(L_8(k), p_8(k)) = (\mathbf{10}, \mathbf{h})$.

Để ý $L_8(\mathbf{e}) = \mathbf{9} = \min\{ L_8(v) \mid v \in V \setminus V_8 \}$ ($\mathbf{e} = v_9$).

Bước 9: $T_9 = (V_9 = V_8 \cup \{ \mathbf{e} \}, E_9 = E_8 \cup \{ \overline{p_8(e)e} = \overline{ge} \})$.

V	b	c	e	f	g	h	i	j	k	T
h	–	–	**(9,g)**	–	–	–	–	–	(10,h)	$\overline{fh}$
e	–	–	–	–	–	–	–	–	**(10,h)**	$\overline{ge}$



$L_8(e,k) = + \infty$ *nên* $(L_9(k), p_9(k))$ *y hệt dòng* 8.

Để ý $L_9(\mathbf{k}) = \mathbf{10} = \min\{ L_9(v) \mid v \in V \setminus V_9 \}$ ($\mathbf{k} = v_{10}$).

Bước 10: $T_{10} = (V_{10} = V_9 \cup \{ \mathbf{k} \}, E_{10} = E_9 \cup \{ \overline{p_9(k)k} = \overline{hk} \})$.

V	**b**	**c**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**T**
e	–	–	–	–	–	–	–	–	**(10,h)**	$\overline{ge}$
k	(1,a)	(5,b)	(9,g)	(6,c)	(7,c)	(7,f)	(5,j)	(3,a)	(10,h)	$\overline{hk}$

Dòng 10 *y hệt dòng* 9.