Session 2: Numeral Systems and Data Storage



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Content

- Bits and their Storage
 - Bits, Gates, Flip-Flop
- Main Memory
- Representing Information as Bit Patterns
 - Text, number, images, sound
- Binary System
- Storing Integers
- Storing Fractions
- Mass Storage



Question

- How computers store data?
 - Number, text, image, sound, video

How computers can mapping data to the real world?



Bits and their storage

- ☐ Bits
- ☐ Gates
- ☐ Flip-flop



Bits

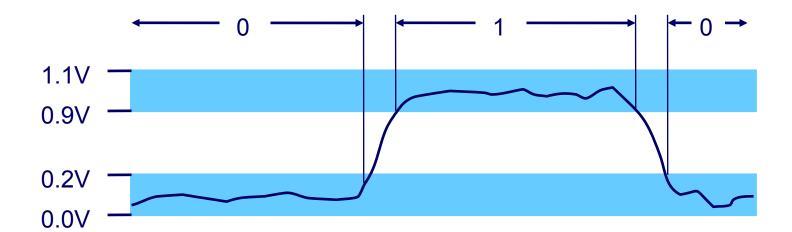
- □ Binary uses two digits: 0 and 1.
- bit (Binary Digit): smallest unit storing information.
- Can be stored in memory (cell) or register.
- Register 1 byte (8 bit) or 1 word (16 bit), etc.



Bits

Why using 2 digits 0 and 1 to encode data?

Electronic implementation





Bits

- Easily to encode:
 - Numeric value : 1 & 0
 - Boolean value : true & false
 - Voltage : high & low
 - Punched card : punched & not punched
- □ Data → encode using binary system to store in computers



Bits – Boolean Operations

- An operation that manipulates one or more true/false values
 - Bit 0 ~ False
 - □ Bit 1 ~ True
- Specific operations : AND, OR, XOR, NOT
- Why Boolean operations?
 - Computers are built by small components
 - These components can process Boolean operations quite fast



Bits – Boolean Operations

The AND operation

$$\frac{\mathsf{AND}}{\mathsf{O}} \overset{1}{\overset{0}{\overset{}{\overset{}{\overset{}{\overset{}{\overset{}{\overset{}}{\overset{}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}}{\overset{}{\overset{}}{\overset$$

The OR operation

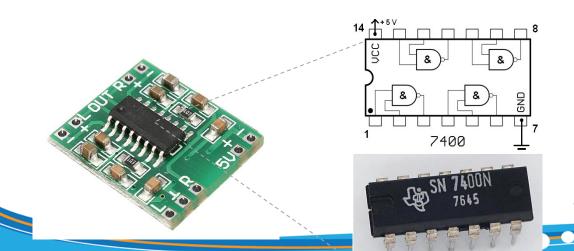
The XOR operation

Source: Computer Science - An Overview, 12e



Gates

- A device that computes a Boolean operation
- Often implemented as (small) electronic circuits:
 - Including: resistor (điện trở), transistor (bòng bán dẫn), Capacitor (tụ điện), diot (điốt), ...
 - 0 & 1 ~ voltage



Nguồn: Wikipedia



A pictorial representation of gates

AND

Inputs Output

Inputs	Output
0 0	0
0 1	0
1 0	0
1 1	1

OR



Inputs	Output
0 0	0
0 1	1
1 0	1
1 1	1

XOR

Inputs Output

Inputs	Output
0 0	0
0 1	1
1 0	1
1 1	0

NOT

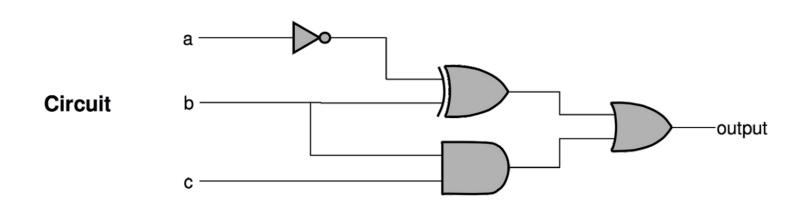
Inputs — Output

Inputs	Output
0	1
1	0

Nguồn: Computer Science - An Overview, 12e



Example – Simple circuit



Truth Table

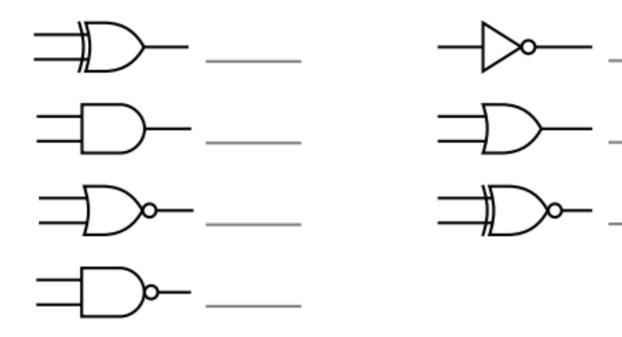
Input a, b, c	Output
000	1
001	1
010	0
011	1
100	0
101	0
110	1
111	1

Source Chun-Jen Tsai, ics12, National Chiao Tung University



Quiz

■ What are the names of these gates?





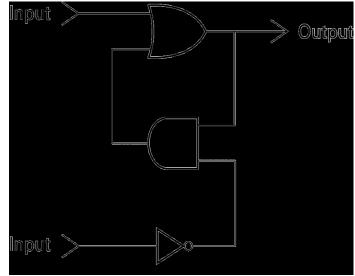
Quiz

□ What input bit patterns will cause the following circuit to produce output of 1?



Flip-Flop

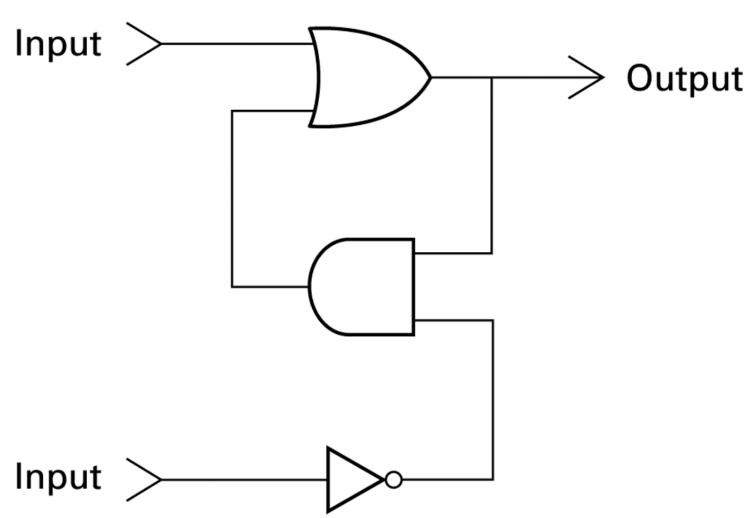
- A circuit is built from gates that can store one bit
- A circuit produces output (or "preserved") 0 or 1, but remain constant until the pulse from another circuit makes it change to another value
 - One input line is used to set its stored value to 1 (output is 1)
 - One input line is used to set its stored value to 0 (output is 0)
 - While both input lines are 0, the most recently stored value is preserved



source: Computer Science - An Overview, 12e



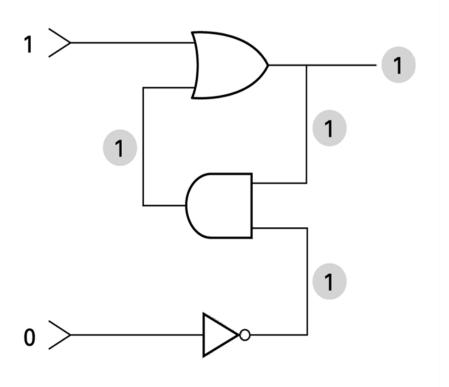
A simple flip-flop circuit

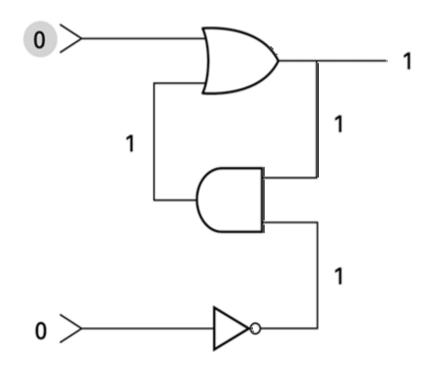




Flip-Flop

Setting the output of a flip-flop to 1



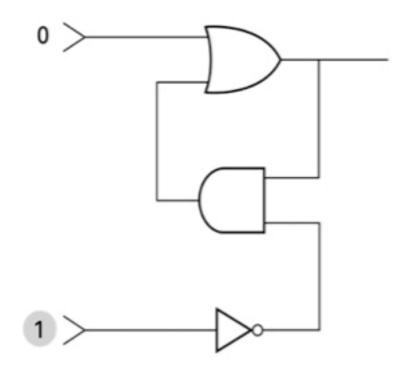


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Flip-Flop

Setting the output of a flip-flop to 0

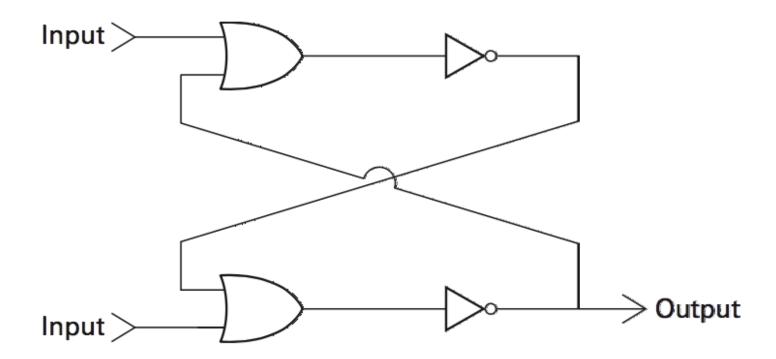


Nguồn: Computer Science - An Overview, 12e



Quiz - Flip-Flop?

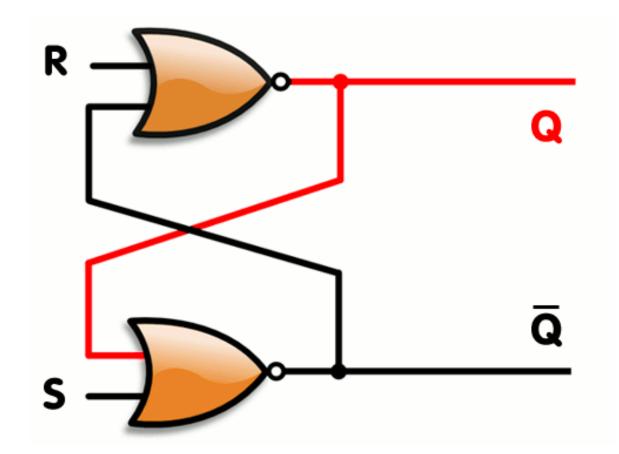
If upper input is 1 and lower input is 0, what is the output?



Source: Computer Science - An Overview, 12e



Quiz-Flip-Flop?



Source: wikipedia



Flip-flop - Activity

What happens when you:

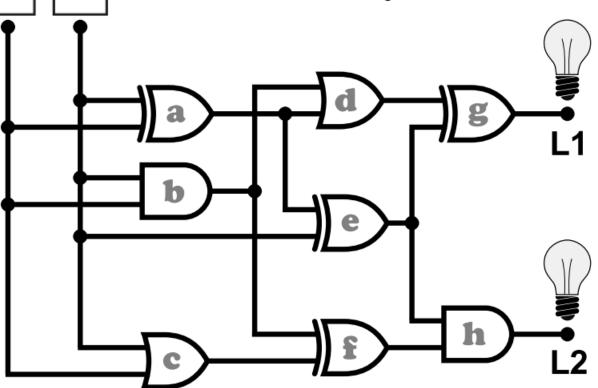
Turn on switch A?

В

Turn on switch B?

Turn on both A and B at the same time?

Fill in the truth table below to figure out the answer.





Hexadecimal Notation

- Hexadecimal notation: A shorthand notation for long bit patterns
 - □ Divides a pattern into groups of four bits each
 - Represents each group by a single symbol

■ Example: 10100011 becomes A3



The hexadecimal coding system

Bit pattern	Hexadecimal representation
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	В
1100	С
1101	D
1110	E
1111	F



Quiz

- What bit patterns are represented by the following hexadecimal patterns?
 - □ 5FD97
 - □ 610A
 - ABCD
 - **1** 0100



MAIN MEMORY



Introduction

- We know
 - How machines encode information into chain of bits
 - Basic storage devices
- □ So
 - To store data, machines need to have a million of circuits (a circuit stores 1 bit)
 - → Place containing these bits is called *Main*Memory



Introduction

- In additional to Flip-flops, machines have other storage devices (called external memory)
 - Magnetic, optical, flash devices
- Storage devices
 - Volatile memory (bộ nhớ khả biến)
 - Requires power to maintain the stored information
 - Non-volatile memory (bộ nhớ bất khả biến)
 - Can retrieve stored information even after having been power cycled



Main memory cells

- Cell: A unit of main memory (typically 8 bits which is one **byte**)
 - Most significant bit: the bit at the left (high-order) end of the conceptual row of bits in a memory cell
 - Least significant bit: the bit at the right (low-order) end of the conceptual row of bits in a memory cell
- Organization of a byte-size memory cell
 - ☐ Size 8 bits (1 byte)
 - Sequence of bits

```
High-order end

| Most | Least | significant | bit | b
```



Main memory address

Address: A "name" that uniquely identifies one cell in the computer's main memory

10111010 Cell 0

- "Names" are actually numbers
- These numbers are assigned consecutively starting at zero
- Numbering the cells in this manner associates an order to the memory cells

Source: Computer Science - An Overview, 12e



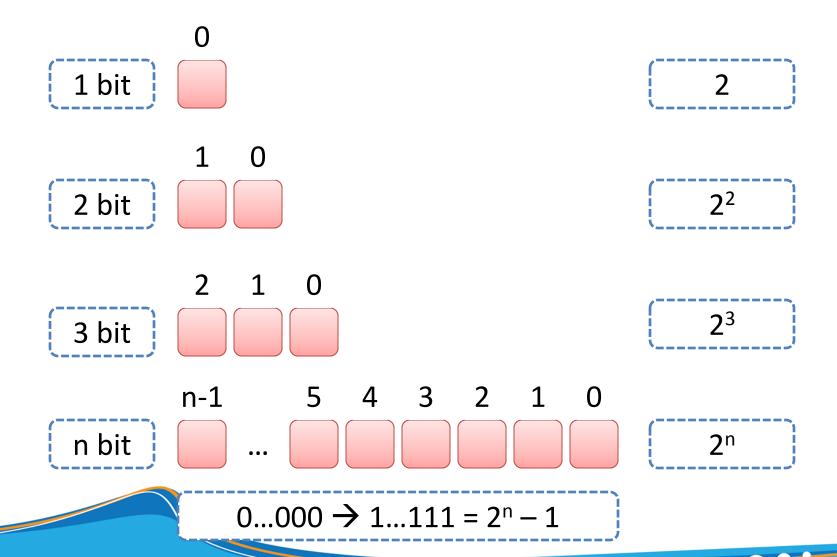
Terminology

- Random Access Memory (RAM)
 - Memory in which individual cells can be easily accessed in any order

- Dynamic Memory (DRAM)
 - □ RAM composed of volatile memory



Measuring memory capacity





Measuring memory capacity

Capacity		Value
Byte	В	8 bit
KiloByte	KB	$2^{10} B = 1024 Byte$
MegaByte	MB	$2^{10} \text{ KB} = 2^{20} \text{ Byte}$
GigaByte	GB	$2^{10} MB = 2^{30} Byte$
TeraByte	ТВ	$2^{10} \text{ GB} = 2^{40} \text{ Byte}$
Peta	РВ	2 ¹⁰ TB = 2 ⁵⁰ Byte
Exabyte	EB	2 ¹⁰ PB= 2 ⁶⁰ Byte



REPRESENTING INFORMATION AS BIT PATTERNS

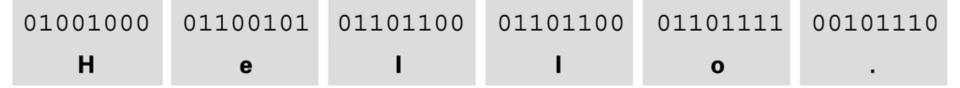


Representing Text

- Each character (letter, punctuation, etc.) is assigned a unique bit pattern
 - ASCII: Uses patterns of 7-bits to represent most symbols used in written English text
 - ISO developed a number of 8 bit extensions to ASCII, each designed to accommodate a major language group
 - □ Unicode: Uses patterns of 16-bits to represent the major symbols used in languages world wide (UTF-8, UTF-16,...)



The message "Hello." in ASCII





Storage - Represent

Storage and processing: bit

- Display/represent: character
 - ⇒ Need to have a map table between the two, do the mapping between numerical values and character values.

ASCII and Unicode.



ASCII

- American Standard Code for Information Interchange.
- □ First edition was published in 1963.
- □ Based on the English alphabet ('a'- 'z', 'A' 'Z').
- ASCII encodes 128 specified characters into seven-bit integers (digits 0 to 9, lowercase letters a to z, uppercase letters A to Z, and punctuation symbols).



ASCII - Characters

- Printing characters
 - ☐ '' (blank): 32 (0x20)
 - □ '0' -> '9': 48 (0x30) -> 57 (0x39)
 - ☐ 'A' -> 'Z': 65 (0x41) -> 90 (0x5A)
 - □ 'a' -> 'z': 97 (0x61) -> 122 (0x7A)
- Non-printing/control characters
 - null: 0
 - ' '(tab): 9
 - enter/ line feed: 10
 - carriage return: 13



ASCII

- ☐ ASCII extent: 256 characters.
 - 128 as the first edition.
 - 128 extent to characters including: Greece ('α', 'β', 'π', ...), currency ('£', '¥', ...), ...

ASCII cannot represent characters in other languages such as Vietnamese, Russian, Japanese, Arabic, etc.



Unicode

- Unicode is a computing industry standard for the consistent encoding, representation, and handling of text expressed in most of the world's writing systems.
- □ Containing 1.114.112 code points, divived into 17 regions, each has 65535 (2¹⁶) code points.



Unicode

There are different in using the Unicode, depending on the storage size of code point

- □ UTF 8: storage size from 1 -> 4 Bytes.
- □ UTF 16: storage size 2 Bytes.
- □ UTF 32: storage size 4 Bytes.



Unicode and Vietnamese

Unicode contains Vietnamese code points is at:

http://vietunicode.sourceforge.net/charset/ v3.htm



Unicode - Font

- □ Each Unicode set has different ways to be represented.
- Unicode is implemented as a digital data file containing a set of graphically related glyphs, characters, or symbols.
- Fonts support Unicode (with Vietnamese) :
 - Times New Roman,
 - Arial,
 - Tahoma,
 - ...



Representing Numeric Values

- Binary notation: uses bits to represent a number in base two
- Limitations of computer representations of numeric values
 - Overflow: occurs when a value is too big to be represented
 - Underflow: occurs when a value is too small to be represented
 - Truncation: occurs when a value cannot be represented accurately



Representing Images

- Bit map techniques
 - □ Pixel: short for "picture element"
 - Encoding
 - RGB
 - Luminance (cường độ sáng) and chrominance (độ đậm/nhạt của màu)
- □ Vector: can zoom in/out → not affected to the quality
 - Scalable
 - TrueType and PostScript

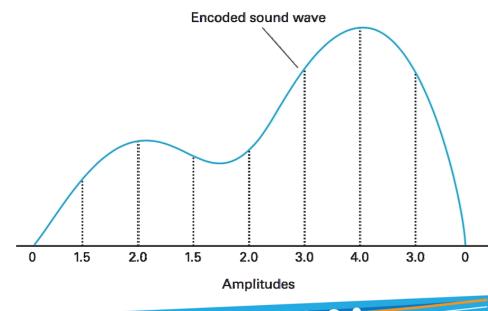
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Representing Sound

- Sampling techniques: get the amplitude of the sound wave at regular intervals and store the series of values
 - Used for high quality recordings
 - Records actual audio
- Sampling rate (Hertz Hz)
 - Telephone: 8,000 samples/second 8 KHz
 - Music: 44,100 samples/second 44,1 KHz

High definition: 16 bits/sample

Stereo: 32 bits/sample





- Suppose a stereo recording of one hour of music is encoded using a sample rate of 44,100 samples per second as discussed.
- What is the size of the encoded version?

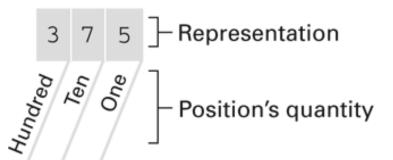


BINARY SYSTEM



Binary system

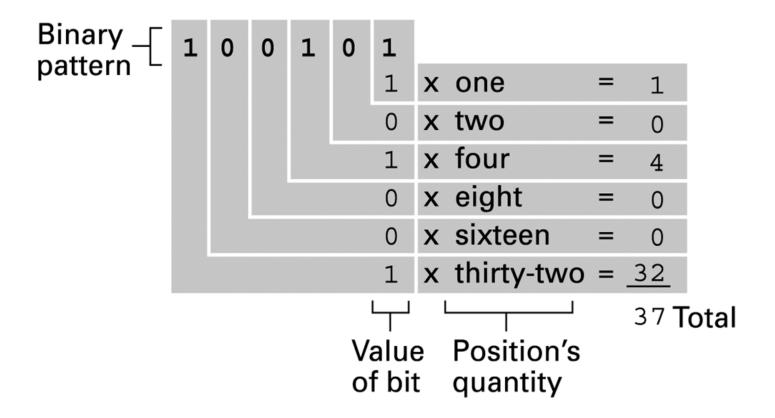
- The traditional decimal system is based on powers of ten.
- The Binary system is based on powers of two
- The base 10 and the binary system
 - a. Base ten system



b. Base two system

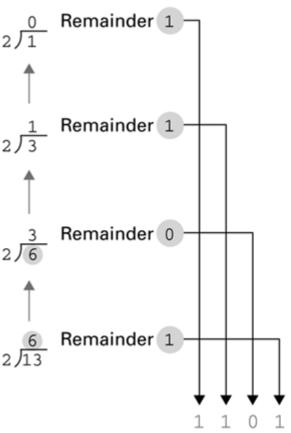


Decoding the binary representing 100101





Base 10 to binary



- **Step 1.** Divide the value by two and record the remainder.
- **Step 2.** As long as the quotient obtained is not zero, continue to divide the newest quotient by two and record the remainder.
- **Step 3.** Now that a quotient of zero has been obtained, the binary representation of the original value consists of the remainders listed from right to left in the order they were recorded.

Binary representation



Binary additional facts

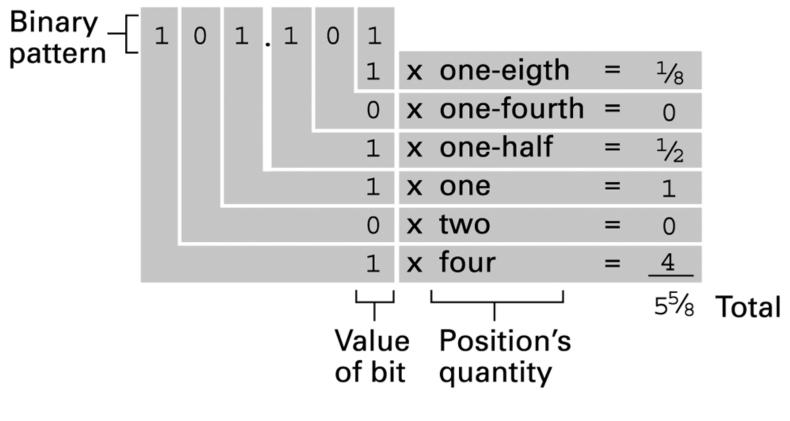
Additional table

+	0	1
0	0	1
1	1	10

$$\begin{array}{r} 1 \\ +1 \\ \hline 10 \end{array}$$



Decoding the binary representation (Binary to base 10)



$$1010.11_2 = 1*2^3 + 0*2^2 + 1*2^1 + 0*2^0 + 1*2^{-1} + 1*2^{-2}$$

$$1010.11_2 = 8 + 0 + 2 + 0 + 0.5 + 0.25 = 10.75_{10}$$



- Convert each of the following base 10 representations to its equivalent binary form:

 - 5

 - 3



- Convert each of the following binary representations to its equivalent base 10 form:
 - 11.01
 - 101.111
 - **10.1**
 - **110.011**
 - 0.101



Express the following values in binary notation:

 $4^{1}/_{2}$

 $2^{3}/_{4}$

 $1^{1}/_{8}$

d. $\frac{5}{16}$

 $e. 5^{5/8}$



- ☐ Binary to base 10
 - **1** 21.125₁₀



STORING INTEGERS

Unsigned Integers

- Representations of quantities are always positive
- Ex: height, weight, ASCII code, etc.
- All bits are used for value
- Max value of 1 unsigned byte:
 - \square 1111 1111₂ = 2⁸ 1 = 255₁₀
- - \square 1111 1111 1111 1111₂ = 2^{16} 1 = 65535_{10}



Signed Integers

- Storing/representing of negative and positive integers
- The leftmost bit is used to represent the sign
- ☐ Ex:
 - □ 0 positive: 0101 0011
 - □ 1 negative: 1101 0011
- Negative integers stored in machine as two's complement or excess notation



Two's complement notation systems

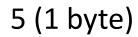
Leftmost bit: sign bit

- One's complement :
 - Changing all the 0s to 1s, all the 1s to 0s.
 - Ex: 0110 and 1001 are complements

- Two's complement: add 1 to the one's complement
 - \square Ex: 1001 + 0001 = 1010



One's and two's complements



0 0 0 0 0 1 0 1

One's complement of 5

1 | 1 | 1 | 1 | 1 | 0 | 1 | 0

+

1

Two's complement of 5

1 | 1 | 1 | 1 | 1 | 0 | 1 | 1

+ 5

Result

1

0

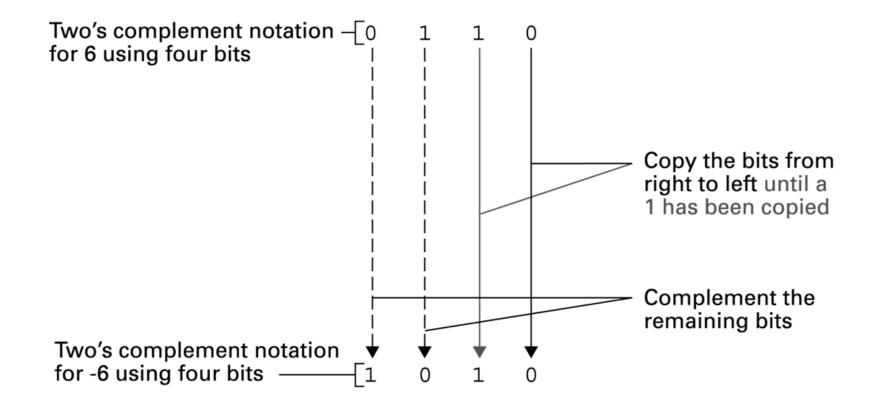
) 0

)[

0



Coding the value -6 in two's complement notation using 4 bits





Two's complement notation systems

a. Using patterns of length three

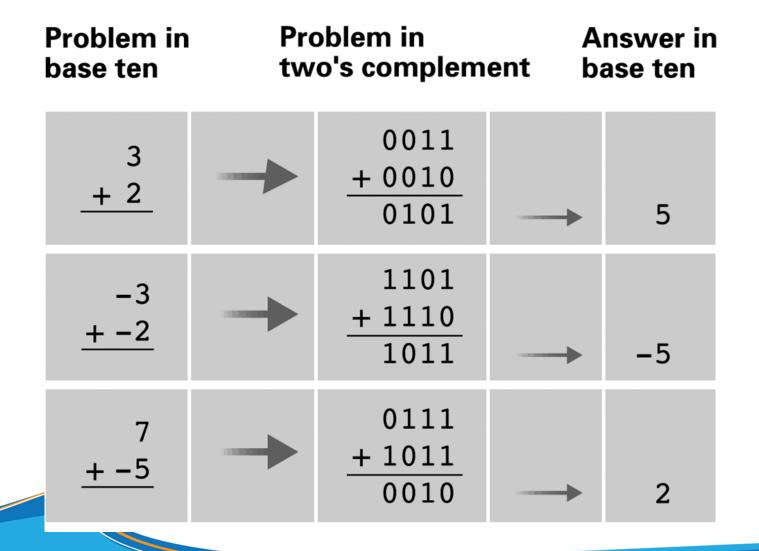
Bit pattern	Value represented
011	3
010	2
001	1
000	0
111	-1
110	-2
101	-3
100	-4

b. Using patterns of length four

Bit pattern	Value represented
0111 0110 0101 0100 0011 0010 0001 0000 1111 1110 1101 1101 1010 1001 1001	7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6 -7 -8



Addition problems converted to two's complement notation





- Convert each of the following two's complement representations to its equivalence base 10 form:
 - **00011**
 - 01111
 - **11100**
 - **11010**
 - **00000**
 - **10000**



- Convert to two's complement form using patterns of 8 bits:
 - **6**
 - **-**6
 - **-17**
 - **1**3
 - □ -1
 - \square 0



- What are the largest and smallest numbers that can be stored if the machine uses bit patterns of the following lengths?
 - □ A. four
 - ☐ B. six
 - C. eight



Min and max of signed integers

N bits	minimum	maximum
8	-2 7 = -128	27 - 1 = +127
16	-215 = -32,768	215 - 1 = +32,767
32	-231 = -2,147,483,648	231 - 1 = +2,147,483,647
64	-263 = - 9,223,372,036,854,775,808	263-1 = +9,223,372,036,854,775,807

Min and max of unsigned integers

n	Minimum	Maximum
8	0	28 - 1 = 255
16	0	216 - 1 = 65,535
32	0	232 - 1 = 4,294,967,295
64	0	264 - 1 = 18,446,744,073,709,551,615



Excess

- Another representation of signed integers
- ☐ Sign bit opposite of two's compement
 - □ 1 positive
 - □ 0 negative
- □ Leftmost bit is 1 → number 0
 - \square 1000 \rightarrow 0 (in 4 bits)
 - \square 100 \rightarrow 0 (in 3 bits)



Excess

Two's complement

Bit	Value
pattern	represented
011	3
010	2
001	1
000	0
111	-1
110	-2
101	-3
100	-4

Excess

Bit pattern	Value represented
111	3
110	2
101	1
100	0
011	-1
010	-2
001	-3
000	-4



Excess

Two's complement

Bit	Value
pattern	represented
0111 0110 0101 0100 0011 0010 0001 0000 1111 1110 1101 1101 1010 1001 1001	7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6 -7 -8

Excess

Bit	Value
pattern	represented
1111 1110 1101 1100 1011 1010 1001 0111 0110 0101 0100 0011 0010 0001	7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6 -7 -8

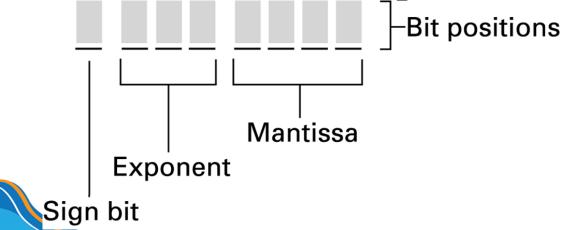


STORING FRACTIONS



Fractions

- Floating-points:
 - Consisting of a sign bit, a mantissa field, and an exponent field.
 - Radix point: used to separate integer part and the fraction part of a number
- Mantissa: also fraction/significant





- 01101011
 - □ Signed bit: 0
 - Exponent: 110
 - Mantissa: 1011

- .1011
- \square 110 = 2 (excess with 3-bit)
- \square 10.11 = 2 $\frac{3}{4}$

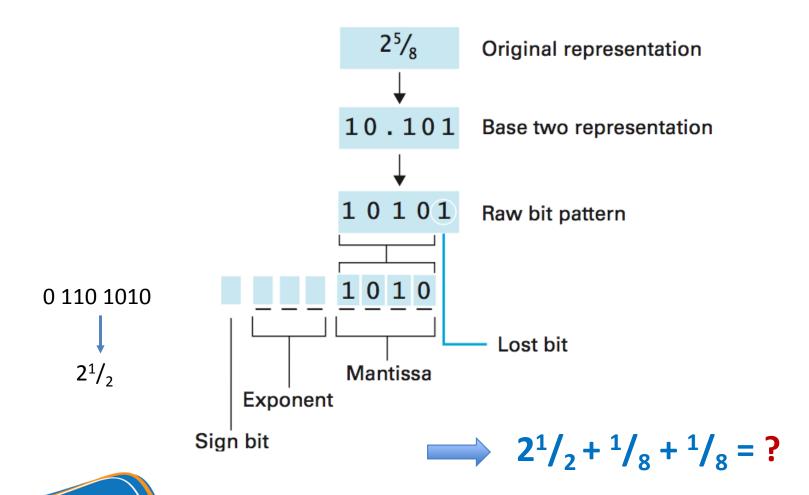
- 00111100
 - □ Signed bit: 0
 - Exponent: 011
 - ☐ Mantissa: 1100

- .1100
- \square 011 = -1 (excess with 3-bit)
- \square .01100 = 3/8

- ☐ Encode 11/8
 - Convert to binary 1.001
 - Mantissa: _ _ _ 1 0 0 1
 - From left to right, start with leftmost bit 1
 - □ Radix point: from .1001 to 1.001
 - $\square \rightarrow$ Exponent: +1 = 101 (excess 3-bit)
 - □ Signed bit: 0
 - □ 1 ½ = 01011001



Encoding the value 2 5∕8→ Truncation error





Floating points

- ☐ Single precision floating point (32 bits)
 - □ 1 bit (sign) + 8 bits (exponent) + 23 bits (mantissa)
 - □ Value: from 10⁻³⁷ to 10³⁸
 - Precision: 7 decimal fraction
- Double precision floating point (64 bits)
 - 15 decimal fraction



Fractions

☐ Single-precision : 32 bits



Double-precision: 64 bits

```
s exp frac

1 11-bits 52-bits
```

Extend-precision : 80 bits (Intel only)

```
s exp frac
```

1 15-bits 63 or 64-bits



IEEE-754 32-bit Single-Precision Floating-Point Numbers

☐ IEEE-754:

$$N = (-1)^S \times 1.F \times 2^E - 127$$

□ S: Sign bit

F: Fraction part

☐ E: Exponent part

 \square 127 = 2 8-1 (excess 8-bit)




```
Sign bit S = 0 \Rightarrow positive number
E = 1000 0000B = 128D
Fraction is 1.11B (with an implicit leading 1) = 1 + 1×2^-1 + 1×2^-2 = 1.75D

The number is +1.75 × 2^(128-127) = +3.5D
```

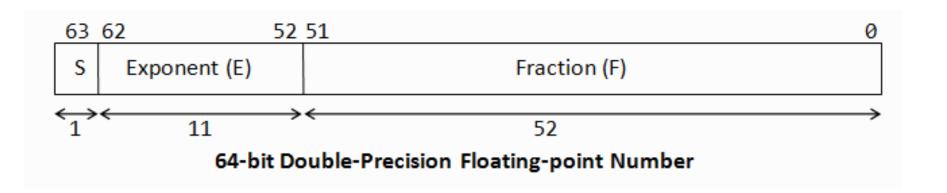



```
Sign bit S = 1 \Rightarrow \text{negative number} E = 0111 \ 1110B = 126D Fraction is 1.1B (with an implicit leading 1) = 1 + 2^-1 = 1.5D The number is -1.5 \times 2^-(126-127) = -0.75D
```



IEEE-754 64-bit Double-Precision Floating-Point Numbers

- □ Exponent: excess − 1023
- Fraction: implicit leading bit (before radix point).
 - \square N = (-1)^S x 1.F x 2^(E-1023)





Min and Max of Floating-Point Numbers

Precision	Min	Max
Single	1.1754 x 10 ⁻³⁸	3.40282 x 10 ³⁸
Double	2.2250 x 10 ⁻³⁰⁸	1.7976 x 10 ³⁰⁸



COMMUNICATION ERRORS



Communication error

- Happening when:
 - Data transferred between components in computer
 - Storing data
- Bits received are not the same with originals
- Reasons:
 - Dirt on disc surface
 - Broken circuit makes reading / writing inaccurate
 - Data transmission line broken
 - Radiation changes the sequence of bits on main memory



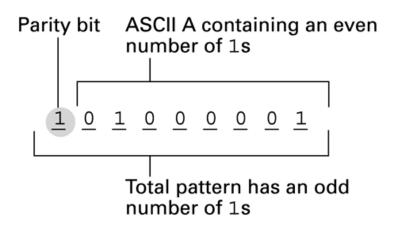
Techniques

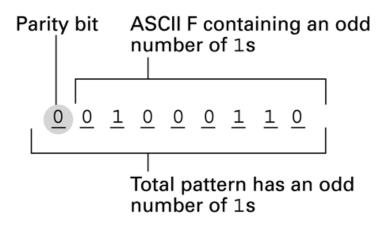
- Parity bits (even vs odd)
- Checkbytes
- Error correcting codes



Parity bits

□ Error detection is based on: odd-number of bits 1 and even-number of bits 1 is found → there must be an error.







Checkbyte

- Set of parity bits
- Each parity bit is scattered in the bit patterns
 - □ For example, a parity bit is associated with every eighth bit in the bit patterns

Checksum



Error-correcting code

- Hamming distance (of two bit patterns):
 - Number of bits in which the patterns differ.

- Example:
 - □ Hamming(00**0000**, 00**1111**) = 4
 - □ Hamming(**10**10**1**100, **01**10**0**100) = 3



Error-correcting code

Code
000000
001111
010011
011100
100110
101001
110101
111010



Error-correcting code

Character	Code	Pattern received	Distance between received pattern and code
А	0 0 0 0 0 0	0 1 0 1 0 0	2
В	0 0 1 1 1 1	0 1 0 1 0 0	4
С	0 1 0 0 1 1	0 1 0 1 0 0	3
D	0 1 1 1 0 0	0 1 0 1 0 0	1
E	1 0 0 1 1 0	0 1 0 1 0 0	3
F	1 0 1 0 0 1	0 1 0 1 0 0	5
G	1 1 0 1 0 1	0 1 0 1 0 0	2
Н	1 1 1 0 1 0	0 1 0 1 0 0	4

D is the answer because it has smallest distance



Quiz

□ The following bytes were originally encoded using odd parity. In which of them do you know that an error has occurred?

a. 100101101

d. 111000000

b. 100000001

e. 011111111

c. 000000000



Quiz

Using the error-correcting code presented before, decode the following messages:

a. 001111 100100 001100

b. 010001 000000 001011

c. 011010 110110 100000 011100



Quiz

Construct a code for the characters A, B, C, D using bit patterns of length five so that the Hamming distance between any two patterns is at least three.



MASS STORAGE



Mass Storage

- On-line versus off-line
- Typically larger than main memory
- Typically less volatile than main memory
- Typically slower than main memory
- Typically lower cost than main memory



Mass Storage Systems

- Magnetic System
 - Disk
 - Tape
- Optical Systems

 - DVD
- Flash Technology
 - Flash Drives
 - Secure Digital (SD) Memory C



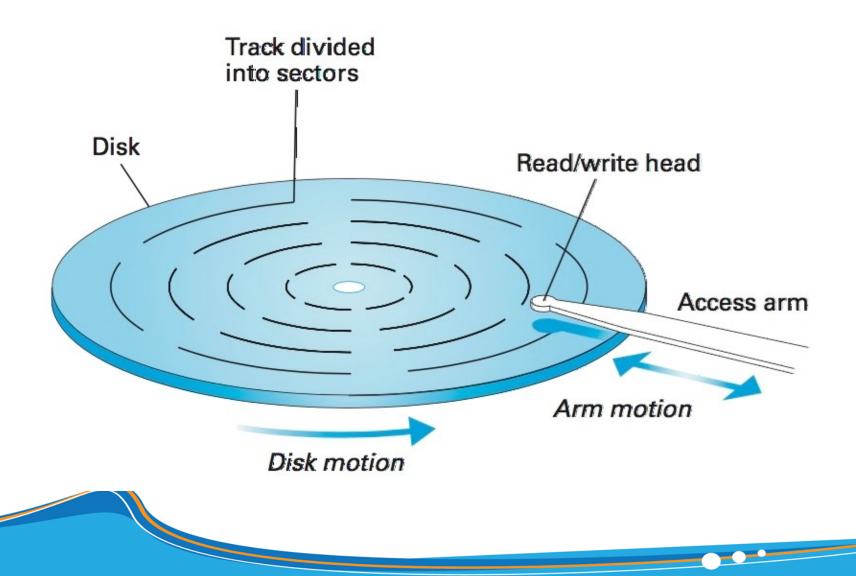








Magnetic disk storage system





Magnetic disk storage system

- Seek time
 - □ Time needed to position the read/write head over the correct track
- Latency
 - □ Time for the beginning of the desired sector to rotate under the read/write head
- Transfer time
 - □ Time for the entire sector to pass under the read/write head and have its contents read into or written from memory

cdio

Magnetic disk storage system

☐ Given:

- □ Rotation speed = 7,200 rev/min=120 rev/sec = 8.33 msec/rev
- ☐ Arm movement time = 0.02 msec to move to an adjacent track
- On average, the read/write head must move about 300 tracks
- Number of tracks/surface = 1,000
- Number of sectors/track = 64
- Number of bytes/sector = 1,024

Seek time

- Best case = 0 msec;
- Worst case = 999*0.02=19.98 msec
- Average case = 300*0.02 = 6 msec

cdio

Magnetic disk storage system

☐ Given:

- □ Rotation speed = 7,200 rev/min=120 rev/sec = 8.33 msec/rev
- ☐ Arm movement time = 0.02 msec to move to an adjacent track
- On average, the read/write head must move about 300 tracks
- Number of tracks/surface = 1,000
- Number of sectors/track = 64
- Number of bytes/sector = 1,024

Latency

- Best case = 0 msec;
- Worst case = 8.33 msec
- Average case = 4.17 msec



Magnetic disk storage system

- ☐ Given:
 - □ Rotation speed = 7,200 rev/min=120 rev/sec = 8.33 msec/rev
 - ☐ Arm movement time = 0.02 msec to move to an adjacent track
 - □ On average, the read/write head must move about 300 tracks
 - Number of tracks/surface = 1,000
 - Number of sectors/track = 64
 - Number of bytes/sector = 1,024
- Transfer time
 - □ 1/64 * 8.33 msec = 4.17 msec



Magnetic tape storage

