THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Simulation 1

Lecture 27

STA 371G

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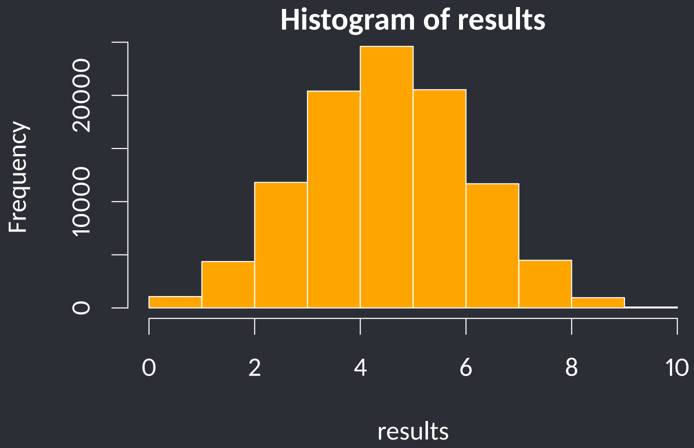
- And what do we do when faced with a scenario that can’t be modeled by decision trees, e.g., predicting the value of a portfolio at retirement age?
- Simulation lets us get a picture of the full distribution of possible outcomes in just about any problem scenario.
- We’ll start with some silly examples today and then delve into business applications next time.

Example 1: Coin flipping

Suppose we flip a coin 10 times. What will the distribution of the number of heads look like?

```r
results <- replicate(100000, {
  flips <- sample(c(0, 1), 10, replace=T)
  sum(flips)
})
hist(results, breaks=10, col='orange')
```



Theory tells us that the outcome of flipping a coin 10 times and counting heads should be a Binomial distribution with expected value $np = 10 \cdot 0.5 = 5$ and SD $\sqrt{np(1-p)} = \sqrt{10 \cdot 0.5 \cdot 0.5} \approx 1.58$.

We can compare our simulated results against the theoretical results:

```
mean(results)

[1] 5.00154

sd(results)

[1] 1.581519
```


LET'S
MAKE
A
DEAL


1
2
3


GSN
CADILLAC


ZONK!!

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- There are three doors: two contain a goat, and one contains…a new car!
- You (the contestant) select one of three doors.
- Without revealing what is behind the selected door, Monty opens one of the other two doors to reveal a goat.

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- Monty Hall used to host the game show *Let's Make a Deal*.
- There are three doors: two contain a goat, and one contains…a new car!
- You (the contestant) select one of three doors.
- Without revealing what is behind the selected door, Monty opens one of the other two doors to reveal a goat.
- Monty then gives you a choice: keep your original door, or switch to the other (unopened) door.

Example 2: Monty Hall and *Let's Make a Deal*

Do you have a better chance of getting the car by switching, or by keeping your original selection—or does it not matter?

```r
doors <- c(1, 2, 3)
results <- replicate(100000, {
  # Randomly select my door and the door with the car.
  car.door <- sample(doors, 1)
  my.door <- sample(doors, 1)
  # If I chose the door with the car, he randomly opens one
  #   of the other two doors.
  # If I chose another door, he opens the remaining door.
  if (car.door == my.door) {
    monty.opens.door <- sample(setdiff(doors, my.door), 1)
  } else {
    monty.opens.door <- setdiff(doors, c(my.door, car.door))
  }
  # Switch doors: select the door that Monty did not open.
  my.door <- setdiff(doors, c(monty.opens.door, my.door))
  my.door == car.door
})
sum(results) / 100000

[1] 0.66824
```

```r
doors <- c(1, 2, 3)
results <- replicate(100000, {
  # Randomly select my door and the door with the car.
  car.door <- sample(doors, 1)
  my.door <- sample(doors, 1)
  # If I chose the door with the car, he randomly opens one
  #   of the other two doors.
  # If I chose another door, he opens the remaining door.
  if (car.door == my.door) {
    monty.opens.door <- sample(setdiff(doors, my.door), 1)
  } else {
    monty.opens.door <- setdiff(doors, c(my.door, car.door))
  }
  # Don't switch; just check to see if I won the car.
  my.door == car.door
})
sum(results) / 100000
```

```
[1] 0.3306
```

Simulating random variables

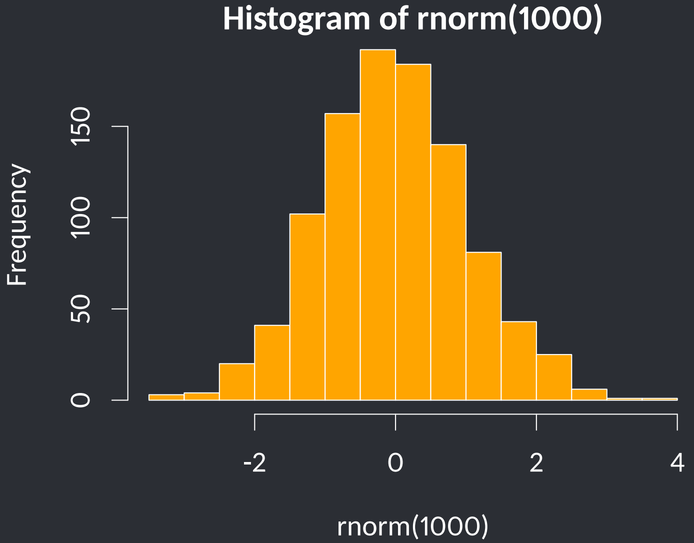
We have seen how the `sample` command can be used to draw from a set of alternatives with equal probability (e.g., flipping a coin). The `rnorm` command can be used to draw randomly from a normal distribution. Let's create 10 random heights, with mean 68 (inches) and SD 4.

```
rnorm(10, 68, 4)

 [1] 68.57607 63.99855 70.53150 65.56610 66.38570
 [6] 69.82433 67.64435 69.96917 72.92480 70.23245
```

Let's check that `rnorm` works as advertised!

```
hist(rnorm(1000), col='orange')
```



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- I want to know how likely it is that I can get 90% or above on my final grade.

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- I just got my score on the first midterm (75%).
- I want to know how likely it is that I can get 90% or above on my final grade.
- This is hard!

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- My best guess is that I’ll get an 80% on Midterm 2, and I’m 95% sure it will be between 70% and 90%

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- I think I can improve on the second midterm, and then even more on the final.
- I'll model my Midterm 2 grade as a normal distribution.
- My best guess is that I'll get an 80% on Midterm 2, and I'm 95% sure it will be between 70% and 90%
- So my Midterm 2 grade should be simulated as a normal distribution with mean 80 and SD 5 (since 95% of a normal distribution is roughly ± 2 SD from the mean).

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- I'll model my Midterm 2 grade as a normal distribution.
- My best guess is that I'll get an 80% on Midterm 2, and I'm 95% sure it will be between 70% and 90%
- So my Midterm 2 grade should be simulated as a normal distribution with mean 80 and SD 5 (since 95% of a normal distribution is roughly ± 2 SD from the mean).
- I think I can improve more on the final; my best guess is that I'll get a 90%, and I'm 95% sure I'll get between 80% and 100%.

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    2. Calculate a final score for the course, and see if it's over 90%.

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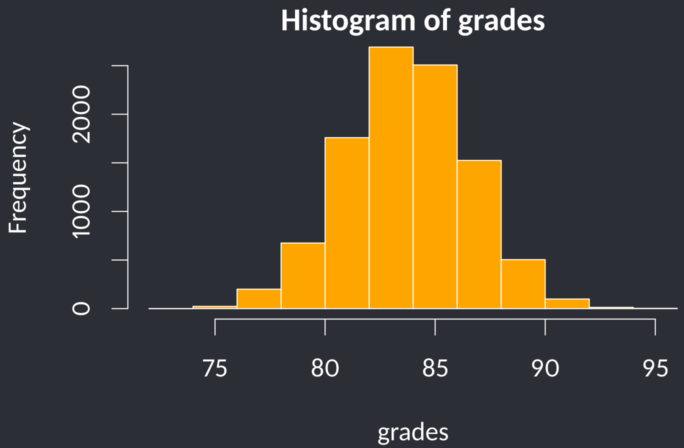
- For each run, we will:
    1. Randomly draw a Midterm 2 score from its normal distribution, and a Final Exam score from its normal distribution.
    2. Calculate a final score for the course, and see if it's over 90%.
- Then we will count the percentage of runs where we got 90%+ for the course. That will be our estimate of the probability of getting an A.

```r
grades <- replicate(10000, {
  midterm1 <- 75
  midterm2 <- rnorm(1, mean=80, sd=5)
  final.exam <- rnorm(1, mean=90, sd=5)
  0.25*midterm1 + 0.25*midterm2 + 0.5*final.exam
})
hist(grades, col='orange')
```



```r
runs <- replicate(10000, {
  midterm1 <- 75
  midterm2 <- rnorm(1, mean=80, sd=5)
  final.exam <- rnorm(1, mean=90, sd=5)
  0.25*midterm1 + 0.25*midterm2 + 0.5*final.exam >= 90
})
sum(runs) / 10000
```

```
[1] 0.0115
```

```r
runs <- replicate(10000, {
  midterm1 <- 75
  midterm2 <- rnorm(1, mean=80, sd=5)
  final.exam <- rnorm(1, mean=90, sd=5)
  0.25*midterm1 + 0.25*midterm2 + 0.5*final.exam >= 90
})
sum(runs) / 10000

[1] 0.0115
```

There's only about a 1.15% chance that I'll get an A.