THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Time Series: Trends and Seasonality

Lecture 26

STA 371G

Predicting beer production over time

Goal: Predict beer production in the US (in millions of gallons)

An autoregressive model for beer production

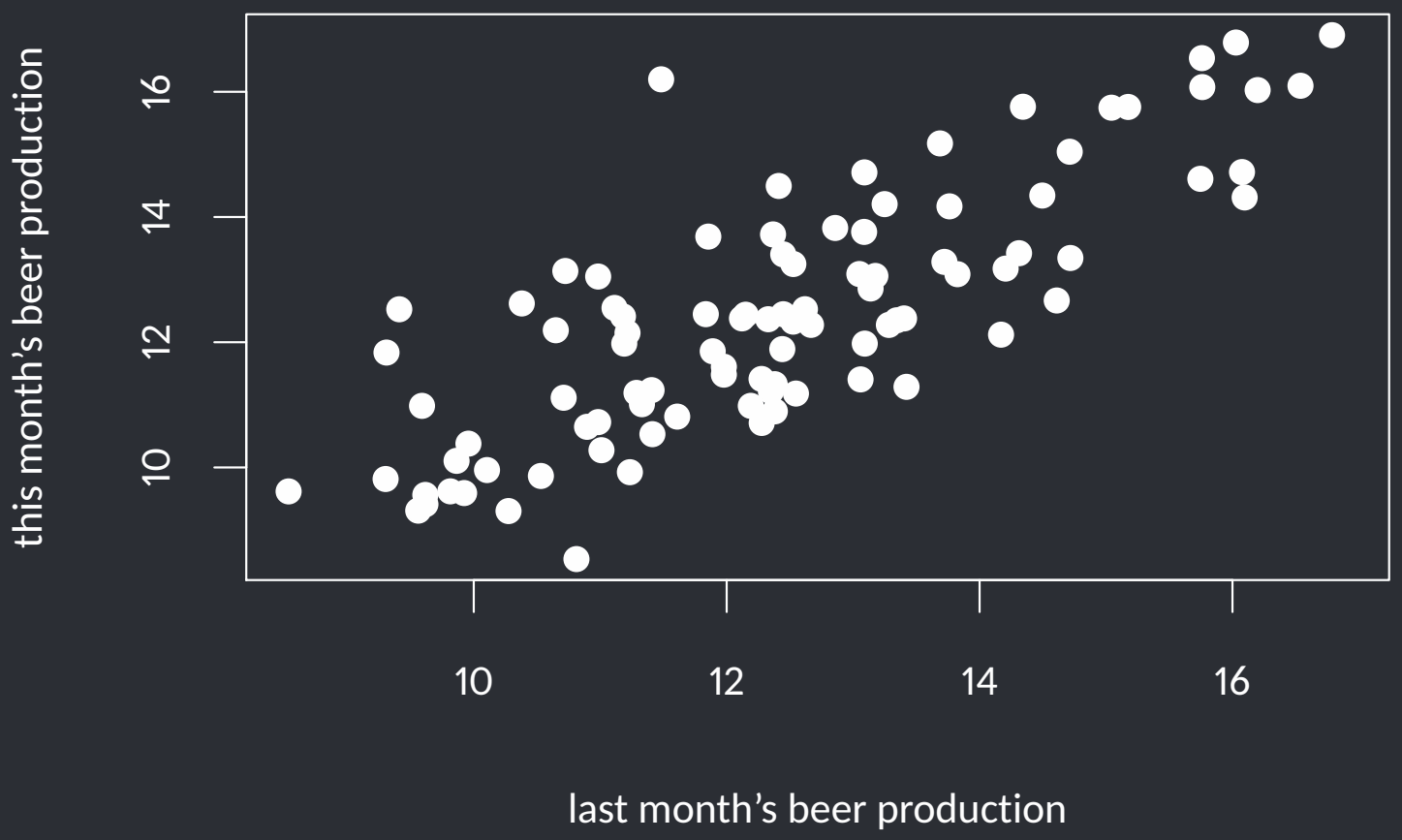
If we believe last month's beer figures are a good prediction for next month's beer figures, we might choose an AR(1) model:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t$$

This means that the next month's beer figures depend only on the current month's beer figures, plus random noise.

Let's look at whether this is a reasonable assumption.

```r
# Convert the data into a time series object (frequency = data pts per year)
beer <- ts(beer.df$beer, start=c(1970,7), frequency=12)
beer_all <- cbind(beer=beer, beerL1=lag(beer, k=-1))
plot(beer ~ beerL1, data=beer_all,
     xlab="last month's beer production",
     ylab="this month's beer production",
     pch=19)
```



```r
model <- lm(beer ~ beerL1, data=beer_all)
summary(model)
```

```
Call:
lm(formula = beer ~ beerL1, data = beer_all)

Residuals:
    Min      1Q  Median      3Q     Max
-2.6018 -0.8428 -0.1902  0.8539  4.5109

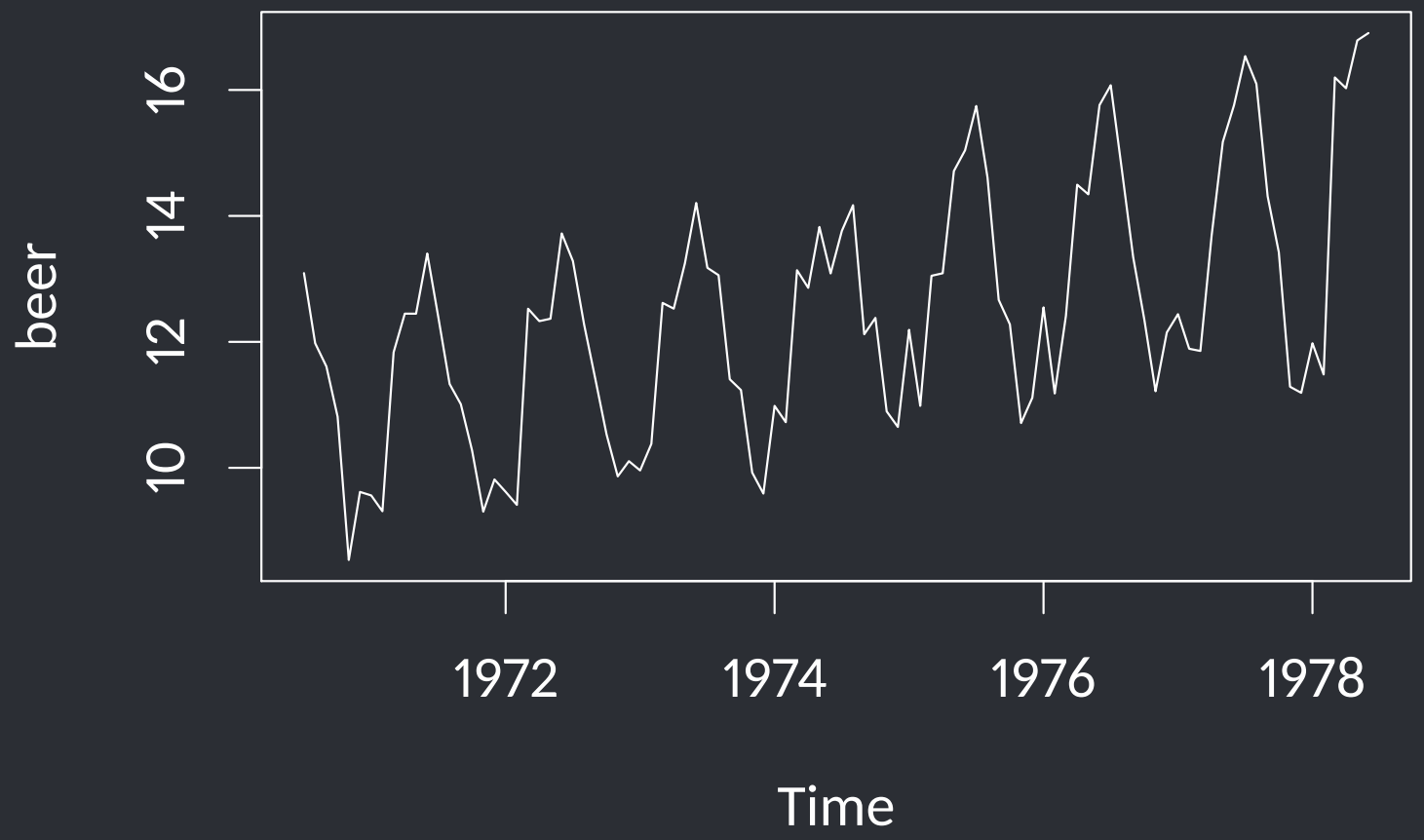
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.25720    0.80557   2.802  0.00618 **
beerL1       0.82136    0.06417  12.800  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.183 on 93 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared:  0.6379,Adjusted R-squared:  0.634
F-statistic: 163.8 on 1 and 93 DF,  p-value: < 2.2e-16
```

Last month's beer production is statistically significant. The R^2 is good, but not amazing—perhaps we can do better!

Hmmm... there seems to be more of a pattern here than in the oil price data from last week!

```
plot(beer)
```



Types of temporal variation

There are several types of temporal variation we might want to predict!

Cyclic variation is *unpredictable* up and down movement, of the sort we saw last week.

- Length of cycle may vary
- Often caused by multiple interacting factors.
- Example: Stock prices vary due to recessions, depressions and recoveries.
- Example: Sales may be affected by fashions.

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If we don't know those factors, often the best we can do is predict based on the last time point... i.e. an autoregressive model.

Trend

A trend is a persistent, overall, upwards or downwards pattern in the data.

- Example: Effects due to population growth (e.g. demand on health services).
- Example: “Moore’s Law” – processor speeds double every two years.
- Could be linear or non-linear – the trend may continue at a constant rate, or accelerate, or level off.

We definitely saw a trend in the beer production numbers!

Seasonal

- Seasonal variation is a regular pattern of up and down fluctuation.
- The length of the cycle is the same (e.g. yearly, monthly)
- Caused by effects such as weather, holidays.
- It is predictable.
- Example: Toy sales increase in December.
- Example: Ice cream sales affected by the weather.

There was clear seasonality in the beer production numbers!

Let's first deal with the upward trend. We can try predicting production from time using a simple linear regression:

```
summary(lm(beer ~ month_count, data=beer.df))



Call:
lm(formula = beer ~ month_count, data = beer.df)

Residuals:
     Min       1Q   Median       3Q      Max
-2.87729 -1.48470  0.00505  1.31704  2.80344

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.576746   0.334920  31.580  < 2e-16 ***
month_count  0.038784   0.005996   6.468 4.42e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.628 on 94 degrees of freedom
Multiple R-squared:  0.308,Adjusted R-squared:  0.3006
F-statistic: 41.84 on 1 and 94 DF,  p-value: 4.415e-09
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Month is a significant predictor! But the R^2 is lower than our AR(1) model, so we need to do better.

Seasonal variation

How can we deal with the seasonal effect?

Seasonal variation

How can we deal with the seasonal effect?

The simplest solution is to treat month as a categorical variable!

- This means that we assume there is some commonality between September 1970, September 1971, September 1972...
- More general, we might want to treat quarters or days of the weeks as categorical variables.

```
Call:
lm(formula = beer ~ month_count + month, data = beer.df)

Residuals:
     Min       1Q   Median       3Q      Max
-2.23574 -0.30407 -0.01724  0.40160  1.65361

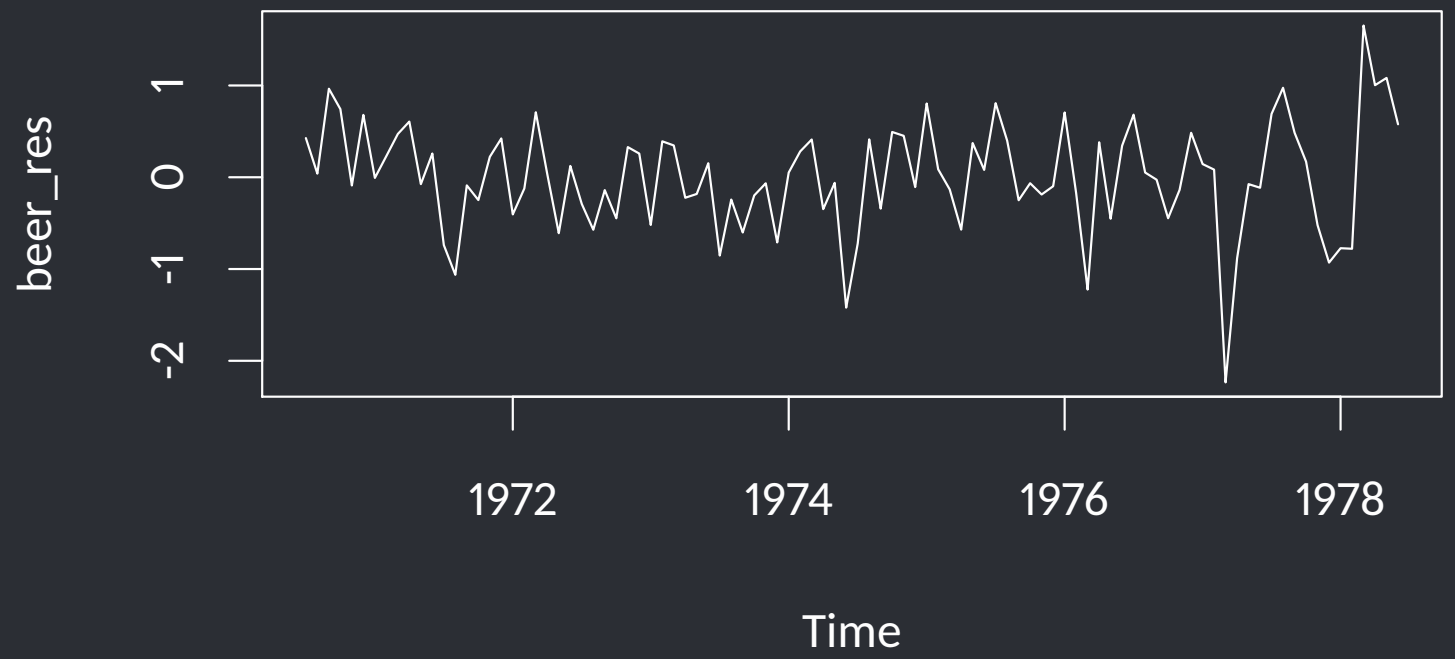
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    11.462493   0.255140  44.926  < 2e-16 ***
month_count     0.037887   0.002351  16.114  < 2e-16 ***
monthAugust     0.400097   0.317257   1.261 0.210801
monthDecember  -2.752326   0.316839  -8.687 2.76e-13 ***
monthFebruary  -2.686726   0.316734  -8.483 7.08e-13 ***
monthJanuary   -2.159214   0.316778  -6.816 1.39e-09 ***
monthJuly       1.164859   0.317405   3.670 0.000428 ***
monthJune       1.227101   0.316734   3.874 0.000213 ***
monthMarch     -0.440613   0.316708  -1.391 0.167874
monthMay        0.642738   0.316708   2.029 0.045618 *
monthNovember  -3.026064   0.316917  -9.548 5.23e-15 ***
monthOctober   -1.544927   0.317013  -4.873 5.20e-06 ***
monthSeptember -0.932040   0.317127  -2.939 0.004263 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6334 on 83 degrees of freedom
Multiple R-squared:  0.9075,Adjusted R-squared:  0.8941
F-statistic: 67.86 on 12 and 83 DF,  p-value: < 2.2e-16
```

- Now that we've modeled seasonality, we've got an R^2 of 0.9075 and an adjusted R^2 of 0.8941... pretty good!
- We've modeled the trend and the seasonality...

- Now that we've modeled seasonality, we've got an R^2 of 0.9075 and an adjusted R^2 of 0.8941... pretty good!
- We've modeled the trend and the seasonality...
- ... maybe there's also a cyclic effect! Let's take a look at the residuals.

```
# Convert the residuals into a time series object
beer_res <- ts(model$residuals, start=c(1970,7), frequency=12)
plot(beer_res)
```



Let's try running an autoregressive model on the residuals:

```
res_all <- cbind(beer_res=beer_res,
                 beer_res_L1=lag(beer_res, k=-1))
model <- lm(beer_res ~ beer_res_L1, data=res_all)
```

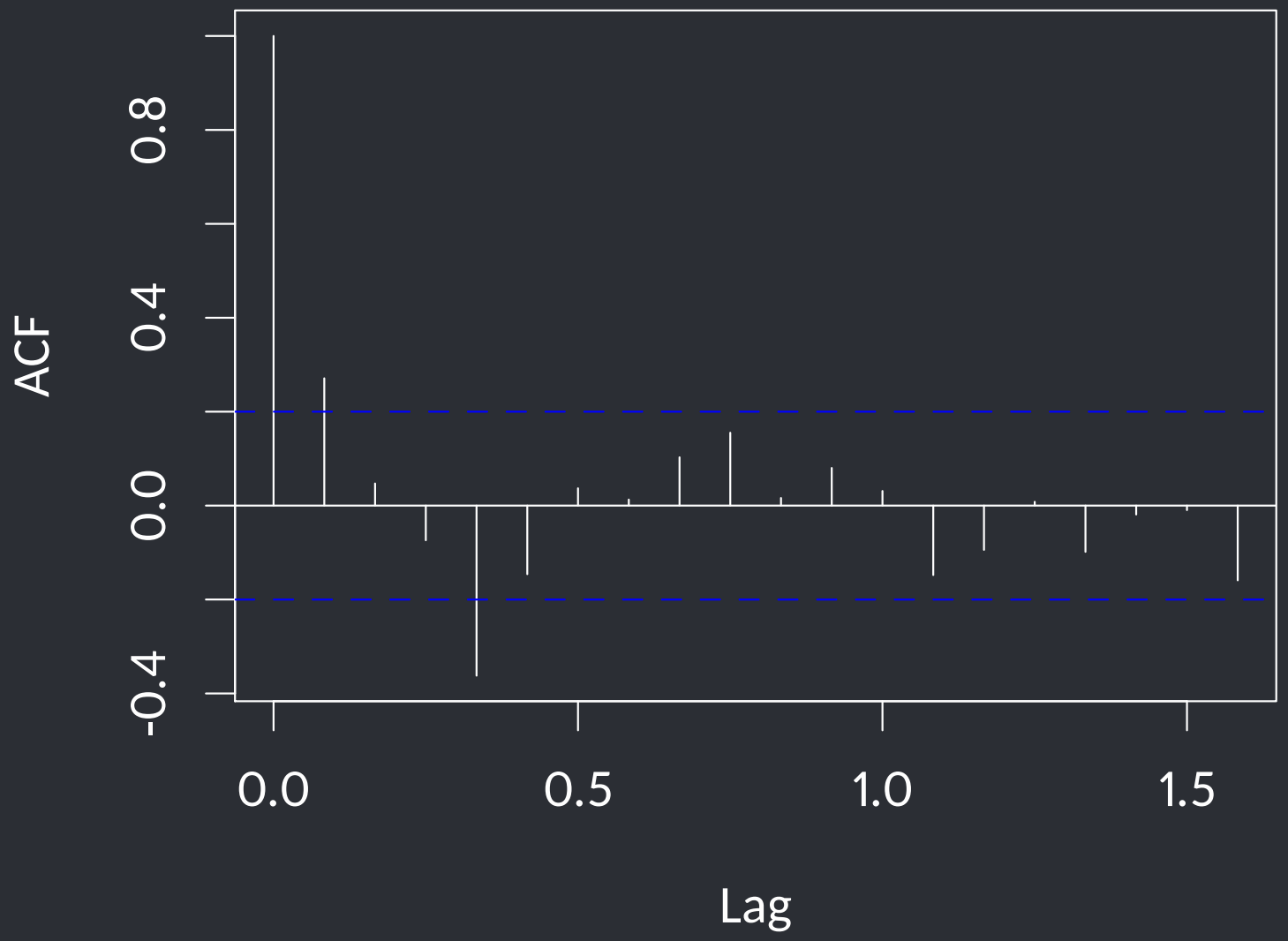
```
Call:
lm(formula = beer_res ~ beer_res_L1, data = res_all)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.25571 -0.34325 -0.00875  0.35091  1.86982 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)   
(Intercept) -0.002829   0.058897  -0.048   0.9618   
beer_res_L1  0.273770   0.099983   2.738   0.0074 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.574 on 93 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared:  0.0746,Adjusted R-squared:  0.06465 
F-statistic: 7.498 on 1 and 93 DF,  p-value: 0.007404
```

Statistically significant… but not really practically significant.



Autocorrelation function agrees… fairly low autocorrelation at lag one.