THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

# Decision Trees 2

## Lecture 7

### STA 371G

# Oil drilling

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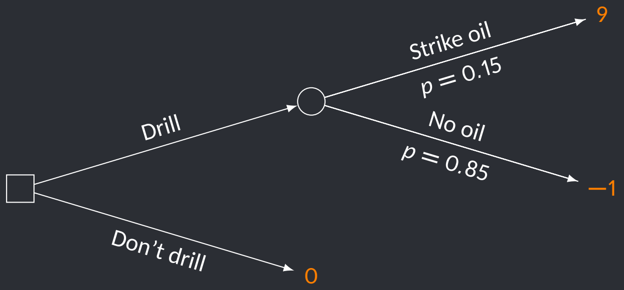
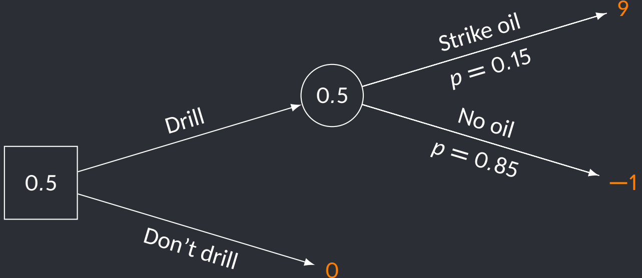
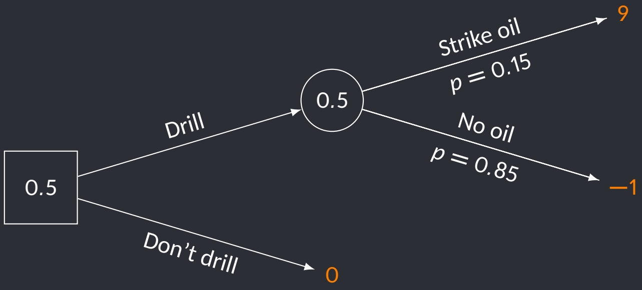
- Suppose you are planning to drill for oil in a newly-discovered field.
- Setting up the drilling equipment costs $1M.
- If you strike oil, you will generate $10M.
- But there's only a 15% chance that a random oil field has oil.


Drill
Don't drill
Strike oil
p = 0.15
No oil
p = 0.85
9
−1
0


0.5
Drill
0.5
Strike oil
$p = 0.15$
9
No oil
$p = 0.85$
−1
Don't drill
0




Without any information, we expect to make $0.5M by deciding to drill.

# How much is perfect information worth?

- Suppose we have a psychic that gives us *perfect information* about whether the field has oil or not.

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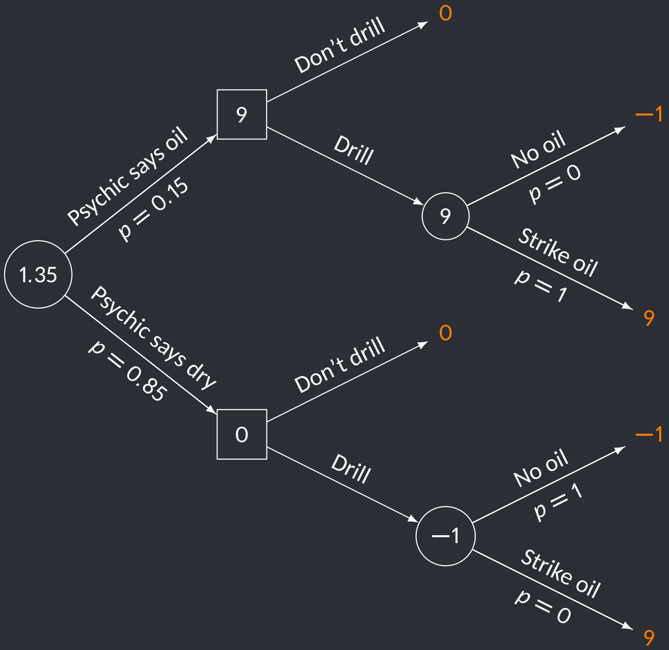
- Suppose we have a psychic that gives us *perfect information* about whether the field has oil or not.
- The psychic is always right!


1.35
Psychic says oil
p = 0.15
Psychic says dry
p = 0.85
9
Don't drill
0
Drill
9
No oil
p = 0
−1
Strike oil
p = 1
9
0
Don't drill
0
Drill
−1
No oil
p = 1
−1
Strike oil
p = 0
9

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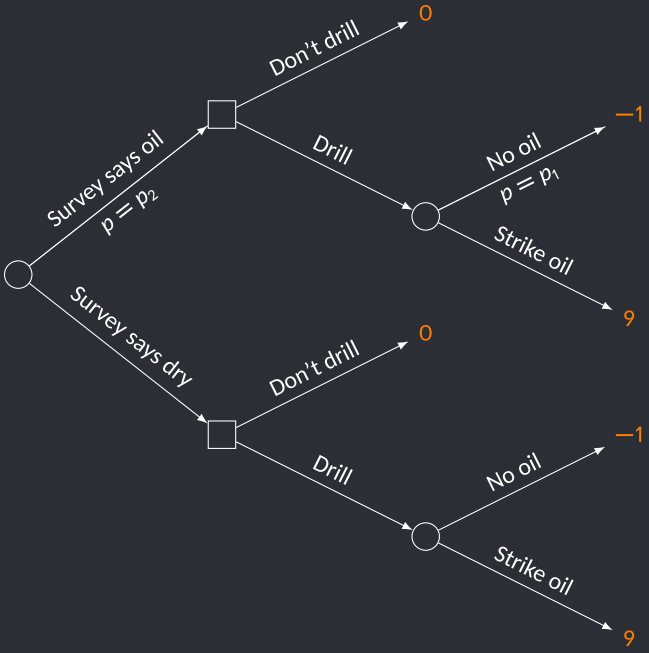
- When a field is oil-rich, the survey will indicate this 60% of the time.
- When a field is dry, the survey will indicate this 80% of the time.
- How much should we be willing to pay for the survey?



Survey says oil
$p = p_2$
Survey says dry
Don't drill
Drill
0
No oil
$p = p_1$
Strike oil
−1
9
Don't drill
Drill
0
No oil
Strike oil
−1
9

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$$P(S|O) = 0.6, \qquad P(\overline{S}|O) = 0.4$$

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$$P(S|\overline{O}) = 0.2,$$

Let's say $O$ is the event that you would strike oil in the oilfield, and $S$ is the event that the survey indicates there is oil present.

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When a field is oil-rich, the survey will indicate this 60% of the time.

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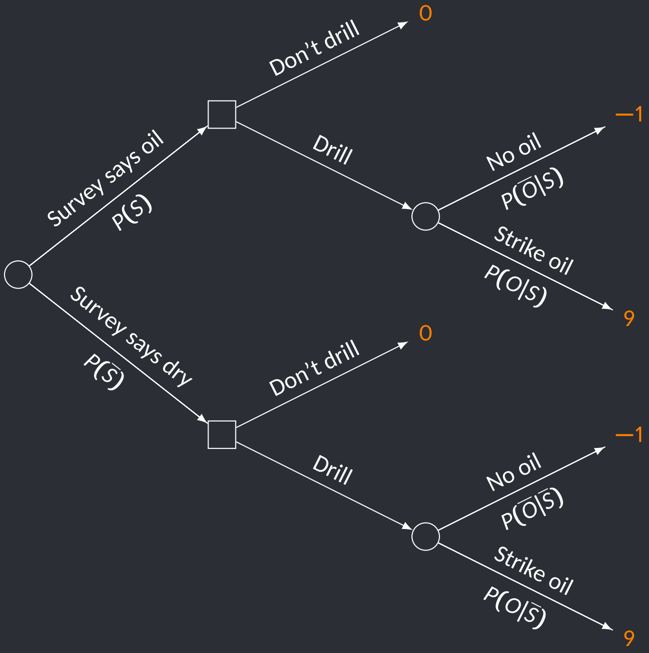
When a field is dry, the survey will indicate this 80% of the time.

$$P(S|\overline{O}) = 0.2, \qquad P(\overline{S}|\overline{O}) = 0.8$$


Survey says oil
P(S)
Survey says dry
P(S̄)
Don't drill
0
Drill
No oil
P(Ō|S)
−1
Strike oil
P(O|S)
9
Don't drill
0
Drill
No oil
P(Ō|S̄)
−1
Strike oil
P(O|S̄)
9

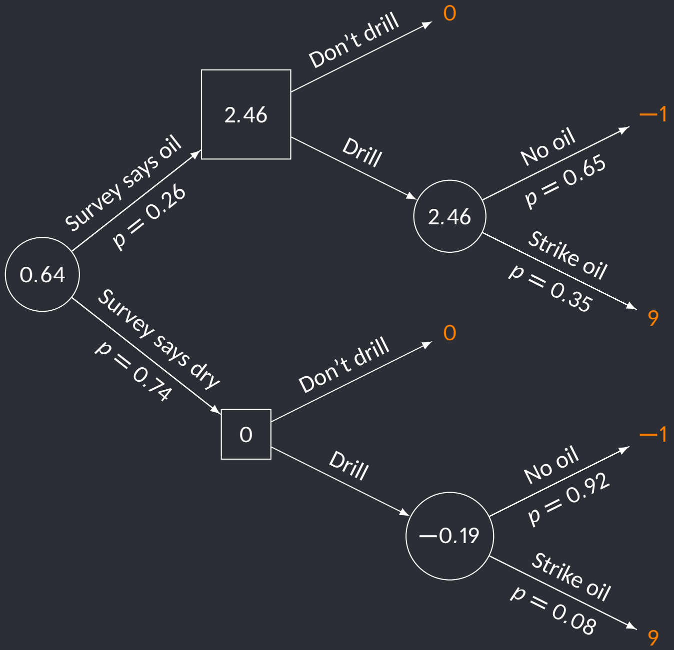
Let's calculate the required probabilities using Bayes' rule:

$$\begin{aligned} P(O|S) &= \frac{P(S|O)P(O)}{P(S|O)P(O) + P(S|\overline{O})P(\overline{O})} \\ &= \frac{(0.6)(0.15)}{(0.6)(0.15) + (0.2)(0.85)} \\ &= 0.35 \end{aligned}$$

$$\begin{aligned} P(\overline{O}|S) &= \frac{P(S|\overline{O})P(\overline{O})}{P(S|\overline{O})P(\overline{O}) + P(S|O)P(O)} \\ &= \frac{(0.2)(0.85)}{(0.2)(0.85) + (0.6)(0.15)} \\ &= 0.65 \end{aligned}$$

We also need to know the probability that the survey will indicate oil:

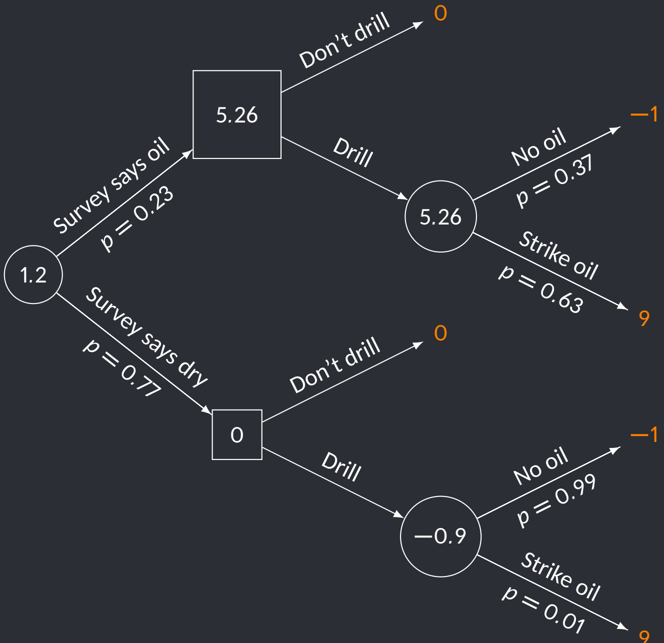
$$
\begin{aligned}
P(S) &= P(S \text{ and } O) + P(S \text{ and } \overline{O}) \\
&= P(S|O)P(O) + P(S|\overline{O})P(\overline{O}) \\
&= (0.6)(0.15) + (0.2)(0.85) \\
&= 0.26
\end{aligned}
$$



0.64
Survey says oil
p = 0.26
2.46
Don't drill
0
Drill
2.46
No oil
p = 0.65
−1
Strike oil
p = 0.35
9
Survey says dry
p = 0.74
0
Don't drill
0
Drill
−0.19
No oil
p = 0.92
−1
Strike oil
p = 0.08
9

- The information from the survey is worth the difference between these two trees ($0.64 - 0.5 = \$0.14M$); this is EVSI (Expected Value of Sample Information)

- The information from the survey is worth the difference between these two trees ($0.64 - 0.5 = \$0.14$M); this is EVSI (Expected Value of Sample Information)
- Suppose we had a better survey, which correctly identifies oil-rich fields 95% of the time, and correctly identifies dry fields 90% of the time.


1.2
Survey says oil
$p = 0.23$
Survey says dry
$p = 0.77$
5.26
Don't drill
0
Drill
5.26
No oil
$p = 0.37$
−1
Strike oil
$p = 0.63$
9
0
Don't drill
0
Drill
−0.9
No oil
$p = 0.99$
−1
Strike oil
$p = 0.01$
9

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  - No survey (or any other kind of information about this oil field) could ever be worth more than \$0.85M.

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  - No survey (or any other kind of information about this oil field) could ever be worth more than \$0.85M.
- With imperfect information (the better survey), the expected value of the tree was \$1.2M. The EVSI (expected value of sample information) is $1.2 - 0.5 = \$0.7$M.
  - It's worth paying up to \$0.7M for this particular survey.