



THE UNIVERSITY OF TEXAS AT AUSTIN  
McCOMBS SCHOOL OF BUSINESS

# Decision Trees 1

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## Lecture 6

STA 371G

# Making Better Decisions



*Decision making is the only way that individuals can purposely exercise any control over their lives, careers, or their surroundings.*

- Ralph Keeney, Making Better Decision Makers, Decision Analysis, vol. 1 No:4, 2004



# Decision Analysis

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- Decision trees: a powerful graphical tool that guides that analysis
- Smaller analyses can be done using pen and paper
- Larger ones require software

## Topics to cover

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- How probabilities are used in the decision-making process
- How early decisions affect later decisions
- How a decision-maker can quantify the value of information
- How attitudes toward risk and uncertainty can affect the analysis

# Elements of a Decision Analysis

All problems have three common elements:

- The decisions available to the decision maker.
- The possible outcomes and the probabilities of these outcomes.
- A value model that provides monetary values for the various outcomes.

Once these elements are defined, the decision maker can find an optimal decision.

## Payoff Tables

A payoff table lists the payoff for each decision outcome pair; positive values are gains and negative values are losses.

	O1	O2	O3
D1	\$10	\$10	\$10
D2	-\$10	\$20	\$30
D3	-\$30	\$30	\$80

- This table shows three possible decisions (D1, D2, and D3) and three possible outcomes (O1, O2, and O3) for each.
- Which decision do you prefer?



## Payoff Tables

We need to know the probability of each outcome to make a good decision!

	O1	O2	O3
D1	\$10	\$10	\$10
D2	-\$10	\$20	\$30
D3	-\$30	\$30	\$80

- Suppose  $P(O1) = 0.3$ ,  $P(O2) = 0.5$ ,  $P(O3) = 0.2$
- Now which decision do you prefer?

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- $E(D3) = ?$



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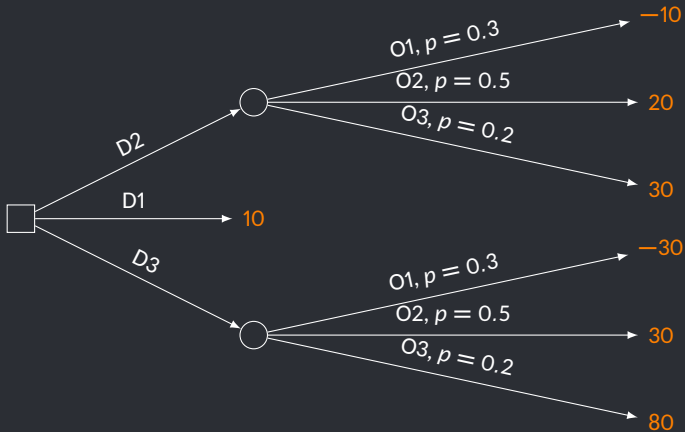
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- Monetary values are shown to the right of the end nodes.
- EVs are calculated through a “rolling-back” process.

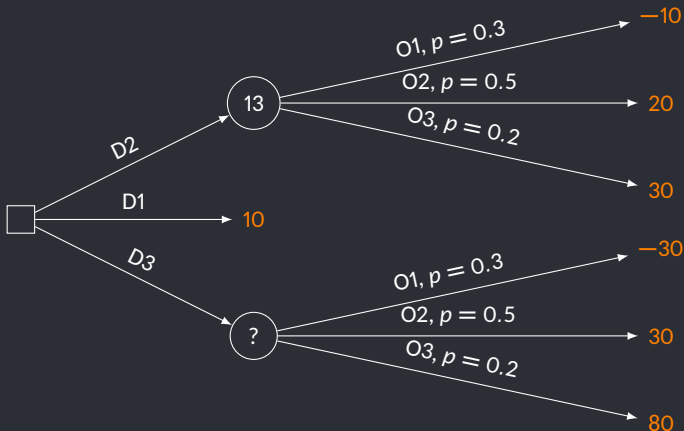
## Example





## Rolling back: Step 1

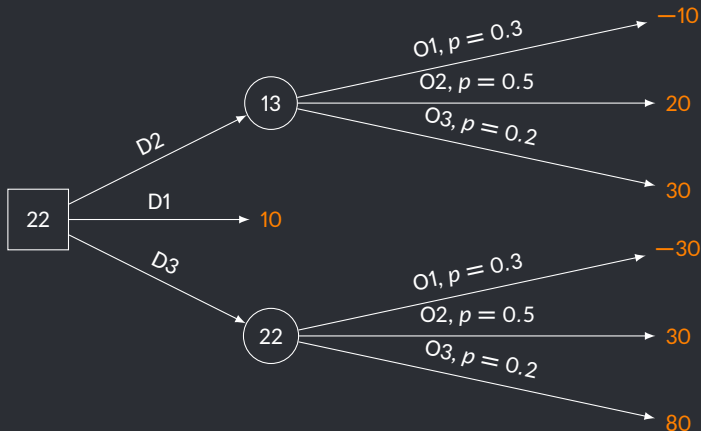
Calculate the expected value at each probability node:



$$E(D_1) = .3(-10) + .5(20) + .2(30) = 13$$

## Rolling back: Step 2

Calculate the maximum at each decision node:



Take decision D3 since  $22 = \max(10, 13, 22)$ .

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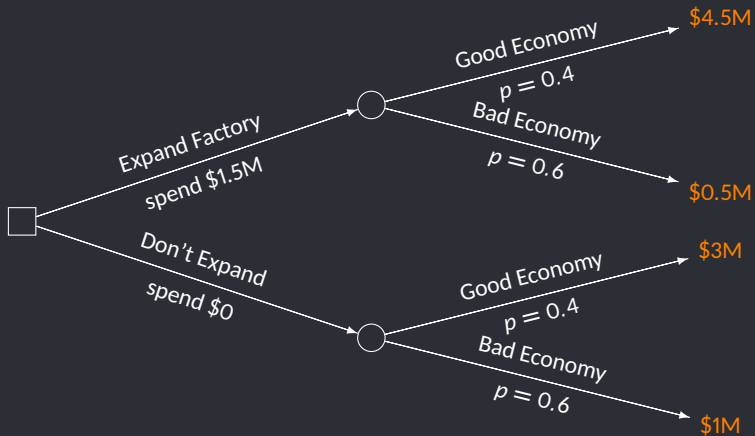
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- If she does nothing and the economy stays good, she expects to earn \$3M in revenue, but if the economy is bad, she expects only \$1M.
- If she expands the factory, she expects to earn \$6M if the economy is good and \$2M if it is bad.

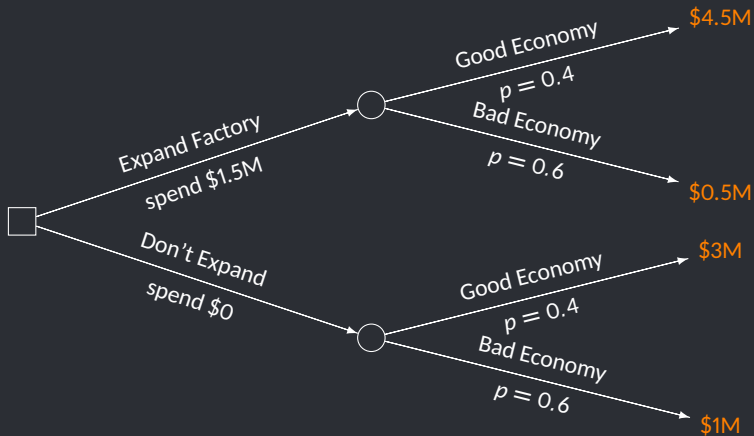
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- The cost of the expansion is \$1.5M.
- If she does nothing and the economy stays good, she expects to earn \$3M in revenue, but if the economy is bad, she expects only \$1M.
- If she expands the factory, she expects to earn \$6M if the economy is good and \$2M if it is bad.
- She estimates that there is a 40 percent chance of a good economy and a 60 percent chance of a bad economy.

Should she expand?

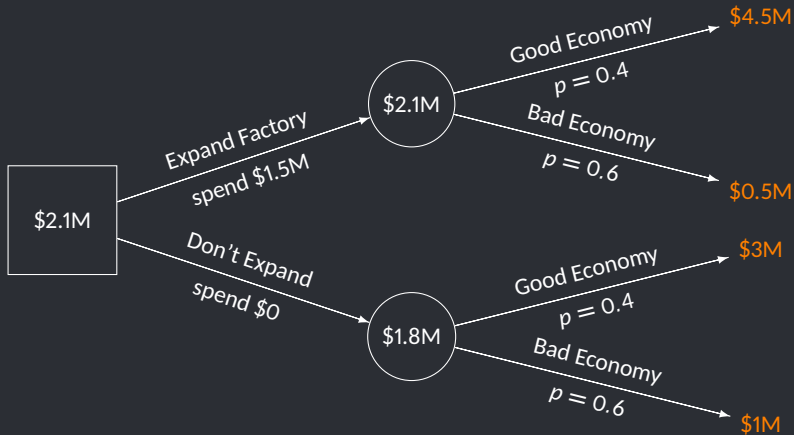




$$E(\text{expand}) = .4(4.5) + .6(0.5) = 2.1$$

$$E(\text{don't expand}) = .4(3) + .6(1) = 1.8$$





$$E(\text{expand}) = (.4(6) + .6(2)) - 1.5 = 2.1$$

$$E(\text{don't expand}) = (.4(3) + .6(1)) = 1.8$$

Since  $\$2.1 > \$1.8$ , she should expand!

## Sequential decisions

She later learns if she expands, she can opt to either:

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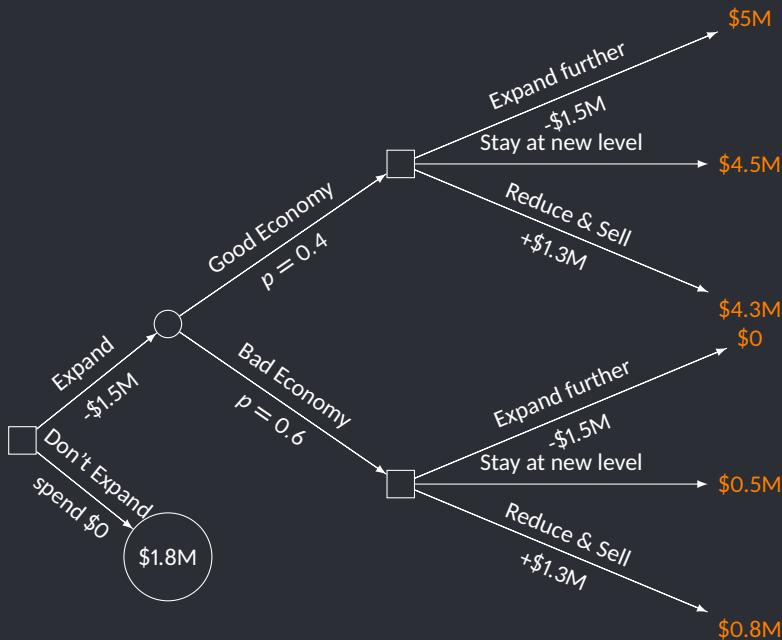
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- (c) do nothing.

How has the decision changed?



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Therefore the value of the option is

$$E(\text{new}) - E(\text{old}) = 2.48 - 2.1 = 0.38,$$

or \$380,000.

## Does this look familiar?

- This method of valuing a real option is used by corporate strategists, management consultants, and bankers.



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- This method of valuing a real option is used by corporate strategists, management consultants, and bankers.
- A further refinement is to take time value of money into account and present value everything.