THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Simple Regression 1

Lecture 9

STA 371G

National Longitudinal Study of Adolescent to Adult Health

Nationally representative sample of US students in grades 7-12 were surveyed in the 1994-95 school year (`http://www.cpc.unc.edu/projects/addhealth`)

Students were followed up on with subsequent in-home interviews four times (most recently 2008)

This is an **awesome** data set, with data on:

- family
- relationships
- health
- military service
- religion
- sex and STDs
- economics
- education
- personality
- criminality
- tobacco
- drugs
- alcohol
- pregnancy
- sleep
- daily activities

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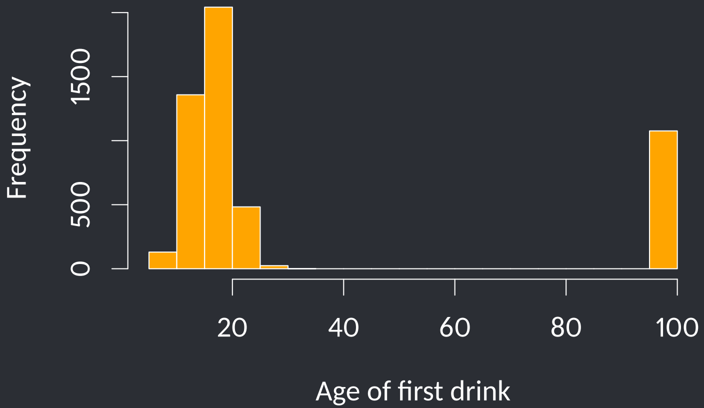
We want to know:

- What is our best **prediction** of alcohol consumption if we know at what age had their first drink?
- How good is that prediction?
- What is the **relationship** between alcohol consumption and age of first drink?

Age of first drink	**Predictor variable**
Number of drinks consumed as adult	**Response variable**

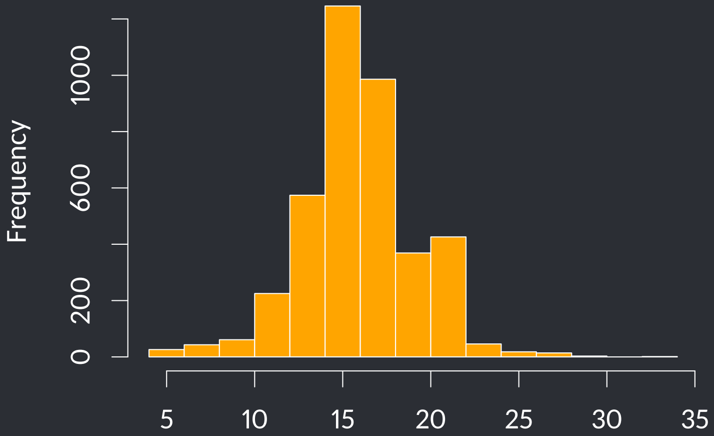
```
> hist(addhealth_public4$h4to34,
+   main='', xlab='Age of first drink',
+   col='orange')
```



Let’s examine our variables

If Q.33 = 1, ask Q.34, else skip to Q.63.			
H4TO34		Num	34. How old were you when you first had an alcoholic drink? By drink, we mean a glass of wine, a can or bottle of beer, a wine cooler, a shot glass of liquor, or a mixed drink, not just sips or tastes from someone else's drink. NOTE: Smallest 5 and largest 5 values are displayed.
Frequency	Percent	Value	Label
56	0.4%	5	5 years
30	0.2%	6	6 years
21	0.1%	7	7 years
71	0.5%	8	8 years
52	0.3%	9	9 years
12014	76.5%	10-31	NOTE: Range of values omitted from display
1	0.0%	32	32 years
2	0.0%	33	33 years
21	0.1%	96	refused
3322	21.2%	97	legitimate skip
111	0.7%	98	don't know

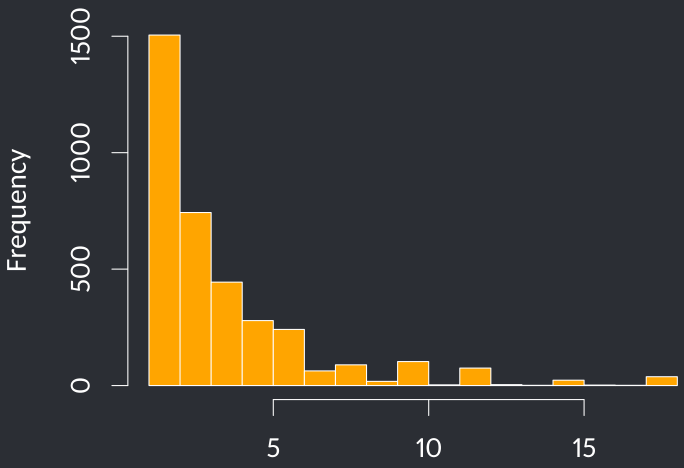
```r
> age <- addhealth_public4$h4to34
> age[age >= 96] <- NA
> hist(age, main='', xlab='', col='orange')
```



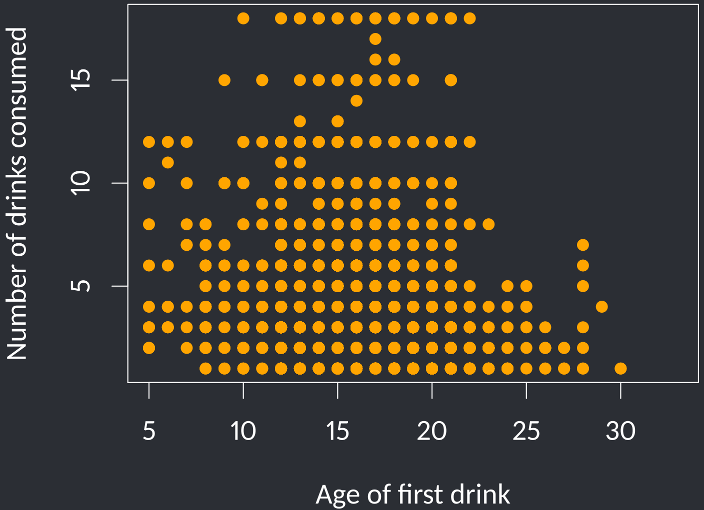
Let’s examine our variables

If Q.35 not equal 0, ask Q.36, else if Q.35 = 0, then skip to Q.43.			
H4TO36		Num	36. Think of all the times you have had a drink during the past 12 months. How many drinks did you **usually** have each time? A 'drink' is a glass of wine, a can or bottle of beer, a wine cooler, a shot glass of liquor, or a mixed drink. NOTE: Smallest 5 and largest 5 values are displayed.
Frequency	Percent	Value	Label
1651	10.5%	1	1 drink
3051	19.4%	2	2 drinks
2274	14.5%	3	3 drinks
1343	8.6%	4	4 drinks
891	5.7%	5	5 drinks
1815	11.6%	6-16	NOTE: Range of values omitted from display
4	0.0%	17	17 drinks
108	0.7%	18	18 drinks
27	0.2%	96	refused
4427	28.2%	97	legitimate skip
110	0.7%	98	don't know

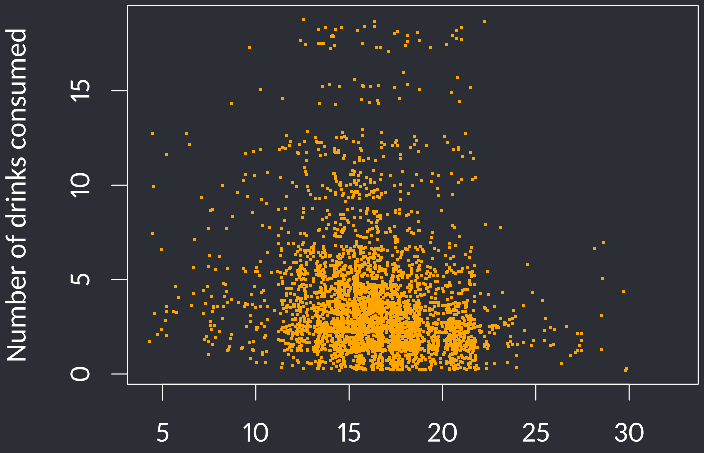
```
> num.drinks <- addhealth_public4$h4to36
> num.drinks[num.drinks >= 96] <- NA
> hist(num.drinks, main='', xlab='How many drinks',
+   col='orange')
```



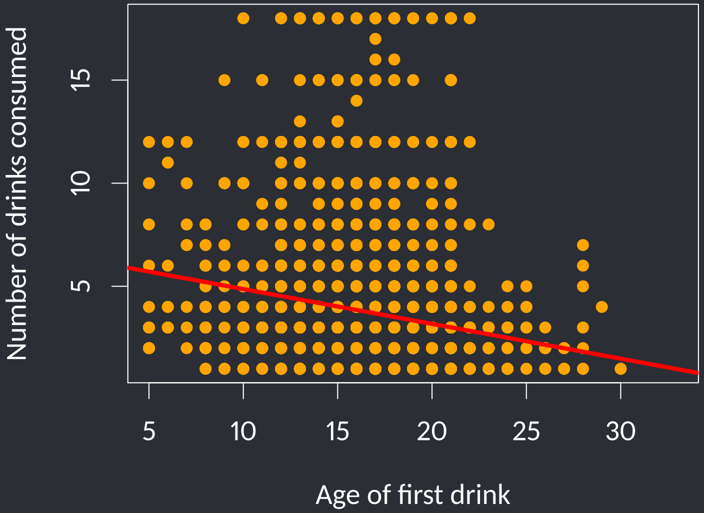
```r
> plot(num.drinks ~ age, pch=16, col='orange',
+   xlab='Age of first drink',
+   ylab='Number of drinks consumed')
```



```r
> plot(jitter(num.drinks, 4) ~ jitter(age, 4),
+   pch=46, col='orange',
+   xlab='Age of first drink',
+   ylab='Number of drinks consumed')
```



The regression line is the line of "best fit" through this plot:



What is linear regression doing?

We model each case ($x_i =$ age for ith person, $y_i =$ number of drinks for ith person) as a linear relationship plus some error:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

β_0 and β_1 are the intercept and slope, respectively.

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We find estimates for β_0 and β_1 in our sample that *minimize* the errors:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

This is the regression (best fit) line.

```
> model <- lm(num.drinks ~ age)
> summary(model)


Call:
lm(formula = num.drinks ~ age)

Residuals:
    Min      1Q  Median      3Q     Max
-4.2035 -1.8528 -0.8528  0.8095 15.1602

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.55417    0.26532   24.70   <2e-16 ***
age         -0.16883    0.01588  -10.63   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.963 on 3600 degrees of freedom
  (2902 observations deleted due to missingness)
Multiple R-squared:  0.03044,Adjusted R-squared:  0.03017
F-statistic:   113 on 1 and 3600 DF,  p-value: < 2.2e-16
```

This translates to a regression line of:

$$\widehat{\text{num drinks}} = 6.55 - 0.17 \cdot \text{age}$$

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Predict number of drinks for age $= 21$:

$$\widehat{\text{num drinks}} = 6.55 - 0.17 \cdot 21 = 3.01$$

Or we can use R to do the work for us:

```
> predict.lm(model, list(age=21))
```

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R^2 quantifies how closely the model fits the data.

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- $R^2 = 1$ when the model yields perfect predictions every time.
- $R^2 = \text{cor}(Y, \hat{Y})^2$, i.e., the squared correlation between the actual and predicted values of Y.

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> model <- lm(num.drinks ~ age)
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Call:
lm(formula = num.drinks ~ age)

Residuals:
   Min      1Q  Median      3Q     Max 
-4.204 -1.853 -0.853  0.810 15.160 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   6.5542     0.2653    24.7   <2e-16 ***
age          -0.1688     0.0159   -10.6   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3 on 3600 degrees of freedom
  (2902 observations deleted due to missingness)
Multiple R-squared:  0.0304,Adjusted R-squared:  0.0302 
F-statistic:  113 on 1 and 3600 DF,  p-value: <2e-16
```

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Is this “significant?”

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- **Statistical significance:** Can we reject the null hypothesis that the correlation between X and Y in the *population* is zero?

In our regression, $R^2 = 0.03$, so $r = \sqrt{0.03} = 0.17$.
Is this "significant?"

- **Statistical significance:** Can we reject the null hypothesis that the correlation between X and Y in the *population* is zero?
- **Practical significance:** Is the correlation in our sample large enough to be meaningful?

The overall null hypothesis for a regression model

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The following are equivalent ways to express the overall null hypothesis:

- $R^2 = 0$ (in the population)
- $\text{cor}(X, Y) = 0$ (in the population)
- $\beta_1 = 0$
- The model has no predictive power
- Predictions from this model are no better than predicting $\overline{Y}$ for every case

Two ways to test the overall null hypothesis

- The *F*-test (tests $H_0 : R^2 = 0$ in the population)
- The *t*-test for the *slope* (β_1) coefficient (tests $H_0 : \beta_1 = 0$)

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- The F-test (tests $H_0 : R^2 = 0$ in the population)
- The t-test for the *slope* (β_1) coefficient (tests $H_0 : \beta_1 = 0$)

Both of these methods are equivalent; the p-values will be exactly the same!

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> summary(model)


Call:
lm(formula = num.drinks ~ age)

Residuals:
   Min     1Q Median     3Q    Max
-4.204 -1.853 -0.853  0.810 15.160

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   6.5542     0.2653    24.7   <2e-16 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

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- Each additional year you wait to start drinking is associated with consuming 0.17 fewer drinks as an adult.

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- There is a **statistically significant** relationship between the age someone starts drinking and how much they drink as an adult.
- Or: People that start drinking earlier in life consume **significantly more** alcohol when they drink as adults.
- Each additional year you wait to start drinking is associated with consuming 0.17 fewer drinks as an adult.
- Is this relationship **practically significant**?

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- Our best estimate for the *effect* of a year's postponement of drinking is 0.17 fewer drinks as an adult
- We can use a confidence interval to give a range of plausible values for what this effect size is in the population

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$$\text{estimate} \pm (\text{critical value})(\text{standard error}).$$

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Recall that the critical value for a 95% confidence interval is the cutoff value that cuts off 95% of the area in the middle of the distribution; the sampling distribution of $\hat{\beta}_1$ is a t-distribution.

```
> n <- nobs(model)
> qt(0.975, n-2)

[1] 1.960623
```

Put a confidence interval on it

R will also calculate confidence intervals for us:

```
> confint(model)

                 2.5 %     97.5 %
(Intercept)  6.0339847  7.0743549
age         -0.1999713 -0.1376959
```

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```
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                 2.5 %     97.5 %
(Intercept)  6.0339847  7.0743549
age         -0.1999713 -0.1376959
```

In other words, we are 95% confident that the effect of each additional year's delay in starting to drink is between 0.14 and 0.2.

Put a confidence interval on it, part 2

We can also put a confidence interval on a prediction!

Two kinds of intervals:

Confidence	Predicting the mean value of Y for a particular X.	Among all people that start drinking at age 21, how many drinks do have on average as adults?
Prediction	Predicting Y for a single new case.	If Bob started drinking at age 21, how many drinks do we think will have as an adult?

Put a confidence interval on it, part 2

```
> predict.lm(model, list(age=21),
+   interval='confidence')

       fit      lwr      upr
1 3.008664 2.83616 3.181167

> predict.lm(model, list(age=21),
+   interval='prediction')

       fit       lwr      upr
1 3.008664 -2.802894 8.820221
```

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> predict.lm(model, list(age=21),
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       fit     lwr      upr
1 3.008664 2.83616 3.181167

> predict.lm(model, list(age=21),
+   interval='prediction')

       fit       lwr      upr
1 3.008664 -2.802894 8.820221
```

Why is the prediction interval wider?