THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Decision Trees 1

Lecture 6

STA 371G

Making Better Decisions

Decision making is the only way that individuals can purposely exercise any control over their lives, careers, or their surroundings.
- Ralph Keeney, Making Better Decision Makers, Decision Analysis, vol. 1 No:4, 2004

Decision Analysis

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- Smaller analyses can be done using pen and paper
- Larger ones require software

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- How probabilities are used in the decision-making process
- How early decisions affect later decisions
- How a decision-maker can quantify the value of information
- How attitudes toward risk and uncertainty can affect the analysis

Elements of a Decision Analysis

All problems have three common elements:

- The decisions available to the decision maker.
- The possible outcomes and the probabilities of these outcomes.
- A value model that provides monetary values for the various outcomes.

Once these elements are defined, the decision maker can find an optimal decision.

Payoff Tables

A payoff table lists the payoff for each decision outcome pair; positive values are gains and negative values are losses.

	O1	O2	O3
D1	$10	$10	$10
D2	-$10	$20	$30
D3	-$30	$30	$80

- This table shows three possible decisions (D1, D2, and D3) and three possible outcomes (O1, O2, and O3) for each.
- Which decision do you prefer?

Payoff Tables

We need to know the probability of each outcome to make a good decision!

	O1	O2	O3
D1	\$10	\$10	\$10
D2	-\$10	\$20	\$30
D3	-\$30	\$30	\$80

- Suppose $P(O1) = 0.3$, $P(O2) = 0.5$, $P(O3) = 0.2$
- Now which decision do you prefer?

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- $E(D3) = ?$

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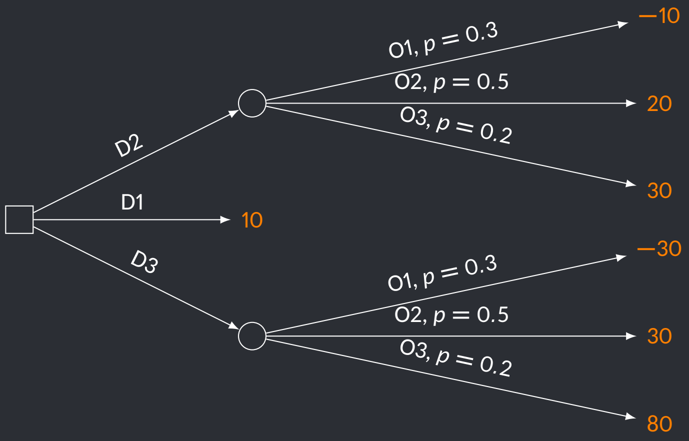
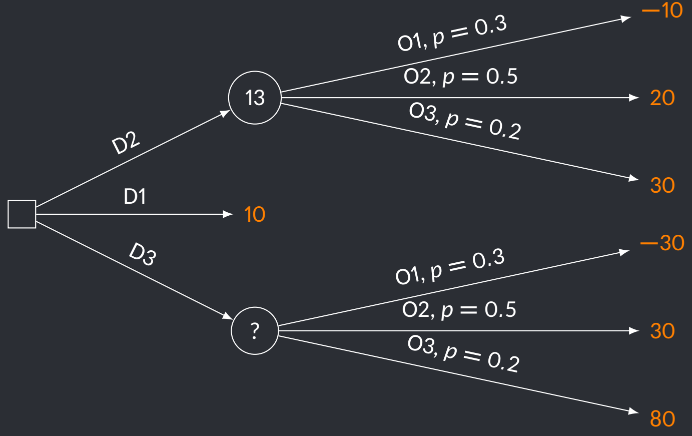
- EVs are calculated through a “rolling-back” process.

Example


D1
10
D2
D3
O1, $p = 0.3$
O2, $p = 0.5$
O3, $p = 0.2$
-10
20
30
O1, $p = 0.3$
O2, $p = 0.5$
O3, $p = 0.2$
-30
30
80

Rolling back: Step 1

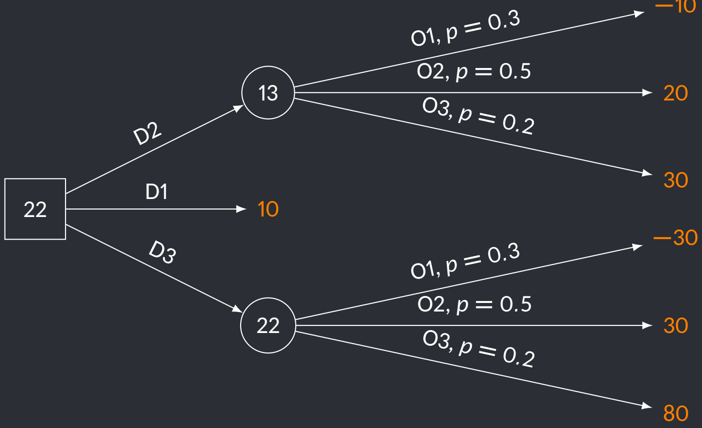
Calculate the expected value at each probability node:



$E(D1) = .3(-10) + .5(20) + .2(30) = 13$ Calculate the node EV

Rolling back: Step 2

Calculate the maximum at each decision node:



Take decision D3 since $22 = \max(10, 13, 22)$.

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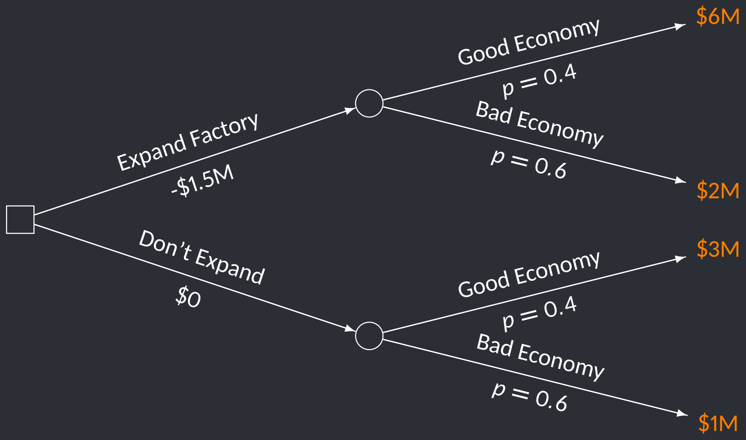
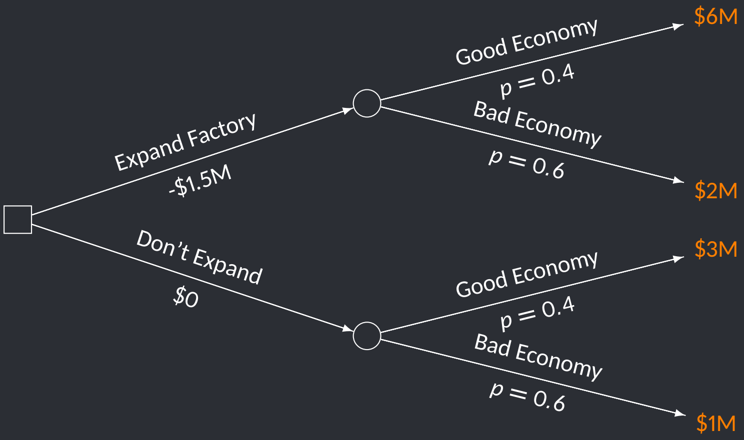
- She estimates that there is a 40 percent chance of a good economy and a 60 percent chance of a bad economy.

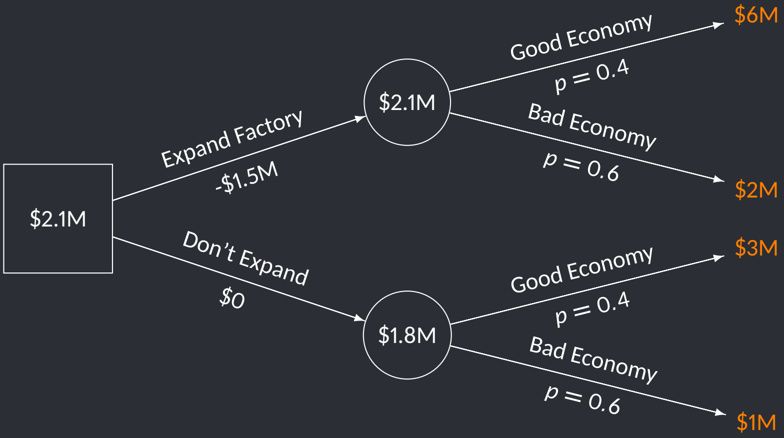
Should she expand?



Expand Factory
-$1.5M
Don't Expand
$0
Good Economy
p = 0.4
Bad Economy
p = 0.6
Good Economy
p = 0.4
Bad Economy
p = 0.6
$6M
$2M
$3M
$1M



$$E(\text{expand}) = (.4(6) + .6(2)) - 1.5 = 2.1$$
$$E(\text{don't expand}) = (.4(3) + .6(1)) = 1.8$$



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Since $2.1 > $1.8, she should expand!

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She later learns if she expands, she can opt to either:

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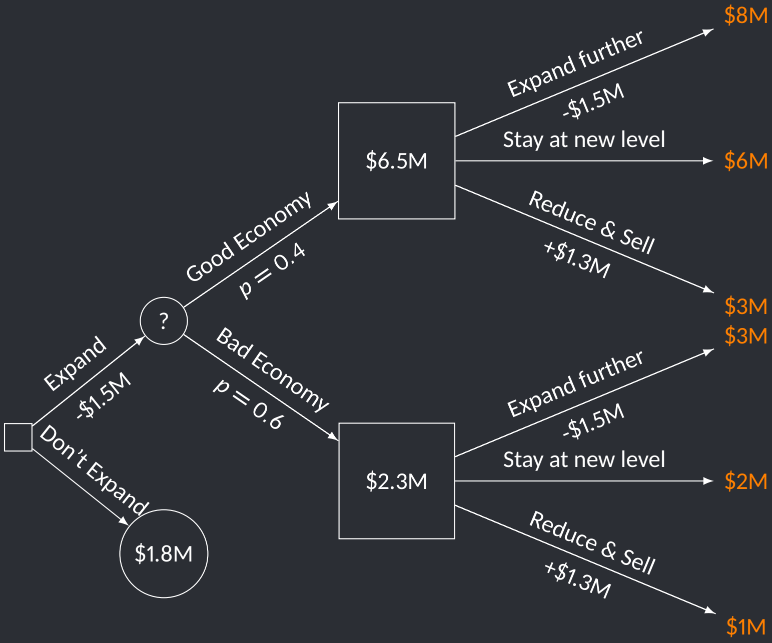
(c) do nothing.

How has the decision changed?



Calculate the new Expansion EV

Expected value of the option

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Therefore the value of the option is

$$E(\text{new}) - E(\text{old}) = 2.48 - 2.1 = 0.38,$$

or \$380,000.

Does this look familiar?

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- A further refinement is to take time value of money into account and present value everything.