THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Simulation 2

Lecture 28

STA 371G

Example 1: Will you have enough money for retirement?

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- Most people (try to) save for retirement throughout their career.
- But how do you know that the money you have saved will last you from retirement age until death?
- Particularly with expected lifespans growing, how do you know that you won't outlive your money?

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- Each year, at the beginning of the year, I put $10,000 into a retirement account.
- The entire account will be invested in a single asset that has normally distributed annual returns, with mean 12% and SD 18%.
- What will the portfolio look like in 30 years?

`for` loops

The `for` command runs a block of code repeatedly:

```
total <- 0
for (j in 1:5) {
  total <- total + j
}
# The total will be 1 + 2 + 3 + 4 + 5
total

[1] 15
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- `for` is similar to `replicate`, but it should be used when you need to keep a running tally of something across iterations.
- Inside the `for` loop you get access to a variable that changes with each iteration (e.g., above $j = 1$ for the first iteration, $j = 2$ for the second iteration, etc).

Let's start by writing R code for the first year:

```
# Start with nothing
account.value <- 0
# Add a $10,000 investment
account.value <- account.value + 10000
# Simulate this year's return
this.years.return <- rnorm(1, mean=.12, sd=.18)
# Apply the return
account.value <- account.value * (1 + this.years.return)
# Examine the account value
account.value
```

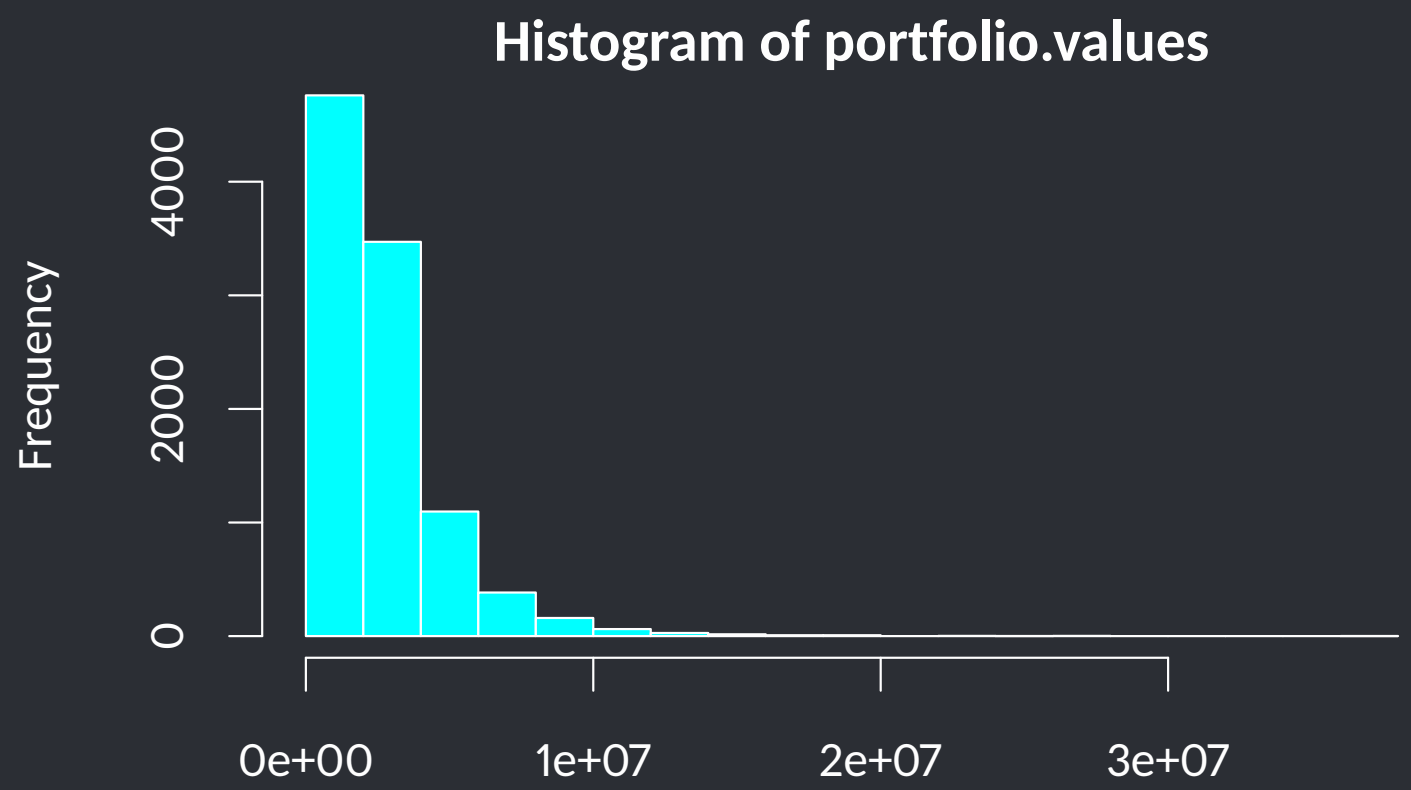
```
[1] 10072.38
```

Now let's repeat the process for 30 years:

```
account.value <- 0
for (year in 1:30) {
  account.value <- account.value + 10000
  this.years.return <- rnorm(1, mean=.12, sd=.18)
  account.value <- account.value * (1 + this.years.return)
}
account.value

[1] 3330261
```

```r
portfolio.values <- replicate(10000, {
  account.value <- 0
  for (year in 1:30) {
    account.value <- account.value + 10000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  account.value
})
hist(portfolio.values, col='cyan')
```



This looks pretty good at first, but there is a wide range of outcomes—let's look at the percentiles:

```
quantile(portfolio.values,
  probs=c(0.05, 0.25, 0.5, 0.75, 0.95))

      5%       25%       50%       75%       95%
 674417.4 1300215.9 2087403.8 3345929.9 6745048.2
```

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That's a 10x spread between the 95th percentile outcome and the 5th percentile outcome!

The expected (average) value of my portfolio is:

```
mean(portfolio.values)

[1] 2683627
```

What if I only contribute $5,000 per year?

```
portfolio.values <- replicate(10000, {
  account.value <- 0
  for (year in 1:30) {
    account.value <- account.value + 5000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  account.value
})
```

Let's look at the percentiles, under this new scenario where I only contribute $5,000 per year:

```
quantile(portfolio.values,
  probs=c(0.05, 0.25, 0.5, 0.75, 0.95))

      5%       25%       50%       75%       95%
 331238.9  661227.0 1047610.8 1697519.4 3368771.5
```

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[1] 1344782
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The expected (average) value of my portfolio is:

```
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[1] 1344782
```

Not contributing that extra $5,000 per year is expected to cost me $1.3M in the value of my retirement account!

How likely is it we'll retire a millionaire?

```
results <- replicate(10000, {
  account.value <- 0
  for (year in 1:30) {
    account.value <- account.value + 10000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  account.value >= 1000000
})
sum(results) / 10000

[1] 0.8591
```

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- Suppose that after I turn 65 (i.e., year 30), I start withdrawing $100,000 each year to live on, and I stop making annual contributions.
- My life expectancy (35 year old, male, married, nonsmoker, works out 3-4 times per week, etc.) is

How likely is it that I'll outlive my money?

- Suppose that after I turn 65 (i.e., year 30), I start withdrawing $100,000 each year to live on, and I stop making annual contributions.
- My life expectancy (35 year old, male, married, nonsmoker, works out 3-4 times per week, etc.) is 95.
- So let's simulate this process, and see how often my money outlives me.

```r
results <- replicate(10000, {
  account.value <- 0
  for (age in 35:64) {
    account.value <- account.value + 10000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  for (age in 65:94) {
    account.value <- account.value - 100000
    if (account.value < 0) {
      account.value <- 0
    } else {
      this.years.return <- rnorm(1, mean=.12, sd=.18)
      account.value <- account.value * (1 + this.years.return)
    }
  }
  account.value > 0
})
sum(results) / 10000
```

```
[1] 0.8079
```

```r
results <- replicate(10000, {
  account.value <- 0
  for (age in 35:64) {
    account.value <- account.value + 10000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  for (age in 65:94) {
    account.value <- account.value - 100000
    if (account.value < 0) {
      account.value <- 0
    } else {
      this.years.return <- rnorm(1, mean=.12, sd=.18)
      account.value <- account.value * (1 + this.years.return)
    }
  }
  account.value > 0
})
sum(results) / 10000
```

```
[1] 0.8079
```

There is a 80.79% chance that I don’t run out of money during retirement.

Example 2: Simulating pricing and demand

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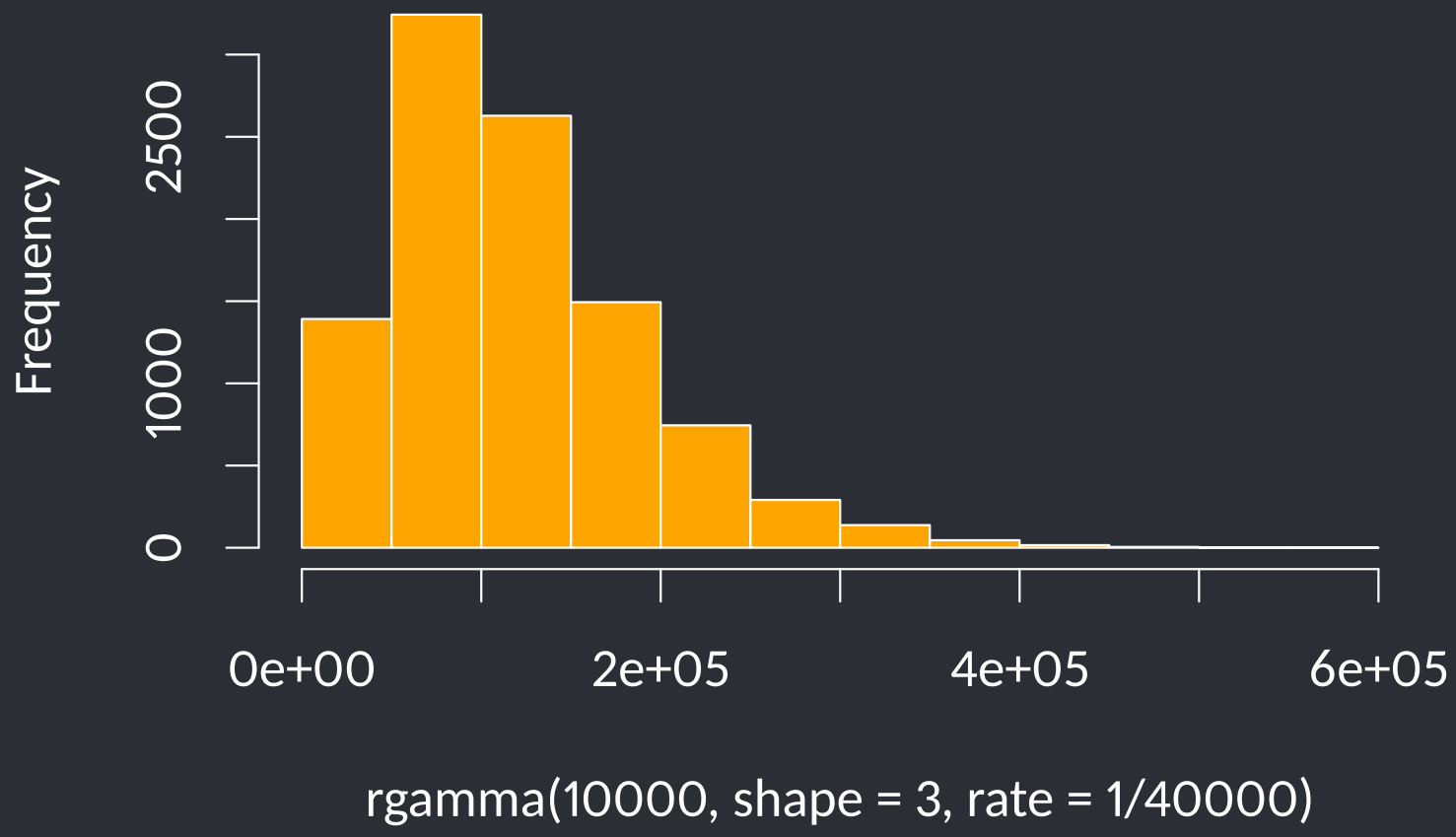
- Let's return to our oil drilling example: you are planning to drill for oil in a newly-discovered field.
- Setting up the drilling equipment costs $1M.
- There's a 45% chance that you strike oil.
- If you strike oil, you will generate money—but how much depends on the price of oil and how much demand there is.
- This is not something we can figure out using a decision tree anymore, because the payoff is randomly drawn from a distribution rather than fixed.

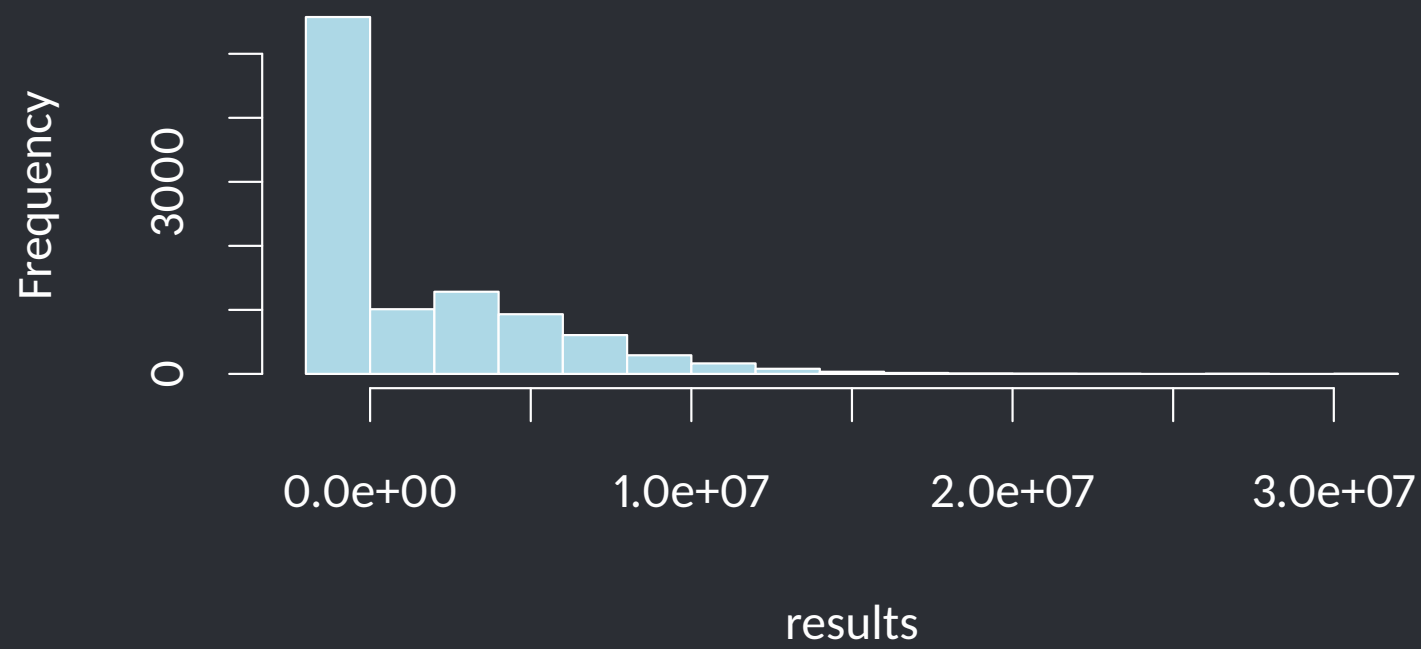
- Suppose that the price of oil is normally distributed, with a mean of $45/barrel and an SD of $8.
- Suppose that the demand (the number of barrels we can sell) has a *gamma distribution* with shape 3 and rate 1/40000:

```
hist(rgamma(10000, shape=3, rate=1/40000),
  col='orange', main='')
```



How much do we make if we decide to drill for oil?

```
results <- replicate(10000, {
  if (runif(1) < 0.45) {
    price <- rnorm(1, mean=45, sd=8)
    demand <- rgamma(1, shape=3, rate=1/40000)
    revenue <- price * demand - 1000000
  } else {
    revenue <- -1000000
  }
  revenue
})
hist(results, col='lightblue', main='')
```



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- It's more realistic to not assume this will necessarily be the case.
- Suppose that production also has a gamma distribution with shape 3 and rate 1/40000.
- How much do we make if we decide to drill for oil?

```r
results <- replicate(10000, {
  if (runif(1) < 0.45) {
    price <- rnorm(1, mean=45, sd=8)
    demand <- rgamma(1, shape=3, rate=1/40000)
    production <- rgamma(1, shape=3, rate=1/40000)
    revenue <- price * min(production, demand) - 1000000
  } else {
    revenue <- -1000000
  }
  revenue
})
hist(results, col='lightblue', main='')
```