THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Multiple Regression 2

Lecture 13

STA 371G

Predicting House prices in the Greater Boston Area

Median house price for each census tract, along with other data.

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Median house price for each census tract, along with other data.
The final model:

```
> model <- lm(MEDV ~ CRIME+ZONE+NOX+ROOM+DIST
+                    +RADIAL+TAX+PTRATIO+LSTAT, data=boston)
```

- MEDV: Median Price (response)
- CRIME: Per capita crime rate
- ZONE: Proportion of large lots
- NOX: Nitrogen Oxide concentration
- DIST: Distance to employment centers
- ROOM: Average # of rooms
- RADIAL: Accessibility to highways
- TAX: Tax rate (per $10K)
- PTRATIO: Pupil-to-teacher ratio
- LSTAT: Proportion of “lower status”

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Is our model useful? Check the R-squared:

```
> summary(model)$r.squared

[1] 0.7282911
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or

$H_0 : R^2 = 0$

$H_1 : R^2 > 0$

Overall Null Hypothesis

Check the P-value for the F-statistic in the summary

```
Residual standard error: 96750 on 496 degrees of freedom
Multiple R-squared:  0.7283,    Adjusted R-squared:  0.7234
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So we can reject the overall null hypothesis!
R-squared was already too big to suspect that it is zero and we already knew some predictors are statistically significant.

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Let's plot the residuals, i.e., discrepancies between the predictions and the data.

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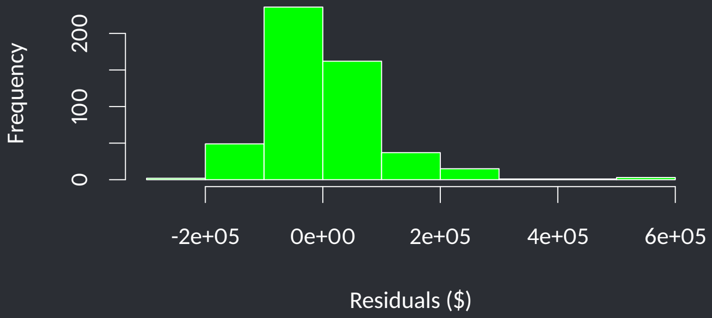
```
> hist(model$residuals, col='green',
+   main='', xlab='Residuals ($)', ylab='Frequency')
```



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By the 2 standard deviation rule, we could estimate that 95% of the time residuals are in [-$192K, $192K] range.

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Again: regression assumptions

Remember the big four:

1. The residuals are independent.
2. *Y* is a linear function of *X*s (except for the errors).
3. The residuals are normally distributed.
4. The variance of *Y* is the same for any value of *X*s (“homoscedasticity”).

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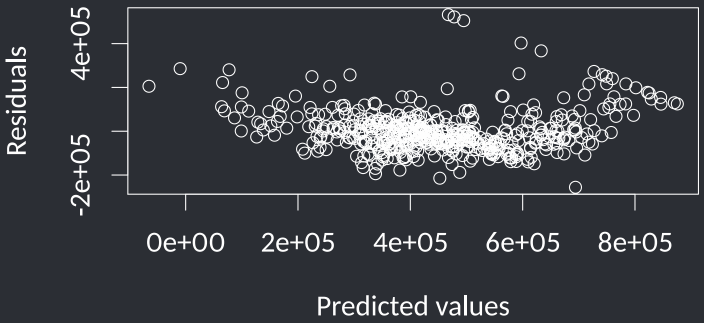
Assumption 2: Linearity

Plot the residuals vs the predicted Y-values and ensure there is no trend:

```
> plot(model$fitted.values, resid(model),
+   xlab='Predicted values', ylab='Residuals')
```



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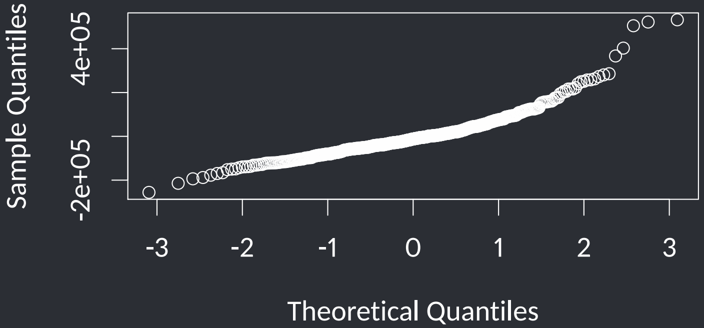
Assumption 3: Normally distributed residuals

Ensure that the Q-Q plot shows a (roughly) straight line:

```
> qqnorm(resid(model), main='')
```



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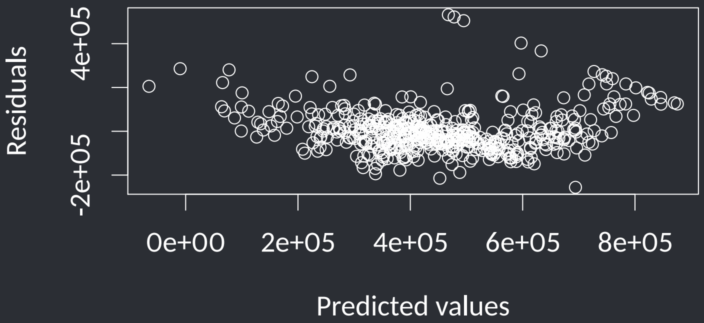
Assumption 4: The variance of Y is the same across

Look for a (roughly) constant vertical "thickness":

```
> plot(model$fitted.values, resid(model),
+   xlab='Predicted values', ylab='Residuals')
```



We have a model. Then what?

Let’s make some predictions.

Model Coefficients

Regression model estimates the coefficients of the predictors.

```
> round(summary(model)$coefficients,2)

              Estimate Std. Error t value Pr(>|t|)
(Intercept)  840065.15   99001.03    8.49        0
CRIME         -2566.08     663.82   -3.87        0
ZONE            922.00     276.22    3.34        0
NOX         -346925.67   71811.32   -4.83        0
ROOM          74242.52    8261.88    8.99        0
DIST         -31049.53    3784.98   -8.20        0
RADIAL         6000.24    1288.14    4.66        0
TAX            -265.33      68.57   -3.87        0
PTRATIO      -19279.75    2627.20   -7.34        0
LSTAT        -11071.73     957.48  -11.56        0
```

Model Coefficients

Let's estimate the median house price in a particular district:

j	Predictor	β_j	X_j	$\beta_j X_j$
0	Intercept	840.07	1	840.07
1	CRIME	-2.57	0.03	-0.0771
2	ZONE	0.92	10	9.2
3	NOX	-346.93	0.5	-173.465
4	ROOM	74.24	4	296.96
5	DIST	-31.05	5	-155.25
6	RADIAL	6	1	6
7	TAX	-0.27	300	-81
8	PTRATIO	-19.28	15	-385.6
9	LSTAT	-11.07	10	-110.7
Price	Estimate	($1000)		342.538

Model Coefficients

Let R do it for us!

```
> predict.lm(model, list(CRIME=0.03, ZONE=10,
+                        NOX=0.5, ROOM=4,
+                        DIST=5,  RADIAL=1,
+                        TAX=300, PTRATIO=15,
+                        LSTAT=10))

       1
343955.2
```

Model Coefficients

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       1
363234.9
```

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Nothing is free. To be able to compansate the new hires, the ISD decides to add $50 more on your tax bill for every $10K of your house price.

So, the tax rate increases to $50 per $10K. How would this affect the median house price?

Confidence intervals

We all know our predictions are not exactly right.

Can we come up with some confidence intervals on our predictions?

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Can we come up with some confidence intervals on our predictions?

Remember the two kinds of intervals:

Confidence	Predicting the mean value of Y for a particular set of X values.	Among all the districts whose predictors are as above, what is the mean value of median house price?
Prediction	Predicting Y for a single new case.	If Springfield has the predictors above, what is the median house price in Springfield?

Confidence intervals

```
> predict.lm(model, list(CRIME=0.03, ZONE=10,
+                        NOX=0.5, ROOM=4,
+                        DIST=5,  RADIAL=1,
+                        TAX=350, PTRATIO=14,
+                        LSTAT=10),
+                        interval = 'confidence')

       fit      lwr      upr
1 349968.4 301948.5 397988.3
```

We can also put a confidence interval on a coefficient to estimate the plausible range of its effect.

```
> confint(model)

                   2.5 %       97.5 %
(Intercept)  645552.0530 1034578.2470
CRIME         -3870.3245   -1261.8439
ZONE            379.2933    1464.7029
NOX         -488017.5640 -205833.7804
ROOM          58009.9148   90475.1248
DIST         -38486.0994  -23612.9585
RADIAL         3469.3548    8531.1305
TAX            -400.0457    -130.6157
PTRATIO      -24441.5728  -14117.9304
LSTAT        -12952.9546   -9190.5075
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Reducing the pupil/teacher ratio (PTRATIO) by one could increase the median house price from $14K to $24K!