

Dracarys

Team Reference Library

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## 多边形与圆面积交

```

1 point ORI;
2 double r;
3 int n;
4 point info[maxn];
5 // 用有向面积, 划分成一个三角形和圆的面积交
6 double area2(point pa, point pb) {
7     if (pa.len() < pb.len()) swap(pa, pb);
8     if (pb.len() < eps) return 0;
9     double a, b, c, B, C, sinB, cosB, sinC, cosC, S, h, theta;
10    a = pb.len();
11    b = pa.len();
12    c = (pb - pa).len();
13    cosB = dot(pb, pb - pa) / a / c;
14    B = acos(cosB);
15    cosC = dot(pa, pb) / a / b;
16    C = acos(cosC);
17    if (a > r) {
18        S = (C/2)*r*r;
19        h = a*b*sin(C)/c;
20        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
21    } else if (b > r) {
22        theta = PI - B - asin(sin(B)/r*a);
23        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
24    } else {
25        S = .5*sin(C)*a*b;
26    }
27    //printf("res = %.4f\n", S);
28    return S;
29 }
30 double area() {
31     double S = 0;
32     for (int i = 0; i < n; ++i) {
33         S += area2(info[i], info[i + 1]) * Sign(cross(info[i], info[i + 1]));
34     }
35     return fabs(S);
36 }

```

## 二维几何

```

1 #include <iostream>
2 #include <cmath>
3 #include <vector>
4
5 using namespace std;
6

```

```

7 const double PI = acos(-1.0);
8 const double EPS = 1e-8;
9
10 int sign(double x)
11 {
12     return x < -EPS ? -1 : x > EPS;
13 }
14
15 double newSqrt(double x)
16 {
17     return x < 0 ? 0 : sqrt(x);
18 }
19
20 struct Point {
21     double x, y;
22     Point(double x = 0, double y = 0) : x(x), y(y) {}
23     Point operator + (const Point &that) const {
24         return Point(x + that.x, y + that.y);
25     }
26     Point operator - (const Point &that) const {
27         return Point(x - that.x, y - that.y);
28     }
29     Point operator * (const double &that) const {
30         return Point(x * that, y * that);
31     }
32     Point operator / (const double &that) const {
33         return Point(x / that, y / that);
34     }
35     Point rotate(const double ang) { // 逆时针旋转 ang 弧度
36         return Point(cos(ang) * x - sin(ang) * y, cos(ang) * y + sin(ang) * x);
37     }
38     Point turn90() { // 逆时针旋转 90 度
39         return Point(-y, x);
40     }
41     double len2() const {
42         return x * x + y * y;
43     }
44     double len() const {
45         return sqrt(x * x + y * y);
46     }
47     Point unit() const {
48         return *this / len();
49     }
50     int operator < (const Point &that) const {
51         int d = sign(x - that.x); if (d) return d < 0;

```

```

52     return sign(y - that.y) < 0;
53 }
54 };
55 double det(Point a, Point b)
56 {
57     return a.x * b.y - b.x * a.y;
58 }
59 double dot(Point a, Point b)
60 {
61     return a.x * b.x + a.y * b.y;
62 }
63 double det(Point s, Point a, Point b)
64 {
65     return (a.x - s.x) * (b.y - s.y) - (b.x - s.x) * (a.y - s.y);
66 }
67
68 struct Line {
69     Point a, b;
70     Line(Point a, Point b) : a(a), b(b) {}
71 };
72
73 Point isLL(const Line &l1, const Line &l2) {
74     double s1 = det(l2.b - l2.a, l1.a - l2.a),
75           s2 = -det(l2.b - l2.a, l1.b - l2.a);
76     return (l1.a * s2 + l1.b * s1) / (s1 + s2);
77 }
78 bool onSeg(const Line &l, const Point &p) { // 点在线段上
79     return sign(det(p - l.a, l.b - l.a)) == 0 && sign(dot(p - l.a, p - l.b)) <= 0;
80 }
81 Point projection(const Line &l, const Point &p) { // 点到直线投影
82     return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
83 }
84 double disToLine(const Line &l, const Point &p) {
85     return abs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
86 }
87 double disToSeg(const Line &l, const Point &p) { // 点到线段距离
88     return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) != 1 ?
89         disToLine(l, p) : min((p - l.a).len(), (p - l.b).len());
90 }
91 Point symmetryPoint(const Point a, const Point b) { // 点 b 关于点 a 的中心对称点
92     return a + a - b;
93 }
94 Point reflection(const Line &l, const Point &p) { // 点关于直线的对称点
95     return symmetryPoint(projection(l, p), p);
96 }

```

```

97 struct Circle {
98     Point o;
99     double r;
100     Circle (Point o = Point(0, 0), double r = 0) : o(o), r(r) {}
101 };
102
103 // 求圆与直线的交点
104 bool isCL(Circle a, Line l, Point &p1, Point &p2) {
105     if (sign(det(l.a - a.o, l.b - a.o) / (l.a - l.b).len()) > 0) return false;
106     Point o = isLL(Line(a.o, a.o + (l.b - l.a).turn90()), l);
107     Point delta = (l.b - l.a).unit() * newSqrt(a.r * a.r - (o - a.o).len2());
108     p1 = o + delta;
109     p2 = o - delta;
110     return true;
111 }
112
113 // 求圆与圆的交面积
114 double areaCC(const Circle &c1, const Circle &c2) {
115     double d = (c1.o - c2.o).len();
116     if (sign(d - (c1.r + c2.r)) >= 0) {
117         return 0;
118     }
119     if (sign(d - abs(c1.r - c2.r)) <= 0) {
120         double r = min(c1.r, c2.r);
121         return r * r * PI;
122     }
123     double x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
124           t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
125     return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
126 }
127
128 // 求圆与圆的交点, 注意调用前要先判定重圆
129 bool isCC(Circle a, Circle b, Point &p1, Point &p2) {
130     double s1 = (a.o - b.o).len();
131     if (sign(s1 - a.r - b.r) > 0 || sign(s1 - abs(a.r - b.r)) < 0) return false;
132     double s2 = (a.r * a.r - b.r * b.r) / s1;
133     double aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
134     Point o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
135     Point delta = (b.o - a.o).unit().turn90() * newSqrt(a.r * a.r - aa * aa);
136     p1 = o + delta, p2 = o - delta;
137     return true;
138 }
139
140 // 求点到圆的切点, 按关于点的左手方向返回两个点
141 bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2)

```

```

142 {
143     double x = (p0 - c.o).len2(), d = x - c.r * c.r;
144     if (d < EPS) return false;
145     Point p = (p0 - c.o) * (c.r * c.r / x);
146     Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
147     p1 = c.o + p + delta;
148     p2 = c.o + p - delta;
149     return true;
150 }
151
152 // 求圆到圆的外共切线, 按关于 c1.o 的左手方向返回两条线
153 vector<Line> extanCC(const Circle &c1, const Circle &c2)
154 {
155     vector<Line> ret;
156     if (sign(c1.r - c2.r) == 0) {
157         Point dir = c2.o - c1.o;
158         dir = (dir * (c1.r / dir.len())).turn90();
159         ret.push_back(Line(c1.o + dir, c2.o + dir));
160         ret.push_back(Line(c1.o - dir, c2.o - dir));
161     } else {
162         Point p = (c1.o * -c2.r + c2.o * c1.r) / (c1.r - c2.r);
163         Point p1, p2, q1, q2;
164         if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
165             if (c1.r < c2.r) swap(p1, p2), swap(q1, q2);
166             ret.push_back(Line(p1, q1));
167             ret.push_back(Line(p2, q2));
168         }
169     }
170     return ret;
171 }
172
173 // 求圆到圆的内共切线, 按关于 c1.o 的左手方向返回两条线
174 vector<Line> intanCC(const Circle &c1, const Circle &c2)
175 {
176     vector<Line> ret;
177     Point p = (c1.o * c2.r + c2.o * c1.r) / (c1.r + c2.r);
178     Point p1, p2, q1, q2;
179     if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
180         ret.push_back(Line(p1, q1));
181         ret.push_back(Line(p2, q2));
182     }
183     return ret;
184 }
185
186 bool contain(vector<Point> polygon, Point p) { // 判断点 p
    ↪ 是否被多边形包含, 包括落在边界上

```

```

187 int ret = 0, n = polygon.size();
188 for(int i = 0; i < n; ++i) {
189     Point u = polygon[i], v = polygon[(i + 1) % n];
190     if (onSeg(Line(u, v), p)) return true;
191     if (sign(u.y - v.y) <= 0) swap(u, v);
192     if (sign(p.y - u.y) > 0 || sign(p.y - v.y) <= 0) continue;
193     ret += sign(det(p, v, u)) > 0;
194 }
195 return ret & 1;
196 }
197
198 vector<Point> convexCut(const vector<Point>&ps, Line l) { // 用半平面 (q1,q2)
    ↪ 的逆时针方向去切凸多边形
199     vector<Point> qs;
200     int n = ps.size();
201     for (int i = 0; i < n; ++i) {
202         Point p1 = ps[i], p2 = ps[(i + 1) % n];
203         int d1 = sign(det(l.a, l.b, p1)), d2 = sign(det(l.a, l.b, p2));
204         if (d1 >= 0) qs.push_back(p1);
205         if (d1 * d2 < 0) qs.push_back(isLL(Line(p1, p2), l));
206     }
207     return qs;
208 }
209 vector<Point> convexHull(vector<Point> ps) { // 求点集 ps 组成的凸包
210     int n = ps.size(); if (n <= 1) return ps;
211     sort(ps.begin(), ps.end());
212     vector<Point> qs;
213     for (int i = 0; i < n; qs.push_back(ps[i++]))
214         while (qs.size() > 1 && sign(det(qs[qs.size()-2], qs.back(), ps[i])) <= 0)
            ↪ qs.pop_back();
215     for (int i = n - 2, t = qs.size(); i >= 0; qs.push_back(ps[i--]))
216         while ((int)qs.size() > t && sign(det(qs[(int)qs.size()-2], qs.back(), ps[i])) <=
            ↪ 0) qs.pop_back();
217     qs.pop_back(); return qs;
218 }
219
220 int main()
221 {
222     Circle c1, c2;
223     c1.o = Point(0, 0); c1.r = 10;
224     c2.o = Point(10, 10); c1.r = 10;
225     Point p1, p2;
226     return 0;
227 }

```

$n \log n$  半平面交

```

1 struct Point {
2     Point norm() const {
3         double l = len();
4         return Point(x / l, y / l);
5     }
6
7     int quad() const {
8         return sign(y) == 1 || (sign(y) == 0 && sign(x) >= 0);
9     }
10 };
11
12 struct Line {
13     bool include(const Point &p) const {
14         return sign(det(b - a, p - a)) > 0;
15     }
16
17     Line push() const { // 将半平面向外推 eps
18         const double eps = 1e-6;
19         Point delta = (b - a).turn90().norm() * eps;
20         return Line(a - delta, b - delta);
21     }
22 };
23
24 bool sameDir(const Line &l0, const Line &l1) {
25     return parallel(l0, l1) && sign(dot(l0.b - l0.a, l1.b - l1.a)) == 1;
26 }
27
28 bool operator < (const Point &a, const Point &b) {
29     if (a.quad() != b.quad()) {
30         return a.quad() < b.quad();
31     } else {
32         return sign(det(a, b)) > 0;
33     }
34 }
35
36 bool operator < (const Line &l0, const Line &l1) {
37     if (sameDir(l0, l1)) {
38         return l1.include(l0.a);
39     } else {
40         return (l0.b - l0.a) < (l1.b - l1.a);
41     }
42 }
43
44 bool check(const Line &u, const Line &v, const Line &w) {

```

```

45     return w.include(intersect(u, v));
46 }
47
48 vector<Point> intersection(vector<Line> &l) {
49     sort(l.begin(), l.end());
50     deque<Line> q;
51     for (int i = 0; i < (int)l.size(); ++i) {
52         if (i && sameDir(l[i], l[i - 1])) {
53             continue;
54         }
55         while (q.size() > 1 && !check(q[q.size() - 2], q[q.size() - 1], l[i])) {
56             q.pop_back();
57         }
58         while (q.size() > 1 && !check(q[1], q[0], l[i])) {
59             q.pop_front();
60         }
61         q.push_back(l[i]);
62     }
63     while (q.size() > 2 && !check(q[q.size() - 2], q[q.size() - 1], q[0])) {
64         q.pop_back();
65     }
66     while (q.size() > 2 && !check(q[1], q[0], q[q.size() - 1])) {
67         q.pop_front();
68     }
69     vector<Point> ret;
70     for (int i = 0; i < (int)q.size(); ++i) {
71         ret.push_back(intersect(q[i], q[(i + 1) % q.size()]));
72     }
73     return ret;
74 }

```

## Delaunay 三角剖分

```

1 /*
2 Delaunay Triangulation 随机增量算法 :
3 节点数至少为点数的 6 倍, 空间消耗较大注意计算内存使用
4 建图的过程在 build 中, 注意初始化内存池和初始三角形的坐标范围 (Triangulation::LOTS)
5 Triangulation::find 返回包含某点的三角形
6 Triangulation::add_point 将某点加入三角剖分
7 某个 Triangle 在三角剖分中当且仅当它的 has_children 为 0
8 如果要找到三角形 u 的邻域, 则枚举它的所有 u.edge[i].tri, 该条边的两个点为 u.p[(i+1)%3],
   ↪ u.p[(i+2)%3]
9 */
10 const int N = 100000 + 5;
11 const int MAX_TRIS = N * 6;
12 const double EPSILON = 1e-6;

```

```

13 const double PI = acos(-1.0);
14 using namespace std;
15
16 struct Point {
17     double x,y;
18     Point() : x(0), y(0) {}
19     Point(double x, double y) : x(x), y(y) {}
20     inline bool operator == (Point const& that) const {
21         return x == that.x && y == that.y;
22     }
23 };
24
25 inline double sqr(double x) { return x*x; }
26 double dist_sqr(Point const& a, Point const& b) {
27     return sqr(a.x-b.x) + sqr(a.y-b.y);
28 }
29 bool in_circumcircle(Point const& p1, Point const& p2, Point const& p3, Point const&
    ↪ p4) {
30     double u11 = p1.x - p4.x;
31     double u21 = p2.x - p4.x;
32     double u31 = p3.x - p4.x;
33     double u12 = p1.y - p4.y;
34     double u22 = p2.y - p4.y;
35     double u32 = p3.y - p4.y;
36     double u13 = sqr(p1.x) - sqr(p4.x) + sqr(p1.y) - sqr(p4.y);
37     double u23 = sqr(p2.x) - sqr(p4.x) + sqr(p2.y) - sqr(p4.y);
38     double u33 = sqr(p3.x) - sqr(p4.x) + sqr(p3.y) - sqr(p4.y);
39     double det = -u13*u22*u31 + u12*u23*u31 + u13*u21*u32 - u11*u23*u32 - u12*u21*u33
    ↪ + u11*u22*u33;
40     return det > EPSILON;
41 }
42 double side(Point const& a, Point const& b, Point const& p) {
43     return (b.x-a.x)*(p.y-a.y) - (b.y-a.y)*(p.x-a.x);
44 }
45
46 typedef int SideRef;
47 struct Triangle;
48 typedef Triangle* TriangleRef;
49 struct Edge {
50     TriangleRef tri;
51     SideRef side;
52     Edge() : tri(0), side(0) {}
53     Edge(TriangleRef tri, SideRef side) : tri(tri), side(side) {}
54 };
55 struct Triangle {

```

```

56     Point p[3];
57     Edge edge[3];
58     TriangleRef children[3];
59     Triangle() {}
60     Triangle(Point const& p0, Point const& p1, Point const& p2) {
61         p[0] = p0; p[1] = p1; p[2] = p2;
62         children[0] = children[1] = children[2] = 0;
63     }
64     bool has_children() const {
65         return children[0] != 0;
66     }
67     int num_children() const {
68         return children[0] == 0 ? 0
69             : children[1] == 0 ? 1
70             : children[2] == 0 ? 2 : 3;
71     }
72     bool contains(Point const& q) const {
73         double a = side(p[0],p[1],q);
74         double b = side(p[1],p[2],q);
75         double c = side(p[2],p[0],q);
76         return a >= -EPSILON && b >= -EPSILON && c >= -EPSILON;
77     }
78 } triange_pool[MAX_TRIS], *tot_triangles;
79 void set_edge(Edge a, Edge b) {
80     if (a.tri) a.tri->edge[a.side] = b;
81     if (b.tri) b.tri->edge[b.side] = a;
82     if (a.tri && b.tri) {
83         assert(a.tri->p[(a.side+1)%3] == b.tri->p[(b.side+2)%3]);
84         assert(a.tri->p[(a.side+2)%3] == b.tri->p[(b.side+1)%3]);
85     }
86 }
87 class Triangulation {
88 public:
89     Triangulation() {
90         const double LOTS = 1e6;
91         the_root = new(tot_triangles++)
    ↪ Triangle(Point(-LOTS,-LOTS),Point(+LOTS,-LOTS),Point(0,+LOTS));
92     }
93     ~Triangulation() {}
94     TriangleRef find(Point p) const {
95         return find(the_root,p);
96     }
97     void add_point(Point const& p) {
98         add_point(find(the_root,p),p);
99     }

```

```

100 private:
101     TriangleRef the_root;
102     static TriangleRef find(TriangleRef root, Point const& p) {
103         for( ; ; ) {
104             assert(root->contains(p));
105             if (!root->has_children()) {
106                 return root;
107             } else {
108                 int flag = true;
109                 for (int i = 0; i < 3 && root->children[i] ; ++i) {
110                     if (root->children[i]->contains(p)) {
111                         root = root->children[i];
112                         break;
113                     }
114                 }
115                 assert(flag&&"point not found");
116             }
117         }
118     }
119     void add_point(TriangleRef root, Point const& p) {
120         TriangleRef tab,tbc,tca;
121         /* split it into three triangles */
122         tab = new(tot_triangles++) Triangle(root->p[0], root->p[1], p);
123         tbc = new(tot_triangles++) Triangle(root->p[1], root->p[2], p);
124         tca = new(tot_triangles++) Triangle(root->p[2], root->p[0], p);
125         set_edge(Edge(tab,0), Edge(tbc,1));
126         set_edge(Edge(tbc,0), Edge(tca,1));
127         set_edge(Edge(tca,0), Edge(tab,1));
128         set_edge(Edge(tab,2), root->edge[2]);
129         set_edge(Edge(tbc,2), root->edge[0]);
130         set_edge(Edge(tca,2), root->edge[1]);
131         root->children[0] = tab;
132         root->children[1] = tbc;
133         root->children[2] = tca;
134         flip(tab,2);
135         flip(tbc,2);
136         flip(tca,2);
137     }
138     void flip(TriangleRef tri, SideRef pi) {
139         TriangleRef trj = tri->edge[pi].tri;
140         int pj = tri->edge[pi].side;
141         if (!trj) return;
142         if (!in_circumcircle(tri->p[0],tri->p[1],tri->p[2],trj->p[pj])) return;
143         assert(tri->p[(pi+2)%3] == trj->p[(pj+1)%3]);
144         assert(tri->p[(pi+1)%3] == trj->p[(pj+2)%3]);

```

```

145         /* flip edge between tri,trj */
146         TriangleRef trk = new(tot_triangles++) Triangle(tri->p[(pi+1)%3], trj->p[pj],
147             ↪ tri->p[pi]);
148         TriangleRef trl = new(tot_triangles++) Triangle(trj->p[(pj+1)%3], tri->p[pi],
149             ↪ trj->p[pj]);
150         set_edge(Edge(trk,0), Edge(trl,0));
151         set_edge(Edge(trk,1), tri->edge[(pi+2)%3]);
152         set_edge(Edge(trk,2), trj->edge[(pj+1)%3]);
153         set_edge(Edge(trl,1), trj->edge[(pj+2)%3]);
154         set_edge(Edge(trl,2), tri->edge[(pi+1)%3]);
155         tri->children[0] = trk; tri->children[1] = trl; tri->children[2] = 0;
156         trj->children[0] = trk; trj->children[1] = trl; trj->children[2] = 0;
157         flip(trk,1);
158         flip(trl,1);
159     }
160 };
161
162 int n;
163 Point ps[N];
164
165 void build()
166 {
167     tot_triangles = triange_pool;
168     cin >> n;
169     for(int i = 0; i < n; ++ i) {
170         int x, y;
171         scanf("%d%d", &x, &y);
172         ps[i].x = x; ps[i].y = y;
173     }
174     random_shuffle(ps, ps + n);
175     Triangulation tri;
176     for(int i = 0; i < n; ++ i) {
177         tri.add_point(ps[i]);
178     }
179 }
180
181 int main()
182 {
183     build();
184     return 0;
185 }

```



## 三维几何操作合并

```

1 struct Point3D {
2     double x, y, z;
3 };
4
5 Point3D det(const Point3D &a, const Point3D &b) {
6     return Point3D(a.y * b.z - a.z * b.y, a.z * b.x - a.x * b.z, a.x * b.y - a.y *
7         ↪ b.x);
8 }
9 // 平面法向量 : 平面上两个向量叉积
10 // 点共平面 : 平面上一点与之的向量点积法向量为 0
11 // 点在线段 ( 直线 ) 上 : 共线且两边点积非正
12 // 点在三角形内 ( 不包含边界, 需再判断是与某条边共线 )
13 bool pointInTri(const Point3D &a, const Point3D &b, const Point3D &c, const Point3D
14     ↪ &p) {
15     return sign(det(a - b, a - c).len() - det(p - a, p - b).len() - det(p - b, p -
16         ↪ c).len() - det(p - c, p - a).len()) == 0;
17 }
18 // 共平面的两点是否在这平面上一条直线的同侧
19 bool sameSide(const Point3D &a, const Point3D &b, const Point3D &p0, const Point3D
20     ↪ &p1) {
21     return sign(dot(det(a - b, p0 - b), det(a - b, p1 - b))) > 0;
22 }
23 // 两点在平面同侧 : 点积法向量符号相同
24 // 两直线平行 / 垂直 : 同二维
25 // 平面平行 / 垂直 : 判断法向量
26 // 线面垂直 : 法向量和直线平行
27 // 判断空间线段是否相交 : 四点共面两线段不平行相互在异侧
28 // 线段和三角形是否相交 : 线段在三角形平面不同侧
29     ↪ 三角形任意两点在线段和第三点组成的平面的不同侧
30 // 求空间直线交点
31 Point3D intersection(const Point3D &a0, const Point3D &b0, const Point3D &a1, const
32     ↪ Point3D &b1) {
33     double t = ((a0.x - a1.x) * (a1.y - b1.y) - (a0.y - a1.y) * (a1.x - b1.x)) /
34         ↪ ((a0.x - b0.x) * (a1.y - b1.y) - (a0.y - b0.y) * (a1.x - b1.x));
35     return a0 + (b0 - a0) * t;
36 }
37 // 求平面和直线的交点
38 Point3D intersection(const Point3D &a, const Point3D &b, const Point3D &c, const
39     ↪ Point3D &l0, const Point3D &l1) {
40     Point3D p = pVec(a, b, c); // 平面法向量
41     double t = (p.x * (a.x - l0.x) + p.y * (a.y - l0.y) + p.z * (a.z - l0.z)) / (p.x *
42         ↪ (l1.x - l0.x) + p.y * (l1.y - l0.y) + p.z * (l1.z - l0.z));
43     return l0 + (l1 - l0) * t;
44 }

```

```

36 // 求平面交线 : 取不平行的一条直线的一个交点, 以及法向量叉积得到直线方向
37 // 点到直线距离 : 叉积得到三角形的面积除以底边
38 // 点到平面距离 : 点积法向量
39 // 直线间距离 : 平行时随便取一点求距离, 否则叉积方向向量得到方向点积计算长度
40 // 直线夹角 : 点积 平面夹角 : 法向量点积
41 // 三维向量旋转操作(绕向量 s 旋转 ang 角度), 对于右手系 s 指向观察者时逆时针
42 // 矩阵版
43 void rotate(const Point3D &s, double ang) {
44     double l = s.len(), x = s.x / l, y = s.y / l, z = s.z / l, sinA = sin(ang), cosA =
45         ↪ cos(ang);
46     double p[4][4] = {cosA + (1 - cosA) * x * x, (1 - cosA) * x * y - sinA * z, (1 -
47         ↪ cosA) * x * z + sinA * y, 0,
48         (1 - cosA) * y * x + sinA * z, cosA + (1 - cosA) * y * y, (1 - cosA) * y * z -
49         ↪ sinA * x, 0,
50         (1 - cosA) * z * x - sinA * y, (1 - cosA) * z * y + sinA * x, cosA + (1 - cosA)
51         ↪ * z * z, 0,
52         0, 0, 0, 1 };
53 }
54 // 计算版 : 把需要旋转的向量按照 s 分解, 做二维旋转, 再回到三维

```

## 三维凸包

```

1 #define SIZE(X) (int(X.size()))
2 #define PI 3.14159265358979323846264338327950288
3 struct Point {
4     Point cross(const Point &p) const
5     { return Point(y * p.z - z * p.y, z * p.x - x * p.z, x * p.y - y * p.x); }
6 } info[1005];
7 int mark[1005][1005], n, cnt;
8 double mix(const Point &a, const Point &b, const Point &c)
9 { return a.dot(b.cross(c)); }
10 double area(int a, int b, int c)
11 { return ((info[b] - info[a]).cross(info[c] - info[a])).length(); }
12 double volume(int a, int b, int c, int d)
13 { return mix(info[b] - info[a], info[c] - info[a], info[d] - info[a]); }
14 struct Face {
15     int a, b, c; Face() {}
16     Face(int a, int b, int c): a(a), b(b), c(c) {}
17     int &operator [] (int k)
18     { if (k == 0) return a; if (k == 1) return b; return c; }
19 };
20 vector <Face> face;
21 inline void insert(int a, int b, int c) { face.push_back(Face(a, b, c)); }
22 void add(int v) {
23     vector <Face> tmp; int a, b, c; cnt++;
24     for (int i = 0; i < SIZE(face); i++) {

```

```

25     a = face[i][0]; b = face[i][1]; c = face[i][2];
26     if (Sign(volume(v, a, b, c)) < 0)
27         mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] = mark[c][a] = mark[a][c] =
        cnt;
28     else tmp.push_back(face[i]);
29 } face = tmp;
30 for (int i = 0; i < SIZE(tmp); i++) {
31     a = face[i][0]; b = face[i][1]; c = face[i][2];
32     if (mark[a][b] == cnt) insert(b, a, v);
33     if (mark[b][c] == cnt) insert(c, b, v);
34     if (mark[c][a] == cnt) insert(a, c, v);
35 }
36 int Find() {
37     for (int i = 2; i < n; i++) {
38         Point ndir = (info[0] - info[i]).cross(info[1] - info[i]);
39         if (ndir == Point()) continue; swap(info[i], info[2]);
40         for (int j = i + 1; j < n; j++) if (Sign(volume(0, 1, 2, j)) != 0) {
41             swap(info[j], info[3]); insert(0, 1, 2); insert(0, 2, 1); return 1;
42         } return 0; }
43 int main() {
44     for (; scanf("%d", &n) == 1; ) {
45         for (int i = 0; i < n; i++) info[i].Input();
46         sort(info, info + n); n = unique(info, info + n) - info;
47         face.clear(); random_shuffle(info, info + n);
48         if (Find()) { memset(mark, 0, sizeof(mark)); cnt = 0;
49             for (int i = 3; i < n; i++) add(i); vector<Point> Ndir;
50             for (int i = 0; i < SIZE(face); ++i) {
51                 Point p = (info[face[i][0]] - info[face[i][1]]).cross(
52                     info[face[i][2]] - info[face[i][1]]);
53                 p = p / p.length(); Ndir.push_back(p);
54             } sort(Ndir.begin(), Ndir.end());
55             int ans = unique(Ndir.begin(), Ndir.end()) - Ndir.begin();
56             printf("%d\n", ans);
57         } else printf("1\n");
58     } }
59 // 求重心
60 double calcDist(const Point &p, int a, int b, int c)
61 { return fabs(mix(info[a] - p, info[b] - p, info[c] - p) / area(a, b, c)); }
62 //compute the minimal distance of center of any faces
63 double findDist() { //compute center of mass
64     double totalWeight = 0; Point center(.0, .0, .0);
65     Point first = info[face[0][0]];
66     for (int i = 0; i < SIZE(face); ++i) {
67         Point p = (info[face[i][0]]+info[face[i][1]]+info[face[i][2]]+first)*.25;
68         double weight = mix(info[face[i][0]] - first, info[face[i][1]]

```

```

69         - first, info[face[i][2]] - first);
70         totalWeight += weight; center = center + p * weight;
71     } center = center / totalWeight;
72     double res = 1e100; //compute distance
73     for (int i = 0; i < SIZE(face); ++i)
74         res = min(res, calcDist(center, face[i][0], face[i][1], face[i][2]));
75     return res; }

```

### 凸包上快速询问

```

1  /*
2   给定凸包, log n 内完成各种询问, 具体操作有 :
3   1. 判定一个点是否在凸包内
4   2. 询问凸包外的点到凸包的两个切点
5   3. 询问一个向量关于凸包的切点
6   4. 询问一条直线和凸包的交点
7   INF 为坐标范围, 需要定义点类大于号
8   改成实数只需修改 sign 函数, 以及把 long long 改为 double 即可
9   构造函数时传入凸包要求无重点, 面积非空, 以及 pair(x,y) 的最小点放在第一个
10 */
11 #include <vector>
12 #include <functional>
13 using namespace std;
14
15 const int INF = 1000000000;
16
17 struct Convex
18 {
19     int n;
20     vector<Point> a;
21     vector<Point> upper, lower;
22     Convex(vector<Point> _a) : a(_a) {
23         n = a.size();
24         int ptr = 0;
25         for(int i = 1; i < n; ++ i) if (a[ptr] < a[i]) ptr = i;
26         for(int i = 0; i <= ptr; ++ i) lower.push_back(a[i]);
27         for(int i = ptr; i < n; ++ i) upper.push_back(a[i]);
28         upper.push_back(a[0]);
29     }
30     int sign(long long x) {
31         return x < 0 ? -1 : x > 0;
32     }
33     pair<long long, int> get_tangent(vector<Point> &convex, Point vec) {
34         int l = 0, r = (int)convex.size() - 2;
35         for( ; l + 1 < r; ) {
36             int mid = (l + r) / 2;

```

```

37     if (sign((convex[mid + 1] - convex[mid]).det(vec)) > 0) r = mid;
38     else l = mid;
39 }
40 return max(make_pair(vec.det(convex[r]), r), make_pair(vec.det(convex[0]), 0));
41 }
42 void update_tangent(const Point &p, int id, int &i0, int &i1) {
43     if ((a[i0] - p).det(a[id] - p) > 0) i0 = id;
44     if ((a[i1] - p).det(a[id] - p) < 0) i1 = id;
45 }
46 void binary_search(int l, int r, Point p, int &i0, int &i1) {
47     if (l == r) return;
48     update_tangent(p, l % n, i0, i1);
49     int sl = sign((a[l % n] - p).det(a[(l + 1) % n] - p));
50     for( ; l + 1 < r; ) {
51         int mid = (l + r) / 2;
52         int smid = sign((a[mid % n] - p).det(a[(mid + 1) % n] - p));
53         if (smid == sl) l = mid;
54         else r = mid;
55     }
56     update_tangent(p, r % n, i0, i1);
57 }
58 int binary_search(Point u, Point v, int l, int r) {
59     int sl = sign((v - u).det(a[l % n] - u));
60     for( ; l + 1 < r; ) {
61         int mid = (l + r) / 2;
62         int smid = sign((v - u).det(a[mid % n] - u));
63         if (smid == sl) l = mid;
64         else r = mid;
65     }
66     return l % n;
67 }
68 // 判定点是否在凸包内, 在边界返回 true
69 bool contain(Point p) {
70     if (p.x < lower[0].x || p.x > lower.back().x) return false;
71     int id = lower_bound(lower.begin(), lower.end(), Point(p.x, -INF)) -
    ↪ lower.begin();
72     if (lower[id].x == p.x) {
73         if (lower[id].y > p.y) return false;
74     } else if ((lower[id] - p).det(lower[id] - p) < 0) return false;
75     id = lower_bound(upper.begin(), upper.end(), Point(p.x, INF), greater<Point>()) -
    ↪ - upper.begin();
76     if (upper[id].x == p.x) {
77         if (upper[id].y < p.y) return false;
78     } else if ((upper[id] - p).det(upper[id] - p) < 0) return false;
79     return true;

```

```

80 }
81 // 求点 p 关于凸包的两个切点, 如果在凸包外则有序返回编号, 多解返回任意一个 否则返回
    ↪ false
82 bool get_tangent(Point p, int &i0, int &i1) {
83     if (contain(p)) return false;
84     i0 = i1 = 0;
85     int id = lower_bound(lower.begin(), lower.end(), p) - lower.begin();
86     binary_search(0, id, p, i0, i1);
87     binary_search(id, (int)lower.size(), p, i0, i1);
88     id = lower_bound(upper.begin(), upper.end(), p, greater<Point>()) -
    ↪ upper.begin();
89     binary_search((int)lower.size() - 1, (int)lower.size() - 1 + id, p, i0, i1);
90     binary_search((int)lower.size() - 1 + id, (int)lower.size() - 1 +
    ↪ (int)upper.size(), p, i0, i1);
91     return true;
92 }
93 // 求凸包上和向量 vec 叉积最大的点, 返回编号, 有多个返回任意一个
94 int get_tangent(Point vec) {
95     pair<long long, int> ret = get_tangent(upper, vec);
96     ret.second = (ret.second + (int)lower.size() - 1) % n;
97     ret = max(ret, get_tangent(lower, vec));
98     return ret.second;
99 }
100 // 求凸包和直线 u,v 的交点, 如果无严格相交返回 false 。如果有则是和 (i,next(i))
    ↪ 的交点, 两个点无序, 交在点上不确定返回两条线段之一。
101 bool get_intersection(Point u, Point v, int &i0, int &i1) {
102     int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
103     if (sign((v - u).det(a[p0] - u)) * sign((v - u).det(a[p1] - u)) < 0) {
104         if (p0 > p1) swap(p0, p1);
105         i0 = binary_search(u, v, p0, p1);
106         i1 = binary_search(u, v, p1, p0 + n);
107         return true;
108     } else {
109         return false;
110     }
111 }
112 };

```

### 圆的面积模板 ( $n^2 \log n$ )

```

1 // Area[i] 表示覆盖次数大于等于 i 的面积
2 struct Tevent {
3     Point p; double ang; int add;
4     Tevent() {}
5     Tevent(const Point &p, double _ang, int _add): p(p), ang(_ang), add(_add) {}
6     bool operator <(const Tevent &a) const {

```

```

7     return ang < a.ang;
8 }
9 } eve[N * 2];
10 int E, cnt, C;
11 Circle c[N];
12 bool g[N][N], overlap[N][N];
13 double Area[N];
14 int cX[N], cY[N], cR[N];
15 bool contain(int i, int j) {
16     return (sign(c[i].r - c[j].r) > 0 || sign(c[i].r - c[j].r) == 0 && i < j) &&
        ↪ c[i].contain(c[j], -1);
17 }
18 int main() {
19     scanf("%d", &C);
20     for (int i = 0; i < C; ++i) {
21         scanf("%d%d%d", cX+i, cY+i, cR+i);
22         c[i].o = Point(cX[i], cY[i]);
23         c[i].r = cR[i];
24     }
25     for (int i = 0; i <= C; ++i) Area[i] = 0;
26     for (int i = 0; i < C; ++i) for (int j = 0; j < C; ++j)
27         overlap[i][j] = contain(i, j);
28     for (int i = 0; i < C; ++i) for (int j = 0; j < C; ++j)
29         g[i][j] = !(overlap[i][j] || overlap[j][i] || c[i].disjunct(c[j], -1));
30     for (int i = 0; i < C; ++i) {
31         E = 0; cnt = 1;
32         for (int j = 0; j < C; ++j) if (j != i && overlap[j][i]) cnt++;
33         for (int j = 0; j < C; ++j) {
34             if (i != j && g[i][j]) {
35                 Point aa, bb;
36                 isCC(c[i], c[j], aa, bb);
37                 double A = atan2(aa.y - c[i].o.y, aa.x - c[i].o.x);
38                 double B = atan2(bb.y - c[i].o.y, bb.x - c[i].o.x);
39                 eve[E++] = Tevent(bb, B, 1);
40                 eve[E++] = Tevent(aa, A, -1);
41                 if (B > A) cnt++;
42             }
43         }
44         if (E == 0) { //cnt 表示覆盖次数超过 cnt
45             Area[cnt] += PI * c[i].r * c[i].r;
46         } else {
47             sort(eve, eve + E);
48             eve[E] = eve[0];
49             for (int j = 0; j < E; ++j) {
50                 cnt += eve[j].add;

```

```

51         Area[cnt] += eve[j].p.det(eve[j + 1].p) * .5;
52         double theta = eve[j + 1].ang - eve[j].ang;
53         if (theta < 0) theta += PI * 2.;
54         Area[cnt] += theta * c[i].r * c[i].r * .5 - sin(theta) * c[i].r * c[i].r *
        ↪ .5;
55     }
56 }
57 }
58 for(int i = 1; i <= C; ++ i) printf("[%d] = %.3f\n", i, Area[i] - Area[i + 1]);
59 }

```

### 三角形的心

```

1 Point inCenter(const Point &A, const Point &B, const Point &C) { // 内心
2     double a = (B - C).len(), b = (C - A).len(), c = (A - B).len(),
3         s = fabs(det(B - A, C - A)),
4         r = s / p;
5     return (A * a + B * b + C * c) / (a + b + c);
6 }
7
8 Point circumCenter(const Point &a, const Point &b, const Point &c) { // 外心
9     Point bb = b - a, cc = c - a;
10    double db = bb.len2(), dc = cc.len2(), d = 2 * det(bb, cc);
11    return a + Point(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc) / d;
12 }
13
14 Point othroCenter(const Point &a, const Point &b, const Point &c) { // 垂心
15     Point ba = b - a, ca = c - a, bc = b - c;
16     double Y = ba.y * ca.y * bc.y,
17         A = ca.x * ba.y - ba.x * ca.y,
18         x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
19         y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
20     return Point(x0, y0);
21 }

```

### 最小覆盖球

```

1 int npoint, nouter; Tpoint pt[200000], outer[4], res; double radius, tmp;
2 void ball() {
3     Tpoint q[3]; double m[3][3], sol[3], L[3], det;
4     int i, j; res.x = res.y = res.z = radius = 0;
5     switch ( nouter ) {
6     case 1: res=outer[0]; break;
7     case 2: res=(outer[0]+outer[1])/2; radius=dist2(res, outer[0]); break;
8     case 3:
9         for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
10        for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=dot(q[i], q[j])*2;

```

```

11     for (i=0; i<2; ++i) sol[i]=dot(q[i], q[i]);
12     if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps) return;
13     L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
14     L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
15     res=outer[0]+q[0]*L[0]+q[1]*L[1];
16     radius=dist2(res, outer[0]);
17     break;
18 case 4:
19     for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol[i]=dot(q[i], q[i]);
20     for (i=0; i<3; ++i) for (j=0; j<3; ++j) m[i][j]=dot(q[i], q[j])*2;
21     det= m[0][0]*m[1][1]*m[2][2]
22         + m[0][1]*m[1][2]*m[2][0]
23         + m[0][2]*m[2][1]*m[1][0]
24         - m[0][2]*m[1][1]*m[2][0]
25         - m[0][1]*m[1][0]*m[2][2]
26         - m[0][0]*m[1][2]*m[2][1];
27     if ( fabs(det)<eps ) return;
28     for (j=0; j<3; ++j) {
29         for (i=0; i<3; ++i) m[i][j]=sol[i];
30         L[j]=( m[0][0]*m[1][1]*m[2][2]
31             + m[0][1]*m[1][2]*m[2][0]
32             + m[0][2]*m[2][1]*m[1][0]
33             - m[0][2]*m[1][1]*m[2][0]
34             - m[0][1]*m[1][0]*m[2][2]
35             - m[0][0]*m[1][2]*m[2][1]
36             ) / det;
37         for (i=0; i<3; ++i) m[i][j]=dot(q[i], q[j])*2;
38     } res=outer[0];
39     for (i=0; i<3; ++i ) res = res + q[i] * L[i];
40     radius=dist2(res, outer[0]);
41 }}
42 void minball(int n) { ball();
43     if ( nouter<4 ) for (int i=0; i<n; ++i)
44         if (dist2(res, pt[i])-radius>eps) {
45             outer[nouter++]=pt[i]; minball(i); --nouter;
46             if (i>0) { Tpoint Tt = pt[i];
47                 memmove(&pt[1], &pt[0], sizeof(Tpoint)*i); pt[0]=Tt;
48             }}
49 int main0(){
50     scanf("%d", &npoint);
51     for (int i=0; i<npoint; i++) scanf("%lf%lf%lf", &pt[i].x, &pt[i].y, &pt[i].z);
52     random_shuffle(pt, pt+npoint); radius=-1;
53     for (int i=0; i<npoint; i++) if (dist2(res, pt[i])-radius>eps)
54         nouter=1, outer[0]=pt[i], minball(i);
55     printf("%.5f\n", sqrt(radius));

```

56 }

## 经纬度求球面最短距离

```

1 //lati 为纬度 longi 为经度 R 为半径
2 double Dist(double lati1, double longi1, double lati2, double longi2, double R) {
3     double pi=acos(-1.0); lati1*=pi/180, longi1*=pi/180, lati2*=pi/180, longi2*=pi/180;
4     double x1=cos(lati1)*sin(longi1), y1=cos(lati1)*cos(longi1), z1=sin(lati1);
5     double x2=cos(lati2)*sin(longi2), y2=cos(lati2)*cos(longi2), z2=sin(lati2);
6     double theta=acos(x1*x2+y1*y2+z1*z2); return(R*theta);
7 }

```

## 长方体表面两点最短距离

```

1 int r;
2 void turn(int i, int j, int x, int y, int z, int x0, int y0, int L, int W, int H) {
3     if (z==0) { int R = x*x+y*y; if (R<r) r=R;
4     } else {
5         if (i>=0 && i<2) turn(i+1, j, x0+L+z, y, x0+L-x, x0+L, y0, H, W, L);
6         if (j>=0 && j<2) turn(i, j+1, x, y0+W+z, y0+W-y, x0, y0+W, L, H, W);
7         if (i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0, x0-H, y0, H, W, L);
8         if (j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0, x0, y0-H, L, H, W);
9     }}
10 int main(){
11     int L, H, W, x1, y1, z1, x2, y2, z2;
12     cin >> L >> W >> H >> x1 >> y1 >> z1 >> x2 >> y2 >> z2;
13     if (z1!=0 && z1!=H) if (y1==0 || y1==W)
14         swap(y1, z1), std::swap(y2, z2), std::swap(W, H);
15     else swap(x1, z1), std::swap(x2, z2), std::swap(L, H);
16     if (z1==H) z1=0, z2=H-z2;
17     r=0x3fffffff; turn(0, 0, x2-x1, y2-y1, z2, -x1, -y1, L, W, H);
18     cout<<r<<endl; return 0;
19 }

```

## 最大团

```

1 // Super Fast Maximum Clique
2 // To Build Graph: Maxclique(Edges, Number of Nodes)
3 // To Get Answer: mcqdyn(AnswerNodes Index Array, AnswerLength)
4 typedef bool BB[N];
5 struct Maxclique {
6     const BB* e; int pk, level; const float Tlimit;
7     struct Vertex{ int i, d; Vertex(int i):i(i),d(0){} };
8     typedef vector<Vertex> Vertices; typedef vector<int> ColorClass;
9     Vertices V; vector<ColorClass> C; ColorClass QMAX, Q;
10     static bool desc_degree(const Vertex &vi, const Vertex &vj){
11         return vi.d > vj.d;

```

```

12 }
13 void init_colors(Vertexes &v){
14     const int max_degree = v[0].d;
15     for(int i = 0; i < (int)v.size(); i++) v[i].d = min(i, max_degree) + 1;
16 }
17 void set_degrees(Vertexes &v){
18     for(int i = 0, j; i < (int)v.size(); i++)
19         for(v[i].d = j = 0; j < int(v.size()); j++)
20             v[i].d += e[v[i].i][v[j].i];
21 }
22 struct StepCount{ int i1, i2; StepCount():i1(0),i2(0){ } };
23 vector<StepCount> S;
24 bool cut1(const int pi, const ColorClass &A){
25     for(int i = 0; i < (int)A.size(); i++) if (e[pi][A[i]]) return true;
26     return false;
27 }
28 void cut2(const Vertexes &A, Vertexes &B){
29     for(int i = 0; i < (int)A.size() - 1; i++)
30         if(e[A.back().i][A[i].i])
31             B.push_back(A[i].i);
32 }
33 void color_sort(Vertexes &R){
34     int j = 0, maxno = 1, min_k = max((int)QMAX.size() - (int)Q.size() + 1, 1);
35     C[1].clear(), C[2].clear();
36     for(int i = 0; i < (int)R.size(); i++) {
37         int pi = R[i].i, k = 1;
38         while(cut1(pi, C[k])) k++;
39         if(k > maxno) maxno = k, C[maxno + 1].clear();
40         C[k].push_back(pi);
41         if(k < min_k) R[j++].i = pi;
42     }
43     if(j > 0) R[j - 1].d = 0;
44     for(int k = min_k; k <= maxno; k++)
45         for(int i = 0; i < (int)C[k].size(); i++)
46             R[j].i = C[k][i], R[j++].d = k;
47 }
48 void expand_dyn(Vertexes &R){// diff -> diff with no dyn
49     S[level].i1 = S[level].i1 + S[level - 1].i1 - S[level].i2;//diff
50     S[level].i2 = S[level - 1].i1;//diff
51     while((int)R.size()) {
52         if((int)Q.size() + R.back().d > (int)QMAX.size()){
53             Q.push_back(R.back().i); Vertexes Rp; cut2(R, Rp);
54             if((int)Rp.size()){
55                 if((float)S[level].i1 / ++pk < Tlimit) degree_sort(Rp);//diff
56                 color_sort(Rp);

```

```

57         S[level].i1++, level++;//diff
58         expand_dyn(Rp);
59         level--;//diff
60     }
61     else if((int)Q.size() > (int)QMAX.size()) QMAX = Q;
62     Q.pop_back();
63 }
64 else return;
65 R.pop_back();
66 }
67 }
68 void mcqdyn(int* maxclique, int &sz){
69     set_degrees(V); sort(V.begin(),V.end(), desc_degree); init_colors(V);
70     for(int i = 0; i < (int)V.size() + 1; i++) S[i].i1 = S[i].i2 = 0;
71     expand_dyn(V); sz = (int)QMAX.size();
72     for(int i = 0; i < (int)QMAX.size(); i++) maxclique[i] = QMAX[i];
73 }
74 void degree_sort(Vertexes &R){
75     set_degrees(R); sort(R.begin(), R.end(), desc_degree);
76 }
77 Maxclique(const BB* conn, const int sz, const float tt = 0.025) \
78 : pk(0), level(1), Tlimit(tt){
79     for(int i = 0; i < sz; i++) V.push_back(Vertex(i));
80     e = conn, C.resize(sz + 1), S.resize(sz + 1);
81 }
82 };

```

### 极大团计数

```

1 //Bool g[][] 为图的邻接矩阵, 图点的标号由 1 至 n
2 void dfs(int size){
3     int i, j, k, t, cnt, best = 0;
4     if (ne[size]==ce[size]){ if (ce[size]==0) ++ans; return; }
5     for (t=0, i=1; i<=ne[size]; ++i) {
6         for (cnt=0, j=ne[size]+1; j<=ce[size]; ++j)
7             if (!g[list[size][i]][list[size][j]]) ++cnt;
8         if (t==0 || cnt<best) t=i, best=cnt;
9     } if (t && best<=0) return;
10    for (k=ne[size]+1; k<=ce[size]; ++k) {
11        if (t>0){ for (i=k; i<=ce[size]; ++i)
12            if (!g[list[size][t]][list[size][i]]) break;
13        swap(list[size][k], list[size][i]);
14    } i=list[size][k]; ne[size+1]=ce[size+1]=0;
15    for (j=1; j<k; ++j)if (g[i][list[size][j]])
16        list[size+1][++ne[size+1]]=list[size][j];
17    for (ce[size+1]=ne[size+1], j=k+1; j<=ce[size]; ++j)

```

```

18     if (g[i][list[size][j]]) list[size+1][++ce[size+1]]=list[size][j];
19     dfs(size+1); ++ne[size]; --best;
20     for (j=k+1, cnt=0; j<=ce[size]; ++j) if (!g[i][list[size][j]]) ++cnt;
21     if (t==0 || cnt<best) t=k, best=cnt;
22     if (t && best<=0) break;
23 }}
24 void work(){
25     ne[0]=0; ce[0]=0;
26     for (int i=1; i<=n; ++i) list[0][++ce[0]]=i;
27     ans=0; dfs(0);
28 }

```

## KM

```

1  // 最小匹配, 自带初始化 n <= m 方案存在 p[] 中
2
3  const int N = 105;
4
5  const int INF = 1000000000; // 严格大于最大边权
6
7  int n, m, a[N][N];
8
9  int u[N], v[N], p[N], fa[N];
10
11 int minv[N];
12
13 bool used[N];
14
15 int km() {
16     for (int i = 0; i < n; ++i) {
17         u[i] = 0;
18     }
19     for (int i = 0; i <= m; ++i) {
20         v[i] = 0;
21         p[i] = n;
22     }
23     for (int i = 0; i < n; ++i) {
24         p[m] = i;
25         int j0 = m;
26         for (int j = 0; j <= m; ++j) {
27             minv[j] = INF;
28             used[j] = false;
29         }
30         do {
31             used[j0] = true;
32             int i0 = p[j0], delta = INF, j1;

```

```

33         for (int j = 0; j < m; ++j) {
34             if (!used[j]) {
35                 int cur = a[i0][j] - u[i0] - v[j];
36                 if (cur < minv[j]) {
37                     minv[j] = cur;
38                     fa[j] = j0;
39                 }
40                 if (minv[j] < delta) {
41                     delta = minv[j];
42                     j1 = j;
43                 }
44             }
45         }
46         for (int j = 0; j <= m; ++j) {
47             if (used[j]) {
48                 u[p[j]] += delta;
49                 v[j] -= delta;
50             } else {
51                 minv[j] -= delta;
52             }
53         }
54         j0 = j1;
55     } while (p[j0] != n);
56     do {
57         int j1 = fa[j0];
58         p[j0] = p[j1];
59         j0 = j1;
60     } while (j0 != m);
61 }
62 return -v[m];
63 }

```

## 最小树形图

```

1 namespace LIUZHU{
2     const int MAXN;
3     int from[MAXN + 10][MAXN * 2 + 10];
4     int n,m;
5     int edge[MAXN + 10][MAXN * 2 + 10];
6     int sel[MAXN * 2 + 10],fa[MAXN * 2 + 10];
7     int vis[MAXN * 2 + 10];
8     const int INF;// INF >= sum( W_ij )
9     int getfa(int x){
10         if(x == fa[x]) return x;
11         return fa[x] = getfa(fa[x]);
12     }

```



```

13 void liuzhu(){ // 1-base: root is 1, answer = (sel[i], i) for i in [2..n]
14     fa[1] = 1;
15     for(int i = 2; i <= n; ++i){
16         sel[i] = 1;
17         fa[i] = i;
18         for(int j = 1; j <= n; ++j) if(fa[j] != i){
19             from[j][i] = i;
20             if(edge[sel[i]][i] > edge[j][i]) sel[i] = j;
21         }
22     }
23     int limit = n;
24     while(1){
25         int prelimit = limit;
26         memset(vis, 0, sizeof(vis));
27         vis[1] = 1;
28         for(int i = 2; i <= prelimit; ++i) if(fa[i] == i && !vis[i]){
29             int j = i;
30             while(!vis[j]){
31                 vis[j] = i;
32                 j = getfa(sel[j]);
33             }
34             if(j == 1 || vis[j] != i) continue;
35             vector<int> C;
36             int k = j;
37             do{
38                 C.push_back(k);
39                 k = getfa(sel[k]);
40             }while(k != j);
41             ++limit;
42             for(int i = 1; i <= n; ++i){
43                 edge[i][limit] = INF;
44                 from[i][limit] = limit;
45             }
46             fa[limit] = vis[limit] = limit;
47             for(int i = 0; i < int(C.size()); ++i){
48                 int x = C[i];
49                 fa[x] = limit;
50                 for(int j = 1; j <= n; ++j){
51                     if(edge[j][x] != INF && edge[j][limit] > edge[j][x] - edge[sel[x]][x]){
52                         edge[j][limit] = edge[j][x] - edge[sel[x]][x];
53                         from[j][limit] = x;
54                     }
55                 }
56             }
57             for(int j = 1; j <= n; ++j) if(getfa(j) == limit){

```

```

58                 edge[j][limit] = INF;
59             }
60             sel[limit] = 1;
61             for(int j = 1; j <= n; ++j){
62                 if(edge[sel[limit]][limit] > edge[j][limit]){
63                     sel[limit] = j;
64                 }
65             }
66         }
67         if(prelimit == limit) break;
68     }
69     for(int i = limit; i > 1; --i){
70         sel[from[sel[i]][i]] = sel[i];
71     }
72 }
73 }

```

### 无向图最小割

```

1 int cost[maxn][maxn], seq[maxn], len[maxn], n, m, pop, ans;
2 bool used[maxn];
3 void Init(){
4     int i, j, a, b, c;
5     for(i=0; i<n; i++) for(j=0; j<n; j++) cost[i][j]=0;
6     for(i=0; i<m; i++){
7         scanf("%d %d %d", &a, &b, &c); cost[a][b]+=c; cost[b][a]+=c;
8     }
9     pop=n; for(i=0; i<n; i++) seq[i]=i;
10 }
11 void Work(){
12     ans=inf; int i, j, k, l, mm, sum, pk;
13     while(pop > 1){
14         for(i=1; i<pop; i++) used[seq[i]]=0; used[seq[0]]=1;
15         for(i=1; i<pop; i++) len[seq[i]]=cost[seq[0]][seq[i]];
16         pk=0; mm=-inf; k=-1;
17         for(i=1; i<pop; i++) if(len[seq[i]] > mm){ mm=len[seq[i]]; k=i; }
18         for(i=1; i<pop; i++){
19             used[seq[l=k]]=1;
20             if(i==pop-2) pk=k;
21             if(i==pop-1) break;
22             mm=-inf;
23             for(j=1; j<pop; j++) if(!used[seq[j]])
24                 if((len[seq[j]]+cost[seq[l]][seq[j]]) > mm)
25                     mm=len[seq[j]], k=j;
26         }
27         sum=0;

```



```

28     for(i=0;i<pop;i++) if(i != k) sum+=cost[seq[k]][seq[i]];
29     ans=min(ans,sum);
30     for(i=0;i<pop;i++)
31         cost[seq[k]][seq[i]]=cost[seq[i]][seq[k]]+cost[seq[pk]][seq[i]];
32     seq[pk]=seq[--pop];
33 }
34 printf("%d\n",ans);
35 }

```

## 带花树

```

1  vector<int> link[maxn];
2  int n,match[maxn],Queue[maxn],head,tail;
3  int pred[maxn],base[maxn],start,finish,newbase;
4  bool InQueue[maxn],InBlossom[maxn];
5  void push(int u){ Queue[tail++]=u;InQueue[u]=true; }
6  int pop(){ return Queue[head++]; }
7  int FindCommonAncestor(int u,int v){
8      bool InPath[maxn];
9      for(int i=0;i<n;i++) InPath[i]=0;
10     while(true){ u=base[u];InPath[u]=true;if(u==start) break;u=pred[match[u]]; }
11     while(true){ v=base[v];if(InPath[v]) break;v=pred[match[v]]; }
12     return v;
13 }
14 void ResetTrace(int u){
15     int v;
16     while(base[u]!=newbase){
17         v=match[u];
18         InBlossom[base[u]]=InBlossom[base[v]]=true;
19         u=pred[v];
20         if(base[u]!=newbase) pred[u]=v;
21     }
22 }
23 void BlossomContract(int u,int v){
24     newbase=FindCommonAncestor(u,v);
25     for (int i=0;i<n;i++)
26         InBlossom[i]=0;
27     ResetTrace(u);ResetTrace(v);
28     if(base[u]!=newbase) pred[u]=v;
29     if(base[v]!=newbase) pred[v]=u;
30     for(int i=0;i<n;i++){
31         if(InBlossom[base[i]]){
32             base[i]=newbase;
33             if(!InQueue[i]) push(i);
34         }
35     }

```

```

36 bool FindAugmentingPath(int u){
37     bool found=false;
38     for(int i=0;i<n;i++) pred[i]=-1,base[i]=i;
39     for (int i=0;i<n;i++) InQueue[i]=0;
40     start=u;finish=-1; head=tail=0; push(start);
41     while(head<tail){
42         int u=pop();
43         for(int i=link[u].size()-1;i>=0;i--){
44             int v=link[u][i];
45             if(base[u]!=base[v]&&match[u]!=v)
46                 if(v==start||((match[v]>=0&&pred[match[v]]>=0))
47                     BlossomContract(u,v);
48             else if(pred[v]==-1){
49                 pred[v]=u;
50                 if(match[v]>=0) push(match[v]);
51                 else{ finish=v; return true; }
52             }
53         }
54     }
55     return found;
56 }
57 void AugmentPath(){
58     int u=finish,v,w;
59     while(u>=0){ v=pred[u];w=match[v];match[v]=u;match[u]=v;u=w; }
60 }
61 void FindMaxMatching(){
62     for(int i=0;i<n;i++) match[i]=-1;
63     for(int i=0;i<n;i++) if(match[i]==-1) if(FindAugmentingPath(i)) AugmentPath();
64 }

```

## 动态最小生成树

```

1  /* 动态最小生成树  $Q(\log Q)^2$ 
2     (qx[i],qy[i]) 表示将编号为 qx[i] 的边的权值改为 qy[i]
3     删除一条边相当于将其权值改为  $\infty$ 
4     加入一条边相当于将其权值从  $\infty$  变成某个值 */
5  const int qsize=maxm+3*maxq;
6  int x[qsize],y[qsize],z[qsize], qx[maxq],qy[maxq],n,m,Q;
7  void init(){
8      scanf("%d%d",&n,&m);
9      for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
10     scanf("%d",&Q);
11     for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i]--; }
12 }
13 int a[maxn],*tz;
14 int find(int x){

```

```

15  int root=x; while(a[root]) root=a[root];
16  int next; while(next=a[x]){ a[x]=root; x=next; }
17  return root;
18 }
19 inline bool cmp(const int &a,const int &b){ return tz[a]<tz[b]; }
20 int kx[maxn],ky[maxn],kt, vd[maxn],id[maxm], app[maxm];
21 bool extra[maxm];
22 void solve(int *qx,int *qy,int Q,int n,int *x,int *y,int *z,int m,long long ans){
23     if(Q==1){
24         for(int i=1;i<=n;i++) a[i]=0;
25         z[ qx[0] ]=qy[0];
26         for(int i=0;i<m;i++) id[i]=i;tz=z;
27         sort(id,id+m,cmp); int ri,rj;
28         for(int i=0;i<m;i++){
29             ri=find(x[id[i]]); rj=find(y[id[i]]);
30             if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
31         }
32         printf("%I64d\n",ans);
33         return;
34     }
35     int ri,rj;
36     //contract
37     kt=0;
38     for(int i=1;i<=n;i++) a[i]=0;
39     for(int i=0;i<Q;i++){
40         ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
41     }
42     int tm=0;
43     for(int i=0;i<m;i++) extra[i]=true;
44     for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
45     for(int i=0;i<m;i++) if(extra[i]) id[tm++]=i;
46     tz=z; sort(id,id+tm,cmp);
47     for(int i=0;i<tm;i++){
48         ri=find(x[id[i]]); rj=find(y[id[i]]);
49         if(ri!=rj){
50             a[ri]=rj; ans += z[id[i]];
51             kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
52         }
53     }
54     for(int i=1;i<=n;i++) a[i]=0;
55     for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
56     int n2=0;
57     for(int i=1;i<=n;i++) if(a[i]==0)
58         vd[i]=++n2;
59     for(int i=1;i<=n;i++) if(a[i])

```

```

60     vd[i]=vd[find(i)];
61     int m2=0, *Nx=x+m, *Ny=y+m, *Nz=z+m;
62     for(int i=0;i<m;i++) app[i]=-1;
63     for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
64         Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ]; Nz[m2]=z[ qx[i] ];
65         app[qx[i]]=m2; m2++;
66     }
67     for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
68     for(int i=1;i<=n2;i++) a[i]=0;
69     for(int i=0;i<tm;i++){
70         ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
71         if(ri!=rj){
72             a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
73             Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
74         }
75     }
76     int mid=Q/2;
77     solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
78     solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
79 }
80 void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
81 int main(){init(); work(); return 0; }

```

## Hopcroft

```

1  int from[1010],wh[1010],g[1010];
2  int num[100010],nxt[100010],tot;
3  int n,m,ans,h,t,q[1010],dx[1010],dy[1010];
4  bool bfs(){
5      bool ret=false;
6      h=0;t=0;
7      for(int i=0;i<n;i++) if(wh[i]==-1) t++, q[t]=i;
8      memset(dx,0,sizeof(dx)), memset(dy,0,sizeof(dy));
9      while(h<=t){
10         for(int i=g[q[h]];i!=0;i=nxt[i])
11             if(dy[num[i]]==0){
12                 dy[num[i]]=dx[q[h]]+1;
13                 if(from[num[i]]==-1) ret=true;
14                 else{
15                     dx[from[num[i]]]=dx[q[h]]+2;
16                     q[++t]=from[num[i]];
17                 }
18             }
19     }
20     return ret;
21 }

```

```

22 bool dfs(int x){
23     for(int i=g[x];i!=0;i=nxt[i]){
24         if(dy[num[i]]==dx[x]+1){
25             dy[num[i]]=0;
26             if(from[num[i]]!=-1||dfs(from[num[i]])){
27                 wh[x]=num[i];from[num[i]]=x;return true;
28             }
29         }
30     }
31     return false;
32 }
33 void hopcroft(){
34     memset(from,-1,sizeof(from)), memset(wh,-1,sizeof(wh));
35     while(bfs()){
36         for(int i=0;i<n;i++){
37             if(wh[i]==-1&&dfs(i)) ans++;
38         }
39     }
40     void insert(int x,int y){ tot++;num[tot]=y;nxt[tot]=g[x];g[x]=tot; }
41     int main(){
42         while(scanf("%d %d",&n,&m)==2){
43             tot=0; memset(g,0,sizeof(g));
44             for(int i=0;i<n;i++){
45                 int x; scanf("%d",&x);
46                 for(int j=0;j<x;j++){
47                     int y; scanf("%d",&y);
48                     y--; insert(i,y);
49                 }
50             }
51             ans=0; hopcroft(); printf("%d\n",ans);
52         }
53     }

```

## 素数判定

```

1 int strong_pseudo_primetest(long long n,int base) {
2     long long n2=n-1,res;
3     int s=0;
4     while(n2%2==0) n2>>=1,s++;
5     res=powmod(base,n2,n);
6     if((res==1)||((res==n-1)) return 1;
7     s--;
8     while(s>=0) {
9         res=mulmod(res,res,n);
10        if(res==n-1) return 1;
11        s--;
12    }

```

```

13     return 0; // n is not a strong pseudo prime
14 }
15 int isprime(long long n) {
16     static LL testNum[]={2,3,5,7,11,13,17,19,23,29,31,37};
17     static LL lim[]={4,0,1373653LL,25326001LL,2500000000LL,2152302898747LL, \
18     3474749660383LL,341550071728321LL,0,0,0,0};
19     if(n<2||n==3215031751LL) return 0;
20     for(int i=0;i<12;++i){
21         if(n<lim[i]) return 1;
22         if(strong_pseudo_primetest(n,testNum[i])==0) return 0;
23     }
24     return 1;
25 }

```

## 启发式分解

```

1 int ansn; LL ans[1000];
2 LL func(LL x,LL n){ return(mod_mul(x,x,n)+1)%n; }
3 LL Pollard(LL n){
4     LL i,x,y,p;
5     if(Rabin_Miller(n)) return n;
6     if(!(n&1)) return 2;
7     for(i=1;i<20;i++){
8         x=i; y=func(x,n); p=gcd(y-x,n);
9         while(p==1) {x=func(x,n); y=func(func(y,n),n); p=gcd((y-x+n)%n,n)%n;}
10        if(p==0||p==n) continue;
11        return p;
12    }
13 }
14 void factor(LL n){
15     LL x;
16     x=Pollard(n);
17     if(x==n){ ans[ansn++]=x; return; }
18     factor(x), factor(n/x);
19 }

```

## 二次剩余

```

1 void calcH(int &t, int &h, const int p) {
2     int tmp = p - 1; for (t = 0; (tmp & 1) == 0; tmp /= 2) t++; h = tmp;
3 }
4 // solve equation x^2 mod p = a
5 bool solve(int a, int p, int &x, int &y) {
6     srand(19920225);
7     if (p == 2) { x = y = 1; return true; }
8     int p2 = p / 2, tmp = power(a, p2, p);

```

```

9  if (tmp == p - 1) return false;
10 if ((p + 1) % 4 == 0) {
11     x = power(a, (p + 1) / 4, p); y = p - x; return true;
12 } else {
13     int t, h, b, pb; calcH(t, h, p);
14     if (t >= 2) {
15         do {b = rand() % (p - 2) + 2;
16             while (power(b, p / 2, p) != p - 1);
17             pb = power(b, h, p);
18         } int s = power(a, h / 2, p);
19         for (int step = 2; step <= t; step++) {
20             int ss = (((long long)(s * s) % p) * a) % p;
21             for (int i = 0; i < t - step; i++) ss = ((long long)ss * ss) % p;
22             if (ss + 1 == p) s = (s * pb) % p; pb = ((long long)pb * pb) % p;
23         } x = ((long long)s * a) % p; y = p - x;
24     } return true;
25 }

```

### Pell 方程

```

1  ULL A,B,p[maxn],q[maxn],a[maxn],g[maxn],h[maxn];
2  int main() {
3      for (int test=1, n; scanf("%d",&n) && n;++test) {
4          printf("Case %d: ",test);
5          if (fabs(sqrt(n)-floor(sqrt(n)+1e-7))<=1e-7) {
6              int a=(int)(floor(sqrt(n)+1e-7)); printf("%d %d\n",a,1);
7          } else {
8              // 求  $x^2 - ny^2 = 1$  的最小正整数根, n 不是完全平方数
9              p[1]=q[0]=h[1]=1;p[0]=q[1]=g[1]=0;
10             a[2]=(int)(floor(sqrt(n)+1e-7));
11             for (int i=2;i;++i) {
12                 g[i]=-g[i-1]+a[i]*h[i-1]; h[i]=(n-sqr(g[i]))/h[i-1];
13                 a[i+1]=(g[i]+a[2])/h[i]; p[i]=a[i]*p[i-1]+p[i-2];
14                 q[i]=a[i]*q[i-1]+q[i-2];
15                 if (sqr((ULL)(p[i]))-n*sqr((ULL)(q[i]))==1){
16                     A=p[i];B=q[i];break; }
17             } cout << A << ' ' << B << endl;
18         }
19     }
20 }

```

### 蔡勒公式

```

1  int zeller(int y,int m,int d) {
2      if (m<=2) y--,m+=12; int c=y/100; y%=100;
3      int w=((c>>2)-(c<<1)+y+(y>>2)+(13*(m+1)/5)+d-1)%7;
4      if (w<0) w+=7; return(w);
5  }

```

### Schreier-Sims

```

1  namespace Schreier_Sims_Algorithm{
2      struct Permutation{
3          vector<int> P;
4          Permutation(){}
5          Permutation(int n){
6              P.resize(n);
7          }
8          Permutation inv()const{
9              Permutation ret(P.size());
10             for(int i = 0; i < int(P.size()); ++i) ret.P[P[i]] = i;
11             return ret;
12         }
13         int &operator [](const int &dn){
14             return P[dn];
15         }
16         void resize(const size_t &sz){
17             P.resize(sz);
18         }
19         size_t size()const{
20             return P.size();
21         }
22         const int &operator [](const int &dn)const{
23             return P[dn];
24         }
25     };
26     Permutation operator *(const Permutation &a, const Permutation &b){
27         Permutation ret(a.size());
28         for(int i = 0; i < (int)a.size(); ++i){
29             ret[i] = b[a[i]];
30         }
31         return ret;
32     }
33
34     typedef vector<Permutation> Bucket;
35     typedef vector<int> Table;
36     typedef pair<int,int> pii;
37     int n, m;
38     vector<Bucket> buckets, bucketsInv;
39     vector<Table> lookupTable;
40
41     int fastFilter(const Permutation &g, bool addToGroup = true){
42         int n = buckets.size();
43         Permutation p;

```

```

44     for(int i = 0; i < n; ++i){
45         int res = lookupTable[i][p[i]];
46         if(res == -1){
47             if(addToGroup){
48                 buckets[i].push_back(p);
49                 bucketsInv[i].push_back(p.inv());
50                 lookupTable[i][p[i]] = (int)buckets[i].size() - 1;
51             }
52             return i;
53         }
54         p = p * bucketsInv[i][res];
55         swap(i1,i2);
56     }
57     return -1;
58 }
59
60 long long calcTotalSize(){
61     long long ret = 1;
62     for(int i = 0; i < n; ++i){
63         ret *= buckets[i].size();
64     }
65     return ret;
66 }
67
68 bool inGroup(const Permutation &g){
69     return fastFilter(g, false) == -1;
70 }
71
72 void solve(const Bucket &gen,int _n){// m perm[0..n - 1]s
73     n = _n, m = gen.size();
74     //clear all
75     vector<Bucket> _buckets(n);
76     swap(buckets, _buckets);
77     vector<Bucket> _bucketsInv(n);
78     swap(bucketsInv, _bucketsInv);
79     vector<Table> _lookupTable(n);
80     swap(lookupTable, _lookupTable);
81 }
82 for(int i = 0; i < n; ++i){
83     lookupTable[i].resize(n);
84     fill(lookupTable[i].begin(), lookupTable[i].end(), -1);
85 }
86 Permutation id(n);
87 for(int i = 0; i < n; ++i){
88     id[i] = i;

```

```

89     }
90     for(int i = 0; i < n; ++i){
91         buckets[i].push_back(id);
92         bucketsInv[i].push_back(id);
93         lookupTable[i][i] = 0;
94     }
95     for(int i = 0; i < m; ++i){
96         fastFilter(gen[i]);
97     }
98     queue<pair<point,point> > toUpdate;
99     for(int i = 0; i < n; ++i){
100         for(int j = i; j < n; ++j){
101             for(int k = 0; k < (int)buckets[i].size(); ++k){
102                 for(int l = 0; l < (int)buckets[j].size(); ++l){
103                     toUpdate.push(make_pair(pii(i,k), pii(j,l)));
104                 }
105             }
106         }
107     }
108     while(!toUpdate.empty()){
109         pii a = toUpdate.front().first;
110         pii b = toUpdate.front().second;
111         toUpdate.pop();
112         int res = fastFilter(buckets[a.first][a.second] * buckets[b.first][b.second]);
113         if(res==-1) continue;
114         pii newPair(res, (int)buckets[res].size() - 1);
115         for(int i = 0; i < n; ++i){
116             for(int j = 0; j < (int)buckets[i].size(); ++j){
117                 if(i <= res){
118                     toUpdate.push(make_pair(pii(i, j), newPair));
119                 }
120                 if(res<=i){
121                     toUpdate.push(make_pair(newPair, pii(i, j)));
122                 }
123             }
124         }
125     }
126 }
127 }

```

## Romberg

```

1 template<class T>
2 double romberg(const T&f,double a,double b,double eps=1e-8){
3     std::vector<double>t; double h=b-a,last,curr; int k=1,i=1;
4     t.push_back(h*(f(a)+f(b))/2); // 梯形

```

```

5  do{ last=t.back(); curr=0; double x=a+h/2;
6    for(int j=0;j<k;++j) curr+=f(x),x+=h;
7    curr=(t[0]+h*curr)/2; double k1=4.0/3.0,k2=1.0/3.0;
8    for(int j=0;j<i;j++){ double temp=k1*curr-k2*t[j];
9      t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1; // 防止溢出
10   } t.push_back(curr); k*=2; h/=2; i++;
11 } while(std::fabs(last-curr)>eps);
12 return t.back();
13 }

```

### 线性规划

```

1  // 求 $\max\{cx \mid Ax \leq b, x \geq 0\}$ 的解
2  typedef vector<double> VD;
3  VD simplex(vector<VD> A, VD b, VD c) {
4    int n = A.size(), m = A[0].size() + 1, r = n, s = m - 1;
5    vector<VD> D(n + 2, VD(m + 1, 0)); vector<int> ix(n + m);
6    for (int i = 0; i < n + m; ++i) ix[i] = i;
7    for (int i = 0; i < n; ++i) {
8      for (int j = 0; j < m - 1; ++j) D[i][j] = -A[i][j];
9      D[i][m - 1] = 1; D[i][m] = b[i];
10     if (D[r][m] > D[i][m]) r = i;
11   }
12   for (int j = 0; j < m - 1; ++j) D[n][j] = c[j];
13   D[n + 1][m - 1] = -1;
14   for (double d; ; ) {
15     if (r < n) {
16       int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
17       D[r][s] = 1.0 / D[r][s]; vector<int> speedUp;
18       for (int j = 0; j <= m; ++j) if (j != s) {
19         D[r][j] *= -D[r][s];
20         if(D[r][j]) speedUp.push_back(j);
21       }
22       for (int i = 0; i <= n + 1; ++i) if (i != r) {
23         for(int j = 0; j < speedUp.size(); ++j)
24           D[i][speedUp[j]] += D[r][speedUp[j]] * D[i][s];
25         D[i][s] *= D[r][s];
26       } r = -1; s = -1;
27       for (int j = 0; j < m; ++j) if (s < 0 || ix[s] > ix[j])
28         if (D[n + 1][j] > EPS || (D[n + 1][j] > -EPS && D[n][j] > EPS)) s = j;
29       if (s < 0) break;
30       for (int i = 0; i < n; ++i) if (D[i][s] < -EPS)
31         if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
32           || (d < EPS && ix[r + m] > ix[i + m])) r = i;
33       if (r < 0) return VD(); // 无边界
34   }

```

```

35  if (D[n + 1][m] < -EPS) return VD(); // 无解
36  VD x(m - 1);
37  for (int i = m; i < n + m; ++i) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
38  return x; // 最优值在 D[n][m]
39 }

```

### FFT

```

1  // double 精度对 $10^9 + 7$  取模最多可以做到 $2^{20}$ 
2
3  const int MOD = 1000003;
4
5  const double PI = acos(-1);
6
7  typedef complex<double> Complex;
8
9  const int N = 65536, L = 15, MASK = (1 << L) - 1;
10
11  Complex w[N];
12
13  void FFTInit() {
14    for (int i = 0; i < N; ++i) {
15      w[i] = Complex(cos(2 * i * PI / N), sin(2 * i * PI / N));
16    }
17  }
18
19  void FFT(Complex p[], int n) {
20    for (int i = 1, j = 0; i < n - 1; ++i) {
21      for (int s = n; j ^= s >>= 1, ~j & s;);
22      if (i < j) {
23        swap(p[i], p[j]);
24      }
25    }
26    for (int d = 0; (1 << d) < n; ++d) {
27      int m = 1 << d, m2 = m * 2, rm = n >> (d + 1);
28      for (int i = 0; i < n; i += m2) {
29        for (int j = 0; j < m; ++j) {
30          Complex &p1 = p[i + j + m], &p2 = p[i + j];
31          Complex t = w[rm * j] * p1;
32          p1 = p2 - t;
33          p2 = p2 + t;
34        }
35      }
36    }
37  }
38 }

```

```

39 Complex A[N], B[N], C[N], D[N];
40
41 void mul(int a[N], int b[N]) {
42     for (int i = 0; i < N; ++i) {
43         A[i] = Complex(a[i] >> L, a[i] & MASK);
44         B[i] = Complex(b[i] >> L, b[i] & MASK);
45     }
46     FFT(A, N);
47     FFT(B, N);
48     for (int i = 0; i < N; ++i) {
49         int j = (N - i) % N;
50         Complex da = (A[i] - conj(A[j])) * Complex(0, -0.5),
51             db = (A[i] + conj(A[j])) * Complex(0.5, 0),
52             dc = (B[i] - conj(B[j])) * Complex(0, -0.5),
53             dd = (B[i] + conj(B[j])) * Complex(0.5, 0);
54         C[j] = da * dd + da * dc * Complex(0, 1);
55         D[j] = db * dd + db * dc * Complex(0, 1);
56     }
57     FFT(C, N);
58     FFT(D, N);
59     for (int i = 0; i < N; ++i) {
60         long long da = (long long)(C[i].imag() / N + 0.5) % MOD,
61             db = (long long)(C[i].real() / N + 0.5) % MOD,
62             dc = (long long)(D[i].imag() / N + 0.5) % MOD,
63             dd = (long long)(D[i].real() / N + 0.5) % MOD;
64         a[i] = ((dd << (L * 2)) + ((db + dc) << L) + da) % MOD;
65     }
66 }

```

## NTT

```

1 //R 是  $2^{n*q+1}$  形质数 p 的原根
2 void NFFT(int P[], int n, int oper) {
3     for (int i = 1, j = 0; i < n - 1; ++i) {
4         for (int s = n; j ^= s >>= 1, ~j & s;);
5         if (i < j) {
6             swap(P[i], P[j]);
7         }
8     }
9     for (int d = 0; (1 << d) < n; ++d) {
10         int m = 1 << d, m2 = m * 2;
11         int unit_p0 = powmod(R, (MOD - 1) / m2);
12         if (oper < 0) {
13             unit_p0 = inverse(unit_p0);
14         }
15         for (int i = 0; i < n; i += m2) {

```

```

16         int unit = 1;
17         for (int j = 0; j < m; ++j) {
18             int &P1 = P[i + j + m],
19                 &P2 = P[i + j];
20             int t = (long long)unit * P1 % MOD;
21             P1 = (P2 - t + MOD) % MOD;
22             P2 = (P2 + t) % MOD;
23             unit = (long long)unit * unit_p0 % MOD;
24         }
25     }
26 }
27 }

```

## FWT

```

1 void FWT(int a[N], int N) {
2     for (int d = 1; d < N; d <= 1) {
3         int d2 = d << 1;
4         for (int i = 0; i < N; i += d2) {
5             int *x = a + i, *y = a + i + d;
6             for (int j = 0; j < d; ++j, ++x, ++y) {
7                 if ((*x += *y) >= MOD) {
8                     *x -= MOD;
9                 }
10                if ((*y = *x - (*y << 1)) < 0) {
11                    if ((*y += MOD) < 0) {
12                        *y += MOD;
13                    }
14                }
15            }
16        }
17    }
18 }
19
20 void xorPow(int a[N], int n, int b[N]) {
21     memset(b, 0, sizeof(int) * N);
22     b[0] = 1;
23     FWT(a, N);
24     FWT(b, N);
25     while(n) {
26         if (n & 1) {
27             dot(b, a, N);
28         }
29         dot(a, a, N);
30         n >>= 1;
31     }

```

```

32 FWT(b, N);
33 norm(b, N);
34 }

```

## 回文串 manacher

```

1 for(int i=1,j=0;i!=(n<<1)-1;++i){
2     int p=i>>1,q=i-p,r=((j+1)>>1)+l[j]-1;
3     l[i]=r<q?0:min(r-q+1,l[(j<<1)-i]);
4     while(p-l[i]!=-1&&q+l[i]!=n&&s[p-l[i]]==s[q+l[i]]) l[i]++;
5     if(q+l[i]-1>r) j=i;
6     a+=l[i];
7 }

```

## 后缀数组 (倍增)

```

1 int rank[MAX_N],height[MAX_N];
2 int cmp(int *x,int a,int b,int d){
3     return x[a]==x[b]&&x[a+d]==x[b+d];
4 }
5 void doubling(int *a,int N,int M){
6     static int sRank[MAX_N],tmpA[MAX_N],tmpB[MAX_N];
7     int *x=tmpA,*y=tmpB;
8     for(int i=0;i<M;++i) sRank[i]=0;
9     for(int i=0;i<N;++i) ++sRank[x[i]=a[i]];
10    for(int i=1;i<M;++i) sRank[i]+=sRank[i-1];
11    for(int i=N-1;i>=0;--i) sa[--sRank[x[i]]]=i;
12    for(int d=1,p=0;p<N;M=p,d<<=1){
13        p=0; for(int i=N-d;i<N;++i) y[p++]=i;
14        for(int i=0;i<N;++i) if(sa[i]>=d) y[p++]=sa[i]-d;
15        for(int i=0;i<M;++i) sRank[i]=0;
16        for(int i=0;i<N;++i) ++sRank[x[i]];
17        for(int i=1;i<M;++i) sRank[i]+=sRank[i-1];
18        for(int i=N-1;i>=0;--i) sa[--sRank[x[y[i]]]]=y[i];
19        swap(x,y); x[sa[0]]=0; p=1;
20        for(int i=1;i<N;++i) x[sa[i]]=cmp(y,sa[i],sa[i-1],d)?p-1:p++;
21    }
22 }
23 void calcHeight(){
24     for(int i=0;i<N;++i) rank[sa[i]]=i;
25     int cur=0; for(int i=0;i<N;++i)
26     if(rank[i]){
27         if(cur) cur--;
28         for(;a[i+cur]==a[sa[rank[i]-1]+cur];++cur);
29         height[rank[i]]=cur;
30     }
31 }

```

## 后缀数组 (DC3)

```

1 // 待排序的字符串放在 r 数组中, 从 r[0] 到 r[n-1], 长度为 n, 且最大值小于 m
2 // 约定除 r[n-1] 外所有的 r[i] 都大于 0, r[n-1]=0
3 // 函数结束后, 结果放在 sa 数组中, 从 sa[0] 到 sa[n-1]
4 #define maxn 10000
5 #define F(x) ((x)/3+((x)%3==1?0:tb))
6 #define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)
7 int wa[maxn],wb[maxn],wv[maxn],wss[maxn]; // 必须这么大
8 int s[maxn*3],sa[maxn*3];
9 int c0(int *r,int a,int b){return r[a]==r[b]&&r[a+1]==r[b+1]&&r[a+2]==r[b+2];}
10 int c12(int k,int *r,int a,int b){
11     if(k==2) return r[a]<r[b]||r[a]==r[b]&&c12(1,r,a+1,b+1);
12     else return r[a]<r[b]||r[a]==r[b]&&wv[a+1]<wv[b+1];
13 }
14 void sort(int *r,int *a,int *b,int n,int m){
15     int i; for(i=0;i<n;i++) wv[i]=r[a[i]];
16     for(i=0;i<m;i++) wss[i]=0; for(i=0;i<n;i++) wss[wv[i]]++;
17     for(i=1;i<m;i++) wss[i]+=wss[i-1];
18     for(i=n-1;i>=0;i--) b[--wss[wv[i]]]=a[i];
19 }
20 void dc3(int *r,int *sa,int n,int m){
21     int i,j,*rn=r+n,*san=sa+n,ta=0,tb=(n+1)/3,tbc=0,p;
22     r[n]=r[n+1]=0;
23     for(i=0;i<n;i++) if(i%3!=0) wa[tbc++]=i;
24     sort(r+2,wa,wb,tbc,m); sort(r+1,wb,wa,tbc,m); sort(r,wa,wb,tbc,m);
25     for(p=1,rn[F(wb[0])]=0,i=1;i<tbc;i++)
26         rn[F(wb[i])]=c0(r,wb[i-1],wb[i])?p-1:p++;
27     if(p<tbc) dc3(rn,san,tbc,p);
28     else for(i=0;i<tbc;i++) san[rn[i]]=i;
29     for(i=0;i<tbc;i++) if(san[i]<tb) wb[ta++]=san[i]*3;
30     if(n%3==1) wb[ta++]=n-1;
31     sort(r,wb,wa,ta,m); for(i=0;i<tbc;i++) wv[wb[i]]=G(san[i])=i;
32     for(i=0,j=0,p=0;i<ta && j<tbc;p++)
33         sa[p]=c12(wb[j]%3,r,wa[i],wb[j])?wa[i++]:wb[j++];
34     for(;i<ta;p++) sa[p]=wa[i++]; for(;j<tbc;p++) sa[p]=wb[j++];}
35 int main(){
36     int n,m=0; scanf("%d",&n);
37     for(int i=0;i<n;i++) scanf("%d",&s[i]),s[i]++,m=max(s[i+1],m);
38     printf("%d\n",m); s[n++]=0; dc3(s,sa,n,m);
39     for(int i=0;i<n;i++) printf("%d ",sa[i]);printf("\n");
40 }

```



## 后缀自动机

```

1 struct State {
2     int length;
3     State *parent,*go[C];
4     State(int length):length(length),parent(NULL){
5         memset(go,0,sizeof(go));
6     }
7     State* extend(State *start,int token){
8         State *p=this;
9         State *np=new State(this->length+1);
10        while(p!=NULL&& p->go[token]==NULL)
11            p->go[token]=np, p=p->parent;
12        if(p==NULL) np->parent=start;
13        else{
14            State *q=p->go[token];
15            if(p->length+1==q->length) np->parent=q;
16            else{
17                State *nq=new State(p->length+1);
18                memcpy(nq->go,q->go,sizeof(q->go));
19                nq->parent=q->parent;
20                np->parent=q->parent=nq;
21                while(p!=NULL&&p->go[token]==q)
22                    p->go[token]=nq, p=p->parent;
23            }
24        }
25        return np;
26    }
27 };

```

## 后缀树 (With Walk)

```

1 int pos;
2
3 struct Node {
4     int l, r;
5
6     Node *suf, *ch[C];
7
8     Node (int l = -1, int r = INF) : l(l), r(r) {
9         suf = 0;
10        memset(ch, 0, sizeof(ch));
11    }
12
13    int len() {
14        return min(r, pos + 1) - l;
15    }

```

```

16 };
17
18 int top;
19
20 Node pool[N << 1];
21
22 Node *root, *nxtSuf, *cur;
23
24 int text[N];
25
26 int remCnt, curP, curLen;
27
28 void init() {
29     top = 0, pos = -1, remCnt = 0, curP = 0, curLen = 0;
30     nxtSuf = NULL;
31     root = cur = new(pool + (top++)) Node(-1, -1);
32 }
33
34 void link(Node *u) {
35     if (nxtSuf) {
36         nxtSuf->suf = u;
37     }
38     nxtSuf = u;
39 }
40
41 bool walk(Node *u) {
42     int len = u->len();
43     if (curLen >= len) {
44         curP += len;
45         curLen -= len;
46         cur = u;
47         return true;
48     }
49     return false;
50 }
51
52 void extend(int c) {
53     text[++pos] = c;
54     nxtSuf = NULL;
55     ++remCnt;
56     while (remCnt) {
57         curP = curLen ? curP : pos;
58         int curE = text[curP];
59         if (!cur->ch[curE]) {
60             cur->ch[curE] = new(pool + (top++)) Node(pos);

```

```

61     link(cur);
62 } else {
63     Node *nxt = cur->ch[curE];
64     if (walk(nxt)) {
65         continue;
66     }
67     if (text[nxt->l + curLen] == c) {
68         ++curLen;
69         link(cur);
70         break;
71     }
72     Node *split = new(pool + (top++)) Node(nxt->l, nxt->l + curLen);
73     cur->ch[curE] = split;
74     split->ch[c] = new(pool + (top++)) Node(pos);
75     nxt->l += curLen;
76     split->ch[text[nxt->l]] = nxt;
77     link(split);
78 }
79 --remCnt;
80 if (cur == root && curLen > 0) {
81     --curLen;
82     curP = pos - remCnt + 1;
83 } else {
84     cur = cur->suf ? cur->suf : root;
85 }
86 }
87 }
88
89 void finish() {
90     nxtSuf = NULL;
91     for (int i = 0; i < top; ++i) {
92         if (pool[i].r == INF) {
93             link(pool + i);
94         }
95     }
96     while (remCnt > 0) {
97         if (curLen) {
98             int curE = text[curP];
99             Node *nxt = cur->ch[curE];
100             if (walk(nxt)) {
101                 continue;
102             }
103             Node *split = new(pool + (top++)) Node(nxt->l, nxt->l + curLen);
104             cur->ch[curE] = split;
105             nxt->l += curLen;

```

```

106         split->ch[text[nxt->l]] = nxt;
107         link(split);
108     } else {
109         link(cur);
110     }
111     --remCnt;
112     if (cur == root && curLen > 0) {
113         --curLen;
114         curP = pos - remCnt + 1;
115     } else {
116         cur = cur->suf ? cur->suf : root;
117     }
118 }
119 if (nxtSuf != root) {
120     link(root);
121 }
122 }
123
124 int buf[N];
125
126 struct Walker {
127     Node *cur, *root;
128     int curP, curLen, pos, totLen;
129
130     Walker(Node* root) : root(root) {
131         cur = root;
132         curP = -1;
133         curLen = 0;
134         totLen = 0;
135         pos = -1;
136     }
137
138     void descend() {
139         while (curLen > 0) {
140             Node *nxt = cur->ch[buf[curP]];
141             int len = nxt->len();
142             if (curLen >= len) {
143                 curP += len;
144                 curLen -= len;
145                 cur = nxt;
146             } else {
147                 break;
148             }
149         }
150     }

```

```

151
152 void walk(int c) {
153     buf[++pos] = c;
154     while (curLen) {
155         Node *nxt = cur->ch[buf[curP]];
156         if (nxt->l + curLen <= ::pos && text[nxt->l + curLen] == c) {
157             ++totLen;
158             ++curLen;
159             break;
160         } else {
161             --totLen;
162             if (cur == root && curLen > 0) {
163                 --curLen;
164                 curP = pos - curLen;
165             } else {
166                 cur = cur->suf;
167             }
168             descend();
169         }
170     }
171     if (curLen == 0) {
172         curP = pos;
173         int curE = buf[curP];
174         while (cur && !cur->ch[curE]) {
175             --totLen;
176             cur = cur->suf;
177         }
178         if (cur) {
179             ++totLen;
180             curLen = 1;
181         } else {
182             cur = root;
183             totLen = 0;
184         }
185     }
186     descend();
187 }
188 };

```

### 后缀树 (With Pop Front)

```

1 int pos;
2
3 int text[N];
4
5 struct Node {

```

```

6     int l, r;
7
8     Node *suf, *ch[C];
9
10    int dgr;
11
12    Node *fa;
13
14    Node (int l = -1, int r = INF) : l(l), r(r) {
15        suf = fa = NULL;
16        memset(ch, 0, sizeof(ch));
17        dgr = 0;
18    }
19
20    Node* addEdge(Node *t) {
21        int c = text[t->l];
22        dgr += !ch[c];
23        ch[c] = t;
24        t->fa = this;
25        return t;
26    }
27
28    int len() {
29        return min(r, pos + 1) - l;
30    }
31 };
32
33 int top;
34
35 Node pool[N << 1];
36
37 Node *root, *nxtSuf, *cur;
38
39 int remCnt, curP, curLen;
40
41 long long size;
42
43 queue<Node*> leaves;
44
45 void init() {
46     top = 0, pos = -1;
47     remCnt = 0, curP = 0, curLen = 0;
48     nxtSuf = NULL;
49     root = cur = new(pool + (top++)) Node(-1, -1);
50     size = 0;

```

```

51 while (leaves.size()) {
52     leaves.pop();
53 }
54 }
55
56 void link(Node *u) {
57     if (nxtSuf) {
58         nxtSuf->suf = u;
59     }
60     nxtSuf = u;
61 }
62
63 bool walk(Node *u) {
64     int len = u->len();
65     if (curLen >= len) {
66         curP += len;
67         curLen -= len;
68         cur = u;
69         return true;
70     }
71     return false;
72 }
73
74 void extend(int c) {
75     text[++pos] = c;
76     nxtSuf = NULL;
77     ++remCnt;
78     while (remCnt) {
79         curP = curLen ? curP : pos;
80         int curE = text[curP];
81         if (!cur->ch[curE]) {
82             leaves.push(cur->addEdge(new(pool + (top++)) Node(pos)));
83             link(cur);
84         } else {
85             Node *nxt = cur->ch[curE];
86             if (walk(nxt)) {
87                 continue;
88             }
89             if (text[nxt->l + curLen] == c) {
90                 ++curLen;
91                 link(cur);
92                 break;
93             }
94             Node *split = new(pool + (top++)) Node(nxt->l, nxt->l + curLen);
95             cur->addEdge(split);

```

```

96         leaves.push(split->addEdge(new(pool + (top++)) Node(pos)));
97         nxt->l += curLen;
98         split->addEdge(nxt);
99         link(split);
100     }
101     --remCnt;
102     if (cur == root && curLen > 0) {
103         curP = pos - (--curLen);
104     } else {
105         cur = cur->suf ? cur->suf : root;
106     }
107 }
108 size += leaves.size();
109 }
110
111 void finish() {
112     nxtSuf = NULL;
113     for (int i = 0; i < top; ++i) {
114         if (pool[i].r == INF) {
115             link(pool + i);
116         }
117     }
118     while (remCnt > 0) {
119         if (curLen) {
120             int curE = text[curP];
121             Node *nxt = cur->ch[curE];
122             if (walk(nxt)) {
123                 continue;
124             }
125             Node *split = new(pool + (top++)) Node(nxt->l, nxt->l + curLen);
126             leaves.push(cur->addEdge(split));
127             nxt->l += curLen;
128             split->addEdge(nxt);
129             link(split);
130         } else {
131             leaves.push(cur);
132             link(cur);
133         }
134         --remCnt;
135         if (cur == root && curLen > 0) {
136             --curLen;
137             curP = pos - remCnt + 1;
138         } else {
139             cur = cur->suf ? cur->suf : root;
140         }

```

```

141 }
142 if (nxtSuf != root) {
143     link(root);
144 }
145 }
146
147 void eraseUp(Node *u) {
148     size -= u->len();
149     int ch = text[u->l];
150     u = u->fa;
151     u->ch[ch] = NULL;
152     --(u->dgr);
153 }
154
155 void erase() {
156     Node *u = leaves.front();
157     leaves.pop();
158     while (u->dgr == 0 && u != cur) {
159         eraseUp(u);
160     }
161     if (u == cur) {
162         if (cur->dgr == 0 && curLen == 0) {
163             int len = u->len();
164             curLen = len;
165             curP = pos - len + 1;
166             cur = cur->fa;
167             eraseUp(u);
168         }
169         if (curLen) {
170             int curE = text[curP];
171             if (!cur->ch[curE]) {
172                 Node *leaf = new(pool + (top++)) Node(pos - curLen + 1);
173                 leaves.push(cur->addEdge(leaf));
174                 size += leaf->len();
175                 --remCnt;
176                 if (cur == root && curLen > 0) {
177                     curP = pos - (--curLen) + 1;
178                 } else {
179                     cur = cur->suf ? cur->suf : root;
180                 }
181                 while (curLen && walk(cur->ch[text[curP]])) {
182                     continue;
183                 }
184             }
185         }
186     }

```

```

186 }
187 }
188
189 int n;
190
191 char s[N], buf[N];
192
193 int ord[N], stop, sord[N << 1];
194
195 void dfs(Node *u) {
196     sord[u - pool] = stop++;
197     for (int i = 0; i < C; ++i) {
198         if (u->ch[i]) {
199             dfs(u->ch[i]);
200         }
201     }
202 }
203
204 void getOrd() {
205     init();
206     for (int i = 0; i < n; ++i) {
207         extend(s[i] - 'a');
208     }
209     finish();
210     stop = 0;
211     dfs(root);
212     int i = 0;
213     while (leaves.size()) {
214         ord[i++] = sord[leaves.front() - pool];
215         leaves.pop();
216     }
217 }

```

#### 字符串最小表示

```

1 std::string find(std::string s) {
2     int i,j,k,l,N=s.length(); s+=s;
3     for(i=0,j=1;j<N;){
4         for(k=0;k<N&&s[i+k]==s[j+k];k++);
5         if(k>=N) break;
6         if(s[i+k]<s[j+k]) j+=k+1;
7         else l=i+k,i=j,j=max(l,j)+1;
8     }
9     return s.substr(i,N);
10 }

```

## 轻重链剖分

```

1 struct Tree(){}*root[N];
2 int father[N],size[N],depth[N];
3 int bfsOrd[N],pathId[N],ordInPath[N],sqn[N];
4 void doBfs(int s){
5     int qh=0,qt=0,*que=bfsOrd; father[s]=-1; depth[s]=0;
6     for(que[qt++]=s;qh<qt;){
7         int u=que[qh++];
8         foreach(iter,adj[u]){
9             int v=*iter; if(v==father[u]) continue;
10            father[v]=u; depth[v]=depth[u]+1; que[qt++]=v;
11        }
12    }
13 }
14 void doSplit(){
15     for(int i=N-1;i>=0;--i){
16         int u=bfsOrd[i]; size[u]=1;
17         foreach(iter,adj[u]){
18             int v=*iter; if(v==father[u]) continue; size[u]+=size[v];
19         }
20     }
21     memset(pathId,-1,sizeof pathId);
22     for(int i=0;i<N;++i){
23         int top=bfsOrd[i],cnt=0;
24         if(pathId[top]!=-1) continue;
25         for(int next,u=top;u!=-1;u=next){
26             sqn[cnt]=val[u]; ordInPath[u]=cnt; pathId[u]=top; ++cnt;
27             next=-1;
28             foreach(iter,adj[u]){
29                 int v=*iter; if(v==father[u]) continue;
30                 if(next<0||size[next]<size[v]) next=v;
31             }
32         }
33         root[top]=new Tree(0,cnt,sqn);
34     }
35 }
36 void prepare(){ doBfs(0); doSplit(); }

```

## KD Tree

```

1 #include <cstdio>
2 #include <vector>
3 #include <iostream>
4 #include <algorithm>
5
6 using namespace std;

```

```

7 // 带插入版本，没有写内存回收，空间复杂度  $n \log n$ ，如果不需要插入可以大大简化
8 // N 为最大点数，D 为每个点的最大维度，d 为实际维度
9 // 以查找最近点为例 ret 为当前最近点的距离的平方，用来剪枝，查询 k 近或 k 远的方法类似
10 // 使用时注意先 initNull
11 const long long INF = (int)1e9 + 10;
12 const int N = 200000 + 10;
13 const int D = 5;
14 const double SCALE = 0.75;
15 struct Point { int x[D]; } buf[N];
16 int d;
17 struct Node {
18     int depth, size;
19     Node *ch[2], *p;
20     Point val, maxv, minv;
21     void set(Node *t, int d) { ch[d] = t; t->p = this; }
22     bool dir() { return this == p->ch[1]; }
23     bool balanced() {
24         return (double)max(ch[0]->size, ch[1]->size) <= (double)size * SCALE;
25     }
26     void update() {
27         size = ch[0]->size + ch[1]->size + 1;
28         for(int i = 0; i < d; ++i) {
29             maxv.x[i] = max(val.x[i], max(ch[0]->maxv.x[i], ch[1]->maxv.x[i]));
30             minv.x[i] = min(val.x[i], min(ch[0]->minv.x[i], ch[1]->minv.x[i]));
31         }
32     }
33 } nodePool[N], *totNode, *null;
34 Node* newNode(Point p, int depth) {
35     Node *t = totNode++;
36     t->ch[0] = t->ch[1] = t->p = null;
37     t->depth = depth;
38     t->val = t->maxv = t->minv = p;
39     t->size = 1;
40     return t;
41 }
42 long long ret;
43 int ctr;
44 int cmp(const Point &a, const Point &b) { return a.x[ctr] < b.x[ctr]; }
45 struct KDTree {
46     Node *root;
47     KDTree() { root = null; }
48     KDTree(Point *a, int n) {
49         root = build(a, 0, n - 1, 0);
50     }
51     Node *build(Point *a, int l, int r, int depth) {

```

```

52     if (l > r) return null;
53     ctr = depth;
54     sort(a + l, a + r + 1, cmp);
55     int mid = (l + r) >> 1;
56     Node *t = newNode(a[mid], depth);
57     t->set(build(a, l, mid - 1, (depth + 1) % d), 0);
58     t->set(build(a, mid + 1, r, (depth + 1) % d), 1);
59     t->update();
60     return t;
61 }
62 void tranverse(Node *t, Point *vec, int &tot) {
63     if (t == null) return;
64     vec[tot++] = t->val;
65     tranverse(t->ch[0], vec, tot);
66     tranverse(t->ch[1], vec, tot);
67 }
68 void rebuild(Node *t) {
69     Node *p = t->p;
70     int tot = 0;
71     tranverse(t, buf, tot);
72     Node *u = build(buf, 0, tot - 1, t->depth);
73     p->set(u, t->dir());
74     for( ; p != null; p = p->p) p->update();
75     if (t == root) root = u;
76 }
77 void insert(Point p) {
78     if (root == null) { root = newNode(p, 0); return; }
79     Node *cur = root, *last = null;
80     int dir = 0;
81     for( ; cur != null; ) {
82         last = cur;
83         dir = (p.x[cur->depth] > cur->val.x[cur->depth]);
84         cur = cur->ch[dir];
85     }
86     Node *t = newNode(p, (last->depth + 1) % d, *bad = null;
87     last->set(t, dir);
88     for( ; t != null; t = t->p) {
89         t->update();
90         if (!t->balanced()) bad = t;
91     }
92     if (bad != null) rebuild(bad);
93 }
94 long long calcEval(Point u, Node *t, int d) {
95     long long l = t->minv.x[d], r = t->maxv.x[d], x = u.x[d];
96     if (x >= l && x <= r) return 0LL;

```

```

97     long long ret = min(abs(x - l), abs(x - r));
98     return ret * ret;
99 }
100 void updateAns(Point u, Point p) {
101     // 在这里更新答案
102 }
103 void query(Node *t, Point p) {
104     if (t == null) return;
105     updateAns(t->val, p);
106     long long evalLeft = calcEval(p, t->ch[0], t->depth);
107     long long evalRight = calcEval(p, t->ch[1], t->depth);
108     if (evalLeft <= evalRight) {
109         query(t->ch[0], p);
110         if (ret > evalRight) query(t->ch[1], p);
111     } else {
112         query(t->ch[1], p);
113         if (ret > evalLeft) query(t->ch[0], p);
114     }
115 }
116 void query(Point p) {
117     query(root, p);
118 }
119 };
120 void initNull() {
121     totNode = nodePool;
122     null = totNode++;
123     null->size = 0;
124     for(int i = 0; i < d; ++i) {
125         null->maxv.x[i] = -INF;
126         null->minv.x[i] = INF;
127     }
128 }

```

## Splay Tree

```

1 // 注意初始化内存池和 null 节点
2 struct Node{
3     int rev,size; Node *ch[2],*p;
4     void set(Node*,int); int dir(); void update(); void relax(); void appRev();
5 } nodePool[MAX_NODE],*curNode,*null;
6 Node *newNode(){
7     Node *t=curNode++; t->rev=0, t->size=1;
8     t->ch[0]=t->ch[1]=t->p=null; return t;
9 }
10 struct Splay{
11     Node *root;

```

```

12 Splay(){ root=newNode(); root->set(newNode(),0); root->update(); }
13 void rot(Node *t){
14     Node *p=t->p; int d=t->dir();
15     p->relax(); t->relax();
16     if(p==root) root=t;
17     p->set(t->ch[!d],d); p->p->set(t,p->dir()); t->set(p,!d);
18     p->update();
19 }
20 void splay(Node *t,Node *f=null){
21     for(t->relax();t->p!=f;){
22         if(t->p->p==f) rot(t);
23         else t->dir()==t->p->dir()? (rot(t->p),rot(t)):(rot(t),rot(t));
24         t->update();
25     }
26 };
27 void initNull(){ curNode=nodePool;null=curNode++;null->size=0; }
28 void Node::set(Node *t,int _d){ ch[_d]=t; t->p=this; }
29 int Node::dir(){ return this==p->ch[1]; }
30 void Node::update(){ size=ch[0]->size+ch[1]->size+1; }
31 void Node::relax(){ if(rev) ch[0]->appRev(), ch[1]->appRev(), rev=false; }
32 void Node::appRev(){ if(this==null) return; rev^=true; swap(ch[0],ch[1]); }

```

## Link Cut Tree

```

1 // 注意初始化 null 节点, 单点的 is_root 初始为 true
2 struct Node{
3     Node *ch[2], *p;
4     int is_root, rev;
5     bool dir();
6     void set(Node*, bool);
7     void update();
8     void relax();
9     void app_rev();
10 } *null;
11 void rot(Node *t){
12     Node *p=t->p; bool d=t->dir();
13     p->relax(); t->relax(); p->set(t->ch[!d],d);
14     if(p->is_root) t->p=p->p, swap(p->is_root,t->is_root);
15     else p->p->set(t,p->dir());
16     t->set(p,!d); p->update();
17 }
18 void splay(Node *t){
19     for(t->relax();!t->is_root;){
20         if(t->p->is_root) rot(t);
21         else t->dir()==t->p->dir()? (rot(t->p),rot(t)):(rot(t),rot(t));
22         t->update();

```

```

23 }
24 void access(Node *t){
25     for(Node *s=null; t!=null; s=t,t=t->p){
26         splay(t);
27         if (t->p == null) { /*TODO*/ }
28         t->ch[1]->is_root=true; s->is_root=false;
29         t->ch[1]=s; t->update();
30     }
31 }
32 bool Node::dir(){ return this==p->ch[1]; }
33 void Node::set(Node *t,bool _d){ ch[_d]=t; t->p=this; }
34 void Node::update(){ }
35 void Node::app_rev(){ if (this == null) return; rev ^= true; swap(ch[0], ch[1]); }
36 void Node::relax() { if(this==null) return; if (rev) { ch[0]->app_rev();
    ↳ ch[1]->app_rev(); rev = false; } }
37 void make_root(Node *u) { access(u); splay(u); u->app_rev(); }

```

## Dominator Tree

```

1 vector<int> prec[N], succ[N];
2
3 vector<int> ord;
4
5 int stamp, vis[N];
6
7 int num[N];
8
9 int fa[N];
10
11 void dfs(int u) {
12     vis[u] = stamp;
13     num[u] = ord.size();
14     ord.push_back(u);
15     for (int i = 0; i < (int)succ[u].size(); ++i) {
16         int v = succ[u][i];
17         if (vis[v] != stamp) {
18             fa[v] = u;
19             dfs(v);
20         }
21     }
22 }
23
24 int fs[N], mins[N];
25
26 int dom[N], sem[N];
27

```



```

28 int find(int u) {
29     if (u != fs[u]) {
30         int v = fs[u];
31         fs[u] = find(fs[u]);
32         if (mins[v] != -1 && num[sem[mins[v]]] < num[sem[mins[u]]]) {
33             mins[u] = mins[v];
34         }
35     }
36     return fs[u];
37 }
38
39 void merge(int u, int v) {
40     fs[u] = v;
41 }
42
43 vector<int> buf[N];
44
45 int buf2[N];
46
47 void mark(int source) {
48     ord.clear();
49     ++stamp;
50     dfs(source);
51     for (int i = 0; i < (int)ord.size(); ++i) {
52         int u = ord[i];
53         fs[u] = u;
54         mins[u] = -1;
55         buf2[u] = -1;
56     }
57     for (int i = (int)ord.size() - 1; i > 0; --i) {
58         int u = ord[i], p = fa[u];
59         sem[u] = p;
60         for (int j = 0; j < (int)prec[u].size(); ++j) {
61             int v = prec[u][j];
62             if (use[v] != stamp) {
63                 continue;
64             }
65             if (num[v] > num[u]) {
66                 find(v);
67                 v = sem[mins[v]];
68             }
69             if (num[v] < num[sem[u]]) {
70                 sem[u] = v;
71             }
72         }

```

```

73         buf[sem[u]].push_back(u);
74         mins[u] = u;
75         merge(u, p);
76         while (buf[p].size()) {
77             int v = buf[p].back();
78             buf[p].pop_back();
79             find(v);
80             if (sem[v] == sem[mins[v]]) {
81                 dom[v] = sem[v];
82             } else {
83                 buf2[v] = mins[v];
84             }
85         }
86     }
87     dom[ord[0]] = ord[0];
88     for (int i = 0; i < (int)ord.size(); ++i) {
89         int u = ord[i];
90         if (~buf2[u]) {
91             dom[u] = dom[buf2[u]];
92         }
93     }
94 }

```

## DancingLinks

```

1 struct node{
2     node *left,*right,*up,*down,*col; int row,cnt;
3 }*head,*col[MAXC],Node[MAXNODE],*ans[MAXNODE];
4 int totNode;
5 void insert(const std::vector<int> &V,int rownum){
6     std::vector<node*> N;
7     for(int i=0;i<(int)V.size();++i){
8         node* now=Node+(totNode++); now->row=rownum;
9         now->col=now->up=col[V[i]], now->down=col[V[i]]->down;
10        now->up->down=now, now->down->up=now;
11        now->col->cnt++; N.push_back(now);
12    }
13    for(int i=0;i<(int)V.size();++i)
14        N[i]->right=N[(i+1)%V.size()], N[i]->left=N[(i-1+V.size())%V.size()];
15 }
16 void Remove(node *x){
17     x->left->right=x->right, x->right->left=x->left;
18     for(node *i=x->down;i!=x;i=i->down)
19         for(node *j=i->right;j!=i;j=j->right)
20             j->up->down=j->down, j->down->up=j->up, --(j->col->cnt);
21 }

```

```

22 void Resume(node *x){
23     for(node *i=x->up;i!=x;i=i->up)
24         for(node *j=i->left;j!=i;j=j->left)
25             j->up->down=j->down->up=j, ++(j->col->cnt);
26     x->left->right=x, x->right->left=x;
27 }
28 bool search(int tot){
29     if(head->right==head) return true;
30     node *choose=NULL;
31     for(node *i=head->right;i!=head;i=i->right){
32         if(choose==NULL||choose->cnt>i->cnt) choose=i;
33         if(choose->cnt<2) break;
34     }
35     Remove(choose);
36     for(node *i=choose->down;i!=choose;i=i->down){
37         for(node *j=i->right;j!=i;j=j->right) Remove(j->col);
38         ans[tot]=i;
39         if(search(tot+1)) return true;
40         ans[tot]=NULL;
41         for(node *j=i->left;j!=i;j=j->left) Resume(j->col);
42     }
43     Resume(choose);
44     return false;
45 }
46 void prepare(int totC){
47     head=Node+totC;
48     for(int i=0;i<totC;++i) col[i]=Node+i;
49     totNode=totC+1;
50     for(int i=0;i<totC;++i){
51         (Node+i)->right=Node+(i+1)%(totC+1);
52         (Node+i)->left=Node+(i+totC)%(totC+1);
53         (Node+i)->up=(Node+i)->down=Node+i;
54     }
55 }

```

### 弦图相关

1. 团数  $\leq$  色数，弦图团数 = 色数
2. 设  $next(v)$  表示  $N(v)$  中最前的点。令  $w^*$  表示所有满足  $A \in B$  的  $w$  中最后的一个点，判断  $v \cup N(v)$  是否为极大团，只需判断是否存在一个  $w$ ，满足  $Next(w) = v$  且  $|N(v)| + 1 \leq |N(w)|$  即可。
3. 最小染色：完美消除序列从后往前依次给每个点染色，给每个点染上可以染的最小的颜色
4. 最大独立集：完美消除序列从前往后能选就选
5. 弦图最大独立集数 = 最小团覆盖数，最小团覆盖：设最大独立集为  $\{p_1, p_2, \dots, p_t\}$ ，则  $\{p_1 \cup N(p_1), \dots, p_t \cup N(p_t)\}$  为最小团覆盖

### 图同构 Hash

$$F_t(i) = (F_{t-1}(i) \times A + \sum_{i \rightarrow j} F_{t-1}(j) \times B + \sum_{j \rightarrow i} F_{t-1}(j) \times C + D \times (i = a)) \bmod P$$

枚举点  $a$  迭代  $K$  次后求得的就是  $a$  点所对应的 hash 值

其中  $K, A, B, C, D, P$  为 hash 参数，可自选

直线下方有多少个格点

```

1 LL solve(LL n,LL a,LL b,LL m){
2     // 计算 for (int i=0;i<n;++i) s+=floor((a+b*i)/m)
3     //n,m,a,b>0
4     if(b==0) return n*(a/m);
5     if(a>=m) return n*(a/m)+solve(n,a%m,b,m);
6     if(b>=m) return (n-1)*n/2*(b/m)+solve(n,a,b%m,m);
7     return solve((a+b*n)/m,(a+b*n)%m,m,b);
8 }

```

### 费用流

```

1 // Q is a priority_queue<PII, vector<PII>, greater<PII> >
2 // for an edge(s, t): u is the capacity, v is the cost, nxt is the next edge,
3 // op is the opposite edge
4 // this code can not deal with negative cycles
5 typedef pair<int,int> PII;
6 struct edge{ int t,u,v; edge *nxt,*op; }E[MAXE],*V[MAXV];
7 int D[MAXN], dist[MAXN], maxflow, mincost; bool in[MAXN];
8 bool modlabel(){
9     while(!Q.empty()) Q.pop();
10    for(int i=S;i<=T;++i){
11        if(in[i]) D[i]=0,Q.push(PII(0,i));
12        else D[i]=inf;
13    }
14    while(!Q.empty()){
15        int x=Q.top().first,y=Q.top().second;
16        Q.pop();
17        if(y==T) break;
18        if(D[y]<x) continue;
19        for(edge *ii=V[y];ii;ii=ii->nxt) if(ii->u){
20            if(x+(ii->v+dist[ii->t]-dist[y])<D[ii->t]){
21                D[ii->t]=x+(ii->v+dist[ii->t]-dist[y]);
22                Q.push(PII(D[ii->t],ii->t));
23            }
24        }
25    }
26    if(D[T]==inf) return false;

```

```

27  for(int i=S;i<=T;++i) if(D[i]>D[T]) dist[i]+=D[T]-D[i];
28  return true;
29  }
30  int aug(int p,int limit){
31  if(p==T) return maxflow+=limit,mincost+=limit*dist[S],limit;
32  in[p]=1; int kk,ll=limit;
33  for(edge *ii=V[p];ii;ii=ii->nxt) if(ii->u){
34      if(!in[ii->t]&&dist[ii->t]+ii->v==dist[p]){
35          kk=aug(ii->t,min(ii->u,ll));
36          ll-=kk,ii->u-=kk,ii->op->u+=kk;
37          if(!ll) return in[p]=0,limit;
38      }
39  }
40  return limit-ll;
41  }
42  PII mincostFlow(){
43  for(int i=S;i<=T;++i) dist[i]=i==T?inf:0;
44  while(!Q.empty()) Q.pop();
45  Q.push(PII(0,T));
46  while(!Q.empty()){
47      int x=Q.top().first,y=Q.top().second;
48      Q.pop();
49      if(dist[y]<x) continue;
50      for(edge *ii=V[y];ii;ii=ii->nxt) if(ii->op->u&&ii->v+x<dist[ii->t){
51          dist[ii->t]=ii->v+x;
52          Q.push(PII(dist[ii->t],ii->t));
53      }
54  }
55  maxflow=mincost=0;
56  do{
57      do{
58          memset(in,0,sizeof(in));
59      }while(aug(S,maxflow));

```

```

60  }while(modlabel());
61  return PII(maxflow,mincost);
62  }

```

### 综合

定理 1: 最小覆盖数 = 最大匹配数

定理 2: 最大独立集  $S$  与 最小覆盖集  $T$  互补

算法:

1. 做最大匹配，没有匹配的空闲点  $\in S$
2. 如果  $u \in S$  那么  $u$  的临点必然属于  $T$
3. 如果一对匹配的点中有一个属于  $T$  那么另外一个属于  $S$
4. 还不能确定的，把左子图的放入  $S$ ，右子图放入  $T$

### 算法结束

上下界无源汇可行流：不用添  $T \rightarrow S$ ，判断是否流量平衡

上下界有源汇可行流：添  $T \rightarrow S$  (下界 0, 上界  $\infty$ )，判断是否流量平衡

上下界最小流：不添  $T \rightarrow S$  先流一遍，再添  $T \rightarrow S$  (下界 0, 上界  $\infty$ ) 在残图上流一遍，答案为  $S \rightarrow T$  的流量值

上下界最大流：添  $T \rightarrow S$  (下界 0, 上界  $\infty$ ) 流一遍，再在残图上流一遍  $S$  到  $T$  的最大流，答案为前者的  $S \rightarrow T$  的值 + 残图中  $S \rightarrow T$  的最大流

Stirling 公式  $n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$

### Stirling 数

第一类： $n$  个元素的项目分作  $k$  个环排列的方法数目

$$s(n, k) = (-1)^{n+k} |s(n, k)|$$

$$|s(n, 0)| = 0, |s(1, 1)| = 1,$$

$$|s(n, k)| = |s(n-1, k-1)| + (n-1) * |s(n-1, k)|$$

第二类： $n$  个元素的集定义  $k$  个等价类的方法数

$$S(n, 1) = S(n, n) = 1, S(n, k) = S(n-1, k-1) + k * S(n-1, k)$$

积分表

<b>Integrals of Rational Functions</b>	$\int \frac{1}{1+x^2} dx = \tan^{-1} x$	$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$	$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln  a^2 + x^2 $	$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a}$	$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln  a^2 + x^2 $
$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}}$	$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, a \neq b$	$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln  a+x $	$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln  ax^2 + bx + c  - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}}$		
<b>Integrals with Roots</b>	$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3} (x \mp 2a) \sqrt{x \pm a}$	$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a}$	$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a \ln [\sqrt{x} + \sqrt{x+a}]$	$\int x \sqrt{x^2 \pm a^2} dx = \frac{1}{3} (x^2 \pm a^2)^{3/2}$	
$\int x \sqrt{ax+b} dx = \frac{2}{15a^2} (-2b^2 + abx + 3a^2 x^2) \sqrt{ax+b}$	$\int \sqrt{x(ax+b)} dx = \frac{1}{4a^{3/2}} \left[ (2ax+b) \sqrt{ax(ax+b)} - b^2 \ln \left  a\sqrt{x} + \sqrt{a(ax+b)} \right  \right]$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln \left  x + \sqrt{x^2 \pm a^2} \right $			
$\int \sqrt{x^3(ax+b)} dx = \left[ \frac{b}{12a} - \frac{b^2}{8a^2 x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln \left  a\sqrt{x} + \sqrt{a(ax+b)} \right $	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$	$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{1}{2} a^2 \ln \left  x + \sqrt{x^2 \pm a^2} \right $			
$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left  x + \sqrt{x^2 \pm a^2} \right $	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}$	$\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2}$	$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$	$\int \sqrt{ax^2 + bx + c} dx = \frac{b+2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac-b^2}{8a^{3/2}} \ln \left  2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right $	
$\int x \sqrt{ax^2 + bx + c} dx = \frac{1}{48a^{5/2}} \left( 2\sqrt{a} \sqrt{ax^2 + bx + c} \times (-3b^2 + 2abx + 8a(c + ax^2)) + 3(b^3 - 4abc) \ln \left  b + 2ax + 2\sqrt{a} \sqrt{ax^2 + bx + c} \right  \right)$	$\int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{\sqrt{a}} \ln \left  2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right $				
$\int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{a} \sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}} \ln \left  2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right $	$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2+x^2}}$	<b>Integrals with Logarithms</b>	$\int \ln(ax+b) dx = \left( x + \frac{b}{a} \right) \ln(ax+b) - x, a \neq 0$		
$\int \frac{\ln x}{x} dx = \frac{1}{2} (\ln x)^2$	$\int \ln(x^2 + a^2) dx = x \ln(x^2 + a^2) + 2a \tan^{-1} \frac{x}{a} - 2x$	$\int \ln(x^2 - a^2) dx = x \ln(x^2 - a^2) + a \ln \frac{x+a}{x-a} - 2x$	$\int x \ln(ax+b) dx = \frac{bx}{2a} - \frac{1}{4} x^2 + \frac{1}{2} \left( x^2 - \frac{b^2}{a^2} \right) \ln(ax+b)$		
$\int \ln(ax^2 + bx + c) dx = \frac{1}{a} \sqrt{4ac - b^2} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left( \frac{b}{2a} + x \right) \ln(ax^2 + bx + c)$	$\int x \ln(a^2 - b^2 x^2) dx = -\frac{1}{2} x^2 + \frac{1}{2} \left( x^2 - \frac{a^2}{b^2} \right) \ln(a^2 - b^2 x^2)$	<b>Integrals with Exponentials</b>			
$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$	$\int x e^{-ax^2} dx = -\frac{1}{2a} e^{-ax^2}$	<b>Integrals with Trigonometric Functions</b>	$\int \sin^3 ax dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a}$	$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$	
$\int \cos^3 ax dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a}$	$\int \cos ax \sin bxdx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b$	$\int \sin^2 ax \cos bxdx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$	$\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x$		
$\int \cos^2 ax \sin bxdx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$	$\int \cos^2 ax \sin ax dx = -\frac{1}{3a} \cos^3 ax$	$\int \sin^2 ax \cos^2 bxdx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$	$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$		
$\int \tan ax dx = -\frac{1}{a} \ln \cos ax$	$\int \tan^2 ax dx = -x + \frac{1}{a} \tan ax$	$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax$	$\int \sec x dx = \ln  \sec x + \tan x  = 2 \tanh^{-1} \left( \tan \frac{x}{2} \right)$	$\int \sec^2 ax dx = \frac{1}{a} \tan ax$	
$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln  \sec x + \tan x $	$\int \sec x \tan x dx = \sec x$	$\int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x$	$\int \sec^n x \tan x dx = \frac{1}{n} \sec^n x, n \neq 0$	$\int \csc x dx = \ln \left  \tan \frac{x}{2} \right  = \ln  \csc x - \cot x  + C$	
$\int \csc^2 ax dx = -\frac{1}{a} \cot ax$	$\int \csc^3 x dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln  \csc x - \cot x $	$\int \csc^n x \cot x dx = -\frac{1}{n} \csc^n x, n \neq 0$	$\int \sec x \csc x dx = \ln  \tan x $	<b>Products of Trigonometric Functions and Monomials</b>	
$\int x \cos x dx = \cos x + x \sin x$	$\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$	$\int x^2 \cos x dx = 2x \cos x + (x^2 - 2) \sin x$	$\int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax$	$\int x \sin x dx = -x \cos x + \sin x$	
$\int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2}$	$\int x^2 \sin x dx = (2 - x^2) \cos x + 2x \sin x$	$\int x^2 \sin ax dx = \frac{2-a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2}$	<b>Products of Trigonometric Functions and Exponentials</b>		
$\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x)$	$\int e^{bx} \sin ax dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax)$	$\int e^{bx} \cos ax dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax)$	$\int x e^x \sin x dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x)$		
$\int x e^x \cos x dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x)$	$\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x)$				

Java

```

1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4
5 public class Main{
6     BufferedReader reader = new BufferedReader(new InputStreamReader(System.in));
7     PrintWriter writer = new PrintWriter(System.out);
8     StringTokenizer tokenizer = null;
9
10    void solve() throws Exception {
11    }
12    void run()throws Exception{
13        try{

```

```

14        while (true) {
15            solve();
16        }
17    }
18    catch(Exception e){
19    }
20    finally{
21        reader.close();
22        writer.close();
23    }
24    }
25    String next()throws Exception{
26        for(;tokenizer == null || !tokenizer.hasMoreTokens();){
27            tokenizer = new StringTokenizer(reader.readLine());

```

```
28     }
29     return tokenizer.nextToken();
30 }
31 int nextInt()throws Exception{
32     return Integer.parseInt(next());
33 }
34 double nextDouble()throws Exception{
35     return Double.parseDouble(next());
36 }
37 BigInteger nextBigInteger()throws Exception{
38     return new BigInteger(next());
39 }
40 public static void main(String args[])throws Exception{
41     (new Main()).run();
42 }
43 }
```

## Vimrc

```
1 \begin{lstlisting}
2 set nu ai ci si mouse=a ts=4 sts=4 sw=4
3
4 nmap<C-A> ggVG
5 vmap<C-C> "+y
6
7 nmap<F3> : vs %<.in <CR>
8 nmap<F5> : !./%< <CR>
9 nmap<F8> : !./%< <%<.in <CR>
10 nmap<F9> : !g++ % -o %< -Wall <CR>
11
12 "nmap<F4> : !gedit % <CR>
13 "autocmd BufNewFile *.cpp 0r ~/temp.cpp
14 "set hlsearch incsearch
15
16 "syntax on
17 "filetype plugin indent on
18 \end{lstlisting}
```