# Dracarys

Team Referrence Library

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上海交通大学 Shanghai Jiao Tong University			Page 1
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#### 多边形与圆面积交

```
double areaCT(Point pa, Point pb, double r) {
      if (pa.len() < pb.len()) swap(pa, pb);</pre>
     if (sign(pb.len()) == 0) return 0;
 3
      double a = pb.len(), b = pa.len(), c = (pb - pa).len();
      double sinB = fabs(det(pb, pb - pa) / a / c),
 5
 6
          cosB = dot(pb, pb - pa) / a / c,
          sinC = fabs(det(pa, pb) / a/b),
          cosC = dot(pa, pb) / a / b;
      double B = atan2(sinB, cosB), C = atan2(sinC, cosC);
 9
      if (a > r) {
10
       S = C / 2 * r * r;
11
12
       h = a * b * sinC / c;
       if (h < r && B < PI / 2) {
13
         S = (acos(h / r) * r * r - h * sqrt(r * r - h * h));
14
15
16
     } else if (b > r) {
       double theta = PI - B - asin(sinB / r * a);
17
18
       S = a * r * sin(theta) / 2 + (C - theta) / 2 * r * r;
19
20
       S = sinC * a * b / 2:
21
     return S:
22
23 | }
```

## 二维几何

```
struct Point {
     Point rotate(const double ang) { // 逆时针旋转 ang 弧度
       return Point(cos(ang) * x - sin(ang) * y, cos(ang) * y + sin(ang) * x);
 3
 4
     Point turn90() { // 逆时针旋转 90 度
 6
       return Point(-y, x);
    }
 8 };
 9 | Point isLL(const Line &11, const Line &12) {
     double s1 = det(12.b - 12.a, 11.a - 12.a).
10
          s2 = -det(12.b - 12.a, 11.b - 12.a):
11
     return (11.a * s2 + 11.b * s1) / (s1 + s2);
12
13 | }
   bool onSeg(const Line &1, const Point &p) { // 点在线段上
     return sign(det(p - 1.a, 1.b - 1.a)) == 0 \&\& sign(dot(p - 1.a, p - 1.b)) <= 0;
15
16 }
17 Point projection(const Line &1, const Point &p) { // 点到直线投影
     return 1.a + (1.b - 1.a) * (dot(p - 1.a, 1.b - 1.a) / (1.b - 1.a).len2());
19 }
```

```
20 double disToLine(const Line &l. const Point &p) {
     return abs(det(p - 1.a, 1.b - 1.a) / (1.b - 1.a).len());
22
   double disToSeg(const Line &l, const Point &p) { // 点到线段距离
     return sign(dot(p - 1.a, 1.b - 1.a)) * sign(dot(p - 1.b, 1.a - 1.b)) != 1 ?
24
       disToLine(1, p) : min((p - 1.a).len(), (p - 1.b).len());
25
26
   Point symmetryPoint(const Point a, const Point b) { // 点 b 关于点 a 的中心对称点
28
     return a + a - b;
29
   Point reflection(const Line &1, const Point &p) { // 点关于直线的对称点
30
     return symmetryPoint(projection(1, p), p);
31
32
   // 求圆与直线的交点
33
   bool isCL(Circle a, Line 1, Point &p1, Point &p2) {
     double x = dot(1.a - a.o, 1.b - 1.a),
36
      y = (1.b - 1.a).len2(),
      d = x * x - y * ((1.a - a.o).len2() - a.r * a.r);
37
     if (sign(d) < 0) return false;</pre>
38
     d = max(d, 0.0);
39
     Point p = 1.a - ((1.b - 1.a) * (x / y)), delta = (1.b - 1.a) * (sqrt(d) / y);
     p1 = p + delta, p2 = p - delta;
41
     return true:
42
43
    // 求圆与圆的交面积
    double areaCC(const Circle &c1, const Circle &c2) {
45
     double d = (c1.o - c2.o).len();
46
     if (sign(d - (c1.r + c2.r)) >= 0) {
47
48
       return 0;
49
     if (sign(d - abs(c1.r - c2.r)) \le 0) {
50
       double r = min(c1.r, c2.r);
51
       return r * r * PI;
52
53
     double x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d).
54
          t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
55
56
     return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1):
57
    // 求圆与圆的交点,注意调用前要先判定重圆
   bool isCC(Circle a, Circle b, Point &p1, Point &p2) {
59
     double s1 = (a.o - b.o).len():
61
     if (sign(s1 - a.r - b.r) > 0 \mid | sign(s1 - abs(a.r - b.r)) < 0) return false;
     double s2 = (a.r * a.r - b.r * b.r) / s1:
63
     double aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
     Point o = (b.o - a.o) * (aa / (aa + bb)) + a.o:
```

```
Point delta = (b.o - a.o).unit().turn90() * newSgrt(a.r * a.r - aa * aa):
 66
      p1 = o + delta, p2 = o - delta;
 67
      return true:
 68 }
    // 求点到圆的切点,按关于点的顺时针方向返回两个点
    bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
      double x = (p0 - c.o).len2(), d = x - c.r * c.r:
      if (d < EPS) return false: // 点在圆上认为没有切点
 72
      Point p = (p0 - c.o) * (c.r * c.r / x);
 73
      Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
 74
      p1 = c.o + p + delta;
 75
 76
      p2 = c.o + p - delta;
 77
      return true;
 78
    // 求圆到圆的外共切线, 按关于 c1.o 的顺时针方向返回两条线
 80
    vector<Line> extanCC(const Circle &c1, const Circle &c2) {
 81
      vector<Line> ret;
 82
      if (sign(c1.r - c2.r) == 0) {
 83
        Point dir = c2.o - c1.o:
 84
        dir = (dir * (c1.r / dir.len())).turn90();
 85
        ret.push_back(Line(c1.o + dir, c2.o + dir));
 86
        ret.push_back(Line(c1.o - dir, c2.o - dir));
 87
      } else {
 88
        Point p = (c1.0 * -c2.r + c2.o * c1.r) / (c1.r - c2.r);
 89
        Point p1, p2, q1, q2;
        if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
 90
          if (c1.r < c2.r) swap(p1, p2), swap(q1, q2);</pre>
 91
          ret.push_back(Line(p1, q1));
 92
          ret.push_back(Line(p2, q2));
 93
        }
 94
 95
 96
      return ret;
97
    // 求圆到圆的内共切线, 按关于 c1.o 的顺时针方向返回两条线
    vector<Line> intanCC(const Circle &c1, const Circle &c2) {
 99
100
      vector<Line> ret:
      Point p = (c1.0 * c2.r + c2.o * c1.r) / (c1.r + c2.r);
101
      Point p1, p2, q1, q2;
102
      if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
103
        ret.push_back(Line(p1, q1));
104
105
        ret.push_back(Line(p2, q2));
106
      return ret:
107
108 }
    bool contain(vector<Point> polygon, Point p) { // 判断点 p
       → 是否被多边形包含,包括落在边界上
```

```
int ret = 0. n = polygon.size();
110
111
       for(int i = 0; i < n; ++ i) {
112
        Point u = polygon[i], v = polygon[(i + 1) % n];
113
        if (onSeg(Line(u, v), p)) return true;
        if (sign(u.y - v.y) \le 0) swap(u, v);
114
        if (sign(p.y - u.y) > 0 \mid | sign(p.y - v.y) \le 0) continue;
115
116
        ret += sign(det(p, v, u)) > 0;
117
118
      return ret & 1;
119
    vector<Point> convexCut(const vector<Point> Line 1) { // 用半平面 (q1,q2)
120
       → 的逆时针方向去切凸多边形
      vector<Point> qs;
121
      int n = ps.size();
122
123
      for (int i = 0; i < n; ++i) {
        Point p1 = ps[i], p2 = ps[(i + 1) % n];
124
        int d1 = sign(det(l.a, l.b, p1)), d2 = sign(det(l.a, l.b, p2));
125
        if (d1 >= 0) qs.push_back(p1);
126
127
        if (d1 * d2 < 0) qs.push_back(isLL(Line(p1, p2), 1));</pre>
128
      return qs;
129
130
     vector<Point> convexHull(vector<Point> ps) { // 求点集 ps 组成的凸包
131
      int n = ps.size(); if (n <= 1) return ps;</pre>
132
      sort(ps.begin(), ps.end());
133
      vector<Point> as:
134
      for (int i = 0; i < n; qs.push_back(ps[i++]))</pre>
135
136
        while (qs.size() > 1 \& sign(det(qs[qs.size()-2],qs.back(),ps[i])) \le 0)
       for (int i = n - 2, t = qs.size(); i \ge 0; qs.push_back(ps[i--]))
137
138
        while ((int)qs.size() > t && sign(det(qs[(int)qs.size()-2],qs.back(),ps[i])) <= 0)</pre>
       139
      qs.pop_back(); return qs;
140
```

# $n \log n$ 半平面交

```
struct Point {

int quad() const { return sign(y) == 1 || (sign(y) == 0 && sign(x) >= 0);}

};

struct Line {

bool include(const Point &p) const { return sign(det(b - a, p - a)) > 0; }

Line push() const{ // 将半平面向外推 eps

const double eps = 1e-6;

Point delta = (b - a).turn90().norm() * eps;
```

```
return Line(a - delta, b - delta):
     }
10
11 }:
12 bool sameDir(const Line &10, const Line &11) { return parallel(10, 11) && sign(dot(10.b
      \hookrightarrow - 10.a, 11.b - 11.a)) == 1; }
13 | bool operator < (const Point &a, const Point &b) {
      if (a.quad() != b.quad()) {
14
       return a.quad() < b.quad();</pre>
15
16
     } else {
17
        return sign(det(a, b)) > 0;
18
19
   bool operator < (const Line &10, const Line &11) {
20
      if (sameDir(10, 11)) {
21
       return 11.include(10.a);
22
     } else {
23
        return (10.b - 10.a) < (11.b - 11.a);
24
25
26 }
   bool check(const Line &u, const Line &v, const Line &w) { return w.include(intersect(u,
      \rightarrow v)): }
28
   vector<Point> intersection(vector<Line> &1) {
      sort(l.begin(), l.end());
29
      deque<Line> q;
30
      for (int i = 0; i < (int)1.size(); ++i) {</pre>
31
        if (i && sameDir(l[i], l[i - 1])) {
32
          continue;
33
       }
34
        while (q.size() > 1 && !check(q[q.size() - 2], q[q.size() - 1], l[i])) q.pop_back();
35
36
        while (q.size() > 1 && !check(q[1], q[0], l[i])) q.pop_front();
        q.push_back(l[i]);
37
38
      while (q.size() > 2 \&\& !check(q[q.size() - 2], q[q.size() - 1], q[0])) q.pop_back();
39
      while (q.size() > 2 \&\& !check(q[1], q[0], q[q.size() - 1])) q.pop_front();
40
      vector<Point> ret:
41
42
      for (int i = 0; i < (int)q.size(); ++i) ret.push_back(intersect(q[i], q[(i + 1) %
      \hookrightarrow q.size()]));
      return ret;
43
44 | }
```

# Delaunay 三角剖分

```
1/*2Delaunay Triangulation 随机增量算法:3节点数至少为点数的 6 倍,空间消耗较大注意计算内存使用4建图的过程在 build 中,注意初始化内存池和初始三角形的坐标范围 (Triangulation::LOTS)
```

```
5 Triangulation::find 返回包含某点的三角形
6 Triangulation::add_point 将某点加入三角剖分
   某个 Triangle 在三角剖分中当且仅当它的 has_children 为 0
8 如果要找到三角形 u 的邻域,则枚举它的所有 u.edge[i].tri, 该条边的两个点为 u.p[(i+1)%3],
      \hookrightarrow u.p[(i+2)\%3]
9
10
   const int N = 100000 + 5. MAX TRIS = N * 6:
   const double EPSILON = 1e-6. PI = acos(-1.0):
   struct Point {
     double x,y; Point():x(0),y(0){} Point(double x, double y):x(x),y(y){}
13
     bool operator ==(Point const& that)const {return x==that.x&&y==that.y;}
14
15
16
   inline double sqr(double x) { return x*x; }
   double dist_sqr(Point const& a, Point const& b){return sqr(a.x-b.x)+sqr(a.y-b.y);}
17
   bool in_circumcircle(Point const& p1, Point const& p2, Point const& p3, Point const& p4)
      - → {
     double u11 = p1.x - p4.x, u21 = p2.x - p4.x, u31 = p3.x - p4.x;
19
     double u12 = p1.y - p4.y, u22 = p2.y - p4.y, u32 = p3.y - p4.y;
20
     double u13 = sqr(p1.x) - sqr(p4.x) + sqr(p1.y) - sqr(p4.y);
21
22
     double u23 = sqr(p2.x) - sqr(p4.x) + sqr(p2.y) - sqr(p4.y);
     double u33 = sqr(p3.x) - sqr(p4.x) + sqr(p3.y) - sqr(p4.y);
23
     double det = -u13*u22*u31 + u12*u23*u31 + u13*u21*u32 - u11*u23*u32 - u12*u21*u33 +
24
      return det > EPSILON;
25
26
   double side(Point const& a, Point const& b, Point const& p) { return (b.x-a.x)*(p.y-a.y)
27
      \hookrightarrow - (b.y-a.y)*(p.x-a.x);}
   typedef int SideRef; struct Triangle; typedef Triangle* TriangleRef;
   struct Edge {
     TriangleRef tri; SideRef side; Edge() : tri(0), side(0) {}
30
31
     Edge(TriangleRef tri, SideRef side) : tri(tri), side(side) {}
32
   struct Triangle {
     Point p[3]; Edge edge[3]; TriangleRef children[3]; Triangle() {}
34
     Triangle(Point const& p0, Point const& p1, Point const& p2) {
35
36
       p[0]=p0;p[1]=p1;p[2]=p2;children[0]=children[1]=children[2]=0;
     }
37
38
     bool has_children() const { return children[0] != 0; }
     int num_children() const {
39
       return children[0] == 0 ? 0
40
41
         : children[1] == 0 ? 1
         : children[2] == 0 ? 2 : 3;
42
43
     bool contains(Point const& q) const {
44
       double a=side(p[0],p[1],q), b=side(p[1],p[2],q), c=side(p[2],p[0],q);
45
```

```
46
        return a >= -EPSILON && b >= -EPSILON && c >= -EPSILON:
     }
47
48
   } triange_pool[MAX_TRIS], *tot_triangles;
49
   void set_edge(Edge a, Edge b) {
     if (a.tri) a.tri->edge[a.side] = b;
50
     if (b.tri) b.tri->edge[b.side] = a;
51
52 }
   class Triangulation {
53
     public:
54
       Triangulation() {
55
56
         const double LOTS = 1e6;
         the_root = new(tot_triangles++)
57
      → Triangle(Point(-LOTS,-LOTS),Point(+LOTS,-LOTS),Point(0,+LOTS));
58
       TriangleRef find(Point p) const { return find(the_root,p); }
59
60
        void add_point(Point const& p) { add_point(find(the_root,p),p); }
61
      private:
62
        TriangleRef the_root;
63
        static TriangleRef find(TriangleRef root, Point const& p) {
64
         for(;;) {
65
            if (!root->has children()) return root;
66
            else for (int i = 0; i < 3 && root->children[i] ; ++i)
67
                if (root->children[i]->contains(p))
68
                  {root = root->children[i]; break;}
69
         }
70
        void add_point(TriangleRef root, Point const& p) {
71
72
         TriangleRef tab, tbc, tca;
          tab = new(tot_triangles++) Triangle(root->p[0], root->p[1], p);
73
          tbc = new(tot_triangles++) Triangle(root->p[1], root->p[2], p);
74
          tca = new(tot_triangles++) Triangle(root->p[2], root->p[0], p);
75
76
          set_edge(Edge(tab,0),Edge(tbc,1));set_edge(Edge(tbc,0),Edge(tca,1));
          set_edge(Edge(tca,0), Edge(tab,1)); set_edge(Edge(tab,2), root->edge[2]);
77
78
          set_edge(Edge(tbc,2),root->edge[0]);set_edge(Edge(tca,2),root->edge[1]);
          root->children[0] = tab; root->children[1] = tbc; root->children[2] = tca;
79
80
         flip(tab,2); flip(tbc,2); flip(tca,2);
81
       }
82
        void flip(TriangleRef tri, SideRef pi) {
83
         TriangleRef trj = tri->edge[pi].tri; int pj = tri->edge[pi].side;
84
          if(!trj||!in_circumcircle(tri->p[0],tri->p[1],tri->p[2],trj->p[pj])) return;
         TriangleRef trk = new(tot_triangles++) Triangle(tri->p[(pi+1)%3], trj->p[pj],
85

    tri->p[pi]);
86
         TriangleRef trl = new(tot_triangles++) Triangle(trj->p[(pj+1)%3], tri->p[pi],
      \hookrightarrow \text{trj->p[pj])};
87
          set_edge(Edge(trk,0), Edge(trl,0));
```

```
88
           set_edge(Edge(trk,1), tri->edge[(pi+2)%3]); set_edge(Edge(trk,2),
        \hookrightarrow \text{trj->edge}[(pj+1)\%3]);
 89
           set_edge(Edge(trl,1), trj->edge[(pj+2)%3]); set_edge(Edge(trl,2),
        \hookrightarrow tri->edge[(pi+1)%3]);
           tri->children[0] = trk; tri->children[1] = trl; tri->children[2] = 0;
 90
           trj->children[0] = trk; trj->children[1] = trl; trj->children[2] = 0;
 91
           flip(trk,1); flip(trk,2); flip(trl,1); flip(trl,2);
 92
 93
         }
 94
 95
     int n; Point ps[N];
     void build(){
       tot_triangles = triange_pool; cin >> n;
 97
       for(int i = 0; i < n; ++ i) scanf("%lf%lf",&ps[i].x,&ps[i].y);
 98
       random_shuffle(ps, ps + n); Triangulation tri;
 99
100
       for(int i = 0; i < n; ++ i) tri.add_point(ps[i]);</pre>
101
```

#### 三维几何操作合并

```
struct Point3D {
    double x, y, z;
3 };
  Point3D det(const Point3D &a, const Point3D &b) {
    return Point3D(a.v * b.z - a.z * b.y, a.z * b.x - a.x * b.z, a.x * b.y - a.y * b.x);
6
  // 平面法向量 : 平面上两个向量叉积
8 // 点共平面 : 平面上一点与之的向量点积法向量为 0
  // 点在线段 ( 直线 ) 上 : 共线且两边点积非正
  // 点在三角形内 ( 不包含边界, 需再判断是与某条边共线 )
  bool pointInTri(const Point3D &a, const Point3D &b, const Point3D &c, const Point3D &p)
12
    return sign(det(a - b, a - c).len() - det(p - a, p - b).len() - det(p - b, p -
     \hookrightarrow c).len() - det(p - c, p - a).len()) == 0;
13 }
   // 共平面的两点是否在这平面上一条直线的同侧
  bool sameSide(const Point3D &a, const Point3D &b, const Point3D &p0, const Point3D &p1)
15
16
    return sign(dot(det(a - b, p0 - b), det(a - b, p1 - b))) > 0;
17
18
  // 两点在平面同侧 : 点积法向量符号相同
  // 两直线平行 / 垂直 : 同二维
  // 平面平行 / 垂直 : 判断法向量
  // 线面垂直 : 法向量和直线平行
  // 判断空间线段是否相交 : 四点共面两线段不平行相互在异侧
  // 线段和三角形是否相交 : 线段在三角形平面不同侧
     → 三角形任意两点在线段和第三点组成的平面的不同侧
```

```
24 // 求空间直线交点
25 Point3D intersection(const Point3D &a0, const Point3D &b0, const Point3D &a1, const
                      → Point3D &b1) {
                  double t = ((a0.x - a1.x) * (a1.y - b1.y) - (a0.y - a1.y) * (a1.x - b1.x)) / ((a0.x - a1.y) + (a1.y - b1.y)) / (a0.x - a1.y) / (a0.x - a1.y) / (a0.y - a1.y) / (a1.y - b1.y) / (a1.y - b1.y)
26
                     \leftrightarrow b0.x) * (a1.y - b1.y) - (a0.y - b0.y) * (a1.x - b1.x));
                   return a0 + (b0 - a0) * t:
27
28 }
29 // 求平面和直线的交点
30 Point3D intersection(const Point3D &a, const Point3D &b, const Point3D &c, const Point3D
                      Point3D p = pVec(a, b, c); // 平面法向量
31
                   double t = (p.x * (a.x - 10.x) + p.y * (a.y - 10.y) + p.z * (a.z - 10.z)) / (p.x * (a.y -
 32
                     \leftrightarrow (11.x - 10.x) + p.y * (11.y - 10.y) + p.z * (11.z - 10.z));
                  return 10 + (11 - 10) * t;
 33
 34
             // 求平面交线 : 取不平行的一条直线的一个交点,以及法向量叉积得到直线方向
             // 点到直线距离 : 叉积得到三角形的面积除以底边
             // 点到平面距离 : 点积法向量
            // 直线间距离 : 平行时随便取一点求距离,否则叉积方向向量得到方向点积计算长度
            // 直线夹角 : 点积 平面夹角 : 法向量点积
            // 三维向量旋转操作(绕向量 s 旋转 ang 角度), 对于右手系 s 指向观察者时逆时针
41 // 矩阵版
 42 void rotate(const Point3D &s, double ang) {
                   double 1 = s.len(), x = s.x / 1, y = s.y / 1, z = s.z / 1, sinA = sin(ang), cosA = sin(ang)
                     double p[4][4] = \{ \cos A + (1 - \cos A) * x * x . (1 - \cos A) * x * y - \sin A * z . (1 - \cos A) \}
44
                     \rightarrow * x * z + SinA * y, 0,
                          (1 - \cos A) * y * x + \sin A * z, \cos A + (1 - \cos A) * y * y, (1 - \cos A) * y * z - \sin A
45
                      \rightarrow * x, 0,
 46
                          (1 - \cos A) * z * x - \sin A * y, (1 - \cos A) * z * y + \sin A * x, \cos A + (1 - \cos A) * z
                      \hookrightarrow * z, 0,
                          0, 0, 0, 1 };
47
48 }
             // 计算版 : 把需要旋转的向量按照 s 分解, 做二维旋转, 再回到三维
```

# 三维凸包

```
1 #define SIZE(X) (int(X.size()))
2 #define PI 3.14159265358979323846264338327950288
3 struct Point {
4    Point cross(const Point &p) const
5    { return Point(y * p.z - z * p.y, z * p.x - x * p.z, x * p.y - y * p.x); }
6 } info[1005];
7 int mark[1005][1005],n, cnt;;
8 double mix(const Point &a, const Point &b, const Point &c)
9 { return a.dot(b.cross(c)); }
```

```
10 double area(int a, int b, int c)
11 | return ((info[b] - info[a]).cross(info[c] - info[a])).length(): }
   double volume(int a, int b, int c, int d)
   { return mix(info[b] - info[a], info[c] - info[a], info[d] - info[a]): }
   struct Face {
14
     int a, b, c; Face() {}
15
16
     Face(int a, int b, int c): a(a), b(b), c(c) {}
    int &operator [](int k)
17
     { if (k == 0) return a; if (k == 1) return b; return c; }
18
19 };
   vector <Face> face:
20
   inline void insert(int a, int b, int c) { face.push_back(Face(a, b, c)); }
   void add(int v) {
23
     vector <Face> tmp; int a, b, c; cnt++;
     for (int i = 0; i < SIZE(face); i++) {</pre>
24
       a = face[i][0]; b = face[i][1]; c = face[i][2];
25
26
       if (Sign(volume(v, a, b, c)) < 0)</pre>
       mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] = mark[c][a] = mark[a][c] = cnt;
27
28
       else tmp.push_back(face[i]);
29
     } face = tmp;
     for (int i = 0; i < SIZE(tmp); i++) {</pre>
30
       a = face[i][0]; b = face[i][1]; c = face[i][2];
31
       if (mark[a][b] == cnt) insert(b, a, v);
       if (mark[b][c] == cnt) insert(c, b, v);
33
       if (mark[c][a] == cnt) insert(a, c, v);
34
   }}
35
   int Find() {
36
     for (int i = 2; i < n; i++) {
37
38
       Point ndir = (info[0] - info[i]).cross(info[1] - info[i]);
       if (ndir == Point()) continue; swap(info[i], info[2]);
39
       for (int j = i + 1; j < n; j++) if (Sign(volume(0, 1, 2, j)) != 0) {
40
         swap(info[i], info[3]); insert(0, 1, 2); insert(0, 2, 1); return 1;
41
   } } return 0; }
   int main() {
43
     for (; scanf("%d", &n) == 1; ) {
44
       for (int i = 0; i < n; i++) info[i].Input();</pre>
45
       sort(info, info + n); n = unique(info, info + n) - info;
       face.clear(); random_shuffle(info, info + n);
47
48
       if (Find()) { memset(mark, 0, sizeof(mark)); cnt = 0;
         for (int i = 3: i < n: i++) add(i): vector<Point> Ndir:
49
50
         for (int i = 0; i < SIZE(face); ++i) {
           Point p = (info[face[i][0]] - info[face[i][1]]).cross(
51
               info[face[i][2]] - info[face[i][1]]);
52
           p = p / p.length(); Ndir.push_back(p);
53
         } sort(Ndir.begin(), Ndir.end());
54
```

```
int ans = unique(Ndir.begin(), Ndir.end()) - Ndir.begin();
55
56
         printf("%d\n", ans);
       } else printf("1\n");
57
58 | } }
59 // 求重心
60 double calcDist(const Point &p, int a, int b, int c)
61 { return fabs(mix(info[a] - p, info[b] - p, info[c] - p) / area(a, b, c)); }
   //compute the minimal distance of center of any faces
   double findDist() { //compute center of mass
64
      double totalWeight = 0; Point center(.0, .0, .0);
65
     Point first = info[face[0][0]];
66
     for (int i = 0; i < SIZE(face); ++i) {</pre>
       Point p = (info[face[i][0]]+info[face[i][1]]+info[face[i][2]]+first)*.25;
67
68
        double weight = mix(info[face[i][0]] - first, info[face[i][1]]
69
            - first, info[face[i][2]] - first);
       totalWeight += weight; center = center + p * weight;
70
     } center = center / totalWeight;
71
      double res = 1e100; //compute distance
72
      for (int i = 0; i < SIZE(face); ++i)</pre>
       res = min(res, calcDist(center, face[i][0], face[i][1], face[i][2]));
74
       return res: }
75
```

## 凸包上快速询问

```
1 /*
     给定凸包, \log n 内完成各种询问, 具体操作有:
2
     1. 判定一个点是否在凸包内
3
     2. 询问凸包外的点到凸包的两个切点
4
     3. 询问一个向量关于凸包的切点
5
6
     4. 询问一条直线和凸包的交点
     INF 为坐标范围,需要定义点类大于号
8
      改成实数只需修改 sign 函数, 以及把 long long 改为 double 即可
9
      构造函数时传入凸包要求无重点,面积非空,以及 pair(x,y) 的最小点放在第一个
10
   const int INF = 1000000000:
11
   struct Convex
12
13 | {
     int n:
14
     vector<Point> a, upper, lower;
15
     Convex(vector<Point> _a) : a(_a) {
16
      n = a.size();
17
18
      int ptr = 0;
      for(int i = 1; i < n; ++ i) if (a[ptr] < a[i]) ptr = i;</pre>
19
      for(int i = 0; i <= ptr; ++ i) lower.push_back(a[i]);</pre>
20
      for(int i = ptr; i < n; ++ i) upper.push_back(a[i]);</pre>
21
      upper.push_back(a[0]);
22
```

```
23
     int sign(long long x) { return x < 0 ? -1 : x > 0 :  }
24
     pair<long long, int> get_tangent(vector<Point> &convex, Point vec) {
25
26
       int l = 0, r = (int)convex.size() - 2;
27
       for(: 1 + 1 < r:) {
28
         int mid = (1 + r) / 2;
         if (sign((convex[mid + 1] - convex[mid]).det(vec)) > 0) r = mid:
29
         else 1 = mid:
30
       }
31
32
       return max(make_pair(vec.det(convex[r]), r), make_pair(vec.det(convex[0]), 0));
33
     void update_tangent(const Point &p, int id, int &i0, int &i1) {
34
       if ((a[i0] - p).det(a[id] - p) > 0) i0 = id;
35
36
       if ((a[i1] - p).det(a[id] - p) < 0) i1 = id;
37
38
     void binary_search(int 1, int r, Point p, int &i0, int &i1) {
       if (1 == r) return;
39
       update_tangent(p, 1 % n, i0, i1);
40
       int sl = sign((a[1 \% n] - p).det(a[(1 + 1) \% n] - p));
41
       for(; 1 + 1 < r; ) {
42
         int mid = (1 + r) / 2:
43
         int smid = sign((a[mid % n] - p).det(a[(mid + 1) % n] - p));
44
         if (smid == sl) l = mid:
45
46
         else r = mid:
47
48
       update_tangent(p, r % n, i0, i1);
49
     int binary_search(Point u, Point v, int 1, int r) {
50
       int sl = sign((v - u).det(a[1 % n] - u));
51
       for(; 1 + 1 < r; ) {
52
         int mid = (1 + r) / 2;
53
         int smid = sign((v - u).det(a[mid % n] - u));
54
         if (smid == sl) l = mid;
55
56
         else r = mid:
       }
57
58
       return 1 % n:
59
     // 判定点是否在凸包内, 在边界返回 true
60
     bool contain(Point p) {
61
62
       if (p.x < lower[0].x || p.x > lower.back().x) return false:
63
       int id = lower_bound(lower.begin(), lower.end(), Point(p.x, -INF)) - lower.begin();
64
       if (lower[id].x == p.x) {
65
         if (lower[id].y > p.y) return false;
       } else if ((lower[id - 1] - p).det(lower[id] - p) < 0) return false;</pre>
66
```

```
67
        id = lower_bound(upper.begin(), upper.end(), Point(p.x, INF), greater<Point>()) -

    upper.begin();
 68
        if (upper[id].x == p.x) {
 69
          if (upper[id].y < p.y) return false;</pre>
        } else if ((upper[id - 1] - p).det(upper[id] - p) < 0) return false;</pre>
 70
        return true:
 71
 72
      // 求点 p 关于凸包的两个切点,如果在凸包外则有序返回编号,多解返回任意一个□ 否则返回
       \hookrightarrow false
      bool get_tangent(Point p, int &i0, int &i1) {
 74
        if (contain(p)) return false;
 75
 76
        i0 = i1 = 0;
        int id = lower_bound(lower.begin(), lower.end(), p) - lower.begin();
 77
 78
        binary_search(0, id, p, i0, i1);
 79
        binary_search(id, (int)lower.size(), p, i0, i1);
 80
        id = lower_bound(upper.begin(), upper.end(), p, greater<Point>()) - upper.begin();
 81
        binary_search((int)lower.size() - 1, (int)lower.size() - 1 + id, p, i0, i1);
 82
        binary_search((int)lower.size() - 1 + id, (int)lower.size() - 1 + (int)upper.size(),
       \hookrightarrow p, i0, i1);
 83
        return true;
 84
      // 求凸包上和向量 vec 叉积最大的点,返回编号,有多个返回任意一个
 85
      int get_tangent(Point vec) {
 86
 87
        pair<long long, int> ret = get_tangent(upper, vec);
 88
        ret.second = (ret.second + (int)lower.size() - 1) % n;
 89
        ret = max(ret, get_tangent(lower, vec));
        return ret.second;
 90
      }
 91
      // 求凸包和直线 u,v 的交点,如果无严格相交返回 false 。如果有则是和 (i,next(i))
       ↔ 的交点,两个点无序,交在点上不确定返回两条线段之一。
      bool get_intersection(Point u, Point v, int &i0, int &i1) {
 93
        int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
 94
        if (sign((v - u).det(a[p0] - u)) * sign((v - u).det(a[p1] - u)) < 0) {
 95
 96
          if (p0 > p1) swap(p0, p1);
          i0 = binary_search(u, v, p0, p1);
 97
 98
          i1 = binary_search(u, v, p1, p0 + n);
          return true;
 99
        } else {
100
          return false;
101
        }
102
103
104 };
```

# 圆的面积模板 $(n^2 \log n)$

```
struct Event {
2
     Point p:
     double ang:
     int delta;
     Event (Point p = Point(0, 0), double ang = 0, double delta = 0) : p(p), ang(ang),

    delta(delta) {}
6 };
   bool operator < (const Event &a, const Event &b) {
     return a.ang < b.ang;</pre>
9
    void addEvent(const Circle &a, const Circle &b, vector<Event> &evt, int &cnt) {
10
      double d2 = (a.o - b.o).len2(),
11
12
           dRatio = ((a.r - b.r) * (a.r + b.r) / d2 + 1) / 2,
13
          pRatio = sqrt(-(d2 - sqr(a.r - b.r)) * (d2 - sqr(a.r + b.r)) / (d2 * d2 * 4));
     Point d = b.o - a.o, p = d.rotate(PI / 2),
14
15
         q0 = a.o + d * dRatio + p * pRatio,
16
         q1 = a.o + d * dRatio - p * pRatio;
     double ang 0 = (q0 - a.o).ang(),
17
18
           ang1 = (q1 - a.o).ang();
     evt.push_back(Event(q1, ang1, 1));
19
     evt.push_back(Event(q0, ang0, -1));
20
     cnt += ang1 > ang0;
21
22
   bool issame(const Circle &a, const Circle &b) { return sign((a.o - b.o).len()) == 0 &&
      \hookrightarrow sign(a.r - b.r) == 0; }
24 bool overlap(const Circle &a, const Circle &b) { return sign(a.r - b.r - (a.o -
      \hookrightarrow b.o).len()) >= 0; }
25 bool intersect(const Circle &a, const Circle &b) { return sign((a.o - b.o).len() - a.r
      \hookrightarrow b.r) < 0; }
26 int C:
   Circle c[N];
28
   double area[N]:
   void solve() {
29
     memset(area, 0, sizeof(double) * (C + 1));
30
     for (int i = 0; i < C; ++i) {
31
32
       int cnt = 1:
       vector<Event> evt:
33
       for (int j = 0; j < i; ++j) if (issame(c[i], c[j])) ++cnt;
34
       for (int j = 0; j < C; ++j) {
35
36
         if (j != i && !issame(c[i], c[j]) && overlap(c[j], c[i])) {
37
            ++cnt;
38
         }
39
       for (int j = 0; j < C; ++j) {
```

```
if (j != i && !overlap(c[j], c[i]) && !overlap(c[i], c[j]) && intersect(c[i],
41
      \hookrightarrow c[i])) {
            addEvent(c[i], c[j], evt, cnt);
42
43
         }
        }
44
        if (evt.size() == 0) {
45
          area[cnt] += PI * c[i].r * c[i].r:
46
        } else {
47
          sort(evt.begin(), evt.end());
48
49
          evt.push_back(evt.front());
          for (int j = 0; j + 1 < (int)evt.size(); ++j) {
50
            cnt += evt[j].delta;
51
            area[cnt] += det(evt[j].p, evt[j + 1].p) / 2;
52
            double ang = evt[j + 1].ang - evt[j].ang;
53
            if (ang < 0) {
54
              ang += PI * 2;
55
56
           }
            area[cnt] += ang * c[i].r * c[i].r / 2 - sin(ang) * c[i].r * c[i].r / 2;
57
58 | }}}}
```

#### 三角形的心

```
1 Point inCenter(const Point &A, const Point &B, const Point &C) { // 内心
     double a = (B - C).len(), b = (C - A).len(), c = (A - B).len(),
       s = fabs(det(B - A, C - A)),
 3
       r = s / p:
 4
     return (A * a + B * b + C * c) / (a + b + c):
 6 | }
   Point circumCenter(const Point &a, const Point &b, const Point &c) { // 外心
 8
     Point bb = b - a, cc = c - a:
     double db = bb.len2(), dc = cc.len2(), d = 2 * det(bb, cc);
     return a - Point(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc) / d;
10
11 | }
12 Point othroCenter(const Point &a, const Point &b, const Point &c) { // 垂心
     Point ba = b - a, ca = c - a, bc = b - c;
13
     double Y = ba.y * ca.y * bc.y,
14
15
          A = ca.x * ba.y - ba.x * ca.y,
16
          x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
          y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
17
18
     return Point(x0, y0);
19 }
```

## 最小覆盖球

```
int nouter; Tpoint outer[4], res; double radius;
void ball() {
   Tpoint q[3]; double m[3][3], sol[3], L[3], det;
```

```
int i,j; res.x = res.y = res.z = radius = 0;
      for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol[i]=dot(q[i], q[i]);</pre>
 5
     for (i=0;i<3;++i) for(j=0;j<3;++j) m[i][j]=dot(q[i],q[j])*2;
      det= m[0][0]*m[1][1]*m[2][2]
     + m[0][1]*m[1][2]*m[2][0]
      + m[0][2]*m[2][1]*m[1][0]
10
      - m[0][2]*m[1][1]*m[2][0]
11
      - m[0][1]*m[1][0]*m[2][2]
12
      -m[0][0]*m[1][2]*m[2][1];
      if ( fabs(det)<eps ) return;</pre>
13
      for (j=0; j<3; ++j) {
14
       for (i=0; i<3; ++i) m[i][j]=sol[i];</pre>
15
16
       L[i]=(m[0][0]*m[1][1]*m[2][2]
17
        + m[0][1]*m[1][2]*m[2][0]
18
        + m[0][2]*m[2][1]*m[1][0]
        - m[0][2]*m[1][1]*m[2][0]
19
        - m[0][1]*m[1][0]*m[2][2]
20
        - m[0][0]*m[1][2]*m[2][1]
21
       ) / det:
22
        for (i=0; i<3; ++i) m[i][j]=dot(q[i], q[j])*2;</pre>
23
24
      } res=outer[0]:
      for (i=0; i<3; ++i ) res = res + q[i] * L[i];</pre>
25
      radius=dist2(res, outer[0]);
26
27
```

## 经纬度求球面最短距离

```
double sphereDis(double lon1, double lat1, double lon2, double lat2, double R) {
   return R * acos(cos(lat1) * cos(lat2) * cos(lon1 - lon2) + sin(lat1) * sin(lat2));
}
```

#### 长方体表面两点最短距离

```
1 int r:
   void turn(int i, int j, int x, int y, int z, int x0, int y0, int L, int W, int H) {
     if (z==0) { int R = x*x+y*y; if (R<r) r=R;
     } else {
       if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x, x0+L, y0, H, W, L);
       if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y, x0, y0+W, L, H, W);
       if(i \le 0 \&\& i \ge -2) turn(i-1, j, x0-z, y, x-x0, x0-H, y0, H, W, L);
       if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0, x0, y0-H, L, H, W);
9
   }}
10
   int main(){
11
    int L, H, W, x1, y1, z1, x2, y2, z2;
     cin >> L >> W >> H >> x1 >> y1 >> z1 >> x2 >> y2 >> z2;
     if (z1!=0 && z1!=H) if (y1==0 || y1==W)
```

```
swap(y1,z1), std::swap(y2,z2), std::swap(W,H);
14
15
     else swap(x1,z1), std::swap(x2,z2), std::swap(L,H);
16
     if (z1==H) z1=0, z2=H-z2;
17
     r=0x3fffffff; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
18
     cout<<r<<endl: return 0:</pre>
19 }
```

## 最大团

```
1 // Super Fast Maximum Clique
 2 // To Build Graph: Maxclique(Edges, Number of Nodes)
 3 // To Get Answer: mcqdyn(AnswerNodes Index Array, AnswserLength)
   typedef bool BB[N];
   struct Maxclique {
     const BB* e; int pk, level; const float Tlimit;
      struct Vertex{ int i, d; Vertex(int i):i(i),d(0){} };
 8
     typedef vector<Vertex> Vertices; typedef vector<int> ColorClass;
      Vertices V: vector<ColorClass> C: ColorClass QMAX. Q:
10
      static bool desc degree(const Vertex &vi. const Vertex &vi){
       return vi.d > vi.d:
11
12
      void init colors(Vertices &v){
13
        const int max degree = v[0].d:
14
       for(int i = 0; i < (int)v.size(); i++)v[i].d = min(i, max_degree) + 1;
15
16
      void set_degrees(Vertices &v){
17
18
       for(int i = 0, j; i < (int)v.size(); i++)</pre>
         for(v[i].d = j = 0; j < int(v.size()); j++)</pre>
19
           v[i].d += e[v[i].i][v[i].i];
20
21
      struct StepCount{ int i1, i2; StepCount():i1(0),i2(0){} };
22
      vector<StepCount> S;
23
      bool cut1(const int pi, const ColorClass &A){
24
       for(int i = 0; i < (int)A.size(); i++) if (e[pi][A[i]]) return true;</pre>
25
26
       return false:
27
      void cut2(const Vertices &A. Vertices &B){
28
29
       for(int i = 0; i < (int)A.size() - 1; i++)
         if(e[A.back().i][A[i].i])
30
           B.push_back(A[i].i);
31
     }
32
      void color_sort(Vertices &R){
33
       int j = 0, maxno = 1, min_k = max((int)QMAX.size() - (int)Q.size() + 1, 1);
34
       C[1].clear(), C[2].clear();
35
       for(int i = 0; i < (int)R.size(); i++) {</pre>
36
37
         int pi = R[i].i, k = 1;
```

```
38
         while(cut1(pi, C[k])) k++:
         if(k > maxno) maxno = k. C[maxno + 1].clear();
39
40
         C[k].push back(pi):
41
         if(k < min_k) R[j++].i = pi;</pre>
42
       if(j > 0) R[j - 1].d = 0;
43
       for(int k = min k: k <= maxno: k++)</pre>
44
         for(int i = 0; i < (int)C[k].size(); i++)
45
46
           R[j].i = C[k][i], R[j++].d = k;
47
48
     void expand_dyn(Vertices &R){// diff -> diff with no dyn
       S[level].i1 = S[level].i1 + S[level - 1].i1 - S[level].i2;//diff
49
       S[level].i2 = S[level - 1].i1;//diff
50
        while((int)R.size()) {
51
52
         if((int)Q.size() + R.back().d > (int)QMAX.size()){
           Q.push_back(R.back().i); Vertices Rp; cut2(R, Rp);
53
           if((int)Rp.size()){
54
              if((float)S[level].i1 / ++pk < Tlimit) degree_sort(Rp);//diff</pre>
55
56
              color_sort(Rp);
              S[level].i1++, level++;//diff
57
58
              expand_dyn(Rp);
              level--://diff
59
60
61
           else if((int)Q.size() > (int)QMAX.size()) QMAX = Q;
62
           Q.pop_back();
63
         }
64
         else return;
65
         R.pop_back();
66
67
68
     void mcqdyn(int* maxclique, int &sz){
69
        set_degrees(V); sort(V.begin(), V.end(), desc_degree); init_colors(V);
       for(int i = 0; i < (int)V.size() + 1; i++) S[i].i1 = S[i].i2 = 0;
70
71
       expand_dyn(V); sz = (int)QMAX.size();
72
       for(int i = 0; i < (int)QMAX.size(); i++) maxclique[i] = QMAX[i];</pre>
73
     void degree_sort(Vertices &R){
74
       set_degrees(R); sort(R.begin(), R.end(), desc_degree);
75
76
     Maxclique(const BB* conn, const int sz, const float tt = 0.025) \
77
78
      : pk(0), level(1), Tlimit(tt){
79
       for(int i = 0; i < sz; i++) V.push_back(Vertex(i));</pre>
80
       e = conn, C.resize(sz + 1), S.resize(sz + 1);
81
82 };
```

```
KM
```

```
1 // 最小匹配、自带初始化 n <= m 方案存在 p□ 中
   const int N = 105:
   const int INF = 1000000000; // 严格大于最大边权
   int n, m, a[N][N];
   int u[N], v[N], p[N], fa[N], minv[N];
 6 bool used[N]:
   int km() {
     memset(u, 0, sizeof(int) * n):
     for (int i = 0; i \le m; ++i) v[i] = 0, p[i] = n;
     for (int i = 0; i < n; ++i) {
10
       p[m] = i;
11
12
       int j0 = m;
       for (int j = 0; j <= m; ++j) minv[j] = INF, used[j] = false;
13
       do {
14
15
         used[j0] = true;
16
         int i0 = p[j0], delta = INF, j1;
         for (int j = 0; j < m; ++j) {
17
18
           if (!used[i]) {
             int cur = a[i0][j] - u[i0] - v[j];
19
20
             if (cur < minv[j]) minv[j] = cur, fa[j] = j0;</pre>
21
             if (minv[j] < delta) delta = minv[j], j1 = j;</pre>
22
           }
         }
23
         for (int j = 0; j \le m; ++j) {
24
25
           if (used[i]) {
26
             u[p[j]] += delta, v[j] -= delta;
           } else {
27
28
             minv[j] -= delta;
           }
29
         }
30
31
         j0 = j1;
       } while (p[j0] != n);
32
33
       do {
         int j1 = fa[j0];
34
35
         p[j0] = p[j1];
36
         j0 = j1;
37
       } while (j0 != m);
38
39
     return -v[m];
40
```

# 最小树形图

```
const int MAXN,INF;// INF >= sum( W_ij )
int from[MAXN + 10][MAXN * 2 + 10],n,m,edge[MAXN + 10][MAXN * 2 + 10];
```

```
3 int sel[MAXN * 2 + 10], fa[MAXN * 2 + 10], vis[MAXN * 2 + 10];
   int getfa(int x){if(x == fa[x]) return x: return fa[x] = getfa(fa[x]):}
   void liuzhu(){ // 1-base: root is 1. answer = (sel[i]. i) for i in [2..n]
     fa[1] = 1:
     for(int i = 2; i \le n; ++i){
       sel[i] = 1; fa[i] = i;
9
       for(int j = 1; j <= n; ++j) if(fa[j] != i)
         if(from[j][i] = i, edge[sel[i]][i] > edge[j][i]) sel[i] = j;
10
11
12
     int limit = n:
     while(1){
13
       int prelimit = limit; memset(vis, 0, sizeof(vis)); vis[1] = 1;
14
       for(int i = 2; i <= prelimit; ++i) if(fa[i] == i && !vis[i]){</pre>
15
16
         int j = i; while(!vis[j]) vis[j] = i, j = getfa(sel[j]);
17
         if(j == 1 || vis[j] != i) continue; vector<int> C; int k = j;
18
         do C.push_back(k), k = getfa(sel[k]); while(k != j);
         ++limit:
19
         for(int i = 1; i <= n; ++i){
20
           edge[i][limit] = INF, from[i][limit] = limit;
21
         }
22
23
         fa[limit] = vis[limit] = limit:
         for(int i = 0; i < int(C.size()); ++i){</pre>
24
25
           int x = C[i], fa[x] = limit;
26
           for(int j = 1; j \le n; ++j)
              if(edge[j][x] != INF && edge[j][limit] > edge[j][x] - edge[sel[x]][x]){
27
28
                edge[j][limit] = edge[j][x] - edge[sel[x]][x];
               from[j][limit] = x;
20
             }
30
31
         for(int j=1;j<=n;++j) if(getfa(j)==limit) edge[j][limit] = INF;</pre>
32
         sel[limit] = 1;
33
         for(int j = 1; j \le n; ++j)
34
           if(edge[sel[limit]][limit] > edge[j][limit]) sel[limit] = j;
35
36
       if(prelimit == limit) break;
37
38
     for(int i = limit; i > 1; --i) sel[from[sel[i]][i]] = sel[i];
39
40
```

## 无向图最小割

```
int cost[maxn] [maxn], seq[maxn], len[maxn], n, m, pop, ans;
bool used[maxn];
void Init(){
int i, j, a, b, c;
```

```
for(i=0;i<n;i++) for(j=0;j<n;j++) cost[i][j]=0;</pre>
 6
      for(i=0:i<m:i++){</pre>
        scanf("%d %d %d",&a,&b,&c); cost[a][b]+=c; cost[b][a]+=c;
 8
 9
      pop=n; for(i=0;i<n;i++) seq[i]=i;
10
    void Work(){
11
      ans=inf; int i,j,k,l,mm,sum,pk;
12
      while(pop > 1){
13
        for(i=1;i<pop;i++) used[seq[i]]=0; used[seq[0]]=1;</pre>
14
        for(i=1;i<pop;i++) len[seq[i]]=cost[seq[0]][seq[i]];</pre>
15
16
        pk=0; mm=-inf; k=-1;
        for(i=1;i<pop;i++) if(len[seq[i]] > mm){ mm=len[seq[i]]; k=i; }
17
18
        for(i=1;i<pop;i++){</pre>
          used[seq[l=k]]=1;
19
          if(i==pop-2) pk=k;
20
          if(i==pop-1) break;
21
          mm=-inf:
22
          for(j=1;j<pop;j++) if(!used[seq[j]])</pre>
23
            if((len[seq[j]]+=cost[seq[1]][seq[j]]) > mm)
24
25
              mm=len[seq[j]], k=j;
26
        }
        sum=0:
27
28
        for(i=0;i<pop;i++) if(i != k) sum+=cost[seq[k]][seq[i]];</pre>
        ans=min(ans,sum);
29
30
        for(i=0:i<pop:i++)</pre>
          cost[seq[k]][seq[i]]=cost[seq[i]][seq[k]]+=cost[seq[pk]][seq[i]];
31
        seq[pk]=seq[--pop];
32
33
      printf("%d\n",ans);
34
35
```

#### 带花树

```
1 | vector<int> link[maxn]:
   int n,match[maxn],Queue[maxn],head,tail;
   int pred[maxn],base[maxn],start,finish,newbase;
   bool InQueue[maxn].InBlossom[maxn]:
   void push(int u){ Queue[tail++]=u; InQueue[u]=true; }
   int pop(){ return Queue[head++]; }
   int FindCommonAncestor(int u,int v){
 8
     bool InPath[maxn];
     for(int i=0;i<n;i++) InPath[i]=0;</pre>
      while(true){ u=base[u]; InPath[u]=true; if(u==start) break; u=pred[match[u]]; }
10
      while(true){ v=base[v];if(InPath[v]) break;v=pred[match[v]]; }
11
     return v;
12
```

```
13 }
    void ResetTrace(int u){
14
15
16
      while(base[u]!=newbase){
17
        v=match[u]:
18
        InBlossom[base[u]]=InBlossom[base[v]]=true;
19
        if(base[u]!=newbase) pred[u]=v;
20
21
22
    void BlossomContract(int u.int v){
     newbase=FindCommonAncestor(u.v):
24
     for (int i=0;i<n;i++)</pre>
25
26
     InBlossom[i]=0;
27
     ResetTrace(u); ResetTrace(v);
28
      if(base[u]!=newbase) pred[u]=v;
     if(base[v]!=newbase) pred[v]=u;
29
     for(int i=0;i<n;++i)</pre>
30
     if(InBlossom[base[i]]){
31
        base[i]=newbase:
32
33
        if(!InQueue[i]) push(i);
34
35
   bool FindAugmentingPath(int u){
36
     bool found=false;
37
38
     for(int i=0;i<n;++i) pred[i]=-1,base[i]=i;</pre>
     for (int i=0;i<n;i++) InQueue[i]=0;</pre>
39
      start=u;finish=-1; head=tail=0; push(start);
40
      while(head<tail){</pre>
        int u=pop();
42
        for(int i=link[u].size()-1;i>=0;i--){
43
          int v=link[u][i];
44
          if(base[u]!=base[v]&&match[u]!=v)
45
46
            if(v==start||(match[v]>=0&&pred[match[v]]>=0))
              BlossomContract(u,v);
47
48
            else if(pred[v]==-1){
              pred[v]=u;
49
              if(match[v]>=0) push(match[v]);
50
              else{ finish=v; return true; }
51
52
       }
53
54
     return found:
55
56
57 | void AugmentPath(){
```

```
int u=finish,v,w;

finit u=finish,v,w;

while(u>=0){ v=pred[u];w=match[v]=u;match[u]=v;u=w; }

void FindMaxMatching(){

for(int i=0;i<n;++i) match[i]=-1;

for(int i=0;i<n;++i) if(match[i]==-1) if(FindAugmentingPath(i)) AugmentPath();
}</pre>
```

# Hopcroft

```
1 // 左侧 N 个点,右侧 K 个点, 1-based,初始化将 matx[],maty[] 都置为 0
 2 int N. K:
   int que[N], dx[N], dy[N], matx[N], maty[N];
   int BFS()
 4
   {
 5
     int flag = 0, qt = 0, qh = 0;
     for(int i = 1; i \le K; ++ i) dy[i] = 0;
     for(int i = 1: i <= N: ++ i) {
       dx[i] = 0:
 9
       if (! matx[i]) que[qt ++] = i;
10
11
     while (qh < qt) {
12
       int u = que[qh ++];
13
       for (Edge *e = E[u]; e; e = e->n)
14
         if (! dy[e->t]) {
15
16
           dv[e->t] = dx[u] + 1;
17
           if (! maty[e->t]) flag = true;
18
           else {
             dx[maty[e->t]] = dx[u] + 2;
19
             que[qt ++] = maty[e->t];
20
           }
21
22
         }
23
     return flag;
24
25
26
   int DFS(int u)
27
28
     for(Edge *e = E[u]; e; e = e->n)
       if (dy[e->t] == dx[u] + 1) {
29
30
         dy[e->t] = 0;
         if (! maty[e->t] || DFS(maty[e->t])) {
31
           matx[u] = e->t; maty[e->t] = u;
32
33
           return true;
         }
34
       }
35
36
     return false;
```

```
37 | 38 | void Hopcroft()
39 | {
40 | while (BFS()) for(int i = 1; i <= N; ++ i) if (! matx[i]) DFS(i);
41 | }
```

#### 素数判定

```
int strong_pseudo_primetest(long long n,int base) {
        long long n2=n-1,res;
3
       int s=0:
        while (n2\%2==0) n2>>=1,s++;
       res=powmod(base,n2,n);
5
       if((res==1)||(res==n-1)) return 1;
       s--;
       while(s \ge 0) {
           res=mulmod(res,res,n);
9
10
           if(res==n-1) return 1;
11
           s--;
       }
12
13
       return 0; // n is not a strong pseudo prime
14
   int isprime(long long n) {
15
16
     static LL testNum[]={2,3,5,7,11,13,17,19,23,29,31,37};
     static LL lim[]={4,0,1373653LL,25326001LL,25000000000LL,2152302898747LL, \
17
18
     3474749660383LL,341550071728321LL,0,0,0,0);
     if(n<2||n==3215031751LL) return 0;
19
     for(int i=0;i<12;++i){</pre>
20
       if(n<lim[i]) return 1;</pre>
21
       if(strong_pseudo_primetest(n,testNum[i])==0) return 0;
22
23
24
     return 1;
25
```

## 启发式分解

```
int ansn; LL ans[1000];
LL func(LL x,LL n){ return(mod_mul(x,x,n)+1)%n; }
LL Pollard(LL n){
    LL i,x,y,p;
    if(Rabin_Miller(n)) return n;
    if(!(n&1)) return 2;
    for(i=1;i<20;i++){
        x=i; y=func(x,n); p=gcd(y-x,n);
        while(p==1) {x=func(x,n); y=func(func(y,n),n); p=gcd((y-x+n)%n,n)%n;}
    if(p==0||p==n) continue;</pre>
```

```
11     return p;
12     }
13     }
14     void factor(LL n){
15         LL x;
16         x=Pollard(n);
17         if(x==n){ ans[ansn++]=x; return; }
18         factor(x), factor(n/x);
19     }
```

## 二次剩余

```
void calcH(int &t, int &h, const int p) {
     int tmp = p - 1; for (t = 0; (tmp & 1) == 0; tmp /= 2) t++; h = tmp;
 3 | }
   // solve equation x^2 \mod p = a
   bool solve(int a, int p, int &x, int &y) {
     srand(19920225):
     if (p == 2) { x = y = 1; return true; }
     int p2 = p / 2, tmp = power(a, p2, p);
     if (tmp == p - 1) return false;
     if ((p + 1) \% 4 == 0) {
10
       x = power(a, (p + 1) / 4, p); y = p - x; return true;
11
     } else {
12
       int t, h, b, pb; calcH(t, h, p);
13
       if (t \ge 2) {
14
         do \{b = rand() \% (p - 2) + 2;
15
16
         } while (power(b, p / 2, p) != p - 1);
         pb = power(b, h, p);
17
18
       } int s = power(a, h / 2, p);
       for (int step = 2; step <= t; step++) {</pre>
19
         int ss = (((long long)(s * s) % p) * a) % p;
20
         for (int i = 0; i < t - step; i++) ss = ((long long)ss * ss) % p;
21
         if (ss + 1 == p) s = (s * pb) % p; pb = ((long long)pb * pb) % p;
22
       x = ((long long)s * a) % p; y = p - x;
23
     } return true;
24
25
```

# Pell 方程

```
ULL A,B,p[maxn],q[maxn],a[maxn],g[maxn],h[maxn];
int main() {
   for (int test=1, n;scanf("%d",&n) && n;++test) {
      printf("Case %d: ",test);
      if (fabs(sqrt(n)-floor(sqrt(n)+1e-7))<=1e-7) {
        int a=(int)(floor(sqrt(n)+1e-7)); printf("%d %d\n",a,1);
      } else {</pre>
```

```
// 求 x^2 - ny^2 = 1 的最小正整数根, n 不是完全平方数
8
         p[1]=a[0]=h[1]=1:p[0]=a[1]=g[1]=0:
9
         a[2]=(int)(floor(sqrt(n)+1e-7));
10
11
         for (int i=2;i;++i) {
12
           g[i]=-g[i-1]+a[i]*h[i-1]; h[i]=(n-sqr(g[i]))/h[i-1];
           a[i+1]=(g[i]+a[2])/h[i]; p[i]=a[i]*p[i-1]+p[i-2];
13
           q[i]=a[i]*q[i-1]+q[i-2];
14
           if (sqr((ULL)(p[i]))-n*sqr((ULL)(q[i]))==1){
15
16
             A=p[i];B=q[i];break; }
17
         } cout << A << ' ' << B <<endl;</pre>
18
```

#### 日期公式

```
int zeller(int y,int m,int d) {
   if (m<=2) y--,m+=12; int c=y/100; y%=100;
   int w=((c>>2)-(c<<1)+y+(y>>2)+(13*(m+1)/5)+d-1)%7;
   if (w<0) w+=7; return(w);
}
int getId(int y, int m, int d) {
   if (m < 3) {y --; m += 12};
   return 365 * y + y / 4 - y / 100 + y / 400 + (153 * m + 2) / 5 + d;
}</pre>
```

# Schreier-Sims

```
vector<int> P:Permutation(){} Permutation(int n){ P.resize(n); }
     Permutation inv()const{
       Permutation ret(P.size());
       for(int i = 0; i < int(P.size()); ++i) ret.P[P[i]] = i;</pre>
       return ret:
     int &operator [](const int &dn){ return P[dn]; }
     void resize(const size t &sz){ P.resize(sz): }
     size_t size()const{ return P.size(); }
10
     const int &operator [](const int &dn)const{ return P[dn]; }
11
12
   Permutation operator *(const Permutation &a, const Permutation &b){
     Permutation ret(a.size()):
14
     for(int i = 0; i < (int)a.size(); ++i) ret[i] = b[a[i]];</pre>
15
16
     return ret;
17 }
18
   typedef vector<Permutation> Bucket;
   typedef vector<int> Table; typedef pair<int,int> pii;
20 int n, m;
```

```
vector<Bucket> buckets. bucketsInv: vector<Table> lookupTable:
   int fastFilter(const Permutation &g, bool addToGroup = true){
      int n = buckets.size():
23
24
     Permutation p:
      for(int i = 0; i < n; ++i){
25
26
        int res = lookupTable[i][p[i]];
27
        if(res == -1){
28
          if (addToGroup) {
29
            buckets[i].push_back(p); bucketsInv[i].push_back(p.inv());
30
           lookupTable[i][p[i]] = (int)buckets[i].size() - 1;
         }
31
          return i;
32
        }
33
        p = p * bucketsInv[i][res]; swap(i1,i2);
34
35
36
     return -1;
37
   long long calcTotalSize(){
38
     long long ret = 1;
40
     for(int i = 0; i < n; ++i) ret *= buckets[i].size();</pre>
     return ret:
41
42 }
   bool inGroup(const Permutation &g){ return fastFilter(g, false) == -1; }
    void solve(const Bucket &gen,int _n){// m perm[0..n - 1]s
44
     n = _n, m = gen.size();
45
46
      {//clear all
        vector<Bucket> _buckets(n); swap(buckets, _buckets);
47
48
        vector<Bucket> _bucketsInv(n); swap(bucketsInv, _bucketsInv);
        vector<Table> _lookupTable(n); swap(lookupTable, _lookupTable);
49
50
      for(int i = 0; i < n; ++i){
51
        lookupTable[i].resize(n);
52
        fill(lookupTable[i].begin(), lookupTable[i].end(), -1);
53
54
      Permutation id(n);
55
56
      for(int i = 0; i < n; ++i) id[i] = i;</pre>
      for(int i = 0; i < n; ++i){
57
58
        buckets[i].push_back(id); bucketsInv[i].push_back(id);
        lookupTable[i][i] = 0;
59
60
61
     for(int i = 0; i < m; ++i) fastFilter(gen[i]);</pre>
62
      queue<pair<point,point> > toUpdate;
      for(int i = 0; i < n; ++i)
63
64
        for(int j = i; j < n; ++j)
          for(int k = 0: k < (int)buckets[i].size(): ++k)</pre>
65
```

```
66
            for(int 1 = 0: 1 < (int)buckets[i].size(): ++1)</pre>
67
              toUpdate.push(make_pair(pii(i,k), pii(j,l)));
68
      while(!toUpdate.empty()){
69
       pii a = toUpdate.front().first, b = toUpdate.front().second;
       toUpdate.pop();
70
        int res=fastFilter(buckets[a.first][a.second]*buckets[b.first][b.second]);
71
72
       if(res==-1) continue:
73
       pii newPair(res, (int)buckets[res].size() - 1);
       for(int i = 0; i < n; ++i)
74
75
         for(int j = 0; j < (int)buckets[i].size(); ++j){</pre>
76
           if(i <= res) toUpdate.push(make_pair(pii(i, j), newPair));</pre>
           if(res <= i) toUpdate.push(make_pair(newPair, pii(i, j)));</pre>
77
78
79
80
```

## 线性规划

```
// 求\max\{cx \mid Ax < b, x > 0\}的解
   typedef vector<double> VD:
   VD simplex(vector<VD> A, VD b, VD c) {
     int n = A.size(), m = A[0].size() + 1, r = n, s = m - 1;
     vector < VD > D(n + 2, VD(m + 1, 0)); vector < int > ix(n + m);
     for (int i = 0; i < n + m; ++ i) ix[i] = i;
     for (int i = 0; i < n; ++ i) {
       for (int j = 0; j < m - 1; ++ j) D[i][j] = -A[i][j];
9
       D[i][m - 1] = 1; D[i][m] = b[i];
       if (D[r][m] > D[i][m]) r = i;
10
11
12
     for (int j = 0; j < m - 1; ++ j) D[n][j] = c[j];
     D[n + 1][m - 1] = -1;
13
     for (double d; ; ) {
14
       if (r < n) {
15
16
         int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
         D[r][s] = 1.0 / D[r][s]; vector<int> speedUp;
17
18
         for (int j = 0; j \le m; ++ j) if (j != s) {
           D[r][j] *= -D[r][s];
19
20
           if(D[r][j]) speedUp.push_back(j);
21
22
         for (int i = 0; i \le n + 1; ++ i) if (i != r) {
           for(int j = 0; j < speedUp.size(); ++ j)</pre>
23
           D[i][speedUp[j]] += D[r][speedUp[j]] * D[i][s];
24
25
           D[i][s] *= D[r][s];
26
       f(s) = -1; s = -1;
       for (int j = 0; j < m; ++ j) if (s < 0 || ix[s] > ix[j])
27
28
         if (D[n + 1][j] > EPS || (D[n + 1][j] > -EPS && D[n][j] > EPS)) s = j;
```

```
if (s < 0) break:
29
30
       for (int i = 0; i < n; ++ i) if (D[i][s] < -EPS)
         if (r < 0 \mid | (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
31
32
              || (d < EPS \&\& ix[r + m] > ix[i + m])) r = i;
       if (r < 0) return VD(); // 无边界
33
34
     if (D[n + 1][m] < -EPS) return VD(): // 无解
35
36
     VD \times (m - 1):
     for (int i = m; i < n + m; ++ i) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
37
     return x; // 最优值在 D[n][m]
38
39 | }
```

# FFT

```
1 / / \text{ double 精度对} 10^9 + 7 取模最多可以做到2^20
 2 const int MOD = 1000003:
   const double PI = acos(-1);
   typedef complex<double> Complex;
   const int N = 65536, L = 15, MASK = (1 << L) - 1;
 6 | Complex w[N];
   void FFTInit() {
     for (int i = 0; i < N; ++i) {
       w[i] = Complex(cos(2 * i * PI / N), sin(2 * i * PI / N));
 9
10
     }
11 }
12
   void FFT(Complex p[], int n) {
     for (int i = 1, j = 0; i < n - 1; ++i) {
13
       for (int s = n; j ^= s >>= 1, ~j & s;);
14
       if (i < j) {
15
         swap(p[i], p[i]);
16
17
       }
18
      for (int d = 0; (1 << d) < n; ++d) {
19
        int m = 1 \ll d, m2 = m * 2, rm = n >> (d + 1);
20
       for (int i = 0; i < n; i += m2) {
21
22
         for (int j = 0; j < m; ++j) {
23
           Complex &p1 = p[i + j + m], &p2 = p[i + j];
24
           Complex t = w[rm * j] * p1;
           p1 = p2 - t;
25
26
           p2 = p2 + t;
27
28 | Complex A[N], B[N], C[N], D[N];
   void mul(int a[N], int b[N]) {
     for (int i = 0; i < N; ++i) {
30
       A[i] = Complex(a[i] >> L, a[i] & MASK);
31
       B[i] = Complex(b[i] >> L, b[i] & MASK);
32
```

```
33
34
     FFT(A, N), FFT(B, N);
     for (int i = 0; i < N; ++i) {
35
36
       int j = (N - i) \% N;
       Complex da = (A[i] - conj(A[j])) * Complex(0, -0.5),
37
38
           db = (A[i] + conj(A[j])) * Complex(0.5, 0),
           dc = (B[i] - conj(B[j])) * Complex(0, -0.5),
39
40
           dd = (B[i] + conj(B[j])) * Complex(0.5, 0);
41
       C[j] = da * dd + da * dc * Complex(0, 1);
       D[j] = db * dd + db * dc * Complex(0, 1);
42
43
     FFT(C, N), FFT(D, N);
44
     for (int i = 0; i < N; ++i) {
45
46
       long long da = (long long)(C[i].imag() / N + 0.5) \% MOD
             db = (long long)(C[i].real() / N + 0.5) % MOD,
47
48
             dc = (long long)(D[i].imag() / N + 0.5) % MOD,
             dd = (long long)(D[i].real() / N + 0.5) % MOD;
49
       a[i] = ((dd << (L * 2)) + ((db + dc) << L) + da) % MOD;
50
51
52
```

# Manacher/ 扩展 KMP

```
void Manacher(char text[], int n, int palindrome[]) {
     palindrome[0] = 1;
     for (int i = 1, j = 0, i < (n << 1) - 1; ++ i) {
       int p = i \gg 1;
4
       int q = i - p;
       int r = (j + 1 >> 1) + palindrome[j] - 1;
       palindrome[i] = r < q ? 0 : min(r - q + 1, palindrome[(j << 1) - i]);
       while (0 <= p - palindrome[i] && q + palindrome[i] < n && text[p - palindrome[i]] ==</pre>

    text[q + palindrome[i]]) {
         palindrome[i] ++;
9
10
       if (q + palindrome[i] - 1 > r) {
11
         j = i;
12
13
14
15
16
   void ExtendedKMP(char *s, int next[]) {
18
       int l = strlen(s), i = 0, j = 0, k = 1;
19
       while (1 + j < 1 & s[j] == s[1 + j]) {
20
           ++j;
21
       }
22
       next[1] = j;
```

```
for (int i = 2: i < 1: ++i) {
23
           int len = k + next[k]. ll = next[i - k]:
24
           if (l1 < len - i) {
25
26
                next[i] = 11:
27
           } else {
28
               j = max(0, len - i);
                while (i + j < l \&\& s[j] == s[i + j]) {
29
30
                    ++j;
               }
31
32
                next[i] = j;
                k = i;
33
           }
34
       }
35
36 }
```

# 后缀数组(倍增)

```
1 int rank[MAX N].height[MAX N]:
    int cmp(int *x,int a,int b,int d){
      return x[a] == x[b] &&x[a+d] == x[b+d];
 3
 4
    void doubling(int *a,int N,int M){
      static int sRank[MAX_N],tmpA[MAX_N],tmpB[MAX_N];
      int *x=tmpA,*y=tmpB;
      for(int i=0;i<M;++i) sRank[i]=0;</pre>
      for(int i=0;i<N;++i) ++sRank[x[i]=a[i]];</pre>
10
      for(int i=1;i<M;++i) sRank[i]+=sRank[i-1];</pre>
      for(int i=N-1;i>=0;--i) sa[--sRank[x[i]]]=i;
11
      for(int d=1,p=0;p<N;M=p,d<<=1){</pre>
12
        p=0; for(int i=N-d;i<N;++i) y[p++]=i;</pre>
13
        for(int i=0; i<N; ++i) if(sa[i]>=d) y[p++]=sa[i]-d;
14
15
        for(int i=0;i<M;++i) sRank[i]=0;</pre>
16
        for(int i=0;i<N;++i) ++sRank[x[i]];</pre>
        for(int i=1;i<M;++i) sRank[i]+=sRank[i-1];</pre>
17
18
        for(int i=N-1;i>=0;--i) sa[--sRank[x[y[i]]]]=y[i];
        swap(x,y); x[sa[0]]=0; p=1;
19
        for(int i=1;i<N;++i) x[sa[i]]=cmp(y,sa[i],sa[i-1],d)?p-1:p++;</pre>
20
21
22
    void calcHeight(){
23
      for(int i=0;i<N;++i) rank[sa[i]]=i;</pre>
24
      int cur=0; for(int i=0;i<N;++i)</pre>
25
26
      if(rank[i]){
        if(cur) cur--;
27
        for(;a[i+cur] == a[sa[rank[i]-1]+cur];++cur);
28
        height[rank[i]]=cur;
29
```

```
30 }
31 }
```

## 后缀自动机

```
struct State {
     int length;
     State *parent,*go[C];
3
     State(int length = 0):length(length),parent(NULL){
       memset(go,0,sizeof(go));
5
6
    }
     State* extend(State*, int token);
   } node_pool[V], *tot_node;
   State* State::extend(State *start,int token){
10
11
     State *np=new(tot_node++) State(this->length+1);
     while(p!=NULL&&p->go[token]==NULL)
12
       p->go[token]=np, p=p->parent;
13
     if(p==NULL) np->parent=start;
14
     else{
15
16
       State *q=p->go[token];
       if(p->length+1==q->length) np->parent=q;
17
18
19
         State *nq=new(tot_node++) State(p->length+1);
         memcpy(nq->go,q->go,sizeof(q->go));
20
21
         nq->parent=q->parent;
22
         np->parent=q->parent=nq;
         while(p!=NULL&&p->go[token]==q)
23
           p->go[token]=nq, p=p->parent;
24
       }
25
26
27
     return np;
28
```

# 后缀树 (With Pop Front)

```
int pos, text[N];
struct Node {
   int l, r;
   Node *suf, *ch[C];
   int dgr;
   Node *fa;
   Node (int l = -1, int r = INF) : l(l), r(r) {
      suf = fa = NULL;
      memset(ch, 0, sizeof(ch));
   dgr = 0;
```

```
11
12
      Node* addEdge(Node *t) {
        int c = text[t->1];
13
14
        dgr += !ch[c];
        ch[c] = t;
15
16
        t->fa = this;
17
       return t:
18
     int len() {
19
20
        return min(r, pos + 1) - 1;
21
22
   };
23
24
   int top;
   Node pool[N << 1], *root, *nxtSuf, *cur;</pre>
26
   int remCnt, curP, curLen;
   long long size;
   queue<Node*> leaves;
28
   void init() {
29
30
     top = 0, pos = -1;
     remCnt = 0, curP = 0, curLen = 0;
31
     nxtSuf = NULL:
32
     root = cur = new(pool + (top++)) Node(-1, -1);
33
     size = 0;
34
      while (leaves.size()) leaves.pop();
35
36
   }
   void link(Node *u) {
37
      if (nxtSuf) nxtSuf->suf = u;
38
      nxtSuf = u;
39
40
   bool walk(Node *u) {
      int len = u->len();
42
      if (curLen >= len) {
43
        curP += len, curLen -= len, cur = u;
44
45
       return true;
     }
46
     return false;
47
48
    void extend(int c) {
49
      text[++pos] = c;
50
      nxtSuf = NULL:
51
      ++remCnt:
52
      while (remCnt) {
53
        curP = curLen ? curP : pos;
54
        int curE = text[curP]:
55
```

```
56
         if (!cur->ch[curE]) {
           leaves.push(cur->addEdge(new(pool + (top++)) Node(pos)));
 57
 58
 59
         } else {
 60
           Node *nxt = cur->ch[curE]:
 61
           if (walk(nxt)) continue;
           if (text[nxt->l + curLen] == c) {
 62
 63
             ++curLen:
            link(cur);
 64
 65
             break;
 66
 67
           Node *split = new(pool + (top++)) Node(nxt->1, nxt->1 + curlen);
 68
           cur->addEdge(split);
 69
           leaves.push(split->addEdge(new(pool + (top++)) Node(pos)));
 70
           nxt->l += curLen;
           split->addEdge(nxt);
 71
           link(split);
 72
        }
 73
         --remCnt:
 74
         if (cur == root && curLen > 0) {
 75
 76
           curP = pos - (--curLen);
 77
        } else {
 78
           cur = cur->suf ? cur->suf : root;
        }
 79
 80
 81
       size += leaves.size():
 82
 83
     void finish() {
 84
       nxtSuf = NULL;
 85
       for (int i = 0; i < top; ++i) if (pool[i].r == INF) link(pool + i);</pre>
 86
       while (remCnt > 0) {
 87
         if (curLen) {
 88
           int curE = text[curP];
 89
           Node *nxt = cur->ch[curE];
           if (walk(nxt)) continue;
 90
 91
           Node *split = new(pool + (top++)) Node(nxt->1, nxt->1 + curLen);
           leaves.push(cur->addEdge(split));
 92
           nxt->l += curLen;
 93
           split->addEdge(nxt);
 94
           link(split);
 95
         } else {
 96
           leaves.push(cur);
 97
           link(cur);
 98
 99
         --remCnt:
100
```

```
if (cur == root && curLen > 0) {
101
102
           --curLen:
103
           curP = pos - remCnt + 1;
104
        } else {
           cur = cur->suf ? cur->suf : root;
105
         }
106
107
      }
      if (nxtSuf != root) link(root);
108
109 }
110
     void eraseUp(Node *&u) {
       size -= u->len();
111
       int ch = text[u->1];
112
      u = u \rightarrow fa;
113
       u \rightarrow ch[ch] = NULL;
114
115
       --(u->dgr);
116 }
     void erase() {
117
       Node *u = leaves.front();
118
       leaves.pop();
119
       while (u->dgr == 0 && u != cur) eraseUp(u);
120
121
       if (u == cur) {
         if (cur->dgr == 0 && curLen == 0) {
122
           int len = u->len();
123
           curLen = len;
124
           curP = pos - len + 1;
125
126
           cur = cur->fa;
           eraseUp(u);
127
         }
128
         if (curLen) {
129
           int curE = text[curP];
130
           if (!cur->ch[curE]) {
131
             Node *leaf = new(pool + (top++)) Node(pos - curlen + 1);
132
             leaves.push(cur->addEdge(leaf));
133
             size += leaf->len();
134
             --remCnt:
135
             if (cur == root && curLen > 0) {
136
               curP = pos - (--curLen) + 1;
137
138
             } else {
               cur = cur->suf ? cur->suf : root;
139
140
             while (curLen && walk(cur->ch[text[curP]])) continue:
141
142 | }}}}
143 | int n;
144 | char s[N], buf[N];
145 int ord[N], stop, sord[N << 1];
```

```
146 void dfs(Node *u) {
       sord[u - pool] = stop++;
147
       for (int i = 0; i < C; ++i) {
148
149
        if (u->ch[i]) {
          dfs(u->ch[i]);
150
        }
151
152
      }
153
154
    void getOrd() {
155
      init();
156
      for (int i = 0; i < n; ++i) extend(s[i] - 'a');
      finish();
157
158
       stop = 0;
       dfs(root);
159
160
      int i = 0;
161
       while (leaves.size()) {
162
        ord[i++] = sord[leaves.front() - pool];
163
        leaves.pop();
164
165 }
```

#### 字符串最小表示

```
std::string find(std::string s) {
   int i,j,k,l,N=s.length(); s+=s;
   for(i=0,j=1;j<N;){
      for(k=0;k<N&&s[i+k]==s[j+k];k++);
      if(k>=N) break;
      if(s[i+k]<s[j+k]) j+=k+1;
      else l=i+k,i=j,j=max(l,j)+1;
}
return s.substr(i,N);
}</pre>
```

## 轻重链剖分

```
struct Tree(){}*root[N];
int father[N],size[N],depth[N];
int bfsOrd[N],pathId[N],ordInPath[N],sqn[N];

void doBfs(int s){
   int qh=0,qt=0,*que=bfsOrd; father[s]=-1; depth[s]=0;
   for(que[qt++]=s;qh<qt;){
     int u=que[qh++];
     foreach(iter,adj[u]){
        int v=*iter; if(v==father[u]) continue;
        father[v]=u; depth[v]=depth[u]+1; que[qt++]=v;
}</pre>
```

```
}
12
13 | }
   void doSplit(){
14
15
      for(int i=N-1;i>=0;--i){
16
        int u=bfsOrd[i]; size[u]=1;
        foreach(iter,adj[u]){
17
18
          int v=*iter; if(v==father[u]) continue; size[u]+=size[v];
       }
19
20
21
      memset(pathId,-1,sizeof pathId);
      for(int i=0;i<N;++i){</pre>
22
        int top=bfsOrd[i],cnt=0;
23
        if(pathId[top]!=-1) continue;
24
        for(int next,u=top;u!=-1;u=next){
25
26
          sqn[cnt]=val[u]; ordInPath[u]=cnt; pathId[u]=top; ++cnt;
          next=-1;
27
28
          foreach(iter,adj[u]){
            int v=*iter; if(v==father[u]) continue;
29
            if(next<0||size[next]<size[v]) next=v;</pre>
30
         }
31
32
        }
33
        root[top] = new Tree(0, cnt, sqn);
34
35
   void prepare(){ doBfs(0); doSplit(); }
```

# Splay Tree

```
// 注意初始化内存池和 null 节点
   struct Node{
 2
     int rev,size; Node *ch[2],*p;
     void set(Node*,int); int dir(); void update(); void relax(); void appRev();
   } nodePool[MAX_NODE],*curNode,*null;
 6 | Node *newNode(){
     Node *t=curNode++; t->rev=0, t->size=1;
 8
     t->ch[0]=t->ch[1]=t->p=null; return t;
 9 3
   struct Splay{
10
     Node *root:
11
     Splay(){ root=newNode(); root->set(newNode(),0); root->update(); }
12
      void rot(Node *t){
13
       Node *p=t->p; int d=t->dir();
14
15
       p->relax(); t->relax();
16
       if(p==root) root=t;
       p->set(t->ch[!d],d); p->p->set(t,p->dir()); t->set(p,!d);
17
18
       p->update();
```

```
19
20
      void splav(Node *t.Node *f=null){
21
       for(t->relax();t->p!=f;)
22
         if(t-p-p==f) rot(t);
         else t->dir()==t->p->dir()?(rot(t->p),rot(t)):(rot(t),rot(t));
23
       t->update();
24
25
     }
26
    void initNull(){ curNode=nodePool;null=curNode++;null->size=0; }
28
   void Node::set(Node *t,int _d){ ch[_d]=t; t->p=this; }
   int Node::dir(){ return this==p->ch[1]; }
   void Node::update(){ size=ch[0]->size+ch[1]->size+1;}
30
   void Node::relax(){ if(rev) ch[0]->appRev(), ch[1]->appRev(), rev=false; }
   void Node::appRev(){ if(this==null) return; rev^=true; swap(ch[0],ch[1]); }
```

#### Link Cut Tree

```
// 注意初始化 null 节点. 单点的 is root 初始为 true
   struct Node{
     Node *ch[2], *p;
     int is_root, rev;
     bool dir();
     void set(Node*, bool):
     void update();
     void relax();
     void app_rev();
10
     *null;
   void rot(Node *t){
11
     Node *p=t->p; bool d=t->dir();
12
     p->relax(); t->relax(); p->set(t->ch[!d],d);
13
     if(p->is_root) t->p=p->p,swap(p->is_root,t->is_root);
14
15
     else p->p->set(t,p->dir());
16
     t->set(p,!d); p->update();
17
18
   void splay(Node *t){
     for(t->relax();!t->is_root;)
19
20
       if(t->p->is_root) rot(t);
21
       else t->dir()==t->p->dir() ?(rot(t->p),rot(t)) : (rot(t),rot(t));
22
     t->update();
23
    void access(Node *t){
24
     for(Node *s=null; t!=null; s=t,t=t->p){
25
26
       splay(t);
       if (t->p == null) { /*TODO*/ }
27
28
       t->ch[1]->is_root=true; s->is_root=false;
       t->ch[1]=s; t->update();
29
```

## Dominator Tree

```
vector<int> prec[N], succ[N];
   vector<int> ord;
   int stamp, vis[N];
 3
   int num[N];
   int fa[N];
   void dfs(int u) {
      vis[u] = stamp;
      num[u] = ord.size();
      ord.push_back(u);
      for (int i = 0; i < (int)succ[u].size(); ++i) {</pre>
10
        int v = succ[u][i]:
11
        if (vis[v] != stamp) {
12
         fa[v] = u;
13
          dfs(v);
14
15
        }
16
17
18
   int fs[N], mins[N], dom[N], sem[N];
   int find(int u) {
19
20
      if (u != fs[u]) {
        int v = fs[u];
^{21}
22
        fs[u] = find(fs[u]);
        if (mins[v] != -1 && num[sem[mins[v]]] < num[sem[mins[u]]]) {</pre>
23
          mins[u] = mins[v]:
24
25
       }
26
      return fs[u];
27
28
   void merge(int u, int v) { fs[u] = v; }
   vector<int> buf[N];
30
   int buf2[N];
   void mark(int source) {
      ord.clear();
33
      ++stamp;
34
```

```
dfs(source):
35
36
     for (int i = 0; i < (int)ord.size(); ++i) {</pre>
       int u = ord[i]:
37
       fs[u] = u, mins[u] = -1, buf2[u] = -1;
38
39
      for (int i = (int) ord.size() - 1; i > 0; --i) {
40
        int u = ord[i], p = fa[u];
41
42
        sem[u] = p;
        for (int j = 0; j < (int)prec[u].size(); ++j) {</pre>
43
         int v = prec[u][j];
44
          if (use[v] != stamp) continue;
45
46
          if (num[v] > num[u]) {
            find(v); v = sem[mins[v]];
47
48
          if (num[v] < num[sem[u]]) {</pre>
49
            sem[u] = v;
50
51
52
        buf[sem[u]].push_back(u);
53
        mins[u] = u;
54
        merge(u, p);
55
        while (buf[p].size()) {
56
         int v = buf[p].back();
57
58
          buf[p].pop_back();
         find(v);
59
60
          if (sem[v] == sem[mins[v]]) {
61
            dom[v] = sem[v];
62
         } else {
63
            buf2[v] = mins[v];
64
65
       }
66
      dom[ord[0]] = ord[0];
67
68
      for (int i = 0; i < (int)ord.size(); ++i) {</pre>
69
        int u = ord[i];
        if (~buf2[u]) {
70
71
          dom[u] = dom[buf2[u]];
72
73
74
```

# DancingLinks

```
struct node{
node *left,*right,*up,*down,*col; int row,cnt;
}*head,*col[MAXC],Node[MAXNODE],*ans[MAXNODE];
```

```
int totNode;
 5
   void insert(const std::vector<int> &V.int rownum){
 6
      std::vector<node*> N:
      for(int i=0:i<int(V.size()):++i){</pre>
 8
        node* now=Node+(totNode++); now->row=rownum;
        now->col=now->up=col[V[i]], now->down=col[V[i]]->down;
 9
10
       now->up->down=now, now->down->up=now;
11
        now->col->cnt++; N.push_back(now);
12
      for(int i=0;i<int(V.size());++i)</pre>
13
        N[i]->right=N[(i+1)%V.size()], N[i]->left=N[(i-1+V.size())%V.size()];
14
15
16
   void Remove(node *x){
      x->left->right=x->right, x->right->left=x->left;
17
18
      for(node *i=x->down;i!=x;i=i->down)
        for(node *j=i->right; j!=i; j=j->right)
19
          j->up->down=j->down, j->down->up=j->up, --(j->col->cnt);
20
21
   void Resume(node *x){
22
      for(node *i=x->up;i!=x;i=i->up)
23
        for(node *j=i->left; j!=i; j=j->left)
24
25
          j->up->down=j->down->up=j, ++(j->col->cnt);
26
     x->left->right=x, x->right->left=x;
27
28
   bool search(int tot){
      if(head->right==head) return true;
29
30
      node *choose=NULL;
      for(node *i=head->right;i!=head;i=i->right){
31
        if(choose==NULL||choose->cnt>i->cnt) choose=i;
32
        if(choose->cnt<2) break;</pre>
33
     }
34
      Remove(choose);
35
36
      for(node *i=choose->down;i!=choose;i=i->down){
37
        for(node *j=i->right; j!=i; j=j->right) Remove(j->col);
38
        ans[tot]=i;
        if(search(tot+1)) return true;
39
        ans[tot]=NULL:
40
        for(node *j=i->left; j!=i; j=j->left) Resume(j->col);
41
42
      Resume(choose):
43
44
      return false:
45
   void prepare(int totC){
     head=Node+totC:
47
     for(int i=0:i<totC:++i) col[i]=Node+i:</pre>
```

```
49     totNode=totC+1;
50     for(int i=0;i<=totC;++i){
51         (Node+i)->right=Node+(i+1)%(totC+1);
52         (Node+i)->left=Node+(i+totC)%(totC+1);
53         (Node+i)->up=(Node+i)->down=Node+i;
54     }
55 }
```

## 环状最长公共子序列

```
int n, a[N << 1], b[N << 1];
   bool has(int i, int j) { return a[(i-1) \% n] == b[(j-1) \% n];}
   const int DELTA[3][2] = \{\{0, -1\}, \{-1, -1\}, \{-1, 0\}\};
   int from[N][N]:
   int solve() {
     memset(from, 0, sizeof(from));
     int ret = 0;
     for (int i = 1: i \le 2 * n: ++ i) {
9
       from[i][0] = 2:
10
       int left = 0, up = 0:
       for (int j = 1; j \le n; ++ j) {
11
12
         int upleft = up + 1 + !!from[i - 1][j];
         if (!has(i, j)) upleft = INT_MIN;
13
         int max = std::max(left, std::max(upleft, up));
14
         if (left == max) {
15
16
           from[i][j] = 0;
17
         } else if (upleft == max) {
18
           from[i][j] = 1;
         } else {
19
           from[i][j] = 2;
20
         }
21
22
         left = max;
       }
23
24
       if (i >= n) {
         int count = 0:
25
26
         for (int x = i, y = n; y;) {
27
           int t = from[x][v]:
28
           count += t == 1:
29
           x += DELTA[t][0]:
30
           y += DELTA[t][1];
31
32
         ret = std::max(ret, count);
33
         int x = i - n + 1, y = 0;
         from[x][0] = 0;
34
         while (y \le n \&\& from[x][y] == 0) y++;
35
         for (; x <= i; ++ x) {
36
```

```
from[x][v] = 0:
37
38
            if (x == i) break:
39
            for (; y <= n; ++ y) {
40
              if (from[x + 1][y] == 2) break;
              if (y + 1 \le n \&\& from[x + 1][y + 1] == 1) {
41
42
               y ++;
43
                break:
     }}}}
44
45
     return ret;
46 | }
```

## 直线下有多少个格点

## 费用流

```
1 // Q is a priority_queue<PII, vector<PII>, greater<PII> >
 2 // for an edge(s, t): u is the capacity, v is the cost, nxt is the next edge,
 3 | // op is the opposite edge
 4 // this code can not deal with negative cycles
   typedef pair<int,int> PII;
 6 struct edge{ int t,u,v; edge *nxt,*op; }E[MAXE],*V[MAXV];
   int D[MAXN], dist[MAXN], maxflow, mincost; bool in[MAXN];
 8 bool modlabel(){
     while(!Q.empty()) Q.pop();
     for(int i=S;i<=T;++i) if(in[i]) D[i]=0,Q.push(PII(0,i)); else D[i]=inf;</pre>
10
      while(!Q.empty()){
11
       int x=Q.top().first,y=Q.top().second; Q.pop();
12
       if(y==T) break; if(D[y]<x) continue;</pre>
13
       for(edge *ii=V[y];ii;ii=ii->nxt) if(ii->u)
14
```

```
if(x+(ii->v+dist[ii->t]-dist[v])<D[ii->t]){
15
16
           D[ii->t]=x+(ii->v+dist[ii->t]-dist[v]):
           Q.push(PII(D[ii->t],ii->t));
17
18
         }
19
     if(D[T]==inf) return false;
20
     for(int i=S:i<=T:++i) if(D[i]>D[T]) dist[i]+=D[T]-D[i]:
21
22
     return true:
23
24
   int aug(int p,int limit){
     if(p==T) return maxflow+=limit,mincost+=limit*dist[S],limit;
25
26
     in[p]=1; int kk,ll=limit;
     for(edge *ii=V[p];ii;ii=ii->nxt) if(ii->u){
27
28
       if(!in[ii->t]&&dist[ii->t]+ii->v==dist[p]){
29
         kk=aug(ii->t,min(ii->u,ll)); ll-=kk,ii->u-=kk,ii->op->u+=kk;
30
         if(!ll) return in[p]=0,limit;
       }
31
32
     return limit-ll;
33
34
35
   PII mincostFlow(){
     for(int i=S;i<=T;++i) dist[i]=i==T?inf:0;</pre>
36
     while(!Q.empty()) Q.pop(); Q.push(PII(0,T));
37
38
     while(!Q.empty()){
       int x=Q.top().first,y=Q.top().second; Q.pop(); if(dist[y]<x) continue;</pre>
39
40
       for(edge *ii=V[y];ii;ii=ii->nxt) if(ii->op->u&&ii->v+x<dist[ii->t]
         dist[ii->t]=ii->v+x,Q.push(PII(dist[ii->t],ii->t));
41
     }
42
     maxflow=mincost=0;
     do{
44
45
       dof
46
         memset(in,0,sizeof(in));
       }while(aug(S,maxflow));
47
     }while(modlabel());
     return PII(maxflow,mincost);
49
50
```

## 积分表

```
\int \frac{1}{1+x^2} dx = \tan^{-1} x \qquad \int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \qquad \int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln|a^2+x^2| \qquad \int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \qquad \int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln|a^2+x^2|
 Integrals of Rational Functions
Integrals with Roots  \int \frac{x}{\sqrt{x\pm a}} dx = \frac{2}{3} (x\mp 2a) \sqrt{x\pm a} \qquad \int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} \qquad \int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a \ln\left[\sqrt{x} + \sqrt{x+a}\right] \qquad \int x \sqrt{x^2 \pm a^2} dx = \frac{1}{3} \left(x^2 \pm a^2\right)^{3/2} 
\int x\sqrt{ax+b}dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b} \qquad \int \sqrt{x(ax+b)}dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right] \qquad \int \sqrt{x^2\pm a^2}dx = \frac{1}{2}x\sqrt{x^2\pm a^2}\pm \frac{1}{2}a^2\ln\left|x + \sqrt{x^2\pm a^2}\right| + \frac{1}{2}a^2\ln\left|x + \sqrt{a(ax+b)}\right| + \frac{1}
\int \sqrt{x^3(ax+b)}dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right| \qquad \int \sqrt{a^2 - x^2}dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1}\frac{x}{\sqrt{a^2 - x^2}} \qquad \int \frac{x^2}{\sqrt{x^2 + a^2}}dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| + \frac{1}{2}a^2 \ln
\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} \int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} \int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} \int \sqrt{ax^2 + bx + c} dx = \frac{b + 2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| = -\sqrt{a^2 - x^2}
 \int x \sqrt{ax^2 + bx + c} = \frac{1}{48a^{5/2}} \left( 2\sqrt{a}\sqrt{ax^2 + bx + c} \right) \times \left( -3b^2 + 2abx + 8a(c + ax^2) \right) \\ +3(b^3 - 4abc) \ln \left| b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + c} \right| 
\int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{a} \sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| \qquad \int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 + bx}} \qquad \text{Integrals with Logarithms} \qquad \int \ln(ax + b) dx = \left(x + \frac{b}{a}\right) \ln(ax + b) - x, a \neq 0
\int \ln(x^2 + a^2) \, dx = x \ln(x^2 + a^2) + 2a \tan^{-1} \frac{x}{a} - 2x
\int \ln(x^2 - a^2) \, dx = x \ln(x^2 - a^2) + a \ln \frac{x+a}{x-a} - 2x
\int x \ln(ax+b) \, dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right) \ln(ax+b) \, dx
\int \ln \left( ax^2 + bx + c \right) dx = \frac{1}{a} \sqrt{4ac - b^2} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} - 2x + \left( \frac{b}{2a} + x \right) \ln \left( ax^2 + bx + c \right)
\int x \ln \left( a^2 - b^2 x^2 \right) dx = -\frac{1}{2} x^2 + \frac{1}{2} \left( x^2 - \frac{a^2}{b^2} \right) \ln \left( a^2 - b^2 x^2 \right)
Integrals with Exponentials
\int x^n e^{ax} \, \mathrm{d}x = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, \mathrm{d}x
\int x e^{-ax^2} \, \mathrm{d}x = -\frac{1}{2a} e^{-ax^2}
\int \cos^3 ax \, dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a}
\int \cos ax \sin bx \, dx = \frac{\cos((a-b)x)}{2(a-b)} - \frac{\cos((a+b)x)}{2(a+b)}, a \neq b
\int \sin^2 ax \cos bx \, dx = -\frac{\sin((2a-b)x)}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin((2a+b)x)}{4(2a+b)}
\int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x
 \int \cos^2 ax \sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \qquad \int \cos^2 ax \sin ax dx = -\frac{1}{3a} \cos^3 ax \qquad \int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \qquad \int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} + \frac{\sin 2bx}{32a} - \frac{\sin 2ax}{16(a+b)} + \frac{\sin 2ax}{32a} + \frac{\sin 2ax}{32a} - \frac{\sin 2ax}{32a} + \frac
 \int \tan ax dx = -\frac{1}{a} \ln \cos ax \qquad \int \tan^2 ax dx = -x + \frac{1}{a} \tan ax \qquad \int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \qquad \int \sec x dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2}\right) \qquad \int \sec^2 ax dx = \frac{1}{a} \tan ax
\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x|
\int \sec x \tan x \, dx = \sec x
\int \sec^2 x \tan x \, dx = \frac{1}{2} \sec^2 x
\int \sec^n x \tan x \, dx = \frac{1}{2} \sec^n x, n \neq 0
\int \csc x \, dx = \ln|\tan \frac{x}{2}| = \ln|\csc x - \cot x| + C
\int \csc^2 ax dx = -\frac{1}{a} \cot ax \int \csc^3 x dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln|\csc x - \cot x| \int \csc^n x \cot x dx = -\frac{1}{n} \csc^n x, n \neq 0 \int \sec x \csc x dx = \ln|\tan x|  Products of Trigonometric Functions and Monomials
 \int x \cos x dx = \cos x + x \sin x \qquad \qquad \int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \qquad \qquad \int x^2 \cos x dx = 2x \cos x + \left(x^2 - 2\right) \sin x \qquad \qquad \int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \qquad \qquad \int x \sin x dx = -x \cos x + \sin x
 \int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \qquad \int x^2 \sin x dx = \left(2 - x^2\right) \cos x + 2x \sin x \qquad \int x^2 \sin ax dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                Products of Trigonometric Functions and Exponentials
\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) \qquad \qquad \int e^{bx} \sin ax dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \qquad \qquad \int e^{bx} \cos ax dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \qquad \qquad \int x e^x \sin x dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x)
  \int xe^x \cos x dx = \frac{1}{2}e^x (x\cos x - \sin x + x\sin x) \quad \int e^x \cos x dx = \frac{1}{2}e^x (\sin x + \cos x)
```

```
Java
```

```
import java.io.*;
   import java.util.*;
   import java.math.*;
   public class Main{
     BufferedReader reader = new BufferedReader(new InputStreamReader(System.in));
     PrintWriter writer = new PrintWriter(System.out);
8
     StringTokenizer tokenizer = null;
9
10
     void solve() throws Exception {
11
      void run()throws Exception{
12
13
       try{
```

```
while (true) {
14
15
           solve():
16
17
18
       catch(Exception e){
19
20
       finallv{
21
         reader.close():
          writer.close():
22
23
24
25
     String next()throws Exception{
26
       for(;tokenizer == null || !tokenizer.hasMoreTokens();){
27
         tokenizer = new StringTokenizer(reader.readLine());
```

```
28
       }
       return tokenizer.nextToken();
29
30
     int nextInt()throws Exception{
31
32
       return Integer.parseInt(next());
33
     double nextDouble()throws Exception{
34
       return Double.parseDouble(next());
35
36
     BigInteger nextBigInteger()throws Exception{
37
       return new BigInteger(next());
38
     }
39
     public static void main(String args[])throws Exception{
40
       (new Main()).run();
41
42
    }
43 }
```

# Vimrc

```
1 \begin{lstlisting}
   set nu ai ci si mouse=a ts=4 sts=4 sw=4
   nmap<C-A> ggVG
   vmap<C-C> "+y
   nmap<F3> : vs %<.in <CR>
   nmap<F5> : !./%< <CR>
   nmap<F8> : !./%< < %<.in <CR>
10 nmap<F9> : !g++ % -o %< -Wall <CR>
11
   "nmap<F4> : !gedit % <CR>
   "autocmd BufNewFile *.cpp Or ~/temp.cpp
   "set hlsearch incseach
14
15
16 "syntax on
   "filetype plugin indent on
18 \end{lstlisting}
```