Dreadnought

Standard Code Library

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#### 二维几何

#### 二维几何基本操作

```
struct Point {
     Point rotate(const double ang) { // 逆时针旋转 ang 弧度
       return Point(cos(ang) * x - sin(ang) * y, cos(ang) * y + sin(ang) * x);
     Point turn90() { // 逆时针旋转 90 度
6
       return Point(-y, x);
    }
8 }:
9 Point isLL(const Line &11, const Line &12) {
     double s1 = det(12.b - 12.a, 11.a - 12.a),
11
          s2 = -det(12.b - 12.a, 11.b - 12.a):
12
     return (11.a * s2 + 11.b * s1) / (s1 + s2);
13 }
   bool onSeg(const Line &1, const Point &p) { // 点在线段上
14
     return sign(det(p - 1.a, 1.b - 1.a)) == 0 && sign(dot(p - 1.a, p - 1.b)) <= 0;
15
16 }
17
   Point projection(const Line &1, const Point &p) { // 点到直线投影
     return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
18
19 }
20 double disToLine(const Line &1, const Point &p) {
21
     return abs(det(p - 1.a, 1.b - 1.a) / (1.b - 1.a).len());
22 }
   double disToSeg(const Line &1, const Point &p) { // 点到线段距离
23
     return sign(dot(p - 1.a, 1.b - 1.a)) * sign(dot(p - 1.b, 1.a - 1.b)) != 1 ?
24
       disToLine(1, p) : min((p - 1.a).len(), (p - 1.b).len());
25
26 }
   Point symmetryPoint(const Point a, const Point b) { // 点 b 关于点 a 的中心对称点
27
     return a + a - b:
28
29 }
   Point reflection(const Line &1, const Point &p) { // 点关于直线的对称点
30
     return symmetryPoint(projection(1, p), p);
31
32 }
33 // 求圆与直线的交点
34 bool isCL(Circle a, Line 1, Point &p1, Point &p2) {
     double x = dot(l.a - a.o, l.b - l.a),
36
      y = (1.b - 1.a).len2(),
      d = x * x - y * ((1.a - a.o).len2() - a.r * a.r);
37
     if (sign(d) < 0) return false;
38
39
     d = max(d, 0.0);
     Point p = 1.a - ((1.b - 1.a) * (x / y)), delta = (1.b - 1.a) * (sqrt(d) / y);
40
41
     p1 = p + delta, p2 = p - delta;
     return true:
42
43 }
44 // 求圆与圆的交面积
45 double areaCC(const Circle &c1, const Circle &c2) {
     double d = (c1.0 - c2.0).len():
47
     if (sign(d - (c1.r + c2.r)) >= 0) {
```

```
48
       return 0:
49
50
     if (sign(d - abs(c1.r - c2.r)) \le 0) {
51
       double r = min(c1.r, c2.r):
52
       return r * r * PI;
53
     double x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d).
54
55
          t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
     return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1):
56
57
   // 求圆与圆的交点, 注意调用前要先判定重圆
   bool isCC(Circle a, Circle b, Point &p1, Point &p2) {
     double s1 = (a.o - b.o).len():
61
     if (sign(s1 - a.r - b.r) > 0 \mid \mid sign(s1 - abs(a.r - b.r)) < 0) return false;
     double s2 = (a.r * a.r - b.r * b.r) / s1;
     double aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
63
     Point o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
     Point delta = (b.o - a.o).unit().turn90() * newSqrt(a.r * a.r - aa * aa);
     p1 = o + delta, p2 = o - delta;
67
     return true:
68
   // 求点到圆的切点,按关于点的顺时针方向返回两个点
   bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
     double x = (p0 - c.o).len2(), d = x - c.r * c.r;
     if (d < EPS) return false; // 点在圆上认为没有切点
73
     Point p = (p0 - c.o) * (c.r * c.r / x);
     Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
75
     p1 = c.o + p + delta;
76
     p2 = c.o + p - delta;
77
     return true;
78
   // 求圆到圆的外共切线, 按关于 c1.o 的顺时针方向返回两条线
   vector<Line> extanCC(const Circle &c1, const Circle &c2) {
     vector<Line> ret;
82
     if (sign(c1.r - c2.r) == 0) {
83
       Point dir = c2.o - c1.o;
84
       dir = (dir * (c1.r / dir.len())).turn90():
85
       ret.push_back(Line(c1.o + dir, c2.o + dir));
86
       ret.push_back(Line(c1.o - dir, c2.o - dir));
87
88
       Point p = (c1.0 * -c2.r + c2.o * c1.r) / (c1.r - c2.r);
89
       Point p1, p2, q1, q2;
90
       if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
91
         if (c1.r < c2.r) swap(p1, p2), swap(q1, q2);
92
         ret.push_back(Line(p1, q1));
93
         ret.push_back(Line(p2, q2));
94
       }
95
     return ret:
97
```

```
98 // 求圆到圆的内共切线, 按关于 c1.o 的顺时针方向返回两条线
    vector<Line> intanCC(const Circle &c1, const Circle &c2) {
     vector<Line> ret:
100
     Point p = (c1.0 * c2.r + c2.0 * c1.r) / (c1.r + c2.r):
101
     Point p1, p2, q1, q2;
102
      if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
103
       ret.push_back(Line(p1, q1));
104
       ret.push_back(Line(p2, q2));
105
106
107
     return ret;
108 }
109 bool contain(vector Point polygon, Point p) { // 判断点 p
      → 是否被多边形包含, 包括落在边界上
     int ret = 0, n = polygon.size();
110
     for(int i = 0; i < n; ++ i) {
111
       Point u = polygon[i], v = polygon[(i + 1) % n];
112
113
       if (onSeg(Line(u, v), p)) return true;
114
       if (sign(u.y - v.y) \le 0) swap(u, v);
115
       if (sign(p.y - u.y) > 0 \mid | sign(p.y - v.y) \le 0) continue;
       ret += sign(det(p, v, u)) > 0;
116
117
     }
118
     return ret & 1;
119 }
120 vector<Point> convexCut(const vector<Point>&ps, Line 1) { // 用半平面 (q1,q2)
      → 的逆时针方向去切凸多边形
     vector<Point> qs;
121
122
     int n = ps.size();
     for (int i = 0; i < n; ++i) {
123
      Point p1 = ps[i], p2 = ps[(i + 1) \% n];
124
       int d1 = sign(det(1.a, 1.b, p1)), d2 = sign(det(1.a, 1.b, p2));
125
       if (d1 \ge 0) qs.push_back(p1);
126
127
       if (d1 * d2 < 0) qs.push_back(isLL(Line(p1, p2), 1));</pre>
128
     }
129
     return qs;
130 }
    vector<Point> convexHull(vector<Point> ps) { // 求点集 ps 组成的凸包
132
      int n = ps.size(); if (n <= 1) return ps;</pre>
133
     sort(ps.begin(), ps.end());
     vector<Point> qs;
134
     for (int i = 0; i < n; qs.push_back(ps[i++]))</pre>
135
136
       while (qs.size() > 1 \&\& sign(det(qs[qs.size()-2],qs.back(),ps[i])) \le 0)
      for (int i = n - 2, t = qs.size(); i \ge 0; qs.push_back(ps[i--]))
137
       while ((int)qs.size() > t && sign(det(qs[(int)qs.size()-2],qs.back(),ps[i])) <= 0)
      139
     qs.pop_back(); return qs;
140 }
```

## $n \log n$ 半平面交

```
struct Point {
     int quad() const { return sign(y) == 1 \mid | (sign(y) == 0 \&\& sign(x) >= 0); }
 3 };
   struct Line {
     bool include(const Point &p) const { return sign(det(b - a, p - a)) > 0; }
     Line push() const{ // 将半平面向外推 eps
       const double eps = 1e-6;
       Point delta = (b - a).turn90().norm() * eps;
       return Line(a - delta, b - delta);
10
11 };
12 bool sameDir(const Line &10, const Line &11) { return parallel(10, 11) && sign(dot(10.b
      \hookrightarrow - 10.a, 11.b - 11.a)) == 1; }
13 bool operator < (const Point &a, const Point &b) {
    if (a.quad() != b.quad()) {
15
       return a.quad() < b.quad();</pre>
16
    } else {
17
       return sign(det(a, b)) > 0;
18
19
   bool operator < (const Line &10, const Line &11) {
20
     if (sameDir(10, 11)) {
       return 11.include(10.a);
23
    } else {
24
       return (10.b - 10.a) < (11.b - 11.a);
25
26
   bool check(const Line &u, const Line &u, const Line &w) { return w.include(intersect(u,
      → v)); }
   vector<Point> intersection(vector<Line> &1) {
     sort(1.begin(), 1.end());
30
     deque<Line> a:
     for (int i = 0; i < (int)1.size(); ++i) {</pre>
31
32
       if (i && sameDir(l[i], l[i - 1])) {
33
         continue:
34
       }
35
       while (q.size() > 1 && !check(q[q.size() - 2], q[q.size() - 1], 1[i])) q.pop_back();
36
       while (q.size() > 1 && !check(q[1], q[0], l[i])) q.pop_front();
37
       q.push_back(l[i]);
38
39
     while (q.size() > 2 && !check(q[q.size() - 2], q[q.size() - 1], q[0])) q.pop_back();
40
     while (q.size() > 2 \&\& !check(q[1], q[0], q[q.size() - 1])) q.pop_front();
41
     vector<Point> ret;
     for (int i = 0; i < (int)q.size(); ++i) ret.push_back(intersect(q[i], q[(i + 1) %
      \hookrightarrow q.size()]));
     return ret;
44
```

#### 三角形的心

```
1 Point inCenter(const Point &A, const Point &B, const Point &C) { // 内心
     double a = (B - C).len(), b = (C - A).len(), c = (A - B).len(),
     s = fabs(det(B - A, C - A)),
     r = s / p;
    return (A * a + B * b + C * c) / (a + b + c);
6 }
   Point circumCenter(const Point &a, const Point &b, const Point &c) { // 外心
     Point bb = b - a. cc = c - a:
     double db = bb.len2(), dc = cc.len2(), d = 2 * det(bb, cc);
     return a - Point(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc) / d;
10
11 }
12 Point othroCenter(const Point &a, const Point &b, const Point &c) { // 垂心
     Point ba = b - a, ca = c - a, bc = b - c;
13
14
     double Y = ba.y * ca.y * bc.y,
15
         A = ca.x * ba.y - ba.x * ca.y,
16
         x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
17
          y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
    return Point(x0, y0);
19 }
```

#### 圆与多边形面积交

```
double areaCT(Point pa, Point pb, double r) {
     if (pa.len() < pb.len()) swap(pa, pb);</pre>
     if (sign(pb.len()) == 0) return 0;
     double a = pb.len(), b = pa.len(), c = (pb - pa).len();
     double sinB = fabs(det(pb, pb - pa) / a / c),
          cosB = dot(pb, pb - pa) / a / c,
          sinC = fabs(det(pa, pb) / a/ b),
          cosC = dot(pa, pb) / a / b;
     double B = atan2(sinB, cosB), C = atan2(sinC, cosC);
     if (a > r) {
10
11
      S = C / 2 * r * r:
12
      h = a * b * sinC / c:
13
      if (h < r && B < PI / 2) {
         S = (acos(h / r) * r * r - h * sqrt(r * r - h * h));
14
15
      }
16
    } else if (b > r) {
      double theta = PI - B - asin(sinB / r * a):
17
18
      S = a * r * sin(theta) / 2 + (C - theta) / 2 * r * r;
19
    } else {
       S = sinC * a * b / 2:
20
21
22
     return S;
23 }
```

# 圆的面积模板 $(n^2 \log n)$

```
struct Event {
     Point p;
     double ang;
    int delta;
     Event (Point p = Point(0, 0), double ang = 0, double delta = 0) : p(p), ang(ang),

    delta(delta) {}
6 }:
   bool operator < (const Event &a. const Event &b) {
     return a.ang < b.ang;</pre>
   void addEvent(const Circle &a, const Circle &b, vector<Event> &evt, int &cnt) {
     double d2 = (a.o - b.o).len2(),
          dRatio = ((a.r - b.r) * (a.r + b.r) / d2 + 1) / 2.
12
13
          pRatio = sqrt(-(d2 - sqr(a.r - b.r)) * (d2 - sqr(a.r + b.r)) / (d2 * d2 * 4));
14
     Point d = b.o - a.o, p = d.rotate(PI / 2),
15
         q0 = a.o + d * dRatio + p * pRatio,
16
         q1 = a.o + d * dRatio - p * pRatio;
17
     double ang 0 = (q0 - a.o).ang(),
          ang1 = (q1 - a.o).ang();
19
     evt.push_back(Event(q1, ang1, 1));
     evt.push_back(Event(q0, ang0, -1));
21
     cnt += ang1 > ang0:
22 }
23 bool issame(const Circle &a. const Circle &b) { return sign((a.o - b.o).len()) == 0 &&
      \hookrightarrow sign(a.r - b.r) == 0: }
24 bool overlap(const Circle &a, const Circle &b) { return sign(a.r - b.r - (a.o -
      \hookrightarrow b.o).len()) >= 0: }
25 bool intersect(const Circle &a, const Circle &b) { return sign((a.o - b.o).len() - a.r -
26 int C:
27 Circle c[N]:
28 double area[N];
29 void solve() {
     memset(area, 0, sizeof(double) * (C + 1));
     for (int i = 0; i < C; ++i) {
32
     int cnt = 1:
33
       vector<Event> evt;
34
       for (int j = 0; j < i; ++j) if (issame(c[i], c[j])) ++cnt;
35
       for (int j = 0; j < C; ++j) {
36
        if (j != i && !issame(c[i], c[j]) && overlap(c[j], c[i])) {
37
           ++cnt:
38
         }
39
40
       for (int j = 0; j < C; ++j) {
         if (j != i \&\& !overlap(c[j], c[i]) \&\& !overlap(c[i], c[j]) \&\& intersect(c[i].
      addEvent(c[i], c[j], evt, cnt);
42
43
44
```

```
if (evt.size() == 0) {
45
46
         area[cnt] += PI * c[i].r * c[i].r;
       } else {
47
         sort(evt.begin(). evt.end());
48
49
         evt.push_back(evt.front());
         for (int j = 0; j + 1 < (int)evt.size(); ++j) {</pre>
50
           cnt += evt[i].delta:
51
52
           area[cnt] += det(evt[j].p, evt[j + 1].p) / 2;
53
           double ang = evt[j + 1].ang - evt[j].ang;
54
           if (ang < 0) {
55
             ang += PI * 2;
56
           }
           area[cnt] += ang * c[i].r * c[i].r / 2 - sin(ang) * c[i].r * c[i].r / 2;
57
58
59
       }
60
61
   }
```

### 凸包快速询问

```
给定凸包, \log n 内完成各种询问, 具体操作有:
     1. 判定一个点是否在凸包内
3
     2. 询问凸包外的点到凸包的两个切点
     3. 询问一个向量关于凸包的切点
     4. 询问一条直线和凸包的交点
6
     INF 为坐标范围,需要定义点类大于号
8
     改成实数只需修改 sign 函数, 以及把 long long 改为 double 即可
9
     构造函数时传入凸包要求无重点,面积非空,以及 pair(x,y) 的最小点放在第一个
10
11 const int INF = 1000000000:
  struct Convex
13 {
14
    int n:
15
    vector<Point> a, upper, lower;
16
    Convex(vector<Point> _a) : a(_a) {
      n = a.size():
17
18
      int ptr = 0;
19
      for(int i = 1; i < n; ++ i) if (a[ptr] < a[i]) ptr = i;</pre>
      for(int i = 0; i <= ptr; ++ i) lower.push_back(a[i]);</pre>
20
21
      for(int i = ptr; i < n; ++ i) upper.push_back(a[i]);</pre>
22
      upper.push_back(a[0]);
23
    }
24
    int sign(long long x) { return x < 0 ? -1 : x > 0; }
25
    pair<long long, int> get_tangent(vector<Point> &convex, Point vec) {
26
      int 1 = 0, r = (int)convex.size() - 2:
27
      for(; 1 + 1 < r; ) {
        int mid = (1 + r) / 2;
28
29
        if (sign((convex[mid + 1] - convex[mid]).det(vec)) > 0) r = mid:
30
        else 1 = mid;
```

```
31
       }
32
       return max(make_pair(vec.det(convex[r]), r), make_pair(vec.det(convex[0]), 0));
33
     void update_tangent(const Point &p, int id, int &i0, int &i1) {
34
35
       if ((a[i0] - p).det(a[id] - p) > 0) i0 = id;
36
       if ((a[i1] - p).det(a[id] - p) < 0) i1 = id;
37
38
     void binary_search(int 1, int r, Point p, int &i0, int &i1) {
39
       if (1 == r) return:
40
       update_tangent(p, 1 % n, i0, i1);
41
       int sl = sign((a[l % n] - p).det(a[(l + 1) % n] - p));
42
       for(; 1 + 1 < r; ) {
43
         int mid = (1 + r) / 2:
44
         int smid = sign((a[mid % n] - p).det(a[(mid + 1) % n] - p));
45
         if (smid == sl) l = mid:
46
         else r = mid;
47
48
       update_tangent(p, r % n, i0, i1);
49
50
     int binary_search(Point u, Point v, int 1, int r) {
51
       int sl = sign((v - u).det(a[1 % n] - u));
52
       for(; 1 + 1 < r; ) {
53
         int mid = (1 + r) / 2:
54
         int smid = sign((v - u).det(a[mid % n] - u));
55
         if (smid == sl) l = mid;
56
         else r = mid:
       }
57
58
       return 1 % n;
59
     // 判定点是否在凸包内, 在边界返回 true
60
61
     bool contain(Point p) {
       if (p.x < lower[0].x || p.x > lower.back().x) return false;
63
       int id = lower_bound(lower.begin(), lower.end(), Point(p.x, -INF)) - lower.begin();
64
       if (lower[id].x == p.x) {
65
        if (lower[id].y > p.y) return false;
       } else if ((lower[id - 1] - p).det(lower[id] - p) < 0) return false;</pre>
       id = lower_bound(upper.begin(), upper.end(), Point(p.x, INF), greater<Point>()) -
67

    upper.begin();

       if (upper[id].x == p.x) {
         if (upper[id].y < p.y) return false;</pre>
69
      } else if ((upper[id - 1] - p).det(upper[id] - p) < 0) return false;</pre>
71
       return true:
72
    }
     // 求点 p
     → 关于凸包的两个切点,如果在凸包外则有序返回编号,共线的多个切点返回任意一个,否则返回
     bool get_tangent(Point p, int &i0, int &i1) {
75
       if (contain(p)) return false;
76
77
       int id = lower_bound(lower.begin(), lower.end(), p) - lower.begin();
```

```
binary search(0, id, p, i0, i1):
79
       binary_search(id, (int)lower.size(), p, i0, i1);
       id = lower_bound(upper.begin(), upper.end(), p, greater<Point>()) - upper.begin();
80
       binary_search((int)lower.size() - 1, (int)lower.size() - 1 + id, p, i0, i1);
81
82
       binary_search((int)lower.size() - 1 + id, (int)lower.size() - 1 + (int)upper.size(),
      \hookrightarrow p, i0, i1);
       return true:
83
84
     // 求凸包上和向量 vec 叉积最大的点,返回编号,共线的多个切点返回任意一个
85
     int get_tangent(Point vec) {
86
87
       pair<long long, int> ret = get_tangent(upper, vec);
       ret.second = (ret.second + (int)lower.size() - 1) % n;
88
       ret = max(ret, get_tangent(lower, vec));
89
90
       return ret.second;
91
     // 求凸包和直线 u,v 的交点,如果无严格相交返回 false.如果有则是和(i,next(i))的交点,
92
      → 两个点无序, 交在点上不确定返回前后两条线段其中之一
     bool get_intersection(Point u, Point v, int &i0, int &i1) {
93
94
       int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
       if (sign((v - u).det(a[p0] - u)) * sign((v - u).det(a[p1] - u)) < 0) {
95
         if (p0 > p1) swap(p0, p1);
96
97
         i0 = binary_search(u, v, p0, p1);
         i1 = binary_search(u, v, p1, p0 + n);
98
99
         return true;
100
       } else {
101
         return false;
102
       }
103
     }
104 };
```

# Delaunay 三角剖分

```
2 Delaunay Triangulation 随机增量算法:
  节点数至少为点数的 6 倍,空间消耗较大注意计算内存使用
  建图的过程在 build 中,注意初始化内存池和初始三角形的坐标范围 (Triangulation::LOTS)
5 Triangulation::find 返回包含某点的三角形
6 Triangulation::add_point 将某点加入三角剖分
  某个 Triangle 在三角剖分中当且仅当它的 has_children 为 0
8 | 如果要找到三角形 u 的邻域,则枚举它的所有 u.edge[i].tri, 该条边的两个点为 u.p[(i+1)%3],
     \hookrightarrow u.p[(i+2)\%3]
10 const int N = 100000 + 5, MAX_TRIS = N * 6;
11
  const double EPSILON = 1e-6, PI = acos(-1.0);
12 struct Point {
    double x,y; Point():x(0),y(0){} Point(double x, double y):x(x),y(y){}
14
    bool operator ==(Point const& that)const {return x==that.x&&y==that.y;}
15 };
16 inline double sqr(double x) { return x*x; }
17 double dist_sqr(Point const& a, Point const& b) {return sqr(a.x-b.x)+sqr(a.y-b.y);}
```

```
18 bool in_circumcircle(Point const& p1, Point const& p2, Point const& p3, Point const& p4)
19
     double u11 = p1.x - p4.x, u21 = p2.x - p4.x, u31 = p3.x - p4.x;
     double u12 = p1.y - p4.y, u22 = p2.y - p4.y, u32 = p3.y - p4.y;
21
     double u13 = sqr(p1.x) - sqr(p4.x) + sqr(p1.y) - sqr(p4.y);
     double u23 = sqr(p2.x) - sqr(p4.x) + sqr(p2.y) - sqr(p4.y);
23
     double u33 = sqr(p3.x) - sqr(p4.x) + sqr(p3.y) - sqr(p4.y);
     double det = -u13*u22*u31 + u12*u23*u31 + u13*u21*u32 - u11*u23*u32 - u12*u21*u33 +
      return det > EPSILON;
26
27
   double side(Point const& a, Point const& b, Point const& p) { return (b.x-a.x)*(p.y-a.y)
      \hookrightarrow - (b.y-a.y)*(p.x-a.x);}
   typedef int SideRef; struct Triangle; typedef Triangle* TriangleRef;
   struct Edge {
     TriangleRef tri; SideRef side; Edge() : tri(0), side(0) {}
31
     Edge(TriangleRef tri, SideRef side) : tri(tri), side(side) {}
32 }:
33
   struct Triangle {
     Point p[3]; Edge edge[3]; TriangleRef children[3]; Triangle() {}
     Triangle(Point const& p0, Point const& p1, Point const& p2) {
36
       p[0]=p0;p[1]=p1;p[2]=p2;children[0]=children[1]=children[2]=0;
37
38
     bool has_children() const { return children[0] != 0; }
     int num_children() const {
40
       return children[0] == 0 ? 0
41
         : children[1] == 0 ? 1
42
         : children[2] == 0 ? 2 : 3;
43
44
     bool contains(Point const& q) const {
45
       double a=side(p[0],p[1],q), b=side(p[1],p[2],q), c=side(p[2],p[0],q);
46
       return a >= -EPSILON && b >= -EPSILON && c >= -EPSILON:
47
   } triange_pool[MAX_TRIS], *tot_triangles;
48
   void set_edge(Edge a, Edge b) {
     if (a.tri) a.tri->edge[a.side] = b;
     if (b.tri) b.tri->edge[b.side] = a;
51
52
53
   class Triangulation {
54
     public:
55
       Triangulation() {
56
         const double LOTS = 1e6;
57
         the_root = new(tot_triangles++)
      → Triangle(Point(-LOTS, -LOTS), Point(+LOTS, -LOTS), Point(0,+LOTS));
58
59
       TriangleRef find(Point p) const { return find(the_root,p); }
60
       void add_point(Point const& p) { add_point(find(the_root,p),p); }
61
     private:
62
       TriangleRef the_root;
63
       static TriangleRef find(TriangleRef root, Point const& p) {
```

```
for(::) {
64
65
            if (!root->has_children()) return root;
66
            else for (int i = 0; i < 3 && root->children[i] ; ++i)
                if (root->children[i]->contains(p))
67
68
                  {root = root->children[i]; break;}
          }
69
        }
70
        void add_point(TriangleRef root, Point const& p) {
71
          TriangleRef tab,tbc,tca;
72
73
          tab = new(tot_triangles++) Triangle(root->p[0], root->p[1], p);
74
          tbc = new(tot_triangles++) Triangle(root->p[1], root->p[2], p);
          tca = new(tot_triangles++) Triangle(root->p[2], root->p[0], p);
75
          set_edge(Edge(tab,0),Edge(tbc,1));set_edge(Edge(tbc,0),Edge(tca,1));
76
77
          set_edge(Edge(tca,0),Edge(tab,1));set_edge(Edge(tab,2),root->edge[2]);
          set_edge(Edge(tbc,2),root->edge[0]);set_edge(Edge(tca,2),root->edge[1]);
78
          root->children[0]=tab;root->children[1]=tbc;root->children[2]=tca;
79
80
          flip(tab,2); flip(tbc,2); flip(tca,2);
81
82
        void flip(TriangleRef tri, SideRef pi) {
83
          TriangleRef trj = tri->edge[pi].tri; int pj = tri->edge[pi].side;
          if(!trj||!in_circumcircle(tri->p[0],tri->p[1],tri->p[2],trj->p[pj])) return;
84
          TriangleRef trk = new(tot_triangles++) Triangle(tri->p[(pi+1)%3], trj->p[pj],
85

    tri->p[pi]);
          TriangleRef trl = new(tot_triangles++) Triangle(trj->p[(pj+1)%3], tri->p[pi],
86
      \hookrightarrow \text{tri->p[pi]};
87
          set_edge(Edge(trk,0), Edge(trl,0));
          set_edge(Edge(trk,1), tri->edge[(pi+2)%3]); set_edge(Edge(trk,2),
88
      \hookrightarrow trj->edge[(pj+1)%3]);
          set_edge(Edge(trl,1), trj->edge[(pj+2)%3]); set_edge(Edge(trl,2),
89
      \hookrightarrow tri->edge[(pi+1)%3]);
          tri->children[0] = trk; tri->children[1] = trl; tri->children[2] = 0;
90
          trj->children[0] = trk; trj->children[1] = trl; trj->children[2] = 0;
91
92
          flip(trk,1); flip(trk,2); flip(trl,1); flip(trl,2);
        }
93
94 };
95 int n; Point ps[N];
96 void build(){
97
      tot_triangles = triange_pool; cin >> n;
      for(int i = 0; i < n; ++ i) scanf("%lf%lf", &ps[i].x, &ps[i].y);
      random_shuffle(ps, ps + n); Triangulation tri;
99
100
      for(int i = 0; i < n; ++ i) tri.add_point(ps[i]);</pre>
101 }
```

## 三维几何 三维几何基本操作

```
struct Point3D {
  double x, y, z;
  };
  Point3D det(const Point3D &a, const Point3D &b) {
```

```
return Point3D(a.v * b.z - a.z * b.v. a.z * b.x - a.x * b.z. a.x * b.v - a.v * b.x):
 6 }
 7 // 平面法向量 : 平面上两个向量叉积
 8 // 点共平面 : 平面上一点与之的向量点积法向量为 0
 9 // 点在线段 ( 直线 ) 上 : 共线且两边点积非正
10 // 点在三角形内 (不包含边界,需再判断是与某条边共线)
11 bool pointInTri(const Point3D &a, const Point3D &b, const Point3D &c, const Point3D &p)
12
             return sign(det(a - b, a - c).len() - det(p - a, p - b).len() - det(p - b, p -
               \rightarrow c).len() - det(p - c, p - a).len()) == 0;
13 }
14 // 共平面的两点是否在这平面上一条直线的同侧
       bool sameSide(const Point3D &a, const Point3D &b, const Point3D &p0, const Point3D &p1)
              return sign(dot(det(a - b, p0 - b), det(a - b, p1 - b))) > 0;
16
17
18 // 两点在平面同侧 : 点积法向量符号相同
19 // 两直线平行 / 垂直 : 同二维
20 // 平面平行 / 垂直 : 判断法向量
21 // 线面垂直 : 法向量和直线平行
22 // 判断空间线段是否相交 : 四点共面两线段不平行相互在异侧
23 // 线段和三角形是否相交 : 线段在三角形平面不同侧
               → 三角形任意两点在线段和第三点组成的平面的不同侧
24 // 求空间直线交点
25 Point3D intersection(const Point3D &a0, const Point3D &b0, const Point3D &a1, const
               → Point3D &b1) {
             double t = ((a0.x - a1.x) * (a1.y - b1.y) - (a0.y - a1.y) * (a1.x - b1.x)) / ((a0.x - a1.x) + (a1.y - b1.y)) / ((a0.x - a1.x) + (a1.y - b1.y)) / ((a0.x - a1.x) + (a1.y - b1.y)) / ((a0.x - a1.y) + (a1.y - a1.y)) / ((a0.x - 
               \rightarrow b0.x) * (a1.y - b1.y) - (a0.y - b0.y) * (a1.x - b1.x));
             return a0 + (b0 - a0) * t;
27
28 }
29 // 求平面和直线的交点
30 Point3D intersection(const Point3D &a, const Point3D &b, const Point3D &c, const Point3D
              \hookrightarrow &10, const Point3D &11) {
             Point3D p = pVec(a, b, c); // 平面法向量
             double t = (p.x * (a.x - 10.x) + p.y * (a.y - 10.y) + p.z * (a.z - 10.z)) / (p.x * (a.z -
              \leftrightarrow (11.x - 10.x) + p.y * (11.y - 10.y) + p.z * (11.z - 10.z));
             return 10 + (11 - 10) * t:
33
34 }
        // 求平面交线 : 取不平行的一条直线的一个交点,以及法向量叉积得到直线方向
36 // 点到直线距离 : 叉积得到三角形的面积除以底边
37 // 点到平面距离 : 点积法向量
38 // 直线间距离: 平行时随便取一点求距离,否则叉积方向向量得到方向点积计算长度
39 // 直线夹角 : 点积 平面夹角 : 法向量点积
40 // 三维向量旋转操作(绕向量 s 旋转 ang 角度), 对于右手系 s 指向观察者时逆时针
41 // 矩阵版
42 void rotate(const Point3D &s, double ang) {
          double 1 = s.len(), x = s.x / 1, y = s.y / 1, z = s.z / 1, sinA = sin(ang), cosA = sin(ang)
               \hookrightarrow \cos(\text{ang});
          double p[4][4] = {CosA + (1 - CosA) * x * x, (1 - CosA) * x * y - SinA * z, (1 - CosA)}
              \hookrightarrow * x * z + SinA * y, 0,
```

#### 三维凸包求重心

```
#define SIZE(X) (int(X.size()))
   #define PI 3.14159265358979323846264338327950288
   struct Point {
     Point cross(const Point &p) const {
       return Point(y * p.z - z * p.y, z * p.x - x * p.z, x * p.y - y * p.x);
    }
7 | } info[1005]:
8 int mark[1005][1005],n, cnt;;
9 double mix(const Point &a, const Point &b, const Point &c) {
     return a.dot(b.cross(c));
11 }
12 double area(int a, int b, int c) {
13
     return ((info[b] - info[a]).cross(info[c] - info[a])).length();
14 }
15 double volume(int a. int b. int c. int d) {
     return mix(info[b] - info[a], info[c] - info[a], info[d] - info[a]);
16
17 }
18 struct Face {
19
     int a. b. c: Face() {}
     Face(int a, int b, int c): a(a), b(b), c(c) {}
20
21
     int &operator [](int k) {
22
       if (k == 0) return a: if (k == 1) return b: return c:
23
    }
24 }:
   vector <Face> face:
   inline void insert(int a, int b, int c) {
     face.push back(Face(a, b, c)):
27
28 }
29
   void add(int v) {
     vector <Face> tmp; int a, b, c; cnt++;
30
31
     for (int i = 0; i < SIZE(face); i++) {
32
       a = face[i][0]; b = face[i][1]; c = face[i][2];
33
       if (Sign(volume(v, a, b, c)) < 0)
34
       mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] = mark[c][a] = mark[a][c] = cnt;
35
       else tmp.push_back(face[i]);
     } face = tmp:
36
37
     for (int i = 0; i < SIZE(tmp); i++) {</pre>
38
       a = face[i][0]; b = face[i][1]; c = face[i][2];
39
       if (mark[a][b] == cnt) insert(b, a, v):
40
       if (mark[b][c] == cnt) insert(c, b, v);
```

```
if (mark[c][a] == cnt) insert(a, c, v);
41
42
43
   }
   int Find() {
     for (int i = 2; i < n; i++) {
       Point ndir = (info[0] - info[i]).cross(info[1] - info[i]);
       if (ndir == Point()) continue; swap(info[i], info[2]);
47
48
       for (int j = i + 1; j < n; j++) if (Sign(volume(0, 1, 2, j)) != 0) {
49
         swap(info[j], info[3]); insert(0, 1, 2); insert(0, 2, 1); return 1;
50
       }
51
     }
52
     return 0;
53
54
   int main() {
     for (; scanf("%d", &n) == 1; ) {
       for (int i = 0; i < n; i++) info[i].Input();</pre>
56
57
       sort(info, info + n); n = unique(info, info + n) - info;
       face.clear(); random_shuffle(info, info + n);
59
       if (Find()) {
         memset(mark, 0, sizeof(mark)); cnt = 0;
60
         for (int i = 3; i < n; i++) add(i); vector<Point> Ndir;
61
62
         for (int i = 0; i < SIZE(face); ++i) {</pre>
           Point p = (info[face[i][0]] - info[face[i][1]]).cross(info[face[i][2]] -
63

    info[face[i][1]]);

64
           p = p / p.length(); Ndir.push_back(p);
65
66
         sort(Ndir.begin(), Ndir.end());
67
         int ans = unique(Ndir.begin(), Ndir.end()) - Ndir.begin();
68
         printf("%d\n", ans);
69
       } else printf("1\n");
70
71 }
   // 求重心
   double calcDist(const Point &p, int a, int b, int c) {
     return fabs(mix(info[a] - p, info[b] - p, info[c] - p) / area(a, b, c));
75
   //compute the minimal distance of center of any faces
   double findDist() { //compute center of mass
     double totalWeight = 0;
79
     Point center(.0, .0, .0);
     Point first = info[face[0][0]];
81
     for (int i = 0; i < SIZE(face); ++i) {</pre>
       Point p = (info[face[i][0]]+info[face[i][1]]+info[face[i][2]]+first)*.25;
82
       double weight = mix(info[face[i][0]] - first, info[face[i][1]] - first,

    info[face[i][2]] - first);

       totalWeight += weight; center = center + p * weight;
85
86
     center = center / totalWeight;
     double res = 1e100; //compute distance
     for (int i = 0; i < SIZE(face); ++i)</pre>
```

```
89    res = min(res, calcDist(center, face[i][0], face[i][1], face[i][2]));
90    return res;
91 }
```

#### 求四点外界球

```
1 // 注意,无法处理小于四点的退化情况
   int nouter; Tpoint outer[4], res; double radius;
   void ball() {
     Tpoint q[3]; double m[3][3], sol[3], L[3], det;
    int i, j; res.x = res.y = res.z = radius = 0;
     for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol[i]=dot(q[i], q[i]);
     for (i=0;i<3;++i) for(j=0;j<3;++j) m[i][j]=dot(q[i],q[j])*2;
     det= m[0][0]*m[1][1]*m[2][2]
    + m[0][1]*m[1][2]*m[2][0]
     + m[0][2]*m[2][1]*m[1][0]
10
11
     - m[0][2]*m[1][1]*m[2][0]
12
     - m[0][1]*m[1][0]*m[2][2]
13
     - m[0][0]*m[1][2]*m[2][1];
14
     if (fabs(det) < eps ) return;</pre>
15
     for (j=0; j<3; ++j) {
      for (i=0; i<3; ++i) m[i][j]=sol[i];</pre>
16
17
       L[j] = (m[0][0]*m[1][1]*m[2][2]
18
       + m[0][1]*m[1][2]*m[2][0]
19
       + m[0][2]*m[2][1]*m[1][0]
20
       - m[0][2]*m[1][1]*m[2][0]
21
       - m[0][1]*m[1][0]*m[2][2]
       - m[0][0]*m[1][2]*m[2][1]
22
23
24
       for (i=0; i<3; ++i) m[i][j]=dot(q[i], q[j])*2;</pre>
     } res=outer[0]:
25
26
     for (i=0; i<3; ++i ) res = res + q[i] * L[i];</pre>
27
     radius=dist2(res, outer[0]);
28 }
```

## 图论

## Hungarian

```
12
       int i0 = m:
13
       for (int j = 0; j \le m; ++j) minv[j] = INF, used[j] = false;
14
15
         used[i0] = true:
16
         int i0 = p[j0], delta = INF, j1;
17
         for (int j = 0; j < m; ++j) {
18
           if (!used[j]) {
19
              int cur = a[i0][j] - u[i0] - v[j];
20
              if (cur < minv[j]) minv[j] = cur, fa[j] = j0;</pre>
21
              if (minv[j] < delta) delta = minv[j], j1 = j;</pre>
22
23
         }
24
         for (int j = 0; j \le m; ++j) {
           if (used[j]) {
25
26
              u[p[j]] += delta, v[j] -= delta;
27
           } else {
28
              minv[j] -= delta;
29
30
         }
31
         j0 = j1;
32
       } while (p[j0] != n);
33
       do {
34
         int j1 = fa[j0];
35
         p[j0] = p[j1];
36
         j0 = j1;
37
       } while (j0 != m);
38
39
     return -v[m];
40
```

# Hopcroft

```
1 // 左侧 N 个点, 右侧 K 个点, 1-based, 初始化将 matx[], matv[] 都置为 0
   int N, K;
   int que[N], dx[N], dy[N], matx[N], maty[N];
   int BFS()
    int flag = 0, qt = 0, qh = 0;
    for(int i = 1; i <= K; ++ i) dy[i] = 0;
    for(int i = 1; i <= N; ++ i) {
      dx[i] = 0;
10
      if (! matx[i]) que[qt ++] = i;
11
    }
12
     while (qh < qt) {
13
      int u = que[qh ++];
14
      for(Edge *e = E[u]; e; e = e->n)
15
        if (! dy[e->t]) {
          dy[e->t] = dx[u] + 1;
16
17
          if (! maty[e->t]) flag = true;
```

```
else {
18
19
              dx[maty[e->t]] = dx[u] + 2;
20
             que[qt ++] = maty[e->t];
21
22
         }
23
24
     return flag;
25
   }
   int DFS(int u)
26
27 | {
28
     for (Edge *e = E[u]; e; e = e->n)
29
       if (dy[e->t] == dx[u] + 1) {
30
         dv[e->t] = 0;
31
         if (! maty[e->t] || DFS(maty[e->t])) {
32
           matx[u] = e->t; maty[e->t] = u;
33
           return true;
34
         }
35
       }
36
     return false;
37
   void Hopcroft()
39
40
     while (BFS()) for(int i = 1; i <= N; ++ i) if (! matx[i]) DFS(i);</pre>
41 }
```

#### 最大团

```
1 // Super Fast Maximum Clique
2 // To Build Graph: Maxclique(Edges, Number of Nodes)
3 // To Get Answer: mcqdyn(AnswerNodes Index Array, AnswserLength)
   typedef bool BB[N]:
   struct Maxclique {
     const BB* e; int pk, level; const float Tlimit;
     struct Vertex{ int i. d: Vertex(int i):i(i).d(0){} }:
     typedef vector<Vertex> Vertices; typedef vector<int> ColorClass;
     Vertices V; vector<ColorClass> C; ColorClass QMAX, Q;
     static bool desc_degree(const Vertex &vi, const Vertex &vj){
10
11
       return vi.d > vj.d;
12
     void init colors(Vertices &v){
13
14
       const int max_degree = v[0].d;
15
       for(int i = 0; i < (int)v.size(); i++) v[i].d = min(i, max_degree) + 1;</pre>
     }
16
17
     void set_degrees(Vertices &v){
      for(int i = 0, j; i < (int)v.size(); i++)</pre>
18
19
         for(v[i].d = j = 0; j < int(v.size()); j++)</pre>
20
           v[i].d += e[v[i].i][v[j].i];
21
     struct StepCount{ int i1, i2; StepCount():i1(0),i2(0){} };
22
23
     vector<StepCount> S;
```

```
bool cut1(const int pi, const ColorClass &A){
25
       for(int i = 0; i < (int)A.size(); i++) if (e[pi][A[i]]) return true;</pre>
26
       return false:
27
28
     void cut2(const Vertices &A, Vertices &B){
       for(int i = 0: i < (int)A.size() - 1: i++)</pre>
30
         if(e[A.back().i][A[i].i])
31
           B.push_back(A[i].i);
32
33
     void color_sort(Vertices &R){
34
       int j = 0, maxno = 1, min_k = max((int)QMAX.size() - (int)Q.size() + 1, 1);
35
       C[1].clear(), C[2].clear();
       for(int i = 0; i < (int)R.size(); i++) {</pre>
36
37
         int pi = R[i].i, k = 1;
38
         while(cut1(pi, C[k])) k++;
39
         if(k > maxno) maxno = k, C[maxno + 1].clear();
40
         C[k].push_back(pi);
41
         if(k < min_k) R[j++].i = pi;</pre>
42
43
       if(j > 0) R[j - 1].d = 0;
44
       for(int k = min_k; k <= maxno; k++)</pre>
45
         for(int i = 0; i < (int)C[k].size(); i++)
46
           R[j].i = C[k][i], R[j++].d = k;
47
48
     void expand_dyn(Vertices &R){// diff -> diff with no dyn
49
       S[level].i1 = S[level].i1 + S[level - 1].i1 - S[level].i2;//diff
50
       S[level].i2 = S[level - 1].i1;//diff
51
       while((int)R.size()) {
52
         if((int)Q.size() + R.back().d > (int)QMAX.size()){
53
           Q.push_back(R.back().i); Vertices Rp; cut2(R, Rp);
           if((int)Rp.size()){
54
55
              if((float)S[level].i1 / ++pk < Tlimit) degree_sort(Rp);//diff</pre>
56
              color_sort(Rp);
57
              S[level].i1++, level++;//diff
58
              expand_dyn(Rp);
59
              level--;//diff
60
61
           else if((int)Q.size() > (int)QMAX.size()) QMAX = Q;
62
           Q.pop_back();
63
64
         else return:
65
         R.pop_back();
66
67
68
     void mcqdyn(int* maxclique, int &sz){
69
       set_degrees(V); sort(V.begin(), V.end(), desc_degree); init_colors(V);
70
       for(int i = 0; i < (int)V.size() + 1; i++)S[i].i1 = S[i].i2 = 0;
71
       expand_dyn(V); sz = (int)QMAX.size();
72
       for(int i = 0; i < (int)QMAX.size(); i++) maxclique[i] = QMAX[i];</pre>
73
```

```
void degree_sort(Vertices &R){
    set_degrees(R); sort(R.begin(), R.end(), desc_degree);
}

Maxclique(const BB* conn, const int sz, const float tt = 0.025) \
    : pk(0), level(1), Tlimit(tt){
    for(int i = 0; i < sz; i++) V.push_back(Vertex(i));
    e = conn, C.resize(sz + 1), S.resize(sz + 1);
}

2}
};</pre>
```

### 最小树形图

```
const int MAXN,INF;// INF >= sum( W_ij )
   int from[MAXN + 10] [MAXN * 2 + 10],n,m,edge[MAXN + 10] [MAXN * 2 + 10];
3 int sel[MAXN * 2 + 10], fa[MAXN * 2 + 10], vis[MAXN * 2 + 10];
   int getfa(int x){if(x == fa[x]) return x; return fa[x] = getfa(fa[x]);}
   void liuzhu(){ // 1-base: root is 1. answer = (sel[i]. i) for i in [2..n]
     fa[1] = 1:
     for(int i = 2; i \le n; ++i){
       sel[i] = 1; fa[i] = i;
       for(int j = 1; j <= n; ++j) if(fa[j] != i)
10
         if(from[i][i] = i, edge[sel[i]][i] > edge[i][i]) sel[i] = j;
11
12
     int limit = n;
     while(1){
13
       int prelimit = limit; memset(vis, 0, sizeof(vis)); vis[1] = 1;
14
15
       for(int i = 2; i <= prelimit; ++i) if(fa[i] == i && !vis[i]){</pre>
         int j = i; while(!vis[j]) vis[j] = i, j = getfa(sel[j]);
16
17
         if(j == 1 || vis[j] != i) continue; vector<int> C; int k = j;
18
         do C.push_back(k), k = getfa(sel[k]); while(k != j);
19
         ++limit:
         for(int i = 1: i \le n: ++i){
20
21
           edge[i][limit] = INF, from[i][limit] = limit;
22
         }
23
         fa[limit] = vis[limit] = limit;
24
         for(int i = 0; i < int(C.size()); ++i){</pre>
           int x = C[i], fa[x] = limit:
25
26
           for(int j = 1; j \le n; ++j)
27
             if(edge[j][x] != INF && edge[j][limit] > edge[j][x] - edge[sel[x]][x]){
               edge[j][limit] = edge[j][x] - edge[sel[x]][x];
28
29
               from[j][limit] = x;
             }
30
31
         }
32
         for(int j=1;j<=n;++j) if(getfa(j)==limit) edge[j][limit] = INF;</pre>
33
         sel[limit] = 1;
34
         for(int j = 1; j \le n; ++j)
35
           if(edge[sel[limit]][limit] > edge[j][limit]) sel[limit] = j;
36
37
       if(prelimit == limit) break:
38
```

```
39     for(int i = limit; i > 1; --i) sel[from[sel[i]][i]] = sel[i];
40 }
```

#### 带花树

```
vector<int> link[maxn];
 2 int n.match[maxn].Queue[maxn].head.tail:
   int pred[maxn],base[maxn],start,finish,newbase;
   bool InQueue[maxn], InBlossom[maxn];
   void push(int u){ Queue[tail++]=u:InQueue[u]=true: }
   int pop(){ return Queue[head++]; }
   int FindCommonAncestor(int u,int v){
     bool InPath[maxn]:
     for(int i=0;i<n;i++) InPath[i]=0;</pre>
     while(true){ u=base[u]; InPath[u]=true; if(u==start) break; u=pred[match[u]]; }
     while(true){ v=base[v];if(InPath[v]) break;v=pred[match[v]]; }
11
12
     return v:
13 }
   void ResetTrace(int u){
15
     int v:
16
     while(base[u]!=newbase){
17
       v=match[u]:
18
       InBlossom[base[u]]=InBlossom[base[v]]=true;
19
       u=pred[v]:
20
       if(base[u]!=newbase) pred[u]=v;
21
22
   void BlossomContract(int u.int v){
     newbase=FindCommonAncestor(u.v):
25
     for (int i=0:i<n:i++)
26
     InBlossom[i]=0:
27
     ResetTrace(u);ResetTrace(v);
     if(base[u]!=newbase) pred[u]=v:
     if(base[v]!=newbase) pred[v]=u;
30
     for(int i=0;i<n;++i)</pre>
31
     if(InBlossom[base[i]]){
32
       base[i]=newbase;
33
       if(!InQueue[i]) push(i);
34
35
   bool FindAugmentingPath(int u){
37
     bool found=false:
     for(int i=0;i<n;++i) pred[i]=-1,base[i]=i;</pre>
39
     for (int i=0;i<n;i++) InQueue[i]=0;</pre>
     start=u:finish=-1: head=tail=0: push(start):
41
     while(head<tail){
42
       int u=pop();
       for(int i=link[u].size()-1:i>=0:i--){
43
44
         int v=link[u][i];
```

```
if (base[u]!=base[v]&&match[u]!=v)
45
46
            if(v==start||(match[v]>=0&&pred[match[v]]>=0))
47
              BlossomContract(u,v);
            else if(pred[v]==-1){
48
49
              pred[v]=u;
              if(match[v]>=0) push(match[v]);
50
              else{ finish=v; return true; }
51
52
           }
       }
53
54
55
     return found;
56
   void AugmentPath(){
57
     int u=finish,v,w;
58
     while(u>=0){ v=pred[u]; w=match[v]; match[v]=u; match[u]=v; u=w; }
59
60
61
   void FindMaxMatching(){
     for(int i=0;i<n;++i) match[i]=-1;</pre>
62
     for(int i=0;i<n;++i) if(match[i]==-1) if(FindAugmentingPath(i)) AugmentPath();</pre>
63
64
   }
```

## **Dominator Tree**

```
vector<int> prec[N], succ[N];
   vector<int> ord:
   int stamp, vis[N];
   int num[N];
   int fa[N]:
   void dfs(int u) {
     vis[u] = stamp;
     num[u] = ord.size();
     ord.push_back(u);
10
     for (int i = 0; i < (int)succ[u].size(); ++i) {</pre>
       int v = succ[u][i]:
11
12
       if (vis[v] != stamp) {
13
        fa[v] = u;
         dfs(v);
14
15
16
17 }
   int fs[N], mins[N], dom[N], sem[N];
   int find(int u) {
19
     if (u != fs[u]) {
20
21
       int v = fs[u];
22
       fs[u] = find(fs[u]);
23
       if (mins[v] != -1 && num[sem[mins[v]]] < num[sem[mins[u]]]) {
24
         mins[u] = mins[v];
25
       }
26
27
     return fs[u];
```

```
28 }
   void merge(int u, int v) { fs[u] = v; }
30 vector<int> buf[N];
31 int buf2[N]:
   void mark(int source) {
     ord.clear():
34
     ++stamp;
35
     dfs(source);
36
     for (int i = 0; i < (int)ord.size(); ++i) {</pre>
37
       int u = ord[i];
38
       fs[u] = u, mins[u] = -1, buf2[u] = -1;
39
     for (int i = (int)ord.size() - 1; i > 0; --i) {
41
       int u = ord[i], p = fa[u];
       sem[u] = p;
42
       for (int j = 0; j < (int)prec[u].size(); ++j) {</pre>
43
44
         int v = prec[u][j];
45
         if (use[v] != stamp) continue;
         if (num[v] > num[u]) {
46
47
            find(v); v = sem[mins[v]];
48
         if (num[v] < num[sem[u]]) {</pre>
49
50
            sem[u] = v;
51
         }
52
53
       buf[sem[u]].push_back(u);
54
       mins[u] = u;
55
       merge(u, p);
56
       while (buf[p].size()) {
         int v = buf[p].back();
57
58
         buf[p].pop_back();
59
         find(v):
60
         if (sem[v] == sem[mins[v]]) {
           dom[v] = sem[v];
61
62
         } else {
63
            buf2[v] = mins[v];
64
       }
65
     dom[ord[0]] = ord[0];
67
     for (int i = 0; i < (int)ord.size(); ++i) {</pre>
69
       int u = ord[i];
       if (~buf2[u]) {
70
71
         dom[u] = dom[buf2[u]];
72
       }
73
    }
74
```

主流

```
1 // Q is a priority_queue<PII, vector<PII>, greater<PII> >
2 // for an edge(s, t): u is the capacity, v is the cost, nxt is the next edge,
3 // op is the opposite edge
4 // this code can not deal with negative cycles
   typedef pair<int,int> PII;
   struct edge{ int t,u,v; edge *nxt,*op; }E[MAXE],*V[MAXV];
   int D[MAXN], dist[MAXN], maxflow, mincost; bool in[MAXN];
   bool modlabel(){
     while(!Q.empty()) Q.pop();
     for(int i=S;i<=T;++i) if(in[i]) D[i]=0,Q.push(PII(0,i)); else D[i]=inf;</pre>
10
11
     while(!Q.empty()){
12
       int x=Q.top().first,y=Q.top().second; Q.pop();
       if(y==T) break; if(D[y]<x) continue;</pre>
13
14
       for(edge *ii=V[y];ii;ii=ii->nxt) if(ii->u)
         if(x+(ii->v+dist[ii->t]-dist[v])<D[ii->t]){
15
           D[ii->t]=x+(ii->v+dist[ii->t]-dist[v]);
16
17
           Q.push(PII(D[ii->t],ii->t));
         }
18
19
20
     if(D[T]==inf) return false;
21
     for(int i=S;i<=T;++i) if(D[i]>D[T]) dist[i]+=D[T]-D[i];
22
     return true:
23 }
   int aug(int p,int limit){
24
     if(p==T) return maxflow+=limit,mincost+=limit*dist[S],limit;
26
     in[p]=1; int kk,ll=limit;
     for(edge *ii=V[p];ii;ii=ii->nxt) if(ii->u){
27
       if(!in[ii->t]&&dist[ii->t]+ii->v==dist[p]){
28
29
         kk=aug(ii->t,min(ii->u,ll)); ll-=kk,ii->u-=kk,ii->op->u+=kk;
30
         if(!ll) return in[p]=0,limit;
       }
31
32
     }
     return limit-ll:
33
34
35 PII mincostFlow(){
     for(int i=S:i<=T:++i) dist[i]=i==T?inf:0:</pre>
37
     while(!Q.empty()) Q.pop(); Q.push(PII(0,T));
38
     while(!Q.empty()){
       int x=Q.top().first,y=Q.top().second; Q.pop(); if(dist[y]<x) continue;</pre>
39
       for(edge *ii=V[v];ii;ii=ii->nxt) if(ii->op->u&&ii->v+x<dist[ii->t]
40
41
         dist[ii->t]=ii->v+x,Q.push(PII(dist[ii->t],ii->t));
     }
42
43
     maxflow=mincost=0:
44
     do{
45
       do{
46
         memset(in,0,sizeof(in));
47
       }while(aug(S,maxflow));
     }while(modlabel()):
48
     return PII(maxflow,mincost);
```

```
50 }
```

#### 无向图最小割

```
int cost[maxn] [maxn], seq[maxn], len[maxn], n, m, pop, ans;
   bool used[maxn];
   void Init(){
     int i,j,a,b,c;
     for(i=0;i<n;i++) for(j=0;j<n;j++) cost[i][j]=0;</pre>
     for(i=0;i<m;i++){</pre>
       scanf("%d %d %d",&a,&b,&c); cost[a][b]+=c; cost[b][a]+=c;
     pop=n; for(i=0;i<n;i++) seq[i]=i;
10
11
   void Work(){
     ans=inf; int i,j,k,l,mm,sum,pk;
13
     while(pop > 1){
       for(i=1;i<pop;i++) used[seq[i]]=0; used[seq[0]]=1;</pre>
14
15
       for(i=1;i<pop;i++) len[seq[i]]=cost[seq[0]][seq[i]];</pre>
16
       pk=0; mm=-inf; k=-1;
       for(i=1;i<pop;i++) if(len[seq[i]] > mm){ mm=len[seq[i]]; k=i; }
17
18
       for(i=1;i<pop;i++){</pre>
19
         used[seq[l=k]]=1;
20
          if(i==pop-2) pk=k;
21
          if(i==pop-1) break;
22
          mm=-inf:
23
          for(j=1;j<pop;j++) if(!used[seq[j]])</pre>
24
            if((len[seq[j]]+=cost[seq[1]][seq[j]]) > mm)
25
              mm=len[seq[j]], k=j;
26
       }
27
       sum=0;
28
        for(i=0;i<pop;i++) if(i != k) sum+=cost[seq[k]][seq[i]];</pre>
29
       ans=min(ans.sum):
30
       for(i=0;i<pop;i++)</pre>
31
          cost[seq[k]][seq[i]]=cost[seq[i]][seq[k]]+=cost[seq[pk]][seq[i]];
32
       seq[pk]=seq[--pop];
33
     printf("%d\n",ans);
35 }
```

# 数论

#### 素数判定

```
int strong_pseudo_primetest(long long n,int base) {
   long long n2=n-1,res;
   int s=0;
   while(n2%2==0) n2>>=1,s++;
   res=powmod(base,n2,n);
```

```
if((res==1)||(res==n-1)) return 1:
       s--;
       while(s \ge 0) {
           res=mulmod(res.res.n):
10
           if(res==n-1) return 1;
11
12
       }
13
       return 0; // n is not a strong pseudo prime
14 }
   int isprime(long long n) {
16
     static LL testNum[]={2,3,5,7,11,13,17,19,23,29,31,37};
     static LL lim[]={4,0,1373653LL,25326001LL,25000000000LL,2152302898747LL,
17
      \rightarrow 3474749660383LL,341550071728321LL,0,0,0,0);
     if(n<2||n==3215031751LL) return 0;
18
     for(int i=0;i<12;++i){</pre>
19
       if(n<lim[i]) return 1;</pre>
20
21
       if(strong_pseudo_primetest(n,testNum[i])==0) return 0;
22
23
     return 1;
24
   }
```

#### 启发式分解

```
int ansn; LL ans[1000];
2 LL func(LL x,LL n){ return(mod_mul(x,x,n)+1)%n; }
3 LL Pollard(LL n){
    LL i,x,y,p;
     if(Rabin Miller(n)) return n:
    if(!(n&1)) return 2;
    for(i=1;i<20;i++){
     x=i; y=func(x,n); p=gcd(y-x,n);
9
      while (p=1) {x=func(x,n); y=func(func(y,n),n); p=gcd((y-x+n)\%n,n)\%n;}
10
      if(p==0||p==n) continue;
11
       return p;
12
    }
13 }
   void factor(LL n){
14
15
     LL x;
16
     x=Pollard(n):
17
     if(x==n){ ans[ansn++]=x; return; }
18
     factor(x), factor(n/x);
19 }
```

#### 直线下整点个数

```
 \begin{bmatrix} 1 \\ // \sum_{i=0}^{n-1} \lfloor \frac{a+bi}{m} \rfloor, & n, m, a, b > 0 \\ \text{LL solve(LL n,LL a,LL b,LL m)} \{ \\ 3 \\ \text{if (b==0) return n*(a/m);} \\ 4 \\ \text{if (a>=m) return n*(a/m)+solve(n,a\%m,b,m);} \end{bmatrix}
```

```
if(b>=m) return (n-1)*n/2*(b/m)+solve(n,a,b%m,m);
return solve((a+b*n)/m,(a+b*n)%m,m,b);
}
```

#### 二次剩余

```
void calcH(int &t, int &h, const int p) {
     int tmp = p - 1; for (t = 0; (tmp & 1) == 0; tmp /= 2) t++; h = tmp;
   // solve equation x^2 \mod p = a
   bool solve(int a, int p, int &x, int &y) {
     srand(19920225);
     if (p == 2) { x = y = 1; return true; }
     int p2 = p / 2, tmp = power(a, p2, p);
     if (tmp == p - 1) return false;
     if ((p + 1) \% 4 == 0) {
11
       x = power(a, (p + 1) / 4, p); y = p - x; return true;
12
13
       int t, h, b, pb; calcH(t, h, p);
14
       if (t >= 2) {
15
        do \{b = rand() \% (p - 2) + 2;
16
        } while (power(b, p / 2, p) != p - 1);
17
         pb = power(b, h, p);
18
       } int s = power(a, h / 2, p);
19
       for (int step = 2; step <= t; step++) {</pre>
20
         int ss = (((long long)(s * s) % p) * a) % p;
21
         for (int i = 0; i < t - step; i++) ss = ((long long)ss * ss) % p;
         if (ss + 1 == p) s = (s * pb) % p; pb = ((long long)pb * pb) % p;
23
       x = ((long long)s * a) % p; y = p - x;
24
     } return true:
25
```

## Pell 方程

```
1 ULL A,B,p[maxn],q[maxn],a[maxn],g[maxn],h[maxn];
2 int main() {
     for (int test=1, n; scanf("%d", &n) && n; ++test) {
      printf("Case %d: ",test);
       if (fabs(sqrt(n)-floor(sqrt(n)+1e-7))<=1e-7) {</pre>
         int a=(int)(floor(sqrt(n)+1e-7)); printf("%d %d\n",a,1);
      } else {
        // 求 x^2 - ny^2 = 1 的最小正整数根, n 不是完全平方数
         p[1]=q[0]=h[1]=1;p[0]=q[1]=g[1]=0;
         a[2]=(int)(floor(sqrt(n)+1e-7));
10
11
         for (int i=2;i;++i) {
12
          g[i]=-g[i-1]+a[i]*h[i-1]; h[i]=(n-sqr(g[i]))/h[i-1];
          a[i+1]=(g[i]+a[2])/h[i]; p[i]=a[i]*p[i-1]+p[i-2];
13
14
           q[i]=a[i]*q[i-1]+q[i-2];
```

```
if (sqr((ULL)(p[i]))-n*sqr((ULL)(q[i]))==1){
          A=p[i];B=q[i];break;
}

cout << A << ' ' << B <<endl;
}

}

20    }

21    }
</pre>
```

### 代数 FFT

```
// double 精度对10^9+7 取模最多可以做到2^{20}
   const int MOD = 1000003:
   const double PI = acos(-1);
   typedef complex<double> Complex;
   const int N = 65536, L = 15, MASK = (1 << L) - 1;
   Complex w[N];
   void FFTInit() {
    for (int i = 0; i < N; ++i) {
       w[i] = Complex(cos(2 * i * PI / N), sin(2 * i * PI / N));
10
11 }
   void FFT(Complex p[], int n) {
     for (int i = 1, j = 0; i < n - 1; ++i) {
13
14
       for (int s = n; j = s >= 1, ~j & s;);
15
       if (i < j) {</pre>
16
         swap(p[i], p[j]);
       }
17
18
19
     for (int d = 0: (1 << d) < n: ++d) {
20
       int m = 1 \ll d, m2 = m * 2, rm = n >> (d + 1);
21
       for (int i = 0; i < n; i += m2) {
22
         for (int i = 0: i < m: ++i) {
23
           Complex &p1 = p[i + j + m], &p2 = p[i + j];
24
           Complex t = w[rm * j] * p1;
25
           p1 = p2 - t;
26
           p2 = p2 + t;
27
28
       }
29
30
   Complex A[N], B[N], C[N], D[N];
32
   void mul(int a[N], int b[N]) {
     for (int i = 0; i < N; ++i) {
33
       A[i] = Complex(a[i] >> L, a[i] & MASK);
34
35
       B[i] = Complex(b[i] >> L, b[i] & MASK);
36
     FFT(A, N), FFT(B, N):
37
     for (int i = 0; i < N; ++i) {
```

```
39
       int i = (N - i) \% N:
40
       Complex da = (A[i] - conj(A[j])) * Complex(0, -0.5),
41
           db = (A[i] + conj(A[j])) * Complex(0.5, 0),
42
           dc = (B[i] - conj(B[j])) * Complex(0, -0.5),
43
           dd = (B[i] + conj(B[j])) * Complex(0.5, 0);
       C[j] = da * dd + da * dc * Complex(0, 1);
44
       D[j] = db * dd + db * dc * Complex(0, 1);
45
46
     FFT(C, N), FFT(D, N);
47
     for (int i = 0; i < N; ++i) {
48
49
       long long da = (long long)(C[i].imag() / N + 0.5) % MOD,
50
             db = (long long)(C[i].real() / N + 0.5) % MOD,
             dc = (long long)(D[i].imag() / N + 0.5) % MOD,
51
52
             dd = (long long)(D[i].real() / N + 0.5) % MOD;
53
       a[i] = ((dd << (L * 2)) + ((db + dc) << L) + da) % MOD;
54
55
```

#### 线性规划

```
1 / /  求\max\{cx | Ax < b, x > 0\}的解
2 typedef vector<double> VD:
   VD simplex(vector<VD> A, VD b, VD c) {
     int n = A.size(), m = A[0].size() + 1, r = n, s = m - 1;
     vector\langle VD \rangle D(n + 2, VD(m + 1, 0)); vector\langle int \rangle ix(n + m);
     for (int i = 0; i < n + m; ++ i) ix[i] = i;
     for (int i = 0; i < n; ++ i) {
       for (int j = 0; j < m - 1; ++ j) D[i][j] = -A[i][j];
       D[i][m - 1] = 1; D[i][m] = b[i];
10
       if (D[r][m] > D[i][m]) r = i;
11
12
     for (int j = 0; j < m - 1; ++ j) D[n][j] = c[j];
13
     D[n + 1][m - 1] = -1:
14
     for (double d; ; ) {
15
       if (r < n) {
         int t = ix[s]: ix[s] = ix[r + m]: ix[r + m] = t:
16
17
         D[r][s] = 1.0 / D[r][s]; vector<int> speedUp;
18
         for (int j = 0; j \le m; ++ j) if (j != s) {
           D[r][j] *= -D[r][s];
19
20
           if(D[r][j]) speedUp.push_back(j);
21
22
         for (int i = 0; i \le n + 1; ++ i) if (i != r) {
23
           for(int j = 0; j < speedUp.size(); ++ j)</pre>
24
           D[i][speedUp[j]] += D[r][speedUp[j]] * D[i][s];
25
           D[i][s] *= D[r][s]:
26
       f(s) = -1; s = -1;
27
       for (int j = 0; j < m; ++ j) if (s < 0 || ix[s] > ix[j])
28
         if (D[n + 1][j] > EPS || (D[n + 1][j] > -EPS && D[n][j] > EPS)) s = j;
29
       if (s < 0) break;
```

```
for (int i = 0: i < n: ++ i) if (D[i][s] < -EPS)
30
31
         if (r < 0 \mid | (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
32
             || (d < EPS \&\& ix[r + m] > ix[i + m])) r = i;
       if (r < 0) return VD(): // 无边界
33
34
     if (D[n + 1][m] < -EPS) return VD(); // 无解
35
36
     VD \times (m - 1):
37
     for (int i = m; i < n + m; ++ i) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
     return x: // 最优值在 D[n][m]
38
39
```

### Schreier-Sims

```
struct Permutation{
     vector<int> P;Permutation(){} Permutation(int n){ P.resize(n); }
     Permutation inv()const{
      Permutation ret(P.size()):
       for(int i = 0; i < int(P.size()); ++i) ret.P[P[i]] = i;</pre>
       return ret;
     int &operator [](const int &dn){ return P[dn]; }
     void resize(const size_t &sz){ P.resize(sz); }
10
     size t size()const{ return P.size(): }
11
     const int &operator [](const int &dn)const{ return P[dn]; }
12 }:
13 Permutation operator *(const Permutation &a, const Permutation &b){
     Permutation ret(a.size());
     for(int i = 0: i < (int)a.size(): ++i) ret[i] = b[a[i]]:</pre>
15
16
     return ret:
17 }
   typedef vector<Permutation> Bucket;
   typedef vector<int> Table; typedef pair<int,int> pii;
20
   int n, m;
   vector<Bucket> buckets, bucketsInv; vector<Table> lookupTable;
   int fastFilter(const Permutation &g, bool addToGroup = true){
23
     int n = buckets.size();
     Permutation p:
24
25
     for(int i = 0; i < n; ++i){
26
       int res = lookupTable[i][p[i]];
       if(res == -1){
27
28
         if (addToGroup) {
29
           buckets[i].push_back(p); bucketsInv[i].push_back(p.inv());
30
           lookupTable[i][p[i]] = (int)buckets[i].size() - 1;
31
         }
32
         return i;
33
       }
34
       p = p * bucketsInv[i][res]; swap(i1,i2);
35
36
     return -1:
37 }
```

```
38 long long calcTotalSize(){
     long long ret = 1;
40
     for(int i = 0; i < n; ++i) ret *= buckets[i].size();</pre>
41
     return ret:
42
   bool inGroup(const Permutation &g){ return fastFilter(g, false) == -1; }
43
   void solve(const Bucket &gen,int _n){// m perm[0..n - 1]s
45
     n = _n, m = gen.size();
46
     {//clear all
47
       vector<Bucket> _buckets(n); swap(buckets, _buckets);
48
       vector<Bucket> _bucketsInv(n); swap(bucketsInv, _bucketsInv);
49
       vector<Table> _lookupTable(n); swap(lookupTable, _lookupTable);
50
51
     for(int i = 0; i < n; ++i){
52
       lookupTable[i].resize(n);
53
       fill(lookupTable[i].begin(), lookupTable[i].end(), -1);
54
55
     Permutation id(n):
     for(int i = 0; i < n; ++i) id[i] = i;
57
     for(int i = 0; i < n; ++i){
58
       buckets[i].push_back(id); bucketsInv[i].push_back(id);
59
       lookupTable[i][i] = 0;
60
61
     for(int i = 0; i < m; ++i) fastFilter(gen[i]);</pre>
     queue<pair<point,point> > toUpdate;
63
     for(int i = 0; i < n; ++i)
64
       for(int j = i; j < n; ++j)
         for(int k = 0; k < (int)buckets[i].size(); ++k)</pre>
65
66
           for(int 1 = 0; 1 < (int)buckets[j].size(); ++1)</pre>
67
              toUpdate.push(make_pair(pii(i,k), pii(j,l)));
68
     while(!toUpdate.empty()){
69
       pii a = toUpdate.front().first, b = toUpdate.front().second;
70
       toUpdate.pop();
71
       int res=fastFilter(buckets[a.first][a.second]*buckets[b.first][b.second]);
72
       if(res==-1) continue:
73
       pii newPair(res, (int)buckets[res].size() - 1);
74
       for(int i = 0; i < n; ++i)
         for(int j = 0; j < (int)buckets[i].size(); ++j){</pre>
75
76
           if(i <= res) toUpdate.push(make_pair(pii(i, j), newPair));</pre>
77
           if(res <= i) toUpdate.push(make_pair(newPair, pii(i, j)));</pre>
78
79
80
```

## 字符串

## 后缀数组 ( 倍增 )

```
int rank[MAX_N],height[MAX_N];
int cmp(int *x,int a,int b,int d){
```

```
return x[a]==x[b]\&\&x[a+d]==x[b+d]:
4 }
   void doubling(int *a,int N,int M){
     static int sRank[MAX_N],tmpA[MAX_N],tmpB[MAX_N];
     int *x=tmpA,*y=tmpB;
     for(int i=0;i<M;++i) sRank[i]=0;</pre>
     for(int i=0;i<N;++i) ++sRank[x[i]=a[i]];</pre>
10
     for(int i=1;i<M;++i) sRank[i]+=sRank[i-1];</pre>
     for(int i=N-1;i>=0;--i) sa[--sRank[x[i]]]=i;
11
12
     for(int d=1,p=0;p<N;M=p,d<<=1){
13
       p=0; for(int i=N-d;i<N;++i) y[p++]=i;</pre>
       for(int i=0;i<N;++i) if(sa[i]>=d) y[p++]=sa[i]-d;
14
       for(int i=0;i<M;++i) sRank[i]=0;</pre>
15
16
       for(int i=0;i<N;++i) ++sRank[x[i]];</pre>
17
       for(int i=1;i<M;++i) sRank[i]+=sRank[i-1];</pre>
18
       for(int i=N-1;i>=0;--i) sa[--sRank[x[y[i]]]]=y[i];
19
       swap(x,y); x[sa[0]]=0; p=1;
20
       for(int i=1;i<N;++i) x[sa[i]]=cmp(y,sa[i],sa[i-1],d)?p-1:p++;</pre>
21
22 }
   void calcHeight(){
23
     for(int i=0;i<N;++i) rank[sa[i]]=i;</pre>
25
     int cur=0: for(int i=0:i<N:++i)</pre>
26
     if(rank[i]){
27
       if(cur) cur--;
28
       for(;a[i+cur] == a[sa[rank[i]-1]+cur];++cur);
29
       height[rank[i]]=cur;
30
    }
31 }
```

#### 后缀自动机

```
struct State {
     int length:
     State *parent,*go[C];
     State(int length = 0):length(length),parent(NULL){
       memset(go,0,sizeof(go));
     State* extend(State*, int token);
   } node_pool[V], *tot_node;
   State* State::extend(State *start,int token){
10
     State *p=this;
     State *np=new(tot_node++) State(this->length+1);
11
12
     while(p!=NULL&&p->go[token]==NULL)
13
      p->go[token]=np, p=p->parent;
14
     if(p==NULL) np->parent=start;
15
     elsef
16
       State *q=p->go[token];
       if(p->length+1==q->length) np->parent=q;
17
18
       else{
```

```
19
         State *ng=new(tot node++) State(p->length+1):
20
         memcpy(nq->go,q->go,sizeof(q->go));
21
         nq->parent=q->parent;
22
         np->parent=q->parent=nq;
23
         while(p!=NULL&&p->go[token]==q)
24
           p->go[token]=nq, p=p->parent;
25
       }
26
     }
27
     return np;
28
```

## Manacher/扩展 KMP

```
void Manacher(char text[], int n, int palindrome[]) {
     palindrome[0] = 1:
     for (int i = 1, j = 0, i < (n << 1) - 1; ++ i) {
       int p = i \gg 1;
       int q = i - p;
       int r = (j + 1 \gg 1) + palindrome[j] - 1;
       palindrome[i] = r < q ? 0 : min(r - q + 1, palindrome[(j << 1) - i]);
       while (0 <= p - palindrome[i] && q + palindrome[i] < n && text[p - palindrome[i]] ==

    text[q + palindrome[i]]) {
         palindrome[i] ++:
10
       }
11
       if (q + palindrome[i] - 1 > r) {
12
         i = i:
13
14
15
16
17
   void ExtendedKMP(char *s, int next[]) {
18
       int l = strlen(s), i = 0, i = 0, k = 1:
19
       while (1 + j < 1 & s[j] == s[1 + j]) {
20
           ++j;
21
       }
22
       next[1] = j;
23
       for (int i = 2; i < 1; ++i) {
24
           int len = k + next[k], ll = next[i - k];
25
           if (11 < len - i) {
26
               next[i] = 11;
27
           } else {
28
               j = max(0, len - i);
29
               while (i + j < 1 && s[j] == s[i + j]) {
30
                    ++j;
31
32
               next[i] = j;
33
               k = i:
34
```

```
35 | }
36 |}
```

### 字符串最小表示

```
std::string find(std::string s) {
   int i,j,k,l,N=s.length(); s+=s;
   for(i=0,j=1;j<N;){
      for(k=0;k<N&&s[i+k]==s[j+k];k++);
      if(k>=N) break;
      if(s[i+k]<s[j+k]) j+=k+1;
      else l=i+k,i=j,j=max(l,j)+1;
   }
   return s.substr(i,N);
}</pre>
```

## 后缀树 (With Pop Front)

```
int pos, text[N];
   struct Node {
     int 1, r;
     Node *suf, *ch[C];
     int dgr;
     Node *fa:
     Node (int l = -1, int r = INF) : l(1), r(r) {
       suf = fa = NULL;
       memset(ch, 0, sizeof(ch));
9
10
       dgr = 0;
11
12
     Node* addEdge(Node *t) {
       int c = text[t->1];
13
14
       dgr += !ch[c];
       ch[c] = t:
15
16
       t->fa = this;
17
       return t;
18
19
     int len() {
20
       return min(r, pos + 1) - 1;
21
     }
22
23
   int top;
25 | Node pool[N << 1], *root, *nxtSuf, *cur;
   int remCnt, curP, curLen;
   long long size;
27
28
   queue<Node*> leaves;
   void init() {
     top = 0, pos = -1;
30
     remCnt = 0, curP = 0, curLen = 0;
```

```
nxtSuf = NULL:
33
     root = cur = new(pool + (top++)) Node(-1, -1);
34
35
     while (leaves.size()) leaves.pop();
36
37
   void link(Node *u) {
     if (nxtSuf) nxtSuf->suf = u:
38
39
     nxtSuf = u;
40
   bool walk(Node *u) {
     int len = u->len();
     if (curLen >= len) {
43
       curP += len, curLen -= len, cur = u;
45
       return true;
    }
46
47
     return false;
48
49
   void extend(int c) {
     text[++pos] = c;
     nxtSuf = NULL;
51
     ++remCnt:
52
53
     while (remCnt) {
       curP = curLen ? curP : pos;
54
55
       int curE = text[curP];
56
       if (!cur->ch[curE]) {
57
         leaves.push(cur->addEdge(new(pool + (top++)) Node(pos)));
58
         link(cur);
59
       } else {
         Node *nxt = cur->ch[curE];
60
         if (walk(nxt)) continue;
61
62
         if (text[nxt->l + curLen] == c) {
63
           ++curLen:
64
           link(cur);
65
           break;
66
67
         Node *split = new(pool + (top++)) Node(nxt->1, nxt->1 + curLen);
68
         cur->addEdge(split);
69
         leaves.push(split->addEdge(new(pool + (top++)) Node(pos)));
70
         nxt->1 += curLen;
71
         split->addEdge(nxt);
72
         link(split);
73
       }
74
       --remCnt:
75
       if (cur == root && curLen > 0) {
76
         curP = pos - (--curLen);
77
78
         cur = cur->suf ? cur->suf : root;
79
80
     size += leaves.size();
```

```
82 }
 83
    void finish() {
      nxtSuf = NULL:
 84
      for (int i = 0; i < top; ++i) if (pool[i].r == INF) link(pool + i);</pre>
 85
      while (remCnt > 0) {
 86
        if (curLen) {
 87
          int curE = text[curP]:
 88
          Node *nxt = cur->ch[curE];
 89
          if (walk(nxt)) continue;
 90
          Node *split = new(pool + (top++)) Node(nxt->1, nxt->1 + curLen);
 91
 92
          leaves.push(cur->addEdge(split));
          nxt->1 += curLen;
 93
           split->addEdge(nxt);
 94
 95
          link(split);
        } else {
 96
          leaves.push(cur);
 97
 98
          link(cur);
 99
        }
100
        --remCnt:
        if (cur == root && curLen > 0) {
101
102
          --curLen:
103
          curP = pos - remCnt + 1;
104
        } else {
           cur = cur->suf ? cur->suf : root;
105
106
        }
107
      }
108
      if (nxtSuf != root) link(root);
109
    void eraseUp(Node *&u) {
110
      size -= u->len();
111
      int ch = text[u->1];
112
113
      u = u \rightarrow fa:
      u \rightarrow ch[ch] = NULL;
114
115
      --(u->dgr);
116 }
    void erase() {
117
      Node *u = leaves.front();
118
      leaves.pop();
119
      while (u->dgr == 0 && u != cur) eraseUp(u);
120
121
      if (u == cur) {
122
        if (cur->dgr == 0 && curLen == 0) {
123
          int len = u->len();
124
          curLen = len:
125
          curP = pos - len + 1;
126
          cur = cur->fa;
127
           eraseUp(u);
128
        }
129
        if (curLen) {
130
          int curE = text[curP]:
131
           if (!cur->ch[curE]) {
```

```
Node *leaf = new(pool + (top++)) Node(pos - curLen + 1);
132
133
            leaves.push(cur->addEdge(leaf));
134
            size += leaf->len();
135
            --remCnt:
136
            if (cur == root && curLen > 0) {
               curP = pos - (--curLen) + 1;
137
138
            } else {
139
               cur = cur->suf ? cur->suf : root;
140
            while (curLen && walk(cur->ch[text[curP]])) continue;
141
142
        }
143
144
145
146
    int n;
    char s[N], buf[N];
    int ord[N], stop, sord[N << 1];</pre>
148
    void dfs(Node *u) {
150
      sord[u - pool] = stop++;
      for (int i = 0; i < C; ++i) {
151
        if (u->ch[i]) {
152
          dfs(u->ch[i]);
153
154
     }
155
156
157
    void getOrd() {
158
      init();
      for (int i = 0; i < n; ++i) extend(s[i] - 'a');</pre>
159
      finish():
160
      stop = 0;
161
162
      dfs(root);
163
      int i = 0:
      while (leaves.size()) {
164
165
        ord[i++] = sord[leaves.front() - pool];
166
        leaves.pop();
167
     }
168
```

# 数据结构

## Splay Tree

```
// 注意初始化内存池和 null 节点
struct Node{
int rev,size; Node *ch[2],*p;
void set(Node*,int); int dir(); void update(); void relax(); void appRev();
} nodePool[MAX_NODE],*curNode,*null;
Node *newNode(){
Node *t=curNode++; t->rev=0, t->size=1;
t->ch[0]=t->ch[1]=t->p=null; return t;
}
```

```
10 struct Splay{
11
     Node *root;
12
     Splay(){ root=newNode(); root->set(newNode(),0); root->update(); }
13
     void rot(Node *t){
14
       Node *p=t->p; int d=t->dir();
15
       p->relax(); t->relax();
16
       if(p==root) root=t;
17
       p->set(t->ch[!d],d); p->p->set(t,p->dir()); t->set(p,!d);
18
       p->update();
19
     }
20
     void splay(Node *t,Node *f=null){
       for(t->relax();t->p!=f;)
21
         if(t->p->p==f) rot(t);
22
23
         else t->dir()==t->p->dir()?(rot(t->p),rot(t)):(rot(t),rot(t));
24
25
    }
26 };
   void initNull(){ curNode=nodePool;null=curNode++;null->size=0; }
27
   void Node::set(Node *t,int _d){ ch[_d]=t; t->p=this; }
   int Node::dir(){ return this==p->ch[1]; }
   void Node::update(){ size=ch[0]->size+ch[1]->size+1;}
   void Node::relax(){ if(rev) ch[0]->appRev(), ch[1]->appRev(), rev=false; }
   void Node::appRev(){ if(this==null) return; rev^=true; swap(ch[0],ch[1]); }
```

#### Link Cut Tree

```
// 注意初始化 null 节点, 单点的 is_root 初始为 true
   struct Node{
     Node *ch[2], *p;
     int is_root, rev;
     bool dir():
     void set(Node*, bool);
     void update();
     void relax():
     void app_rev();
10 } *null;
11 | void rot(Node *t){
     Node *p=t->p; bool d=t->dir();
12
13
     p->relax(); t->relax(); p->set(t->ch[!d],d);
14
     if(p->is_root) t->p=p->p,swap(p->is_root,t->is_root);
15
     else p->p->set(t,p->dir());
     t->set(p,!d); p->update();
16
17 }
18 void splay(Node *t){
     for(t->relax();!t->is_root;)
19
20
       if(t->p->is_root) rot(t);
21
       else t->dir()==t->p->dir() ?(rot(t->p),rot(t)) : (rot(t),rot(t));
     t->update();
22
23 }
24 void access(Node *t){
```

```
for(Node *s=null; t!=null; s=t,t=t->p){
26
      splay(t);
27
      if (t->p == null) { /*TODO*/ }
28
      t->ch[1]->is root=true: s->is root=false:
29
      t->ch[1]=s; t->update();
30
31
  bool Node::dir(){ return this==p->ch[1]; }
  void Node::set(Node *t,bool _d){ ch[_d]=t; t->p=this; }
  void Node::update(){ }
  void Node::app_rev(){ if (this == null) return; rev ^= true; swap(ch[0], ch[1]); }
36 void Node::relax() { if(this==null) return; if (rev) { ch[0]->app_rev();
     37 void make_root(Node *u) { access(u); splay(u); u->app_rev(); }
```

#### 轻重链剖分

```
struct Tree(){}*root[N];
   int father[N],size[N],depth[N];
   int bfsOrd[N],pathId[N],ordInPath[N],sqn[N];
   void doBfs(int s){
     int qh=0,qt=0,*que=bfs0rd; father[s]=-1; depth[s]=0;
     for(que[qt++]=s;qh<qt;){</pre>
       int u=que[qh++];
       foreach(iter,adj[u]){
         int v=*iter; if(v==father[u]) continue;
10
         father[v]=u; depth[v]=depth[u]+1; que[qt++]=v;
11
       }
12
    }
13 }
   void doSplit(){
15
     for(int i=N-1;i>=0;--i){
16
       int u=bfsOrd[i]: size[u]=1:
17
       foreach(iter,adj[u]){
18
         int v=*iter; if(v==father[u]) continue; size[u]+=size[v];
19
       }
20
21
     memset(pathId,-1,sizeof pathId);
     for(int i=0:i<N:++i){</pre>
22
23
       int top=bfsOrd[i],cnt=0;
24
       if(pathId[top]!=-1) continue;
25
       for(int next,u=top;u!=-1;u=next){
26
         sqn[cnt]=val[u]; ordInPath[u]=cnt; pathId[u]=top; ++cnt;
27
         next=-1;
28
         foreach(iter.adi[u]){
29
           int v=*iter; if(v==father[u]) continue;
30
           if(next<0||size[next]<size[v]) next=v;</pre>
31
32
       }
```

```
33     root[top]=new Tree(0,cnt,sqn);
34    }
35 }
36 void prepare(){ doBfs(0); doSplit(); }
```

#### 综合

# ${\bf Dancing Links}$

```
1 struct node
     node *left,*right,*up,*down,*col; int row,cnt;
3 | }*head,*col[MAXC],Node[MAXNODE],*ans[MAXNODE];
   int totNode:
   void insert(const std::vector<int> &V,int rownum){
     std::vector<node*> N:
     for(int i=0;i<int(V.size());++i){</pre>
8
       node* now=Node+(totNode++); now->row=rownum;
9
       now->col=now->up=col[V[i]], now->down=col[V[i]]->down;
10
       now->up->down=now, now->down->up=now;
11
       now->col->cnt++; N.push_back(now);
12
13
     for(int i=0;i<int(V.size());++i)</pre>
14
       N[i] \rightarrow right = N[(i+1)\%V.size()], N[i] \rightarrow left = N[(i-1+V.size())\%V.size()];
15 }
16
   void Remove(node *x){
     x->left->right=x->right, x->right->left=x->left;
17
     for(node *i=x->down;i!=x;i=i->down)
18
19
       for(node *j=i->right; j!=i; j=j->right)
20
          j->up->down=j->down, j->down->up=j->up, --(j->col->cnt);
21 }
22
   void Resume(node *x){
23
     for(node *i=x->up;i!=x;i=i->up)
24
       for(node *j=i->left; j!=i; j=j->left)
25
         j->up->down=j->down->up=j, ++(j->col->cnt);
     x->left->right=x, x->right->left=x;
26
27
28
   bool search(int tot){
     if(head->right==head) return true:
29
30
     node *choose=NULL;
31
     for(node *i=head->right;i!=head;i=i->right){
       if(choose==NULL||choose->cnt>i->cnt) choose=i;
32
33
       if(choose->cnt<2) break;</pre>
34
     Remove(choose):
35
36
     for(node *i=choose->down;i!=choose;i=i->down){
37
       for(node *j=i->right; j!=i; j=j->right) Remove(j->col);
38
       ans[tot]=i:
39
       if(search(tot+1)) return true;
40
       ans[tot]=NULL:
41
       for(node *j=i->left; j!=i; j=j->left) Resume(j->col);
42
```

```
43
     Resume(choose):
44
     return false;
45
46
   void prepare(int totC){
47
     head=Node+totC;
     for(int i=0;i<totC;++i) col[i]=Node+i;</pre>
     totNode=totC+1:
49
50
      for(int i=0;i<=totC;++i){</pre>
        (Node+i)->right=Node+(i+1)%(totC+1);
51
52
       (Node+i)->left=Node+(i+totC)%(totC+1);
53
        (Node+i)->up=(Node+i)->down=Node+i;
54
    }
55
```

#### 日期公式

```
int zeller(int y,int m,int d) {
   if (m<=2) y--,m+=12; int c=y/100; y%=100;
   int w=((c>>2)-(c<<1)+y+(y>>2)+(13*(m+1)/5)+d-1)%7;
   if (w<0) w+=7; return(w);
}
int getId(int y, int m, int d) {
   if (m < 3) {y --; m += 12};
   return 365 * y + y / 4 - y / 100 + y / 400 + (153 * m + 2) / 5 + d;
}</pre>
```

#### 环状最长公共子序列

```
1 int n. a[N << 1]. b[N << 1]:
2 bool has(int i, int i) { return a[(i - 1) % n] == b[(i - 1) % n];}
3 const int DELTA[3][2] = \{\{0, -1\}, \{-1, -1\}, \{-1, 0\}\}\};
   int from[N][N]:
   int solve() {
     memset(from, 0, sizeof(from));
     int ret = 0:
     for (int i = 1; i \le 2 * n; ++ i) {
       from[i][0] = 2;
       int left = 0, up = 0;
10
11
       for (int j = 1; j \le n; ++ j) {
12
         int upleft = up + 1 + !!from[i - 1][j];
13
         if (!has(i, j)) upleft = INT_MIN;
14
         int max = std::max(left, std::max(upleft, up));
15
         if (left == max) {
16
           from[i][i] = 0:
17
         } else if (upleft == max) {
18
           from[i][j] = 1;
19
         } else {
20
           from[i][j] = 2;
```

```
}
21
22
         left = max;
23
       }
24
       if (i \ge n) {
25
         int count = 0;
26
         for (int x = i, y = n; y;) {
27
           int t = from[x][v]:
28
           count += t == 1;
29
           x += DELTA[t][0];
30
           y += DELTA[t][1];
31
32
         ret = std::max(ret, count);
33
         int x = i - n + 1, y = 0;
34
         from[x][0] = 0;
35
         while (y \le n \&\& from[x][y] == 0) y++;
         for (; x <= i; ++ x) {
36
37
           from[x][y] = 0;
38
           if (x == i) break:
39
           for (; y <= n; ++ y) {
40
             if (from[x + 1][y] == 2) break;
             if (y + 1 \le n \&\& from[x + 1][y + 1] == 1) {
41
42
               y ++; break;
43
44
           }
45
46
47
48
     return ret;
49
```

#### 经纬度球面距离

```
double sphereDis(double lon1, double lat1, double lon2, double lat2, double R) {
   return R * acos(cos(lat1) * cos(lat2) * cos(lon1 - lon2) + sin(lat1) * sin(lat2));
}
```

## 长方体表面两点最短距离

```
int r;
void turn(int i, int j, int x, int y, int z,int x0, int y0, int L, int W, int H) {
   if (z==0) { int R = x*x+y*y; if (R<r) r=R;
   } else {
      if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x, x0+L, y0, H, W, L);
      if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y, x0, y0+W, L, H, W);
      if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0, x0-H, y0, H, W, L);
      if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0, x0, y0-H, L, H, W);
   }
}
int main(){
```

```
int L, H, W, x1, y1, z1, x2, y2, z2;
13
     cin >> L >> W >> H >> x1 >> y1 >> z1 >> x2 >> y2 >> z2;
     if (z1!=0 && z1!=H) if (y1==0 || y1==W)
15
          swap(y1,z1), std::swap(y2,z2), std::swap(W,H);
     else swap(x1,z1), std::swap(x2,z2), std::swap(L,H);
     if (z1==H) z1=0, z2=H-z2;
17
     r=0x3fffffff:
18
19
     turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
20
     cout<<r<<endl:
21
```

#### 其他

#### 简易积分表

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2| \int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a}$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}|$$

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \tan^{-1} \frac{x}{\sqrt{a^2-x^2}}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{1}{\sqrt{a^2 + a^2}} dx = \sin |x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} \int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} \int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$$

$$\int \sqrt{ax^2 + bx + c} dx = \frac{b + 2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln |2ax + b + 2\sqrt{a(ax^2 + bx + c)}|$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a} \sin 2ax \int \sin^3 ax dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a}$$

$$\int \tan ax dx = -\frac{1}{a} \ln \cos ax \int \tan^2 ax dx = -x + \frac{1}{a} \tan ax$$

$$\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax$$

$$\int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \int x^2 \sin ax dx = \frac{2-a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2}$$

## 常用结论

#### 弦图

设 next(v) 表示 N(v) 中最前的点 . 令 w\* 表示所有满足  $A\in B$  的 w 中最后的一个点 ,判断  $v\cup N(v)$  是否为极大团 ,只需判断是否存在一个  $w\in w*$ ,满足 Next(w)=v 且  $|N(v)|+1\le |N(w)|$  即可 .

### 五边形数

$$\prod_{n=1}^{\infty} (1-x^n) = \sum_{n=0}^{\infty} (-1)^n (1-x^{2n+1}) x^{n(3n+1)/2}$$

### 重心

半径为 r ,圆心角为  $\theta$  的扇形重心与圆心的距离为  $\frac{4r\sin(\theta/2)}{3\theta}$ 半径为 r ,圆心角为  $\theta$  的圆弧重心与圆心的距离为  $\frac{4r\sin^3(\theta/2)}{3(\theta-\sin(\theta))}$ 

## 第二类 Bernoulli number

$$B_m = 1 - \sum_{k=0}^{m-1} {m \choose k} \frac{B_k}{m-k+1}$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m {m+1 \choose k} B_k n^{m+1-k}$$

## Stirling 数

第一类:n 个元素的项目分作 k 个环排列的方法数目

$$\begin{aligned} s(n,k) &= (-1)^{n+k} |s(n,k)| \\ |s(n,0)| &= 0 \\ |s(1,1)| &= 1 \\ |s(n,k)| &= |s(n-1,k-1)| + (n-1) * |s(n-1,k)| \end{aligned}$$

第二类:n 个元素的集定义 k 个等价类的方法数

$$S(n,1) = S(n,n) = 1$$
  
 
$$S(n,k) = S(n-1,k-1) + k * S(n-1,k)$$

## 三角公式

$$\begin{split} &\sin(a\pm b) = \sin a \cos b \pm \cos a \sin b - \cos(a\pm b) = \cos a \cos b \mp \sin a \sin b \\ &\tan(a\pm b) = \frac{\tan(a) \pm \tan(b)}{1 \mp \tan(a) \tan(b)} - \tan(a) \pm \tan(b) = \frac{\sin(a\pm b)}{\cos(a) \cos(b)} \\ &\sin(a) + \sin(b) = 2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2}) - \sin(a) - \sin(b) = 2\cos(\frac{a+b}{2})\sin(\frac{a-b}{2}) \\ &\cos(a) + \cos(b) = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2}) - \cos(a) - \cos(b) = -2\sin(\frac{a+b}{2})\sin(\frac{a-b}{2}) \\ &\sin(na) = n\cos^{n-1}a\sin a - \binom{n}{3}\cos^{n-3}a\sin^3 a + \binom{n}{5}\cos^{n-5}a\sin^5 a - \dots \\ &\cos(na) = \cos^n a - \binom{n}{2}\cos^{n-2}a\sin^2 a + \binom{n}{4}\cos^{n-4}a\sin^4 a - \dots \end{split}$$

## Java 读入优化

```
import java.io.*;
import java.util.*;
import java.math.*;
public class Main{
```

```
BufferedReader reader = new BufferedReader(new InputStreamReader(System.in)):
     PrintWriter writer = new PrintWriter(System.out);
     StringTokenizer tokenizer = null;
     void solve() throws Exception {
     void run()throws Exception{
10
11
12
         while (true) {
13
           solve();
16
       catch(Exception e){
17
18
       finally{
19
         reader.close();
20
         writer.close();
21
22
23
     String next()throws Exception{
24
       for(;tokenizer == null || !tokenizer.hasMoreTokens();){
25
         tokenizer = new StringTokenizer(reader.readLine());
26
27
       return tokenizer.nextToken();
28
29
     int nextInt()throws Exception{
30
       return Integer.parseInt(next());
31
32
     double nextDouble()throws Exception{
33
       return Double.parseDouble(next());
34
35
     BigInteger nextBigInteger()throws Exception{
36
       return new BigInteger(next());
37
38
     public static void main(String args[])throws Exception{
39
       (new Main()).run();
40
    }
41
```

#### Vimrc

```
set nu si mouse=a sw=4 ts=4 sts=4
set mp=g++\ -02\ -Wall\ --std=c++11\ -Wno-unused-result\ %:r.cpp\ -o\ %:r
nmap <F2> :vs %:r.in <CR>
nmap <F8> :!time ./%:r < %:r.in <CR>
nmap <F9> :w <CR> :make<CR>
```

| n   | log10 n | n!        | n C(n/2)  | LCM(1n)   | Pn        | Bn         |
|-----|---------|-----------|-----------|-----------|-----------|------------|
| 2   | 0.30    | 2         | 2         | 2         | 2         | 2          |
| 3   | 0.48    | 6         | 3         | 6         | 3         | 5          |
| 4   | 0.60    | 24        | 6         | 12        | 5         | 15         |
| 5   | 0.70    | 120       | 10        | 60        | 7         | 52         |
| 6   | 0.78    | 720       | 20        | 60        | 11        | 203        |
| 7   | 0.85    | 5040      | 35        | 420       | 15        | 877        |
| 8   | 0.90    | 40320     | 70        | 840       | 22        | 4140       |
| 9   | 0.95    | 362880    | 126       | 2520      | 30        | 21147      |
| 10  | 1       | 3628800   | 252       | 2520      | 42        | 115975     |
| 11  |         | 39916800  | 462       | 27720     | 56        | 678570     |
| 12  |         | 479001600 | 924       | 27720     | 77        | 4213597    |
| 15  |         |           | 6435      | 360360    | 176       | 1382958545 |
| 20  |         |           | 184756    | 232792560 | 627       |            |
| 25  |         |           | 5200300   |           | 1958      |            |
| 30  |         |           | 155117520 |           | 5604      |            |
| 40  |         |           |           |           | 37338     |            |
| 50  |         |           |           |           | 204226    |            |
| 70  |         |           |           |           | 4087968   |            |
| 100 |         |           |           |           | 190569292 |            |