Design and simulation of a piezoelectric cantilever to detect various fluids

MAJOR PROJECT REPORT

SEMESTER- I

FOUR-YEAR UNDERGRADUATE PROGRAMME
(DESIGN YOUR DEGREE)

SUBMITTED TO

UNIVERSITY OF JAMMU, JAMMU

SUBMITTED BY THE TEAM: **LEONARD EULER**

TEAM MEMBERS	**ROLL NO**
Divya Verma	DYD-23-04
Diya Rani	DYD-23-05
Kashvi Vaid	DYD-23-09
Paridhi Mahajan	DYD-23-14
Sarnish Kour	DYD-23-18
Vidhita Arora	DYD-23-22

UNDER THE GUIDANCE OF

Prof. K. S. CHARAK, Dr. JATINDER MANHAS, Dr. SUNIL KUMAR BHOUGAL &
Dr. SANDEEP ARYA

Submitted on: ____ February, 2024

CERTIFICATE

The work embodied in this report entitled "**Design and simulation of a piezoelectric cantilever to detect various fluids**" has been done by Team Leonhard Euler including group members- Divya Verma, Diya Rani, Kashvi Vaid, Paridhi Mahajan, Sarnish Kour and Vidhita Arora as a Major Project for Semester I of Four-Year Undergraduate Programme (Design Your Degree). This work was carried out under the guidance of Mentor Prof. K.S. Charak, Dr. Jatinder Manhas, Dr. Sunil Kumar Bhougal and Dr. Sandeep Arya for the partial fulfillment of the award of the Design Your Degree, Four Year Undergraduate Programme, University of Jammu, Jammu & Kashmir. This project report has not been submitted anywhere else.

Signature of Students:

Signature of Mentors:

1. Prof. K.S. Charak
2. Dr. Jatinder Manhas
3. Dr. Sandeep Arya
4. Dr. Sunil Kumar Bhougal

Prof. Alka Sharma

Director, SIIEDC, University of Jammu

ACKNOWLEDGMENT

We avail this opportunity to acknowledge all those who helped and guided us during our work. First and foremost, we thank Almighty God from the depth of our hearts for generating enthusiasm and granting our spiritual strength to successfully pass through this challenge. Words are too meager to express our esteem indebtedness and wholehearted sense of gratitude towards our research guide and supervisor Prof. K.S. Charak, Department of Mathematics, Dr. Jatinder Manhas, Department of Computer Application and IT, Dr Sunil Kumar Bhougal, Department of Statistics and Dr. Sandeep Arya, Department of Physics, University of Jammu, Jammu. It is our pleasure beyond words to express our deep sense of feelings for their inspiring guidance, generous encouragement, and well-versed advice. They provided us with undaunted encouragement and support despite their busy schedules. Really, fortunate we are and we feel extremely honored for the opportunity conferred upon us to work under their perpetual motivation.

We are extremely fortunate to have an opportunity to express our heartiest gratitude to Prof. Alka Sharma, Director, SIIEDC, University of Jammu, Jammu, and member of the advisory committee for her valuable advice and generous help and for providing us all the necessary facilities from the department. We are highly thankful to all respected mentors for their immense help during the conduct of this major project. The words are small trophies to express our feelings of affection and indebtedness to our friends who helped us throughout the major project, and whose excellent company, affection, and cooperation helped us carrying out our work with joy and happiness.

We acknowledge all the people, mentioned or not mentioned here, who have silently wished and gave fruitful suggestions and helped us in achieving the present goal.

INDEX

CHAPTER- 1
INTRODUCTION TO PIEZOELECTRIC CANTILEVERS

1.1 Introduction

This chapter delves into the intricate design and simulation of a piezoelectric cantilever system tailor-made for fluid testing applications. The comprehensive coverage of topics includes the fundamentals of piezoelectricity, cantilever beam mechanics, various piezoelectric materials, and their wide-ranging applications. Additionally, the chapter outlines the specific objectives, problem statement, and anticipated results of the proposed project.

1.2 Cantilever

The term "cantilever " originally had the name "cantlapper" combining "cant" meaning "slope" and "lever" from the Latin word "levare," meaning "to raise." Over time, the term evolved to "cantilever" [1]. A cantilever beam, a crucial structural element in engineering and construction, is characterized by its horizontal extension with support on only one end. The projecting end, beyond the support point, is aptly termed the cantilever. Commonly employed in construction for bearing balconies, roofs, and other overhangs, cantilever beams also find application in bridges to extend decks over waterways or obstacles. Typically featuring fixed support on one side, cantilever beams are designed to be statically stable. This fixed support ensures the capability to withstand forces and moments in all directions.

The conventional representation of a cantilever beam involves modeling it with the support located at the left end and the cantilevered portion extending toward the right. This structural configuration demands precise engineering analysis and design to guarantee stability and strength, making cantilever beams indispensable in various architectural and civil engineering applications [2].

1.2.1 Origin

Frank Lloyd Wright is credited with popularizing cantilevers in modern architectural construction. He famously utilized them in the design of the Robie House in Chicago, completed in 1909. However, the first cantilever bridge is attributed to German engineer Gottfried Heinrich Gerber, who constructed one over the Main River in Germany in 1867 [3].

1.2.2 Types of Cantilevers

The three main types of cantilever bridges include the **simple, the through, and the box girder cantilever bridges.**

- **The simple cantilever bridge**: A simple span cantilever bridge is a type of bridge that stretches over a gap using sections that stick out from each side. These sections, called cantilever arms, support the bridge's deck. One good thing about this bridge is that it can be used in different places and can cover distances that other bridges might struggle with. Plus, it's not too costly or tough to construct. But there are some downsides. It can't handle really heavy stuff because the arms and their connections to the ground can only take so much weight. Also, it doesn't protect against sideways forces like wind or earthquakes. These bridges work best for short distances and lighter loads. They're good for places where you don't need to carry super heavy things or go far.
- **The through cantilever bridge**: A cantilever through the bridge is like two long arms reaching out and joining together in the middle. They support each other and make a big open space without any pillars in the water below. These bridges are great for going over rivers or really wide areas. They're strong and can handle heavy stuff because of how they're built. And because there are no pillars in the water, they don't mess with the natural flow of the river or lake. The cool thing about these bridges is that they can be made in lots of different sizes and shapes, so they're really versatile. Additionally, they're usually cheaper to build because they don't need as much support. But building them is tricky and takes a lot of time and effort. You have to make sure everything fits together perfectly, which can be a big job. And if there are any mistakes, it can cost a lot to fix them.
- **The box girder cantilever bridge**: A cantilever truss bridge is a special kind of bridge made with cantilevers. Cantilevers are like beams sticking out from one end. These bridges are used to cross big gaps like rivers or canals. Truss bridges are great for making cantilever bridges because they're really strong. They're made of triangles, usually with steel beams or rods. These triangles make the bridge extra sturdy, so it can stretch across long distances. A cantilever truss bridge has two main parts: the cantilever arm, which is a strong beam connected to the bridge's main structure, and the truss, which is the triangle-shaped

framework that supports the bridge. The truss has vertical columns called cantilever arms and horizontal beams connecting them to support the arm [4].

1.2.3 Applications

Cantilever beams find a wide range of applications in engineering and architecture due to their unique design. Some common applications include:

- **Bridges:** Cantilever beams play a crucial role in building bridges, particularly for spanning obstacles such as rivers or valleys. They are frequently integrated with other structural components to form various cantilever bridge designs.

Fig. 1.1 Cantilever application in bridges [5]

Fig.1.2 Cantilever application in Building [6]

- **Buildings:** Cantilever beams are employed in architectural design to produce captivating and visually striking building elements. They serve to support balconies, overhangs, and even entire floors that protrude beyond the main structure of the building.
- **Signage and Billboards:** Cantilever beams are frequently used to support large signage and billboards. This design choice enables a sleek and minimalist appearance, as support structures can be concealed from view.

Fig.1.3 Cantilever application in Billboard [7]

Fig.1.4 Cantilever application in Stadium [8]

- **Stadiums and Auditoriums:** Cantilever beams play a vital role in building stadium roofs and auditorium canopies, ensuring unobstructed views for spectators across expansive seating areas.

Fig.1.5 Cantilever application in Auditorium [9]

Fig.1.6 Cantilever application in Airplane [10]

- **Mechanical Engineering:** Cantilever beams are integral components in mechanical systems, providing support for loads and facilitating the creation of mechanisms such as cantilevered arms in machining equipment [11].

1.2.4 Factors Responsible for Cantilever Beam

The maximum span of a cantilever beam is influenced by several factors

- **Cantilever depth:** The depth of the cantilever affects its ability to support loads over a distance without bending or breaking.
- **Load size, type, and placement:** The size, type, and where the load is placed on the beam also affect its maximum span. Heavier loads or unevenly distributed loads can decrease the span.
- **Material type and quality:** The type and quality of the material used for the beam impact its strength and ability to resist bending or breaking.

Generally, small cantilever beams have spans limited to 2 to 3 meters. However, by increasing the depth of the beam or incorporating steel or prestressed structural elements, the span can be extended. If the structure effectively transfers the moments generated by the cantilever to the ground, longer spans can be achieved. The potential for long-span cantilever beams can be explored through detailed analysis and design of the structure, ensuring safety and stability [12].

1.2.5 Advantages and Disadvantages of Cantilever Beams

Some important advantages of cantilever beams are [13]:

- Cantilever beams do not require support on the opposite side.
- The negative bending moment created in cantilever beams helps to counteract the positive bending moments created.
- Cantilever beams can be easily constructed.

Some important disadvantages of cantilever beams are:

- Cantilever beams are subjected to large deflections.
- Cantilever beams are subjected to larger moments.
- Strong fixed support or a backspin is necessary to keep the structure stable.

1.2.6 Cantilever Beam Design Parameters

When designing a cantilever structure, several important factors should be considered:

- **Loads:** The cantilever must be able to support the applied loads, including the weight of the structure itself and any additional loads such as wind, snow, and seismic loads. The loads should be analyzed and distributed appropriately throughout the structure.
- **Strength and stiffness:** The cantilever must be strong and stiff enough to resist deflection, buckling, and other types of failure. The properties of the materials used, such as the modulus of elasticity and yield strength, will affect the strength and stiffness of the structure.
- **Stress concentration:** The stress concentration at the fixed end of the cantilever must be taken into account in the design to prevent failure. Stress concentration can be reduced by using larger cross-sections or by using fillets or rounded corners.
- **Deflection:** The deflection of the cantilever under load should be analyzed to ensure that it remains within acceptable limits, both for structural safety and also for aesthetic reasons.
- **Durability:** The structure should be designed to last for the intended service life with minimal maintenance required. This includes considering factors such as corrosion, fatigue, and the effects of weathering [14].

1.2.7 Working of Cantilever

When a cantilever beam experiences vertical loads, it bends downward, forming a convex shape upward. Whether it's a point load, uniform load, or varying load, the beam bends in this manner, resulting in tension at the upper fiber and compression at the lower fibers. Consequently, to reinforce the concrete beam effectively, the main reinforcement is typically placed in the upper fiber, where the tensile stress is highest.

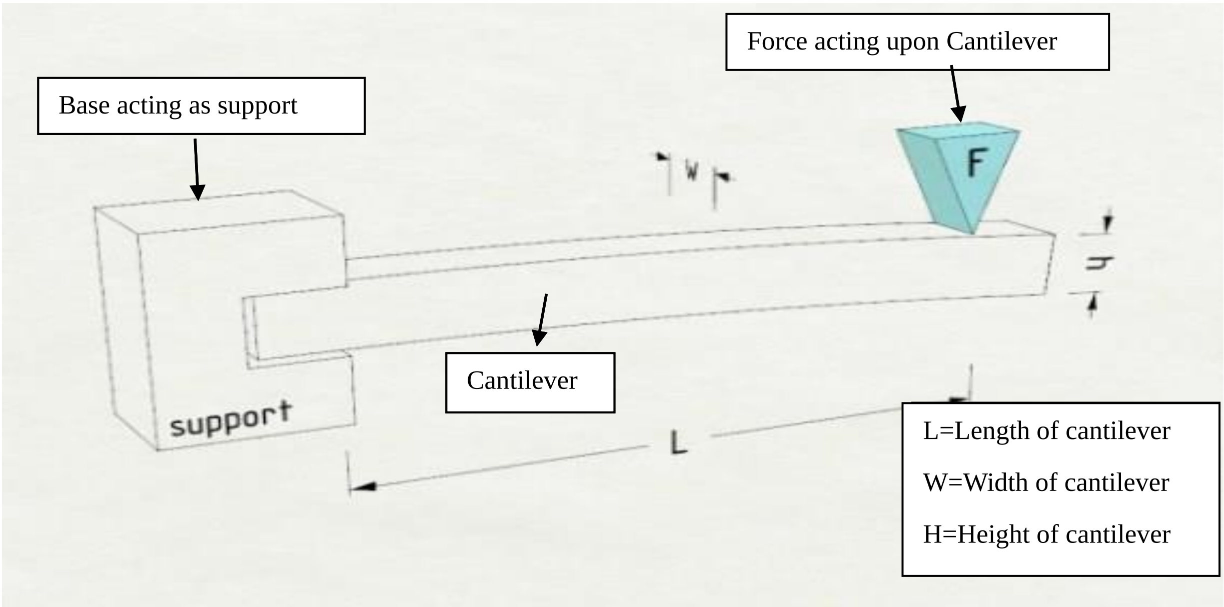

Fig.1.7 Schematic showing the working of cantilever [15].

A cantilever is essentially a rigid structure anchored at one end. This means that any forces acting on the cantilever are transmitted to the fixed end. As long as the fixed end remains stable, the cantilever stays in place, making it possible to support weight even at its far end. While no object is perfectly rigid and a cantilever will bend to some extent, sturdy materials are chosen to minimize this bending under normal conditions. Conceptually, a cantilever design resembles a lever, with the fixed end acting as the fulcrum. However, unlike a lever, a cantilever remains stationary. The restraints at the fixed end keep the entire cantilever beam in position, ensuring that any forces applied to the free end are counteracted. Despite its rigidity, every material exhibits some elasticity, however slight, so forces on the free end cause shear stresses, resulting in bending or deflection. Deflection, the bending response of the cantilever, varies depending on factors such as where the force is applied along the beam. Although a cantilever behaves as if it were rotating around its fulcrum when it deflects, it doesn't rotate. Instead, it exhibits a moment of inertia, akin to a rotating object, but remains fixed in place [16].

1.3 Piezoelectric Material

The term Piezoelectric originates from the Greek word “piezein,” which means to press or squeeze, aptly describing the process of generating electricity through pressure. Piezoelectricity is a fascinating phenomenon that occurs in certain materials, wherein mechanical stress induces an electrical charge, or conversely, an applied electric field causes mechanical deformation.

Here's a breakdown of how it works:

- **Microscopic Mechanism**: At the heart of piezoelectricity lies the behavior of the crystal structure of certain materials, particularly those lacking inversion symmetry. When mechanical stress is applied to such materials, it causes a displacement of positive and negative charge centers within the crystal lattice.
- **Electric Polarization:** This displacement of charge centers leads to the creation of an electric polarization within the material. Essentially, the material becomes polarized, with positive and negative charges separated along a certain axis.
- **Electric Potential (Voltage) Generation:** As a result of this polarization, an electric potential, or voltage, develops across the material. This voltage can be harnessed for various applications.
- **Converse Piezoelectric Effect:** Conversely, when an electric field is applied to the piezoelectric material, it causes a mechanical deformation. This phenomenon is known as the converse piezoelectric effect. Essentially, the material changes its shape in response to the electric field [17].

1.3.1 Types of Piezoelectric Material [18]

- **CRYSTALS**

a) **Quartz (SiO_2)**: Quartz is a naturally occurring crystal with a well-defined and strong piezoelectric effect. It's widely used in electronic devices, sensors, and timing applications.

b) **Topaz:** Topaz exhibits piezoelectric properties and finds applications in various electronic devices and scientific instruments.

c) **Tourmaline:** Tourmaline is another crystal known for its piezoelectric properties and is used in certain electronic applications.

d) **Rochelle Salt (Potassium Sodium Tartrate, $KNaC_4H_4O_6 \cdot 4H_2O$):** Although Rochelle Salt has strong piezoelectric properties, its water solubility and low-temperature stability limit its industrial use.

e) **Gallium Orthophosphate ($GaPO_4$):** Similar to quartz, Gallium Orthophosphate exhibits strong piezoelectric properties but offers higher temperature stability.

f) **Lead Titanate ($PbTiO_3$):** Lead titanate is a piezoelectric material commonly used in sensors, actuators, and ultrasound transducers.

- **CERAMICS**

a) **Lead Zirconate Titanate (PZT, $Pb[Zr_xTi_{1-x}]O_3$):** PZT is a synthetic ceramic with one of the most significant piezoelectric effects. It's widely used in sensors, actuators, and medical ultrasound transducers.

b) **Barium Titanate ($BaTiO_3$):** Barium titanate is utilized not only for its piezoelectric properties but also in capacitors and nonlinear optics applications.

c) **Zinc Oxide (ZnO):** Single crystals of zinc oxide with the Wurtzite structure exhibit piezoelectricity and are used in various electronic and sensor applications.

- **POLYMERS**

a) **Polyvinylidene Fluoride (PVDF):** PVDF is a thermoplastic polymer known for its piezoelectric properties. It's used in flexible sensors, actuators, and energy-harvesting applications.

b) **Polyvinylidene Fluoride-Tri fluoroethylene (P(VDF-TrFE**)): This copolymer of PVDF enhances the piezoelectric effect and is used in high-performance applications.

c) **Poly L-lactic Acid (PLLA):** PLLA is a biodegradable polymer with piezoelectric characteristics, making it suitable for medical applications.

d) **Polyurea:** Polyurea is a polymer known for its piezoelectric response under specific conditions.

- **NATURAL ORGANIC MATERIALS**

a) **Collagen:** Found in bones and tendons, collagen exhibits natural piezoelectric properties and plays a role in bone remodeling and repair.

b) **Cellulose:** Certain forms of cellulose, especially in its crystalline form, exhibit piezoelectric effects.

c) **Glycine:** Glycine, an amino acid, displays piezoelectricity in specific crystalline forms.

d) **DNA:** DNA molecules exhibit slight piezoelectricity due to their helical structure.

These are just some examples of piezoelectric materials, each with its specific advantages and applications .

1.3.2 Applications

Piezoelectricity has numerous practical applications across industries [19]:

- **Sensors and Actuators:** Used in devices like accelerometers, vibration sensors, and precision motion actuators.
- **Medical Devices:** Utilized in ultrasound imaging, where it helps in generating and detecting sound waves.
- **Consumer Electronics:** Found in microphones, headphones, and quartz watches.
- **Energy Harvesting:** Collects ambient mechanical energy and converts it into electrical energy for various purposes.
- **Automotive Industry:** Used in knock sensors for engine management systems, among other applications.
- **Military and Aerospace:** Applied in sonar, guidance systems, and vibration monitoring.

1.3.3 Biological Role

Piezoelectricity also plays a crucial role in biological processes:

- **Bone Remodeling and Growth:** Bone tissue generates electrical potentials when subjected to mechanical stress, aiding in bone remodeling and growth.
- **Tendon Function:** Tendons exhibit piezoelectric properties, which may contribute to repair, growth, and signaling within the tissue.
- **Dental Applications:** Piezoelectric properties of dental tissues have applications in understanding tooth mechanics and improving dental restorations.
- **Hearing Mechanisms:** Certain biological materials in the ear utilize piezoelectric properties to convert mechanical vibrations into electrical signals for hearing.
- **Cell and Tissue Mechanics:** Piezoelectricity influences cellular processes such as migration, division, and communication, as well as tissue responses to mechanical stress.

1.3.4 Advantages of Piezoelectric Materials [20]

- Piezoelectric materials operate effectively across a wide range of temperatures.
- Their low carbon footprint positions them as an excellent alternative to fossil fuels.
- Their inherent characteristics make them highly efficient energy harvesters.
- Unused vibrational energy can be harnessed to generate sustainable power.
- Piezoelectric materials are reusable, contributing to their sustainability.

1.3.5 Limitations of Piezoelectric Materials [21]

- Devices utilizing piezoelectric materials can inadvertently capture unwanted vibrations.

- Resistance and durability constraints limit their effectiveness in energy harvesting applications, particularly on pavements and roads.
- Mismatch in stiffness between piezoelectric materials and pavement substrates presents challenges.
- Despite their potential, piezoelectric devices require further research to fully realize their capabilities and applications.

1.3.6 Key properties of piezoelectric materials

- **Piezoelectric Effect:** The most fundamental property of piezoelectric materials is their ability to convert mechanical energy into electrical energy (direct effect) or vice versa (inverse effect).
- **Crystal Structure:** Piezoelectricity is primarily observed in crystals lacking a center of symmetry. The most common piezoelectric materials are crystals such as quartz, Rochelle salt, and tourmaline. However, synthetic materials like lead zirconate titanate (PZT) are widely used due to their superior piezoelectric properties.
- **Electromechanical Coupling Coefficient:** This coefficient represents the efficiency with which a piezoelectric material converts mechanical energy into electrical energy or vice versa. It quantifies the strength of the piezoelectric effect.
- **Piezoelectric Constants:** These constants describe the relationship between mechanical stress or strain and the resulting electric field or voltage. They are essential parameters for characterizing the piezoelectric behavior of a material.
- **Dielectric Properties:** Piezoelectric materials possess dielectric properties, meaning they can store electrical energy when subjected to an electric field. This property is crucial for their application in sensors, actuators, and transducers.
- **Frequency Response:** The piezoelectric effect is highly dependent on frequency. Piezoelectric materials have a characteristic resonance frequency at which their response to mechanical or electrical excitation is most pronounced.
- **Temperature Stability:** The piezoelectric properties of materials can be temperature-dependent. Some materials exhibit stable piezoelectric behavior over a wide temperature range, while others may experience a significant decrease in performance at elevated temperatures.

- **Mechanical Compliance:** This property describes the ability of a piezoelectric material to deform in response to an applied electric field. It influences the sensitivity and response time of piezoelectric devices.
- **Fatigue and Aging:** Piezoelectric materials can experience fatigue and aging effects when subjected to repeated mechanical or electrical loading. These effects can degrade the material's performance over time.
- **Poling:** Piezoelectric materials often require a poling process, which involves applying a strong electric field to align the electric dipoles within the crystal structure. Poling enhances the piezoelectric properties and ensures uniformity in the material's response.

1.4 Description

The project aims to design and simulate a piezoelectric cantilever system capable of testing various fluids and sensing methods. The cantilever will be equipped with sensors to detect changes in the fluid properties, such as viscosity, density, and composition. The primary objective is to develop a versatile platform that can accommodate different types of fluids and accurately sense their characteristics. This project will involve exploring various design parameters, sensor configurations, and testing methodologies to optimize the performance of the piezoelectric cantilever system.

1.5 Problem Statement

In various scientific and engineering endeavors, the ability to accurately test and analyze fluid properties is essential for understanding complex systems and optimizing processes. Traditional methods often involve cumbersome setups and limited sensitivity, hindering advancements in fields such as microfluidics, biomedical engineering, and environmental monitoring. To address these challenges, this project focuses on leveraging COMSOL Multiphysics, a powerful simulation software, to design, simulate, and optimize a piezoelectric cantilever system for fluid testing.The problem at hand revolves around the design and performance of the piezoelectric cantilever within COMSOL Multiphysics. This includes:

- **Geometry and Material Selection:** Designing the geometry of the cantilever and selecting appropriate piezoelectric materials to maximize sensitivity and response.
- **Fluid-Structure Interaction Modeling:** Simulating the interaction between the fluid and the cantilever to accurately predict the mechanical deformation and resulting electrical signals.

- **Electrical Circuit Simulation:** Modeling the electrical circuitry of the piezoelectric sensor to analyze its response to mechanical stimuli and fluid properties.
- **Parameter Optimization:** Iteratively adjust design parameters such as cantilever dimensions, material properties, and operating conditions to optimize sensitivity and performance.
- **Validation and Verification:** Comparing simulation results with experimental data to validate the accuracy and reliability of the COMSOL model.

Through this project, we aim to provide a comprehensive understanding of the piezoelectric cantilever system's behavior under different fluid conditions and optimize its performance for various applications. The use of COMSOL Multiphysics facilitates efficient simulation and analysis, enabling researchers and engineers to explore design alternatives and iterate toward superior solutions.

1.6 Research Motivation

Understanding fluid behavior is vital across various fields, yet traditional testing methods often face limitations in sensitivity, accuracy, and versatility. This research explores the innovative use of piezoelectric cantilevers to address these challenges, offering enhanced sensitivity and adaptability in fluid testing applications.

- **Sensitivity Enhancement:** Conventional methods struggle with detecting small changes in fluid properties, especially in dynamic environments. Piezoelectric cantilevers offer superior sensitivity, enabling the detection of minute variations in density, viscosity, and composition.
- **Versatility and Adaptability:** Existing techniques may lack versatility across different fluid types and conditions. Piezoelectric cantilevers, with customizable designs, provide a flexible platform suitable for applications from biomedical diagnostics to environmental monitoring.
- **Simulation-Driven Optimization**: Leveraging advanced simulation tools like COMSOL Multiphysics allows for rapid prototyping and optimization of cantilever designs. This approach minimizes experimental iterations, reducing time and costs while enhancing performance.

1.7 Objectives

- To gain a comprehensive understanding of the mechanical and piezoelectric properties of cantilever structures, focusing on their behavior within the COMSOL Multiphysics simulation environment.

- To Investigate the sensing mechanism of piezoelectric cantilevers within COMSOL, exploring how variations in fluid properties affect the cantilever's mechanical deformation and piezoelectric response.
- To develop a detailed COMSOL simulation model that accurately represents the behavior of a piezoelectric cantilever when exposed to different fluid environments, incorporating parameters such as material properties, geometry, and boundary conditions.
- To design and optimize the simulation setup to enhance the sensitivity and selectivity of the piezoelectric cantilever sensor for detecting and distinguishing between various fluids, ensuring robust performance across a wide range of operating conditions.
- To Implement the designed sensor model in COMSOL Multiphysics to systematically simulate and evaluate its performance when interacting with different fluids, enabling the identification of unique signatures indicative of fluid characteristics.

1.8 Expected Outcomes

- A validated COMSOL Multiphysics model of the piezoelectric cantilever system for fluid testing.
- Insights into the influence of design parameters on the system's sensitivity and performance.
- Optimization guidelines for maximizing the effectiveness of the piezoelectric cantilever in fluid testing applications.

1.9 Concluding remarks

In conclusion, the design and simulation of piezoelectric cantilever systems offer promising avenues for advancing fluid testing and analysis. Leveraging the unique properties of cantilever beams and materials like Lead Zirconate Titanate (PZT), these systems enable highly sensitive and precise detection of fluid properties. COMSOL Multiphysics facilitates efficient modeling and optimization, ensuring the cantilever systems achieve optimal performance. Experimental validation enhances confidence in simulation results, validating the accuracy and reliability of the model. Piezoelectric cantilever systems find applications in diverse fields, promising to revolutionize fluid testing methodologies with unprecedented accuracy. As research progresses, exploration of new materials and configurations will drive innovation in this critical area of study.

CHAPTER 2

LITERATURE REVIEW

2.1 History of Cantilever

A cantilever, also known as a fixed-end beam, is a structural method where a beam is supported at only one end, creating a stable support. This approach, alongside post-and-beam construction and arch construction, forms one of the fundamental structural methods. The practicality of cantilevers in architecture became widespread with the advent of steel, as its strength combined with cement is essential for effective cantilever construction. Buildings employing cantilevers feature an internal skeleton, allowing walls to hang like curtains rather than serving as load-bearing supports for ceilings. This design flexibility enables architects, such as Frank Lloyd Wright, to create interiors with almost arbitrary designs for utility or aesthetic preference [22]

He was a renowned architect, notably utilized the cantilever system in the construction of the Robie House in Chicago in 1906. By incorporating steel and concrete, Wright extended the roof of the Robie House beyond its support by 20 ft (6 m). This innovation, coupled with Wright's emphasis on integrating buildings with their natural surroundings, gave rise to the Prairie School of architecture. Before cantilevers found use in buildings, they were employed in bridge construction. Heinrich Gerber constructed the first cantilever bridge in Germany in the late 1800s, drawing inspiration from ancient Chinese bridges. Cantilever bridges eliminated the need for supports in the middle, enabling them to span deep ravines or rivers. This engineering breakthrough allowed for the construction of bridges across wide bodies of water and valleys with fewer and more widely spaced supports, transforming the realms of architecture and civil engineering [23].

2.2 History of Piezoelectric Cantilever [24]

The first experimental demonstration of a connection between macroscopic piezoelectric phenomena and crystallographic structure was published in 1880 by Pierre and Jacques Curie. Their experiment consisted of a conclusive measurement of surface charges appearing on specially prepared crystals (tourmaline, quartz, topaz, cane sugar, and Rochelle salt among them) which were subjected to mechanical stress. These results were a credit to the Curies' imagination and perseverance, considering that they were obtained with nothing more than tinfoil, glue, wire, magnets, and a jeweler's saw. In the scientific circles of the day, this effect was considered quite a "discovery," and was quickly dubbed as "piezoelectricity" to distinguish it from other areas of scientific phenomenological experience such as "contact electricity" (friction-generated static electricity) and "pyroelectricity" (electricity generated from crystals by heating).

The Curie brothers asserted, however, that there was a one-to-one correspondence between the electrical effects of temperature change and mechanical stress in a given crystal, and that they had used this correspondence not only to pick the crystals for the experiment but also to determine the cuts of those crystals. To them, their demonstration was a confirmation of predictions that followed naturally from their understanding of the microscopic crystallographic origins of pyroelectricity (i.e., from certain crystal asymmetries). The Curie brothers did not, however, predict that crystals exhibiting the direct piezoelectric effect (electricity from applied stress) would also exhibit the converse piezoelectric effect (stress in response to applied electric field). This property was mathematically deduced from fundamental thermodynamic principles by Lippmann in 1881. The Curies immediately confirmed the existence of the "converse effect," and continued on to obtain quantitative proof of the complete reversibility of electro-elastic-mechanical deformations in piezoelectric crystals.

At this point, after only two years of interactive work within the European scientific community, the core of piezoelectric applications science was established: the identification of piezoelectric crystals based on asymmetric crystal structure, the reversible exchange of electrical and mechanical energy, and the usefulness of thermodynamics in quantifying complex relationships among mechanical, thermal and electrical variables.In the following 25 years (leading up to 1910), much more work was done to make this core grow into a versatile and complete framework that defined completely the 20 natural crystal classes in which piezoelectric effects occur and defined all 18 possible macroscopic piezoelectric coefficients accompanying a rigorous thermodynamic treatment of crystal solids using appropriate tensorial analysis. In 1910 Voigt's "Lehrbuch der Kristallphysik" was published, and it became the standard reference work embodying the understanding that had been reached.During the 25 years that it took to reach Voigt's benchmark, however, the world was not holding its breath for piezoelectricity. A science of such subtlety as to require tensorial analysis just to define relevant measurable quantities paled by comparison to electro-magnetism, which at the time was maturing from a science to a technology, producing highly visible and amazing machines. Piezoelectricity was obscure even among crystallographers; the mathematics required to understand it was complicated; and no publicly visible applications had been found for any of the piezoelectric crystals.The first serious application work on piezoelectric devices took place during World War I. In 1917, P. Langevin and French co-workers began to perfect an ultrasonic submarine detector. Their transducer was a mosaic of thin quartz crystals glued between two steel plates (the composite having a resonant frequency of about 50 KHz), mounted in a housing suitable for submersion. Working past the end of the war, they did achieve their goal of emitting a high-frequency "chirp" underwater and measuring depth by timing the return echo. The strategic importance of their achievement was not overlooked by any industrial nation, however, and since that time the development of sonar transducers, circuits, systems, and materials has never ceased.

2.2.1 First Generation Applications with Natural Piezoelectric Crystals: 1920 – 1940

The success of sonar stimulated intense development activity on all kinds of piezoelectric devices, both resonating and non-resonating. Some examples of this activity include:

Megacycle quartz resonators were developed as frequency stabilizers for vacuum-tube oscillators, resulting in a ten-fold increase in stability.A new class of materials testing methods was developed based on the propagation of ultrasonic waves. For the first time, elastic and viscous properties of liquids and gases could be determined with comparative ease, and previously invisible flaws in solid metal structural members could be detected. Even acoustic holographic techniques were successfully demonstrated.

Also, new ranges of transient pressure measurement were opened up permitting the study of explosives and internal combustion engines, along with a host of other previously unmeasurable vibrations, accelerations, and impacts. In fact, during this revival following World War I, most of the classic piezoelectric applications with which we are now familiar (microphones, accelerometers, ultrasonic transducers, bender element actuators, phonograph pick-ups, signal filters, etc.) were conceived and reduced to practice. It is important to remember, however, that the materials available at the time often limited device performance and certainly limited commercial exploitation [25].

2.2.2 Second Generation Applications with Piezoelectric Crystals: 1940 - 1965

During World War II, in the U.S., Japan and the Soviet Union, isolated research groups working on improved capacitor materials discovered that certain ceramic materials (prepared by sintering metallic oxide powders) exhibited dielectric constants up to 100 times higher than common cut crystals. Furthermore, the same class of materials (called ferroelectrics) were made to exhibit similar improvements in piezoelectric properties. The discovery of easily manufactured piezoelectric ceramics with astonishing performance characteristics naturally touched off a revival of intense research and development into piezoelectric devices.The advances in materials science that were made during this phase fall into three categories:

- Development of the barium titanate family of piezoceramics and later the lead zirconate titanate family.
- The development of an understanding of the correspondence of the perovskite crystal structure to electro-mechanical activity.
- The development of a rationale for doping both of these families with metallic impurities in order to achieve desired properties such as dielectric constant, stiffness, piezoelectric coupling coefficients, ease of poling, etc.

All of these advances contributed to establishing an entirely new method of piezoelectric device development - namely, tailoring a material to a specific application. Historically speaking, it had always been the other way around.

This "lock-step" material and device development proceeded the world over, but was dominated by industrial groups in the U.S. who secured an early lead with strong patents. The number of applications worked on was staggering, including the following highlights and curiosities:

- **Powerful sonar** - based on new transducer geometries (such as spheres and cylinders) and sizes achieved with ceramic casting.
- **Ceramic phono cartridge** - cheap, high signal elements simplified circuit design.
- **Piezo ignition systems** - single-cylinder engine ignition systems which generated spark voltages by compressing a ceramic "pill".
- **Sonobuoy** - sensitive hydrophone listening/radio transmitting buoys for monitoring ocean vessel movement.
- **Small, sensitive microphones** - became the rule rather than the exception.
- **Ceramic audio tone transducer** - small, low power, low voltage, audio tone transducer consisting of a disc of ceramic laminated to a disc of sheet metal.
- **Relays -** snap action relays were constructed and studied, and at least one piezo relay was manufactured

It is worth noting that during this revival, especially in the U.S., device development was conducted along with piezo material development within individual companies. As a matter of policy, these companies did not communicate. The reasons for this were threefold: first, the improved materials were developed under wartime research conditions, so the experienced workers were accustomed to working in a "classified" atmosphere; second, post-war entrepreneurs saw the promise of high profits secured by both strong patents and secret processes; and third, the fact that by nature piezoceramic materials are extraordinarily difficult to develop, yet easy to replicate once the process is known.

From a business perspective, the market development for piezoelectric devices lagged behind the technical development by a considerable margin. Even though all the materials in common use today were developed by 1970, at that same point in time only a few high-volume commercial applications had evolved (phono cartridges and filter elements, for instance). Considering this fact with hindsight, it is obvious that while new material and device developments thrived in an atmosphere of secrecy, new market development did not - and the growth of this industry was severely hampered.

2.2.3 Developments: 1965 - 1980

In contrast to the "secrecy policy" practiced among U.S. piezoceramic manufacturers at the outset of the industry, several Japanese companies and universities formed a "competitively cooperative" association, established as the Barium Titanate Application Research Committee, in 1951. This association set an organizational precedent for successfully surmounting not only technical challenges and manufacturing hurdles but also for defining new market areas. Beginning in 1965 Japanese commercial enterprises began to reap the benefits of steady applications and materials development work which began with a successful fish-finder test in 1951. From an international business perspective, they were "carrying the ball," i.e., developing new knowledge, new applications, new processes, and new commercial market areas coherently and profitably.

Persistent efforts in materials research had created new piezoceramic families that were competitive with Vernitron's PZT, but free of patent restrictions. With these materials available, Japanese manufacturers quickly developed several types of piezoceramic signal filters, which addressed needs arising in television, radio, and communications equipment markets; and piezoceramic igniters for natural gas/butane appliances.As time progressed, the markets for these products continued to grow, and other similarly valuable ones were found. Most notable were audio buzzers (smoke alarms, TTL compatible tone generators), air ultrasonic transducers (television remote controls and intrusion alarms), and SAW filter devices (devices employing Surface Acoustic Wave effects to achieve high-frequency signal filtering).By comparison to the commercial activity in Japan, the rest of the world was slow, even declining. Globally, however, there was still much pioneering research work taking place as well as device invention and patenting.

2.3 Applications [26]

- **Fluid Dynamics and Research Institutions:**

  Research institutions focusing on fluid dynamics can benefit from the detailed analysis of different liquids. The piezoelectric cantilever's ability to measure fluid properties can contribute to fundamental research in fluid mechanics.

- **Oil and Gas Industry:**

  The project's insights into the behavior of fluids like heptane and octane can be invaluable in the oil and gas industry. It may aid in the characterization of hydrocarbons and their derivatives, crucial for exploration, extraction, and refining processes.

- **Biomedical Engineering:**

  In biomedical applications, the project's piezoelectric cantilever could be employed in biosensors for detecting and analyzing fluid samples. This could have applications in medical diagnostics and research, offering a new tool for studying biological fluids.

- **Pharmaceuticals and Chemical Processing:**

  Industries involved in pharmaceuticals and chemical processing can use the technology to assess the properties of various solvents. This could enhance the quality control processes in drug manufacturing and chemical synthesis.

- **Automotive Research and Development:**

  Automotive companies can leverage the understanding of fluid properties, particularly the octane rating, to improve fuel formulations and enhance engine performance. The project may contribute to advancements in fuel efficiency and emissions control.

- **Environmental Monitoring Agencies:**

  The ability to analyze fluids, including antifreeze agents like ethylene glycol, can find applications in environmental monitoring. Monitoring the behavior of such fluids in different conditions may be crucial for assessing environmental impact.

- **Materials Engineering:**

  The project's use of specific materials in the cantilever design, such as structural steel and PZT 5A, has implications for materials engineering. It may inspire the development of new materials with tailored mechanical and piezoelectric properties.

- **Quality Control in Industrial Processes:**

  Industries involving the use of various liquids in manufacturing processes can implement the piezoelectric cantilever system for real-time quality control. This can ensure consistent fluid properties and enhance the overall quality of end products.

- **Renewable Energy:**

  Understanding the fluid dynamics using the piezoelectric cantilever can be relevant in renewable energy applications. It might contribute to optimizing the efficiency of fluid-based systems, such as those used in hydropower or thermal energy storage.

- **Aerospace Industry:**

  In aerospace, where fluid dynamics play a critical role, the project's findings may contribute to the design and optimization of fluid systems within aircraft and spacecraft, potentially improving efficiency and safety .

2.4 Concluding Remarks

In conclusion, the historical evolution of cantilevers, from their architectural use pioneered by Frank Lloyd Wright to their application in bridge construction, has laid the foundation for innovative structural design. Meanwhile, the journey of piezoelectric cantilevers traces back to the pioneering work of Pierre and Jacques Curie in 1880, evolving into a crucial component of various technological advancements. The literature review highlights the trajectory of piezoelectricity, from its early experimental stages to its applications during World War I and the subsequent developments in the mid-20th century. The strategic importance of piezoelectric devices, especially in sonar technology, propelled their evolution, paving the way for the first and second generations of applications.

CHAPTER-3

METHODOLOGY

3.1 Introduction

In this chapter, we investigate the design and simulation of a piezoelectric cantilever, aiming to explore its potential applications in fluid testing. This innovative approach holds promise for enhancing our understanding of fluid dynamics and opens avenues for advanced sensing technologies. Through comprehensive simulations, we seek to elucidate the performance and capabilities of the proposed piezoelectric cantilever in evaluating various fluid properties.

In our work, we utilized COMSOL Multiphysics software for the design and simulation of the piezoelectric cantilever. COMSOL Multiphysics is a simulation platform that provides fully coupled Multiphysics and single-physics modeling capabilities. The Model Builder includes all of the steps in the modeling workflow — from defining geometries, material properties, and the physics that describe specific phenomena to performing computations and evaluating the results.

3.2 Structural Configuration

The model consists of three main components: the base (block 1), the cantilever beam itself (block 2), and the container (block 3). Each component was designed using specific materials to achieve the desired functionality:

- **Base (Block 1):** We employed structural steel as the material for the base of the cantilever. Structural steel offers excellent strength and durability, providing a stable foundation for the cantilever system.
- **Cantilever Beam (Block 2):** The cantilever portion of the device was fabricated using PZT 5A, which is Lead Zirconate Titanate, a commonly used piezoelectric material known for its high sensitivity to mechanical stress and excellent electromechanical properties. PZT 5A enables the conversion of mechanical energy from fluid interactions into electrical signals, facilitating the detection and analysis of fluid properties.
- **Container (Block 3):** The container housing the fluid specimens was designed to accommodate five different liquids -

  a) Ethylene glycol

  b) Diethyl ether

c) Octane
d) Heptane
e) Toluene

Each liquid offers distinct properties and behaviors, allowing for a comprehensive evaluation of the cantilever's performance in different fluid environments.

3.3 Fluid Properties

The five liquids selected for testing in our study exhibit diverse characteristics, contributing to the robustness of our analysis:

a) **Ethylene Glycol:** A viscous liquid commonly used as an antifreeze agent due to its high boiling point and low freezing point. Ethylene glycol is known for its thermal stability and compatibility with various materials.
   - Chemical formula: $C_2H_6O_2$
   - Structure: It's a simple diol, meaning it has two hydroxyl (-OH) groups attached to a two-carbon chain. Its structure can be represented as $HO\text{-}CH_2\text{-}CH_2\text{-}OH$.
   - Physical properties: Colorless, odorless, viscous liquid. Miscible with water in all proportions. Has a high boiling point (197°C) and a low freezing point (-13°C).
   - Applications: Antifreeze in car engines, coolant in industrial processes, deicer, and a component in the production of polyester fibers.

b) **Diethyl Ether:** A volatile and highly flammable liquid with a characteristic ether-like odor. Diethyl ether is commonly used as a solvent and as a starting fluid for diesel engines due to its low ignition temperature.
   - Chemical formula: $C_4H_{10}O$
   - Structure: An ether, meaning it has two ethyl groups ($-CH_2\text{-}CH_3$) attached to an oxygen atom (O). Its structure can be represented as $CH_3\text{-}CH_2\text{-}O\text{-}CH_2\text{-}CH_3$.
   - Physical properties: Colorless, volatile, flammable liquid with a strong, sweet odor. Soluble in water to a limited extent. Has a low boiling point (34.6°C) and a low freezing point (-116°C).
   - Applications: Anesthetic, solvent, and starting material for the production of other chemicals.

c) **Octane:** A hydrocarbon compound primarily found in gasoline, contributing to its combustion properties. Octane rating is a measure of a fuel's resistance to knocking or pinging during combustion in an internal combustion engine.

- Chemical formula: C_8H_{18}
- Structure: An alkane, but unlike heptane, it can have branched chains. There are 18 different structural isomers of octane, but the most common ones are n-octane (straight-chain) and iso-octane (branched-chain).
- Physical properties: Colorless, flammable liquid. Insoluble in water. Has a slightly higher boiling point (125°C) and freezing point (-57°C) than heptane.
- Applications: The primary component of gasoline. Its octane rating, a measure of its anti-knocking properties, is crucial for engine performance.

d) **Heptane:** Another hydrocarbon compound commonly used as a solvent and as a standard reference for octane ratings. Heptane exhibits similar combustion characteristics to octane but with a lower resistance to knocking.

- Chemical formula: C_7H_{16}
- Structure: A straight-chain alkane, meaning it only has carbon-carbon and carbon-hydrogen single bonds. Its structure can be represented as CH_3-$(CH_2)_5$-CH_3.
- Physical properties: Colorless, flammable liquid. Insoluble in water. Has a low boiling point (98°C) and a low freezing point (-90°C).
- Applications: A component of gasoline, solvent, and cleaning agent.

e) **Toluene:** A volatile aromatic hydrocarbon solvent with various industrial applications, including in the production of explosives, dyes, and pharmaceuticals. Toluene is known for its high-octane rating and solvent properties.

- Chemical formula: C_7H_8
- Structure: An aromatic hydrocarbon, meaning it has a benzene ring (a six-membered carbon ring with alternating single and double bonds). It has a methyl group (-CH_3) attached to the ring.
- Physical properties: Colorless, flammable liquid with a distinctive odor. Insoluble in water. Has a moderate boiling point (111°C) and a low freezing point (-95°C).

TABLE 3.1: MATERIAL PARAMETERS FOR DIFFERENT FLUIDS

Parameters	Liquids				
	Diethyl Ether	Heptane	Octane	Toluene	Ethylene Glycol
Density (kg/m^3)	713	684	703	867	1090
Dynamic Viscosity (Pascal)	0.000223	0.000376	0.00051	0.000550	0.0162
Young's Modulus (pa)	1	1	1	1	1
Poisson's Ratio	1	1	1	1	1
Thermal Conductivity (w/m K)	0.130	0.140	0.147	0.151	0.258

TABLE 3.2: PIEZOELECTRIC MATERIAL CONSTANTS

ELASTICITY MATRIX:					
S_{11}	S_{12}	S_{13}	0	0	0
S_{12}	S_{11}	S_{13}	0	0	0
S_{13}	S_{13}	S_{33}	0	0	0
0	0	0	S_{44}	0	0
0	0	0	0	S_{44}	0
0	0	0	0	0	S_{66}

We incorporated the concept of elasticity matrices and material constants to accurately model the mechanical behavior of the piezoelectric cantilever. The elasticity matrix serves as a fundamental tool in our finite element simulations, allowing us to predict the deformation and stress distribution within the cantilever structure under various fluid interactions. By introducing the elasticity matrix, we establish a mathematical framework that links applied stress to the resulting strain, providing crucial insights into the mechanical response of the cantilever.

3.4 Mathematical Model

Piezoelectric systems are used in finite element analysis to break down a continuous system into smaller, manageable parts. They connect mechanical behavior with electrical charges using certain constants and considering a beam fixed at one end with a length labeled as L. The equations involve stress, strain, and electric fields.

$$\varepsilon = S^E\sigma + d_{/} E \tag{1}$$

$$D = d\sigma + \eta E^{/} \tag{2}$$

Where E is a strain vector, S^E is a compliance or elasticity matrix, is a stress vector, d is a piezoelectric coupling coefficient matrix, $d^{/}$ is the transpose of piezoelectric coupling coefficient matrix E^t is the electric field vector, D is the electric displacement vector, η is the electric

Material Constants	Values
S_{11}	1.20346×10^{11}
S_{12}	7.51791×10^{10}
S_{13}	7.50901×10^{10}
S_{33}	1.10867×10^{11}
S_{44}	2.10526×10^{10}
S_{66}	2.25734×10^{10}

permittivity matrix. Elasticity helps determine how a material responds to stress and strain, essentially how stiff or flexible it is. Hooke's law describes this relationship-

$$\sigma = C\varepsilon \tag{3}$$

$$\varepsilon = S\sigma \tag{4}$$

Here, C is mechanical stiffness and S is compliance.

Hooke's law which connects stress and strain through a stiffness value called Young's Modulus (E), can be written in another way.

$$\sigma = E\varepsilon \tag{5}$$

The elasticity matrix for isotropic materials can be expressed as

$$S^E = \frac{E}{(1+\upsilon)(1-2\upsilon)}\begin{Bmatrix} 1-\upsilon & \upsilon & \upsilon & 0 & 0 & 0 \\ \upsilon & 1-\upsilon & \upsilon & 0 & 0 & 0 \\ \upsilon & \upsilon & 1-\upsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\upsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\upsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\upsilon \end{Bmatrix}$$

I. **Deflection Formula**: The deflection (bending) of a tiny beam is determined by a formula:

Deflection (δ) = $\frac{FL^3}{3EI}$

- F is the force applied at the end of the beam.
- L is the length of the beam.
- E is the material's stiffness (Young's modulus).
- I represent how the beam's cross-sectional area is distributed.

II. **Beam Stiffness**: The stiffness of the beam is determined by another formula:

Stiffness (k) = $\frac{3EI}{L^3}$

- This tells us how resistant the beam is to bending.

III. **Calculating I:** The value of I can be found using the formula:

I = $\frac{wt^3}{12}$

- w is the width of the beam.
- t is the thickness of the beam.

IV. **Relationship between Deflection and Stress:** The deflection happens because of stress differences in the beam's top and bottom surfaces. Stoney's equation helps us calculate this difference:

Difference = $\frac{1}{R} = 6 * \frac{(1-\nu)}{Et^2} (\Delta\sigma_1 - \Delta\sigma_2)$

- R is the beam's curvature due to stress.
- E is the material's stiffness.
- t is the beam's thickness
- $\Delta\sigma_1$ and $\Delta\sigma_2$ are stresses acting on top and bottom surfaces of the beam.

The stresses acting on the top and bottom surfaces of a beam. To estimate the deflection of the beam, we use a formula that involves the beam's free-end deflection.

$$\frac{1}{R} = \frac{2\delta 1}{L^2 R} = \frac{2\delta}{L^2}$$

There is the beam's free end deflection. This deflection happens because the free end of the micro-cantilever beam bends when it's vibrated by a piezoelectric material. The amount of bending depends on the vibrational amplitude, which is highest at the resonant frequency.

V. **Resonant frequency:** The resonant frequency of the micro-cantilever beam can be calculated using a formula that involves a resonance constant and the density of the beam material.

$$f_0 = \frac{\lambda^2 t}{2\pi L^2} \sqrt{\frac{E}{12p}}$$

3.5 Sequential Steps

From the File menu, choose New.

New

In the New window, click Model Wizard.

MODEL WIZARD

i. In the model wizard window, click 3D.

ii. In the select physics tree, select Structural Mechanics > Electromagnetics Structure Interaction> Piezoelectricity> Piezoelectricity, Solid.

iii. Click Add.

iv. In the Select Physics tree, select AC/DC> Electrical Circuit(cir).

v. Click Add.

vi. Click Study.

vii. In the Select Study tree, select General Studies> Frequency Domain.

viii. Click Done.

GLOBAL DEFINITIONS

Parameters I

1. In the **Model Build**er window, under Global Definitions click Parameters I.
2. In the settings window for parameters, locate the parameters sections.
3. In the table, enter the following settings:

Name	Expression	Value	Description
Acc	1	I	Acceleration(g)
R_load	12[kohm]	12000	Load Resistance
w_plate	14[mm]	0.014m	Out of Plane Dimension

FORMATION OF GEOMETRY

1. In the Model Builder window, under Component 1 (comp1) click Geometry 1.
2. In the Settings window for Geometry, locate the Units section.
3. From the Length unit list, choose mm.

Block 1

1. In the Geometry toolbar, click block.
2. In the setting window for block, locate the size section.
3. In the Width text field, type 1.
4. In the Height text field, type 1.
5. In the length text field, type 1.

Block 2

1. In the Geometry toolbar, click block.

2. In the setting window for block, locate the size section.
3. In the Width text field, type 1.
4. In the Height text field, type 0.01.
5. In the length text field, type 5.

Block 3

1. In the Geometry toolbar, click block.
2. In the setting window for block, locate the size section.
3. In the Width text field, type 0.3.
4. In the Height text field, type 0.3.
5. In the length text field, type 0.3.

Union I

1. In the Geometry toolbar, click Booleans and Partitions and choose Union.
2. Select the objects r1 and r2 only.
3. In the Settings window for Union, locate the Union section.
4. Click Zoom to Selection.
5. 5lick in the Graphics window and then press Ctrl+A to select all objects.
6. Clear the Keep interior boundaries check box.

SELECTION OF MATERIAL

1. In the Home toolbar, click Add Material to open the Add Material window.
2. Go to the Add Material window.
3. In the tree, select Piezoelectric>Lead Zirconate Titanate (PZT-5A).
4. Click Add to Component in the window toolbar.
5. In the tree, select Built-in>Structural steel.
6. Click Add to Component in the window toolbar.
7. In the Home toolbar, click Add Material to close the Add Material window.

MATERIALS

Structural steel (mat1)

Select domain 1 only.

SOLID MECHANICS (SOLID)

Linear Elastic Material

In the Model Builder window, under Component 1 (comp1)>Solid Mechanics (solid) click Linear Elastic Material I.

Damping I

1. In the Physics toolbar, click Attributes and choose Damping.
2. In the Settings window for Damping, locate the Damping Settings section.
3. From the Damping type list, choose Isotropic loss factor.
4. From the ηs list, choose User defined. In the associated text field, type 0.001

Piezoelectric Material I

1. In the Model Builder window, under Component 1 (comp1)>Solid Mechanics (solid) click Piezoelectric Material I.
2. In the Settings window for Piezoelectric Material, locate the Domain Selection section.
3. From the Selection list, choose Manual.
4. Select Domain 2.

Mechanical Damping I

1. In the Physics toolbar, click Attributes and choose Mechanical Damping.
2. In the Settings window for Mechanical Damping, locate the Damping Settings section.
3. From the Damping type list, choose Isotropic loss factor.
4. From the ηs list, choose User defined. In the associated text field, type 0.001.

Body Load I

1. In the Physics toolbar, click Domains and choose Body Load.
2. In the Settings window for Body Load, locate the Domain Selection section.
3. From the Selection list, choose All domains.
4. Locate the Force section. Specify the FV vector as

0	X
-solid.rho*g_const*acc	y
0	z

Fixed Constraint I

1. In the Physics toolbar, click Boundaries and choose Fixed Constraint.
2. Select Block 3 only.

ELECTROSTATICS (ES)

1. In the Model Builder window, under Component 1 (comp1) click Electrostatics (es).
2. Select Domain 2.

Ground I

1. In the Physics toolbar, click Boundaries and choose Ground.
2. Select Boundaries 5, 7 and 17 only.

Terminal I

1. In the Physics toolbar, click Boundaries and choose Terminal.
2. Select Boundary 2 only.
3. In the Settings window for Terminal, locate the Terminal section.
4. From the Terminal type list, choose Circuit.

ELECTRICAL CIRCUIT (CIR)

In the Model Builder window, under Component 1 (comp1) click Electrical Circuit (cir).

Resistor I (R1)

1. In the Electrical Circuit toolbar, click Resistor.
2. In the Settings window for Resistor, locate the Node Connections section.
3. In the table, enter the following settings:

Label	Node names
n	0

4. locate the device parameters section. In the R text field, type R_load.

External I-terminal 1 (term I)

1. In the Electrical Circuit toolbar, click External I-terminal.
2. In the Settings window for External I-terminal, locate the Node Connections section.
3. In the Node name text field, type 1.
4. Locate the External Terminal section. From the V list, choose Terminal voltage (es/ term1).

The high aspect ratio of the modelled geometry makes this problem numerically challenging. There is only a moderate range of mesh sizes where the result is reliable within a few percent. Outside of this range, with the mesh either too coarse or too fine, the result is not reliable.

MESH 1

Free Triangular I

In the Mesh toolbar, click Free Triangular.

Size

1. In the Model Builder window, click Size.
2. In the Settings window for Size, locate the Element Size section.
3. Click the Custom button.
4. Locate the Element Size Parameters section. In the Maximum element size text field, type 0.02.
5. In the Minimum element size text field, type 0.002.

FREQUENCY RESPONSE

1. In the Model Builder window, click Study 1.
2. In the Settings window for Study, type Frequency Response in the Label text field.

Step 1: Frequency Domain

1. In the Model Builder window, under Frequency Response click Step 1: Frequency Domain.
2. In the Settings window for Frequency Domain, locate the Study Settings section.
3. In the Frequencies text field, type range (62,1,80).

Disable direct solver error checking which is too stringent in this case.

Solution I (sol I)

1. In the Study toolbar, click Show Default Solver.
2. In the Model Builder window, expand the Solution 1 (sol1) node.

DEFINITIONS

Define a nonlocal integration coupling to calculate mechanical power input later when plotting results.

Integration 1 (intop1)

1. In the Model Builder window, expand the Frequency Response>Solver Configurations> Solution 1 (sol1)>Stationary Solver 1 node.
2. Right-click Component 1 (comp1)>Definitions and choose Nonlocal Couplings> Integration.
3. In the Settings window for Integration, locate the Source Selection section.
4. From the Selection list, choose All domains.

FREQUENCY RESPONSE

Solution 1 (sol1)

1. In the Model Builder window, under Frequency Response>Solver Configurations> Solution 1 (sol1)>Stationary Solver 1 click Direct.
2. In the Settings window for Direct, click to expand the Error section.

3. From the Check error estimate list, choose No.
4. In the Home toolbar, click Compute.

RESULTS

Frequency Response: Von Mises Stress

1. In the Home toolbar, click Add Plot Group and choose 3D Plot Group.
2. In the Settings window for 1D Plot Group, type Frequency Response: Von Mises Stress in the Label text field.
3. Click to expand the Title section. From the Title type list, choose Manual.
4. In the Title text area, type Frequency Response: Von Mises Stress.

CHAPTER-4

RESULTS AND DISCUSSION

4.1 Results and Discussions

The following results and discussions reflect, how density of different liquids affects the deflection of a cantilever beam –

4.1.1 Heptane

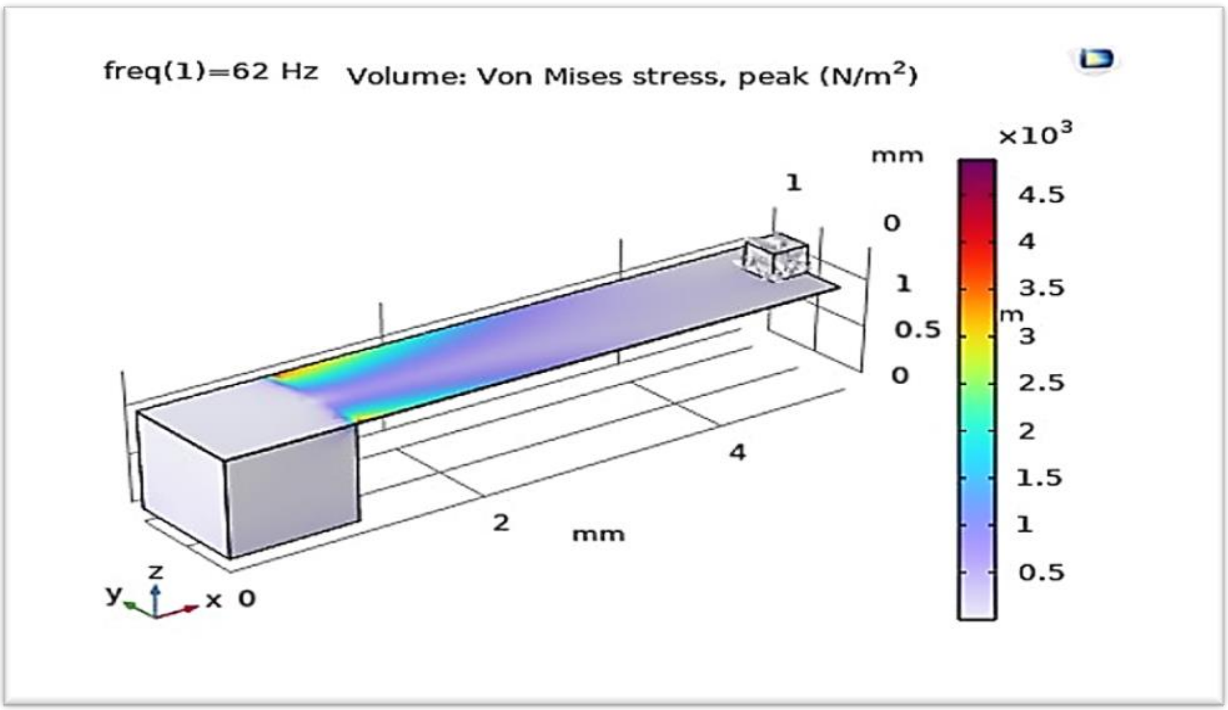

Fig. 4.1 Von mises stress of Heptane

Result

This graph shows a contour plot of the Von Mises stress experienced by a cantilever beam subjected to a frequency of 80 Hz. The stress is highest at the fixed end (left) of the beam and decreases towards the free end (right). The maximum stress is 4.5 N/m^2.

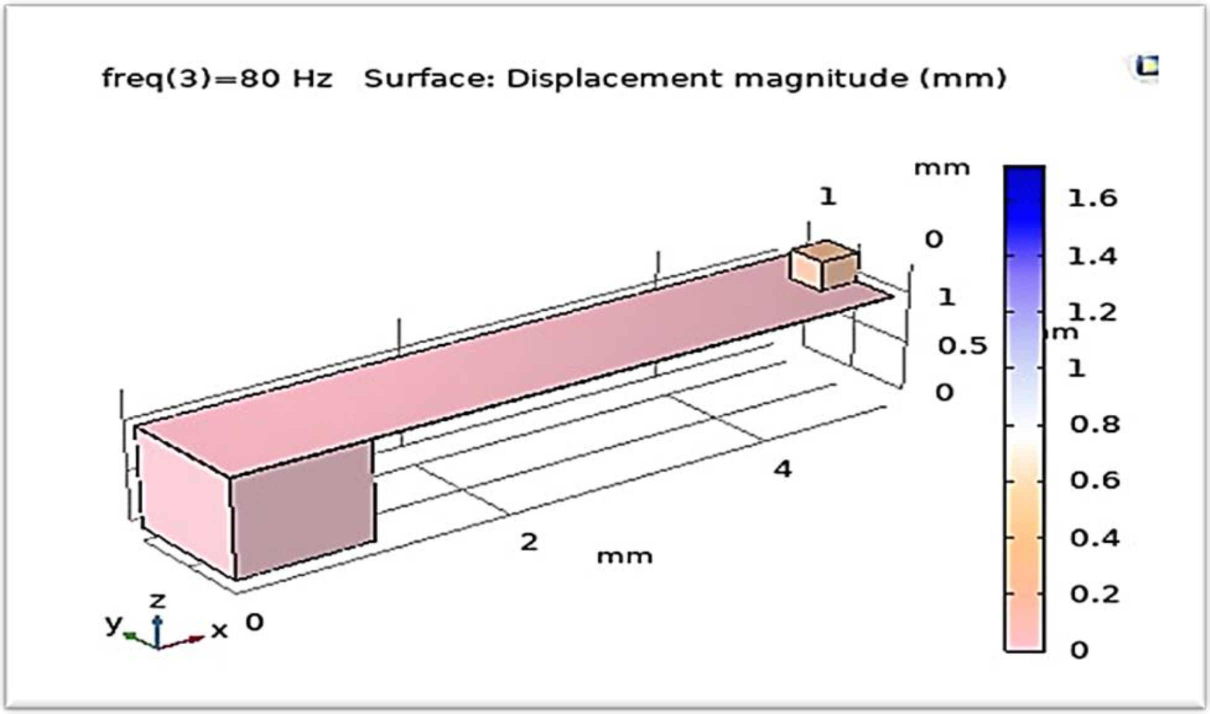

Fig. 4.2 Displacement magnitude of Heptane

Result

This graph shows a surface plot of the displacement magnitude of the beam under the same conditions. The displacement is greatest in the center of the beam and decreases towards the edges. The maximum displacement is about 1.6 mm.

Discussion

The two graphs are related in that they show the effects of the same force (the frequency of 80 Hz) on the cantilever beam. The stress graph shows how much force is being exerted on the beam at different points, while the displacement graph shows how much the beam is bending under that force.

The results of the graphs are consistent with what we would expect for a cantilever beam under vibration. Cantilever beams experience the most stress at the fixed end and the most displacement in the centre, as seen in the first and second graphs, respectively. This is because the fixed end is unable to move, so it must absorb all of the force from the vibrations. The centre of the beam, on the other hand, is free to move, so it deflects more easily.The fact that the stress is highest at the fixed end of the beam is consistent with engineering principles. This is the location where the beam is mounted and thus cannot deform as much, leading to higher stress concentration. As we move towards the free end, the beam has more freedom to deform, so the stress reduces.

The results of these graphs can be used to understand the behavior of beams under vibration. They can also be used to improve the design of beams that are subjected to vibration, such as in microelectromechanical systems (MEMS) devices.

4.1.2 Toluene

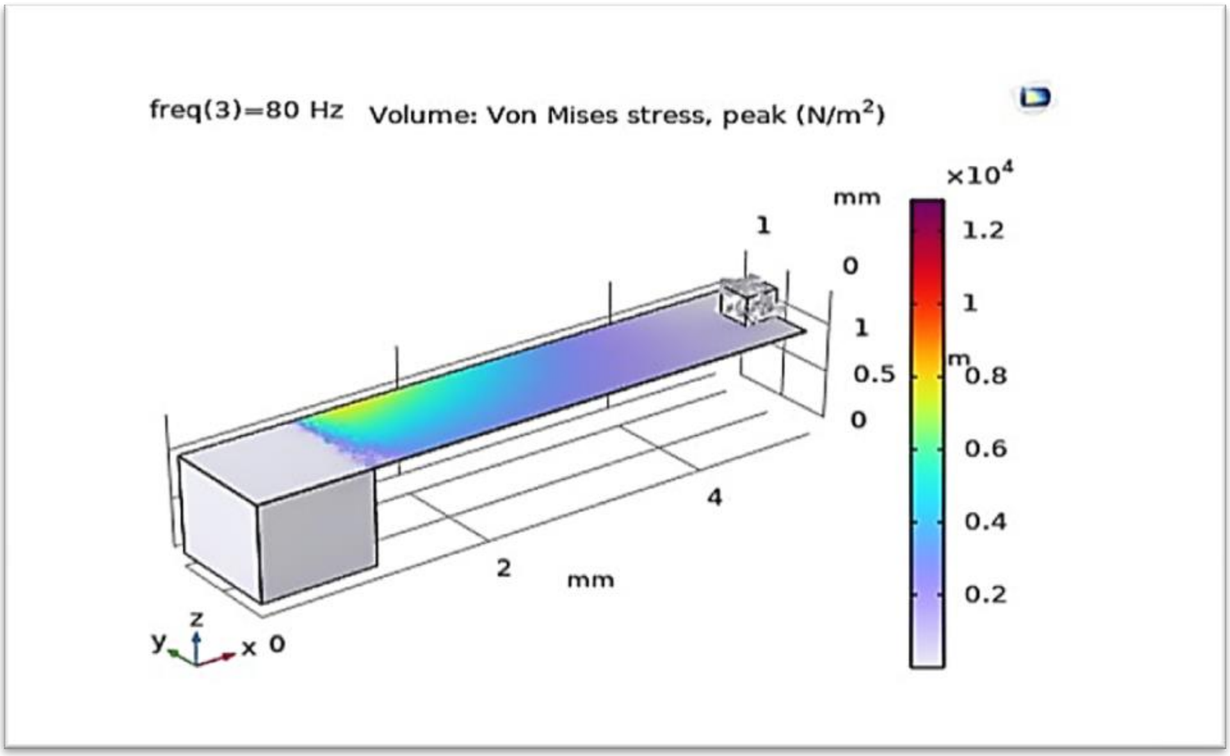

Fig. 4.3 Von Mises Stress of Toluene

Result

This graph depicts a contour plot of the Von Mises stress experienced by a structure, likely a microcantilever beam, subjected to a frequency of 80 Hz. The stress is highest at the clamped end (left) of the structure and tapers off towards the free end (right). The maximum stress is 1.2x10^4 N/m².

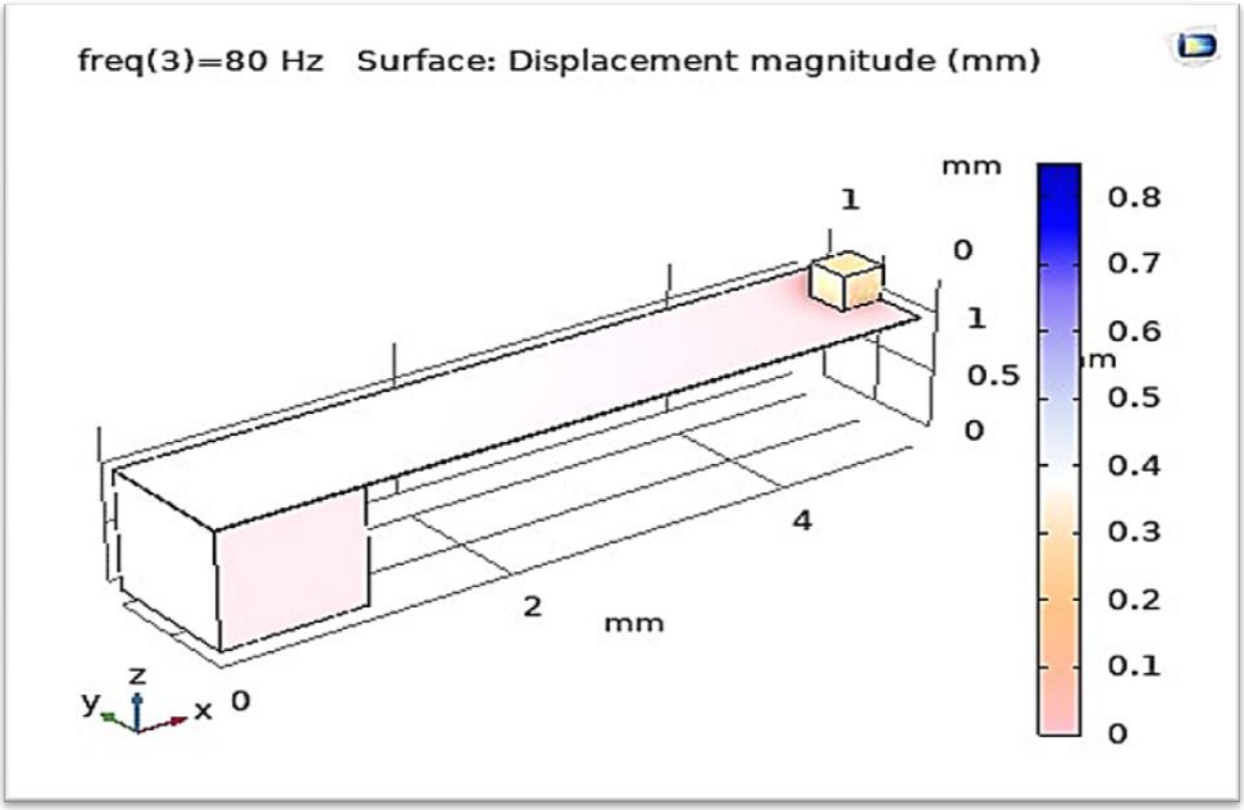

Fig.4.4 Displacement magnitude of Toluene

Result

This graph showcases a surface plot of the displacement magnitude of the structure under the same conditions. The displacement is the greatest near the center of the structure and diminishes towards the edges. The maximum displacement is roughly 0.8 mm.

Discussions

The two graphs are interrelated as they illustrate the effects of the same external force (the 80 Hz frequency) on the structure. The stress graph visualizes the distribution of force exerted on the structure at various points, while the displacement graph demonstrates how much the structure bends under that force. The results align with what we would expect for a beam undergoing vibration. Beams generally experience the most significant stress at the clamped end, as observed in the first graph. This is because the clamped end is stationary and has to absorb all the vibrational force. In contrast, the free end has more freedom to move, and hence, experiences less stress. The second graph's higher displacement in the center aligns with this understanding. Since the center has less constraint compared to the ends, it deflects more readily under the applied force.

The stress concentration at the clamped end is a crucial engineering principle. This is the anchoring point where the structure cannot deform as much, leading to higher stress accumulation. As we move towards the free end, the structure has more leeway to deform, and consequently, the stress lessens.

4.1.3 Diethyl Ether

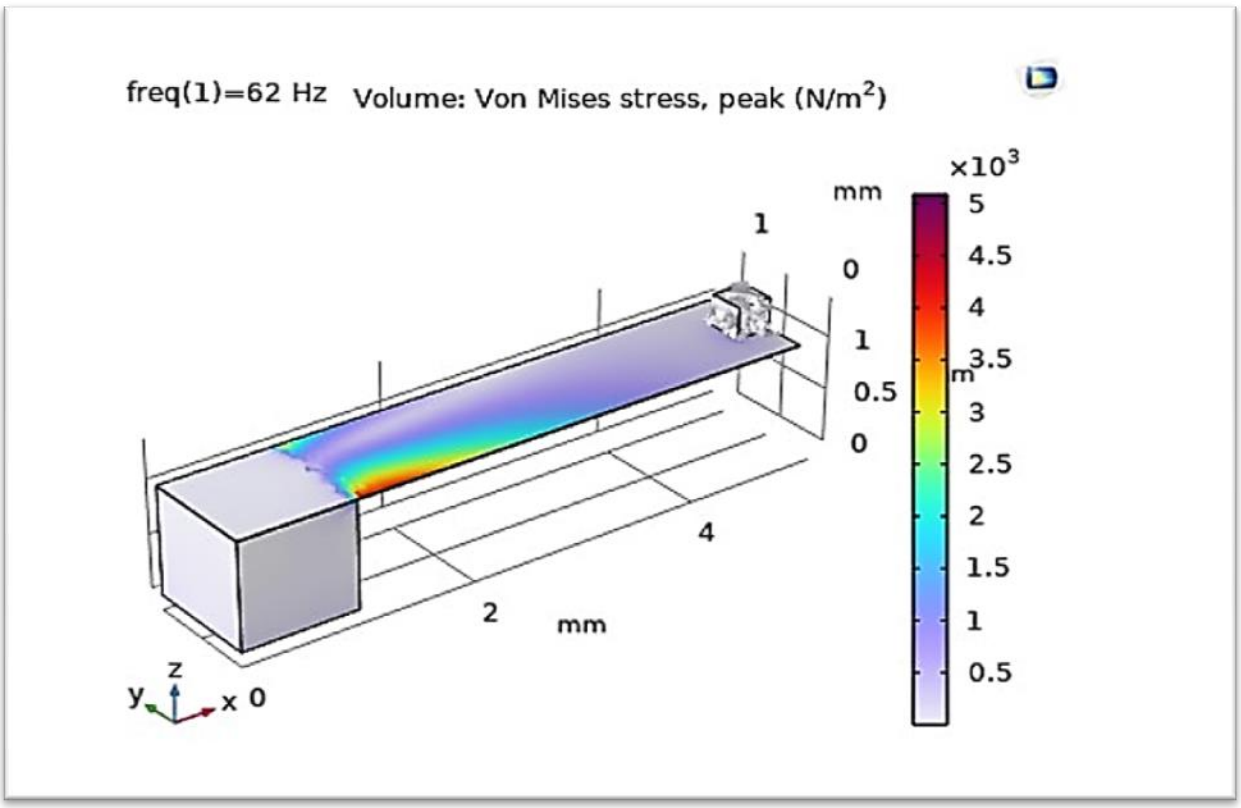

Fig.4.5 Von mises stress of Diethyl Ether

Result

The first graph shows a contour plot of the Von Mises stress experienced by a sample of diethyl ether subjected to a frequency of 62 Hz. The stress is highest at the bottom left corner of the sample and decreases towards the top right corner. The maximum stress is 5 N/m^2.

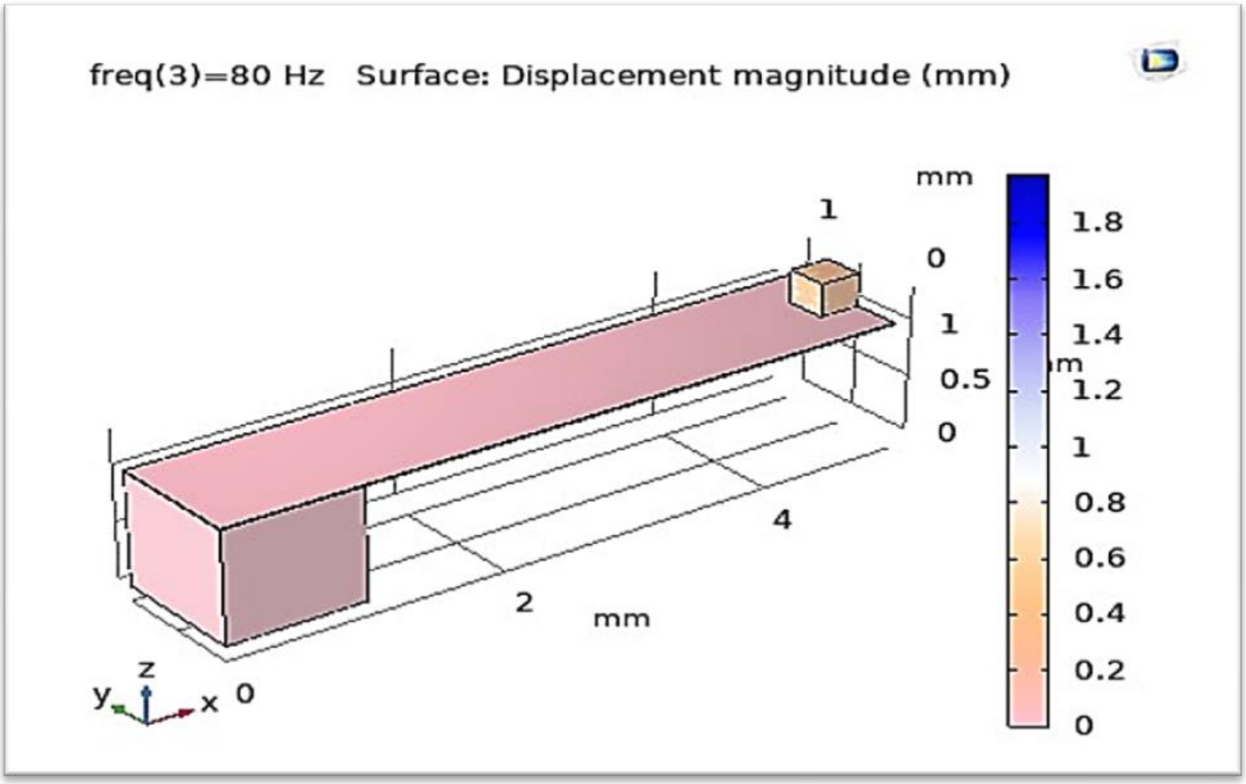

Fig.4.6 Displacement Magnitude of Diethyl Ether

Result

The second graph shows a surface plot of the displacement magnitude of the sample under the same conditions. The displacement is greatest in the center of the sample and decreases towards the edges. The maximum displacement is about 1.8 mm.

Discussion

The two graphs are related in that they show the effects of the same force (the frequency of 62 Hz) on the sample of diethyl ether. The stress graph shows how much force is being exerted on the sample at different points, while the displacement graph shows how much the sample is deforming under that force. The results of the graphs are consistent with what we would expect for a fluid like diethyl ether. Fluids are known to be relatively incompressible, so they do not deform much under stress. This is reflected in the small displacement values seen in the second graph. However, fluids can still transmit stress, and this is reflected in the stress values seen in the first graph. The fact that the stress is highest at the bottom left corner of the sample is likely due to the way the sample is being vibrated. The sample is likely being supported at the bottom left corner, and this is where the vibrations are being introduced. The vibrations then travel through the sample and cause it to deform.

The results of these graphs can be used to understand the behavior of diethyl ether under stress. They can also be used to improve the design of processes that involve diethyl ether, such as mixing or transportation.

4.1.4 Ethylene Glycol

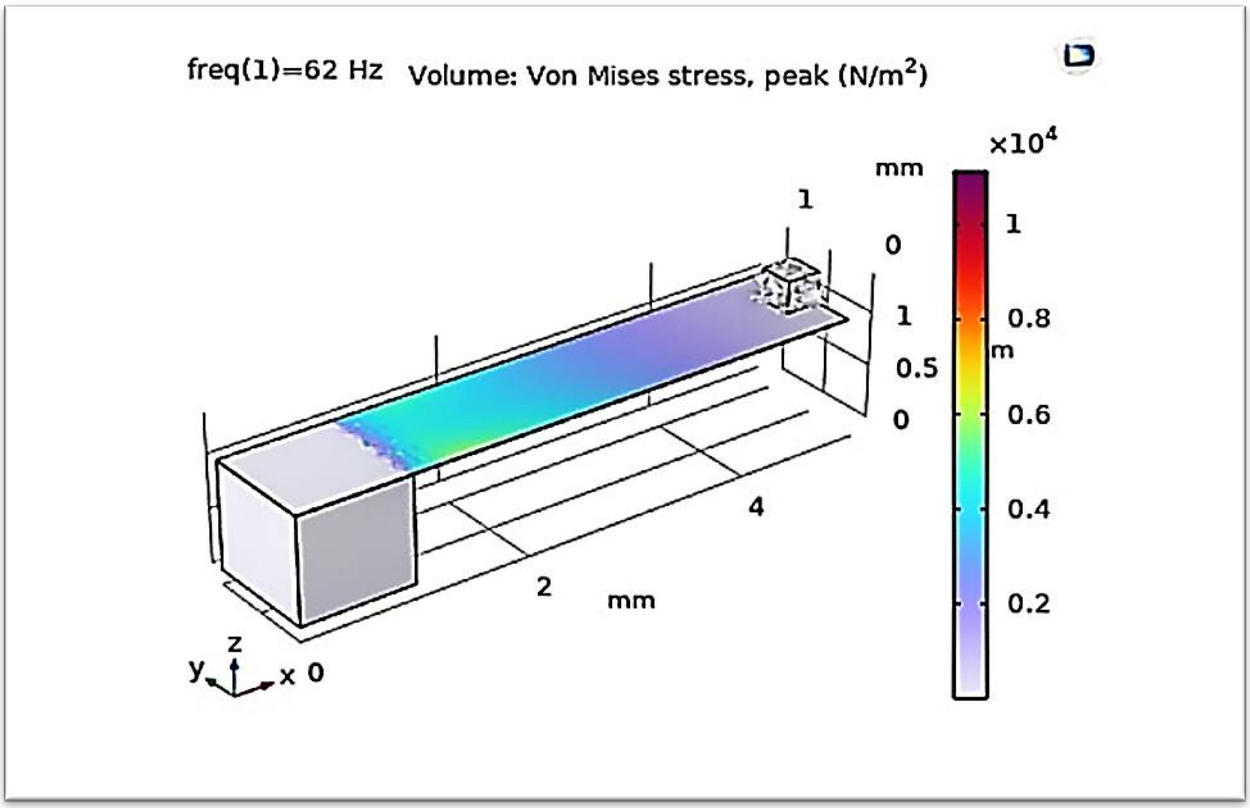

Fig. 4.7 Von Mises Stress of Ethylene Glycol

Result

This graph shows a contour plot of the Von Mises stress experienced by a cantilever beam subjected to a frequency of 62 Hz. The stress is highest at the fixed end (left) of the beam and decreases towards the free end (right). The maximum stress is 1x10^4 N/m^2.

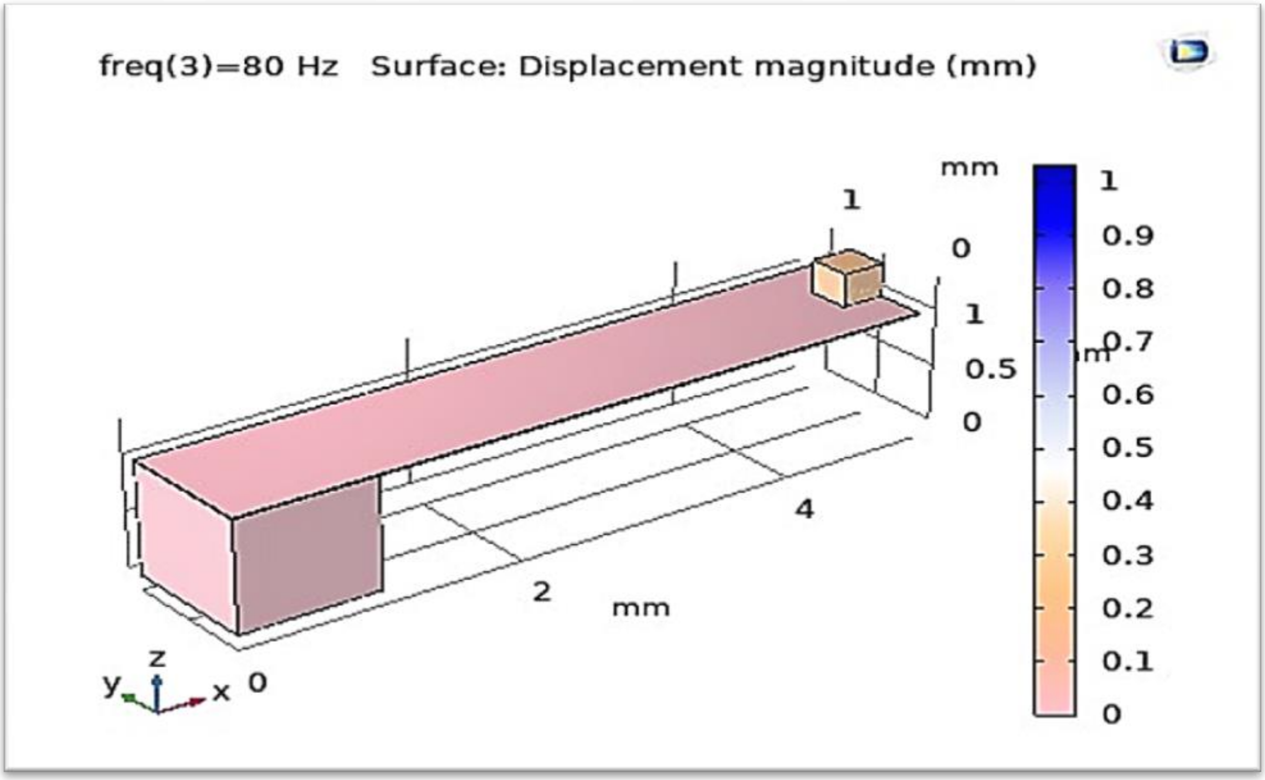

Fig.4.8 Displacement Magnitude of Ethylene Glycol

Result

This graph shows a surface plot of the displacement magnitude of the beam under the same conditions. The displacement is greatest in the center of the beam and decreases towards the edges. The maximum displacement is about 1 mm.

Discussion

The two graphs are related in that they show the effects of the same force (the frequency of 62 Hz) on the cantilever beam. The stress graph shows how much force is being exerted on the beam at different points, while the displacement graph shows how much the beam is bending under that force.

The results of the graphs are consistent with what we would expect for a cantilever beam under vibration. Cantilever beams experience the most stress at the fixed end and the most displacement in the center, as seen in the first and second graphs, respectively. This is because the fixed end is unable to move, so it must absorb all of the force from the vibrations. The center of the beam, on the other hand, is free to move, so it deflects more easily.

4.1.5 Octane

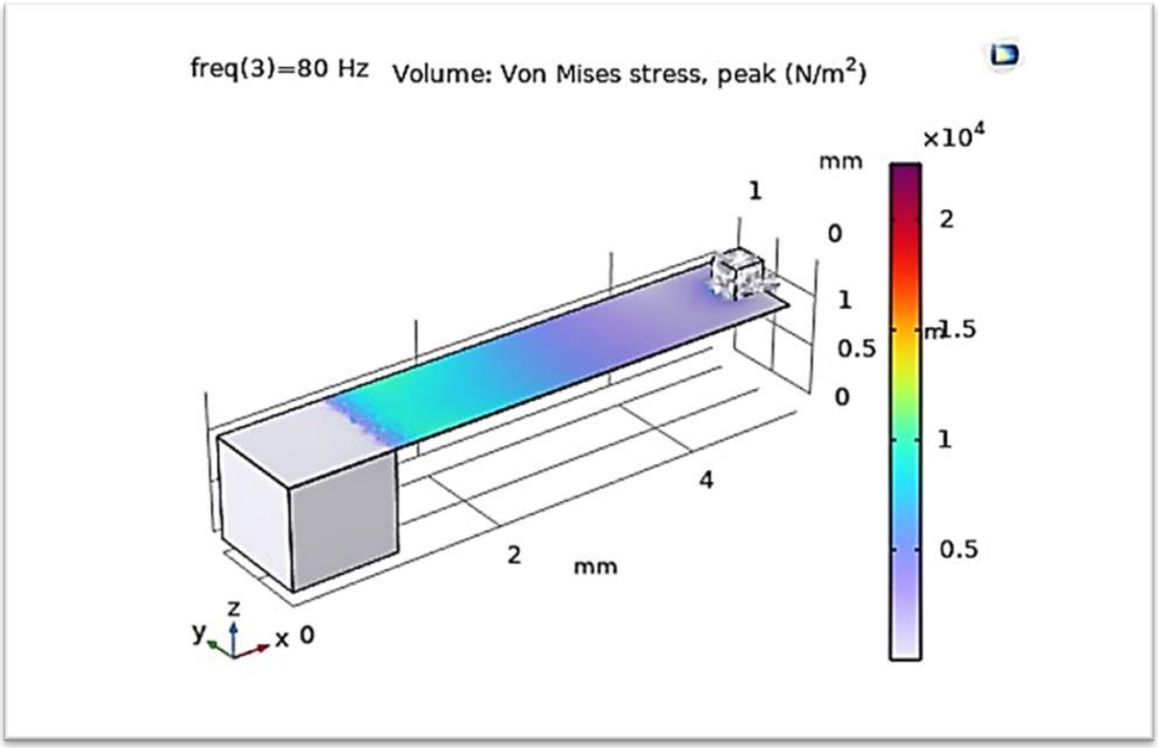

Fig.4.9 Von Mises Stress of Octane

Result

This graph shows a contour plot of the Von Mises stress experienced by an object subjected to a frequency of 80 Hz. The stress is highest at the fixed end (left) of the object and decreases towards the free end (right). The maximum stress is 2x10^4 N/m^2

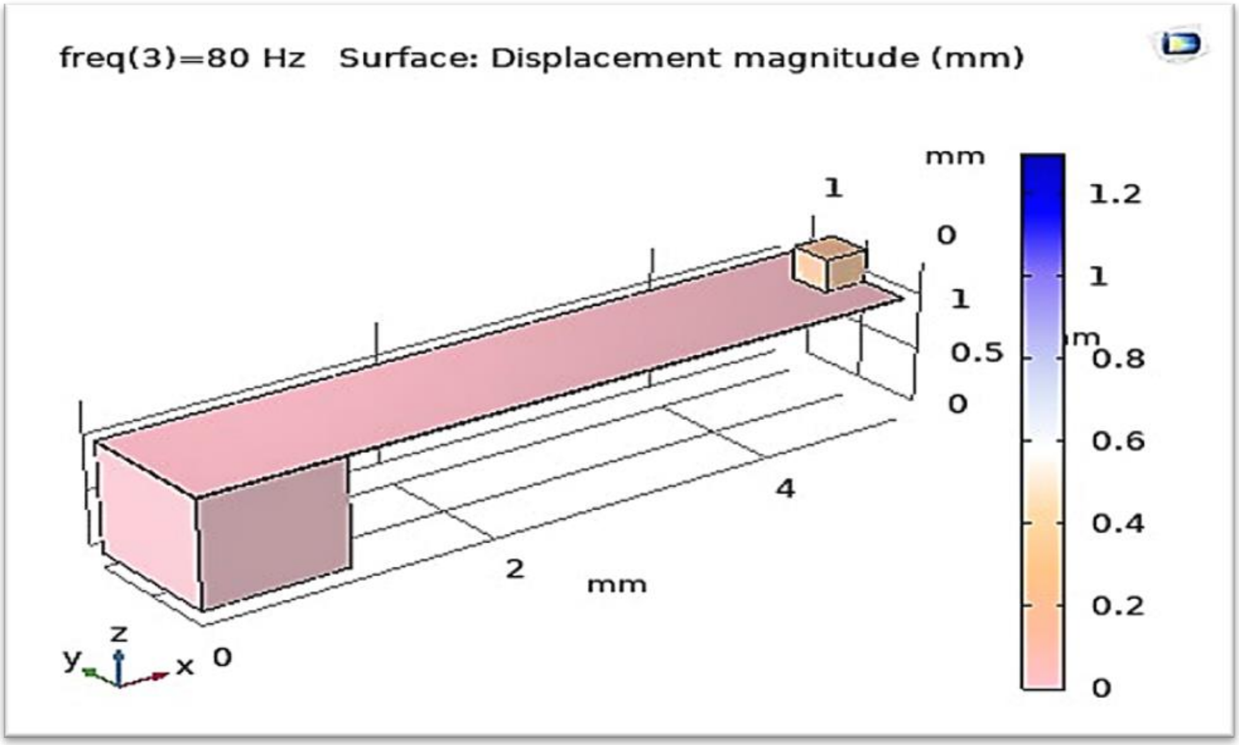

Fig. 4.10 Displacement Magnitude of Octane

Result

This graph shows a surface plot of the displacement magnitude of the object under the same conditions. The displacement is greatest in the center of the object and decreases towards the edges. The maximum displacement is about 1.2 mm.

Discussion

The two graphs are related in that they show the effects of the same force (the frequency of 80 Hz) on the object. The stress graph shows how much force is being exerted on the object at different points, while the displacement graph shows how much the object is bending under that force. The results of the graphs are consistent with what we would expect for a beam under vibration. Beams tend to experience the most stress at the fixed ends and the most displacement in the center. This is because the fixed ends are unable to move, so they must absorb all of the force from the vibrations. The center of the beam, on the other hand, is free to move, so it deflects more easily. The fact that the stress is highest at the fixed end of the object is likely due to the way it is being vibrated. The object is likely being supported at the fixed end, and this is where the vibrations are being introduced. The vibrations then travel through the object and cause it to deform.

The results of these graphs can be used to understand the behavior of objects under vibration. They can also be used to improve the design of objects that are subjected to vibration, such as buildings and bridges.

4.2 Comparison between the liquids-based Stress as a Force Reactant:

TABLE 4.1 VON MISES STRESS OF VARIOUS FLUIDS

LIQUIDS	VON MISES STRESS
Diethyl ether	5
Heptane	4.5
Octane	2
Toluene	1.2
Ethylene glycol	1

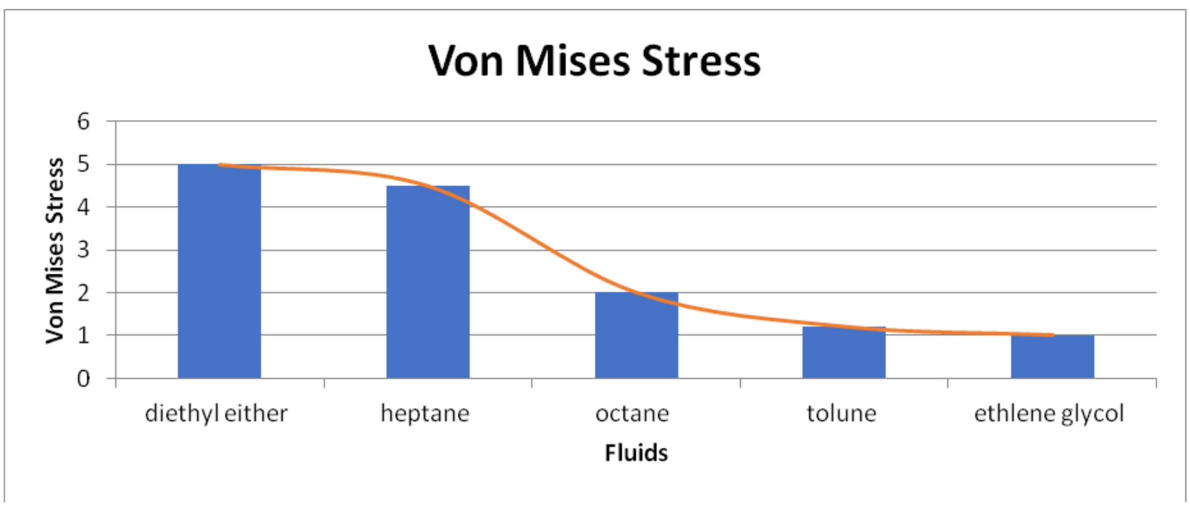

Fig. 4.11 Bar graph of Von Mises Stress of Various Fluids

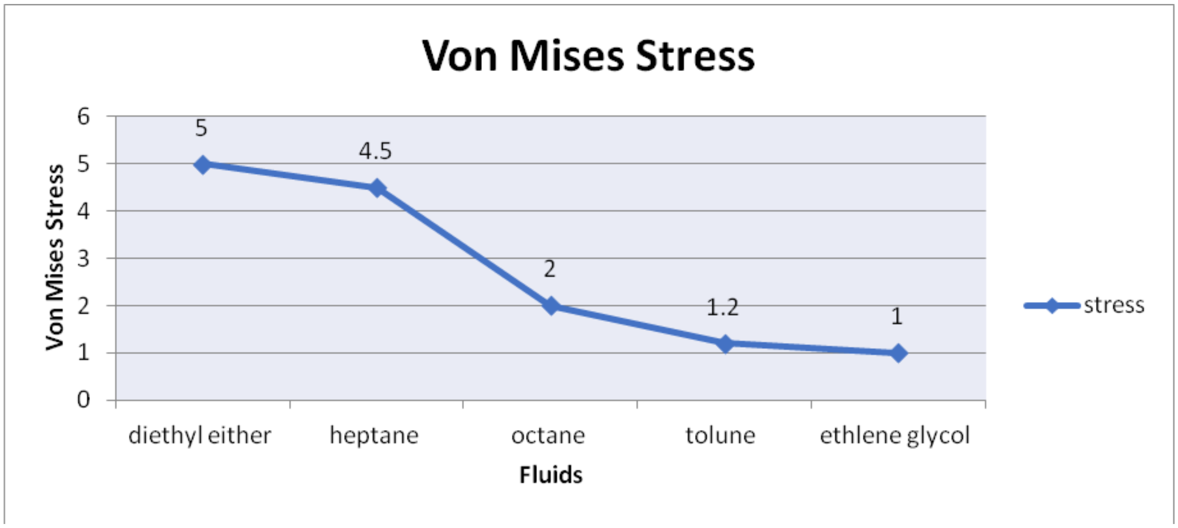

Fig. 4.12 Line Graph of Von Mises Stress Of Various Fluids

4.3. Comparison between the liquids-based Dynamic Viscosity as a Force Reactant:

TABLE 4.1: VON MISES STRESS OF VARIOUS FLUIDS

LIQUIDS	**DYNAMIC VISCOSITY (pa)**
Diethyl ether	0.000223
Heptane	0.000376
Octane	0.00051
Toluene	0.00055
Ethylene glycol	0.0162

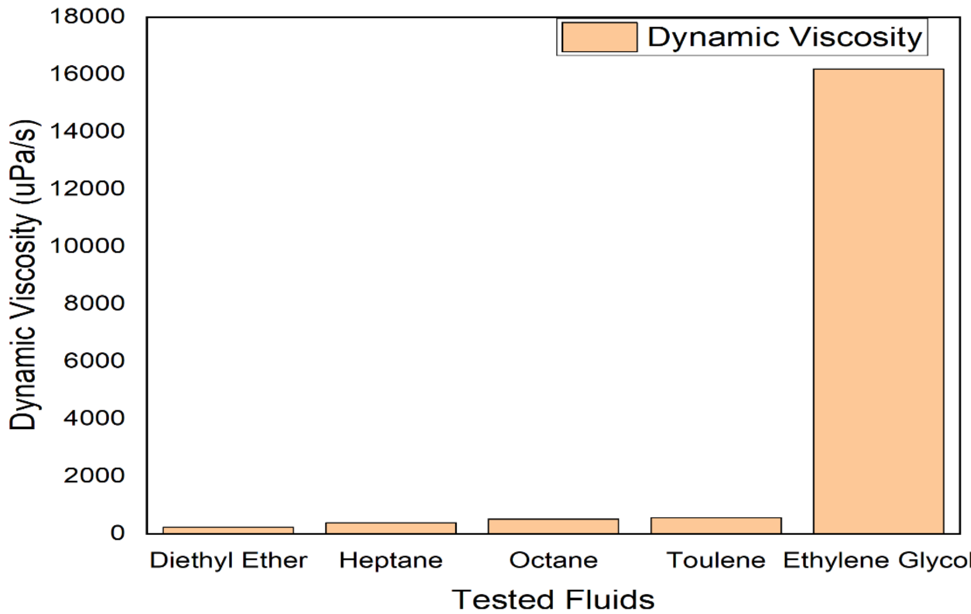

Fig. 4.13. Bar Graph of Dynamic viscosity of Various Fluids

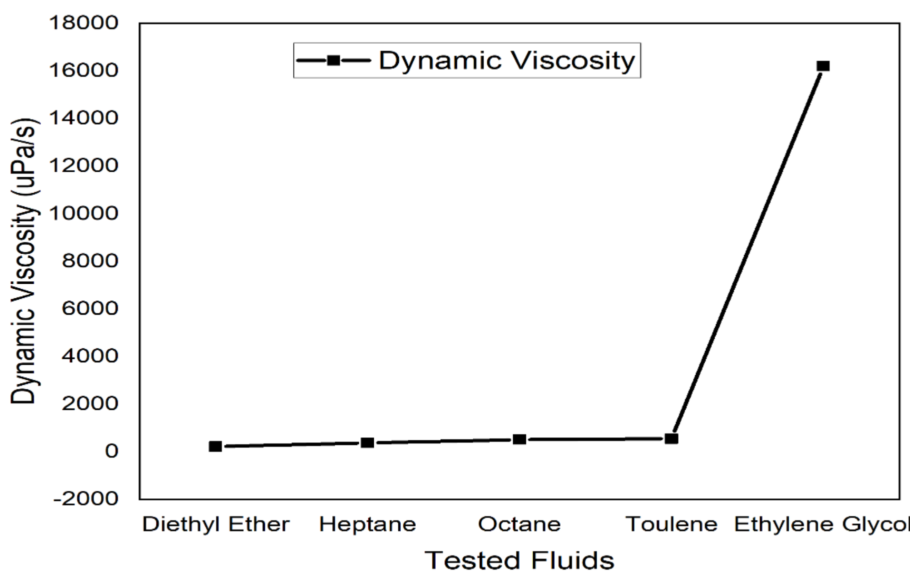

Fig. 4.14. Line Graph of Dynamic viscosity of Various Fluids

CHAPTER-5

KEY TAKEAWAYS AND FINAL THOUGHTS

5.1 Conclusion

In exploring the intricate realm of piezoelectric cantilever systems for fluid testing applications, we embark on a journey at the forefront of scientific inquiry and technological innovation. Piezoelectricity, a phenomenon discovered by the Curie brothers in 1880, forms the foundational principle upon which these systems operate. From its nascent experimental stages to its pivotal role in World War I and subsequent technological developments, piezoelectricity has evolved into a cornerstone of modern engineering. The historical evolution of cantilevers mirrors humanity's quest for architectural excellence and engineering ingenuity. From Frank Lloyd Wright's pioneering architectural designs to their utilization in bridge construction, cantilevers have demonstrated remarkable versatility and utility. However, it is their application in piezoelectric systems that unveils a new chapter in their storied history, one defined by innovation and scientific discovery. The design and simulation of piezoelectric cantilever systems herald a new era in fluid testing and analysis, offering unprecedented levels of sensitivity and precision. Leveraging the unique properties of materials like Lead Zirconate Titanate (PZT), these systems hold immense potential for revolutionizing our understanding of fluid dynamics. By integrating advanced modelling techniques and material selection, researchers can explore the performance and capabilities of these cantilever systems in evaluating various fluid properties. Central to this endeavour is the utilization of COMSOL Multiphysics, a powerful software tool that facilitates efficient modelling and optimization. Through COMSOL, researchers can analyse the mechanical behaviour of cantilevers under different fluid interactions, ensuring optimal performance and reliability. Moreover, experimental validation enhances confidence in simulation results, validating the accuracy and efficacy of the model. The heart of these piezoelectric cantilever systems lies in their intricate design and composition. Comprising structural steel bases, PZT piezoelectric cantilever beams, and containers accommodating diverse fluids, these systems embody the epitome of interdisciplinary engineering. Each component plays a crucial role in ensuring the effectiveness and reliability of the cantilever system, from structural stability to piezoelectric responsiveness. Material selection is paramount in optimizing the performance of piezoelectric cantilever systems. Structural steel provides the necessary stability and support, while PZT 5A offers exceptional piezoelectric properties essential for sensing and detection. The inclusion of fluid parameters such as density, viscosity, and thermal conductivity further enhances our understanding of fluid-cantilever interactions, enabling comprehensive analysis and predictive modelling. Mathematical modelling and simulation serve as the cornerstone of this research endeavour, enabling researchers to derive equations governing cantilever deflection, stiffness, and resonance frequency. By employing elasticity matrices and material constants, researchers can accurately model the mechanical behaviour of piezoelectric cantilevers, providing valuable insights into their response to different fluid environments. The simulations conducted reveal a wealth of information regarding the cantilever's performance and capabilities. Insights gleaned from these simulations serve as the basis for further experimentation and optimization, guiding researchers toward unlocking the full potential of piezoelectric cantilever systems in fluid testing applications. One of the key findings from the analysis pertains to the consistency of results with the cantilever beam theory. The observed deflection patterns align well with theoretical predictions, validating the accuracy and reliability of the model. This consistency lends credibility to the analysis, reinforcing the efficacy of piezoelectric cantilever systems in fluid testing applications. Moreover, the influence of liquid density on cantilever behaviour emerges as a significant factor. A comparison of different liquids based on their respective stresses reveals intriguing insights into the mechanical response of the cantilever. For instance, liquids exerting higher stresses result in larger maximum displacements, indicating greater flexibility or deformability of the cantilever under these conditions. Furthermore, the influence of liquid properties, such as viscosity and surface tension, underscores the complexity of fluid-cantilever interactions. Variations in displacement under different liquids highlight the nuanced effects of fluid properties on cantilever performance, offering valuable insights for future research and design optimization. These findings have profound implications for the design and optimization of piezoelectric cantilever systems. Understanding how different liquids affect cantilever deflection can inform material selection and structural modifications, allowing researchers to tailor cantilever systems to specific application requirements. Thicker or more robust designs may be warranted for applications involving liquids that induce higher stresses and greater deflections. However, it is essential to acknowledge the limitations of the analysis conducted. Factors such as material properties and manufacturing tolerances, though not considered in this study, could potentially impact the actual displacement of the cantilever. Addressing these limitations and incorporating additional factors into future research endeavours will be critical for ensuring the accuracy and reliability of cantilever models in real-world applications. Looking ahead, the future holds immense promise for piezoelectric cantilever systems in fluid testing applications. Experimental validation and refinement of the model will be paramount in realizing their full potential and facilitating advancements in fluid dynamics research. Moreover, continued exploration of new materials and configurations will drive innovation in this critical area of study, paving the way for groundbreaking discoveries and technological breakthroughs.

Thus, the exploration of piezoelectric cantilever systems for fluid testing applications represents a convergence of scientific inquiry, engineering innovation, and technological advancement. Through meticulous research, advanced modelling techniques, and experimental validation, we stand poised to unlock the full potential of these systems, revolutionizing our understanding of fluid dynamics and facilitating the development of advanced sensing technologies. As we continue to push the boundaries of scientific knowledge and technological capability, the journey of piezoelectric cantilevers unfolds as a testament to human ingenuity and the relentless pursuit of excellence in engineering and science.

5.2 Future Scope

- **Make the Simulations Even More Accurate**: We'll work on improving the computer models we use to mimic how the cantilevers behave in real life. This means making them better at understanding how fluids affect the cantilevers and making sure they match up well with actual experiments.
- **Test the Models in Real Life**: We'll keep doing experiments to check if our computer models are right. This helps us trust the results we get from the computer and make sure they match what happens in the real world.
- **Make the Cantilevers Work Even Better**: We'll try to find the best designs for the cantilevers. This means figuring out the best shapes, materials, and sizes to make them work really well for testing different fluids.

➢ References

[1] Vocabulary.com. Cantilever. Retrieved from-
https://www.vocabulary.com/dictionary/cantilever

[2] Allthescience.org. What is a cantilever bridge? Retrieved from-
https://www.allthescience.org/what-is-a-cantilever-bridge.htm

[3] Housing.com. Cantilever beam. Retrieved from- https://housing.com/news/cantilever-beam/

[4] Theconstructor.org. Cantilever beams. Retrieved from- https://theconstructor.org/structural-engg/cantilever-beams/167474/?amp=1

[5] "1971 Concrete Bridge" by Hugo-90 is licensed under CC BY 2.0.https://live.staticflickr.com/7053/6980712643_463dce0921_b.jpg

[6]] "Lego Building" by Bon Stormer is licensed under CC BY 2.0.
https://openverse.org/image/6cc4ef31-2d6b-4a44-abf9-1b48829cae0e?q=buildings

[7] "Billboard Study #10" by Daveness_98 is licensed under CC BY 2.0.
https://openverse.org/image/8828046d-6af1-4ab0-b745-be7a272f24fe?q=billboard

[8] "2010 FIFA World Cup Stadiums - South Africa" by South African Tourism is licensed under CC BY 2.0. https://openverse.org/image/3b41fa37-5b33-42f6-a313-e7e86eab3850?q=stadium

[9] "ARCH608 Kansas City Opera House - Auditorium Interior Design" by FHKE is licensed under CC BY-SA 2.0.https://openverse.org/image/854ea2bd- 345b-417e-be8e-b5f85ec84096?q=auditorium0

[10] "Airplane" by Andrew Malone is licensed under CC BY 2.0.
https://openverse.org/image/06fe65b6-ba9d-456a-afc3-d08e0a0e8f09?q=airplane0

[11] Medium.com. Understanding cantilever beams design applications and building structures. Published by SAMJADESBUILDINGCONSTRUCTION. Retrieved from-
https://medium.com/@SAMJADESBUILDINGCONSTRUCTION/understanding-cantilever-beams-design-applications-and-building-structures-98bf1dbdc824

[12] Mechanicalboost.com. Cantilever beam. Retrieved from-
https://mechanicalboost.com/cantilever-beam/#google_vignette

[13] Theconstructor.org. Cantilever beam. Retrieved from- https://theconstructor.org/structural-engg/cantilever-beams/167474/?amp=1

[14] Skyciv.com. Cantilever beam. Retrieved from-https://skyciv.com/docs/tutorials/beam-tutorials/cantilever-beam/

[15] Constructor.org. Cantilever beam. Figure Retrieved from-
https://theconstructor.org/structural-engg/cantilever-beams/167474/?amp=1
[16] Constructor.org. Cantilever beam. Retrieved from-https://theconstructor.org/structural-engg/cantilever-beams/167474/?amp=1
[17] Study.com. Cantilever structure design examples. Retrieved from-
https://study.com/learn/lesson/cantilever-structure-design-examples.html
[18] Science notes.org. Piezoelectricity and the piezoelectric effect. Retrieved from-
https://sciencenotes.org/piezoelectricity-and-the-piezoelectric-effect/#google_vignette
[19] Elprocus.com. What are piezoelectric materials, types, properties and characteristics. Retrieved from-https://www.elprocus.com/what-are-piezo-electric-materials-types-properties-and-characteristics/
[20] Elprocus.com. What is a piezoelectric material. Retrieved from-
https://www.elprocus.com/what-is-a-piezoelectric-material-working/
[21] Elprocus.com. What is a piezoelectric material. Retrieved from-
https://www.elprocus.com/what-is-a-piezoelectric-material-working/
[22] Historyofbridges.Com. Cantilever bridge. Retrieved from-
http://www.historyofbridges.com/facts-about-bridges/cantilever-bridge/
[23] Britannica.com. Cantilever. Retrieved from-
https://www.google.com/url?q=https://www.britannica.com/technology/cantilever&sa=U&sqi=2&ved=2ahUKEwi55u-e0qiEAxWv-zgGHUfQDcAQFnoECDAQAQ&usg=AOvVaw2sblD0NeVvOG9u4XJcmkFC
[24] Piezo.com. History of piezoelectricity. Retrieved from-
https://piezo.com/pages/history-of-piezoelectricity
[25] Link.springer.com. Piezoelectric crystals and ceramics. Retrieved from-
https://www.google.com/url?q=https://link.springer.com/chapter/10.1007/978-1-4757-0468-6_2&sa=U&sqi=2&ved=2ahUKEwi7grKt4aiEAxVCyDgGHcWDAmMQFnoECB0QAQ&usg=AOvVaw07nUcy2rqNkJCAhMQOmfcn
[26] Encyclopedia.com. Applications of Cantilever. Retrieved from-
https://www.encyclopedia.com/science-and-technology/technology/technology-terms-and-concepts/cantilever#:~:text=The%20first%20cantilever%20bridge%20was,span%20deep%20ravines%20or%20rivers.

[27] Moheimani, S.R. and Fleming, A.J., 2006. Piezoelectric transducers for vibration control and damping. Springer Science & Business Media.