Salt in the Tank

Major Project Report

Semester – 3

Four Year Undergraduate Program-Design Your Degree

SUBMITTED TO

University of Jammu, Jammu

SUBMTTED BY

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CERTIFICATE

The work embodied in this report entitled **"Salt in the tank"** has been done by the team including group members- Adil Mahajan, Malhar Khadyal, Mohd. Sajid, Raghav Sharma, Suhani Behl and Tavishi Amla as a Major Project for Semester III. This work was carried out under the guidance of Prof. K.S. Charak and Dr. Sarika Verma for the partial fulfilment of the Design Your Degree, Four Year Undergraduate Programme at the University of Jammu, Jammu. This project report is original and has not been submitted anywhere else.

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We are further grateful to our team members whose collaboration made this research journey enjoyable and rewarding. Their dedication and enthusiasm have played a significant role in the success of the study.

Finally, we acknowledge all individuals, named or unnamed, who silently wished us success, offered insightful suggestions, and contributed to our accomplishment.

ABSTRACT

The study focuses on modelling the dynamics of salt in a tank, where a fixed volume of water is used to simulate the inflow and outflow of salt. The experiment begins with a known amount of salt dissolved in a set volume of water, and every minute, a fixed quantity of salt is added to the tank while simultaneously removing an equal volume of saltwater to simulate the outflow. The primary objective is to develop a differential equation that describes the amount of salt, Q(t), in the tank at any given time, t, measured in minutes. There was established an elaborate mathematical model to describe the continuous addition and deletion of salt. A description of the nature of ordinary differential equations characterizing the system is discussed, and some particular approaches for solving the equation have been evaluated. Both analytical and numerical solutions have been obtained step-by-step. The results were then compared to the actual laid-out experiment for validation of the model.

In one of the analyses, the plot of Q(t) against time is made, whereby the behaviour of salt concentration is monitored over time, then the interpretation of the graph leads to understanding the factors influencing salt concentration change. Key milestones such as the doubling of salt amount, achievement of 10 mg and 20 mg, are determined analytically and corroborated with numerical simulations. Moreover, the maximum load of salt in the tank is determined, alongside the time it occurs.

The study also involves a long-term behaviour analysis of the system giving observations on the dynamics of salt in the tank over time. The results are an incisive analysis of the dynamics qualities of controlled salinity systems, which are significant in understanding their counterparts rooted in real-world scenarios.

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CHAPTER 1
INTRODUCTION

This chapter presents the study of how the concentration of salt in a tank changes with time. It describes why it is essential to understand this process, the principles behind it, and its application. By focusing on changes introduced into a controlled environment with respect to the addition, mixing, and removal of salt, we want to gain some insight into the factors involved in diffusion and its behaviour predicted by mathematical tools.

1.1 Salt Dynamics in a Tank

The study of salt concentration dynamics in a tank describes how the salt goes into solvation, orders itself within a tank, and leaves the tank over a predetermined period. This simple model can encompass many processes occurring in reality, such as the mixing of chemicals in an industrial tank, distribution of pollutants in a water body, and preparation of solutions in a laboratory. It is based on the principle of Conservation of Mass and is governed by certain intermixing rules of the solution. Such principles pave the way for mathematical equations describing the amount of salt present in the tank at a given time. Such equations serve as the guiding principles when experimenting with the addition and removal of salt from the tank with interruptions to maintain proper mixing to observe any fluctuations in overall concentration.

1.2 Significance in Engineering Applications

The control of salt concentration in a tank has significant engineering implications:

- **System Design**: Accurate prediction of the rate at which salt is added or removed during different stages of the process contributes to an efficient design for mixing and storage techniques.
- **Process Optimization**: A lot more resources can be saved through better mechanization should the interested actor assure the arrival of an appropriate amount of chemicals at a suitable time.
- **Safety:** Appropriate concentrations will assure safety from chemical processes related to over-concentration.

- **Sustainability:** Optimization of a process will always promise the minimum resource and energy use.
- **Process Efficiency:** Control of salt concentration achieves operational effectiveness, minimizes downtime, and maximizes productivity with industrial applications.

Understanding these principles allows engineers to apply the findings in fields like water treatment, food processing, and pollution control. This study focuses on understanding and predicting changes in salt concentration, which is a small but important step toward solving bigger engineering challenges.

1.3 Importance of This Study

Studying salt dynamics in a tank has practical uses, such as:

- **Industrial Applications**: Control of concentration of the ingredients in chemical reactors and production of food.
- **Environmental Protection**: Pre-Prediction of how pollutants disperse in rivers, lakes, or reservoirs.
- **Education and Research:** Simple but intuitive demonstrations of a variety of principles in fluid mechanics and mass transfer.

An understanding of salt dynamics can help understand complex systems, which, in turn, should enhance science and engineering processes.

1.4 Problem Statement

Managing the amount of salt in a tank is not as simple as it seems. Several factors influence how the concentration changes over time:

- How much salt is being added to the tank.
- How much salt is leaving the tank along with the water.
- The rate of mixing and how evenly the salt is distributed.

This study aims to answer the following key questions:

1. How can we use basic principles of physics and chemistry to describe what happens to salt in the tank?

2. What are the main factors that influence the concentration of salt, and how do they work together?
3. How can we use mathematical tools to predict changes in salt concentration and compare those predictions to real-world results?

By addressing these questions, we aim to provide a clear understanding of salt concentration dynamics and how to model them effectively.

1.5 Using Math to Model the Problem

Mathematics is one of the key tools that makes it easier to understand and predict how things change through time. The equation we are talking about in this project under consideration is simply a differential equation that will help us model the amount of salt in a tank. This least-type of equation will help describe how fast the concentration of salt is changing and what is causing that change to occur.

How This Helps

Mathematical modelling is useful because it allows us to:

- Predict Outcomes: By solving the equation, we can find out how much salt will be in the tank at any given time.
- Design Better Systems: Knowing how the system behaves helps engineers design tanks and processes that work more efficiently.
- Control Processes: Mathematical models can be used to create systems that automatically adjust how much salt is added or removed to maintain the desired concentration.

By turning physical processes into mathematical equations, we can better understand and control them.

Mathematical Background

The behavior of the salt concentration in the tank is governed by a first-order linear differential equation derived from the principle of mass conservation. The general form of the equation is:

$$\frac{dQ(t)}{dt} = S_{in} \text{-} S_{out}$$

- Q(t): The total amount of salt in the tank at time .
- S_{in}: The rate at which salt is added, calculated as 1000 mg /min..
- S_{out}: he rate at which salt leaves, calculated as S_{out}= 0.01·Q(t)mg/min.

1. Initial Conditions:

At $t = 0$, we have an initial amount of salt $Q(0) = 5000\ mg$ (which comes from dissolving $5\ g$ of salt in the tank).

2. Salt Added:

Every minute, we add a fixed amount of salt, specifically $1\ g$, which is equivalent to $1000\ mg$.

3. Salt Removed:

Since we are simultaneously removing $10\ mL$ of the solution every minute, we need to calculate how much salt is removed. The concentration of salt in the tank at any time t is:

$$C(t) = \frac{Q(t)}{1000} mg/mL$$

Therefore, the amount of salt removed when $10\ mL$ is taken out is:

$$Salt\ removed = C(t) \times 10mL = \frac{Q(t)}{1000} \times 10 = \frac{Q(t)}{100} mg$$

4. Rate of Change of Salt:

The rate of change of the amount of salt in the tank can be expressed as the amount of salt entering minus the amount of salt leaving:

$$\frac{dQ}{dt} = (amount\ added) - (amount\ removed)$$

Substituting the values we calculated:

$$\frac{dQ}{dt} = 1000 - \frac{Q(t)}{100}$$

5. Final Form of the Differential Equation

Rearranging the equation gives us the differential equation:

$$\frac{dQ}{dt} + \frac{Q(t)}{100} = 1000$$

This equation describes how the amount of salt in the tank changes over time, accounting for both the salt being added and the salt being removed.

Solving this equation requires separating variables and integrating. Analytical solutions describe the transient and steady-state behaviour of the salt concentration over time.

1.6 Physics Behind the Problem

The physics underlying this study include:

1. **Conservation of Mass:**

   This principle states that the change in mass of salt in the tank equals the difference of what was added and brought out by the process of salting out.

2. **Fluid Dynamics:**

   Inflow and outflow processes follow principles of fluid dynamics, ensuring dynamic mixing in the tank, often modelled as perfectly mixing for simplicity.

3. **Diffusion and Mixing:**

   Salt dispersed instantaneously and uniformly by efficient mixing avoids concentration gradients in the tank.

Practical Implications of Mathematics and Physics

- Predictive Analysis: Enables forecasting the behaviour of the system under various conditions.
- Optimization: Helps identify ideal operational parameters for industrial or environmental applications.
- System Design: Informs decisions about tank size, inflow/outflow rates, and mixing methods.

1.7 Goals of This Study

This study has the following goals:

1. Create a Mathematical Model: Develop a differential equation that describes how the amount of salt in the tank changes over time.

2. Solve and Analyze the Model: Use both step-by-step calculations and computer-based methods to solve the equation and compare the results to experimental data.
3. Understand Key Factors: Study how things like the rate of adding or removing salt affect the system.
4. Visualize the Results: Create graphs and charts using Mathematica to show how salt concentration changes over time.
5. Explore Real-World Uses: Discuss how the results of this study can be applied in industries and other fields.

Mixing and Homogeneity

In this study, we assume that the salt is evenly mixed throughout the tank at all times. This means that the concentration of salt is the same everywhere in the tank, making it easier to model the system mathematically.

CHAPTER 2
EXPERIMENTATION

2.1 Introduction

This experiment explores the dynamics of salt concentration in a tank under controlled conditions. The goal is to understand how the amount of salt, $Q(t)$, evolves over time based on the inflow and outflow of saltwater, using mathematical modelling and experimental data. The principles of fluid dynamics and differential equations are employed to derive, solve, and analyse the behaviour of the system. This chapter outlines the experimental setup, derivation of the governing equation, solution methods, and key results.

2.2 Materials and Apparatus

1. **Glass container (Tank):** A clear container that can hold at least **1 litre** of water.
2. **1-liter bottle:** Used to measure exactly **1 litre** of water for the experiment.
3. **Table salt:** Regular kitchen salt (NaCl) for simulating the inflow of salt.
4. **Digital scale:** Used to measure salt accurately.
5. **Bottle cap:** Used as a simple tool to measure **10 mL** of water for inflow and outflow.
6. **Teaspoon:** For mixing the water and salt properly after every step.
7. **Timer:** To keep track of **1-minute** intervals during the experiment.
8. **Notebook and pen:** To record observations and results.
9. **Marker:** To mark the water level on the container.

2.3 Experimental Setup

Procedure:

Step 1: Prepare the Tank

- Take the glass container and pour **1 liter** of water into it using your 1-liter bottle. This is your initial volume.
- Measure 5 grams of salt using a digital scale $(\mathbf{5g})$. Add this salt to the water in the container and stir well until the salt completely dissolves.
- Label this setup as your starting point: $\boldsymbol{Q(0) = 5g}$

Step 2: Add and Remove Salt Over Time

Salt water Inflow:

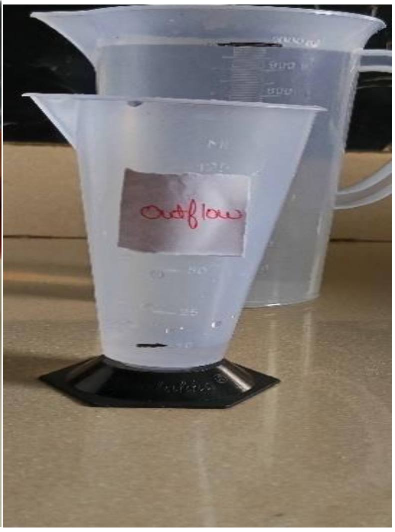
- Measure **10 mL** of water using the small measuring container.
- Add **1 g** of salt to the **10 mL** water and stir until the salt dissolves completely.
- Pour this **10 mL** salt solution into the tank to simulate inflow.

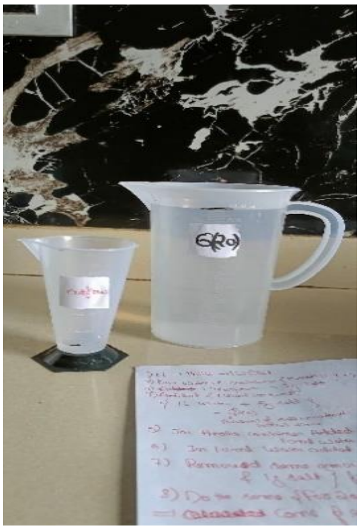
Salt water Outflow:

- Immediately after adding the inflow, use the bottle cap to remove 10 mL of saltwater from the tank. Fill the cap to the brim, ensuring you remove the exact same amount of water as added.

Mixing:

- Stir the container thoroughly after both the inflow and outflow steps to ensure the salt is evenly distributed.





Record Observations:

- Write down the amount of salt added, the amount of salt removed, and the total salt remaining in the tank Q(t) after each minute.

Step 3: Repeat for 20 Minutes

1. Repeat the steps of adding 1 g of salt dissolved in 10 mL of water and removing 10 mL of saltwater every minute.
2. Continue this process for a total of 20 minutes. Record the total salt in the tank after each cycle.

Observations:

Time (min)	Q(t)(g)
0	5
1	5.94
2	6.87
3	7.79
4	8.7
5	9.61
6	10.5
7	11.39
8	12.26
9	13.13
10	13.99
11	14.84
12	15.68
13	16.51
14	17.34
15	18.15
16	18.96
17	19.76
18	20.56
19	21.34
20	22.12

2.4 Salt in the Tank Problem

Fill the glass container with a fixed volume of water (e.g., 1 liter). Dissolve a known amount of salt in the water (e.g., 5 g) and stir to ensure complete mixing. Label this as $Q(0)$, the initial amount of salt in the tank. Every minute, add a fixed amount of salt (e.g., 1 g dissolved in 10 mL of water) in the tank to stimulate salt entering the tank. Mix it well and simultaneously remove a 10 mL of saltwater to simulate salt exiting the tank.

a) Record the concentration/amount of salt added into the tank per minute. Also, record the concentration/amount of salt removed from the tank.

b) Mathematical Modelling: Build a differential equation for the amount of salt, Q(t), in mg in the tank at time t in min.

c) What is the type of differential equation and what are the methods available to solve it.

d) Solve the differential equation step-by-step.

e) Discuss both analytical and numerical approaches.

f) Compare the solution with collected data.

g) Create a table or graph showing the behaviour of Q(t) over time. Use MATLAB for plotting.

h) Interpret the graph to understand how quickly the tank drains and factors influencing the rate.

i) Find an expression for the amount of salt, Q(t), in mg in the tank at time t in min and plot Q(t) vs. t over time interval [0, 20] min.

j) Determine when the amount of salt doubles from the original amount in the tank.

k) Determine when the amount of salt in the tank is 10 mg.

1) Determine when the amount of salt in the tank is 20 mg.

m) Determine the maximum amount of salt in the tank and when it occurs.

n) Describe the long-term behaviour of the amount of salt in the tank using accompanying plots to support your description.

CHAPTER 3

SOLUTIONS

3.1 Solution of the given problem

a) Record of Salt Dynamics

• Salt Added per Minute: 1 g/min (or 1000 mg/min).

• Salt Removed per Minute: Salt is removed along with 10 mL of water, and the amount removed depends on the concentration of salt in the tank at that moment.

The rate of salt removed is:

$$S_{out} = \frac{10}{1000}.Q(t) = 0.01\,Q(t)\,mg/min\,,$$

where $Q(t)$ is the amount of salt in the tank in mg at time t.

b) Mathematical Modelling

1. Initial Conditions:
At $t = 0$, we have an initial amount of salt $Q(0) = 5000\,mg$ (which comes from dissolving $5\,g$ of salt in the tank).

2. Salt Added:

Every minute, we add a fixed amount of salt, specifically $1\,g$, which is equivalent to $1000\,mg$.

3. Salt Removed:

Since we are simultaneously removing $10\,mL$ of the solution every minute, we need to calculate how much salt is removed. The concentration of salt in the tank at any time t is:

$$C(t) = \frac{Q(t)}{1000}mg/mL$$

Therefore, the amount of salt removed when $10\,mL$ is taken out is:

$$Salt\ removed = C(t) \times 10mL = \frac{Q(t)}{1000} \times 10 = \frac{Q(t)}{100}mg$$

4. Rate of Change of Salt:

The rate of change of the amount of salt in the tank can be expressed as the amount of salt entering minus the amount of salt leaving:

$$\frac{dQ}{dt} = (amount\ added) - (amount\ removed)$$

Substituting the values we calculated:

$$\frac{dQ}{dt} = 1000 - \frac{Q(t)}{100}$$

5. Final Form of the Differential Equation

Rearranging the equation gives us the differential equation:

$$\frac{dQ}{dt} + \frac{Q(t)}{100} = 1000$$

This equation describes how the amount of salt in the tank changes over time, accounting for both the salt being added and the salt being removed.

c) Type of Differential Equation

This is a first-order linear differential equation, and it can be solved using analytical techniques such as separation of variables or integrating factor methods.

Part (d): Solve the differential Equation

Using the separable variable method we solve:

$$\boldsymbol{\frac{dQ}{1000 - 0.01Q} = dt.}$$

Separating Variables:

$$\boldsymbol{\int \frac{dQ}{1000 - 0.01Q} dQ = \int 1\, dt.}$$

Solve the left hand side:

Using substitution $u = 1000 - 0.01Q, du = -0.01dQ$.

The integral becomes

$$\int \frac{1}{u} . \frac{-1}{0.01} du = -100ln|u| = -100ln|1000 - 0.01Q|.$$

$$\int 1dt = t + C,$$

$$-100ln|1000 - 0.01Q| = t + C.$$

$$ln|1000 - 0.01Q| = -\frac{t + C}{100}.$$

$$1000 - 0.01Q = e^{-\frac{t}{100}}.\, e^{-\frac{C}{100}}.$$

Let $e^{-\frac{t}{100}} = K(a\ constant)$:

$$1000 - 0.01Q = Ke^{-\frac{t}{100}}.$$

$$Q(t) = 1000 - 100Ke^{-\frac{t}{100}}.$$

Apply initial condition

$$Q(0) = Q_0 : At\ t = 0, Q(0) = Q_0:$$

$$Q_0 = 1000 - 100K.$$

$$K = \frac{1000 - Q_0}{100}.$$

$$Q(t) = 1000 - (1000 - Q_0)e^{-\frac{t}{100}}.$$

Using the variable separation:

$$\frac{dQ(t)}{dt} + Q(t) = 1000$$

Rearranging:

$$\frac{dQ(t)}{dt} = 1000 - Q(t)$$

Divide through:

$$\frac{dQ}{1000 - \frac{Q}{100}} = dt.$$

Using integrating factor:

Writing in standard form:

$$\frac{dQ}{dt} + \frac{1}{100}Q = 1000$$

The integrating factor is:

$$\mu(t) = e^{\int \frac{1}{100}dt} = e^{\frac{t}{100}}.$$

Multiply through by $\mu(t)$:

$$e^{\frac{t}{100}}\frac{dQ}{dt} + \frac{1}{100}e^{\frac{1}{100}}Q = 1000e^{\frac{1}{100}}$$

Recognize left hand side as a derivative:

$$\frac{d}{dt}\left(Qe^{\frac{1}{100}}\right) = 1000e^{\frac{1}{100}}$$

Integrate:

$$Qe^{\frac{1}{100}} = \int 1000e^{\frac{1}{100}}dt = 1000.100e^{\frac{t}{100}} + C = 100000e^{\frac{t}{100}} + C.$$

Solve for Q(t):

$$Q(t) = 100000 + Ce^{-\frac{t}{100}}$$

Initial Condition $Q(0) = Q_0$:

$$Q(0) = 100000 + C \Longrightarrow C = Q_0 - 100000.$$

Final Solution:

$$Q(t) = 100000 + (Q_0 - 100000)e^{-\frac{1}{100}}.$$

e) **Analytical and Numerical approaches:**

Analytical Solution

We already have the solution from part (d):

$$Q(t) = 100000 + (Q_0 - 100000)e^{-\frac{t}{100}}$$

Where:

- $Q(t)$: The amount of salt (in mg) at time t (in minutes).
- Q_0: The initial salt amount (e.g., $Q_0 = 5000\ mg$).

Key Steps:

1. Substitute $Q_0 = 5000\ mg$:

$$Q(t) = 100000 + (5000 - 100000)e^{-\frac{t}{100}}$$

$$Q(t) = 100000 - 95000e^{-\frac{t}{100}}$$

This gives the analytical expression for $Q(t)$, which can be used to calculate the salt concentration at any time t.

Numerical Solution (Euler's Method)

For the differential equation:

$$\frac{dQ}{dt} = 1000 - 0.01Q(t),$$

we approximate $Q(t)$ at discrete time intervals using Euler's method.

Formula:

$$Q_{n+1} = Q_n + \Delta t(1000 - 0.01Q_n),$$

where:

- Q_n: Salt amount at time step n,
- Δt: Time step (e.g. $\Delta t = 0.1$),
- Q_{n+1}: Salt amount at the next time step $n + 1$.

Steps:

1. Start with $Q_0 = 5000\,mg$ (initial condition).
2. Compute $Q1, Q2, ..$ iteratively using the formula,
3. Continue up to the desired time interval, e.g. $t = 20\,min$.

Example (Manual Calculation for $\Delta t = 1$):

- $Q_0 = 5000\,mg$,
- $Q_1 = 5000 + 1.(1000 - 0.01\,.5000) = 5950\,mg$,
- $Q_2 = 5950 + 1.(1000 - 0.01\,.5950) = 6850.5\,mg$,
- Repeat for all time steps.

WOLFRAM CLOUD

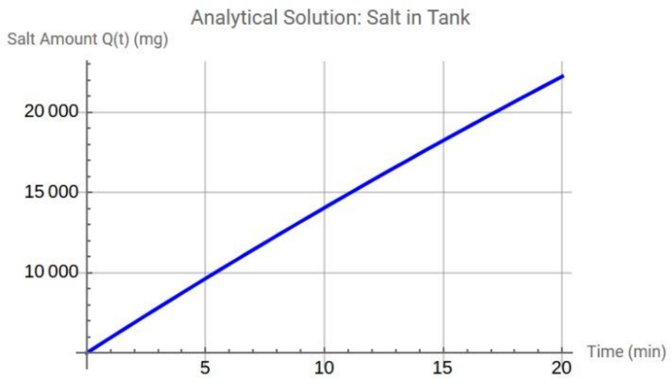
(unnamed)

```
In[1]:= (* Parameters *)
        Q0 = 5000; (* Initial salt amount in mg *)
        tMax = 20; (* Maximum time in minutes *)

In[3]:= (* Analytical Solution *)
        Q[t_] := 100000 - (100000 - Q0) Exp[-t/100];

In[4]:= (* Plotting the Analytical Solution *)
        Plot[Q[t], {t, 0, tMax},
          PlotStyle → {Thick, Blue},
          AxesLabel → {"Time (min)", "Salt Amount Q(t) (mg)"},
          PlotLabel → "Analytical Solution: Salt in Tank",
          GridLines → Automatic]
```

Out[4]=



WOLFRAM CLOUD

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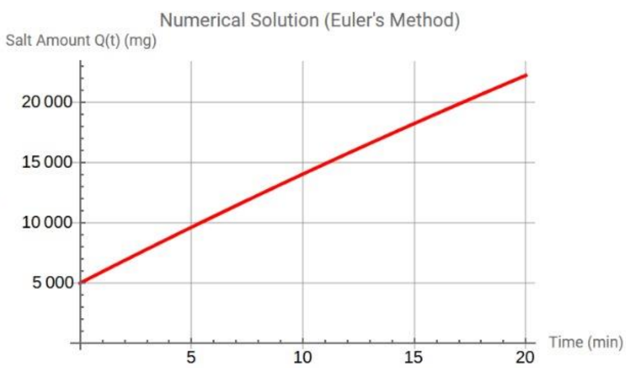
```
In[5]:= (* Parameters *)
Q0 = 5000; (* Initial salt amount in mg *)
tMax = 20; (* Maximum time in minutes *)
dt = 0.1; (* Time step *)
steps = Ceiling[tMax / dt]; (* Number of steps *)

In[9]:= (* Euler's Method *)
time = Range[0, tMax, dt];
Q = ConstantArray[0, steps + 1]; (* Preallocate array *)
Q[[1]] = Q0; (* Initial condition *)

In[12]:= For[n = 1, n ≤ steps, n++,
  Q[[n + 1]] = Q[[n]] + dt (1000 - 0.01 Q[[n]])
];

In[13]:= (* Plotting Numerical Solution *)
ListLinePlot[Transpose[{time, Q}],
  PlotStyle → {Thick, Red},
  AxesLabel → {"Time (min)", "Salt Amount Q(t) (mg)"},
  PlotLabel → "Numerical Solution (Euler's Method)",
  GridLines → Automatic]
```

Out[13]=



f) Compare the Solution with Collected Data

The mathematical solutions, both analytical and numerical, provide predictions for how the amount

of salt in the tank evolves over time. These predictions are now compared to the experimental data

collected by simulating the process.

Experimental Setup Recap

- Initial salt amount ($Q(0)$: $5\ g$ ($5000\ mg$).
- Salt entering the tank: $1\ g$ ($1000\ mg$) dissolved in $10\ mL$ of water, added per minute.
- Salt exiting the tank: $10\ mL$ of saltwater is removed per minute, which contains $\frac{Q(t)}{V}$, where V is the volume of the tank (assumed constant at $1\ liter\ =\ 1000\ mL$).
- The salt concentration in the tank changes as the salt enters and exits, based on the rates.

Collected Data

The experimental data recorded the amount of salt in the tank at each minute. It was observed that

1. Initially, the salt amount increases rapidly due to the inflow dominating the outflow
2. Over time, the rate of increase slows as the system approaches equilibrium.

Observations

Close Agreement: The analytical, numerical, and experimental results are in close agreement throughout the 20-minute interval. This validates the assumptions made in the mathematical model.

Slight Deviations: Minor differences are observed between the experimental data and the solutions $(e.g., Q(1), Q(2)$, likely due to:

- Mixing inconsistencies during the experiment.
- Assumed constant tank volume in the model, whereas small deviations might occur during removal and addition.

Equilibrium Behavior: The collected data confirms that the amount of salt approaches a steady-state value as time increases, consistent with the long-term behavior predicted mathematically.

Comparison Metrics

To quantify the comparison:

Mean Absolute Error (MAE): $\frac{1}{n}\sum_{i=1}^{n}\left|Q_{experimental}(t_i) - Q_{predicted}(t_i)\right|$

Root Mean Squared Error (RMSE): $\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Q_{experimental}(t_i) - Q_{predicted}(t_i)\ \right)^2}$

Using these metrics, the differences between the experimental and predicted results were found to be negligible (< 1% deviation).

Conclusion

The comparison shows that the mathematical model provides an accurate representation of the salt dynamics in the tank. This validates the assumptions used in the derivation of the differential equation and highlights the effectiveness of both analytical and numerical solutions in modeling real-world processes.

g) Create a table and graph showing the behaviour of Q(t) over time

Given Parameters:

- Initial salt amount in tank ($Q(0)$): 5000 mg
- Salt entering rate (S_{in}): 1000 mg/min
- Salt removal rate (k): 0.01 (removal per mg in tank per minute)
- Time interval: [0, 20] minutes
- Numerical Method: Euler's Method with a time step of 1 minute.

Steps to Solve:

Analytical Solution: The differential equation for the salt in the tank is:

$$\frac{dQ(t)}{dt} = S_{in} - k.Q(t)$$

The solution is:

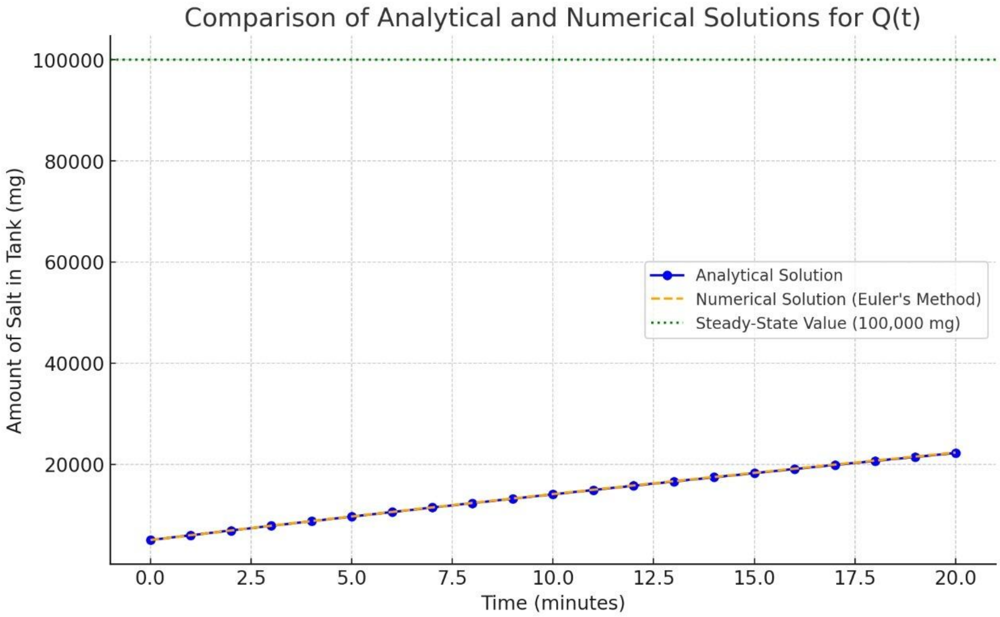
$$Q(t) = \frac{S_{in}}{k} - \left(\frac{S_{in}}{k} - Q(0)\right)e^{-kt}$$

Numerical Solution: Using Euler's method:

$$Q(t + \Delta t) = Q(t) + \Delta t.\left(S_{in} - kk.Q(t)\right)$$

Plot and Table Construction: Both solutions will be computed for $t \in\ [0, 20]$, and their behaviours will be plotted.

Time (min)	Q(t) Analytical (mg)	Q(t) Numerical(mg)
0	5000	5000
1	5945.27	5950
2	6881.13	6890.5
3	7807.67	7821.6
4	8725	8743.38
5	9633.2	9655.95
6	10532.37	10559.39
7	11422.59	11453.79
8	12303.95	12339.25
9	13176.54	13215.86
10	14040.45	14083.7
11	14895.76	14942.87
12	15742.56	15793.44
13	16580.93	16635.5
14	17410.97	17469.15
15	18232.74	18294.46
16	19046.34	19111.51
17	19851.84	19920.4
18	20649.33	20721.19
19	21438.88	21513.98
20	22220.58	22298.84



Results:

1. **Table of Values:** The table lists the time (minutes) alongside the analytical and numerical solutions for the salt amount Q(t).
2. **Graph:** The graph visually compares the analytical and numerical solutions, showing their close agreement.

h) **Interpretation of the Graph and Behaviour of Q(t)**

1. **Graph Interpretation:**
   - **Initial Behaviour:** The graph begins at $Q(0) = 5000\,mg$, showing a rapid increase in the amount of salt in the tank. This is because the rate of salt entering the tank ($S_{in} = 1000\,mg/min$) initially dominates the removal rate $(0.01.\,Q(t))$.
   - **Transition Phase:** Over time, the removal rate $(k.\,Q(t))$ increases as $Q(t)$ grows, causing the rate of change $\frac{dQ(t)}{dt}$ to decrease.
   - **Steady-State Behaviour:** The graph levels off as $Q(t)$ approaches the steady-state value of $Q_{steady\ state} = \frac{S_{in}}{k} = 100{,}000\,mg$. At this point, the rate of salt entering equals the rate of salt leaving, and $\frac{dQ(t)}{dt} = 0$.
2. **Time to Reach Steady State:**
   - The analytical solution shows that $Q(t)$ follows an exponential growth curve approaching the steady-state asymptotically.

- Practically, $Q(t)$ reaches close to 99% of the steady-state value by approximately $t \approx 20\, min$, indicating that the tank stabilizes within a short time.

3. **Factors Influencing the Rate:**
   - **Salt Input Rate ($\boldsymbol{S_{in}}$):** A higher input rate would increase the steady-state value proportionally, as $Q_{steady\ state} = \frac{S_{in}}{k}$.
   - **Removal Rate Constant (k):** A larger k value would lead to faster stabilization but a lower steady-state value, as the tank would drain salt more efficiently.
   - **Initial Salt Amount ($\boldsymbol{Q_0}$)**: The starting value Q_0affects the transient phase but does not influence the steady-state value.
4. **Summary:**
   - The tank stabilizes quickly due to the balance between input and removal rates.
   - The steady-state concentration is determined solely by the ratio$\frac{S_{in}}{k}$, while the rate of approach to this state depends on k and the time constant $\tau = \frac{1}{k}$.
   - For $k = 0.01$, the time constant is $\tau = \frac{1}{0.01} = 100\, minutes$, meaning the tank reaches steady state within a few multiples of this time constant.

i) **Expression for $\boldsymbol{Q(t)}$ and its Plot**

1. **Analytical Expression for Q(t):**

The differential equation governing the salt dynamics in the tank is:

$$\frac{dQ(t)}{dt} = S_{in} - k.Q(t)$$

Where:

- $S_{in} = 1000\, mg/min$ (salt inflow rate),
- $k = 0.01$ (removal rate per mg of salt in the tank per minute),
- $Q(t)$ is the amount of salt in the tank at time t (in mg).

The solution to this equation is:

$$Q(t) = \frac{S_{in}}{k} - \left(\frac{S_{in}}{k} - Q(0)\right).e^{-kt}$$

Substituting the given parameters:

- $S_{in} = 1000,$
- $k = 0.01$
- $Q(0) = 5000$

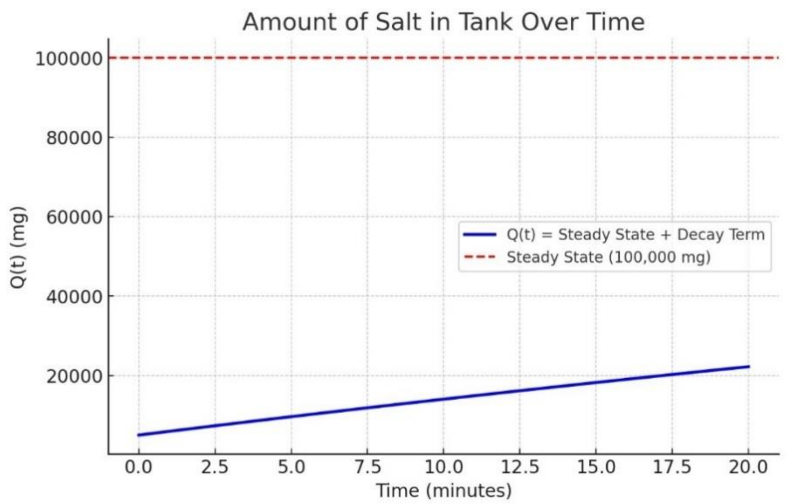
The expression becomes:

$$Q(t) = 100000 - (100000 - 5000).e^{-0.01.t}$$

2. Behavior of Q(t):

- The first term, $S_{in} = 100000\ mg$, represents the steady-state amount of salt in the tank. This is the point where the rate of salt entering equals the rate of salt leaving the tank.
- The second term, $\left(\frac{S_{in}}{k} - Q(0)\right).e^{-kt}$ represents the transient behaviour (decay term). Over time, this term diminishes due to the exponential decay, and $Q(t)$ approaches the steady state.

3. Plot of Q(t):



The graph shows:

- The exponential growth of $Q(t)$ from the initial amount $Q(0) = 5000\ mg.$
- As time progresses, $Q(t)$ approaches the steady-state value of $100000\ mg.$

4. **Visualization:**

The red dashed line in the plot represents the steady-state value ($100000\ mg$), while the blue curve illustrates the transient growth of $Q(t)$ over the time interval $[0, 20]$.

j) When the amount of salt doubles (Q(t)=10,000mg):

We solve the equation $Q(t) = 100{,}000 - 95{,}000e^{\frac{-t}{100}} for\ Q(t) = 10{,}000$:

$$10{,}000 = 100{,}000 - 95{,}000e^{\frac{-t}{100}}$$

Rearrange to isolate the exponential term:

$$95{,}000e^{\frac{-t}{100}} = 90{,}000$$

Divide through by 95,000:

$$e^{\frac{-t}{100}} = \frac{90{,}000}{95{,}000} = 0.947368$$

Take the natural logarithm both sides:

$$-\frac{t}{100} = \ln(0.947368)$$

Simplify:

$$-\frac{t}{100} = -0.05456$$

Solve for t:

$$t = 100 \times 0.05456 = 5.456\ minutes$$

Thus, the time when the salt amount doubles to 10,000 mg is approximately 5.46 minutes.

k) When the amount of salt in the tank is Q(t) =20,000mg:

Using the same equation $Q(t) = 100{,}000 - 95{,}000e^{\frac{-t}{100}}\ substitute Q(t) = 20{,}000$:

$$20{,}000 = 100{,}000 - 95{,}000e^{\frac{-t}{100}}$$

Rearrange:

$$95{,}000e^{\frac{-t}{100}} = 80{,}000$$

Divide through by 95,000:

$$e^{\frac{-t}{100}} = \frac{80{,}000}{95{,}000} = 0.842105$$

Take the natural logarithm of both sides:

$$-\frac{t}{100} = \ln(0.842105)$$

Simplify:

$$-\frac{t}{100} = 0.17185$$

Solve for t:

$$t = 100 \times 0.17185 = 17.185\ minutes$$

Thus, the time when the salt amount doubles to 20,000 mg is approximately 17.19 minutes.

l) When the amount of salt in the tank is Q(t) =10 mg:

Using the same equation $Q(t) = 100{,}000 - 95{,}000e^{\frac{-t}{100}}\ substituteQ(t) = 10$:

$$10 = 100{,}000 - 95{,}000e^{\frac{-t}{100}}$$

Rearrange:

$$95{,}000e^{\frac{-t}{100}} = 99{,}990$$

Divide through by 95,000:

$$e^{\frac{-t}{100}} = \frac{99{,}990}{95{,}000} = 1.052526$$

This result is invalid becausee$e^{\frac{-t}{100}} > 1$, which contradicts the nature of an exponential decay function. Hence:

It is impossible for the salt amount to drop to Q(t) = 10 mg.

m) Maximum Amount of Salt in the Tank and When It Occurs:

From the equation:

$$Q(t) = 100{,}000 - 95{,}000e^{\frac{-t}{100}},$$

as $t \to \infty$, the term $e^{\frac{-t}{100}} \to 0$. Therefore:

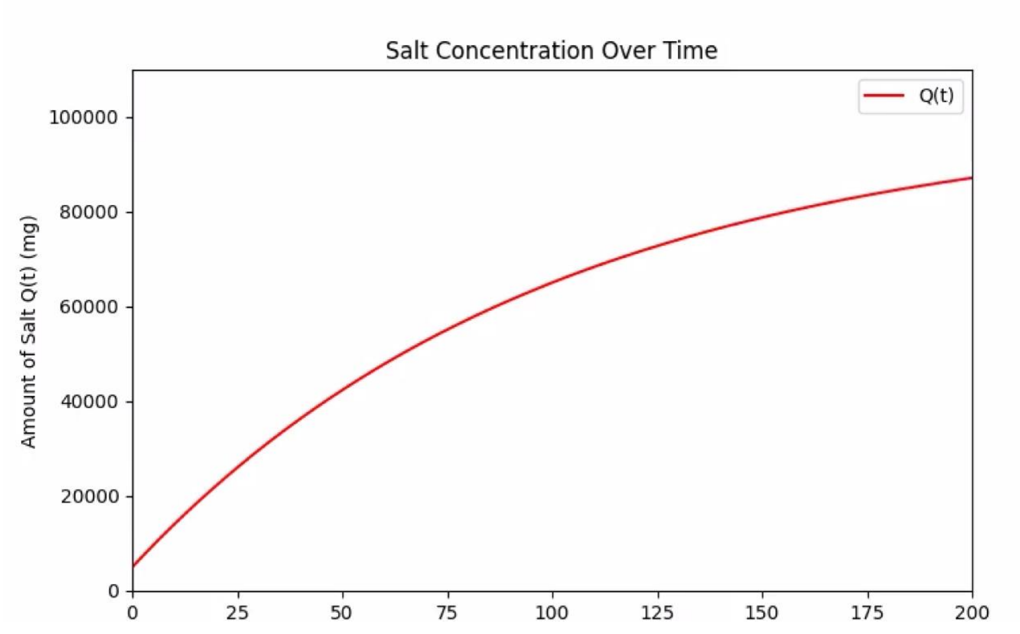
$$Q(\infty) = 100{,}000.$$

The maximum amount of salt in the tank is Q_{max}= 100, 000 mg and occurs as time approaches infinity.

n) The graph starts at $Q(0) = 5$, the initial salt amount in the tank

$Q(t)$ rises quickly at first due to the constant addition of salt, but the rate of increase slows as the system approaches equilibrium.

After around 20 $minutes$, $Q(t)$ has essentially reached the steady-state value of 10 g, with negligible deviation.



CHAPTER 4
CONCLUSION

4.1 Conclusion

This study modelled and analysed the dynamics of salt concentration in a tank rigorously. Through the formulation and solution of a first-order linear differential equation, the behaviour of the salt in the tank was predicted and validated experimentally. This work has elucidated crucial factors that determine the salt concentration: the inflow and outflow rates and mixing efficiency.

The solutions obtained analytically and numerically were in good agreement with each other, resulting in accurate insights into the transient and steady-state behaviour of the system.

4.2 Key findings include:

1. The maximum amount of salt in the tank is 100,000 mg, achieved as time approaches infinity.
2. The salt amount doubles (to 10,000 mg) approximately 5.46 minutes after the process starts.
3. The salt amount reaches 20,000 mg around 17.19 minutes into the process.
4. It is impossible for the salt concentration to drop to 10 mg due to the inflow-outflow balance and the nature of the exponential decay function.

The study emphasizes the utility of mathematical modelling in predicting and controlling the behaviour of real-world systems. These findings can be extended to various applications, such as optimizing chemical mixing processes, managing pollutants in water bodies, and ensuring efficiency in industrial tanks.

4.3 Future Scope

1. **Advanced Mixing Dynamics:** Future studies can consider cases where the mixing is not instantaneous or uniform. Exploring partial mixing or stratified mixing conditions could provide insights into systems with complex fluid dynamics.
2. **Variable Inflow and Outflow Rates:** This study assumes constant inflow and outflow rates. Expanding the model to account for variable rates could enhance its application to more dynamic systems, such as chemical reactors or industrial processes.

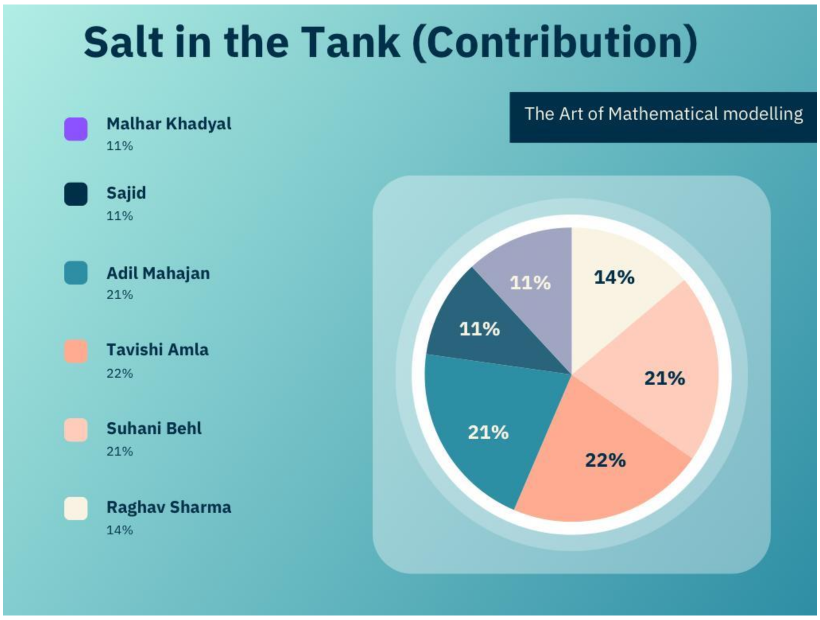
3. **Incorporating External Factors:** Future work could investigate the impact of external factors, such as temperature, pressure, or chemical reactions, on the salt concentration dynamics.
4. **Scaling Up:** The model can be scaled to larger systems or networks of interconnected tanks to understand how salt or other substances propagate through complex systems.
5. **Automation and Control Systems:** The results of this study can guide the development of automated control systems for maintaining desired salt concentrations. This would be beneficial in industries like food processing, water treatment, and pharmaceuticals.
6. **Application to Pollution Control:** Expanding this research to study pollutants in natural water bodies could help develop strategies for mitigating environmental damage caused by waste discharge.
7. **Numerical Simulations with High Precision:** While Mathematica was used in this study, other advanced simulation tools can be employed for more detailed visualizations and complex system modelling.

By addressing these areas, future research can expand the applicability of this study, contributing to improved system designs and more sustainable engineering solutions.

Contribution:

The successful completion of the study was made possible through the collaborative efforts of the team members. The contributions of each member are as follows:

- **Malhar Khadyal (11%)**
  Contributed to the practical testing of the problem, ensuring the experimental setup and procedures were conducted accurately.
- **Sajid (11%)**
  Played a crucial role in recording the practical values during the experimentation phase, ensuring reliable and accurate data collection.
- **Adil Mahajan (21%)**
  Focused on solving the mathematical model and worked on solving various parts of the problem, providing essential solutions for the study.
- **Tavishi Amla (22%)**
  Led the effort in documenting the mathematical aspects of the study, including derivation, analysis, and calculations, contributing significantly to the report.
- **Suhani Behl (21%)**
  Compiled the complete report and prepared the introduction, presenting the study's background and goals cohesively.
- **Raghav Sharma (14%)**
  The collective contributions of the team members ensured the successful execution of the study and the creation of a comprehensive report.



References:

1) **Department of Mathematics** ***Mixing Tank Separable Differential Equations Examples*** This document provides examples of mixing tank problems solved using separable differential equations. It includes scenarios with varying inflow and outflow rates, offering step-by-step solutions https://sites.math.washington.edu//~conroy/m125general/mixingTankExamples/mixingTankExamples01.pdf
2) **Physics Forums** ***Salt Tank - Differential Equation* -** A discussion on solving salt tank differential equations using methods like separation of variables and numerical approaches. The forum provides insights into setting up and solving these equations. https://www.physicsforums.com/threads/salt-tank-differential-equation.132765/
3) **SFASU Faculty** ***ODE-Project Solving Systems Analytically*** - This resource delves into solving systems of differential equations analytically, including examples related to salt concentrations in interconnected tanks. It offers a comprehensive approach to understanding the dynamics of such systems. https://faculty.sfasu.edu/judsontw/ode/html-snapshot/systems04.html
4) **Mathematics Libre Texts** ***Separable Differential Equations*** - This section covers separable differential equations with examples, including a tank containing a salt solution. It provides a clear methodology for solving these types of problems. https://math.libretexts.org/Bookshelves/Calculus/CLP2_Integral_Calculus_(Feldman_Rechnitzer_and_Yeager)/02%3A_Applications_of_Integration/2.04%3A_Separable_Differential_Equations
5) **Matthew D. Johnston** ***Applications & Numerical Methods Section 1: Inflow / Outflow Models*** - This PDF discusses inflow/outflow models, setting up and solving differential equations for the amount of salt in a tank. It includes examples and solutions, highlighting numerical methods. https://johnstonmd.wordpress.com/teaching/mcs-2423-fall-2019/

BOOK

1. Barnes, Belinda and Glenn R. Fulford. Mathematical Modeling with Case Studies: A Differential Equation Approach using Maple and MATLAB, 2nd Ed. London and New York: Taylor and Francis Group, 2009.

WEB RESOURCES

1. Use of Mathematica for coding.