MATHEMATICAL MODELING

PROJECT REPORT

MINOR -3

FOUR-YEAR UNDERGRADUATE PROGRAMME

(DESIGN YOUR DEGREE)

SUBMITTED TO UNIVERSITY OF JAMMU, JAMMU

SUBMITTED BY: TEAM-3

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Certificate

This is to certify that the project report titled "Propagation of the Word Facetious" has been successfully completed by the following students: Avichal Badyal, Ishaan Uppal, Radhey Sharma, Sarnish Kour and Shubham Sharma, as part of their academic curriculum for the 3rd semester of The Art of Mathematical Modelling, University of Jammu, Jammu.

The project has been carried out under the esteemed guidance of Dr. K.S. Charak and Dr. Sarika Manhas, whose invaluable support and mentorship have been instrumental in its completion. This report is submitted in partial fulfillment of the requirements of the program Design Your Degree and has not been submitted elsewhere for any other purpose.

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1. Introduction and History of the Word "Facetious"

Introduction

The word "facetious" holds a unique place in the English language, often used to describe humor or wit that is inappropriate or sarcastic. It reflects a lighthearted approach to communication, even in serious contexts. In this project, the word "facetious" serves as a model to explore the mathematical quantification of word popularity over time.

History

The term "facetious" originates from the Latin word facetiæ, which translates to "witty or amusing." It entered the English lexicon in the 16th century, gaining prominence during the Renaissance. Writers of that era used it to describe jesting remarks and playful expressions. Its peak usage occurred in the 18th and 19th centuries, as evidenced by historical texts and linguistic data. However, its popularity has since declined, which this report examines using mathematical modeling.

(a)Quantifying word's populariuty

The popularity of a word can be quantified by the frequency of its occurrence in a given corpus of text, such as books, articles, or online content, within a specific time period. A common method is to express this frequency as a percentage of the total number of words in the corpus.

Mathematically:

Popularity of word =

$$\left(Number\ of\ instances\ of\ the\ word - Total\ number\ of\ words\ in\ the\ corpus\right) \times\ 100$$

This formula allows us to express the popularity of a word as a **percentage** relative to the total word count in the corpus, enabling meaningful comparisons across different corpora and time periods.

(b)Impact of Corpus Size on world Popularity

Yes, changing the size of the corpus affects the **absolute frequency** of the word but does not change the **relative popularity** if expressed as a percentage. For example:

1. **Case 1:** Suppose a word appears once in a sentence of 10 words:

Popularity = $\left(1-10\right) \times 100 = 10\%$

2. **Case 2:** If the word appears once in a paragraph of 100 words:

Popularity = $\left(1-100\right) \times 100 = 1\%$

Thus, while the **absolute frequency** changes with the size of the corpus, expressing popularity as a **percentage** normalizes this effect and allows for meaningful comparisons across different corpora.

(c) Popularity as a function of Time

Case 1:

$$\text{Popularity} = \left(\frac{1}{10}\right) \times 100 = 10\%$$

Case 2:

$$\text{Popularity} = \left(\frac{1}{100}\right) \times 100 = 1\%$$

Let the popularity $P(t)$ be a function of time t, where t is in years. The word "facetious" first appeared in print at $t = 0$. The popularity $P(t)$ at any time t can be defined using an exponential model as:

$P(t) = P^0 * e^{kt}$

Here:

- $P(t)) \, is \, the \, popularity \, at \, time \, t,$
- $P^0 is \, the \, initial \, popularity \, at \, time \, t = 0,$
- k is a proportionality constant that determines whether the popularity increases ($k > 0$) or decreases ($k < 0$) over time.

Reasonableness of the function:

This function is reasonable because it normalizes the occurrence of the word by expressing the popularity in relative terms (as a percentage of the total number of words). Additionally, it allows us to model both growth and decay in popularity, which aligns with the observed data trends.

(b) Understanding the Rate of Change $\left(\frac{dP}{dt}\right)$

The derivative $\frac{dP}{dt}$ represents the **rate of change of the popularity** of the word "facetious" with respect to time. It shows how fast or slow the popularity of the word is changing at any given point in time.

- $\boldsymbol{If \frac{dP}{dt} > 0}$
- This indicates that the popularity of the word is increasing over time.
- $\boldsymbol{If \frac{dP}{dt} < 0}$

This indicates that the popularity of the word is decreasing over time.

Mathematically:

The differential equation representing the rate of change of popularity is:

$$\frac{dP}{dt} = kP$$

Where:

- $\frac{dP}{dt}$ is the rate of change of popularity.
- k is a constant that determines whether the popularity increases or decreases.
    - If $k > 0$, the popularity increases.
    - If $k < 0$, the popularity decreases.

(c) Factors influencing dp/dt

Several factors can influence the rate of change in the popularity of a word $\left(\frac{dP}{dt}\right)$

1. **Cultural trends:** Shifts in cultural interests can either increase or decrease word usage over time.

If cultural trends promote the word's usage, the rate of change will be positive, meaning *k>0k > 0*k>0 in the equation $\frac{dP}{dt} = kP$

2. **Media influence:** The popularity of certain words can increase dramatically due to exposure in books, articles, movies, or social media.

This factor can cause sudden spikes in the popularity curve, which can be modeled as temporary increases in *k*.

3. **Technological changes:** The introduction of new technologies can make some words obsolete and introduce new terms.

This leads to long-term trends where *k* becomes negative for old words and positive for new ones.

4. **Education and literacy:** Changes in education systems and literacy rates affect word usage patterns.

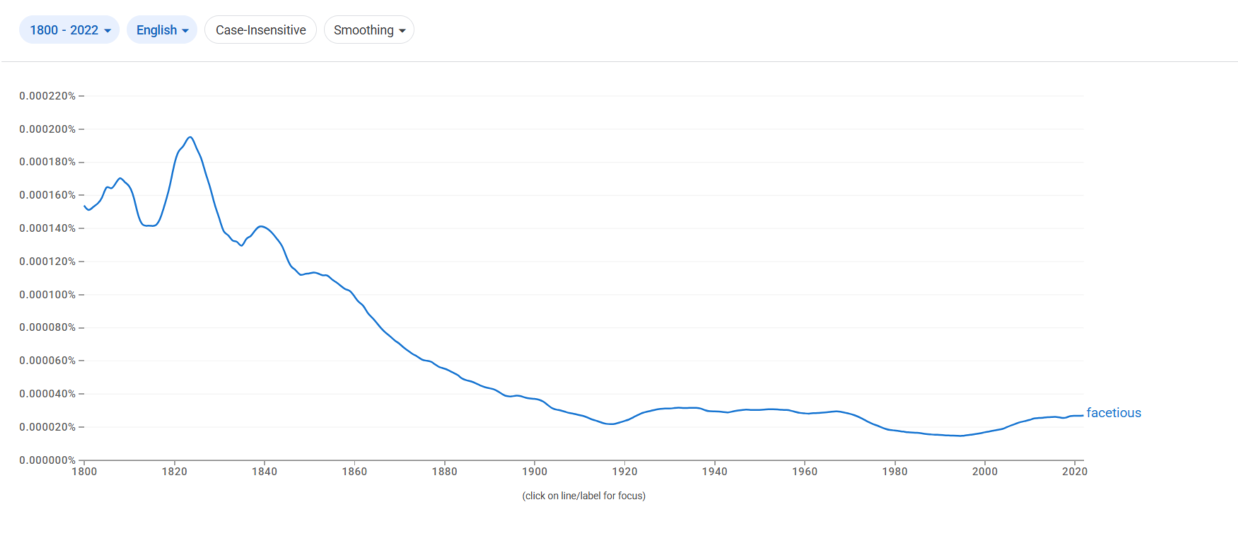
Increased literacy can result in a gradual increase in word usage, meaning a positive kkk over time.

Mathematically, these factors can be modeled by adjusting the proportionality constant kkk in the differential equation:

$\frac{dP}{dt} = kP$ Where:

- $k > 0$ indicates an increasing trend in popularity.
- $k < 0$ indicates a decreasing trend in popularity.

(d)Scatter plot of Google Ngrams data



Google Ngram.

The scatter plot represents the usage percentage of the word "facetious" over time. The general trend, based on Google Ngrams data, shows a steady decline in popularity after the early 19th century.

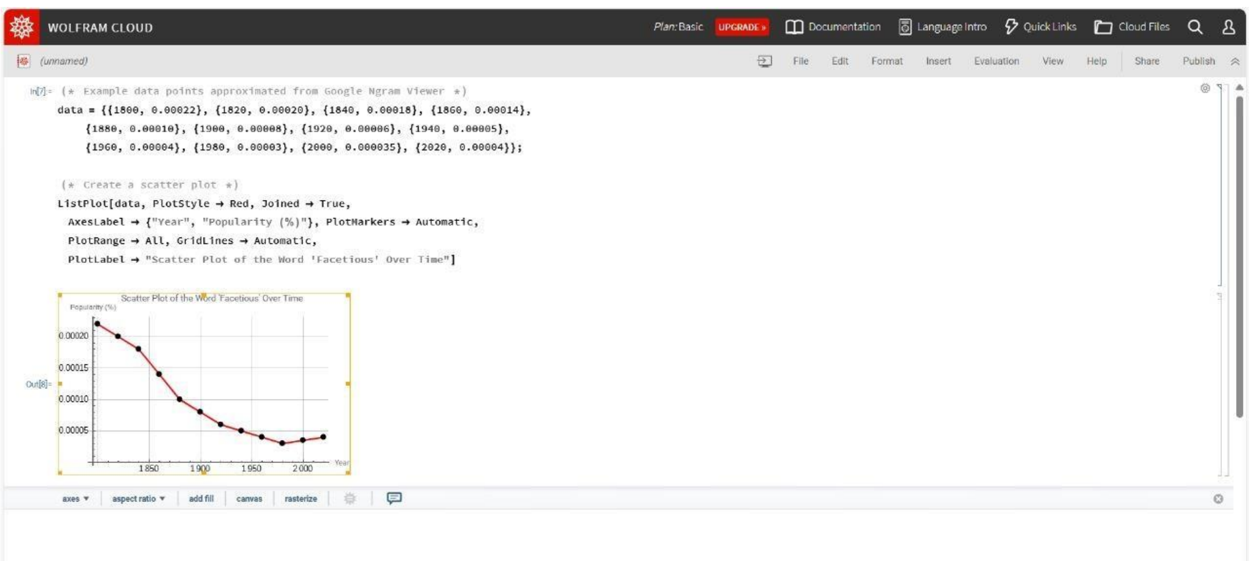
Mathematically, the trend can be described using an exponential decay model:

$$P(t) = P_0 * e^{kt}$$

Here:

- $P(t)$ is the popularity at time *t*,
- P^0 is the initial popularity,
- $k < 0$ is a negative constant indicating the rate of exponential decay.

Based on the graph, $k < 0$ because the word's usage decreases over time, following a typical exponential decay curve.



Wolfram Mathematica

(e) Factor influencing $\frac{dP}{dt}$

Based on the scatter plot, one primary factor influencing the rate of change in popularity $\left(\frac{dP}{dt}\right)$ appears to be **cultural relevance** and **changing language patterns**. The decline in popularity after the 1800s could be due to the word becoming less relevant in common discourse. Mathematically, the factor influencing the rate of change can be modeled using the differential equation:

$$\frac{dP}{dt} = kP$$

Here:

- If $k > 0$, external factors are promoting the word's usage, causing its popularity to increase.
- If $k < 0$, external factors (such as reduced cultural relevance) are causing the word's popularity to decline over time, leading to an exponential decay.

Given the observed data, $k < 0$ during this period, indicating a declining trend in the popularity of the word "facetious."

(f) Differential equation for $\frac{dp}{dt}$

Assume that the rate of change of popularity is proportional to the current popularity.
The differential equation can be written as:

$$\frac{dP}{dt} = kP$$

where:

- $\frac{dP}{dt}$ represents the rate of change of popularity with respect to time.
- k is a proportionality constant that determines whether the popularity increases or decreases over time.

If $k > 0$, the popularity increases over time.

If $k = 0$, the popularity remains constant over time.

If $k < 0$, the popularity decreases over time (this is the case for the word "facetious" based on the observed trend).

(g)Interpretation of k

1. **$If\ k > 0$**:
2. The popularity increases exponentially over time because the rate of change is positive.
3. Mathematically:
4. $P(t) = P^0 * e^{kt}$, where $k > 0$.
5. **$If\ k = 0$**:

The popularity remains constant over time because the rate of change is zero.

Mathematically:

$$P(t) = P^0.$$

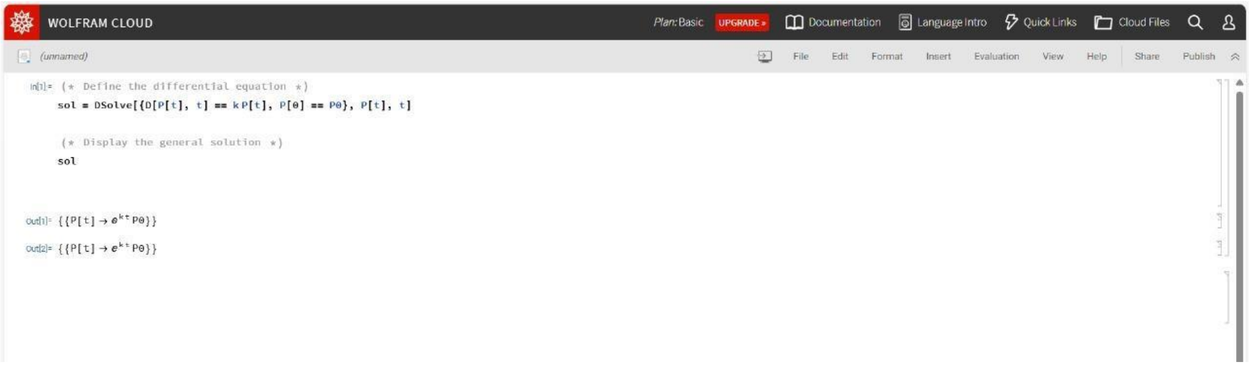
6. **$If\ k < 0$**:

The popularity decreases exponentially over time because the rate of change is negative.

Mathematically:

$$P(t) = P^0 * e^{kt}, where\ k < 0$$

Given that the popularity of "facetious" decreased significantly after the 1800s, we expect $k < 0$ for this period, indicating a decaying exponential trend.



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(h) Scenario Analysis: $k = 0.5$

If $k = 0.5$, the popularity would increase exponentially over time, as the rate of change is directly proportional to its current value.

Mathematically:

$$P(t) = P^0 * e^{0.5t}$$

where P_0 is the initial popularity at time $t = 0$.

(i) Scenario Analysis: $k < 0$

If $k < 0$, the popularity decreases exponentially over time, indicating a decline in usage, as observed in the given data.

Mathematically:

$$P(t) = P^0 * e^{kt}$$

where P^0 is the initial popularity at time $t = 0$, and k is a negative constant.

(j) Solving the differential equation

The differential equation we want to solve is:

$$\frac{dP}{dt} = kP$$

Step 1: Separate variables

We separate the variables P and *t*:

$$\frac{dP}{P} = k\ dt$$

Step 2: Integrate both sides

Integrating both sides gives:

$$\int \left(\frac{1}{P}\right) dP = \int k\ dt$$

$$\ln|P| = kt + C\ (where\ C\ is\ the\ constant\ of\ integration)$$

Step 3: Solve for $P(t)$

Exponentiating both sides:

$$P(t) = e^{kt + C}$$

We can rewrite the expression as:

$$P(t) = P^0 * e^{kt}$$

where P^0 is the initial popularity at time $t = 0$.

Example:

$Suppose\ the\ initial\ popularity\ at\ time\ t = 0\ (say, the\ year\ 1800) is\ given\ by\ P0 = 0.00022\% P^0 = 0.00022\backslash\% P0 = 0.00022\%. If\ the\ proportionality\ constant\ k = -0.005k = -0.005k = -0.005\ (since\ popularity\ decreases\ over\ time),$

$the\ popularity\ at\ any\ later\ time\ t\ can\ be\ predicted\ using:$

$$P(t) = 0.00022 * e^{-0.005t}$$

This equation can be used to estimate the popularity of the word "facetious" over different years.

(k) Runge-Kutta method of order 4

The Runge-Kutta method of order 4 (RK4) is a numerical method for solving ordinary differential equations. It provides an accurate approximation by using intermediate points to estimate the slope.

RK4 Algorithm:

For a differential equation of the form:

$$\frac{dy}{dx} = f(x, y)$$

with an initial condition $y(x_0) = y_0$, the RK4 method updates the solution iteratively as follows:

$$y_{\mathrm{n}}{}^{+1} = y_{\mathrm{n}} + \left(\frac{1}{6}\right) * (k^1 + 2k^2 + 2k^3 + k^4)$$

where:

$$k^1 = h * f(x_{\mathrm{n}}, y_{\mathrm{n}})$$

$$k_2 = h * f\left(x_{\mathrm{n}} + \frac{h}{2}, y_{\mathrm{n}} + \frac{k_1}{2}\right)$$

$$k^3 = h * f\left(x_{\mathrm{n}} + \frac{h}{2}, y_{\mathrm{n}} + \frac{k_2}{2}\right)$$

$$k^4 = h * f(x_{\mathrm{n}} + h, y_{\mathrm{n}} + k^3)$$

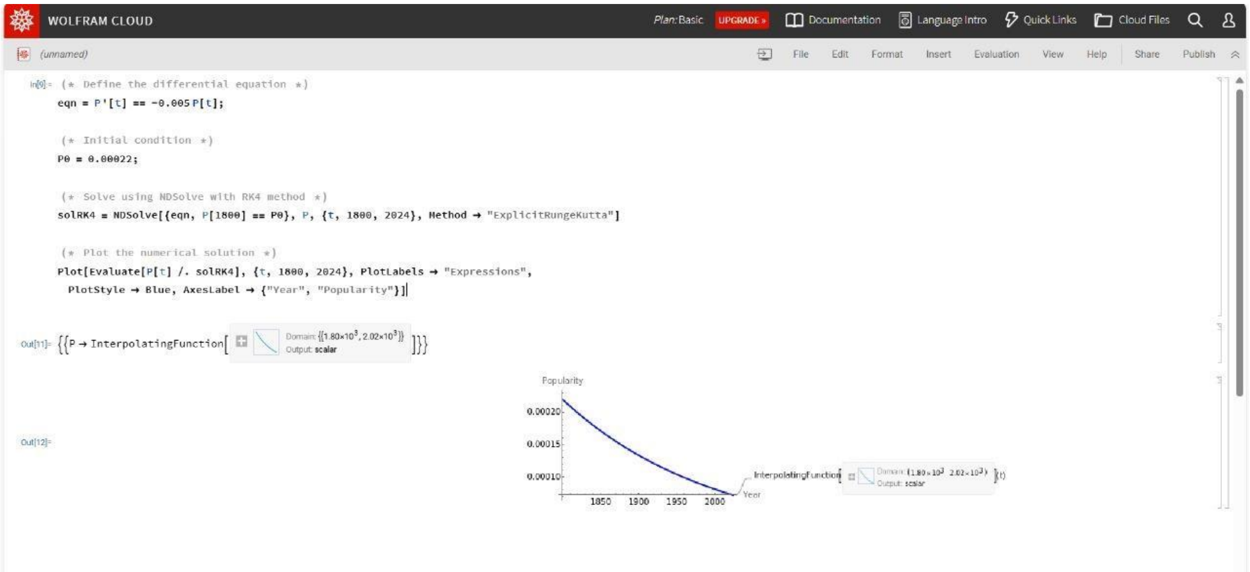
Here, h is the step size, and y_n represents the approximate solution at the nth step.

Example:

Solve the differential equation:

$$\frac{dP}{dt} = -0.005P$$

numerically for $P(1800) = 0.00022$ from the year 1800 to 2024 with a step size of 1 year.



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(l) General shape of the graph

Given the exponential model:

$$P(t) = P^0 * e^{kt}$$

where P_0 is the initial popularity and k is the proportionality constant.

1. **If $k > 0$:**
2. The graph shows an **increasing exponential curve**, meaning the popularity increases rapidly over time.

3. **If $k = 0$:**

The graph becomes a **horizontal line**, indicating that the popularity remains constant over time.

4. **If $k < 0$:**

The graph shows a **decaying exponential curve**, meaning the popularity decreases over time.

Since in this case $k < 0$, the general shape of the graph is a **decaying exponential curve**. As $t \to \infty, P(t) \to 0$, meaning the popularity of the word approaches zero in the long run.

(m) Model validity as $t \to \infty$

The exponential model $P(t) = P^0 * e^{kt}$ remains valid as long as the proportionality constant k accurately reflects the factors influencing word popularity. However, as time progresses, external factors such as changes in language, culture, media influence, and technology may render the model less accurate.

If $k < 0$ (as in this case), the model predicts that the popularity will asymptotically approach zero as $t \to \infty$.

Mathematically:
As $t \to \infty, P(t) \to 0$ since $e^{kt} \to 0$ for $k < 0$.

References:-

1) The Impact of Differential Equations in Our Everyday Lives (9 examples!) - mathodics.com7

2) Real-World Applications Of Differential Equations - Number Dyslexia

3) What Are Differential Equations - Solving Methods and Examples