"Analyzing the Dynamics of Tank Draining: A Mathematical Approach"

MAJOR PROJECT REPORT

Semester-3

FOUR-YEAR UNDERGRADUATE PROGRAMME

(DESIGN YOUR DEGREE)

SUBMITTED TO

UNIVERSITY OF JAMMU, JAMMU

SUBMITTED BY:

Team Members	**Roll No.**
Divya Verma	DYD-23-04
Diya Rani	DYD-23-05
Gourav Sharma	DYD-23-06
Paridhi Mahajan	DYD-23-14
Vidhita Arora	DYD-23-22
Manjot Singh	DYD-23-23

UNDER THE GUIDANCE OF

Prof K.S Charak

Professor

Department of Mathematics

Dr. Sarika Verma

Associate Professor

Department of Mathematics

Submitted on: ____ January, 2025

CERTIFICATE

The work embodied in this report entitled **"Analyzing the Dynamics of Tank Draining: A Mathematical Approach"** has been done by the team including group members- Divya Verma, Diya Rani, Gourav Sharma, Paridhi Mahajan, Vidhita Arora and Manjot Singh as a Major Project for Semester 3 of Four-Year Undergraduate Programme (Design Your Degree). This work was carried out under the guidance of Mentor Prof K.S Charak and Dr. Sarika Verma for the partial fulfilment for the award of the Design Your Degree, Four Year Undergraduate Programme, University of Jammu, Jammu and Kashmir. This project report has not been submitted anywhere else.

Signature of Students:

1. Divya Verma

2. Diya Rani

3. Gourav Sharma

4. Paridhi Mahajan

5. Vidhita Arora

6. Manjot Singh

Signature of Mentor:

1. Prof K.S Charak

Prof. Alka Sharma
Director, SIIEDC, University of Jammu

2. Dr. Sarika Verma

ACKNOWLEDGMENT

We avail this opportunity to acknowledge all those who helped and guided us during the course of our work. First and foremost, we thank Almighty God from the depth of our heart for generating enthusiasm and granting our spiritual strength to successfully pass through this challenge. Words are too meager to express our esteem indebtedness and whole hearted sense of gratitude towards our mentor Prof. K.S Charak, Department of Mathematics and Dr. Sarika Verma, Department of Mathematics, University of Jammu, Jammu. It is our pleasure beyond words to express our deep sense of feelings for their inspiring guidance, generous encouragement and well versed advice. They provided us undaunted encouragement and support in spite of their busy schedules. Really, fortunate we are and we feel extremely honoured for the opportunity conferred upon us to work under their perpetual motivation.

We are extremely fortunate for having an opportunity to express our heartiest gratitude to Prof. Alka Sharma, Director of SIIEDC (Design your Degree), University of Jammu, Jammu and member of advisory committee for her valuable advice and generous help and providing us all the necessary facilities from the department. We are highly thankful to all respected mentors for their immense help during the conduct of this major project. The words are small trophies to express our feelings of affection and indebtedness to our friends who helped us throughout the major project, whose excellent company, affection and co-operation helped us in carrying out our research work with joy and happiness.

We acknowledge all the people, mentioned or not mentioned here, who have silently wished and gave fruitful suggestions and helped us in achieving the present goal.

ABSTRACT

This project investigates the fluid drainage dynamics from a tank, emphasizing mathematical modelling and its engineering applications. Utilizing foundational principles such as Conservation of Mass and Torricelli's Theorem, the study derives a governing differential equation to describe the fluid height's time-dependent behaviour. Analytical and numerical methods were applied to solve and validate the model, supported by experimental data affirming the theoretical predictions. Additionally, the study incorporates computational analysis performed in **Mathematica**, where code was developed to simulate the drainage process and generate graphical insights. The research highlights the influence of key factors, including tank geometry, outlet size, and fluid properties, on drainage rates. Practical applications of this study extend to designing efficient irrigation systems, optimizing industrial fluid storage and release, and managing wastewater. Moreover, the project provides a basis for enhancing sustainability in fluid management by minimizing resource wastage and improving system reliability. These insights contribute to advancing fluid mechanics and its role in addressing real-world challenges in engineering and environmental systems.

INDEX

CHAPTER 1

INTRODUCTION

This chapter delves into the fundamentals of tank draining, exploring its engineering significance, theoretical principles, and practical applications. It establishes the foundation for understanding the dynamics of fluid flow during the draining process and its implications in various real-world scenarios.

1.1 Draining of Tank

The draining of a tank is a fundamental problem in fluid mechanics, combining the principles of physics with practical engineering applications. It provides a classic example of how fluid flow is governed by basic physical laws and serves as a cornerstone for designing systems where fluid storage and controlled release are crucial. Understanding the dynamics of fluid drainage is essential in diverse fields, including industrial manufacturing, irrigation systems, and environmental management. At its core, the draining process is dictated by the principles of **Conservation of Mass** and **Torricelli's Theorem**, two pivotal laws in fluid dynamics. Conservation of Mass ensures that the rate at which fluid exits the tank is directly tied to the reduction in fluid volume, while Torricelli's Theorem relates the velocity of the outflowing fluid to the height of the liquid in the tank. These principles, when combined, allow for the derivation of a differential equation that captures the behaviour of the fluid's height over time.

This study also connects theoretical principles with real-world scenarios, as the draining of tanks is integral to processes such as wastewater management, chemical storage operations, and even emergency scenarios like controlling flood discharge. By modelling and analysing the process mathematically, we aim to derive actionable insights that improve efficiency and safety in these applications.

1.2 Significance in Engineering Applications

Draining a tank involves the flow of liquid under gravity or external pressure. This process is governed by principles of fluid mechanics, including the conservation of mass and energy. The study of tank draining is significant in engineering for the following reasons:

- **System Design:** Accurate predictions of flow rates and discharge times are essential for designing efficient storage and drainage systems.

- **Safety:** Properly designed drainage systems reduce the risk of overflows, structural failures, and accidents.
- **Sustainability:** Optimizing the draining process ensures minimal wastage of resources and energy.
- **Process Efficiency:** In industries, precise control over draining processes enhances productivity and reduces downtime.

1.3 Problem Statement

Efficiently managing the drainage of a tank is a challenge that requires precise prediction of fluid dynamics over time. Factors such as tank geometry, outlet size, and fluid properties influence the rate of drainage, making it necessary to derive mathematical models that accurately describe the system. This study seeks to address the following questions:

1. How can the principles of physics be applied to derive an accurate model for tank drainage?
2. What are the key variables and parameters that govern the draining process, and how do they influence the rate of drainage?
3. How can analytical and numerical methods be used to solve and validate the model?

By addressing these questions, the study aims to provide a comprehensive understanding of the tank drainage process, with practical applications in engineering and environmental systems.

1.4 Role of Differential Equation in Modelling such Systems

Differential equations play a crucial role in modelling dynamic systems, particularly those governed by rate of change. In the context of tank drainage, the behaviour of the system, specifically how the height of the fluid $h(t)$ changes over time, can be accurately described using a differential equation. These equations bridge the physical laws governing fluid flow with mathematical tools for analysis and prediction.

Practical Applications

The use of differential equations in modelling tank draining systems has practical implications:

- **System Design:** Helps engineers optimize tank dimensions and outlet sizes for desired drainage rates.
- **Predictive Analysis**: Enables accurate prediction of how long it takes to drain a tank under specific conditions.

- **Control Systems**: Provides the foundation for designing automated systems that monitor and control fluid levels in reservoirs, irrigation setups, or industrial tanks.

By translating physical phenomena into mathematical expressions, differential equations serve as indispensable tools for understanding, analysing, and solving real-world problems. In this study, they form the backbone of the model for tank drainage and are key to deriving insights into its behaviour.

1.5 Goals

1. **To derive a mathematical model based on physics principles:**

   Formulate a differential equation for the height of fluid (h) with respect to time (t) using Conservation of Mass and Torricelli's Theorem.

2. **To solve and analyse the governing equation:**

   Explore analytical and numerical methods to solve the differential equation and validate the model against experimental data.

3. **To investigate key parameters influencing the drainage process:**

   Examine the effects of tank geometry, outlet size, and gravitational forces on the rate of drainage.

4. **To provide visual insights into the process:**

   Use MATLAB to generate graphical plots showing how $h(t)$ evolves over time and interpret these results for practical applications.

5. **To highlight the applications and future scope of tank drainage analysis:** Identify real-world scenarios where this study can be extended to address more complex challenges.

1.6 Physics Behind the Problem

1. **Conservation of Mass**

   The Conservation of Mass is one of the most fundamental principles in physics, ensuring that the total mass of a closed system remains constant. In the context of tank drainage, this principle ensures that the rate at which fluid exits the tank matches the rate at which the fluid volume decreases. Mathematically, it can be expressed as:

$$\text{Rate of Outflow} = -\frac{dV}{dt}$$

Here, V is the volume of the fluid, and the negative sign indicates that the volume is decreasing over time.

Applications:

- Design of irrigation systems to control water flow.
- Modelling of industrial tanks used in chemical and food industries to maintain precise outflow rates.
- Managing outflow in reservoirs to prevent overflow or optimize resource usage.

2. **Torricelli's Theorem**

   Torricelli's Theorem relates the velocity of fluid exiting the tank to the height of the fluid column. It is derived from Bernoulli's principle and states:

$$v = \sqrt{2gh(t)}$$

where (v) is the velocity of the fluid at the outlet, (g) is the acceleration due to gravity, and $h(t)$ is the height of the fluid at time (t). This theorem will be utilized later in our project.

Applications:

- Predicting discharge rates in dam spillways and flood control systems.
- Designing nozzles and outlet valves for precise control of fluid speed.
- Engineering fuel tanks in aerospace and automotive industries to regulate flow dynamics.

CHAPTER 2

EXPERIMENTATION

2.1 Introduction

The experiment aims to study the rate of water drainage from a cylindrical bottle through a small hole at the bottom. By analysing the relationship between the height of the water column and time, the experiment demonstrates the principles of fluid mechanics, specifically Torricelli's law and its application to derive a differential equation. This chapter provides a detailed explanation of the experiment, derivation of the governing equation, and experimental results.

2.2 Material and Apparatus

1. **Bottle**: A 2-liter cylindrical plastic bottle with a diameter of **10 cm**.
2. **Jet Hole**: A small circular hole of diameter **0.45 cm** made at the lower part of the bottle.
3. **Water**: Used as the fluid medium.
4. **Ruler**: To measure the water height.
5. **Timer**: To record the time taken for water to drain.

2.3 Experimental Setup

1. The bottle was filled with water to initial height: **12 cm.**
2. The hole was drilled at the bottom of the bottle to ensure water could flow out freely due to gravity.
3. The diameter of the bottle (**10 cm**) and the jet hole (**0.45 cm**) remained constant across all trials.
4. The time taken was recorded at different heights: **12 cm, 10 cm, 8 cm, 6 cm, 4 cm, 2 cm and 0 cm.**

2.4 Variables

- **Independent Variable**: Time t (in seconds).
- **Dependent Variable**: Height of the water column $h(t)$ (in cm) at time t.
- **Controlled Variables**:
    - Diameter of the bottle (10 cm).
    - Diameter of the jet hole (0.45 cm).

2.5 Key Assumptions

- **Cross-sectional Area of the Bottle** remains constant.
- **Cross-sectional Area of the Jet Hole** is constant and small relative to the bottle.
- The flow is driven by **gravitational potential energy** converting into **kinetic energy**.
- Neglecting surface tension effects at higher water heights (though it affects at lower levels).
- The system follows the **Torricelli theorem** to derive velocity of water exiting the hole

2.6 Steps Performed

1. **Preparation:**
   - Take a plastic bottle and cut off the top to create an open container.
   - Measure and record the **diameter** of the bottle to determine its cross-sectional area.
2. **Creating the Drain Hole:**
   - Drill or carefully make a **small hole at a fixed height above the bottle's base** for water drainage.
   - Ensure the hole is smooth and uniform for consistent water flow.
3. **Marking Water Levels:**
   - Measure and mark height levels **from the hole upwards** at 12 cm, 10 cm, 8 cm, 6 cm, 4 cm, 2 cm and 0 cm.
4. **Filling the Bottle:**
   - Pour water into the bottle up to the **12 cm mark above the hole**.
5. **Starting the Timer:**
   - Begin the timer as soon as water starts flowing out of the hole.
   - Record the time taken for the water level to drop to each marked height (10 cm, 8 cm, etc.).
6. **Recording Observations:**
   - Note the time taken for the water to drain at each height level.
   - Record the total time taken for the water to completely drain.
7. **Analysis:**
   - Examine the relationship between water height and drainage time to study how pressure impacts the speed of water flow (Torricelli's Theorem).

Fig. 1 Experimental Setup

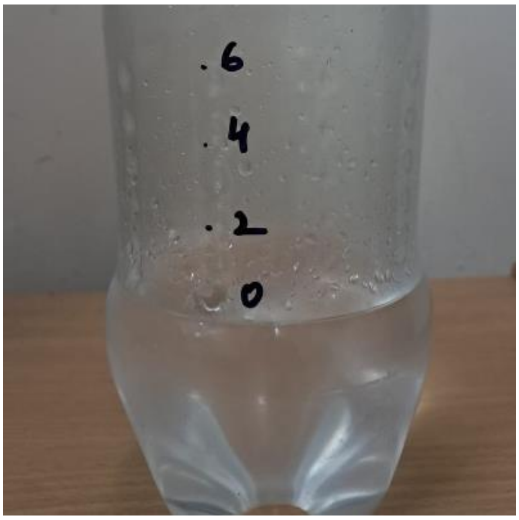


Fig. 2 Experiment in Progress

2.7 Dataset

The recorded times for the water levels to reach specific heights are as follows:

Height of water: h (cm)	**Time taken: t (s)**
12	0
10	6
8	13
6	21
4	30
2	41
0	67

2.8 Theory and Derivation of the Differential Equation

1. Volume of Water in the Bottle

- The volume of water, V_w , is the product of the bottle's **cross-sectional area** (A_b) and the height of water (h):

$$V_w = A_b . h(t).$$

- The rate of change of the volume of water in the bottle is influenced by the rate at which water exits the bottle. This can be expressed mathematically by differentiating the volume of water (V_w) with respect to time (t):

$$\frac{d}{dt}(V_w) = \frac{d}{dt}\left(A_b.h(t)\right) = -Q$$

  Here, -Q = the flow of rate of water leaving the bottle, marked by a -ve sign representing decrease in the volume.

- Since the bottle's cross-sectional area (A_b) is constant, the equation simplifies to:

$$A_b.\frac{dh(t)}{dt} = -Q$$

$Where,$ $\frac{dh(t)}{dt}$ = the rate of change of water height over time.

2. Flow Rate from the Hole

- The rate at which water exits the bottle (volume flow rate) is given by:

$$Q = A_j.v$$

  where A_j is the cross-sectional area of the jet hole, and v is the velocity of water exiting the hole.

3. Velocity of Water (Torricelli's Theorem)

- According to the conservation of energy:

$$\text{Potential Energy (PE)} = \text{Kinetic Energy (KE)}$$

- Potential energy of water at height h(t):

$$\text{PE} = mgh(t)$$

- Kinetic energy of water exiting with velocity v(t):

$$\text{KE} = \frac{1}{2}mv^2(t)$$

Equating PE and KE:

$$mgh(t) = \frac{1}{2}mv^2(t).$$

Cancelling m:

$$gh(t) = \frac{1}{2}v^2(t)$$

We get, Velocity $v(t)$:

$$v = \sqrt{2gh(t)}$$

where:

- g: gravitational acceleration (9.8 m/s^2)
- $h(t)$: height of water above the jet hole at time t

4. Substituting Flow Rate into Volume Change

- The rate at which water exits the bottle is equal to the rate of change of volume:

$$A_b.\frac{dh(t)}{dt} = -A_j.\sqrt{2gh(t)}.$$

5. Rearrange and Simplify

- Divide through by A_b :

$$\frac{dh(t)}{dt} = -\frac{A_j}{A_b}.\sqrt{2g}\sqrt{h(t)}.$$

- Group constants into K:

$$K = \frac{A_j}{A_b}.\sqrt{2g}$$

Simplifying K by substituting these expressions:

$$A_j = \frac{\pi d_j^2}{4}, \ A_b = \frac{\pi d_b^2}{4}.$$

Here, Ratios of these areas:

$$\frac{A_j}{A_b} = \frac{\frac{\pi d_j^2}{4}}{\frac{\pi d_b^2}{4}} = \frac{d_j^2}{d_b^2}.$$

Therefore, simplified form of K:

$$K = \frac{d_j^2}{d_b^2} . \sqrt{2g}$$

Resulting in the final differential equation:

$$\frac{dh(t)}{dt} = -K . \sqrt{h(t)}.$$

CHAPTER 3

SOLUTIONS

The following chapter explores how liquid drains from a tank, ***modelled using a first-order differential equation***, through the experiment of a bottle draining water. Both analytic and numeric methods are used to solve the equation. The analytic solution provides a formula to predict the drainage time, while the numeric solution confirms the results through calculations. Practical examples, such as a 2-liter bottle draining from different starting heights, demonstrate how the initial liquid height affects the total time it takes to empty. This study highlights the connection between fluid flow, mathematical modelling, and real-world experiments.

The differential equation derived from the setup or experiment is:

$$\frac{dh(t)}{dt} = -K.\sqrt{h(t)}$$

describing the rate of change of the height h***(t)*** of liquid in a tank as it drains over time.

Here:

1. $h(t)$ is the height of the liquid in the tank at any given time t.
2. $\frac{dh(t)}{dt}$ represents the rate of change of the height with respect to time, which means how fast the liquid level is dropping.
3. K is a constant that determines the rate at which the tank drains. The larger the value of K, the faster the liquid drains.
4. $\sqrt{h(t)}$ indicates that the rate of change is proportional to the square root of the height. This is a key feature and suggests that the rate of drainage is not constant but depends on the current height of the liquid.
5. The negative sign on the right-hand side, $-K\sqrt{h(t)}$, indicates that the height of the liquid is decreasing over time. This reflects the draining process essentially, as time passes, the height $h(t)$ of the liquid goes down, meaning the liquid is being drained from the tank.

3.1 Solving the Differential Equation

To analyze how the height of the liquid in the tank, $h(t)$, evolves over time during the draining process, it is necessary to solve the differential equation that governs the rate of change of the height. This requires finding an ***analytic solution*** to the equation, which describes the rate of drainage and provides a formula for predicting the height of the liquid at any given time.

$$\frac{dh(t)}{dt} = -K\sqrt{h(t)}$$

1. **The first step in solving the differential equation is separating the variables.**

The goal is to rearrange the terms so that all the $h(t)$ dependent terms are on one side of the equation and all the t dependent terms are on the other side.

$$\frac{1}{\sqrt{h(t)}}dh = -K.dt$$

This separates the equation into two parts:

- The left side has only $h(t)$ and $dh(t)$,
- The right side has only t and dt.

2. Since the variables are separated, **the second step is to integrate both sides of the equation** which would be:

$$\int \frac{1}{\sqrt{h}}dh = -\int K.dt$$

Here, h is treated as an independent variable on the left side, while t is the independent variable on the right side. This allows to integrate each side with respect to its corresponding variable without confusion.

Using initial condition, the equation can be solved

- At $t = 0$ the height of the fluid is h_0, the initial height of the liquid in the tank.
- At any later time t, the height of the fluid is denoted as $h(t)$.

This ensures the solution reflects how the fluid height changes over time, starting from the known initial condition $h(0) = h_0$.

3. After setting the initial condition ($h(0) = h_0$) **we integrate both sides with limits**:

$$\int_{h_0}^{h(t)} \frac{1}{\sqrt{h}}dh = -\int_0^t K.dt$$

This indicates the comparison between the change in height from its initial state to its state at time t. The left side is integrated with respect to h from the initial height h_0 to the current height $h(t)$, while the right side is integrated with respect to time from *0* to t.

$$\int_{h_0}^{h(t)} h^{\frac{-1}{2}}\, dh = -K \int_{0}^{t} dt$$

Now let's proceed with evaluating both sides of the integral to understand how the fluid height h(t) changes over time.

$$2\sqrt{h}\,\Big|_{h_0}^{h(t)} = -Kt\,\Big|_{0}^{t}$$

Left Side:

- This expression shows the evaluation of the integral of $\frac{1}{\sqrt{h}}dh$ from the initial height h_0 to the current height $h(t)$.
- Here, $2\sqrt{h\ (t)}$ reflects the square root of the height at time $\boldsymbol{t}$**.**

Right Side:

- The right side evaluates the integral of $-K$ with respect to time from 0 to t. The result is $-Kt$, indicating the total change over the given time interval.
- The negative sign indicates that the height is decreasing over time.

4. The resulting equation after evaluating the integral is:

$$2\left(\sqrt{h(t)} - \sqrt{h_0}\right) = -Kt$$

This equation shows that the change in the square root of ($\sqrt{h(t)}$ - $\sqrt{h_0}$) over time is proportional to time (t). The negative sign indicates that $\sqrt{h(t)}$ is decreasing as t increases. The factor k controls how quickly this decrease happens. The quantity h(t) diminishes over time, and its rate of decrease depends on the square root of the difference from its initial value h_0. Further Solving we get:

$$\sqrt{h(t)} = \sqrt{h_0} - \frac{1}{2}Kt.$$

Therefore

$$h(t) = \left(\sqrt{h_0} - \frac{1}{2}Kt\right)^2.$$

The above equation describes how the height ***h(t)*** *of liquid in a tank decreases over time. Initially, at t=0, the height is* h_0*, and as time progresses,* ***h(t)*** *decreases quadratically, slowing down as the tank empties. The rate of change depends on the square root of the height, meaning the drainage is faster when the tank is fuller and slows as* ***h(t)*** *approaches* ***zero.***

After deriving the equation for the height of the liquid over time, we now focus on calculating the total time required for the liquid to completely drain from the tank, starting from an initial height h_0. The time t_{final}, which represents the total drainage time, is obtained from the equation for h_t.

The equation for t_{final} is given by:

$$t_{final} = \frac{2\sqrt{h_0}}{K} = \frac{2d_b^2}{d_j^2\sqrt{2g}}\sqrt{h_0}$$

Where:

- d_b is the diameter of the bottle.
- d_j is the diameter of the hole at the bottom of the bottle.
- g is the acceleration due to gravity.
- h_0 is the initial height of the liquid in the bottle.

This equation represents the total time required for the liquid to fully drain out of the tank, starting from the initial height $\boldsymbol{h_0}$. The time depends on several factors, including the size of the bottle, the size of the hole, the gravitational constant, and the initial height of the liquid. Through this equation, we can predict how long it will take for the liquid to drain based on given parameters. This time equation plays a crucial role in understanding the dynamics of fluid drainage in a controlled setup like the draining bottle experiment.

3.2 Experimental Values:

1. Diameter for the:

2-lt Bottle	0.1m
Jet Hole *(*A small circular hole made at the lower part of the bottle)	0.0045m

2. Water Heights and Times**:**

Water Height (m)	**Experimental Time (s)**
0.12	0
0.1	6
0.08	13
0.06	21
0.04	30
0.02	41
0	67

The table above presents recorded times for the water levels to reach specific heights

Let us now determine the total drainage time of the bottle using the theoretical approach. By applying the formula for t_{final} and substituting the given parameters, we aim to calculate the exact

Given are the values for ***calculating the total drainage time*** of the bottle:

Diameter of the Bottle	**Diameter of jet hole**	**Initial Height or h_0**	***g* acceleration due to gravity**
0.1 m	0.0045 m	0.12 m	9.8 m/s^2

Using the formula for $t_{final} = \frac{2d_b^2}{d_j^2\sqrt{2g}}\sqrt{h_0}$ and substituting the above values :

$$t_{\text{final}} = \frac{2.(0.1m)^2}{(0.0045m)^2\sqrt{2.(9.8m/s)^2}}\sqrt{0.12}$$

t_{final} = 76 ± 0.4 seconds

The (plus-minus) sign in t_{final} = 76 ± 0.4 seconds indicates the uncertainty in the measurement, accounting for possible deviations due to instrument precision or experimental error. It shows the value could range from 75.6 to 76.4 seconds.

Therefore:

Initial Height or ***h_0 (m)***	Experimental Time (s)	Calculated Drainage Time (s)
0.12	67	76 ± 0.4

The calculated total drainage time of **76 ± 0.4 seconds** corresponds to an initial height of **0.12m.** Using the theoretical formula and substituting the given parameters, the result accurately reflects the expected time for the liquid to completely drain from the bottle. This value provides a clear reference for the total drainage duration based solely on the bottle diameter, jet hole diameter, initial height, and gravitational acceleration

3.3 Numerical Solution for the Given Equation

Given the parameters:

- n = 6
- Initial height (h_0) = 0.12
- Initial time (t_0) = 0
- Final time (t_{final}) = 67
- Step size (m) = 11.1
- Constant (K) = 0.008

Tabulated Results

Below is the computed table based on the given data:

n	Time (t)	Height (h)	Actual Height
0	0	0.12	0.12
1	11.1	0.089	0.1
2	22.2	0.062	0.08
3	33.3	0.040	0.06
4	44.4	0.022	0.04
5	55.5	0.009	0.02
6	66.6	0.007	0

Calculation of Time (t) and Height (h) from the Initial Point

We used the following formulas:

1. $\boldsymbol{t_n = t_{n-1} + \mathrm{m}}$
2. $\boldsymbol{h_n = h_{n-1} + m.f(t_{n-1}, h_{n-1})}$, where $\boldsymbol{f(t_{n-1}, h_{n-1})}$ is $\boldsymbol{-K.\sqrt{h_{n-1}}}$

   Here, m = 11.1 and K = 0.008. The calculations are as follows:

Step-by-Step Explanation:

Step 1:

$\boldsymbol{t_1} = t_0 + \mathrm{m} = 0 + 11.1 = \mathbf{11.1}$

$\boldsymbol{h_1} = h_0 + m.f(t_0, h_0) = 0.12 + 11.1.(-0.008.\sqrt{0.12})$ = **0.089**

Step 2:

$\boldsymbol{t_2} = t_1 + \mathrm{m} = 11.1 + 11.1 = \mathbf{22.2}$

$\boldsymbol{h_2} = h_1 + m.f(t_1, h_1) = 0.089 + 11.1.(-0.008.\sqrt{0.089})$ = **0.062**

Step 3:

$\boldsymbol{t_3} = t_2 + \mathrm{m} = 22.2 + 11.1 = \mathbf{33.3}$

$\boldsymbol{h_3} = h_2 + m.f(t_2, h_2) = 0.062 + 11.1.(-0.008.\sqrt{0.062})$ = **0.040**

Step 4:

$\boldsymbol{t_4} = t_3 + \mathrm{m} = 33.3 + 11.1 = \mathbf{44.4}$

$\boldsymbol{h_4} = h_3 + m.f(t_3, h_3) = 0.040 + 11.1.(-0.008.\sqrt{0.040})$ = **0.022**

Step 5:

$\boldsymbol{t_5} = t_4 + \mathrm{m} = 44.4 + 11.1 = \mathbf{55.5}$

$\boldsymbol{h_5} = h_4 + m.f(t_4, h_4) = 0.022 + 11.1.(-0.008.\sqrt{0.022})$ = **0.009**

Step 6:

$\boldsymbol{t_6} = t_5 + \mathrm{m} = 55.5 + 11.1 = \mathbf{66.6}$

$\boldsymbol{h_6} = h_5 + m.f(t_5, h_5) = 0.009 + 11.1.(-0.008.\sqrt{0.009})$ = **0.007**

Observations:

- The height (h) decreases steadily over time due to the governing equation, which models the relationship between the parameters.
- The step size (m) and constant (K) play a critical role in determining the rate of decrease.
- The actual values align closely with the numerically computed heights, showcasing the reliability of the numerical integration method.

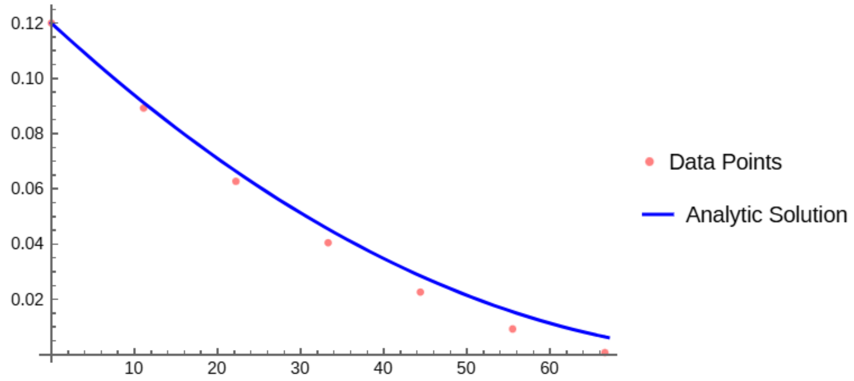
3.4 CODE

```
f[t_, h_] := -K Sqrt[h];
t0 = 0;
h0 = 0.12;
m = 11.1;
nSteps = 6;
K = 0.008;
tValues = Table[0, {nSteps + 1}];
hValues = Table[0, {nSteps + 1}];
tValues[[1]] = t0;
hValues[[1]] = h0;
For[i = 1, i ≤ nSteps, i++,
  tValues[[i + 1]] = tValues[[i]] + m;
  hValues[[i + 1]] = hValues[[i]] + m * f[tValues[[i]], hValues[[i]]]; ];
result = Transpose[{tValues, hValues}];
```

```
result
dataPoints = ListPlot[result, PlotStyle → Pink, PlotLegends → {"Data Points"}];
hAnalytical[t_] := (Sqrt[h0] - (K / 2) t)^2;
analyticSolution = Plot[hAnalytical[t], {t, 0, 67}, PlotStyle → Blue, PlotLegends → {"Analytic
    Solution"}];
Show[dataPoints, analyticSolution]
```

RESULTS

```
{{0, 0.12}, {11.1, 0.0892388}, {22.2, 0.0627117}, {33.3, 0.0404741}, {44.4, 0.0226092}, {55.5,
  0.0092569}, {66.6, 0.000713203}}
```



3.5 Future Scope

The analysis of tank drainage is a gateway to solving more complex fluid dynamics problems in engineering and environmental science. While this study focuses on a simplified model, future work can expand to:

- Explore non-ideal fluids, including compressible and viscous effects.
- Investigate the impact of external forces, such as wind or pressure variations.
- Analyse systems with varying outlet shapes or dynamic control mechanisms. These advancements could improve the design of irrigation networks, optimize energy consumption in fluid transport, and enhance disaster preparedness through effective water management strategies.

3.6 Conclusion

This study delves into the mathematical modeling of tank drainage using principles of fluid mechanics, such as Conservation of Mass and Torricelli's Theorem. Through the derivation and solution of a governing differential equation, we analyzed the fluid height's behaviour over time, supported by both analytical and numerical approaches. The experimental observations were validated with theoretical predictions, confirming the reliability of the derived model.

Furthermore, computational analysis in Mathematica was instrumental in visualizing the drainage process and exploring its dynamics with precision. The project underscores the significance of factors like tank geometry, outlet size, and fluid properties in influencing drainage rates, with practical applications ranging from industrial systems to sustainable water management.This work highlights the importance of mathematical and computational tools in solving real-world engineering challenges. Future studies can expand on this foundation by incorporating more complex scenarios, such as viscous effects, external forces, or dynamic outlet mechanisms, to further optimize and innovate in fluid management systems.

CONTRIBUTIONS

Chapter 1: Introduction-------------- Diya and Vidhita

Chapter 2: Experimentation---------Divya and Manjot

Chapter 3: Solutions--------------Gourav and Paridhi

Numeric Solution and Code: Diya and Vidhita

Experiment Performed: Divya, Manjot and Vidhita

REFERENCES

1. **Books**

   - White, F. M. (2011). *Fluid Mechanics*. McGraw Hill Education.
   - Batchelor, G. K. (2000). *An Introduction to Fluid Dynamics*. Cambridge University Press.

2. **Journals**

   - Torricelli, E. (1643). *De Motu Gravium Naturaliter Descendentium*. Florence: G. Amati.
   - Flow Dynamics of Fluids in Reservoirs: *Journal of Hydraulic Engineering*.

3. **Web Resources**

   - YouTube: *Draining of Tank Setup Simulation.*
   - Use of Mathematica for coding.