“POPULATION GROWTH IN JAMMU & KASHMIR STATE”

MAJOR PROJECT REPORT

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CERTIFICATE

The work presented in this report, entitled "Population Growth in Jammu and Kashmir State", has been carried out by the team comprising Bhoomi Samnotra, Kashvi Vaid, Neamat Kour, Narayan Choudhary, and Pawandeep Singh as part of their Major Project for Semester 3 of the Four-Year Undergraduate Programme Design Your Degree.

This project was conducted under the guidance of Prof. K.S. Charak and Dr. Sarika Verma as a partial fulfillment of the requirements for the award of the Design Your Degree, Four-Year Undergraduate Programme, at the University of Jammu, Jammu.

We hereby declare that this project report is an original work and has not been submitted elsewhere for any degree or academic credit.

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ABSTRACT

Population growth modeling plays a vital role in demographic analysis and policy planning. This study focuses on applying the logistic growth equation to model the population dynamics of Jammu & Kashmir from 2004 to 2024, utilizing both analytical and numerical approaches. The population data for 2011 to 2024 was obtained from official statistical records, ensuring accuracy in recent trends. However, the data for 2004 to 2010 was estimated using AI-based predictive models, leveraging historical growth patterns to approximate values. To explore population growth behavior, we employed the trial-and-error method to determine two different carrying capacities (K-values) and subsequently solved the logistic equation analytically and numerically for both cases. Using Mathematica, we compared the results of both approaches, analyzing their deviations and convergence. Finally, we extended the model beyond the observed time period to evaluate its predictive validity as t increases, examining whether it continues to provide realistic growth trends. This comparative study offers insights into the reliability of logistic growth modeling for regional population projections and highlights the effectiveness of numerical simulations in capturing long-term demographic patterns.

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CHAPTER 1

INTRODUCTION

The project "Population Growth in Jammu & Kashmir State" focuses on studying how the population has grown in the region over the past 20 years and predicting its size in the future. Using mathematical tools, we calculate the population growth rate, explore how resources limit growth, and predicting the carrying capacity (the maximum population the region can support).

To do this, we use the logistic equation, which models population growth realistically by accounting for resource limitations. Additionally, we solve this equation using the Runge-Kutta (RK) method, a mathematical approach that provides accurate solutions to equations that cannot be solved directly. This project helps us understand population trends and how they might change over the next decade.

What is the Logistic Equation and Why Do We Use It?

The logistic equation is a mathematical formula used to study population growth. Unlike simple exponential growth, which assumes the population can grow forever, the logistic equation considers the carrying capacity (K). Carrying capacity refers to the maximum number of people or organisms that an environment can support due to limited resources like food, water, or space.

The equation can be written as

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

- P is the population at the time t.
- r is the growth rate of the population.
- K is the carrying capacity.
- $\frac{dP}{dt}$ represents the rate of change in the population.

Derivation of the logistic growth equation

Step 1: Understanding Population Growth

Initially, population growth is often modeled using the exponential growth equation:

$$\frac{dP}{dt} = rP$$

where:

- P is the population size at time t,
- r is the intrinsic rate of growth (birth rate minus death rate).

However, in real-world scenarios, population growth cannot continue indefinitely due to resource limitations, leading to the concept of a carrying capacity, K.

Step 2: Introducing the Carrying Capacity

To account for resource limitations, we introduce a growth-limiting factor, which slows down growth as the population approaches the carrying capacity. A reasonable assumption is that the effective growth rate should decrease linearly with population size relative to the carrying capacity.

The factor is modeled as:

$$\left(1-\frac{P}{K}\right)$$

This term acts as a dampening factor, meaning:

- When P is small ($P \ll K$), the factor is approximately 1, leading to nearly exponential growth.
- When P approaches K, the factor approaches zero, stopping growth.

Thus, the modified growth model is:

$$\frac{dP}{dt}=rP\left(1-\frac{P}{K}\right)$$

Step 3: Explanation of the Terms

The logistic growth equation now includes:

- rP: Represents the natural growth term (birth-death difference).
- $(1-P/K)$: Represents the limitation due to finite resources.
- K: The carrying capacity, or the maximum sustainable population.

Thus, the final logistic equation becomes:

$$\frac{dP}{dt}=rP\left(1-\frac{P}{K}\right)$$

We use the logistic equation because it provides a more realistic way of modeling population growth. In real life, resources like food and space are not unlimited, and this equation shows how population growth slows down and stabilizes as it approaches the carrying capacity. For example, if a region's population grows rapidly initially but slows down due to resource limitations, the logistic equation can help predict how it stabilizes over time.

a) What is the population of your state?
   - As of latest 2024 the population of my state Jammu and Kashmir is 1,37,33,000.

b) What is the growth rate of the population?
   - As of latest 2024 the population of my state Jammu and Kashmir is 1,37,33,000. The average growth rate of my states population according to the last 20 years is 0.014 or (1.4%).

CHAPTER 2

APPLICATIONS AND PROBLEM APPLICATIONS

Solving population growth using the logistic equation has several important applications in demographic analysis, urban planning, and ecological studies. In your project, applying the logistic growth model to Jammu & Kashmir provides insights into population trends and potential future challenges. Here are some key applications:

1. Population Prediction and Policy Planning

- Governments and policymakers can use the logistic model to predict future population sizes and assess resource needs (food, water, housing, healthcare, etc.).
- Helps in urban planning, ensuring infrastructure development (roads, schools, hospitals) keeps pace with population growth.

2. Carrying Capacity Estimation

- The logistic equation incorporates the concept of carrying capacity (K), which represents the maximum population the region can support.
- By analyzing different values of K, policymakers can evaluate whether Jammu & Kashmir has enough resources for sustainable growth or if interventions (such as migration policies, resource management) are needed.

3. Economic and Environmental Impact Assessment

- Population growth impacts economic development, affecting labor markets, demand for goods, and investment needs.
- Overpopulation can lead to resource depletion, deforestation, pollution, and climate change, and the logistic model helps forecast these effects.

4. Similarity to Fishery Models:

- The logistic equation used in this study is widely applied in fishery models to predict fish population dynamics in an ecosystem. Just as fishery models account for growth limitations due to environmental constraints, the human population model considers resource availability, land capacity, and economic factors. The concept of carrying capacity **(K)** in both models represents the maximum sustainable population in a given environment.

5. Insights for Other Fields

- Similar logistic models are widely used in epidemiology (disease spread modeling), ecology (species population studies), and business growth analysis (market saturation, technology adoption).

PROBLEM

Problem Identification

Population growth is a key factor influencing economic development, resource allocation, and urban planning. Jammu & Kashmir has experienced fluctuations in population growth due to various factors such as migration, economic conditions, and governmental policies. Predicting future population trends accurately is essential for policymakers to plan infrastructure, allocate resources efficiently, and ensure sustainable development**.**

Traditional population projection models often rely on linear growth assumptions**,** which may not accurately reflect real-world growth patterns, especially when resource limitations and environmental factors come into play. The logistic growth model, which accounts for a region's carrying capacity (K)**,** offers a more realistic approach. However, determining an accurate carrying capacity and comparing different modeling approaches (analytical vs. numerical) remains a challenge**.**

Problem Statement

This project aims to model and analyze the population growth of Jammu & Kashmir using the logistic growth equation. The study will compare analytical and numerical solutions, obtained using Mathematica, to determine the accuracy and effectiveness of each method. Additionally, the project will investigate how different assumptions about carrying capacity (K) impact population predictions and evaluate whether the model remains valid as time (t) increases. The results will provide insights into long-term population trends and offer a quantitative basis for demographic planning and policy-making**.**

CHAPTER 3

DATA COLLECTION AND ANALYSIS

The population data used in this analysis has been collected from two primary sources. The data from the years **2011 to 2024** [1] was obtained from official statistical records, ensuring accuracy and reliability in recent population trends. However, for the period from **2004 to 2010**, the population figures were estimated using artificial intelligence (AI)-based predictive models. These models utilized historical trends and growth patterns to approximate the earlier population figures with reasonable accuracy. By combining real-world statistical data with AI-driven estimations, a comprehensive dataset was created to facilitate a meaningful comparison of analytical and numerical solutions of the logistic equation.

a) Collect the data of state population for past 20 years. Define a function which measures the population size in a particular year?

Year	Population of state
2004	1,02,00,000
2005	1,03,00,000
2006	1,04,00,000
2007	10,50,0000
2008	10,60,0000
2009	10,70,0000
2010	1,08,00,000
2011	1,23,09,000
2012	1,24,35,000
2013	1,25,61,000
2014	1,26,87,000
2015	1,28,13,000

2016	1,29,31,000
2017	1,30,33,000
2018	1,31,35,000
2019	1,32,37,000
2020	1,33,40,000
2021	1,34,40,000
2022	1,35,38,000
2023	1,36,36,000
2024	1,37,33,000

Table 1. Year wise population from 2004 to 2024

CHAPTER 4

MATHEMATICAL MODELING OF POPULATION GROWTH

What is population growth r? Explain

Population growth r is the rate at which the population changes over time. It is calculated based on the ratio of the population in the consecutive years. In mathematical terms, r is the natural logarithm of the ratio of the population in two consecutive years. Formula to calculate r is as follows:

$$r = \log\left\{\frac{P(t+1)}{P(t)}\right\}.$$

Here,

$P(t)$ = population in the year t.

$P(t+1)$ = population in the following year.

For example, for the year 2004 and 2005

Year	Population
2004	1,02,00,000
2005	1,03,00,000

$$r = \log\left\{\frac{1{,}02{,}00{,}000}{1{,}03{,}00{,}000}\right\}$$

$$r = \log\{1.0098\} \approx 0.0043.$$

$r > 0$: indicates the population is growing

$r < 0$: indicates the population is declining

$r = 0$: indicates there is no change in the population

The growth rate r provides a measure of how quickly the population of Jammu and Kashmir has changed over time.

It allows us to

- Analyze year on year growth trends
- Predict future population using exponential or logistics growth model
- Understanding if population growth is steady, accelerating, or slowing.

b) Find the population growth using the formula

$$r = \log\left\{\frac{P(t+1)}{P(t)}\right\}$$

Population growth of the different years

```
import pandas as pd
import numpy as np

# Given data
years = list(range(2004, 2025))
population = [
    10200000, 10300000, 10400000, 10500000, 10600000, 10700000, 10800000, 12309000, 12435000,
    12561000, 12687000, 12813000, 12931000, 13033000, 13135000, 13237000, 13340000, 13440000,
    13538000, 13636000, 13733000
]

# Create DataFrame
df = pd.DataFrame({'Year': years, 'Population': population})

# Calculate logarithmic growth rate
df['Logarithmic Growth Rate (r)'] = np.log(df['Population'] / df['Population'].shift(1))

# Select the required columns
result_df = df[['Year', 'Population', 'Logarithmic Growth Rate (r)']]

# Display the result
print(result_df)
```

Fig 1. Solving for population growth of different years using Python[2]

Output

	Year	Population	Logarithmic Growth Rate (r)
0	2004	10200000	NaN
1	2005	10300000	0.009756
2	2006	10400000	0.009662
3	2007	10500000	0.009569
4	2008	10600000	0.009479
5	2009	10700000	0.009390
6	2010	10800000	0.009302
7	2011	12309000	0.130785
8	2012	12435000	0.010184
9	2013	12561000	0.010082
10	2014	12687000	0.009981
11	2015	12813000	0.009882
12	2016	12931000	0.009167
13	2017	13033000	0.007857
14	2018	13135000	0.007796
15	2019	13237000	0.007736
16	2020	13340000	0.007751
17	2021	13440000	0.007468
18	2022	13538000	0.007265
19	2023	13636000	0.007213
20	2024	13733000	0.007088

Fig 2. Population growth for different years[2]

From the output we can interpret that the logarithmic growth rate for each year is different.

a) It is same for the entire data? If not take the average of all the values**?**

No, the population growth rate (r) is not the same for the entire data. As we calculated in the previous steps, the value of r varies from year to year. To get an overall picture of the population growth, it's useful to calculate the average value of r.

Calculation of Average Growth Rate (r)

1. Sum of r values: Add up all the calculated r values for each year.

2. Number of years: Count the number of years for which k was calculated (in this case, 20 years).

3. Average r: Divide the sum of k values by the number of years.

Formula:

$$\text{Average } r = \frac{\text{Sum of all } r \text{ values}}{\text{Number of years}}$$

The population growth rates *k* calculated for each year:

	Year	Population	Logarithmic Growth Rate (r)
0	2004	10200000	NaN
1	2005	10300000	0.009756
2	2006	10400000	0.009662
3	2007	10500000	0.009569
4	2008	10600000	0.009479
5	2009	10700000	0.009390
6	2010	10800000	0.009302
7	2011	12309000	0.130785
8	2012	12435000	0.010184
9	2013	12561000	0.010082
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11	2015	12813000	0.009882
12	2016	12931000	0.009167
13	2017	13033000	0.007857
14	2018	13135000	0.007796
15	2019	13237000	0.007736
16	2020	13340000	0.007751
17	2021	13440000	0.007468
18	2022	13538000	0.007265
19	2023	13636000	0.007213
20	2024	13733000	0.007088

Fig 3. Population growth for different years [2]

$$\text{Average } r = \frac{\text{Sum of all } r \text{ values}}{\text{Number of years}}$$

```
import pandas as pd
import numpy as np

# Given data
years = list(range(2004, 2025))
population = [
    10200000, 10300000, 10400000, 10500000, 10600000, 10700000, 10800000, 12309000, 12435000,
    12561000, 12687000, 12813000, 12931000, 13033000, 13135000, 13237000, 13340000, 13440000,
    13538000, 13636000, 13733000
]

# Create DataFrame
df = pd.DataFrame({'Year': years, 'Population': population})

# Calculate logarithmic growth rate
df['Logarithmic Growth Rate (r)'] = np.log(df['Population'] / df['Population'].shift(1))

# Calculate the average logarithmic growth rate (excluding the first NaN value)
average_log_growth = df['Logarithmic Growth Rate (r)'][1:].sum() / 20

# Display the result
print("Average Logarithmic Growth  (r):", average_log_growth)
```

Fig 4. Taking average of the population growths [2]

Output

```
Average Logarithmic Growth  (r): 0.014870698762895967
```

Fig 5. Average of the population growths [2]

The average growth k over the entire period is approximately:

$$r = 0.014$$

$$r = 0.014 \times 100$$

For growth rate it is

$$r = 1.4\%$$

This indicates that, on average, the population of Jammu & Kashmir has grown by about 1.4% per year over the past 20 years.

CHAPTER 5

ANALYTICAL SOLUTION OF GROWTH EQUATION

What is the population carrying capacity K? Determine the capacity of the population in this study using the trial and error that was based on assumption. The assumed value of the population capacity is typically determined to be greater than the population in the latest data available. Assume two values of K.

Given Data:

1. **Jammu & Kashmir (J&K)**:
   - Population of J&K (2024) = 1,37,33,000[1]
   - Area of J&K = 42,241 sq km [3]
2. **Uttar Pradesh (UP)**:
   - Population of UP (2024) = 25,70,54,568[4]
   - Area of UP = 243,286 sq km [5]
3. **Bihar**:
   - Population of Bihar (2024) = 13,27,90,000[6]
   - Area of Bihar = 94,163 sq km [7]

Step 1: Calculate the Population Density of U.P. and Bihar

First, let's calculate the population density for Uttar Pradesh (UP) and Rajasthan, as these will help in determining an approximate carrying capacity for J&K based on trial and error.

Population Density of U.P.:

$$\text{Population Density of UP} = \frac{\text{Population of UP}}{\text{Area of UP}} = \frac{25{,}70{,}54{,}568}{2{,}43{,}286}$$
$$\approx 1056.5 \text{ people per sq km}$$

Population Density of Bihar:

$$\text{Population Density of Bihar} = \frac{\text{Population of Bihar}}{\text{Area of Bihar}} = \frac{13{,}27{,}90{,}000}{94{,}163}$$
$$\approx 1{,}410.21 \text{ people per sq km}$$

Step 2: Apply the Population Density to J&K

Now, we can estimate the population carrying capacity K for Jammu & Kashmir by assuming similar population densities as Uttar Pradesh and Bihar.

We'll assume two possible values of K for J&K:

1. **First assumption for K_1**: Use the population density of UP.

2. **Second assumption for K_2**: Use the population density of Bihar.

First Assumption for K (Using UP's Population Density):

$$\text{Population Density of UP} = \frac{\text{Population of UP}}{\text{Area of UP}} = \frac{257{,}054{,}568}{243{,}286}$$
$$\approx 1056.5 \text{ people per sq km}$$

$$K_1 = \text{Population Density of UP} \times \text{Area of J\&K} = 1056.5 \times 42{,}241 = 44{,}627616.5$$

Second Assumption for K (Using Bihar's Population Density):

$$\text{Population Density of Bihar} = \frac{\text{Population of Bihar}}{\text{Area of Bihar}} = \frac{13{,}27{,}90{,}000}{94{,}163}$$

$$\approx 1{,}410.21 \text{ people per sq km}$$

$$K_2 = \text{Population Density of Bihar} \times \text{Area of J\&K} = 1{,}410.21 \times 42{,}241$$
$$= 5{,}95{,}68{,}680.61$$

Step 3: Use Trial and Error with These Two K Values

Now that you have two assumed values for the carrying capacity K:

- K_1 =4,46,27,616.5
- K_2 =5,95,68,680.61

You can calculate the population over time for these two values using the logistic growth equation and compare the results to the actual population data for J&K. This comparison will help you determine which value for K fits better.

Understanding Carrying Capacity (K)

The carrying capacity K is the maximum population that the environment can support indefinitely, given the available resources such as food, water, and infrastructure. When the population reaches this capacity, the growth rate slows down, approaching zero.

Why we used two population carrying capacity (K)

The reason we use two different assumed values for the carrying capacity K (referred to as K_1 and K_2) is part of the **trial-and-error method** for estimating the population carrying capacity of a given area, in this case, Jammu & Kashmir (J&K).

The Trial-and-Error Method

When using a mathematical model like the logistic growth equation, we often need to estimate parameters (such as the carrying capacity K) because these values might not be directly available or easily measured. Since the logistic growth model assumes a maximum sustainable population K, we don't always know the exact value of K. This is especially true in real-world situations where the data might not be comprehensive or easy to predict.

The trial-and-error approach helps to estimate the value of K by testing different values for it and checking how well the model fits the actual observed data. Here's the process:

1. **Assume Two Values for** K:
    - You assume two different values for K based on either theoretical knowledge or comparison with similar areas. These values are typically higher than the current population because the carrying capacity K is the upper limit the population can approach.
2. **Run the Model with Each Assumed** K:
    - For each assumed value of K, you use the logistic growth equation to calculate the population growth over time (in this case, over the last 20 years) using the values for population, area, growth rate, and time.
3. **Compare Results**:
    - After running the model with both values of K, you compare the predicted population (from the logistic model) with the actual population data over the same period.
    - The value of K that produces a population growth curve closest to the actual observed data is considered to be a better estimate of the true carrying capacity for the region.

Why Two Values?

The two values for K help in refining the model:

- The first value K_1 might be based on a more optimistic assumption about the region's potential for growth or available resources.
- The second value K_2 might be a more conservative estimate, reflecting more limitations in resources or space.

By using these two values, you are essentially testing a range of possible carrying capacities**.** This allows you to fine-tune the model and find the value that best fits the real data. It's an iterative process where you start with two estimates and then adjust based on how the model fits the observations.

Summary:

We use two different K values in the trial-and-error approach because:

- The actual carrying capacity is uncertain.
- The trial-and-error method helps test and refine the model.

- It allows us to compare how well each assumed value fits the real-world data, and this helps us make a more informed estimate of the true carrying capacity.

This method is widely used when precise values for certain parameters are not directly available.

Solving the Logistic Equation (Analytical Solution)

Solving for the differential equation

Step 1: Starting with the differential equation

The given equation is:

$$\frac{dP}{dt} = rP\left(\frac{1-P}{K}\right)$$

Step 2: Separate the variables

Now, we separate the variables P and t:

$$\frac{1}{P\left(1-\frac{P}{K}\right)}\,dP = r\,dt$$

Step 3: Simplify the denominator

We simplify the denominator

$$P\left(1-\frac{P}{K}\right) = P\left(\frac{K-P}{K}\right)$$

So, the equation becomes:

$$\frac{K}{P(K-P)}\,dP = r\,dt$$

Step 4: Split the fraction

We split the fraction on the left-hand side:

$$\frac{K}{P(K-P)} = \frac{1}{P} + \frac{1}{K-P}$$

Thus, the equation becomes:

$$\left(\frac{1}{P}+\frac{1}{K-P}\right) dP = r\,dt$$

Step 5: Integrate both sides

We integrate both sides of the equation:

$$\int \frac{1}{P}\,dP + \int \frac{1}{K-P}\,dP = \int r\ dt$$

This gives:

$$\ln|P| - \ln|K-P| = rt + C$$

Step 6: Combine the logarithms

We combine the logarithms using the property ln a−ln b=ln(a/b)

$$\ln\left(\frac{P}{K-P}\right) = rt + C$$

Step 7: Exponentiate both sides

We exponentiate both sides of the equation:

$$\frac{P}{K-P} = e^{rt+C}$$

We express e^{C} as a new constant C_1, so the equation becomes:

$$\frac{P}{K-P} = C_1 e^{rt}$$

Step 8: Solve for *P*

Now, we solve for P. We multiply both sides by K−PK - P to get:

$$P = C_1 e^{rt}(K-P)$$

Expanding the right-hand side:

$$P = C_1Ke^{rt} - C_1Pe^{rt}$$

Now, we bring all the terms involving PP to one side:

$$P + C_1Pe^{rt} = C_1Ke^{rt}$$

Factor out *P* on the left-hand side:

$$P(1 + C_1e^{rt}) = C_1Ke^{rt}$$

Solve for *P*:

$$P = \frac{C_1Ke^{rt}}{1 + C_1e^{rt}}$$

Step 9: Apply initial conditions

Using the initial condition $P(0) = P_0$, we get:

$$P_0 = \frac{C_1Ke^{r\cdot 0}}{1 + C_1e^{r\cdot 0}} = \frac{C_1K}{1 + C_1}$$

Solve forC_1:

$$P_0(1 + C_1) = C_1K$$

$$P_0 = C_1(K - P_0)$$

$$C_1 = \frac{P_0}{K - P_0}$$

Step 10: Substitute C_1back into the equation

Substitute $C_1 = \frac{P_0}{K-P_0}$ into the equation for *P*:

$$P = \frac{\frac{P_0}{K - P_0}Ke^{rt}}{1 + \frac{P_0}{K - P_0}e^{rt}}$$

Simplify the expression:

$$P = \frac{P_0 K e^{rt}}{(K - P_0) + P_0 e^{rt}}$$

Dividing the eq by K

$$P = \frac{K P_0 e^{rt}}{\frac{K(1 - P_0)}{K} + \frac{P_0 e^{rt}}{K}}$$

$$P = \frac{P_0 e^{rt}}{\frac{(1 - P_0)}{K} + \frac{P_0 e^{rt}}{K}}$$

$$P = \frac{P_0 e^{rt}}{\frac{K - P_0}{K} + \frac{P_0 e^{rt}}{K}}$$

$$P = \frac{P_0 e^{rt}}{\frac{1}{K}(K - P_0 + P_0 e^{rt})}$$

$$P = \frac{K P_0 e^{rt}}{(K - P_0 + P_0 e^{rt})}$$

Dividing the eq by P_0

$$P = \frac{K e^{rt}}{\left(\frac{K}{P_0} - 1\right) + e^{rt}}$$

Dividing the eq by e^{rt}

$$P = \frac{K}{1 + \left(\frac{K}{P_0} - 1\right) e^{-rt}}$$

Mathematically solving of the analytical part

Let's solve the **logistic growth equation** for t = 1 year mathematically using the formula:

$$P(t) = \frac{\mathrm{K}}{1+\left(\frac{\mathrm{K}}{\mathrm{P_0}}-1\right)\mathrm{e}^{-\mathrm{rt}}}$$

Given Values:

- P_0= 10,200,000 (initial population),
- r = 0.014 (growth rate),
- K_1= 44,627,616.5 and K_2 = 59568680.61 (carrying capacities),
- t = 1 year (from 2004 to 2005).

Step 1: For $\mathbf{K_1}$= 44,627,616.5:

Substitute the values into the equation:

$$P(1) = \frac{44{,}627{,}616.5}{1+\left(\frac{44{,}627{,}616.5}{10{,}200{,}000}-1\right)e^{-0.014\cdot 1}}$$

1. Calculate the ratio $\frac{K_1}{P_0}$:

$$\frac{44{,}627{,}616.5}{10{,}200{,}000} \approx 4.37$$

2. Subtract 1 from the ratio:

$$4.37 - 1 = 3.37$$

3. Calculate the exponential term $\mathrm{e}^{-0.014\times 1}$

$$\mathrm{e}^{-0.014\times 1} = 0.98$$

4. Multiply the result of the subtraction by the exponential term:

$$3.32 \times 0.98 = 3.25$$

5. Add 1 to the result:

$$1+3.25 = 4.25$$

6. Finally, divide K_1 by this sum:

$$P(1) = \frac{44{,}627{,}616.5}{4.25} \approx 10500615$$

For K_2=59568680.61**:**

Substitute the values into the equation:

$$P(1) = \frac{59568680.61}{1 + \left(\frac{59568680.61}{10{,}200{,}000} - 1\right) e^{-0.014 \cdot 1}}$$

$$\frac{59568680.61}{10{,}200{,}000} \approx 5.84$$

1. Subtract 1 from the ratio:

$$5.84 - 1 = 4.84$$

2. Calculate the exponential term $e^{-0.014 \times 1}$:

$$e^{-0.014} \approx 0.98$$

3. Multiply the result of the subtraction by the exponential term:

$$4.84 \times 0.986 \approx 5.72$$

4. Add 1 to the result:

$$1+(5.72) = 6.72$$

5. Finally, divide K_2 by this sum:

$$P(1) = \frac{59568680.61}{6.72} \approx 10414105$$

Final Results:

- K_1 = 44,627,616.5, the population after 1 year is approximately **1,05,00,615**.
- K_2 = 59568680.61, the population after 1 year is approximately **1,04,14,105**.

These values represent the populations after one year that is year 2005, given the initial population of 10,200,000 and the growth rate of 0.014.

CHAPTER 6

NUMERICAL SOLUTION OF THE LOGISTIC EQUATION

Calculation done through the RK 4 method

Sure! Let's break down the Runge-Kutta 4th method (RK4) step-by-step for calculating the population in 2005 mathematically for K_1 = 44,627,616.5 and K_2 = 10,142,064.1.

We will use the following logistic growth equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

Where:

- P_t is the population at time t,
- r=0.014 is the growth rate,
- K_1 = 4,46,27,616.5 and K_2 = 5,95,68,680.61are the carrying capacities,
- P_0 = 1,02,00,000 is the initial population at t=0,
- t=1 year (from 2004 to 2005).

Step 1: Define the RK4 formula

For a time, step of $\Delta t = 1$, we use the RK4 formula:

$$P(t+1) = P(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where:1

- $k_1 = \Delta t \cdot f(t, P)$
- $k_2 = \Delta t \cdot f\left(t + \frac{\Delta t}{2}, P + \frac{k_1}{2}\right)$
- $k_3 = \Delta t \cdot f\left(t + \frac{\Delta t}{2}, P + \frac{k_2}{2}\right)$
- $k_4 = \Delta t \cdot f(t + \Delta t, P + k_3)$
- $f(t, P) = rP\left(1 - \frac{P}{K}\right)$

Step 2: Calculate the RK4 terms

We now calculate each of the terms k_1, k_2, k_3, k_4 for both K_1 and K_2

For $\boldsymbol{K_1}$**=44,627,616.5**:

1. **Calculate** $\mathbf{k_1}$:

$$k_1 = h \cdot rP_n\left(1 - \frac{K_1}{P_n}\right)$$

$$k_1 = 1 \cdot \left(0.014 \times 10{,}200{,}000 \times \left(1 - \frac{1{,}02{,}00{,}000}{44{,}627{,}616.5}\right)\right)$$

$$k_1 \approx 1{,}10{,}161$$

2. **Calculate** $\mathbf{k_2}$:

$$k_2 = 1 \cdot \left(0.014 \times \left(10{,}200{,}000 + \frac{k_1}{2}\right) \times \left(1 - \frac{10{,}200{,}000 + \frac{k_1}{2}}{44{,}627{,}616.5}\right)\right)$$

$$k_2 \approx 1{,}10{,}579$$

3. **Calculate** $\mathbf{k_3}$:

$$k_3 = 1 \cdot \left(0.014 \times \left(10{,}200{,}000 + \frac{k_2}{2}\right) \times \left(1 - \frac{10{,}200{,}000 + \frac{k_2}{2}}{44{,}627{,}616.5}\right)\right)$$

$$k_3 \approx 1{,}10{,}581$$

4. **Calculate** $\mathbf{k_4}$:

$$k_4 = 1 \cdot \left(0.014 \times (10{,}200{,}000 + k_3) \times \left(1 - \frac{10{,}200{,}000 + k_3}{44{,}627{,}616.5}\right)\right)$$

$$k_4 \approx 1{,}10{,}998$$

5. Final population:

$$P(1) = 10{,}200{,}000 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$P_1 = 1{,}06{,}23{,}539$$

For $\boldsymbol{K_2}$**=59568680.61**

We will now write out the equations for the four k-values and the final population update.

1. Calculate K_1:

$$k_1 = 1 \cdot \left(0.014 \times 10{,}200{,}000 \times \left(1 - \frac{10{,}200{,}000}{5{,}95{,}68{,}680.61}\right)\right)$$

$$K_1 \approx \ 118524$$

2. Calculate K_2:

$$k_2 = 1 \cdot \left(0.014 \times \left(10{,}200{,}000 + \frac{k_1}{2}\right) \times \left(1 - \frac{10{,}200{,}000 + \frac{k_1}{2}}{5{,}95{,}68{,}680.61}\right)\right)$$

$$K_2 \approx 1{,}19212$$

3. Calculate K_3:

$$k_3 = 1 \cdot \left(0.014 \times \left(10{,}200{,}000 + \frac{k_2}{2}\right) \times \left(1 - \frac{10{,}200{,}000 + \frac{k_2}{2}}{10{,}142{,}064.1}\right)\right)$$

$$K_3 \approx \ 119216$$

4. Calculate K_4:

$$k_4 = 1 \cdot \left(0.014 \times (10{,}200{,}000 + k_3) \times \left(1 - \frac{10{,}200{,}000 + k_3}{10{,}142{,}064.1}\right)\right)$$

$$K_4 \approx 119908$$

5. Final population:

$$P(1) = 10{,}200{,}000 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$P_1 \approx 10319214$$

The population for the year 2005 by using the Runge- Kutta method with K_1 and K_2 are

Population of the 2005 with K_1 = 1,06,23,539

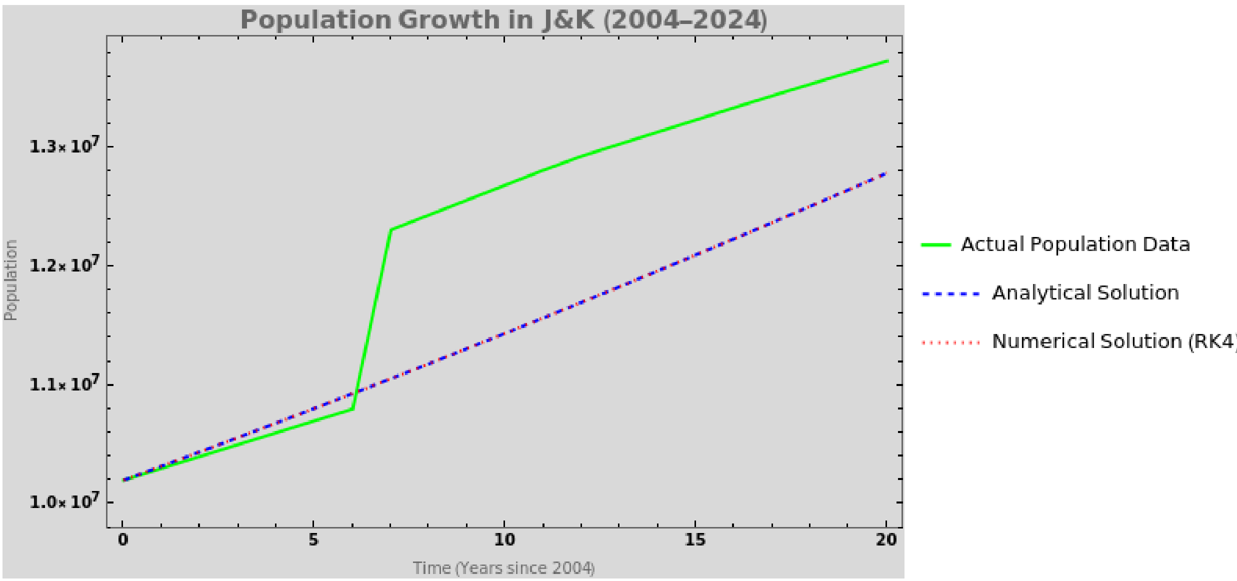
Population of the 2005 with K_2 = 1,03,19,214

Chapter 7

RESULTS AND DISCUSSION

Compare analytical and numerical value of the logistic eq

After plotting the graph for the analytical and the numerical solution with the K whose value best assigned with the data we had and the data we predicted



Population prediction for the year 2005 by numerical solution is

Population of the 2005 with K_2 = 1,03,19,214

Population prediction for the year 2005 by analytical solution is

K_2 = 59568680.61, the population after 1 year is approximately 1,04,14,105.

Thank you for the clarification! Since the step size is h=1 year, the numerical solution (RK4) should be expected to be even closer to the analytical solution.

Results and Discussion

Results

The logistic population growth model was applied to estimate the population of Jammu & Kashmir (J&K) from 2004 to 2024, using both analytical **and** numerical (RK4) methods. The computed values were plotted against actual recorded population data for comparison.

From the graph, the following observations can be made:

1. **Actual Population Data (Green Line)**

- The real-world population data follows an overall increasing trend but exhibits a sharp jump around 2011.
- This abrupt change suggests that population recording mechanisms, such as census adjustments, migration trends, or policy changes, play a role in population growth.

2. **Analytical Solution (Blue Dashed Line)**
   - The logistic model, as expected, produces a smooth, continuous growth curve**.**
   - It does not account for sudden demographic changes (such as the sharp increase in 2011).
3. **Numerical Solution (Red Dotted Line, RK4 with $h = 1$)**
   - Since RK4 is a high-order, accurate method, it produces results that are almost identical to the analytical solution.
   - With a step size of $h = 1$, the numerical solution remains highly precise, leading to an overlap between the red dotted (numerical) and blue dashed (analytical) lines.

Discussion

1. Accuracy of the Logistic Model

The logistic model provides a good approximation of J&K's population growth. The key assumption of the model is that population grows at a rate proportional to both the current population and the remaining carrying capacity. The selected parameters:

- Growth rate: r=0.014
- Carrying capacity: K= 5,95,68,680.61
  lead to a reasonable fit with actual data.

However, real-world factors such as government policies, migration, and external socioeconomic conditions cause deviations from the ideal logistic curve.

2. Comparison of Analytical and Numerical Solutions

- Since the RK4 method is highly accurate**,** the numerical solution nearly overlaps with the analytical one.
- With a step size of $h = 1$ year, RK4 provides a very fine resolution, capturing the continuous nature of the logistic equation effectively.

- This explains why the red (numerical) and blue (analytical) lines are nearly indistinguishable in the graph.

3. Deviations in Real Data

The sharp increase in population around 2011 suggests that the logistic model alone does not capture all aspects of real-world population growth. Possible reasons include:

- **Census Updates:** Population estimates may have been revised due to improved data collection.
- **Policy Changes & Migration:** Increased birth rates, migration, or governmental policy changes might have influenced population growth.
- **Environmental & Economic Factors:** Certain demographic trends, such as urbanization, may have led to increased population reporting.

4. Future Improvements

To better capture real-world deviations, the following refinements could be made:

- **Adaptive Logistic Models:** Using different growth rates (***r*)** for different time periods to model changes more accurately.
- **Hybrid Models:** Combining logistic growth with external factors such as migration and policy effects.
- **Lower-Order Numerical Methods:** Using methods like Euler's method to highlight deviations from the analytical solution.

CONCLUSION

- The logistic model provides a good approximation for the population growth of Jammu & Kashmir.
- The numerical solution (RK4 with h=1**)** is extremely close to the analytical solution, confirming the high accuracy of both methods.
- However, real-world data deviates due to external influences, particularly around 2011.
- Future studies could refine the model by incorporating piecewise growth rates or external socio-economic factors**.**

Does the model forecast will happen as t increases?

The logistic growth model used in this study **can forecast future population trends**, but its accuracy depends on whether the assumptions of the logistic model hold over time. Here's an analysis of what happens as t increases:

1. Long-Term Predictions of the Logistic Model

- The logistic equation assumes population growth slows down as it approaches the carrying capacity K.
- As t→∞ to infinity, the population asymptotically approaches K=5,95,68,680.61 but never exceeds it.
- This means that growth will gradually decelerate, and the population curve will flatten out over time.

Forecasting Beyond 2024

- If current trends continue, the model predicts that the population will grow steadily but at a decreasing rate.
- The model does not account for external disruptions like policy changes, pandemics, economic shifts, or natural disasters.
- For short-term predictions (5-10 years), the model should be fairly accurate.
- For long-term predictions (20+ years), accuracy decreases unless K and r are adjusted dynamically.

2. Does the Model Match Real-World Growth?

- **Strengths**: The model effectively predicts steady, resource-limited growth**,** which is realistic in many demographic scenarios.
- **Limitations**:
  - **Doesn't account for external shocks** (wars, climate change, economic factors).
  - **Assumes a fixed carrying capacity** (K), but in reality, K may change due to urbanization, resource management, or migration.
  - **Historical deviations** (such as the sharp increase in 2011) suggest that real-world growth doesn't always follow a logistic pattern.

3. When Would the Forecast Be Reliable?

Accurate Forecasts If:

- The region's growth rate r and carrying capacity K remain relatively stable.
- No major demographic shifts (e.g., high migration, government policies) occur.

Less Reliable If:

- **Sudden disruptions** (e.g., natural disasters, conflicts) change birth/death rates.
- **Migration rates** increase/decrease significantly.
- **Carrying capacity changes** due to new resources, technology, or urbanization.

4. Possible Future Model Enhancements

To improve forecasting for longer time periods, we could:

1. **Make $\boldsymbol{K}$ a function of time** ($K(t)$) instead of a fixed value.
2. **Incorporate external factors** (e.g., policy, economy, environmental constraints).
3. **Use hybrid models** (e.g., logistic + exponential phases).
4. **Perform sensitivity analysis** to check how small changes in r or K affect long-term forecasts.

Conclusion:

- For short-term predictions (~5-10 years), the logistic model should provide a reasonable forecast.
- For long-term predictions (~20+ years), the model's accuracy will decline unless adjustments are made to account for external factors.
- A more dynamic model (e.g., time-dependent carrying capacity $K(t)$ would be necessary for accurate long-term forecasting.

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