There are **FOUR** questions. **Answer all of them.** Show full simulation/tabulations wherever necessary. Figures in the right-hand margin indicate full marks. **Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.**

1. (a) Derive the best-case and the worst-case running-time equations for the [2+2] following function sum1 and express those in Big-Oh (O) notation. Also provide the best-case and the worst-case examples of the arrays A and B with $n = 4$ and $m = 5$ for the function sum1.

```
function sum1(A, B):
   1.        m = A.length
   2.        n = B.length
   3.        s = 0; i = 1;
   4.        while i<=m do
   5.              s = s + A[i]
   6.              i = i+1
   7.        end
   8.        for j=1 to n do
   9.              if B[j]<0 then
  10.                    return s
  11.              s = s + B[j]
  12.        end
  13.        return s
```

(b) Derive the exact-cost equation for the running-time of the following function and **prove that it is in** $O(n)$. [2+2]

GREEDY-ACTIVITY-SELECTOR (s, f)
```
1   n ← length[s]
2   A ← {a_1}
3   i ← 1
4   for m ← 2 to n
5       do if s_m ≥ f_i
6          then A ← A ∪ {a_m}
7               i ← m
8   return A
```

2. (a) Given an array $A = \{-2, 3, -1, 2, -4, 4\}$, find the maximum-sum continuous [3] subarray using divide-and-conquer approach. You must show the recursion tree and clearly mention left, right and crossing sum for each tree node.

(b) Given an array of integers $A = \{1, 3, -5, 2, -3, -2\}$, find the Maximum and [2] Minimum using divide-and-conquer. Show the necessary steps to support your answer.

(c) Explain with an example how merge sort is performed using divide-and-conquer. [2]

3. (a) You are given the following table containing symbols and their frequencies: [3+1]

Symbol	A	B	C	D	+
Frequency	40	10	20	15	15
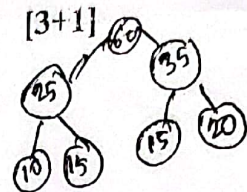
I. Build the Huffman code tree and find the codeword for each character.
II. Decode 100010111001010 using the Huffman code that you generated.

(b) You are given the arrival and the departure times of eight trains for a railway platform, and each one is in the format: [arrival time, departure time). Only one train can use the platform at a time. Suppose that you have got the following train-use requests for the next day. [3]

{ [8, 12), [6, 9), [11, 14), [2, 7), [1, 7), [12, 20), [7, 12) , [13, 19) }

Find the maximum number of trains that can use the platform without any collision by using *earliest departure time*.

4. (a) What is Optimal Substructure? Show at least 2 valid differences between the Greedy approach and the Dynamic Programming approach. [1]

(b) Suppose, *Crimson Cup Coffee Shop* charges **50 BDT** (Bangladesh Taka) for each cup of small cream latte with **an additional vat of 3% for any purchase.** You bought **2 cups of small cream latte** and gave the cashier **110 taka**. The cashier has got a huge supply of **1 taka, 2 taka, and 5 taka** coins available in the cashbox. You don't want to carry many coins, so you asked her to return the change using a **minimum number of coins.** [3]

Determine how many coins she should return in this scenario by applying the Dynamic Programming Approach.

(c) Two infamous thieves, Denver and Nairobi, planned to rob the famous Louvre Museum. Before the scene, they both agreed on the fact on the fact that **none of them will break any item** as all the items in the Louvre are too precious, and taking a fraction of any item won't sell in the black market. **If it fits in the bag as a whole, they will take it, otherwise, leave it as it is.** [2+2]
Both of them arrived at the Louvre with **an empty knapsack weighing a total of 5 kg.** Despite the fact that both thieves are experts in their fields, they take slightly different approaches.
Denver believes he will use a **Dynamic Programming Approach** to rob the items in the most efficient manner possible. **Nairobi**, on the other hand, believes that if she chooses the **Greedy Approach,** she will make the most money.
The objects in the Louvre Museum are listed below.

Objects	Jewelry	Sculpture	Paintings
Profit	7	9	6
Weight	3	5	4

I. What is the maximum profit Denver can make using his strategy?
II. Does Nairobi's belief remain valid after the robbery? Prove it.

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