

Homework 24 - MATH 791

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Problem 1:

Write out addition and multiplication tables for the field $\mathbb{Z}_2(\alpha)$ is problem 5 of the previous assignment.

Solution:

Admitted

Problem 2:

Now let $p(x)$ and α be as in problems 1 and 2 from Homework 23. Determine whether or not $\mathbb{Z}_3(\alpha)$ the splitting field for $p(x)$ over \mathbb{Z}_3 .

Solution:

Admitted

Problem 3:

Let $p, q \in \mathbb{Z}$ be distinct prime numbers. Show that $[\mathbb{Q}(\sqrt{p}, \sqrt{q}) : \mathbb{Q}] = 4$.

Solution:

Admitted

Problem 4:

Let $n \geq 2$ and set $\epsilon := e^{\frac{2\pi i}{n}}$.

- (i) Show that $\mathbb{Q}(\epsilon)$ is the splitting field for $x^n - 1$ over \mathbb{Q} .
- (ii) If $n = p$ is prime, find $[\mathbb{Q}(\epsilon) : \mathbb{Q}]$. Hint: first make an educated guess for the minimal polynomials of ϵ over \mathbb{Q} , then show that the function $\phi : \mathbb{Q}[x] \rightarrow \mathbb{Q}[x]$ given by $\phi(f(x)) = f(x + 1)$ is an automorphism, and then apply/consider Eisenstein's criterion.

Solution:

Admitted