Homework 25 - MATH 791

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Problem 1:
Consider $\alpha := 1 + \sqrt{2} + \sqrt{3} + \sqrt{6} \in \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Find a polynomial $p(x) \in \mathbb{Q}[x]$ such that $p(\alpha) = 0$. Determine if the polynomial you found is the minimal polynomial for α over $\mathbb{Q}$.
Solution:

Admitted

Problem 2:
Prove that for any field L containing $\mathbb{Z}_p$ and $a, b \in L$, then $(a+b)^p = a^p + b^p$.
Solution:

Admitted

Problem 3:
Let x, y be indeterminates over the field $\mathbb{Z}_2$. Set $F := \mathbb{Z}_2(x^2, y^2)$ and $K := \mathbb{Z}_2(x, y)$. Set $E := \mathbb{Z}(x, y^2)$. Prove that $[E : F] = 2$ and $[K : E] = 2$.
Conclude that $[K : F] = 4$.
Solution:

Admitted

Problem 4:
In the notation of problem 3, prove that $\alpha^2 \in F$, for all $\alpha \in K$.
Solution:

Admitted

Problem 5:
Use the previous two problems to show that for $F \subseteq K$ as in problem 3, there does **not** exist $\alpha \in K$ such that $K = F(\alpha)$. Conclude that there are infinitely many intermediate field $F \subsetneq E \subsetneq K$.
Solution:

Admitted