Homework 23 - MATH 791

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Problem 1:
Show that $p(x) = x^3 + x^2 + 2x + 1$ is irreducible over $\mathbb{Z}_3$.
Solution:

Admitted

Problem 2:
For $p(x)$ as in the previous problem, from class we know that there is a field K containing $\mathbb{Z}_3$ and $\alpha \in K$ such that $p(\alpha) = 0$.

(i) How many elements are in the field $\mathbb{Z}_3(\alpha)$?

(ii) In the field $\mathbb{Z}_3(\alpha)$ calculate $A \cdot B$ and A^{-1} for $A := 1 + 2\alpha + \alpha^2$ and $B := 2 + \alpha + 2\alpha^2$

Solution:

Admitted

Problem 3:
Give an example of a field with 125 elements.
Solution:

Admitted

Problem 4:
Fix a prime p. Assume that for all $n \geq 1$, there exists an irreducible polynomial in $\mathbb{Z}_p[x]$ having degree n. Show that for all primes p and $n \geq 1$, there exists a field with p^n elements.
Solution:

Admitted

Problem 5:
Let $\alpha \in K \supseteq \mathbb{Z}_2$ be a root of $x^2 + x + 1$. Show that $\mathbb{Z}_2(\alpha)$ is the splitting field for $x^2 + x + 1$.
Solution:

Admitted