Homework 26 - MATH 791

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Problem 1:
Construct a field K with 16 elements, and identify explicitly a subfield with 4 elements. Hint: start by finding an irreducible polynomial of degree four over $\mathbb{Z}_2$.
Solution:

Admitted

Problem 2:
For K as in problem 1, is there a subfield of K with 8 elements?
Solution:

Admitted

Problem 3:
Let K be a field with p^m elements, with p prime and $m \geq 1$. Let $\sigma : K \to K$ be given by $\sigma(\alpha) = \alpha^p$, for all $\alpha \in K$. Show that σ is an automorphism of K fixing $\mathbb{Z}_p$. We call σ the *Frobenius automorphism* of K.
Solution:

Admitted

Problem 4:
For K and σ as in problem 3, what is $\sigma^j(\alpha)$, for $j \geq 1$ and $\alpha \in K$? What is σ^m?
Solution:

Admitted

Problem 5:
Let K and σ be as in problem 3. Supposed $n \mid m$. Show that $F := \{\alpha \in K \mid \sigma^n(\alpha) = \alpha\}$ is the unique subfield of K containing p^n elements.
Solution:

Admitted