Philosophy 160 Homework
Test the following arguments for validity using trees for predicate logic.

1. ~∃xFx, ~Fc → ∀y~Gy ├ ∀y∀z~(Fy v Gz)

√~∃xFx
√~Fc → ∀y~Gy
√~∀y∀z~(Fy v Gz)
√∃y~∀z~(Fy v Gz)
√~∀z~(Fa v Gz)
√∃z~~(Fa v Gz)
√~~(Fa v Gb)
√(Fa v Gb)
∀x~Fx

Fa	Gb
~Fa	~Fa
x	

Under Gb / ~Fa:

~~Fc	∀y~Gy
~Fc	~Fc
x	~Gb
	x

Valid

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2. ∀x(Fx → (Gx v Tx)), ~Ga v ~Ta ├ ~Fa

∀x(Fx → (Gx v Tx))
√~Ga v ~Ta
Fa
√Fa → (Ga v Ta)

~Fa   √ (Ga v Ta)
X

Ga   Ta

~Ga   ~Ta   ~Ga   ~Ta
x   x

Invalid

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3. ~∃xFx, ~∃xGx ├ ~∃x(Fx v Gx)

√~∃xFx
√~∃xGx
√∃x(Fx v Gx)
√∀x~Fx
∀x~Gx

Fa	Ga
~Fa	~Fa
x	~Ga
	x

Valid

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4. $\forall x\forall y(Fxy \leftrightarrow Fyx) \vdash \sim\exists x\exists y(\sim Fxy \;\&\; \sim Fyx)$

$\forall x\forall y(Fxy \leftrightarrow Fyx)$
$\surd\exists x\exists y(\sim Fxy \;\&\; \sim Fyx)$
$\surd\exists y(\sim Fay \;\&\; \sim Fya)$
$\surd\sim Fab \;\&\; \sim Fba$
$\sim Fab$
$\sim Fba$
$\forall y(Fay \leftrightarrow Fya)$
$\forall y(Fby \leftrightarrow Fyb)$
$\surd Fab \leftrightarrow Fba$
$\surd Faa \leftrightarrow Faa$
$\surd Fba \leftrightarrow Fab$
$\surd Fbb \leftrightarrow Fbb$

Note the necessity of these two instantiations as both x and y must be instantiated to every name in the branch. It is fairly obvious that they have no power to produce contradictions and you can avoid this in a valid argument. But invalidity will not have been totally demonstrated in their absence.

Fab	~Fab
Fba	~Fba
x	

Fba	~Fba
Fab	~Fab
x	

Faa	~Faa
Faa	~Faa

Fbb	~Fbb	Fbb	~Fbb
Fbb	~Fbb	Fbb	~Fbb

Invalid

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5. $\forall x {\sim}\exists y {\sim}(Fxy \rightarrow Fyx),\ \exists x Fxa \vdash {\sim}\forall x {\sim}Fax$

$\forall x {\sim}\exists y {\sim}(Fxy \rightarrow Fyx)$

$\surd \exists x Fxa$

$\forall x {\sim}Fax$

Fba

${\sim}\exists y {\sim}(Fay \rightarrow Fya)$

$\surd {\sim}\exists y {\sim}(Fby \rightarrow Fyb)$

$\forall y {\sim}{\sim}(Fby \rightarrow Fyb)$

$\surd {\sim}{\sim}(Fba \rightarrow Fab)$

${\sim}{\sim}(Fbb \rightarrow Fbb)$

$Fba \rightarrow Fab$

${\sim}Fba$	Fab
x	${\sim}Fab$
	x

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6. $\vdash \sim\forall x\exists yFxy \rightarrow (\exists x\sim Fxa \;\&\; \exists x\sim Fxb)$

$\sim(\sim\forall x\exists yFxy \rightarrow (\exists x\sim Fxa \;\&\; \exists x\sim Fxb))$

$\surd\sim\forall x\exists yFxy$

$\surd\sim(\exists x\sim Fxa \;\&\; \exists x\sim Fxb)$

$\surd\exists x\sim\exists yFxy$

$\surd\sim\exists yFcy$

$\forall y\sim Fcy$

$\surd\sim\exists x\sim Fxa$	$\surd\sim\exists x\sim Fxb$
$\forall x\sim\sim Fxa$	$\forall x\sim\sim Fxb$
$\sim\sim Fca$	$\sim\sim Fcb$
$\sim Fca$	$\sim Fca$
x	$\sim Fcb$
	x

Valid

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7. ∀x∃y(Cxy → Cyx) ├ Cab → Cba

∀x∃y(Cxy → Cyx)
√~(Cab → Cba)
Cab
~Cba
√∃y(Cay → Cya)
√∃y(Cby → Cyb)
Cac → Cca
Cbd → Cdb
√∃y(Ccy → Cyc)
√∃y(Cdy → Cyd)
(Cce → Cec)
(Cdf → Cfd)
.
.

This pattern will repeat forever. The argument is invalid, but the machinery itself will not demonstrate this. The question then becomes how to prove that this is invalid. The answer is that you need to do a semantic proof by providing a model that contains a counterexample, i.e., an instance in which the premises are true and the conclusion false.

Proof: Interpret C as the relation of choosing. Then the premise asserts that for every named object *k* in the model, there exists an object *m* (which may or may not be identical to *k*) such that if k chooses m, then m chooses k. Suppose a model containing only objects a,b and d in which the following is true:

a chooses b
a chooses d
d chooses a
Nothing else chooses anything.

This models makes the premise true. It is true for every object that chooses nothing, since each of these makes the antecedent of the premise false. It is true for every object *k* that chooses some *m*, since a chooses d and d chooses a. But the conclusion is false because a chooses b and b doesn not choose a. QED.

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8. $\sim\exists x\exists yGxy \vdash \exists xGxx \rightarrow \sim\forall yFy$

$\surd\sim\exists x\exists yGxy$

$\surd\sim(\exists xGxx \rightarrow \sim\forall yFy)$

$\surd\exists xGxx$

$\surd\sim\sim\forall yFy$

$\forall yFy$

Gaa

$\forall x\sim\exists yGxy$

$\surd\sim\exists yGay$

$\forall y\sim Gay$

$\sim Gaa$

x

Valid

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9. ${\sim}\exists x(Fx \,\&\, Gx) \vdash \forall x(Fx \rightarrow {\sim}Gx)$

$$\surd{\sim}\exists x(Fx \,\&\, Gx)$$
$$\surd{\sim}\forall x(Fx \rightarrow {\sim}Gx)$$
$$\surd\forall x{\sim}(Fx \,\&\, Gx)$$
$$\surd\exists x{\sim}(Fx \rightarrow {\sim}Gx)$$
$$\surd{\sim}(Fa \rightarrow {\sim}Ga)$$
$$Fa$$
$$\sim\sim Ga$$
$$\surd{\sim}(Fa \,\&\, Ga)$$

${\sim}Fa$	${\sim}Ga$
x	x

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10. ∃x(Fx → Fb) ├ Fc → Fb

√∃x(Fx → Fb)
√~(Fc → Fb)
Fc
~Fb
Fa → Fb

~Fa | Fb
 | X

Invalid

11. Fa & ~Fb ├ ~a=b

√Fa & ~Fb
a=b
Fa
~Fb
~Fa
x

~Fa results from substituting a for b in ~Fb according to the rule for identity.

12. ~∃x∃y x=y ├ P

√~∃x∃y x=y
~P
∀x~∃y x=y
~∃y a=y
∀y~a=y
~a=a
x

Here we insantiate the universal to a and close in accord with the rule for negated identity.

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Test the following arguments for validity using trees for predicate logic.

13. ∀x(Fx → Gx), ∃x∃y((Fx & Fy) & ~x=y) ├ ∃x∃y((Gx & Gy) & ~x=y)

∀x(Fx → Gx)
√∃x∃y((Fx & Fy) & ~x=y)
√~∃x∃y((Gx & Gy) & ~x=y)
√∃y((Fa & Fy) & ~a=y)
√(Fa & Fb) & ~a=b)
√(Fa & Fb)
~a=b
Fa
Fb

~Fa　　　　　　　　Ga
x

∀x~∃y((Gx & Gy) & ~x=y)
√~∃y((Ga & Gy) & ~a=y)
√~∃y((Gb & Gy) & ~b=y)
∀y~((Ga & Gy) & ~a=y)
∀y~((Gb & Gy) & ~b=y)
√~((Ga & Ga) & ~a=a)
√~((Ga & Gb) & ~a=b)
~((Gb & Gb) & ~b=b)
~((Gb & Ga) & ~b=a)

These four formulas result from instantating the two universals above to both a and b as required.

√~(Ga & Ga)　　　　　　　　√ ~~a=a
　　　　　　　　　　　　　　a=a

~Ga　　~Ga

√~(Ga & Gb)　√ ~~a=b　√~(Ga & Gb)　√~~a=b　√ ~(Ga & Gb)　√~~a=b

~Ga　~Gb　a=b　~Ga　~Gb　a=b　~Ga　~Gb　a=b

To finish this tree we must decompose the last two unchecked negated conjunctions above. However, no branches have closed to this point and it is clear from the content of these formulas that they have no capacity to generate contradictions. So we will just stop here and declare it obviously invalid.