Chapter 15

Active Filter Circuits

Text: *Electric Circuits* by J. Nilsson and S. Riedel

Prentice Hall

EEE 117 Network Analysis

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Preview

This chapter will introduce the use of OpAmps as a frequency selective circuit which can also have *Gain.*

In the passive circuits of Chapter 14, the maximum magnitude of a transfer function was 1.

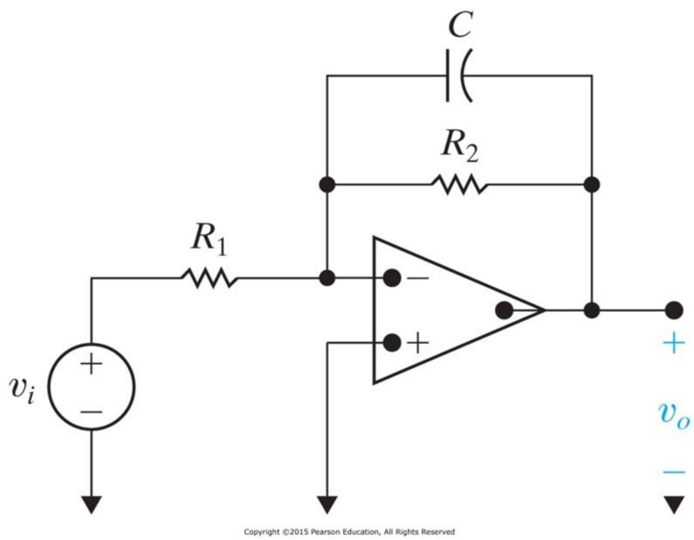
Active circuits can be designed to both amplify and attenuate an input signal by some significant amount.

Since OpAmps provide power to the output, the output signal is not as sensitive to *loading* as the passive circuits.

This brief introduction will be limited to first-order circuits, i.e. simple low-pass and high-pass active circuits.

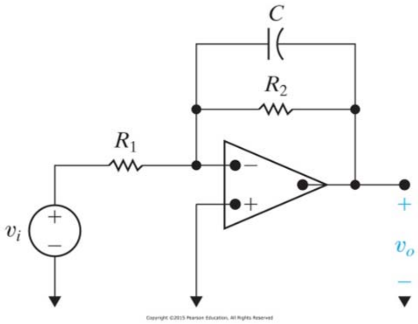
Let's start with a first-order low-pass filter.

Recall that it is usually convenient to analyze an opamp circuit with node analysis.



Using node analysis, find the transfer function $H(s) = \dfrac{V_o(s)}{V_i(s)}$

Section 15.1 First-Order Low-Pass and High-Pass Filters



The node equation at node v_n is

$$\frac{v_n - V_i}{R_1} + i_n + \frac{v_n - V_0}{R_2 \parallel Z_c} = 0$$

$$\Rightarrow v_n\left(\frac{1}{R_1} + \frac{1}{R_2 \parallel Z_c}\right) + V_i\left(\frac{-1}{R_1}\right) + V_0\left(\frac{-1}{R_2 \parallel Z_c}\right) + i_n = 0$$

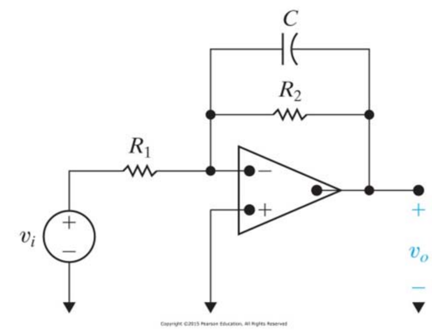
Recall the ideal opamp assumptions:

$v_n = v_p$ $\quad$ $i_n = i_p = 0$ $\quad$ $-V_{CC} \le V_0 \le +V_{CC}$ where V_{CC} is the power supplies

In this circuit, $v_n = v_p = 0$. Thus

$$\frac{V_0}{R_2 \parallel Z_c} = V_i\left(\frac{-1}{R_1}\right) \qquad \Rightarrow \frac{V_0}{V_i} = \frac{-\left(R_2 \parallel Z_c\right)}{R_1}$$

The last result can be compactly expressed as $\dfrac{V_0}{V_i} = -\dfrac{Z_{feedback}}{Z_{input}}$



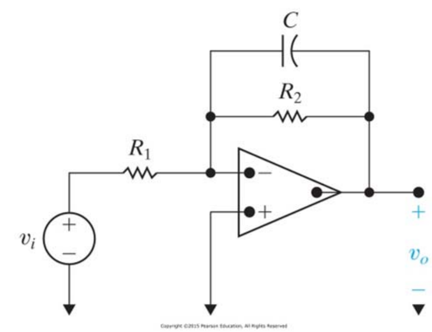
Now simplify the transfer function H(s)

$$H(s)=\frac{V_0(s)}{V_i(s)}=\frac{-\left(R_2 \parallel Z_c\right)}{R_1}=\frac{-\left(\dfrac{R_2\dfrac{1}{sC}}{R_2+\dfrac{1}{sC}}\right)}{R_1}=\left(-\frac{R_2}{R_1}\right)\frac{\dfrac{1}{sC}}{R_2+\dfrac{1}{sC}}$$

Continue to simplify into a form typically found in a Laplace transform table.

$$H(s)=\frac{V_0(s)}{V_i(s)}=\left(-\frac{R_2}{R_1}\right)\frac{1}{sR_2C+1}=\left(-\frac{R_2}{R_1}\right)\frac{\frac{1}{R_2C}}{s+\frac{1}{R_2C}}$$

Where the gain is $K=\dfrac{R_2}{R_1}$ and the *corner frequency is* $\omega_c=\frac{1}{R_2C}$



The transfer function has a similar form to the passive filter but with a "settable" gain in the passband of

$$K = \frac{R_2}{R_1}$$

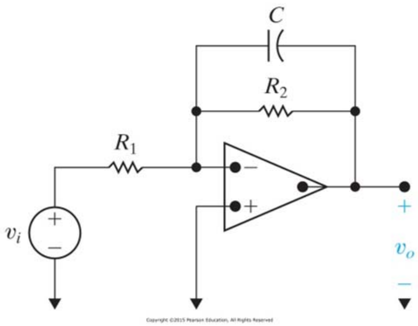
The last equation stated the gain as a positive magnitude where the "-" component found in the circuit analysis is carried by the associated phase of ±180°.

Note that the corner frequency can be set *independently* of the passband gain by adjusting R_2 and/or C.

$$\omega_c = \frac{1}{R_2 C}$$

The circuit in the figure above is an active low-pass filter.

Example P15.4_9ed



Design an opamp based low-pass filter with a cutoff frequency of 2,500 Hz and a passband gain of 5.   Let C = 10 nF.

The corner frequency was given in Hz. ω_C in radians is

$$\omega_C = 2\pi f = 2\pi(2,500Hz) = 5,000\pi \tfrac{rad}{\text{sec}}$$

We can now find the feedback resistor R_2.

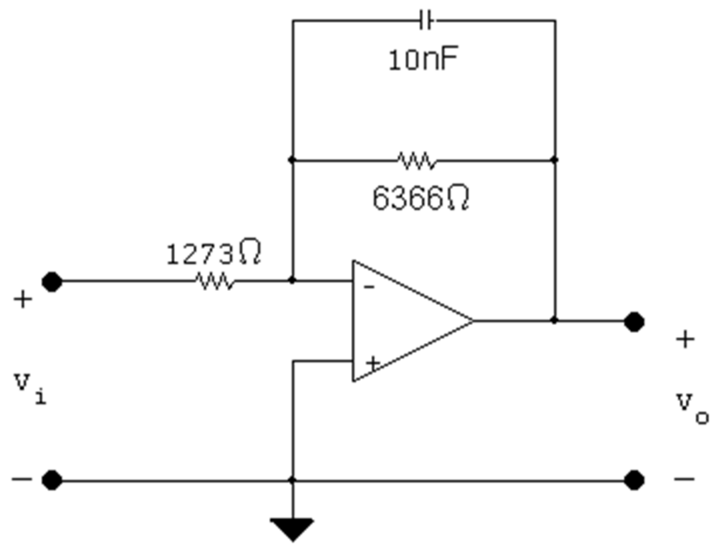
$$\omega_C = \frac{1}{R_2 C} \Rightarrow R_2 = \frac{1}{\left[5,000\pi \tfrac{rad}{\text{sec}}\right](10nF)} = 6,366.198\ \Omega$$

The input resistor is found from the desired gain K.

$$K = \frac{R_2}{R_1} = 5 \Rightarrow R_1 = \frac{R_2}{5} = \frac{6,366.198\Omega}{5} = 1,273.24\ \Omega$$

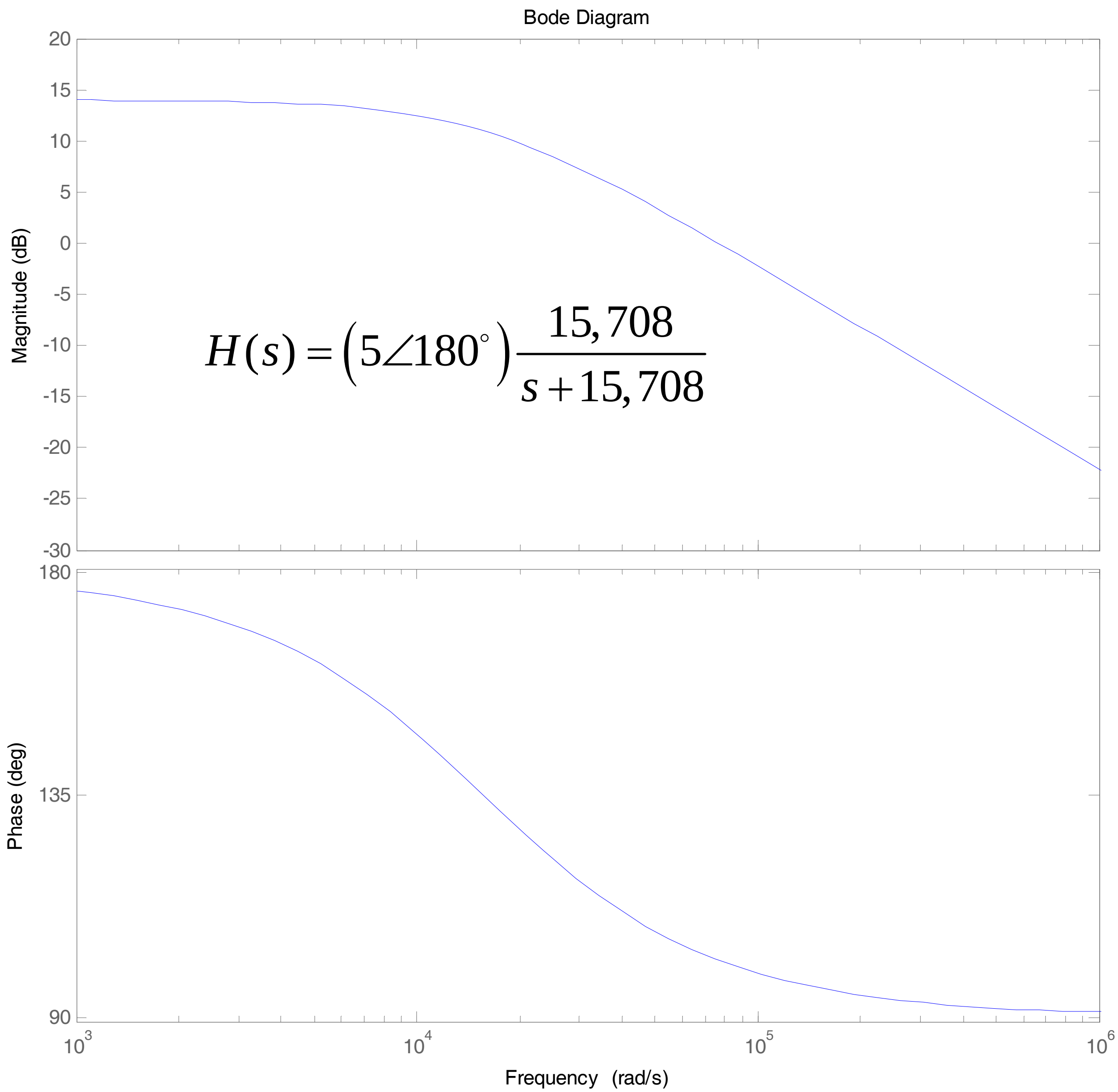
Example P15.4_9ed

The low-pass filter is then



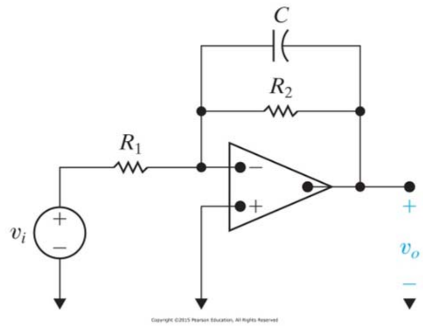
$$H(s) = \frac{V_0}{V_i} = (-5)\frac{15,708}{s+15,708}$$

A Matlab generated Bode plot is shown next.



Note that Matlab lets the phase run from “+”180° rather than “-”180 °

Example P15.4_9ed



We completed the design of the opamp based low-pass filter with very specific calculated values for R_1 and R_2 with an assumed C = 10 nF.

It is folly to expect circuits to be built to such exact element values without extremely compelling reasons. So we usually build to commonly available parts such as 10% tolerance resistors.

For example, 10% tolerance resistors come in "decade multiples" of the following

10 12 15 18 22 27 33 39 47 56 68 82

Thus the closest 10% tolerance resistor to 1,273.24 Ω calculated value is 1.2 kΩ.

Thus a ±10% tolerance resistor of 1.2 kΩ nominal value has a range of

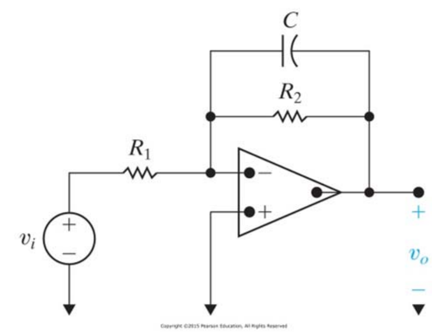
$$1{,}080\,\Omega \le 1{,}200\,\Omega \le 1{,}320\,\Omega$$

The closest 10% tolerance resistor for the 6,366.198 Ω is 6.8 kΩ.

This ±10% tolerance resistor 6.8 kΩ is thus between

$$6{,}120\,\Omega \le 6{,}800\,\Omega \le 7{,}480\,\Omega$$

Example P15.4_9ed



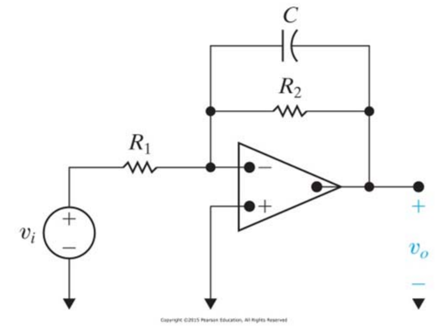
So let's now calculate a range of gains depending on the permissible values of the resistors.

$$K_{smallest} = \frac{R_{2,smallest}}{R_{1,\text{largest}}} = \frac{6{,}120\,\Omega}{1{,}320\,\Omega} = 4.64$$

$$K_{\text{largest}} = \frac{R_{2,\text{largest}}}{R_{1,smallest}} = \frac{7{,}480\,\Omega}{1{,}080\,\Omega} = 6.93$$

The designer must determine what this opamp is "feeding" and what is allowable in the range of gain.

Example P15.4_9ed



Capacitors also come in a range of tolerances.

A common capacitor tolerance range is 10% of pF
1   1.1   1.2   1.3   1.5   1.6   1.8   2.0   2.2   2.4   2.7 3.0 ...

So at the 10% tolerance 10 nF capacitor (which is 10,000 pF) is

$$9nF \leq 10nF \leq 11nF$$

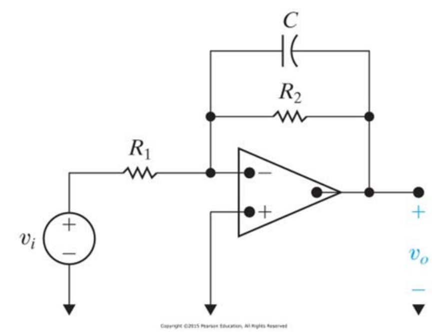
Recall that the corner frequency ω_c is given by $\omega_c = \frac{1}{R_2 C}$

So the corner frequency ω_c can vary from the specified value of 15,708 rad/sec by

$$\omega_{c,smallest} = \frac{1}{R_{2,\text{largest}} C_{\text{largest}}} = \frac{1}{7,480\Omega(11nF)} = 12,154 \frac{rad}{\text{sec}}$$

$$\omega_{c,\text{largest}} = \frac{1}{R_{2,smallest} C_{\text{smallest}}} = \frac{1}{6,120\Omega(9nF)} = 18,155 \frac{rad}{\text{sec}}$$

Active Filter Limitations



The analysis just completed assumed the opamp is ideal.

But real opamps have limits in the output typically given as the short-circuit current output of the opamp.

In the lab, you likely used the venerable 741 opamp whose data sheets says the *continuous short-circuit current output* is 15 mA.

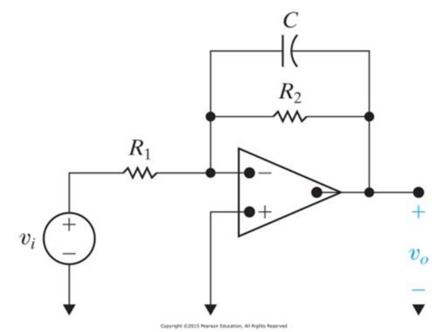
What limits does this output current limitation place on the low-pass filter?

What is the smallest resistor we can place across the voltage v_o?

$$15mA \geq \frac{v_0}{R_o} \Rightarrow R_{o,\min} \geq \frac{v_0}{15mA}$$

Not all that output current is necessarily available to the output resistor. Simulate this circuit and track the currents (on your own).

Active Filter Limitations

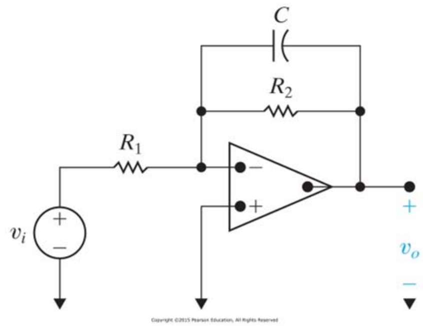


The opamp may not be able to provide the full power input voltage to the output (which a so called rail-to-rail opamp can do).

So the short-circuit current output of the opamp may also be influenced by when the opamp enters saturation due to voltage considerations.

So even with active circuits, you must be aware of the device limitations in your design.

Active Filter Limitations – LED example



Design an output indicator that an opamp is in saturation.

The approach I will take is to light up an LED when the opamp just enters saturation.

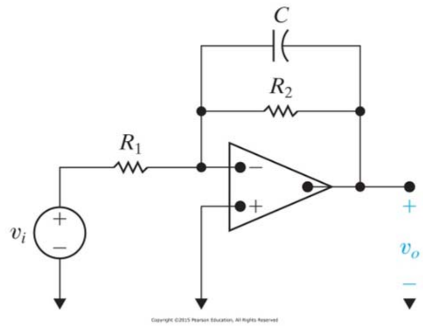
We found an LED that desires 16 mA of current at full brightness. But it will give dim light well below that level of current. The LED forward voltage is 2 V.

We will use an LM741 opamp with power supply rails at ±15V. The “output voltage swing” is listed in the data sheet as ±13V for a load resistance $R_L \geq 2$ kΩ.

So it seems that we can assume the opamp is in saturation at or before the output voltage reaches ±13V.

LED means Light Emitting Diode. Emphasis on Diode. You will learn about diodes in a later course (which will include how to properly wire up an LED for this inverting opamp example).

Active Filter Limitations – LED example



What should be the value of the current limiting resistor needed to protect the LED?

Full brightness will be at a load current of 16 mA. Let's limit the current (arbitrarily) to 10 mA.

What should be the value of the current limiting resistor needed to protect the LED?

$$R_L = \frac{v_0 - V_{LED}}{10mA} = \frac{13V - 2V}{10mA} = \frac{11V}{10mA} = 1{,}100\,\Omega$$

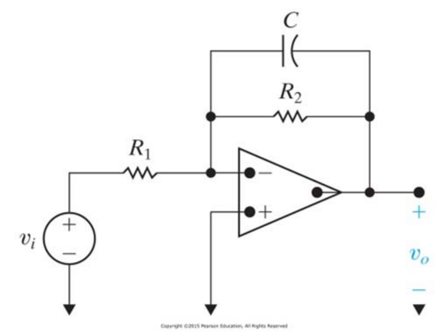
The closet 10% tolerance resistor is either 1,000 Ω or 1,200 Ω.

Since our goal is to barely light up the LED and to limit the OpAmp current to less than 16 mA, Let's pick the 1.2k Ω resistor.

Thus a ±10% tolerance resistor of 1.2 kΩ nominal value has a range of

$$1{,}080\,\Omega \le 1{,}200\,\Omega \le 1{,}320\,\Omega$$

Active Filter Limitations – LED example



The current through the LED can then vary as

$$\frac{11V}{1{,}320\Omega} \le I_{LED} = \frac{11V}{1{,}080\Omega}$$

$$8.33\, mA \le I_{LED} = 10.19\, mA$$

Note that the data sheet listed the output voltage with a load resistor $R_L \geq 2$ kΩ.
We may find the opamp goes into saturation a lower voltage than suggested by this analysis!

Testing the circuit will reveal if this level of current creates a sufficiently bright enough LED just as the opamp enters saturation.

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