**#1**

**Explain the difference between (P → Q) in classical propositional logic and □(P → Q) in Leibnizian modal logic with specific reference to the truth conditions of each. Is there any <u>valid</u> formula A in classical logic for which □A may sometimes be false in Leibnizian modal logic? Explain why or why not, and explain how the derivation system of Leibnizian modal logic reflects your answer.**

Classical propositional logic is the most thoroughly researched branch of propositional logic, which studies logical operators and connectives that are used to produce complex statements whose truth-value depends entirely on the truth-values of the simpler statements making them up, and in which it is assumed that every statement is either true or false and not both. However, there are other forms of propositional logic in which other truth-values are considered, or in which there is consideration of connectives that are used to produce statements whose truth-values depend not simply on the truth-values of the parts, but additional things such as their necessity, possibility or relatedness to one another. Truth-functional propositional logic does not analyze the parts of simple statements, and only considers those ways of combining them to form more complicated statements that make the truth or falsity of the whole dependent entirely on the truth or falsity of the parts, in effect, it does not matter what meaning we assign to the individual statement letters like 'P', 'Q' and 'R', etc., provided that each is taken as either true or false (and not both).

Leibniz's modal logic of concepts lacks the operator of *disjunction*. Although this is by and large correct, it doesn't imply any defect or any incompleteness of the system *L1* because the operator A∨B may simply be introduced by *definition*:

DISJ 1 A∨B =df ~(~A ~B).

On the background of the above axioms of *negation* and *conjunction*, the standard laws for *disjunction*, for example

DISJ 2 A∈(A∨B)
DISJ 3 B∈(A∨B)
DISJ 4 A∈C ∧ B∈C → (A∨B)∈C

Simpler terms:

Classical propositional logic is the most thoroughly researched branch of propositional logic, and truth-values depends on truth-values of simpler statements. While Leibnizian lacks disjunction but it does not mean it is incomplete, and necessity does not always come out true.

EX: the first says that if P then q. the second one says that it is necessary for p to imply q. necessary A is true if A is true in all possible worlds, so if there exist a world where A is false then necessary A is not true. An example would be all the things in our world, tvs, shoes, clothes, etc. all these things we understand what they refer to. By Leibnizian modal, these names refer to the same entities in all worlds. But that's not necessarily the case, something we call a tv here can be something completely different in another world. Since this is true we can say that saying it is necessary for p to imply q is false because it it's a tv in this world, that doesn't necessarily imply it's a tv in another.

**#2**

**Characterize Meinongian free logic as an extension of modal predicate logic, explaining what it attempts to accomplish and how it does so. Your explanation should refer to a problem inherent in both classical predicate logic and Leibnizian modal logic regarding necessary existence. Explain how this is dealt with both in the semantics and in the derivation system.**

Meinongian free logic is a way for us to make reference to the possible worlds. Therefore, it's an extension of modal logic, the extension allowing it to say things that can not be said in classical modal predicate logic.

In predicate logic, models are defined in such a way that every name used necessarily refers to something in the model. Leibnizian modal logic's problem is that there are worlds where the entity exist and doesn't exist but even so it still is present in the model. Both of them contain the entity in the model as a existing entity somewhere and that's what causes the problem. Meinong free logic attempts to acknowledge the realm of the nonexistent. Meignongian semantics permits a proposition of the form Pa to be true even when a fails to denote. But it also restricts the use of universal and existential quantifiers. So saying “Of all x, x is fun” saying ”C is fun” is not permitted unless it is a member of the domain of existing objects. Every model has a inner and outer domain. Inner domain is relative to a world and contains only objects existing in that world, outer domain is not relative to the world and includes both existing and non existing objects.

**#3**

**Explain mathematical induction by reference to the Sorites Paradox, being clear what is missing from the Sorites argument that prevents it from being deductively valid. Prove the validity of mathematical induction in English.(This proof exists in the lecture notes in 5 discrete steps. Replicate them here.)**

In its primary sense inductive reasoning involves drawing a conclusion that is not deductively implied by the premises. The idea that there can be reasoning but it is not deductive.

Induction is drawing a conclusion that is not deductively implied by the premises. In mathematical induction, the inductive first step is to show that a conditional holds true at any arbitrary point k, then it must also hold true at k+1. And the next step is basis step: for k=1, the assertion is that the sum of the first one positive integers of N^2. The sum of the first one positive integer is 1, which is equal to 1^2. Sorites argument is perfectly valid but there's a problem with the predicate being used. The predicate in question must be defined well enough to yield a propositional function.(unambiguously map Hx to the values true or false for every permissible argument of x.). Heapiness is a notoriously fuzzy predicate that does not satisfy this requirement and is what generates the paradox.

Ex) Every time I have tasted sugar, it has been sweet. Thus, the next time I taste sugar, it will be sweet. -- This is good reasoning but not deductively valid.

1. Assume that some propositional function Pn has the value True for both the basis step n=1 and the inductive step, but is not true for all values of n.
2. Then we can define a set S that contains all values of n for which Pn is false.
3. We know from the well-ordering principle that S is well, ordered. Hence, S has a least member. Call this least member b.

4. By the definition of S, Pb is False.
5. Now consider the integer (b-1) immediately preceding b. We know that P(b-1) is True. (Otherwise it would be a member of S.)

But, as we have assumed the inductive step is True, it follows that Pb must be True as well. Hence, Pb is both True and False. QED.

**#4**

**Explain Russell's Paradox and the Burali-Forti Paradox in a way that shows their fundamental similarity. Be clear about the contradiction implied in each. Explain what fundamental change was required to deal with them.**

**1. Russell's Paradox states that you can't believe that no set is a member of itself because if you do then you are saying that set has no property which means that it's not the case that it is not a member of itself, meaning it is a member of itself. But if you believe it is a member of itself that you are say that the set has a property which is saying that it is not a member of itself. Burali-Forti Paradox is as follows: consider the set of all ordinals and call it uppercase omega. If we let O the property of being an ordinal, then this appears as this:**

**Ω={x: Ox}**

**Ordinals are defined in such a way that each set of ordinals has an immediate successor. This implies that Ω itself has a successor, which it does not by definition. Hence, Ω both does and does not have a successor. Both depend on formulating intuitively well-constructed sets that in their own ways purport to contain everything, and imply a contradiction when the property in question is applied to that very set. Using Axiom of specification helps block both paradoxes.**

Russell's Paradox is to let the domain be the set of all sets and define S to be the following set: R={x|x∉x}, then R∈R <-> R∉R, A contradiction. Burali-Forti Paradox is to let omega be a set that contains all ordinal numbers. Omega = {x: Ox}, O= being ordinal

They are similar in a sense both depend on formulating intuitively well-constructed sets in their own ways purport to contain everything, and imply a contradiction when the property in question is applied to that very set.

**#5**

**Evaluate the following in a way that demonstrates a clear grasp of all three concepts and by making reference to at least one established proof: If a formal system S is both sound and complete, then it is decidable.**

**So say we wanted to evaluate D->V, well a logical system is sound iff every sequent that can be derived is valid. Looking at it D implying V is sound so far so yes. If there were contradictions it would not be sound. Next a logical system is complete iff every valid sequent is derivable. The valid sequents from the equation are derivable. Lastly a logical**

**system is decidable iff there is an effective method for determining for any proposition whither it is valid or invalid. In propositional logic, the best way is to use refutation trees.**

Completeness:

An formal system is said to be complete if for each sentence. S either S can be proven in the system or it can be disproven in the system.

Soundness:

Soundness is the converse of completeness. A formal system is sound if it only proves things that are appropriate to prove. In a case like first order logic, a perhaps weak system would be sound if each of the sentences provable in it is true in all models.

Decidability

A set of strings is decidable if there is an algorithm for determining which strings belong to it. A property of a string is decidable if the set of strings having the property is decidable.

Proof: I cannot think of a proof to reference...

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**1. f: P → Z where f is a function mapping philosophy students to the number of computer science students they know by name.**

• This is not injective because more than one philosophy student may know the same number of computer science students.
• This is not surjective, since the codomain of the function is the integers, which includes negative numbers. So we know that there are members of the codomain not mapped onto by members of the domain.

2. **g: P X C → Z where g is a function mapping ordered pairs of students to the absolute difference in their heights rounded to the nearest centimeter.**

• This is not injective because it is possible for two pairs of students to differ in height by the same amount.
• This is not surjective for the same reason as in 1 above.

3. **f: P X C → C where f is a function mapping ordered pairs of students to the computer science student in the pair.**

• This cannot be deemed injective because as long a C has more than one member, P X C will consist of multiple ordered pairs containing the same computer science student. Therefore this function gives multiple ordered pairs the same value.
• This is surjective, since the codomain consists entirely of computer science students in the class and every computer science student will be mapped onto at least once.

4. **g: P X C → A, where A = {0,1,2,3,4} and g is a function mapping ordered pairs of students onto the cardinality of the subset of D = {broccoli, asparagus, cauliflower, spinach} consisting of vegetables enjoyed by both members of the pair.**

• This is not injective since multiple pairs of students may like the same number of vegetables in D,
• This is not surjective, since there may be a number of vegetables not liked by any ordered pair of students.

5. **g ∘ f, f: C → L is a function that maps computer science students in the classroom to their cell phone numbers and g: L → Z: is a function that maps cell phone numbers to the number of philosophy students who have access to that number. (Assume everybody has one and only one cell phone.)**

• f is bijective, hence injective, but g is not injective. Hence g ∘ f may map multiple computer science students onto the same number of philosophy students, making g ∘ f not injective.
• g ∘ f is not surjective, since the codomain is Z .

Compute the first four members of the sequence {an} in each of the following:
6. an = n2 – n
{0, 2, 6, 12} (assuming the relation is defined on Z+ }
7. an = an-1 + 7, a0 = 3
{3, 10, 17, 24}
8. an = an-1 + 2(an-2), a0 = 0, a1 = 2
{0, 2, 2, 6}
9. an = a2n-1 + 2, a0 = 1
{1, 3, 11, 123}
10. an = (an-1 · an-2) + 3, a0 = 1, a1 = 3
{1, 3, 6, 21}

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***Set Theory 4 HW:***

**#1**

**Determine whether f(x,y) = x/y is (a) injective and (b) surjective from Z+ x Z+ to Q+, i.e., from the positive integers to the positive rationals. Prove your result.**

The function is not injective but it is surjective.

Proof:

a. f(x,y) = x/y is injective iff xy(f(x,y) = f(u,w) → (x,y) = (u,w)). Counterexample: f(2,2) = f(3,3), but (2,2) ≠ (3,3). Hence, not injective.

b. Assume for indirect proof that f(x,y) = x/y is not surjective. Then xy(x/y Q+ & (x,y)  Z+ x Z+ ).

However, xy(x/yQ+→ (xZ+ and yZ+ ) .

Hence (x,y)Z+ x Z+ , contradicting the assumption. Hence the function is surjective.

**#2**

**Prove that a fully occupied Hilbert's Hotel can accommodate 0 new guests without evicting any current guests.**

Proof: To make room for an infinite number of guests, every current occupant of the hotel may be moved according to the formula an = 2n. This will result in the current occupants being moved into all of the even numbered rooms, with 0 odd rooms left for the new guests.

**#3**

**Is it true that for any sets A and B, (A  B → |B| > |A|)? Prove your hypothesis.**

It is not true.

Proof:

Assume that A is the set of even integers Z+e and B is the set of integers Z.

Then A  B because x(xA → xB) & y(yB & yA), namely all the odd integers.

But |A| = |B| since an=2n is a bijective function f: B → A; i.e., it assigns every integer to one and only one even number

**#4**

**Suppose that A and B are sets and that there is a surjective function from A to B. If A is countable, does it follow that B is countable? Prove your hypothesis.**

Assume, contrary to hypothesis, that there is a surjective function f: A → B in which A is countable and B is uncountable. Then |B| > |A|

Since the function is surjective, that there is a mapping that assigns to every element of B some element of A.

The mapping must be either bijective or surjective. If the mapping is bijective, then by definition |A| = |B|, which by assumption is not the case.

If the mapping is surjective but not bijective, then one or more elements of A are mapped to some element of B.

In this case there is a set C such that C  A for which f: C→ B is bijective. Since |C|  |A|, it follows that |B|  |A|, which by assumption is not the case. QED

**#5**

**Use the Schröder–Bernstein theorem to prove that (0,1) and [0,1] have the same cardinality. (Note that these are real number intervals, not ordered pairs. Consult notes or text if you are unclear on what is being requested.)**

Proof:

We first show there is an injective function from (0,1) to [0,1]. f(x) = x is such a function, since every element of the interval (0,1) is an element of the interval [0,1],

We next show that there is an injective function from [0,1] to (0,1). f(x) = $x$+1 3 is such a function. Hence, by the Schröder–Bernstein |(0,1)| = |(0,1)|