

SOLUTIONS - EXERCISES ON PUSH-DOWN AUTOMATA

1. Construct an PDA that accepts the language $\{ a^i b^j \mid 0 \leq i \leq j \}$

$\delta(q_0, a, z) = (q_0, az)$ read a, push a
 $\delta(q_0, a, a) = (q_0, aa)$
 $\delta(q_0, b, a) = (q_1, \lambda)$ read b, pop a
 $\delta(q_1, b, a) = (q_1, \lambda)$
 $\delta(q_1, \lambda, z) = (q_f, z)$ end of string, stack empty
 $\delta(q_1, b, z) = (q_1, z)$ check the additional b's, if any

2. Construct an PDA that accepts the language $\{ a^i c^j b^i \mid i, j \geq 0 \}$

$\delta(q_0, a, z) = (q_0, az)$ read a, push a
 $\delta(q_0, a, a) = (q_0, aa)$

 $\delta(q_0, c, z) = (q_1, z)$ read c, push nothing (just a or z back)
 $\delta(q_0, c, a) = (q_1, a)$
 $\delta(q_1, c, z) = (q_1, z)$
 $\delta(q_1, c, a) = (q_1, a)$

 $\delta(q_0, b, a) = (q_2, \lambda)$ read b, pop a
 $\delta(q_1, b, a) = (q_2, \lambda)$
 $\delta(q_2, b, a) = (q_2, \lambda)$

 $\delta(q_0, \lambda, z) = (q_f, z)$ end of string, stack empty
 $\delta(q_1, \lambda, z) = (q_f, z)$
 $\delta(q_2, \lambda, z) = (q_f, \lambda)$

3. Construct an PDA that accepts the language $\{ w \mid w \in \{ a, b \}^* \text{ and } w \text{ has the same number of } a\text{'s and } b\text{'s} \}$.

In q_0 read an a or a b and push it on the stack. As long as the same symbol appears continue to push onto the stack. If the stack becomes empty repeat the process. At the end of the string if the stack is empty the string is accepted.

$\delta(q_0, \lambda, z) = (q_f, z)$ end of string, stack empty

 $\delta(q_0, a, z) = (q_0, az)$ reading a, stack empty, push a
 $\delta(q_0, a, a) = (q_0, aa)$ reading a, a on stack. push a
 $\delta(q_0, b, a) = (q_0, \lambda)$ reading b, a on stack, pop a

 $\delta(q_0, b, z) = (q_0, bz)$ reading b, stack empty, push b
 $\delta(q_0, a, b) = (q_0, \lambda)$ reading a, b on stack, pop b
 $\delta(q_0, b, b) = (q_0, bb)$ reading b, b on stack, push b

4. Construct an PDA that accepts the language $\{ w \mid w \in \{ a, b \}^* \text{ and } w \text{ has twice as many } a\text{'s as } b\text{'s} \}$

$\delta(q_0, a, z) = (q_1, z)$ reading a, stack empty, don't push a
 $\delta(q_1, a, z) = (q_0, az)$ reading second a, push a

$\delta(q_0, a, a) = (q_1, a)$
 $\delta(q_1, a, a) = (q_0, aa)$

reading first a, ignore that a
 reading second a, push it

$\delta(q_0, b, z) = (q_0, bz)$
 $\delta(q_1, b, z) = (q_1, bz)$
 $\delta(q_0, b, b) = (q_0, bb)$
 $\delta(q_1, b, b) = (q_0, bb)$

reading b, stack empty, push b
 same thing
 reading b, b on stack, push b
 same thing

$\delta(q_0, b, a) = (q_0, \lambda)$
 $\delta(q_1, b, a) = (q_1, \lambda)$

reading b, a on stack, pop a
 same thing

$\delta(q_0, a, b) = (q_1, b)$
 $\delta(q_1, a, b) = (q_0, \lambda)$

reading a, b on stack, pop b
 same thing

$\delta(q_0, \lambda, z) = (q_f, z)$

end of string, stack empty

5. Construct an PDA that accepts the language $\{a^{i+j}b^i c^j \mid i, j > 0\}$

$\delta(q_0, a, z) = (q_0, az)$
 $\delta(q_0, a, a) = (q_0, aa)$

read a, push a
 same thing

$\delta(q_0, b, a) = (q_1, \lambda)$
 $\delta(q_1, b, a) = (q_1, \lambda)$

read b, pop a, change state
 read b, pop a

$\delta(q_1, c, a) = (q_2, \lambda)$
 $\delta(q_2, c, a) = (q_2, \lambda)$

read c, pop a, change state
 read c, pop a

$\delta(q_2, \lambda, z) = (q_f, z)$

end of string, stack empty

6. Given the following PDA describe the language it accepts.

$Q = \{q_0, q_1, q_2\}$
 $\Sigma = \{a, b\}$
 $\Gamma = \{A, B\}$
 $F = \{q_1, q_2\}$
 $z = B$

$\delta(q_0, a, B) = \{(q_0, AB)\}$
 $\delta(q_0, \lambda, B) = \{(q_1, \lambda)\}$
 $\delta(q_0, b, A) = \{(q_2, \lambda)\}$
 $\delta(q_0, a, A) = \{(q_0, AA)\}$
 $\delta(q_2, b, A) = \{(q_2, \lambda)\}$
 $\delta(q_2, \lambda, A) = \{(q_2, \lambda)\}$
 $\delta(q_2, \lambda, B) = \{(q_1, \lambda)\}$

The language $\{a^i b^j \mid 0 \leq j \leq i\}$

7. Show the sequence of moves for the PDA in 6 on strings aab, aba, abb, aabb, aaab.

$(q_0, aab, B) \vdash (q_0, ab, AB) \vdash (q_0, b, AAB) \vdash (q_2, \lambda, AB)$
 $\vdash (q_2, \lambda, B) \vdash (q_1, \lambda, \lambda)$

$(q_0, aba, B) \vdash (q_0, ba, AB) \vdash (q_0, a, B) \vdash (q_0, \lambda, AB)$ crashes

$(q_0, abb, B) \vdash (q_0, bb, AB) \vdash (q_2, b, B)$ crashes

$(q_0, aabb, B) \vdash (q_0, abb, AB) \vdash (q_0, bb, AAB) \vdash$
 $(q_2, b, AB) \vdash (q_2, \lambda, B) \vdash (q_1, \lambda, \lambda)$

$(q_0, aaab, B) \vdash (q_0, aab, AB) \vdash (q_0, ab, AAB) \vdash$
 $(q_0, b, AAAB) \vdash (q_2, \lambda, AAB) \vdash (q_2, \lambda, AB) \vdash$
 $(q_2, \lambda, B) \vdash (q_1, \lambda, \lambda)$

The first two strings, "aab" and "aba." I believe "aab" should end up in (q_1, λ, λ) , and it also seems to skip one instance of q_2 (though that may have been intentional since the string is λ by that point). For the string "aba," i think it should end up in (q_2, a, B) before it crashes.