Philosophy 160 Practice In-class Test 1 Name: Sol Yushin

1. (5 pts.) Use a tree to determine whether the following argument is valid or invalid. Clearly indicate what property of the tree justifies your answer.

$\forall x\exists y(Cxy \rightarrow Cyx) \vdash Cab \rightarrow Cba$

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$$
\begin{gathered}
\forall x\exists y(Cxy \rightarrow Cyx)\\
\surd{\sim}(Cab \rightarrow Cba)\\
Cab\\
{\sim}Cba\\
\surd\exists y(Cay \rightarrow Cya)\\
\surd\exists y(Cby \rightarrow Cyb)\\
Cac \rightarrow Cca\\
Cbd \rightarrow Cdb\\
\surd\exists y(Ccy \rightarrow Cyc)\\
\surd\exists y(Cdy \rightarrow Cyd)\\
(Cce \rightarrow Cec)\\
(Cdf \rightarrow Cfd)\\
.\\
.
\end{gathered}
$$

Explanation: Invalid. The tree will not terminate and all non terminating trees in predicate logic are invalid.

2. (5pts.) Derive the following in propositional logic. Use only basic rules (hypothetical and non hypothetical), not equivalences.

$S \to T \vdash T \vee {\sim}S$

1.	$S \to T$	A
2.	$\sim(T \vee \sim S)$	H ~I
3.	S	H ~I
4.	T	1,3→E
5.	$(T \vee \sim S)$	4, vI
6.	$(T \vee \sim S) \,\&\, \sim(T \vee \sim S)$	2, 5 &I
7.	$\sim S$	3-6, ~I
8.	$T \vee \sim S$	7, vI
9.	$(T \vee \sim S) \,\&\, \sim(T \vee \sim S)$	2,8 &I
10.	$\sim\sim(T \vee \sim S)$	2-9 ~I
11.	$T \vee \sim S$	10 DN

3. (5pts.) Derive the following in predicate logic.

$\exists x \forall y(y=x \leftrightarrow Gy) \vdash \forall x Gx \rightarrow \forall x \forall y\, x=y$

1.	$\exists x \forall y(y=x \leftrightarrow Gy)$	A
2.	$\quad \forall x Gx$	H
3.	$\quad\quad \forall y(y=a \leftrightarrow Gy)$	H, $\exists$E
4.	$\quad\quad b=a \leftrightarrow Gb$	3, $\forall$E
5.	$\quad\quad Gb \rightarrow b=a$	4, $\leftrightarrow$E
6.	$\quad\quad Gb$	2, $\forall$E
7.	$\quad\quad b=a$	5,6 $\rightarrow$E
8.	$\quad\quad c=a \leftrightarrow Gc$	3, $\forall$E
9.	$\quad\quad Gc$	2, $\forall$E
10.	$\quad\quad Gc \rightarrow c=a$	8, $\leftrightarrow$E
11.	$\quad\quad c=a$	9,10 $\rightarrow$E
12.	$\quad\quad c=b$	7,11 =E
13.	$\quad c=b$	1, 3-12 $\exists$E
14.	$\quad \forall y\, c=y$	13, $\forall$I
15.	$\quad \forall x \forall y x=y$	14, $\forall$I
16.	$\forall x Gx \rightarrow \forall x \forall y\, x=y$	2-15 $\rightarrow$I

4. (5 pts. ½ pt. each.) Evaluate each of the following predicate logic formulas as either true or false in the model. by writing either T or F beneath the formula. Study the definition of the model carefully. If you believe the formula is not a wff, write "~wff," make the minimal correction that would render it a wff, and evaluate it on that basis.

p	q	9	z
1	3	r	e
a	j	8	2
f	b	5	m

Universe = All and only numbers and letters in the grid to the left.
Names = all the letters and numbers in the grid.

One place predicates
- N = number
- L = letter
- V = vowel
- C = consonant
- E = even
- O = odd
- P = prime

Two place predicates
- A = above
- U = under
- R = right of
- F= left of
- G = greater than
- S = subsequent to

Notes: Spatial predicates do not imply "directly". E.g., Apm is true even though p is not directly above m.

Of the numbers n the grid, only 2, 3 and 5 are prime.

G is a relation between numbers only.

S is a relation between letters only. Sea is true because e comes after a in the alphabet, not the grid.

1. $\forall x((Nx \rightarrow \sim Lx) \rightarrow \forall y(Uyx \vee Ayx))$

False

m satisfies the antecedent condition but 5 is neither above nor below m.

2. $\exists x\exists y(Gxy \leftrightarrow \sim Ryx)$

True

This is satisfied by x=9, y=3

3. $\forall x((Nx \,\&\, \sim\exists y(Ny \,\&\, Ryx)) \rightarrow Axe)$

False

The only thing that satisfies the antecedent condition is 2, but 2 is below e.

4. $\forall y(Vy \rightarrow \forall x(\sim x=y \rightarrow Nx))$

False

This says there is at most one vowel in the grid, but there are two.

5. $\forall x(((Px \,\&\, Ox) \vee Lx) \rightarrow (x=3 \vee x=e))$

False

The antecedent condition is satisfied by every letter but not every letter is identical to 3 or e.

6. $\forall x(Px \rightarrow \exists y\exists z(((Ly \,\&\, Lz) \,\&\, (Ayx \,\&\, Uzx))))$

False

This says that every prime number has a letter above it and a letter under it. 5 has no letter under it.

7. $\forall x((Nx \,\&\, Ox) \rightarrow \sim\exists y((Ly \,\&\, Vy) \,\&\, Ayx))$

False

This says that no odd number has a vowel above it. e is above 5.

8. $\exists x\exists y\exists w((Sxy \,\&\, Syw) \,\&\, (Fxy \,\&\, Fyw))$

True

This is satisfied by x=p, y=j, w=e

9. $\forall x((Px \,\&\, \sim Ox) \rightarrow \exists y(Ey \,\&\, (Fyx \leftrightarrow Fxy)))$

False

No two (or one) things can satisfy the consequent condition of each being to the left of each other. Since the consequent is always false, the conditional is false.

10.

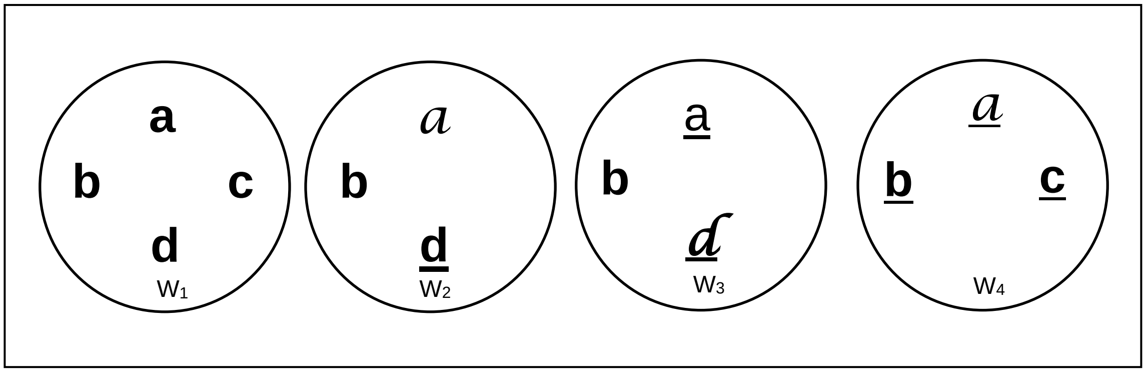
$\forall x\forall y(((Vx \,\&\, Vy) \,\&\, (\sim x=y \,\&\, Syx)) \rightarrow \forall w(Nw \rightarrow (\sim Fwy \,\&\, \sim Rwx)))$

False

The antecedent condition is satisfied by any two non identical vowels where one is subsequent to the other. This is only e and a, respectively. The consequent requires of any number that it be neither to the left of e nor to the right of a. 3 is to the left of e (and also to the right of a.)

Philosophy 160 Test 2 Name: ___________________________

5. (5pts.) The model $\mathcal{V}$ below depicts a universe of four possible worlds $\mathcal{W}_{\mathcal{V}} = \{w_1, w_2, w_3, w_4\}$. The styles of the name letters indicate the properties of the <u>distinct</u> objects to which they refer, designated B, U and S for Bold, Underlined and Script. The table below supplies exactly the same information. Check it if you are uncertain about a visual interpretation. Notice that these are not mutually exclusive properties.



$\mathcal{V}(B,w_1) = \{a,b,c,d\}$	$\mathcal{V}(B,w_2) = \{b,d\}$	$\mathcal{V}(B,w_3) = \{b,d\}$	$\mathcal{V}(B,w_4) = \{b,c\}$
$\mathcal{V}(U,w_1) = \{\ \}$	$\mathcal{V}(U,w_2) = \{d\}$	$\mathcal{V}(U,w_3) = \{a,d\}$	$\mathcal{V}(U,w_4) = \{a,b,c\}$
$\mathcal{V}(S,w_1) = \{\ \}$	$\mathcal{V}(S,w_2) = \{a\}$	$\mathcal{V}(S,w_3) = \{d\}$	$\mathcal{V}(S,w_4) = \{a\}$

(1/2 pt. each) Evaluate the following as true, false or no truth value of the model above in Leibnizian modal logic. Write your answers as T or F or N directly beneath the corresponding formula. If you believe the formula is not a wff, write "~wff," make the minimal correction that would render it a wff, and evaluate it on that basis.

1. $\mathcal{V}(Ba, w_1)$
True

2. $\mathcal{V}((\sim Sa \rightarrow Ud), w_3)$
True

3. $\mathcal{V}((\sim\exists xBx \rightarrow \sim\exists x\ x{=}c), w_3)$
N (because c does not exist at w_3)

4. $\mathcal{V}(\sim\Diamond\exists x(Bx \,\&\, Ux \,\&\, Sx), w_4)$
False

5. $\mathcal{V}(\sim\Diamond(Bd \,\&\, Ua), w_3)$
False

6. $\mathcal{V}((\Box Bd \rightarrow \Diamond Ua), w_2)$
True

7. $\mathcal{V}((\Box\sim\forall xBx \rightarrow Sa), w_3)$
True

8. $\mathcal{V}(\Box(\Diamond Sd \rightarrow \forall xBx), w_1)$
False

9. $\mathcal{V}(\Box\Diamond Bd, w_2)$
True

10. $\mathcal{V}((\Diamond(Sa \,\&\, Ua) \leftrightarrow \Box\sim(Sa \,\&\, Ba)), w_2)$
True

6. Extra credit: 2 pt.

Finish the following semantic rules from Leibnizian modal logic

- Given any Leibnizian valuation $\mathcal{V}$ for any world *w* in $\mathcal{W}_{\mathcal{V}}$

If *P* is a two-place predicate and *a and b* are names whose extensions $\mathcal{V}(a) \in \mathcal{D}_w$ and $\mathcal{V}(b) \in \mathcal{D}_w$ then:

$\mathcal{V}\Box(Pab, w) = T$ iff the $\mathcal{V}(a,b) \in \mathcal{V}(P)$ in every world w in $\mathcal{W}_{\mathcal{V}}$

$\mathcal{V}\Diamond(Pab, w) = F$ iff $\mathcal{V}(a,b) \notin \mathcal{V}(P)$ of any world w in $\mathcal{W}_{\mathcal{V}}$