Philosophy 160 Takehome test 2 Name: Sol Ushion

1. (3 pts.) Derive the following in Leibnizian modal logic. If you use the N or TI rules, first derive the relevant theorems. Then for your justification write “N, derived above” or “TI, derived above”

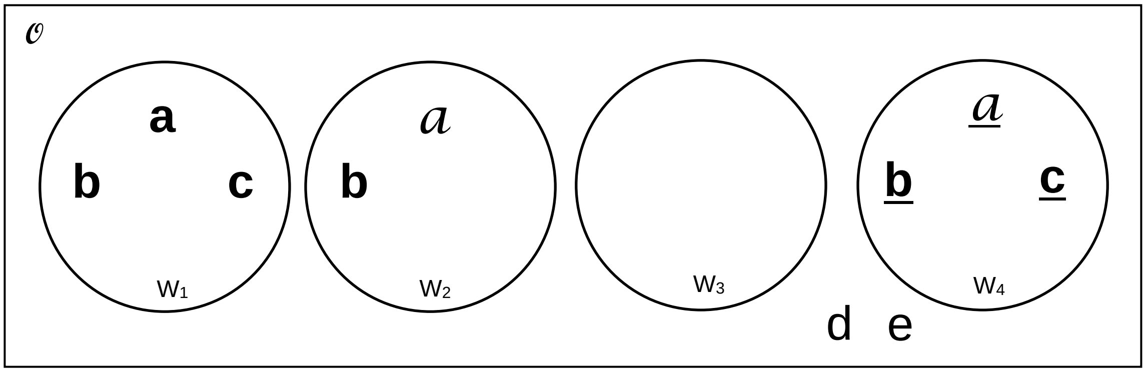
□(P v Q) ├ ◊~P → ◊(Q v M)

1.	□(P v Q)	A
2.	◊~P	H →I
3.	□~(Q v M)	H~I
4.	□(~(Q v M) → ~Q)	N
5.	□~(Q v M) → □~Q	4, K
6.	□~Q	2,5 →E
7.	□(~Q → ((P v Q) → P))	N
8.	□~Q → □((P v Q) → P)	7,K
9.	□(P v Q) → □P	6,8 →E
10.	□P	1,9→E
11.	~◊~P	10, Dual
12.	S&~S	2,11 Con
13.	~□~(Q v M)	3-12, ~I
14.	◊(Q v M)	13 Dual
15.	◊~P → ◊(Q v M)	

Easily proven using →I and DM

Easily proven using →I twice and DS

2. (5pts.) The model $\mathcal{V}$ below depicts a universe of four possible worlds $\mathcal{W}_{\mathcal{V}} = \{w_1, w_2, w_3, w_4\}$ as well as a nonempty outerdomain $\mathcal{O}$. Interpret the predicates in the same way as problem 1. For properties of outer domain objects, you must refer to the valuations below the model.



$\mathcal{V}(B,w_1) = \{a,b,c,d\}$	$\mathcal{V}(B,w_2) = \{b\}$	$\mathcal{V}(B,w_3) = \{d\}$	$\mathcal{V}(B,w_4) = \{b,c\}$
$\mathcal{V}(U,w_1) = \{d\}$	$\mathcal{V}(U,w_2) = \{d,e\}$	$\mathcal{V}(U,w_3) = \{d,e\}$	$\mathcal{V}(U,w_4) = \{a,b,c,d,e\}$
$\mathcal{V}(S,w_1) = \{d, e\}$	$\mathcal{V}(S,w_2) = \{a,d\}$	$\mathcal{V}(S,w_3) = \{d\}$	$\mathcal{V}(S,w_4) = \{a,d,e\}$

(1/2 pt. each) Evaluate the following with respect to the model above in Meinongian free logic. Write your answer as T or F in the space provided on the left.

True 1. $\mathcal{V}(\forall x(Bx \rightarrow Ux), w_3)$

False 2. $\mathcal{V}(\exists x(Bx \rightarrow Ux), w_3)$

False 3. $\mathcal{V}((\Box Bd \ \& \ \sim\exists x\ d=x), w_3)$

False 4. $\mathcal{V}(\Diamond\sim Ud, w_4)$

False 5. $\mathcal{V}(\Box(\exists x\ x=b \rightarrow (Bb \ \& \ \sim Bu)), w_1)$

False 6. $\mathcal{V}(\Diamond\sim(Ud \rightarrow \Diamond(Sa \ \& \ Ua)), w_1)$

False 7. $\mathcal{V}((\exists x(x=d \vee x=e) \leftrightarrow \Box Ud), w_3)$

False 8. $\mathcal{V}((\Box Ud \rightarrow \Box\exists x\ x=x), w_2)$

False 9. $\mathcal{V}((\sim\forall x Ux \ \& \ \Diamond Ud), w_4)$

False 10. $\mathcal{V}(\Box(\forall x Sx \leftrightarrow \Box a=a), w_3)$

4. (3pts.) Derive the following in Meinongian free logic. (Be sure each quantifier rule cited begins with F) Any use of N or TI should be done as in problem 1.

$\vdash \Diamond\Box\forall x(\exists y(Fxy \;\&\; {\sim}x=y) \rightarrow \exists w\exists z(Fwz \;\&\; {\sim}w=z))$

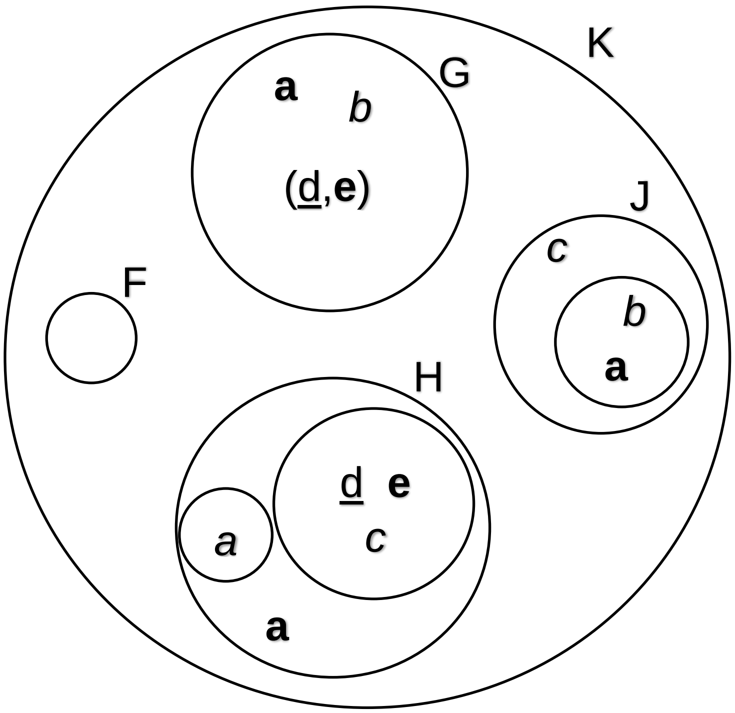
$\vdash \forall x(\exists y(Fxy \;\&\; {\sim}x=y) \rightarrow \exists w\exists z(Fwz \;\&\; {\sim}w=z))$

1.	$\exists x\, x=a$	H F∀I
2.	$\exists y(Fay \;\&\; {\sim}a=y)$	H →I
3.	$(Fab \;\&\; {\sim}a=b) \;\&\; \exists y\, y=b$	H F∃E
4.	$Fab \;\&\; {\sim}a=b$	3, &E
5.	$\exists y\, y=b$	3, &E
6.	$\exists z(Faz \;\&\; {\sim}a=z)$	F∃I 4,5
7.	$\exists w\exists z(Fwz \;\&\; {\sim}w=z))$	F∃I 1,6
8.	$\exists w\exists z(Fwz \;\&\; {\sim}w=z))$	3, 3-7 F∃E
9.	$\exists y(Fay \;\&\; {\sim}a=y) \rightarrow \exists w\exists z(Fwz \;\&\; {\sim}w=z))$	2-8 →I
10.	$\forall x(\exists y(Fxy \;\&\; {\sim}x=y) \rightarrow \exists w\exists z(Fwz \;\&\; {\sim}w=z))$	F∀I 1-9

$\vdash \Diamond\Box\forall x(\exists y(Fxy \;\&\; {\sim}x=y) \rightarrow \exists w\exists z(Fwz \;\&\; {\sim}w=z))$

1.	$\Box\forall x(\exists y(Fxy \;\&\; {\sim}x=y) \rightarrow \exists w\exists z(Fwz \;\&\; {\sim}w=z))$	N(derived above)
2.	$\Box\Diamond\Box\forall x(\exists y(Fxy \;\&\; {\sim}x=y) \rightarrow \exists w\exists z(Fwz \;\&\; {\sim}w=z))$	1, B
3.	$\Diamond\Box\forall x(\exists y(Fxy \;\&\; {\sim}x=y) \rightarrow \exists w\exists z(Fwz \;\&\; {\sim}w=z))$	2, T

5. (5pts.) Consider the following model.



Universe: Objects, sets of objects and ordered pairs of objects to the left. Every circle represents a set, including the largest circle.

Names of objects that are not sets: a,b,c,d,e.

Names of sets: F,G,H, J, K (located outside to upper right of circle.) Not all sets are named.

One-place predicates

B= bold
S = script
U=underlined
T= set

Two-place predicates

All standard set-theoretic relations

Clarification: Only objects, not sets, belong to the extension of one-place predicates.

(1/2 pt. each) Evaluate the propositions below as true or false in the model above. Solve the identities. You may identify sets by name (when they have them) or by roster notation.

1. **False** $\forall x (x \in K \rightarrow \exists y\, y \in x)$

2. **True** $\forall x(x \in F \rightarrow x \subseteq F)$

3. **True** $\exists x(x \in H \,\&\, (Ux \vee Bx))$

4. $|G \times (F \cap H)| =$ **0**

5. $\mathcal{P}(J) =$ **{{c},{{b,a}}, ∅, J}**

6. **True** $\forall x \forall y \forall z((Tx \,\&\, Ty) \rightarrow ((x \in y \,\&\, y \in z) \rightarrow z{=}K))$

7. **True** $\exists x(x \in \mathcal{P}(G) \leftrightarrow x \in J)$

8. $\{x \in H \mid \exists y\, y \in x\} =$ **{{c, d, e}, {a}}**

9. **True** K - H = {F, G, J, H}

10. $\{x \in K \mid \exists y(y \in x \,\&\, (By \vee Sy))\} =$ **{G, J, H}**