Chapter 6

Inductance and Capacitance

And

Mutual Inductance

Text: *Electric Circuits*, 9th Edition, by J. Nilsson and S. Riedel

Prentice Hall

Engr 17 Introductory Circuit Analysis

Instructor: Russ Tatro

Section 6.4

Mutual Inductance

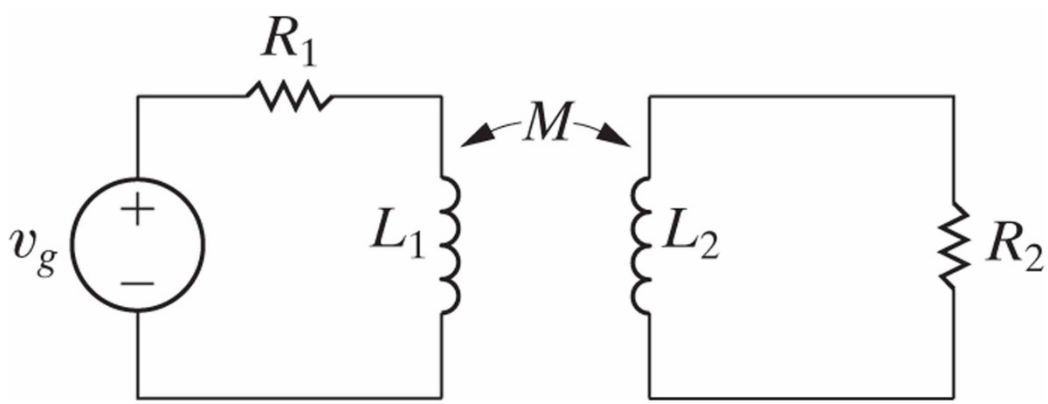
Mutual Inductance

We previously looked at the effect of a moving charge (current) creating a magnetic field which is called inductance.

Since the effect was the current in a single circuit, it should be properly called *self-inductance.*

This section looks at the situation where the magnetic field of two or more circuits are linked.

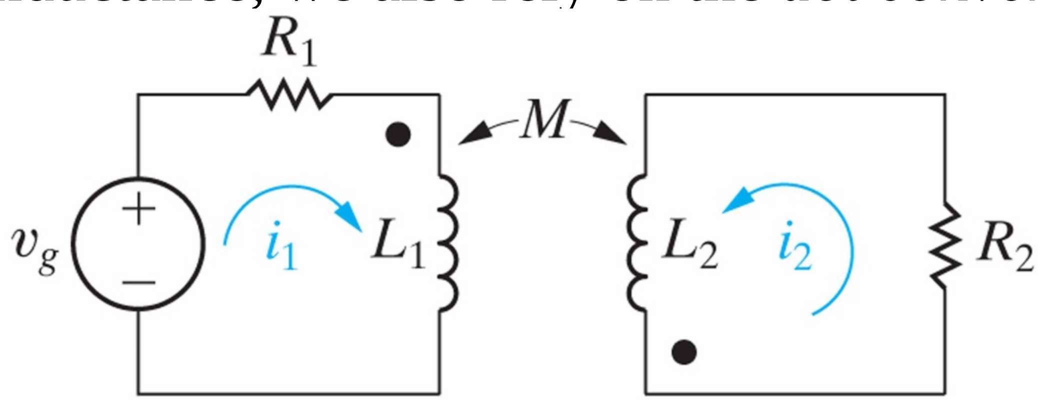
When the magnetic field is linked among circuits by a time varying current then there is *mutual inductance.*



Mutual Inductance

We rely on the passive sign convention to determine voltage versus current direction.

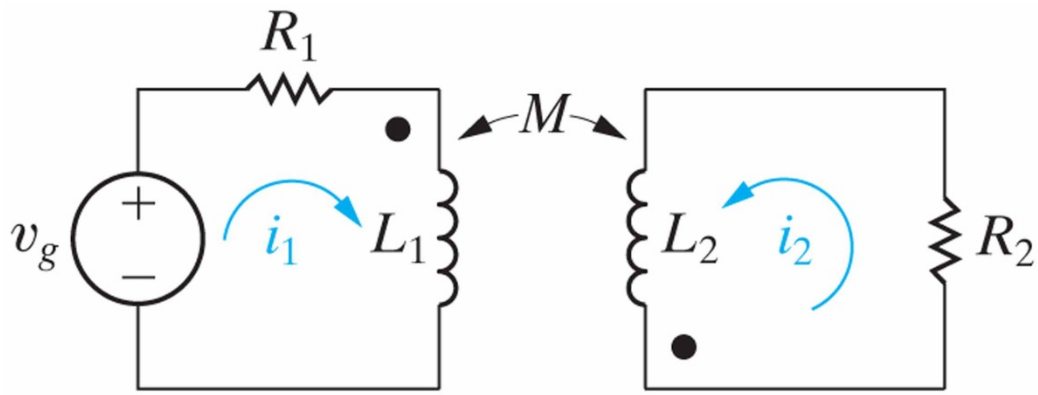
For mutual inductance, we also rely on the *dot convention*..



The polarity of mutually induced voltage depends on the way the coils are wound in relation to the reference direction of coil currents.

So *dots* are placed on the terminals to carry the polarity information schematically (rather than knowing righty or lefty directions).

Dot Convention



Dot Convention

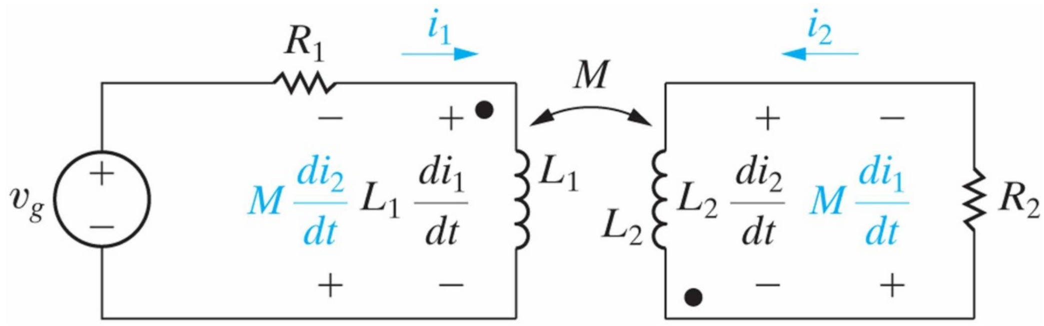
When the reference direction for a current enters the dotted terminal of a coil,

the reference polarity of the voltage that it induces in the other coil is positive at its (the other coil) dotted terminal.

In this course, we will use the dots. Figuring out how to correctly place the dots is left to other courses (such as EEE 130).

Mutual Inductance

In a mutual inductance circuit, we have new voltages to now track.



KVL on the left hand side gives us

$$-v_g + i_1 R_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0$$

KVL on the right hand side gives us

$$-L_2 \frac{di_2}{dt} - i_2 R_2 + M \frac{di_1}{dt} = 0$$

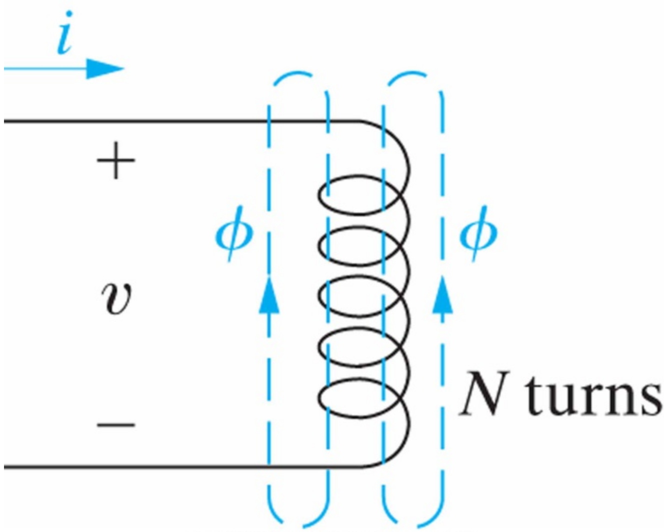
Or equally (multiply last result by -1)

$$L_2 \frac{di_2}{dt} + i_2 R_2 - M \frac{di_1}{dt} = 0$$

Closer Look at Mutual Inductance (briefly)

The voltage induced by the magnetic field surrounding a current carrying conductor is described by Faraday's Law:

$$v = \frac{d\lambda}{dt}$$



Where λ is called the magnetic flux linkage – measured in weber-turns.

The flux linkage λ is the product of the magnetic field (ϕ) and the number of turns linked (N)

$$\lambda = N\phi$$

Closer Look at Mutual Inductance (briefly)

The magnetic field strength per unit volume depends on the material in that space - described by the *permeance* of the material.

The text has more details, but the important message is the magnetic coupling M may not be perfect.

Thus we introduce a coefficient of coupling *k* which can be used to better model a specific physical circumstance.

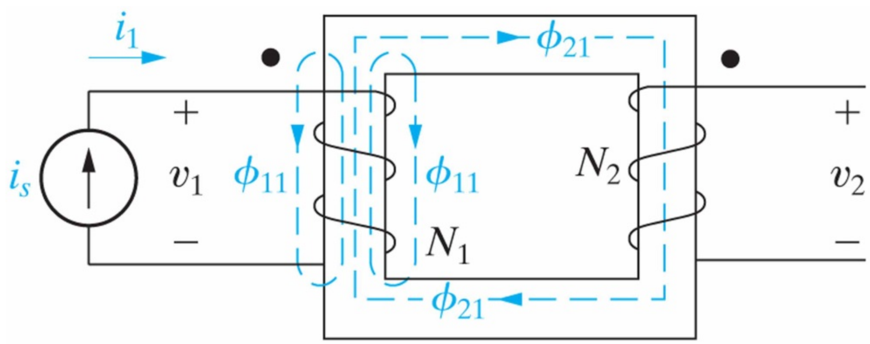
$$M = k\sqrt{L_1 L_2}$$

Where $0 \le k \le 1$.

In general, *k* is found by experimental measurement.

Closer Look at Mutual Inductance (briefly)

We thus have the following view of a mutually coupled circuit:

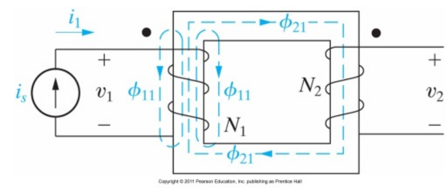


$$M = k\sqrt{L_1 L_2}$$

The energy stored in these two coils (self-inductance) and the energy stored in the coupled magnetic fields (mutual inductance) has the form:

$$w(t) = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

Example – Problem 6.46 (9^{th} edition)



Two magnetically coupled coils have:

$$L_1 = 60mH \qquad L_2 = 9.6mH \qquad M = 22.8mH$$

What is the coefficient of coupling?

$$M = k\sqrt{L_1 L_2} \qquad \Rightarrow k = \frac{M}{\sqrt{L_1 L_2}} = \frac{22.8mH}{\sqrt{(60mH)(9.6mH)}} = 0.95$$

What is the largest value of M?

The largest value of M occurs when k = 1 (perfect coupling).

$$k(where\ k = 1) = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow M = \sqrt{L_1 L_2} = \sqrt{(60mH)(9.6mH)} = 24.0mH$$

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