

SOLUTIONS - EXERCISES ON REGULAR EXPRESSIONS AND FINITE AUTOMATA

Give regular expressions representing the following languages on $\{a, b\}$ or $\{0, 1\}$:

1. all strings

$$(a + b)^*$$

2. strings which begin and end with the same letter

$$a(a + b)^*a + b(a + b)^*b + a + b$$

3. strings which begin and end with a different letter

$$a(a + b)^*b + b(a + b)^*a$$

4. strings with at least two occurrences of ab

$$(a + b)^*ab(a + b)^*ab(a + b)^*$$

5. strings with exactly two occurrences of ab

$$b^*a^*ab b^*a^*ab b^*a^* \quad \text{or} \quad b^*a^+b^+a^+b^+a^*$$

6. strings with at most two occurrences of ab

$$b^*a^*(ab + \lambda)b^*a^*(ab + \lambda)b^*a^* \quad \text{or} \quad b^*a^* + b^*a^+b^+a^* + b^*a^+b^+a^+b^+a^*$$

7. strings with exactly one 0 and at least one 1

$$1^*(01 + 10)1^*$$

8. strings that contain exactly two pairs of consecutive 1's (111 represents two pairs - there may be isolated 1's)

$$\text{First look at strings not containing the substring 11: } 0^*(10^+)^*1 + 0^*(10^+)^*$$

$$\text{or } 0^*(10^+)^*(1 + \lambda)$$

$$\text{or } (0 + 10)^*(1 + \lambda)$$

$$\text{Hence: } (0 + 10)^*(11(0 + 01)^*011 + 111)(0 + 01)^* \text{ or}$$

$$0^*(10^+)^*11(0^+1)^*1(0^+1)^*0^* \quad \text{or}$$

$$0^*(10^+)^*(11(0^+1)^*0^+11 + 111)(0^+1)^*0^*$$

9. $L = \{a^n \mid n \text{ is even}\}$

$$(aa)^*$$

10. $L = \{a^n \mid n \text{ is odd}\}$

$$(aa)^* a$$

11. $L = \{ \text{strings on } \{a, b\} \text{ of odd length} \}.$

$$((a+b)(a+b))^*(a+b)$$

12. $L = \{a^n b^m \mid n + m \text{ is odd}\}$

$$(aa)^* (b+a) (bb)^*$$

13. strings that contain ab

$$(a+b)^* ab (a+b)^*$$

14. strings that do not contain ab

$$b^* a^*$$

15. strings that start with ab

$$ab (a+b)^*$$

16. strings that finish with ab

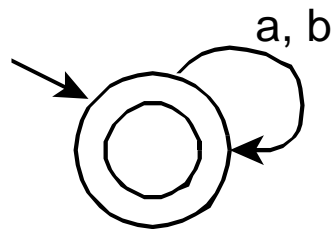
$$(a+b)^* ab$$

17. strings which contain a 1 in the third position from the end

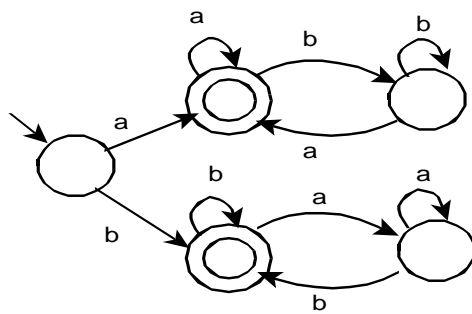
$$(0+1)^* 1 (0+1) (0+1)$$

Now for each language give an automata which accepts that language.

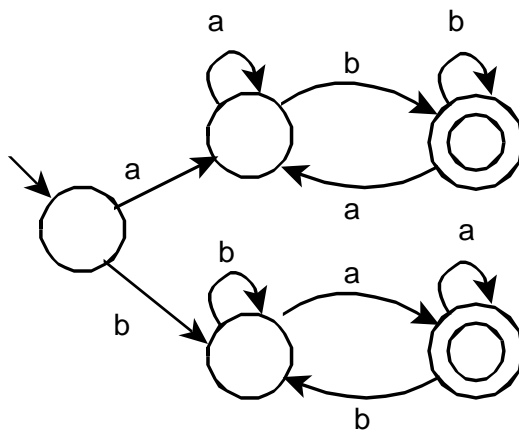
1. all strings



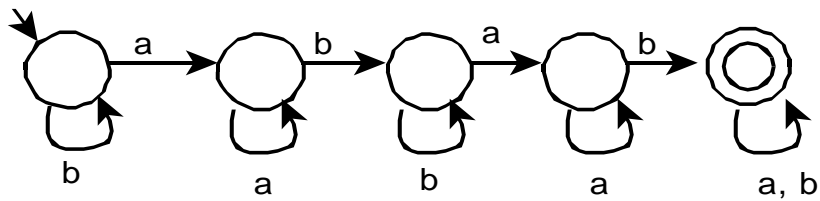
2. strings which begin and end with the same letter



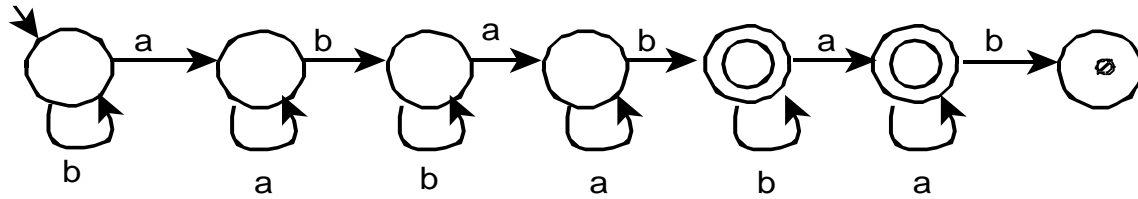
3. strings which begin and end with a different letter(assume ϵ is not in the language)



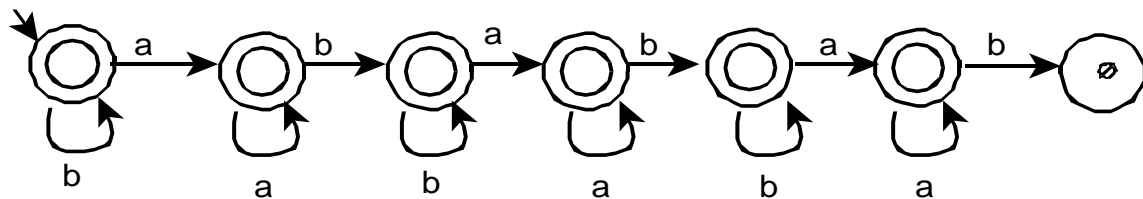
4. strings with at least two occurrences of ab.



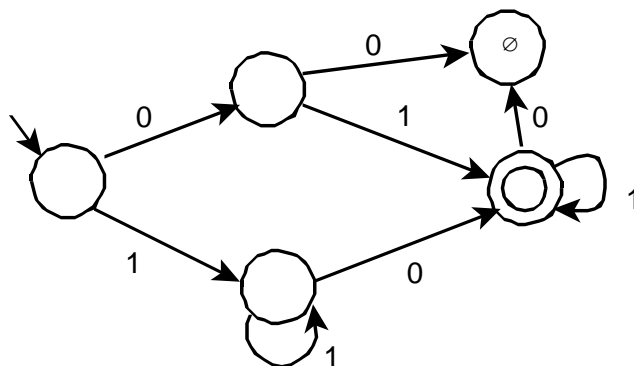
5. strings with exactly two occurrences of ab.



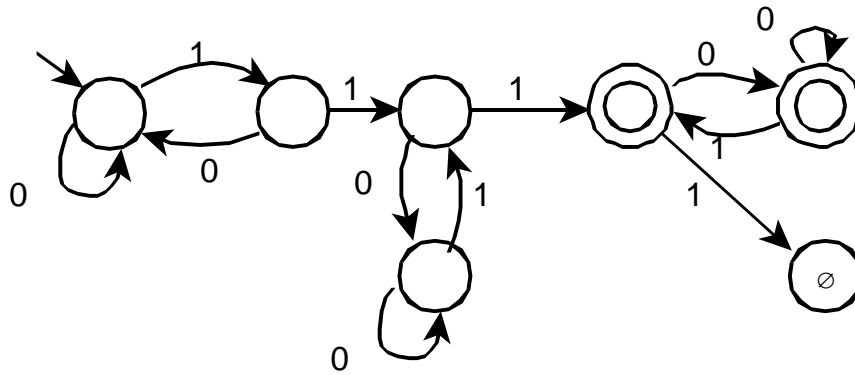
6. strings with at most two occurrences of ab.



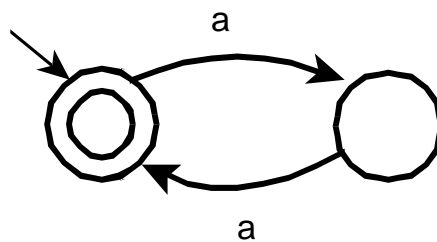
7. strings with exactly one 0 and at least one 1.



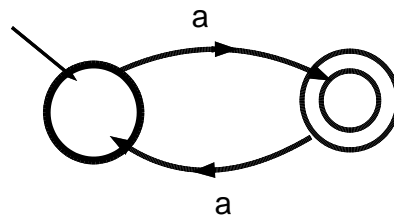
8. strings that contain exactly two pairs of consecutive 1's (111 represents two pairs - there may be isolated 1's).



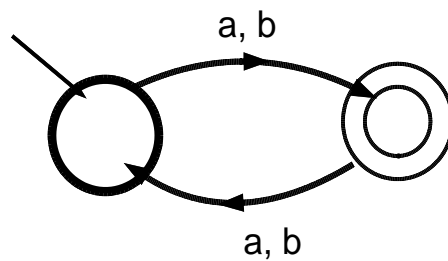
9. $L = \{a^n \mid n \text{ even} \}$. (assume alphabet is $\{a\}$)



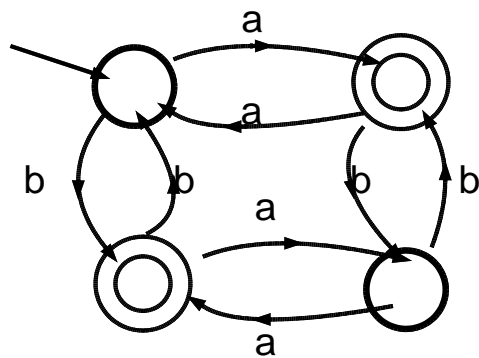
10. $L = \{a^n \mid n \text{ odd} \}$. (same)



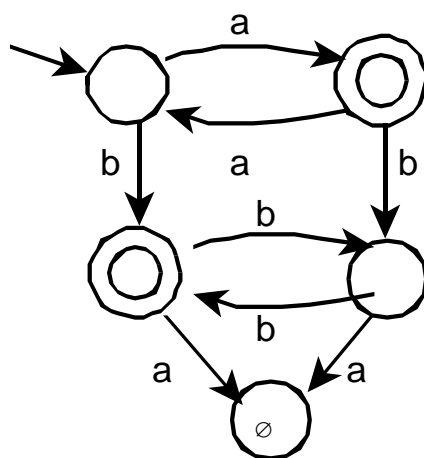
11. $L = \{ \text{strings on } \{a, b\} \text{ of odd length} \}$.



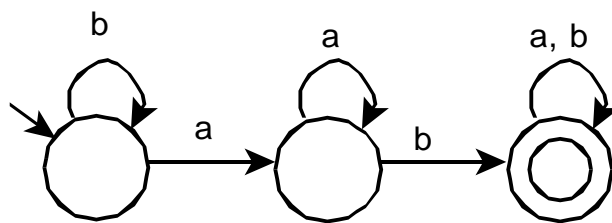
11a. $L = \{ \text{strings on } \{a, b\} \text{ where the number of } a \text{ is odd and the number of } b \text{ is even} \}$



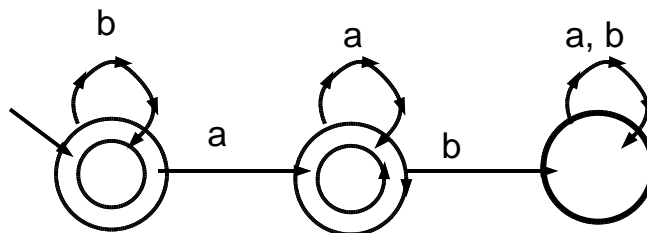
12. $L = \{a^n b^m \mid n + m \text{ is odd}\}.$



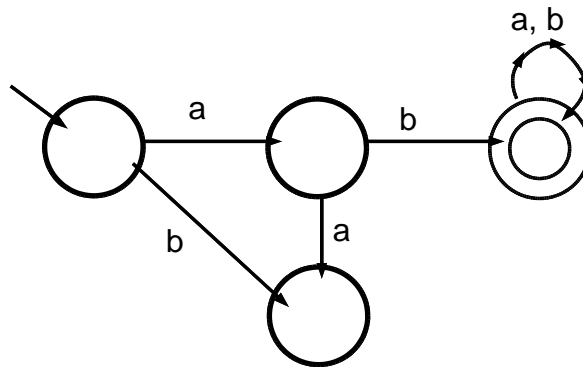
13. strings that contain ab



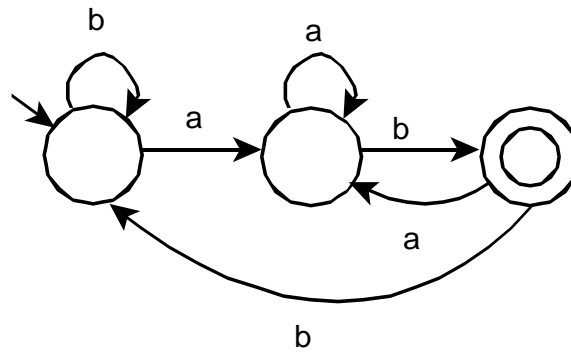
14. strings that do not contain ab



15. strings that start with ab



16. strings that finish with ab



17. strings which contain a 1 in the third position from the end
the end of these strings is one of: 100, 101, 110, 111

