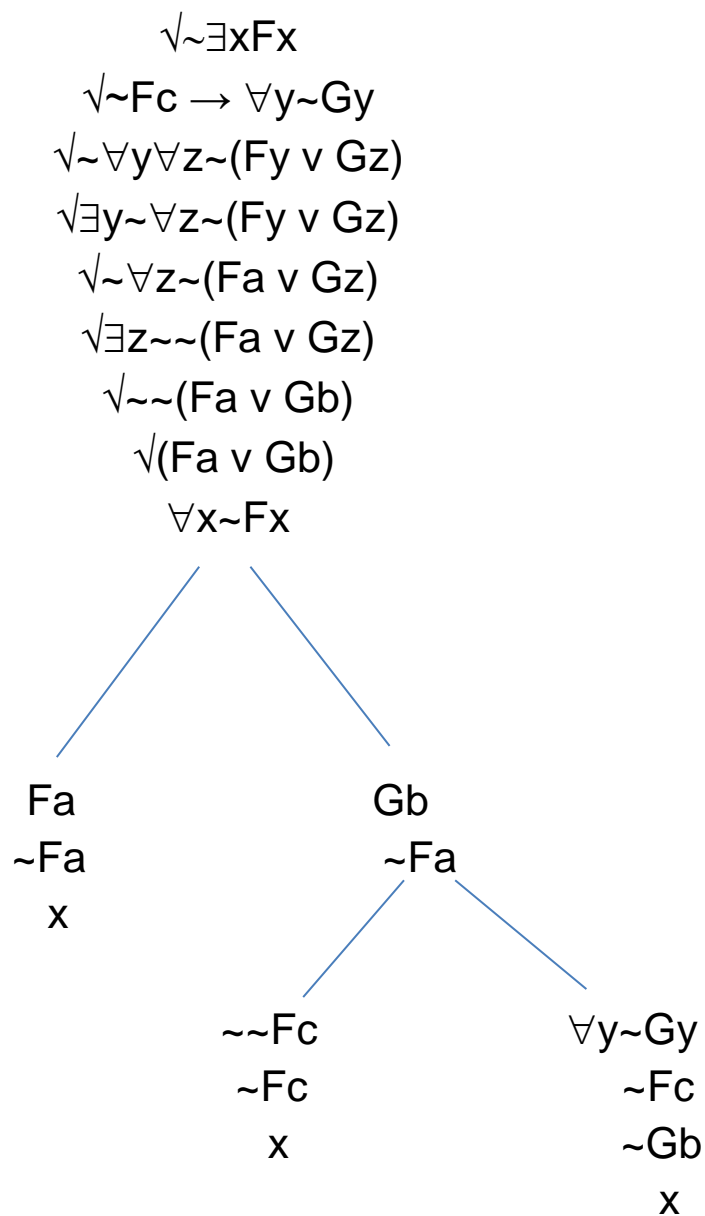


Test the following arguments for validity using trees for predicate logic.

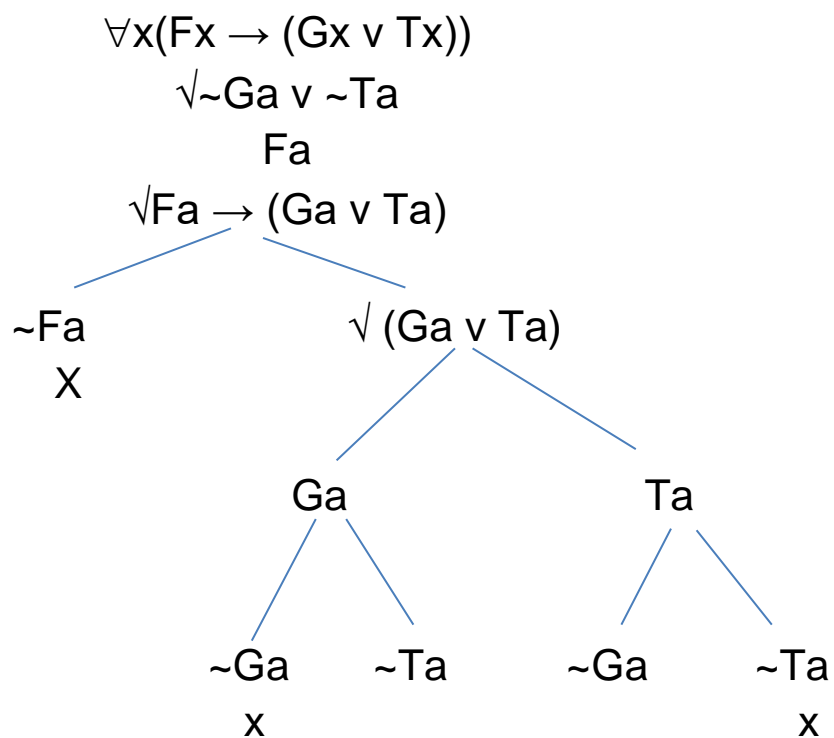
$$1. \sim\exists xFx, \sim Fc \rightarrow \forall y\sim Gy \vdash \forall y\forall z\sim(Fy \vee Gz)$$



Valid

Test the following arguments for validity using trees for predicate logic.

2. $\forall x(Fx \rightarrow (Gx \vee Tx)), \sim Ga \vee \sim Ta \vdash \sim Fa$

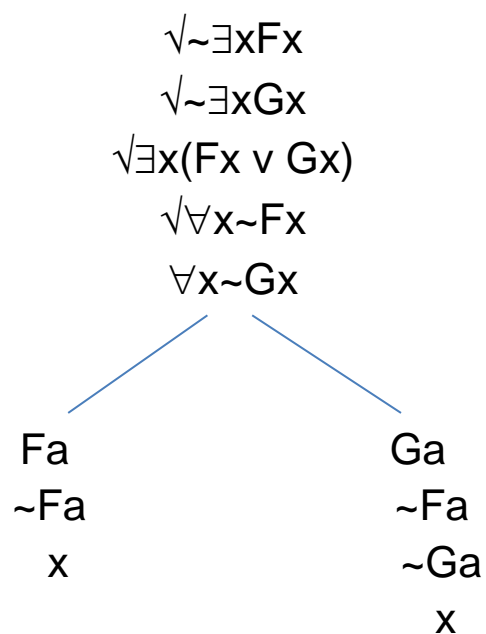


Invalid

Philosophy 160 Homework

Test the following arguments for validity using trees for predicate logic.

3. $\sim\exists xFx, \sim\exists xGx \vdash \sim\exists x(Fx \vee Gx)$



Valid

Test the following arguments for validity using trees for predicate logic.

$$4. \forall x \forall y (Fxy \leftrightarrow Fyx) \vdash \sim \exists x \exists y (\sim Fxy \& \sim Fyx)$$

$$\forall x \forall y (Fxy \leftrightarrow Fyx)$$

$$\sqrt{\exists x \exists y (\sim Fxy \& \sim Fyx)}$$

$$\sqrt{\exists y (\sim Fay \& \sim Fya)}$$

$$\sqrt{\sim Fab \& \sim Fba}$$

$$\sim Fab$$

$$\sim Fba$$

$$\forall y (Fay \leftrightarrow Fya)$$

$$\forall y (Fby \leftrightarrow Fyb)$$

$$\sqrt{Fab \leftrightarrow Fba}$$

$$\sqrt{Faa \leftrightarrow Faa}$$

$$\sqrt{Fba \leftrightarrow Fab}$$

$$\sqrt{Fbb \leftrightarrow Fbb}$$

$$Fab$$

$$\sim Fab$$

$$Fba$$

$$\sim Fba$$

$$x$$

$$Fba$$

$$\sim Fba$$

$$Fab$$

$$\sim Fab$$

$$x$$

$$Faa$$

$$\sim Faa$$

$$Faa$$

$$\sim Faa$$

$$Fbb$$

$$Fbb$$

$$\sim Fbb$$

$$\sim Fbb$$

$$Fbb$$

$$Fbb$$

$$\sim Fbb$$

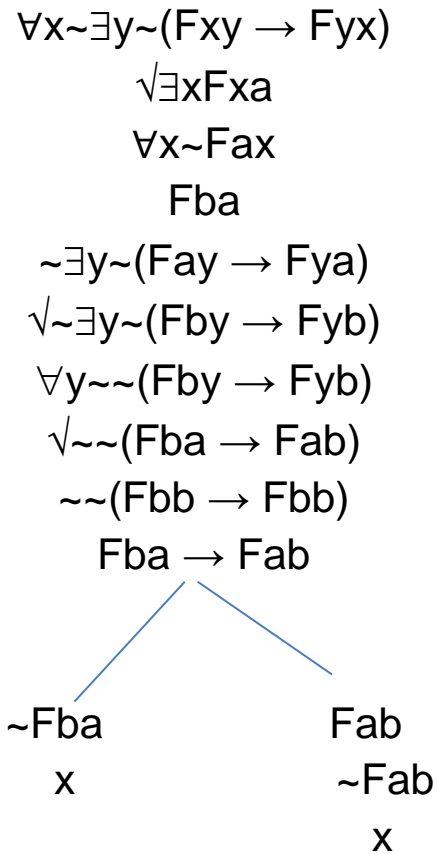
$$\sim Fbb$$

Note the necessity of these two instantiations as both x and y must be instantiated to every name in the branch. It is fairly obvious that they have no power to produce contradictions and you can avoid this in a valid argument. But invalidity will not have been totally demonstrated in their absence.

Invalid

Test the following arguments for validity using trees for predicate logic.

5. $\forall x \sim \exists y \sim (Fxy \rightarrow Fyx), \exists x Fxa \vdash \sim \forall x \sim Fax$

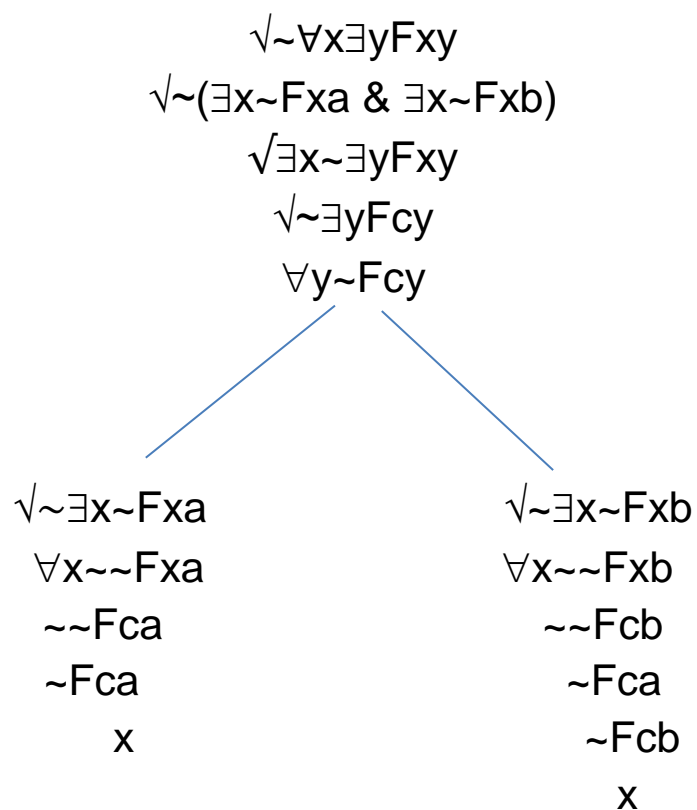


Valid

Test the following arguments for validity using trees for predicate logic.

6. $\vdash \sim \forall x \exists y Fxy \rightarrow (\exists x \sim Fxa \ \& \ \exists x \sim Fxb)$

$\sim(\sim \forall x \exists y Fxy \rightarrow (\exists x \sim Fxa \ \& \ \exists x \sim Fxb))$



Valid

Philosophy 160 Homework

Test the following arguments for validity using trees for predicate logic.

7. $\forall x \exists y (Cxy \rightarrow Cyx) \vdash Cab \rightarrow Cba$

$\forall x \exists y (Cxy \rightarrow Cyx)$

$\sqrt{\sim}(Cab \rightarrow Cba)$

Cab

$\sim Cba$

$\sqrt{\exists y (Cay \rightarrow Cyx)}$

$\sqrt{\exists y (Cby \rightarrow Cyb)}$

$Cac \rightarrow Cca$

$Cbd \rightarrow Cdb$

$\sqrt{\exists y (Ccy \rightarrow Cyc)}$

$\sqrt{\exists y (Cdy \rightarrow Cyd)}$

$(Cce \rightarrow Cec)$

$(Cdf \rightarrow Cfd)$

.

.

This pattern will repeat forever. The argument is invalid, but the machinery itself will not demonstrate this. The question then becomes how to prove that this is invalid. The answer is that you need to do a semantic proof by providing a model that contains a counterexample, i.e., an instance in which the premises are true and the conclusion false.

Proof: Interpret C as the relation of choosing. Then the premise asserts that for every named object k in the model, there exists an object m (which may or may not be identical to k) such that if k chooses m , then m chooses k . Suppose a model containing only objects a,b and d in which the following is true:

a chooses b

a chooses d

d chooses a

Nothing else chooses anything.

This models makes the premise true. It is true for every object that chooses nothing, since each of these makes the antecedent of the premise false. It is true for every object k that chooses some m , since a chooses d and d chooses a. But the conclusion is false because a chooses b and b doesn't choose a.

QED.

Test the following arguments for validity using trees for predicate logic.

$$8. \sim \exists x \exists y Gxy \vdash \exists x Gxx \rightarrow \sim \forall y Fy$$

$$\sqrt{\sim \exists x \exists y Gxy}$$

$$\sqrt{\sim (\exists x Gxx \rightarrow \sim \forall y Fy)}$$

$$\sqrt{\exists x Gxx}$$

$$\sqrt{\sim \sim \forall y Fy}$$

$$\forall y Fy$$

$$Gaa$$

$$\forall x \sim \exists y Gxy$$

$$\sqrt{\sim \exists y Gay}$$

$$\forall y \sim Gay$$

$$\sim Gaa$$

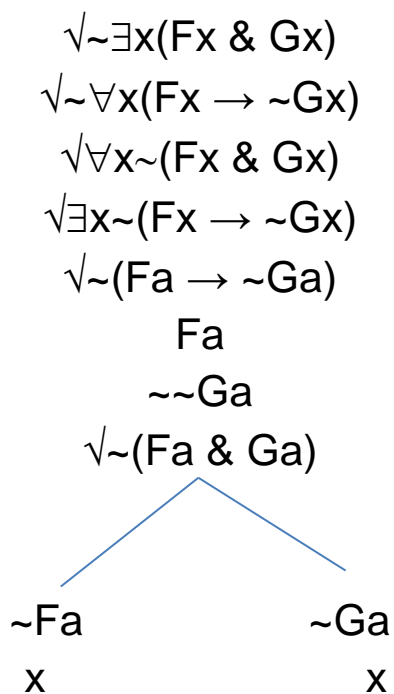
$$x$$

Valid

Philosophy 160 Homework

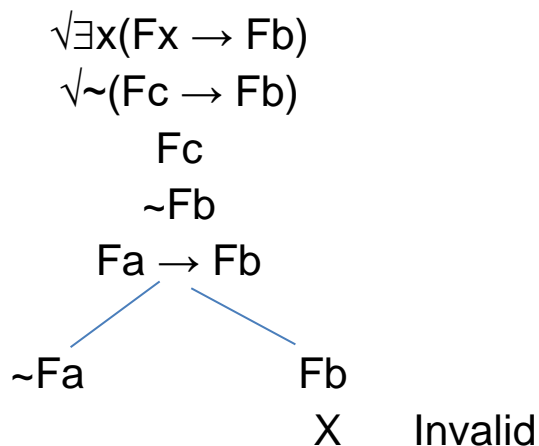
Test the following arguments for validity using trees for predicate logic.

9. $\sim\exists x(Fx \& Gx) \vdash \forall x(Fx \rightarrow \sim Gx)$

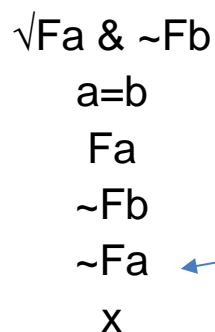


Test the following arguments for validity using trees for predicate logic.

$$10. \exists x(Fx \rightarrow Fb) \vdash Fc \rightarrow Fb$$

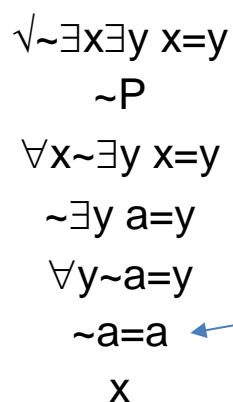


$$11. Fa \& \sim Fb \vdash \sim a=b$$



$\sim Fa$ results from substituting a for b in $\sim Fb$ according to the rule for identity.

$$12. \sim \exists x \exists y x=y \vdash P$$

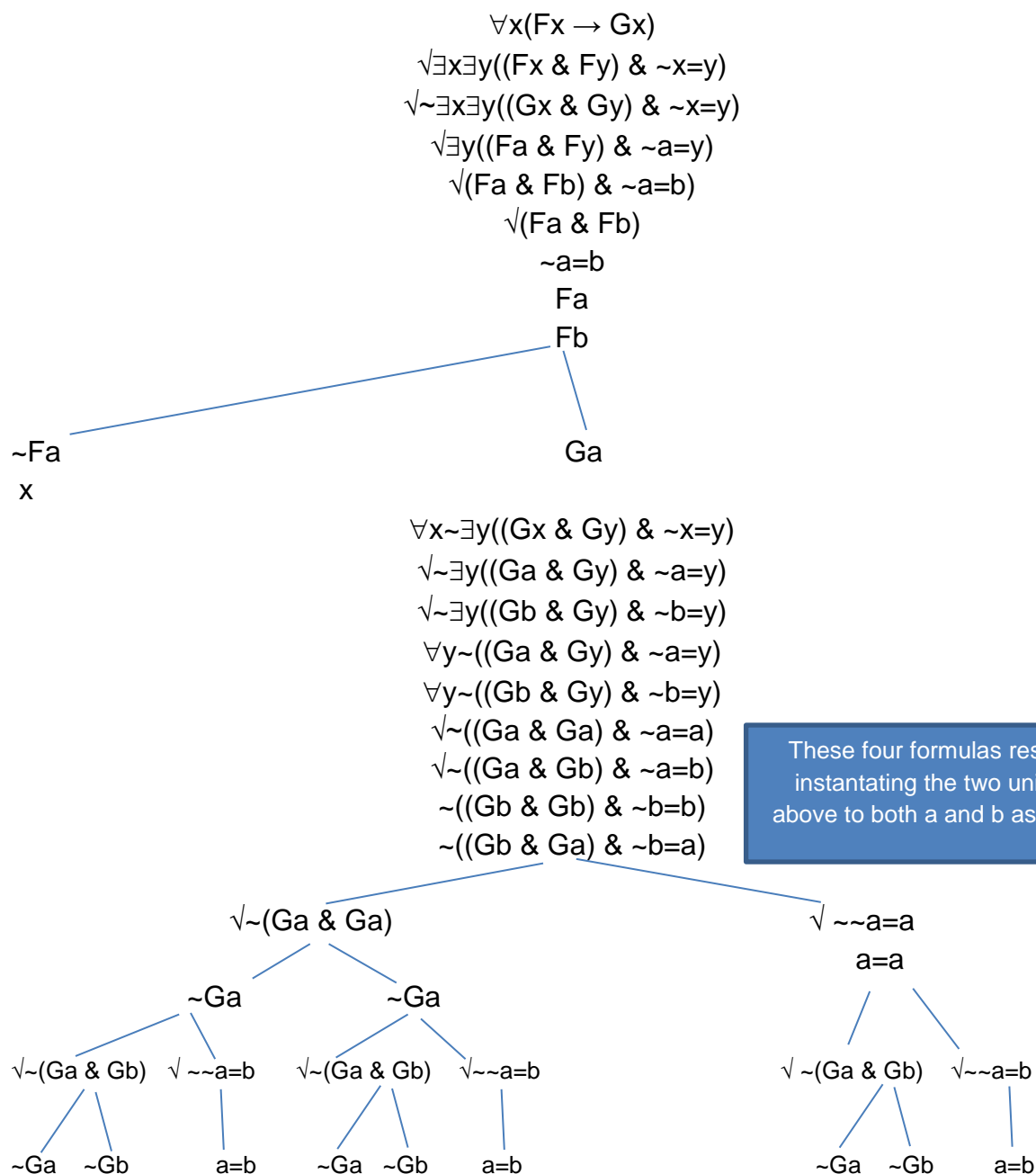


Here we instantiate the universal to a and close in accord with the rule for negated identity.

Philosophy 160 Homework

Test the following arguments for validity using trees for predicate logic.

13. $\forall x(Fx \rightarrow Gx), \exists x\exists y((Fx \& Fy) \& \sim x=y) \vdash \exists x\exists y((Gx \& Gy) \& \sim x=y)$



To finish this tree we must decompose the last two unchecked negated conjunctions above. However, no branches have closed to this point and it is clear from the content of these formulas that they have no capacity to generate contradictions. So we will just stop here and declare it obviously invalid.