160 Homework 2
Propositional calculus, basic non hypothetical rules.

Directions: Prove that each of these arguments is valid using only the following rules from the propositional calculus. Turn in the blue ones.

~E, →E, &I, &E, vI, vE, ↔E, ↔I

Important tip. The requirement to cite multiple lines in the justification of some rules is not necessarily the requirement to cite different multiple lines. The structure of some rules, such as →E will entail this, but other rules like &I and vE can sometimes involve citing the same line. Two examples:

P ⊢ P & P

1. P   A
2. P & P   1,1 &I

(P→Q) v (P → Q), (P → Q) → (M & T) ⊢ M & T

1. (P→Q) v (P → Q)   A
2. (P → Q) → (M & T)   A
3. M & T   1,2,2 vE

1. P, (P v Q) ↔ S, (S v T) ↔ M ⊢ M & P

1. P   A
2. (P v Q) ↔ S   A
3. (S v T) ↔ M   A
4. (P v Q) → S   2, ↔E
5. P v Q   1, vI
6. S   4,5 →E
7. (S v T) → M   3, ↔E
8. S v T   6, vI
9. M   7,8 →E
10. M & P   1,9 &I

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2. P & S, P ↔ R, S → T, T → R ⊢ (R v (S → K)) v T

1. P & S	A
2. P ↔ R	A
3. S → T	A
4. T → R	A
5. S	1, &E
6. T	3,5 →E
7. (R v (S → K)) v T	6, vI

3. P → Q, Q → P, (P ↔ Q) → (R →S), ~~R ⊢ S

1. P → Q	A
2. Q → P	A
3. (P ↔ Q) → (R →S)	A
4. ~~R	A
5. R	4, ~E
6. P ↔ Q	1,2 ↔I
7. R → S	3,6 →E
8. S	5,7 →E

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4. (P & M) ↔ (T & R), ~~T & ~~R ⊢ (P & T) & (R & T)

1. (P & M) ↔ (T & R)	A
2. ~~T & ~~R	A
3. ~~T	2, &E
4. ~~R	2, &E
5. T	3, ~E
6. R	4, ~E
7. T & R	5,6 &I
8. (T & R) → (P & M)	1, ↔E
9. P & M	7,8 →E
10. P	9, &E
11. M	9, &E
12. (P & T)	5,10 &I
13. (R & T)	5,6 &I
14. (P & T) & (R & T)	12,13 &I

5. P v S, P → (M v N), S → (M v N) ⊢ (S ↔ P) v (K v (M v N))

1. P v S	A
2. P → (M v N)	A
3. S → (M v N)	A
4. M v N	1, 2, 3 vE
5. K v (M v N)	4, vI
6. (S ↔ P) v (K v (M v N))	5, vI

6. L v (T & S), L ↔ (J → Q), (J → Q) ↔ (T & S), ~~J ⊢ Q

1. L v (T & S)	A
2. L ↔ (J → Q)	A
3. (J → Q) ↔ (T & S)	A
4. ~~J	A
5. (T & S) → (J → Q)	3, ↔E
6. L → (J → Q)	2, ↔E
7. J → Q	1,5,6 vE
8. J	4, ~E
9. Q	7,8 →E

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7. T → (R v S), (R v S) → T, (T ↔ (R v S)) → (P v M), P → T,
   M → T ⊢ T v (R v S)

1. T → (R v S)   A
2. (R v S) → T   A
3. (T ↔ (R v S)) → (P v M)   A
4. P → T   A
5. M → T   A
6. T ↔ (R v S)   1,2 ↔I
7. P v M   3,6 →E
8. T   7,4,5 vE
9. T v (R v S)   8, vI