Predicate logic translation practice

Directions: Translate the following into predicate logic. Use the first letters of names and predicates. If predicates or names have the same first letter, then use the second letter for one of them, as shown in the first exercise.

Solutions: When more than one formula is given, they are logically equivalent and both correct.

1. Anyone who dates Donald is desperate.

   $\forall x(Dxd \rightarrow Ex)$

2. There are no cats.

   $\sim\exists xCx$

   $\forall x\sim Cx$

3. Cats are robots.

   $\forall x(Cx \rightarrow Rx)$

   $\sim\exists x(Cx \,\&\, \sim Rx)$

4. If cats are robots, then there are no cats.

   $(\forall x(Cx \rightarrow Rx) \rightarrow \sim\exists yCy)$

   $(\sim\exists x(Cx \,\&\, \sim Rx) \rightarrow \forall x\sim Cx)$

5. Mable loves cats.

   $\forall x(Cx \rightarrow Lmx)$

   $\sim\exists x(Cx \,\&\, \sim Lmx)$

6. Everyone loves Mable.

   $\forall xLxm$

   $\sim\exists x\sim Lxm$

7. Mable only loves cats.

   $\forall x(Lmx \rightarrow Cx)$

   $\sim\exists x(Lmx \,\&\, \sim Cx)$

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8. Mable loves Playstation.

Lmp

9. Someone walks.

$\exists xWx$

$\sim\forall x\sim Wx$

10. Someone walks but does not talk.

$\exists x(Wx \ \&\ \sim Tx)$

$\sim\forall x(Wx \rightarrow Tx)$

11. Jon’s hairdresser knows all of his secrets.

$\exists x(Hxj \ \&\ \forall y(Syj \rightarrow Kxy))$

Translation: There exists an x such that x is a hairdresser of John and for all y, if y is a secret of John, x knows y.

12. Matilda gave Bob a job.

$\exists x(Jx \ \&\ Gmbx)$

Note: G is a three place relation meaning. Gabc means, “a gave b to c.”

13. If a toaster isn’t working then it’s not plugged in.

$\forall x((Tx \ \&\ \sim Wx) \rightarrow \sim Px)$

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14. Janice works for Yelp!.

    Wje

    Note: y is a variable in predicate logic, so use ‘e’ for the name Yelp!

15. Maelynn avoids Todd.

    Amt

16. Maelynn avoids toads.

    $\forall x(Tx \rightarrow Amx)$

17. Dogs are better than cats, but cats are better than jellyfish.

    $\forall x \forall y \forall z((Dx \ \& \ Cy \ \& \ Jz) \rightarrow (Bxy \ \& \ Byz))$

    Note: When strings of conjunctions and strings of disjunctions of <u>atomic wffs or subwffs</u> are present, it is conventional to drop parentheses as above.

    It is also conventional to drop the two outermost parentheses, which will be done in the remaining problems. Remember, this only means dropping parentheses that both begin and end a formula. It would not allow you, for example, to drop any parentheses from the above formula.

18. There is some cat that is worse than a jellyfish.

    $\exists x \exists y(Cx \ \& \ Jy \ \& \ Wxy)$

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19. There is a barber that shaves himself.

$\exists x(Bx \,\&\, Sxx)$

20. Not all mosquitoes carry malaria.

$\sim\forall x\forall y((Mx \,\&\, Ay) \rightarrow Cxy)$

$\exists x\exists y(Mx \,\&\, Ay \,\&\, \sim Cxy)$

21. There is nothing Jerry won't do to get laid.

$\sim\exists x(Ax \,\&\, Lxj \,\&\, \sim Djx)$

Translation: It is not the case that there exists an x such that x is an action and x lays Jerry and Jerry does not do x.

22. The only dog in the yard is a collie.

Note: This sentence involves the definite description "the yard" which complicates things. The following translation would be a good first pass:

$\forall x\forall y((Dx \,\&\, Yy \,\&\, Ixy) \rightarrow Cx)$

Translation: For all x and y, if x is a dog and y is a yard and x is in y, then x is a collie. Note that this is a weak interpretation that does not imply that would be true even if there are no dogs in the yard. A stronger interpretation would be:

$\exists x\exists y((Dx \,\&\, Yy \,\&\, Ixy) \,\&\, (\forall z(Dz \,\&\, Izy) \rightarrow Cz))$

Translation: There exists an x and a y such that x is a dog and y is a yard and x is in y, and for all z, if z is a dog in the yard, it is a collie. Note that this translation requires that there is at least one dog in at least one yard but adds that any dog in <u>any</u> yard is a collie. This is <u>much too strong</u> an interpretation. The reason is because the statement only concerns one particular yard, not every yard. This is established by the definition description "the yard."

One way to recast this statement in English is as follows:

> There is one and only one dog in the yard, and it is a collie.

This, however, does not tell us how to deal with the definition description. We noted in class that Bertrand Russell suggested that definition descriptions should be treated as follows:

The king is dead

$$\exists x((Kx \,\&\, (\forall y Ky \rightarrow x=y)) \,\&\, Dx)$$

Translation: There is one and only one king and he is dead. The problem with this as a realistic interpretation is that we rarely mean anything so extreme. There may be many kings, and we rely on the context of utterance for people to understand which king "the king" is picking out. The same holds true with "the yard." Nobody who utters this is really saying that there is one and only one yard.

One way to deal with this is to define a universe of discourse. We can say that universe of discourse is "things in the yard" and this will mean that we can simply leave out the expression "in the yard" since it now understood that we are talking about things in the yard. Under these conditions, the translation can simply be:

$$\exists x((Dx \,\&\, \forall y(Dy \rightarrow y=x)) \,\&\, Cx)$$

Translation: There is one and only one dog and it is a collie. Where the phrase "in the yard" is not translated but understood to be defining the universe of discourse.

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23. The only cat in the basement is Tootsie.

Universe of discourse: Things in the basement.

$Ct \mathbin{\&} \forall y(Cy \rightarrow y=x)$

Since we are now only talking about things in the basement, when we say that Tootsie is a cat, it implies that Tootsie is in the basement, and the rest of the formula tells us that she is the only cat.

24. There is a barber who only shaves himself.

$\exists x((Bx \mathbin{\&} Sxx) \mathbin{\&} \forall y(Sxy \rightarrow x=y))$

Translation: There is a x who is a barber and shaves himself and for all y, if x shaves y, then x is y.

24. You can’t divide by zero.

Universe of discourse: Numbers

$\forall x{\sim}Dxe$

Translation: For all x it is not the case that x divides by zero. Note that here the letter name ‘e’ is being used to stand for the number zero. Numbers like 0,1,2,3 are not names in predicate logic, but objects which have to be named. Though we can easily extend predicate logic to permit numbers as names if we want to.

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25. Shantell lied under oath.

$\exists x(Ox \ \& \ Usx \ \& \ Ls)$

Translation: There exists an x such that x is an oath and Shantell is under it and Shantell lied. This is about the best we can do. Our translation does not actually capture the implication that Shantell lied <u>while under oath</u>. In other words, the predicate logic statement would be true if Shantell were under oath at one point in time and lied at some other point in time. To capture tense in predicate logic (past, present, future) we must move on to something called <u>temporal logic</u>, which is beyond our scope.