Probability and Statistics for Engineering and the Sciences (8th Edition)

Chapter 4, Problem 40E (5 Bookmarks) Show all steps: ON

Problem

The article “Monte Carlo Simulation—Tool for Better Understanding of LRFD” (*J. of Structural Engr.,* 1993: 1586–1599) suggests that yield strength (ksi) for A36 grade steel is normally distributed with μ = 43 and σ = 4.5.

a. What is the probability that yield strength is at most 40? Greater than 60?

b. What yield strength value separates the strongest 75% from the others?

Step-by-step solution

Step 1 of 3

The given is dealing about the Z-scores. A *Z*-score indicates how many standard deviations an element is from the mean. And a *Z*-score is a numerical measurement of a value’s relationship to the mean in a group of values. *Z*-scores may also be positive or negative.

Let the random variable *X* represents the yield strength for A36 grade steel.

Given that the random variable *X* follows normal distribution with mean $\mu = 43$and standard deviation $\sigma = 4.5$.

(a)

Probability that yield strength is at most 40 is calculated as follows:

$$\begin{aligned} P(X \le 40) &= P\left(\frac{X-\mu}{\sigma} \le \frac{40-43}{4.5}\right) \\ &= P(Z \le -0.67) \\ &= 1 - P(Z \le 0.67) \\ &= 1 - 0.7486 \qquad \text{From normal tables} \\ &= \boxed{0.2514} \end{aligned}$$

Therefore, the probability that yield strength is at most 40 is 0.2514.

Comment

Step 2 of 3

Probability that yield strength greater than 60 is calculated as follows:

$$\begin{aligned} P(X > 60) &= 1 - P(X \le 60) \\ &= 1 - P\left(\frac{X-\mu}{\sigma} \le \frac{60-43}{4.5}\right) \\ &= 1 - P(Z \le 3.78) \\ &= 1 - \Phi(3.78) \\ &= 1 - 1 \\ &= \boxed{0} \end{aligned}$$

Therefore, the probability that yield strength greater than 60 is 0

Comment

Step 3 of 3

(b)

The Z-score is given by,

$$Z = \frac{x - \mu}{\sigma}$$

Yield strength value separates the strongest 75% from the others

Here, the objective is to find the value of Z which separates the above 75%.

Therefore, the area to the left of z is 25%.

$$P(X \le x) = 0.25$$

Therefore,

$$Z_{0.25} = \frac{x - \mu}{\sigma}$$

$$x = \mu + Z_{0.25}\sigma$$

$$x = 43 + (-0.67)(4.5) \quad (\text{Using the excel function NORMSINV}(0.25))$$

$$= \boxed{39.985}$$

Comments (1)

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Recommended solutions for you in Chapter 4

Chapter 4, Problem 39E

a. If a normal distribution has μ = 30 and σ = 5, what is the 91st percentile of the distribution? b. What is the 6th percentile of the distribution? c. The width of a line etched on an integrated circuit chip is normally distributed with...

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Chapter 4, Problem 5E

A college professor never finishes his lecture before the end of the hour and always finishes his lectures within 2 min after...

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