Philosophy 160
Test 3

Name :

1. (2pts.) Derive the following in propositional logic.

$S \rightarrow (R \,\&\, Q),\ P \rightarrow ({\sim}R \,\&\, {\sim}T) \vdash (P \,\&\, S) \rightarrow N$

1.	$S \rightarrow (R \,\&\, Q)$	A
2.	$P \rightarrow ({\sim}R \,\&\, {\sim}T)$	A
3.	$(P \,\&\, S)$	H
4.	P	3, &E
5.	S	3, &E
6.	$R \,\&\, Q$	1,5 →E
7.	${\sim}R \,\&\, {\sim}T$	2,4 →E
8.	R	6, &E
9.	${\sim}R$	7, &E
10.	N	8,9 CON
11.	$(P \,\&\, S) \rightarrow N$	3-10, →I

2. (2 pts.) Derive the following in predicate logic.

$\forall x \forall y({\sim}Pxy \rightarrow {\sim}x{=}y) \vdash \exists x \exists y Pxy$

1.	$\forall x \forall y({\sim}Pxy \rightarrow {\sim}x{=}y)$	A
2.	$\forall y({\sim}Pay \rightarrow {\sim}a{=}y)$	1, ∀E
3.	${\sim}Paa \rightarrow {\sim}a{=}a$	2, ∀E
4.	${\sim}\exists x \exists y Pxy$	H ~I
5.	$\forall x {\sim}\exists y Pxy$	4, QE
6.	${\sim}\exists y Pay$	5, ∀E
7.	$\forall y {\sim}Pay$	6, QE
8.	${\sim}Paa$	7, ∀E
9.	${\sim}a{=}a$	3,8 →E
10.	$a{=}a$	=I
11.	$P \,\&\, {\sim}P$	9,10 CON
12.	${\sim}{\sim}\exists x \exists y Pxy$	4-11 ~I
13.	$\exists x \exists y Pxy$	12, DN

3. Translate and prove: (2pts.)

Whoever fooled Ezekiel is a logician.
There are no logicians besides Mitch.
So Mitch fooled Ezekiel.

Translation:

$\exists xFxe \mathbin{\&} \forall y(Fye \to Ly)$
$Lm \mathbin{\&} \forall x(Lx \to x{=}m)$
$\vdash Fme$

1.	$\exists xFxe$	A
2.	$\forall y(Fye \to Ly)$	A
3.	Lm	A
4.	$\forall x(Lx \to x{=}m)$	A
5.	$\quad\vert\ Fae$	H $\exists$E
6.	$\quad\vert\ Fae \to La$	2, $\forall$E
7.	$\quad\vert\ La$	5,6 $\to$E
8.	$\quad\vert\ La \to a{=}m$	4, $\forall$E
9.	$\quad\vert\ a{=}m$	7,8 $\to$E
10.	$\quad\vert\ Fme$	5,9 =E
11.	Fme	5-10 $\exists$E

4. (2pts.) Derive in Leibnizian modal logic.

$\Box(\sim Q \to \sim P), \Box P \vdash \Diamond Q$

1.	$\Box(\sim Q \to \sim P)$	A
2.	$\Box P$	A
3.	$\Box\sim Q \to \Box\sim P$	1, K
4.	$\quad\vert\ \Box\sim Q$	H ~I
5.	$\quad\vert\ \Box\sim P$	3,4 $\to$E
6.	$\quad\vert\ \sim P$	T
7.	$\quad\vert\ P$	2, T
8.	$\quad\vert\ P \mathbin{\&} \sim P$	6,7 &I
9.	$\sim\Box\sim Q$	4-8, ~I
10.	$\Diamond Q$	9. Dual

5. (2 pts.) Derive in Meinongian free logic

$\exists x\square Fx, \forall x\square(Fx \rightarrow Gx) \vdash \exists x\square Gx$

1.	$\exists x\square Fx$	A
2.	$\forall x\square(Fx \rightarrow Gx)$	A
3.	$\mid\ \square Fa \,\&\, \exists x\, x{=}a$	H (F∃E)
4.	$\mid\ \exists x\, x{=}a$	3, &E
5.	$\mid\ \square(Fa \rightarrow Ga)$	2,4 F∀E
6.	$\mid\ \square Fa$	3, &E
7.	$\mid\ \square Fa \rightarrow \square Ga$	5, K
8.	$\mid\ \square Ga$	6,7 →E
9.	$\mid\ \exists x\square Gx$	4,8 F∃I
10.	$\exists x\square Gx$	1, 3-9 F∃E

6. (2pts.) Prove using natural deduction.

$\mathcal{P}(A) \subseteq \mathcal{P}(B) \vdash A \subseteq B$

1.	$\mathcal{P}(A) \subseteq \mathcal{P}(B)$	A
2.	$\mid\ a \in A$	H →I
3.	$\mid\ \forall x(x \in \mathcal{P}(A) \rightarrow x \in \mathcal{P}(B))$	1, def ⊆
4.	$\mid\ \{a\} \in \mathcal{P}(A)$	2, def $\mathcal{P}$
5.	$\mid\ \{a\} \in \mathcal{P}(A) \rightarrow \{a\} \in \mathcal{P}(B)$	3, ∀E
6.	$\mid\ \{a\} \in \mathcal{P}(B)$	4,5 →E
7.	$\mid\ a \in B$	6, def $\mathcal{P}$
8.	$a \in A \rightarrow a \in B$	2-7, →I
9.	$\forall x(x \in A \rightarrow x \in B)$	8, ∀I
10.	$A \subseteq B$	9, def ⊆

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7. (5 pts. 1/5 pt. each) Evaluate the following formulas with respect to the model provided. Most evaluations are truth values. Of these, some may be necessary truths or necessary falsehoods which would be true or false in any model. Some formulas define sets, which you should indicate using roster notation, i.e., members listed inside of brackets. Write your answers in the space on the left. If there is not enough room, write them on the right. Note: The bold **X** signifies Cartesian product. Two vertical bars |x| always refers to cardinality.

F 1. $a \in a$

F 2. $c \subseteq d$

F 3. $(2,1) \notin a \mathbf{X} b$

F 4. $\emptyset \in \emptyset$

T 5. $\forall x((x \subseteq \emptyset) \rightarrow x \notin x)$

T 6. $\mathcal{V}(\sim\Diamond a \subseteq b, w_2)$

F 7. $\mathcal{V}(\{1,2,3,4\} = (a \cap b), w_2)$

{b,d} 8. $\mathcal{V}(O, w_1)$

Bad 9. $\mathcal{V}(\Diamond Nbc \rightarrow \exists x(x \in d \ \& \ Vx), w_2)$

T 10. $\mathcal{V}(\sim\Box Scb, w_2)$

T 11. $\mathcal{V}(\forall x\ (Nax \rightarrow \sim|a| = |x|), w_2)$

T 12. $\mathcal{V}(\Box\forall x(\sim x=a \rightarrow |x| < |a|), w_3)$

∅ 13. $\mathcal{D}w_2 \cap (a \cup b)$

F 14. $\mathcal{V}(O,w_3) \in \mathcal{D}w_1$

15. $\mathcal{P}(a) \cap \mathcal{P}(b) =$ {∅,{1},{2},{3}, {1,2}, {2,3}, {1,3}, b}

{b} 16. $\{x|\ x \in \mathcal{D}w_4 \ \& \ Ox\}$

F 17. $\mathcal{V}(\{x|\ x \in \mathcal{D}w_3\} \subset \{y|\ y \in \mathcal{D}w_4\})$

F 18. $\mathcal{V}(\forall x \forall y |x \cap y| = 2, w_2)$

T 19. $\mathcal{V}(b \subseteq c \rightarrow \Box\emptyset \in \mathcal{P}(a), w_2)$

F 20. $\mathcal{V}(\Box\forall y \forall z\ |y \mathbf{X} z| > 1, w_1)$

21. $\mathcal{D}w_3 \mathbf{X} \mathcal{D}w_3$ {(a,a), (a,d), (d,a), (d,d)}

F 22. $\mathcal{V}(\Box\forall x \exists y(Syx \rightarrow |y| < |x|), w_3)$

F 23. $\mathcal{V}(\Diamond\forall x \forall y((Vx \lor Vy) \rightarrow Sxy), w_1)$

T 24. $\mathcal{V}(\mathcal{D}w_2 \in \mathcal{P}(\mathcal{D}w_1))$

F 25. $\mathcal{V}(\Diamond\exists x \exists y(Vx \ \& \ Oy \rightarrow x=y), w_1)$

8. (2 pts. ½ point each.) Calculate the sequences or summations for the first four members of each of the following. Begin with n=1 in each case.

a. $a_n = (2n - 2)^2$

(0, 4, 16, 64)

b. $a_n = (a_{n-1})^2 + n$, $a_0 = 0$

(1, 3, 12, 148)

c. $a_n = a_{n-1} \cdot (a_{n-2} + 2)$, $a_0 = 1$, $a_1 = 3$

(3, 9, 45, 495, 23,265) **(Starting with n= 1 or n=2 both acceptable.)**

d. $\sum_{j=1}^{4} (a_{j-1} + 2)$ **=** **4 + 6 + 8 + 10 = 28**

a_o=2

9. (2 pts. 1/5 pt. each.) For 1-6, state whether each is (a) injective, (b) surjective, (c) bijective or (d) a function but neither of the preceding and not a function. If a function is bijective, you must say it is bijective to get credit. Use the blank provided on the left, and the symbols I, S, B, F, ~F respectively. For 9 & 10 you must answer the "How do you know?" question correctly to get credit.

$\mathbb{R}$ = real numbers
$\mathbb{Q}$ = rational numbers
$\mathbb{N}$ = {0, 1, 2, 3….}
$\mathbb{Z}$ = all integers {…-3, -2, -1, 0, 2, 3….}
+, - superscripts indicate restriction to positive or negative numbers, 0 being neither.
(a,b) = open interval, meaning a and b are not included.
[a,b] = closed interval, meaning a and b are included.

F 1. f: $\mathbb{N} \rightarrow \mathbb{Z}^+$ $f(x) = x^2$

~F 2. f: $\mathbb{Z}^+ \rightarrow \mathbb{Z}$ $f(x) = \sqrt{2}$

F/S 3. f: $\mathbb{Z}^+$ **x** $\mathbb{Z}^+ \rightarrow \mathbb{N}$ $f(x,y) = x + y$ **(Nothing maps on to 0; accept S)**

F 4. f: $\mathbb{N} \rightarrow \mathbb{Q}^+$ $f(x) = \frac{1}{x}$

F 5. f: $\mathbb{Z}^+ \rightarrow \mathbb{Z}^+$, g: $\mathbb{Z}^+ \rightarrow \mathbb{Z}^+$, $f \circ g$ when $g(x) = x^2$ and $f(x) = x$

F 6. f: $\mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ **x** $\mathbb{Z}^+$ $f(x) = (x, x+1)$ **(Nothing maps on to (n, ≤n))**

Y 7. Is there a bijective function from the real number intervals (0,1) → (0,1) ?

Y 8. Is there a bijective function from the real number intervals (0,1) → (0,1] ?

Y 9. Is there a bijective function f: $\mathbb{Z}^+ \rightarrow \mathbb{Q}^+$ How do you know?

Cantor showed there is a one-to-one correspondence between the positive integers and the ratioinal numbers.

N 10. Is there a bijective function f: $\mathbb{Z}^+ \rightarrow \mathbb{R}$? How do you know?

Cantor showed there can be no one-to-one correspondence between the positive integers and the reals.

Essay questions: (2 pts each.)

Either do 2 of the 3 essay questions selected or do all 3 and line out one of proofs 1-6 above to be ungraded. You must line out one of the proofs if you choose this option. Otherwise your third question below will not be graded.

Essay 1: Number ____

Essay 2: Number ____

Essay 3: Number ____

Essay questions

1. Explain the difference between (P → Q) in classical propositional logic and □(P → Q) in Leibnizian modal logic with specific reference to the truth conditions of each. Is there any valid formula *A* in classical logic for which □*A* may sometimes be false in Leibnizian modal logic? Explain why or why not, and explain how the derivation system of Leibnizian modal logic reflects your answer.

2. Characterize Meinongian free logic as an extension of modal predicate logic, explaining what it attempts to accomplish and how it does so. Your explanation should refer to a problem inherent in both classical predicate logic and Leibnizian modal logic regarding necessary existence. Explain how this is dealt with both in the semantics and in the derivation system.

3. Explain the steps involved in (weak) mathematical deduction as it would apply to the claim that $\sum_{k=1}^{n} 2k - 1 = n^2$. Prove that mathematical induction itself is deductively valid. Your proof may be in English (i.e., it needn't be an explicit natural deduction proof) but it must be explicit.

4. Summarize Cantor's proof that the positive integers can not be placed into one-to-one correspondence with the reals and explain what unanswered question this raises about the relation between the cardinality of these two sets.

5. Explain why it is not accurate to say that all well-ordered sets can be put into one-to-one correspondence with the natural numbers in a way that makes essential use of the distinction between cardinal numbers and ordinal numbers.

6. Summarize the logical basis of (a) Russell's Paradox and (b) the Burali-Forti Paradox and identify the way in which they are similar. What sort of revision to set theory was made to deal with them both?

7. Explain the difference between the soundness, completeness and decidability of a formal system. In each case provide an example of a system that has the property and a system that lacks it. State Church's theorem and Gödel's incompleteness theorem. Explain (a) why the first does not imply the second and (b) why both results are compatible with the completeness of predicate logic.

Model for problem # 7

The model $\mathcal{V}$ below depicts a universe of four possible worlds $\mathcal{W}_{\mathcal{V}}$ = {w_1, w_2, w_3, w_4}. The constituents of the worlds are <u>sets</u>, defined in the table below. Since sets are defined by their membership, the membership of a set does not vary across worlds.

The constituents of these worlds have relative positions designated by the two-place predicates N, S, E, W for north, south, east and west. So, e.g., $\mathcal{V}$(Sba,w_1) = T. (Note that "south of" does <u>not</u> mean "due south of"; i.e., b,c and d are all south of a in w_1. The same goes for the other relations.)

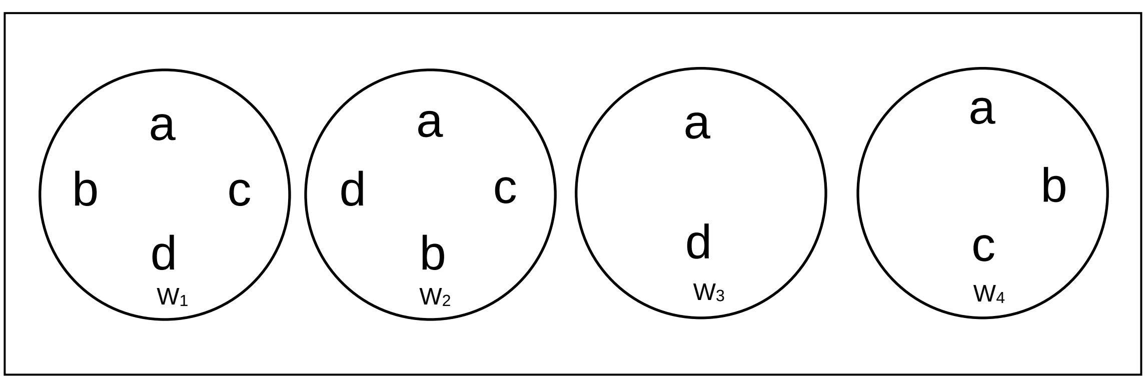
Additionaly, we will speak of the sets as having either an even or an odd <u>number</u> of members, designated by the one-place predicates V and O. Note that this is a property of the <u>sets</u>, not the members of the sets. You will not see formulas attributing these properties to the members of the sets themselves.

Two-place predicates do not meaningfully apply to set operations. For example, we will not speak of the intersection of two sets as having a relative position. However, one-place predicates do apply to set operations. So, for example, $\mathcal{V}$(E(a∩b), w_1) = F

As the members of the sets are integers, the usual mathematical relations apply to them.

Remember that set theoretic notation applies to both the domains of the worlds and the values of the predicates in those worlds. For example, $\mathcal{D}w_3$ ={a,d}. $\mathcal{V}$(E,w_1) = {a,c}

Finally, you will be asked to evaluate both formulas that specify a relation to a world and formulas that do not. In the latter case, we will drop the $\mathcal{V}$ notation and explicitly ask you to determine the truth and falsity of formulas or to compute their values. For example, a $\subseteq$ b = F. a ∩ b = {1, 2, 3}



a = {1, 2, 3, 4}	b = {1, 2, 3}	c = {1, 2}	d = {1}