Engr 17 - CSUS Instructor – Russ Tatro

Chapter 9 Sinusoidal Steady-State Analysis – Homework Solutions
James W. Nilsson and Susan A. Riedel, *Electric Circuits*, 6th Edition, 2001

Problems 3 a thru h, 6 a & b, 9, 14, 16, 23, 29, 36, 39, 45, 47, 48, 49

3
a) 170 V
b) 60 Hz
c) 376.99 rads/sec
d) –1.05 rads
e) –60°
f) 16.67 msec
g) t = 2.78 msec
h) v(t) = 170 sin (120 π t) Volts

6
a) y = 483.8 cos (300 t – 48.49°)
b) y = 120.5 cos (377 t + 4.8°)

9
a) Zc = -j 20 Ω and Z_L = j 2 Ω. Redraw the circuit with all the phasor info
b) **V** = 46.5 < 34.46° Volts
c) $v_{ss}(t)$ = 46.5 cos (50,000 t + 34.46°) Volts

14
$i_{ss}(t)$ = 1.5 cos (5,000 t + 36.87°) mA

16
a) f = 5/π ≅ 1.59 Hz
b) $i_{ss}(t)$ = 50 cos (10 t) mA

23
Zab = 50 < -53.15° Ω in polar form

29
a) Z = 5 < 72° Ω
b) T = 50 µs

36
Z_{TH} = 3,500 – j 12,000 Ω and also redraw the equivalent circuit

39
Z_{TH} = 50 – j 25 Ω and I_{Norton} = 6.4 – j 4.8 Amp
Redraw the circuit in the Norton equiv form.

45
Ig = 3 < -90° Amp (what is this in rectangular form?)

47
Vo = 15.81 < 18.4° Volts

48 Voss = 11.31 cos (5,000 t - 45°) Volts

49 Voss = 12 cos (5,000 t) Volts

Engr 17 Chapter 9 Homework

9.3 $v = 170\cos(120\pi t - 60^\circ)$ Volts

a) $V_m = \underline{170V}$

b) $f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = \underline{60\ Hz}$

c) $\omega = 120\pi$ rads/sec $= \underline{376.99 \text{ rads/sec}}$

d) $\phi = -60^\circ = -60^\circ\left(\frac{\pi}{180^\circ}\right) = -\frac{\pi}{3} = -1.05$ rads

e) $\phi = -60^\circ$

f) $T = \frac{1}{f} = \frac{1}{60} = 16.67$ ms

g) find $t \geq 0$ for $v = 170V$ ie $\cos(0) = 1$

$120\pi t - \frac{\pi}{3} = 0 \Rightarrow 120\pi t = \frac{\pi}{3} \Rightarrow t = \frac{1}{360}$

$\underline{t = 2.78\text{ ms}}$

h) shift left $\frac{125}{18}$ ms $= \frac{.125}{18}$ sec

$v = 170\cos\left[120\pi\left(t + \frac{.125}{18}\right) - \frac{\pi}{3}\right]$

$= 170\cos\left(120\pi t + 120\pi\left(\frac{.125}{18}\right) - \frac{\pi}{3}\right)$

$= 170\cos\left(120\pi t + \frac{15\pi}{18} - \frac{\pi}{3}\right)$

$= 170\cos\left(120\pi t + \frac{9\pi}{18}\right)$ $\frac{9\pi}{18} = \frac{\pi}{2} = 90^\circ$

$= 170\cos(120\pi t + 90^\circ)$

$\underline{v = 170\sin(120\pi t)\text{ Volts}}$

9.6

a) $y = 100\cos(300t + 45°) + 500\cos(300t - 60°)$

$Y = 100\angle 45° + 500\angle -60°$

$= 100(\cos 45 + j\sin 45°) + 500[\cos(60°) - j\sin(60°)]$

$= 70.7 + j\,70.7 + 250 - j\,433$

$= 320.7 - j\,362.3$

$= \sqrt{320.7^2 + 362.3^2}\angle \tan^{-1}\left(\frac{-362.3}{320.7}\right)$

$= 483.8\angle -48.49°$

$y = \mathcal{P}^{-1}\{Y\} = \underline{\underline{483.8\cos(300t - 48.49°)}}$

b) $y = 250\cos(377t + 30°) - 150\sin(377t + 140°)$

$Y = 250\angle 30° - 150\angle(140° - 90°)$

$= 250\angle 30° - 150\angle 50°$

$= (216.5 + j\,125) - (96.42 + j\,114.91)$

$= 120.5\angle 4.8°$

$y = \underline{\underline{120.5\cos(377t + 4.8°)}}$

c) $y = 60\cos(100t + 60°) - 120\sin(100t - 125°) + 100\cos(100t + 90°)$

$Y = 60\angle 60° - 120\angle -125° - 90° + 100\angle 90°$

$= 152.88\angle 32.94°$

$\underline{\underline{y = 152.88\cos(100t + 32.94°)}}$

9.6.

d) $y = 100\cos(\omega t + 40^\circ) + 100(\cos\omega t + 160^\circ)$
$+ 100\cos(\omega t - 80^\circ)$

$Y = 100\angle 40^\circ + 100\angle 160^\circ + 100\angle -80^\circ$

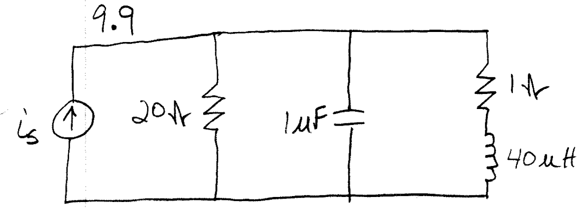
$= (76.6 + j64.279) + (-93.969 + j34.20) + (17.36 - j98.48)$

$= -0.009 + j0.079$

$= 0.08\angle -83.5^\circ)$ pretty much = zero

$\underline{\underline{y \approx 0}}$

9.9



$i_s = 20\cos(50{,}000t - 20^\circ)\,A$

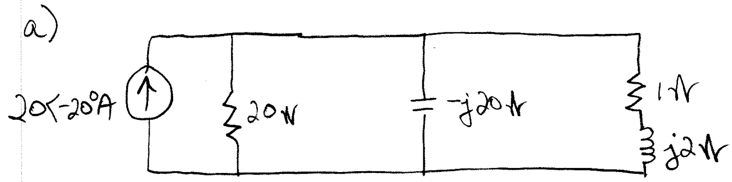
$\underline{I} = 20\angle{-20^\circ}\ A$

$Z_{R_1} = 20\,\Omega \qquad Z_{R_2} = 1\,\Omega$

$Z_C = jX_C = j\,\dfrac{-1}{\omega C} = j(-20)\,\Omega$

$Z_L = j\omega L = j2\,\Omega$

a)



b) $Z_{eq} = 20 \| (-j20) \| (1+j2)$

so $\dfrac{1}{Z_{eq}} = \dfrac{1}{20} + j\dfrac{1}{20} + \dfrac{1}{1+j2} = Y_{eq}$ (in terms of admittance)

$Y_{eq} = 0.05 + j\,0.05 + \dfrac{1-j2}{(1+j2)(1-j2)}$

$= 0.05 + j\,0.05 + 0.5 - j\,0.4 = 0.25 - j\,0.35\ S$

$Z_{eq} = \dfrac{1}{Y_{eq}} = \dfrac{1}{0.25 - j\,0.35} = (1.351 + j\,1.892)\,\Omega$

$= 2.32\angle 54.46^\circ\ \Omega$

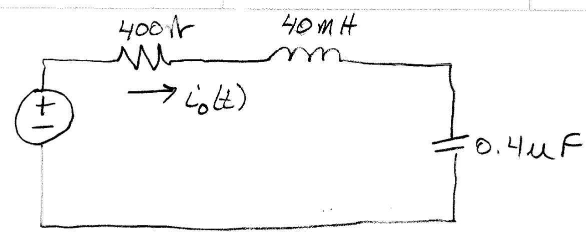
so $V = \mathbf{I}\,Z_{eq} = (20\angle{-20^\circ}A)(2.32\angle 54.46^\circ\,\Omega)$

$= \underline{46.5\angle 34.46^\circ \text{ volts}}$

9.9

c) $v_{ss} = 46.5 \cos(50{,}000t + 34.46°)$ Volts

9.14



$v_s = 750 \cos 5{,}000t$ mV

find $i_{ss}(t)$

$Z_L = j\omega L = j(5000)(40mH) = j\,200\,\Omega$

$Z_C = j\left(\frac{-1}{\omega C}\right) = -j\,500\,\Omega$

$Z_{eq} = 400\,\Omega + j\,200\,\Omega - j\,500\,\Omega = (400 - j\,300)\,\Omega$

$= 500 \angle -36.87°$

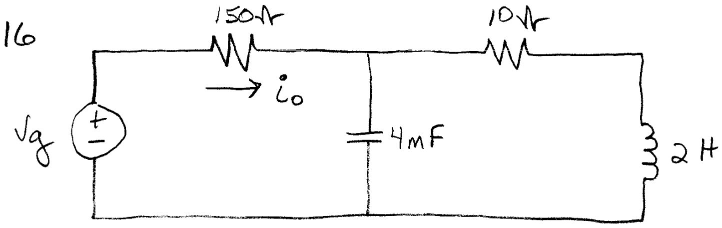
$\mathbf{V} = 750 \angle 0°$

$\mathbf{I} = \frac{\mathbf{V}}{Z} = \frac{750\angle 0° \text{ mV}}{500 \angle -36.87° \,\Omega} = 1.5 \angle 36.87° \text{ mA}$

so $i_{ss}(t) = \underline{1.5 \cos(5000t + 36.87°) \text{ mA}}$

Note that current <u>leads</u> the voltage

9.16



ω changed until v_g "in phase" with i_o
(remember in class we showed this means the <u>behavior</u> of the circuit is purely resistive)

a)
$$Z_{eq} = 150\,\Omega + \left[j\left(\frac{-1}{\omega C}\right) \parallel (10\,\Omega + j\omega 2H) \right]$$

$$= 150 + \frac{1}{j\left(\frac{-1}{\omega C}\right)} + \frac{1}{10 + j\omega 2}$$

$$= 150 + j\omega(4\times10^{-3}) + \frac{10 - j\omega 2}{100 + 4\omega^2}$$

= real (only) when imaginary terms cancel.

so $\cancel{j}\,\cancel{\omega}(4\times10^{-3}) = \dfrac{\cancel{j}\,\cancel{\omega}2}{100+4\omega^2}$

$$100 + 4\omega^2 = 500 \Rightarrow \omega^2 = 100$$

$$\omega = 10 \text{ rad/sec} = 2\pi f$$

$$f = \frac{10}{2\pi} = \frac{5}{\pi} \approx \underline{1.59 \text{ Hz}}$$

9.16

b) Given $v_g = 10 \cos \omega t$ volts at $\omega = 10$ rad/s (part a)

find i_{ss}

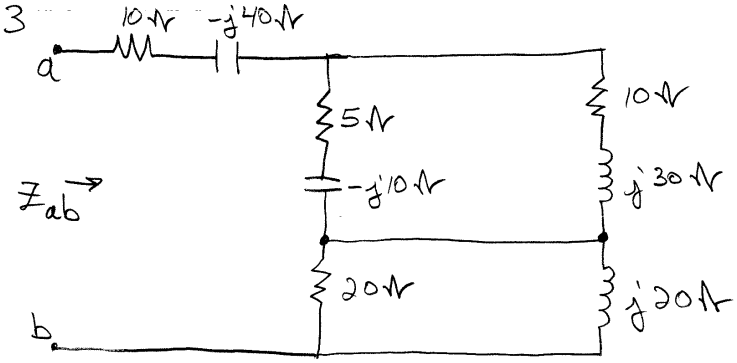
$$Z_{eq} = 150 + \left[\frac{10}{10+4\omega^2}\right]^{-1}$$ we have found were the imaginary terms cancel — so j terms ignored.

$$= 150 + \left[\frac{10}{500}\right]^{-1} = 150 + 50 = 200\,\Omega$$

$$I = \frac{V}{Z_{eq}} = \frac{10\angle 0}{200\angle 0} = 50\angle 0^\circ \text{ mA}$$

$$i_{ss} = 50 \cos 10t \text{ mA}$$

9.23



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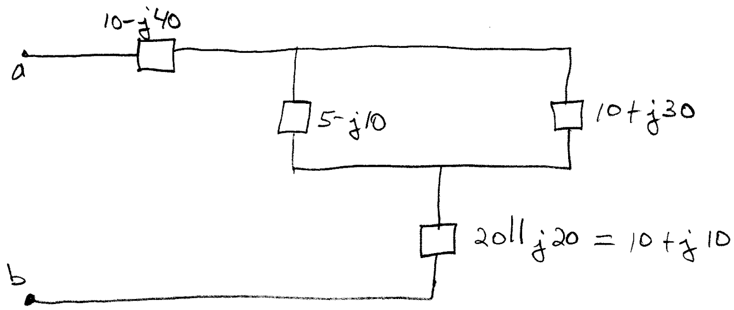
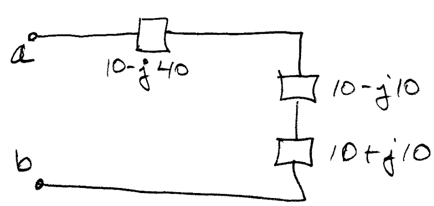


$$20 \| j20 = \frac{20(j20)}{20+j20} = \frac{j400}{20(1+j)} = \frac{j20}{1+j}\,\frac{(1-j)}{(1-j)} = \frac{20+j20}{2} = 10+j10$$

$$(5-j10) \| 10+j30 = \frac{(5-j10)(10+j30)}{(5-j10)+(10+j30)} = \frac{350+j50}{15+j20}$$

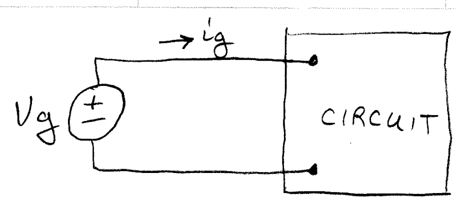
$$= \frac{6250-j6250}{625} = 10-j10$$



$$Z_{ab} = (10-j40) + (10-\cancel{j10}) + (10+\cancel{j10}) = \underline{(30-j40)\,\Omega}$$

$$= \underline{50 \angle -53.15^\circ\ \Omega}$$

9.29



$V_g = 150 \cos(8000\pi t + 20°)$ Volts

$i_g = 30 \sin(8000\pi t + 38°)$ Amps

a) $Z_{circuit} = \dfrac{V}{I} = \dfrac{150\angle 20°}{30\angle 38°-90°} = \dfrac{150\angle 20°}{30\angle -52°} = \underline{\underline{5\angle 72°}}$

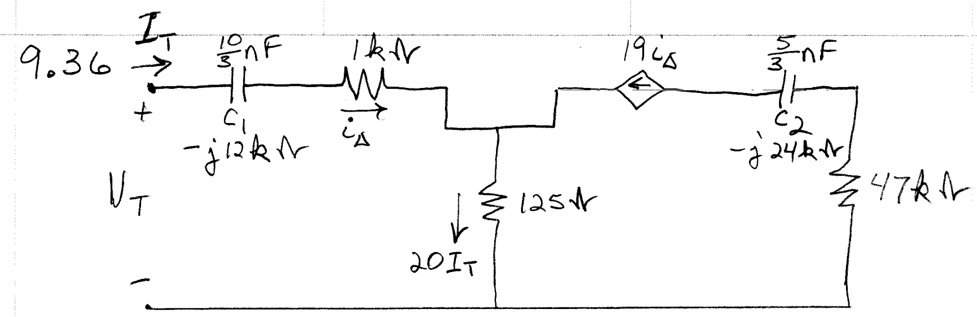
b) Current lags voltage by $72°$ = how many μ sec?

$2\pi f = 8000\pi \Rightarrow f = 4000\,Hz \Rightarrow T = \frac{1}{f} = 250\mu s$

so every $T = 250\mu$ sec the angular freq ω goes thru 2π (or equivalently goes thru $360°$)

thus $t = 72° \left(\dfrac{250\mu\,sec}{360°}\right) = \underline{\underline{50\,\mu\,sec}}$

9.36



$\omega = 25{,}000$ rad/sec

Note $i_\Delta = I_{Test}$

find Z_{th}

$Z_{C_1} = j\left(\frac{-1}{\omega C_1}\right) = -j12\text{ k}\Omega$

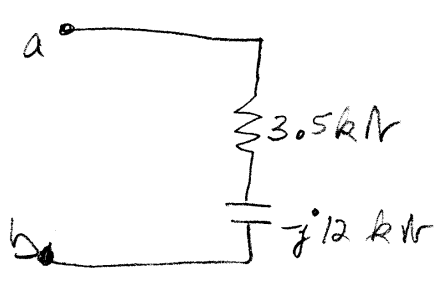
$Z_{C_2} = j\left(\frac{-1}{\omega C_2}\right) = -j24\text{ k}\Omega$

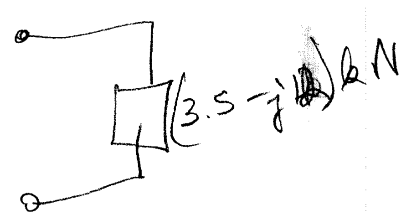
$V_T = I_T\,(1-j12)\text{ k}\Omega + 20 I_T\,(.125\text{ k}\Omega)$

$= I_T\left[(1-j12) + 2.5\right]\text{ k}\Omega$

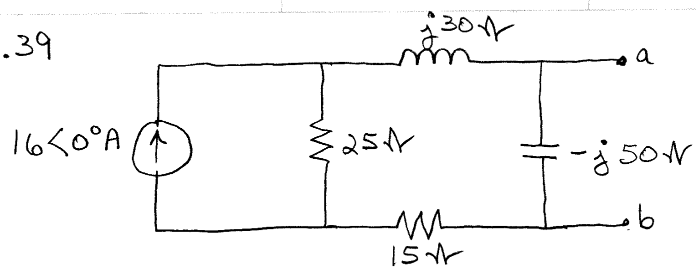
$= I_T\,(3.5-j12)\text{ k}\Omega$

$Z_{Th} = \frac{V_T}{I_T} = (3.5-j12)\text{ k}\Omega = (3{,}500 - j12{,}000)\,\Omega$



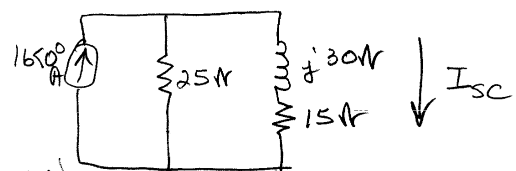


9.39



find the Norton Equiv.

$I_{Norton} = I_{short\ circuit}$



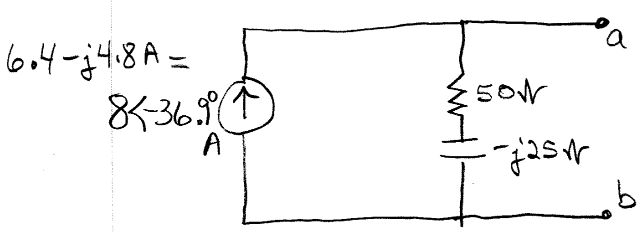
$$I_{SC} = \frac{25}{25+15+j30}(16\angle 0°)$$

by current divider rule

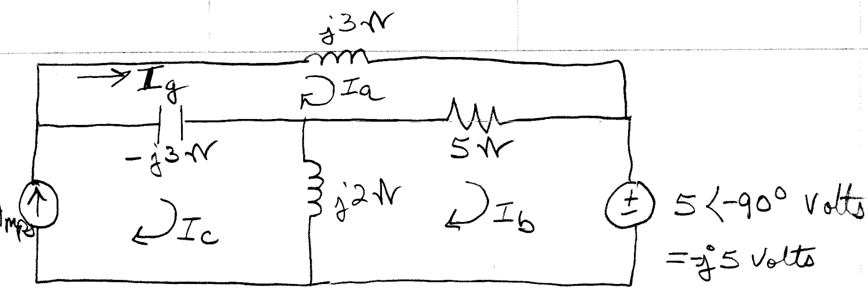
$$= \frac{25}{40+j30}(16\angle 0°) = 6.4 - j4.8 \text{ Amp}$$

$$Z_{Th} = (-j50) \,||\, (j30+25+15)$$

$$= \frac{(-j50)(40+j30)}{-j50+40+j30} = 50 - j25\ \Omega$$



9.45



find the phasor I_g by mesh current analysis

Note that $I_c = 5\angle 0^\circ$

<u>loop a</u>

$$I_a(j3\Omega) + (5\Omega)(I_a - I_b) - j3\Omega(I_a - 5\angle 0^\circ) = 0$$

$$I_a(5) - I_b(5) + \underbrace{(j3\Omega)(5\angle 0^\circ)}_{j15\text{ Volt}} = 0$$

$$I_a(5) + I_b(-5) = -j15\text{ Volt}$$

<u>loop b</u>

$$(j2\Omega)(I_b - 5) + 5\Omega(I_b - I_a) - j5V = 0$$

$$I_a(-5) + I_b(5 + j2\Omega) - j10V - j5V = 0$$

$$I_a(-5) + I_b(5 + j2\Omega) = j15V$$

<u>sub in from loop a</u>

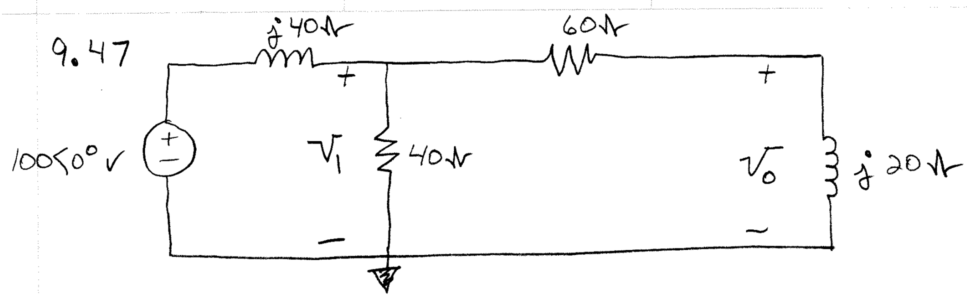
$$I_a(-5) + (I_a + j3V)(5 + j2\Omega) = j15V$$

$$I_a\left[(\cancel{-5+5})^{0} + j2\right] + \cancel{j15} - 6 = \cancel{j15}$$

$$I_a(j2) = 6$$

$$I_a = \frac{6}{j2} = -j3\text{ Amp} = I_g = \underline{3\angle -90^\circ\text{ Amp}}$$

9.47



find V_0 by node voltage method

$$\frac{V_1}{40\Omega} + \frac{V_1 - 100}{j40\Omega} + \frac{V_1}{60 + j20\Omega} = 0$$

notice I write only V_1 (V_0 then found later)

do the math & $V_1 = 30 - j40$ Volts

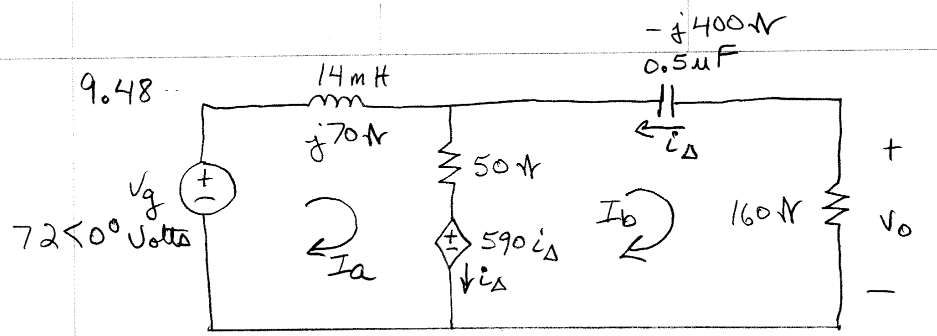
$$V_0 = \frac{j20}{60 + j20}(V_1) \quad \text{by voltage divider rule}$$

$$= \frac{j20}{60 + j20}(30 - j40V) = \frac{j}{3 + j}(30 - j40\text{ V})$$

$$= \frac{1 + 3j}{10}(30 + j40) = -9 + j13 \text{ Volts}$$

$$= \underline{\underline{15.81 \angle -55.3^\circ \text{ Volts}}}$$

9.48



$v_g = 72\cos 5000t$ volts   Note $I_b = -i_\Delta$

find $v_{o_{ss}}$ by mesh-current method

$Z_L = j\omega L = j(5000)(14\times10^{-3}H) = j70\,\Omega$

$Z_C = j\left(\frac{-1}{\omega C}\right) = j\left(\frac{-1}{5000\cdot 0.5\times10^{-6}}\right) = -j400\,\Omega$

loop a

$72 = (50 + j70)I_a - 50 I_b + 590(-I_b)$

loop b

$(-50)I_a - 590(-I_b) + (50+160-j400)I_b = 0$

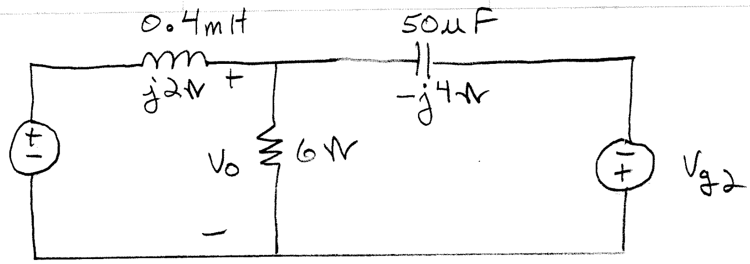
Solve these equations

$I_b = (50 - j50)\ mA$

$V_o = (160\,\Omega)\,I_b = 8 - j8$ volts $= 11.31\angle -45^\circ$

$v_{o_{ss}} = 11.31\cos(5000t - 45^\circ)$ Volts

9.49



$v_{g1} = 10\cos(5000t + 53.13°)$ Volts $\quad V_{g1} = 10\angle 53.13° = 6 + j8$ Volts

$v_{g2} = 8\sin 5000t = 8\cos(5000t - 90°)$ Volts $\quad V_{g2} = 8\angle -90° = -j8$ volts

find $v_{o_{ss}}$ by the node voltage method

$Z_L = j\omega L = j2\,\Omega$

$Z_C = j\left(\frac{-1}{\omega C}\right) = -j4\,\Omega$

Be careful here - note the source polarity!

$$\frac{V_o - (6+j8)}{j2\,\Omega} + \frac{V_o}{6} + \frac{V_o + (-j8)}{-j4} = 0$$

Solving this equation gives us

$V_o = 12\angle 0$

so $v_{o_{ss}} = 12\cos(5000t)$ Volts