Philosophy 160
HW 4 Derivations using →I Solutions

Derive the following using any of the first eight non-derived rules as well as the rule of →I.

1. P & Q, Q → (S → T), P → (T →R) ├ S → R
2. Q → (R & S) ├ P → (Q → R)
3. R v R ├ R
4. P v T, P ↔ T ├ P & T
5. P → Q, R → S ├ (P v R) → (Q v S)
6. (P v Q) → (S & M), (Q v P) ├ (M & S)
7. R v ~R ├ (R v R) v (~R v R)

1. P & Q, Q → (S → T), P → (T →R) ├ S → R

1.	P & Q	A
2.	Q → (S → T)	A
3.	P → (T →R)	A
4.	S	H (Get R!)
5.	P	1, &E
6.	T →R	3,5 →E
7.	Q	1, &E
8.	S → T	2,7 →E
9.	T	4,8 →E
10.	R	6,9 →E
11	S → R	4-10, →I

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2. $Q \rightarrow (R \,\&\, S) \vdash P \rightarrow (Q \rightarrow R)$

1.	$Q \rightarrow (R \,\&\, S)$	A
2.	$\quad P$	H (Get $Q \rightarrow R$)
3.	$\quad\quad Q$	H (Get R!)
4.	$\quad\quad R \,\&\, S$	1,3 →E
5.	$\quad\quad R$	4, &E
6.	$\quad Q \rightarrow R$	3-5 →I
7.	$P \rightarrow (Q \rightarrow R)$	2-6, →I

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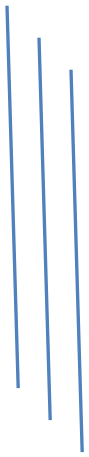
3. R v R ⊢ R

1.	R v R	A
2.	| R	H (Get R → R)
3.	R → R	2-2, →I
4.	R	1, 3, 3 vE



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4. P v T, P ↔ T ⊢ P & T

1.	P v T	A
2.	P ↔ T	A
3.	P	H
4.	P → T	2, ↔E
5.	T	3,4 →E
6.	P & T	3,5 &I
7.	P →(P & T)	3-6, →I
8.	T	H
9.	T → P	2, ↔E
10.	P	8,9 →E
11.	P & T	8,10 &I
12.	T → (P &T)	8-11 →I
13.	P & T	1, 7, 12 vE

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5. $P \rightarrow Q, R \rightarrow S \vdash (P \vee R) \rightarrow (Q \vee S)$

1.	$P \rightarrow Q$	A
2.	$R \rightarrow S$	A
3.	$P \vee R$	H
4.	P	H
5.	Q	1,4 →E
6.	$Q \vee S$	5, vI
7.	$P \rightarrow (Q \vee S)$	4-6 →I
8.	R	H
9.	S	2,8 →E
10.	$Q \vee S$	9 vI
11.	$R \rightarrow (Q \vee S)$	8-10 →I
12.	$Q \vee S$	3,7,11 vE
13.	$(P \vee R) \rightarrow (Q \vee S)$	3-12, →I

Strategy:

Hypothesize P v R for →I

then derive

P → (Q v S) and

R → (Q v S)

By two more applications of →I.

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6. (P v Q) → (S & M), (Q v P) ├ (M & S)

1.	(P v Q) → (S & M)	A
2.	(Q v P)	A
3.	| Q	H
4.	| P v Q	3, vI
5.	Q → (P v Q)	3-4, →I
6.	| P	H
7.	| P v Q	6, vI
8.	P → (P v Q)	6-7, →I
9.	P v Q	2, 5, 8 vE
10.	S & M	1,9 →E
11.	S	10, &E
12.	M	10, &E
13.	M & S	11,12 &I

Strategy:

Derive

(P v Q)

from

(Q v P)

using vE.

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7. R v ~R ⊢ (R v R) v (~R v R)

1.	R v ~R	A
2.	| R	H
3.	| R v R	2, vI
4.	| (R v R) v (~R v R)	3, vI
5.	R → ((R v R) v (~R v R))	2-4, →I
6.	| ~R	H
7.	| ~R v R	6, vI
8.	| (R v R) v (~R v R)	7, vI
9.	~R → ((R v R) v (~R v R))	6-8, →I
10.	(R v R) v (~R v R)	1,5,9 vE

Strategy:

Demonstrate the entire disjunction twice, once from R, and again from ~R for vE.

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