Philosophy 160
Russell and Burali-Forti

Answer the following questions. I prefer that you type your answers, but if you do not have easy access to a printer you may write them neatly. If you write them on a different sheet of paper, write the question out above each answer.

1. Summarize Russell's paradox.

Russell's paradox results within naïve set theory. Russell defined the following set:

$$U = \{x| \ x \notin x\}$$

Russell then asked whether $U \in U$. It can easily be demonstrated that $U \in U$ iff $U \notin U$, which implies a contradiction.

2. How is Russell's paradox resolved?

Russell's paradox resulted in the recognition of the need to axiomatize set theory. The axiom that blocks the paradox is the axiom of specification, or restricted comprehension. It essentially says that for a set to exist it must be possible to create a subset of all its members with an arbitrary property P. This blocks Russell's paradox because in order for U to exist it would have to be possible to construct a subset of U as follows:

$$\{x| \ x \in U \ \& \ Px\}$$

We can prove by Russell's reasoning that this set also implies a contradiction when we assert that U either belongs to this set or it does not. But instead of being a paradox, this simply implies that this set does not exist.

3. Summarize the Burali-Forti paradox.

The Burali-Forti paradox relates to ordinal numbers. On the theory of ordinals, there must be an ordinal that is a successor of every set of ordinals. This means that there must be a successor to the entire set of ordinals: $\Omega = \{x| \ Ox\}$. By definition the successor of the set does not belong to that set. Hence the successor to the the entire set of ordinals both is and is not a member of Ω.

4. Identify a deep similarity between the two paradoxes.

One deep similarity is that both paradoxes arise through an attempt to define sets that purport to contain everything.

5. How is the Burali-Forti paradox resolved?

This paradox is also thwarted in axiomatic set theory by the axiom of comprehension. If we define the subset of Ω $B= \{x| x\in\Omega \ \& \ Px\}$, we will generate a contradiction when considering whether the successor to Ω is a member of B or not.

However, there are some who believe that Ω should be permitted to exist and there are some alternative developments of set theory that do so.