Note: Only the first three problems are solved completely below. Solutions to 7-10 are provided on this lecture video as well as these corresponding slides. Solution outlines to 11-13 are below.

1. P → Q ├ ~Q → ~P

1.	P → Q	A
2.	│ ~Q	H →I
3.	│ │ P	H ~I
4.	│ │ Q	1,3 →E
5.	│ │ Q & ~Q	2,4 &I
6.	│ ~P	3-5, ~I
7.	~Q → ~P	2-6, →I

2. P & Q ├ ~(P → ~Q)

1.	P & Q	A
2.	│ P → ~Q	H
3.	│ P	1, &E
4.	│ ~Q	2,3 →E
5.	│ Q	1, &E
6.	│ Q & ~Q	4,5 &I
7.	~(P →~Q)	2-6, ~I

3. (P & Q) → (R v S), ~(R v S) ├ ~(P & Q)

1. (P & Q) → (R v S)	A
2. ~(R v S)	A
3. │ P & Q	H
4. │ R v S	1,3 →E
5. │ (R v S) & ~(R v S)	2,4 &I
6. ~(P & Q)	3-5, ~I

Philosophy 160
HW5 Derivations using →I & ~I Solutions

11. ~(P → Q) ├ P & ~Q

Outline of proof.

Since you are trying to prove a conjunction, consider proving each conjunct independently by ~I.

For the first leg, hypothesize ~P. From here it's not clear what to do, but you know you need a contradiction, and the only formula you can contradict is the assumption, which means that you want to prove (P → Q).

Proceed by →I, hypothesizing P. This give you a contradiction to work with, and so you can derive Q by hypothesizing ~Q for ~I. This will allow you to show (P → Q) which contradiction the assumption, giving you ~~P and thus P.

Proceed similarly to prove ~Q, by hypothesizing Q for ~I and deriving (P →Q) once again.

Philosophy 160
HW5 Derivations using $\rightarrow$I & ~I Solutions

12. $S \leftrightarrow T, T \vee S \vdash \sim(T \rightarrow \sim S)$

Outline of proof.

Since you have a disjunction, you will want to use vE to derive the conclusion twice, each time by ~I.

So you first show $T \rightarrow \sim(T \rightarrow \sim S)$, by hypothesizing T for $\rightarrow$I and then $(T \rightarrow \sim S)$ for ~I.

By involving the biconditional, you will arrive at a contradiction in a few steps.

Repeat the procedure for the second disjunct S, then draw the conclusion by vE.

13. $Q \rightarrow R \vdash {\sim}Q \vee R$

Outline of proof.

Hypothesize ~(~Q v R) for ~I.

The problem now is to generate a contradiction which takes some creativity.

If you look at your hypothesis, you'll see it affords you the opportunity to prove Q by hypothesizing ~Q and using vI to derive a contradiction.

But if you have Q, you have R by →E, and then you can use vI again to contradiction the original hypothesis.