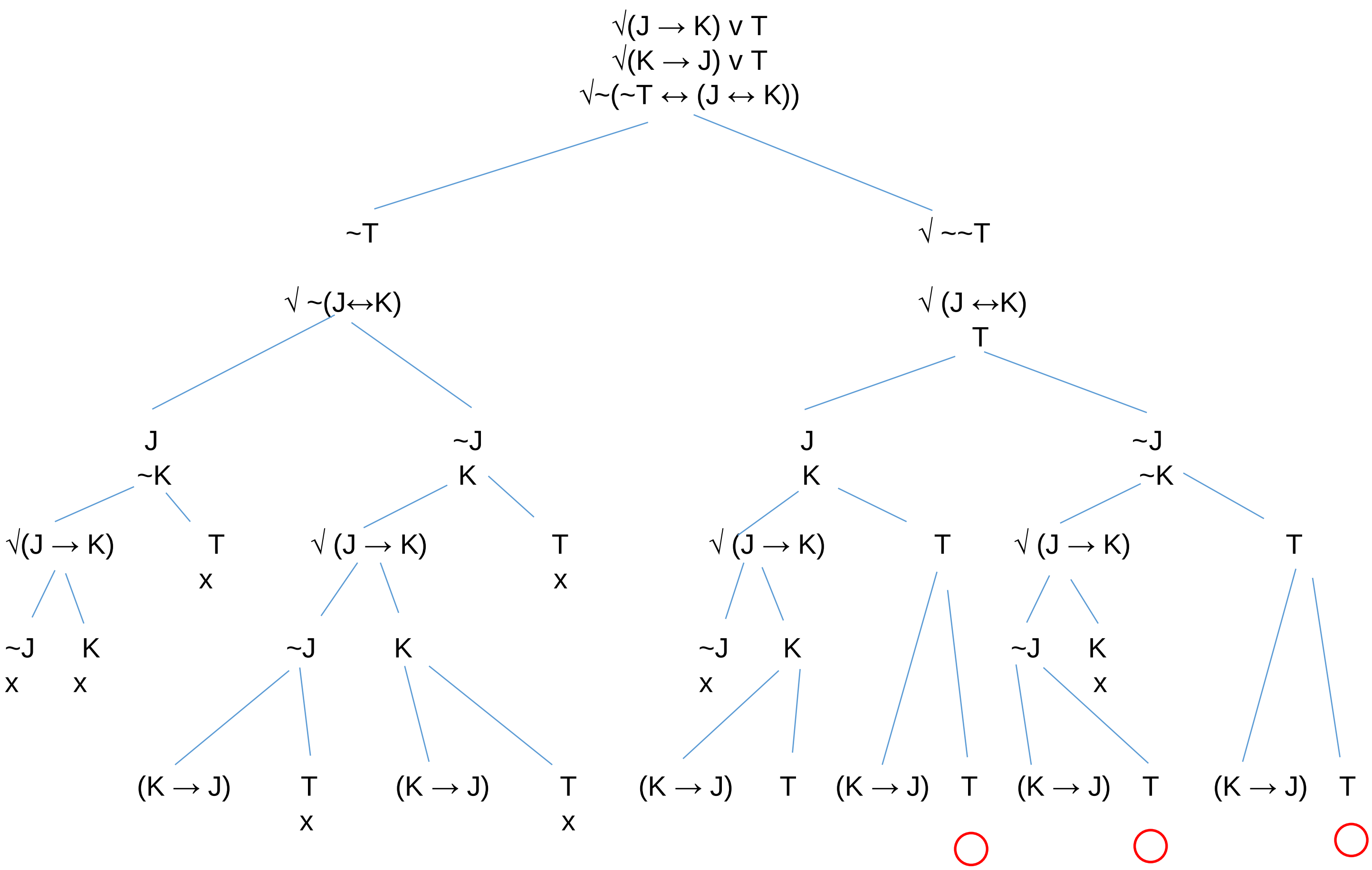
Philosophy 160 Test 1 Name: ___________________________

1. (5pts.) Use a tree to determine whether the following argument is valid or invalid. <u>Clearly explain</u> what property of the tree justifies your answer: (3pts.)

$(J \rightarrow K) \vee T,\ (K \rightarrow J) \vee T \ \vdash \sim T \leftrightarrow (J \leftrightarrow K)$



Explanation: Invalid. Negating the conclusion does not result in contradictions on every branch. The branches with the red circles under them are fully decomposed and do not contain a contradiction.

2. Use the tree method to determine whether the following is a tautology, a contradiction, or a contingent formula. Display <u>all and only</u> the trees that are necessary to confirm your conclusion. So do this on scratch paper first. If you display a tree that is irrelevant you will lose points. <u>Clearly explain</u> what property of the tree(s) justify your answer: (3pts.)

$$\sim((\sim Q \,\&\, \sim M) \rightarrow (Q \leftrightarrow M)) \rightarrow \sim(\sim(R \rightarrow S) \rightarrow (W \rightarrow (R \,\&\, (\sim S \vee M))))$$

$$\surd\sim(\sim((\sim Q \,\&\, \sim M) \rightarrow (Q \leftrightarrow M)) \rightarrow \sim(\sim(R \rightarrow S) \rightarrow (W \rightarrow (R \,\&\, (\sim S \vee M)))))$$

$$\surd\sim((\sim Q \,\&\, \sim M) \rightarrow (Q \leftrightarrow M))$$

$$\sim\sim(\sim(R \rightarrow S) \rightarrow (W \rightarrow (R \,\&\, (\sim S \vee M))))$$

$$\surd(\sim Q \,\&\, \sim M)$$

$$\surd\sim(Q \leftrightarrow M)$$

$$\sim Q$$

$$\sim M$$

Q	$\sim Q$
$\sim M$	M
x	x

Explanation: This is a tautology because negating it produces a tree in which all branches close.

(This occurs in such short order because the antecedent of the formula is a contradiction.)

3. Derive the following in propositional logic using only the 8 non derived rules and the two hypothetical rules. (3pts)

$$(P \vee S) \& (L \vee G) \vdash ((P \rightarrow \sim S) \& (\sim S \rightarrow \sim L)) \rightarrow (\sim G \rightarrow \sim P))$$

1.	(P v S) & (L v G)	A
2.	P v S	1, &E
3.	L v G	1, &E
4.	| (P → ~S) & (~S → ~L)	H→I
5.	| P → ~S	4, &E
6.	| ~S →~L	4, &E
7.	| | ~G	H →I
8.	| | | P	H →I
9.	| | | ~S	5,8 →I
10.	| | | ~L	6,9 →I
11.	| | | | L	H →I
12.	| | | | | P	H ~I
13.	| | | | | L & ~L	10,11 &I
14.	| | | | ~P	12-13, ~I
15.	| | | L → ~P	11-14, →I
16.	| | | | G	H→I
17.	| | | | | P	H~I
18.	| | | | | G & ~G	7,16 &I
19.	| | | | ~P	17-18 ~I
20.	| | | G →~P	16-19 →I
21.	| | | ~P	3,15,20 vE
22.	| | P → ~P	8-21 →I
23.	| | | S	H →I
24.	| | | | P	H ~I
25.	| | | | ~S	5,24 →E
26.	| | | | S & ~S	23,25 &I
27.	| | | ~P	24-26 ~I
28.	| | S → ~P	23-27 →I
29.	| | ~P	2, 22, 28 vE
30.	| ~G → ~P	7-29 →I
31.	((P → ~S) & (~S → ~L)) → (~G → ~P))	4-30 →I

4. If the following is a tautology, derive it as a theorem in propositional logic. If it is a contradiction, derive it's negation as a theorem in propositional logic. Use only the 8 non derived rules and the two hypothetical rules. (3pts.)

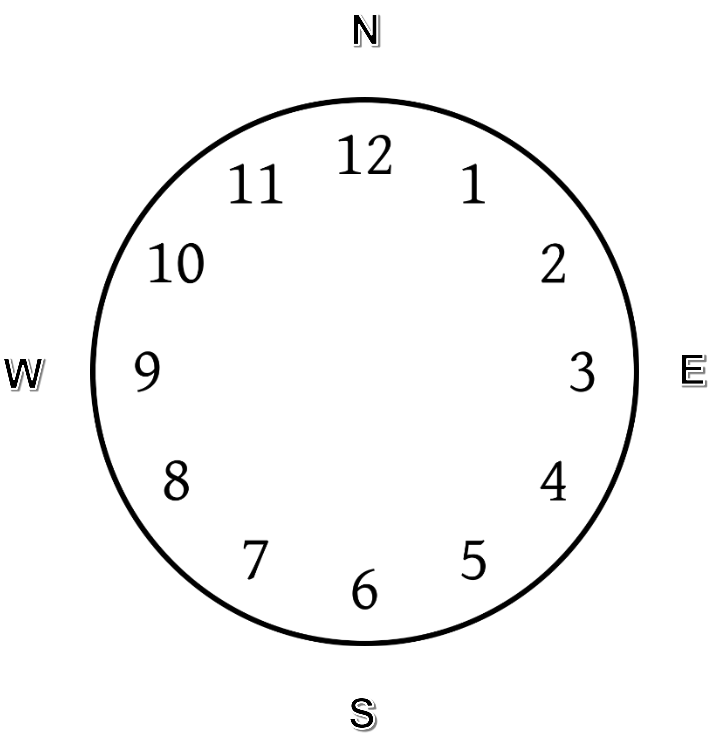
$$\sim(S \rightarrow (P \rightarrow S)) \rightarrow (S \,\&\, (P \,\&\, \sim S))$$

1.	$\sim(S \rightarrow (P \rightarrow S))$	H →I
2.	S	H →I
3.	P	H →I
4.	$S \,\&\, P$	2,3 &I
5.	S	4, &E
6.	$P \rightarrow S$	3-5 →I
7.	$S \rightarrow (P \rightarrow S)$	2-6 →I
8.	$\sim(S \,\&\, (P \,\&\, \sim S))$	H ~I
9.	$\sim(S \rightarrow (P \rightarrow S) \,\&\, (S \rightarrow (P \rightarrow S))$	1,7 &I
10.	$\sim\sim(S \,\&\, (P \,\&\, \sim S))$	8-9, ~I
11.	$(S \,\&\, (P \,\&\, \sim S))$	10 ~E
12.	$\sim(S \rightarrow (P \rightarrow S)) \rightarrow (S \,\&\, (P \,\&\, \sim S))$	1-11 →I

This is an easy proof if you have one insight, which is that (S→(P →S)) is a theorem. So just derive it. Then when you assume it's negation for →I you have an immediate contradiction.

Philosophy 160 Test 1 Name: ___________________________

5. Evaluate the following formulas as true or false in the model by writing either T or F under the formulas. Study the model carefully. Reminder 1: When evaluating universally quantified expressions, remember that they refer to all entities in the model. Hence, ∃x∀yLxy = F, because no number is less than itself. Reminder 2: Be sure you understand the truth conditions of the conditional. If you do, then it will be clear why, e.g., $\forall x(Sxx \rightarrow G_{34}) = T$. For this formula to be false, there would have to be a value of x which made the antecedent true and the consequent false. But since nothing is south of itself, this is not the case. (3 pts, .3 each)



Universe = Numbers 1-12.The circle is not part of the universe. The directional indicators are not part of the universe. No other numbers are part of the universe.

Names = Numerals 1-12. We here extend predicate logic to include numerals as names. Hence ∃x x=1, ~2=3, G_{32} are all wffs.

1-place predicates

P = prime
V = even
O = odd

2-place predicates

N = north of
S = south of
E = east of
W = west of
D = divisible by
G = greater than
L = less than

Clarifications:

1. Directional predicates are not mutually exclusive. e.g., the value of $(S_{512}\ \&\ E_{512}) = T$

2. Directional predicates do not mean “completely”. e.g. $N_{121} = T$ even though 12 is not completely north of 1.

3. D means “divisible into whole numbers without remainder.” Hence $D_{67} = F$, $D_{63} = T$

4. 1 is not a prime number.

1. $\forall x(Wx_4 \rightarrow Gx_1)$
False. 1 is west of 4.

2. ~∃x∃y(Sxy & Dxy)
False. 8 is south of 2 and divisible by 2.

3. ∃x∀y(~Exy → x=y)
True. 3 is east of everything but itself.

4. ∀x(Px → ∀y(Dxy → x=y))
False, every prime number is divisible by itself and 1. This is just a property of primes.

5. ∀x(Px → ∃y(Nyx & Gyx))
True. 12 is north of every prime.

6. $\forall x\forall y(Dxy \rightarrow (\sim Gxy \vee \forall z Lz_1))$
False e.g., 12 is divisible by, but 12>2 and no number is less than 1.

7. ∀x∀y∀z((Nxy & Nyz) → Szx)
True This is entailed by the meaning of North and South.

8. ∃x∃y((Ox & Vy) & (Dyx ↔ Px))
True This is satisfied by, e.g., x=3 and y =12.

9. $\exists x((Px \leftrightarrow Vx)\ \&\ \forall y(\sim Gyx \rightarrow (y=x \vee y=1)))$
True The first conjunct is satisfied by 2 and the second conjunct is satisfied by every number, as every number that is not greater than 2 is 2 or 1.

10. ∃x∃y((Dyx & Sxy) & ∃z∃u(~u=z & (Dxz & Dzu)))
True 12 is divisible by 6 and 6 is south of 12. 3 ≠1 and 6 is divisible by 3 and 3 is divisible by 1.