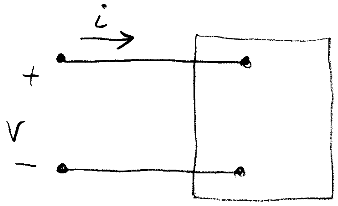
Engr 17 - CSUS Instructor – Russ Tatro

Chapter 10 Sinusoidal Steady-State Power Calculations
James W. Nilsson and Susan A. Riedel, *Electric Circuits*, 6th Edition, 2001

Lecture Notes

Please see your textbook for the figures and problems cited in the lectures.

Chap 10 – Sinusoidal steady-state Power



Instantaneous Power

$$p = vi = V_m \cos(\omega t + \theta_{voltage})$$
$$\cdot\, I_m \cos(\omega t\ \theta_i)$$

For convenience we write this as (current passing thru maximum)

$$p = V_m \cos(\omega t + \theta_v - \theta_i) \cdot I_m \cos(\omega t)$$

$$= V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t$$

Now use the trig identity

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

& let $\alpha = \omega t + \theta_v - \theta_i$

& $\beta = \omega t$

& re-write Power as

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$

Now lets expand the 2nd term by Trig Identities again

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$p = \underbrace{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)}_{P}$$
$$+ \underbrace{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)}_{P} \cos 2\omega t$$
$$- \underbrace{\frac{V_m I_m}{2} \sin(\theta_v - \theta_i)}_{Q} \sin 2\omega t$$

So the Power can have a "DC" component (1st Term) & the freq of the instantaneous power is twice that of the input current & voltage.

Negative Power indicates energy is being extracted (from capacitor and/or inductor) for that instant

See fig 10.2 on page 491

Sec 10.2 Average Power

From the above eqn we see that

$$\text{power} = P + P\cos 2\omega t - Q \sin 2\omega t$$

where $P =$ the average power – delivers work

$Q \triangleq$ the reactive power

Recall the average is given by $P = \int_{t_0}^{t_0+T} p \, dt$ as well, but easier to use Trig form above.

Power for Purely resistive circuit

voltage & current are in phase

that is $\theta_v = \theta_i$

so $\cos(\theta_v - \theta_i) = 1$

$\sin(\theta_v - \theta_i) = 0$

$p = P + P\cos 2\omega t$ = instantaneous real power in watts

see fig 10.3 → real power is always positive since no energy can be extracted from a purely resistive network (all energy dissipated as heat)

Power for Purely Inductive circuit

current lags voltage by exactly 90°

$\theta_i = \theta_v - 90°$ or $\theta_v - \theta_i = 90°$

thus $\cos(\theta_v - \theta_i) = 0$

$\sin(\theta_v - \theta_i) = 1$

thus

power $= -Q\sin 2\omega t$ reactive power in "VAR" volt-amp reactive

Power is continually exchanged between the inductor's magnetic field & the source.

average power = 0

Inductors "demand" (absorb) magnetizing VARs

Power for Purely Capacitive circuits

current leads the voltage

$\theta_i = \theta_v + 90° \Rightarrow \theta_v - \theta_i = -90°$

$\cos -90° = 0 \quad \sin -90° = -1$ so negatives cancel

thus power $= Q \sin 2\omega t$

avg power $= 0$

energy is continually exchanged between the source & the capacitors electric field.

Capacitors "furnish" (deliver) magnetizing VARs

The Power Factor

the angle $\theta_v - \theta_i$ is called the "Power factor Angle"

$pf = \cos(\theta_v - \theta_i) \stackrel{d}{=}$ Power Factor abbrev. pf

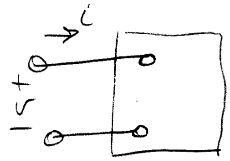
$rf = \sin(\theta_v - \theta_i) \stackrel{d}{=}$ Reactive Factor abbrev rf

we add "lagging" power factor for inductive load (current lags voltage)

& "leading" power factor for capacitive load (current leads voltage)

Ex 10.1 pg 494

for our block diagram



$v = 100 \cos(\omega t + 15^\circ)$ volts

$i = 4 \sin(\omega t - 15^\circ)$ Amps

$= 4 \cos(\omega t - 15^\circ - 90^\circ) = 4 \cos(\omega t - 105^\circ)$ A

$$P = \frac{V_m I_M}{2} \cos(\theta_v - \theta_i) = \frac{100 \cdot 4}{2} \cos[15 - (-105)]$$

$= 200 \cos 120^\circ = -100$ watts delivering $\overset{\text{avg}}{\text{power}}$

$$Q = \frac{V_M I_M}{2} \sin(\theta_v - \theta_i) = \frac{100 \cdot 4}{2} \sin 120^\circ = 173.21 \text{ VAR}$$

absorbing magnetizing VARs

D.E. 10.2 pg 495

find the power factor for the ex 10.1

we shift to $-i$ to keep pf positive so

$180^\circ - 105^\circ = 75^\circ$

$pf = \cos(15 - 75) = \cos(-60) = 0.5$ lagging

$rf = \sin(-60) = -0.866$

read section on appliance ratings on your own.

recall that $P_{effective} = V_{rms} I_{rms}$

so if computer takes 300W = P_{eff}

then $I_{eff} = \frac{P_{eff}}{V_{rms}} = \frac{300W}{115 V_{rms}} = 2.6$ Amps

Plug in a laser printer (my HP 3100) takes 135W

so $I_{eff} = \frac{135W}{115 V_{rms}} = 1.2$ Amp

Combined = 3.8 Amp

Notes on D.E. 10.2 (Text answers are wrong!)

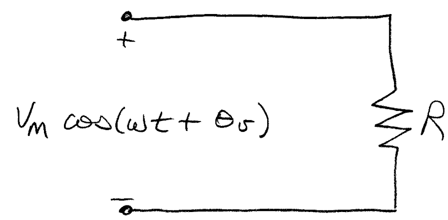
Voltage is $+15^\circ$ & current is -15°

so Voltage peaks 30° before the current

so current is lagging

$pf = 0.5$ lagging
$rf = -0.866$ } as given in class

Sec 10.3 The RMS Value & Power calculations



find the average Power delivered to the resistor

$$P_{avg} = \frac{1}{T} \int_{t_0}^{t_0+T} \frac{V_m^2 \cos^2(\omega t + \theta_v)}{R} \, dt$$

$$= \frac{1}{R} \left[\quad V_{rms}^2 \quad \right]$$

It also directly follows that

$$P_{avg} = I_{rms}^2 R$$

rms is often called the "effective value" of the sinusiodal voltage or current.

The Avg Power found by the rms is equal to a DC source at the rms value over the same time interval.

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$= V_{eff} I_{eff} \cos(\theta_v - \theta_i)$$

Reactive Power $Q = V_{eff} I_{eff} \sin(\theta_v - \theta_i)$

Ex 10.3 pg 499

$V_m = 625V$ sinusoidal voltage

a) find avg power delivered to 50Ω load.

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{625V}{\sqrt{2}} = 441.94V$$

$$P = \frac{V_{rms}^2}{R} = \frac{(441.94V)^2}{50\Omega} = 3906.25 \text{ Watt}$$

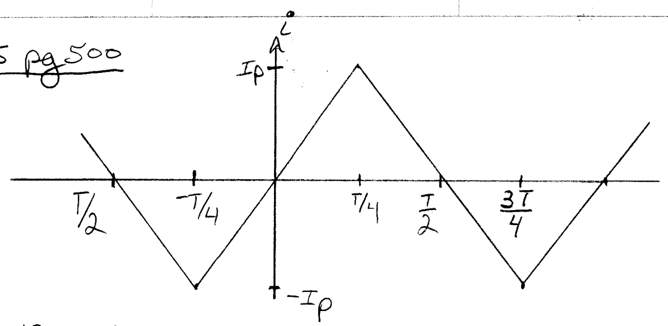
b) repeat by calculating current I_m

$$I_m = \frac{V_m}{R} = \frac{625V}{50\Omega} = 12.5A$$

$$I_{rms} = \frac{12.5}{\sqrt{2}} = 8.84A$$

$$P = I_{rms}^2 R = (8.84A)^2 (50\Omega) \doteq 3906.25 \text{ Watt}$$

D.E. 10.5 pg 500



$I_{peak} = 180$ mA

find the average power delivered to a 5 kΩ resistor.

$$I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2\,dt}$$

$$= \sqrt{\frac{1}{T}\,4\int_0^{T/4} i^2\,dt}$$

where $i = \frac{4}{T} I_p t$ for $0 \le t \le \frac{T}{4}$

so $\int_{t_0}^{t_0+T} i^2\,dt = 4\int_0^{T/4} \frac{16 I_p^2}{T^2} t^2\,dt$

integral $= (4)\frac{16 I_p^2}{T^2}\int_0^{T/4} t^2\,dt = \frac{4\cdot 16 I_p^2}{T^2}\left(\frac{t^3}{3}\Big|_0^{T/4}\right)$

$$= \cancel{\frac{4\cdot 16 I_p^2}{T^2}}\left(\frac{T^{\cancel{3}}}{\cancel{4^3}\cdot 3}\right) = \frac{I_p^2\,T}{3}$$

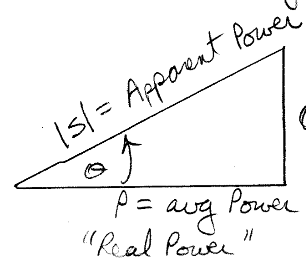
$$I_{rms} = \sqrt{\frac{1}{T}\left(\frac{I_p^2\,T}{3}\right)} = \frac{I_p}{\sqrt{3}}$$

so $I_{eff} = \frac{.180}{\sqrt{3}}$ A

$P_{avg} = I_{eff}^2 R = \left(\frac{.180}{\sqrt{3}}\right)^2 (5\,k\Omega) = \underline{54 \text{ watts}}$

Sec 10.4 Complex Power

Complex Power $\stackrel{d}{=} S = P + jQ$ V·A (Volt Amps)



where $\tan\theta = \frac{Q}{P}$ (from the geometry)

recall that
$$\frac{Q}{P} = \frac{\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)}{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}$$
$$= \tan(\theta_v - \theta_i)$$

Therefore our θ in the right triangle above is equal to $\theta_v - \theta_i$

$$\theta = \theta_v - \theta_i = \text{Power factor angle}$$

so all the right triangle tools apply.

Apparent Power $\stackrel{d}{=} |S| = \sqrt{P^2 + Q^2}$ in V·A (Volt Amps)

this apparent power is the volt·Amps required to supply the avg power.

when power factor angle $\theta = 0°$

then $pf = 1$ & $rf = 0$

$pf = \cos\theta$ $\quad rf = \sin\theta$

so when $\theta > 0°$ you must supply more volt·Amps than the average power used by the device!

Ex 10.4

$V_{rms} = 240\,V$

$P_{avg} = 8\,kW$

$pf = 0.8$ lagging (inductive load)

a) find the complex power S of the load.

$S = P + jQ$ (Q is positive)

$\cos\theta = 0.8$ so $\sin\theta = 0.6$ $\quad \theta = 36.87°$

$P = |S|\cos\theta \Rightarrow |S| = \frac{P}{\cos\theta} = \frac{8\,kW}{0.8} = 10\,kVA$

$Q = (10\,kVA)\underbrace{\sin\theta}_{0.6} = 6\,kVA$

so $S = 8 + j6\ kVA$

recall for inductor $Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$

$\theta_v - \theta_i = +90°$ for inductor so Q positive

b) Now find the impedance of the load.

P_{avg} (given) $= 8\ kW$

$= V_{eff}\, I_{eff} \cos(\theta_v - \theta_i)$

$= (240\ V_{rms})\, I_{eff}\, (0.8)$

so $I_{eff} = \dfrac{8\ kW}{(240\ V_{rms})(0.8)} = 41.67$ Amps

θ is positive $= 36.87°$ (inductive load)

$|Z| = \dfrac{|V_{eff}|}{|I_{eff}|} = \dfrac{240}{41.67} = 5.76$

thus

$Z = 5.76 \angle 36.87°\ \Omega = 4.608 + j\,3.456\ \Omega$

Next Time – Sec 10.5 Power Calculations.

Sec 10.5 Power Calculations

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \qquad \text{eqn } 10.10$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \qquad \text{eqn } 10.11$$

$$S = P + jQ \qquad \text{eqn } 10.23$$

$$= \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + j \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2} \left[\cos(\theta_v - \theta_i + j \sin(\theta_v - \theta_i) \right]$$

$$= \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = \frac{1}{2} V_m I_m \angle(\theta_v - \theta_i)$$

$$= \left[\frac{V_m}{\sqrt{2}} \quad \frac{I_m}{\sqrt{2}} \right] \angle(\theta_v - \theta_i)$$

$$= V_{eff} I_{eff} \angle(\theta_v - \theta_i)$$

$$= (V_{eff} e^{j\theta_v})(I_{eff} e^{-j\theta})$$

$$= \mathbf{V}_{eff} \quad \mathbf{I}^*_{eff}$$

* complex conjugate

Now to prove the last equation.

Recall $\cos(-\theta) = \cos\theta$

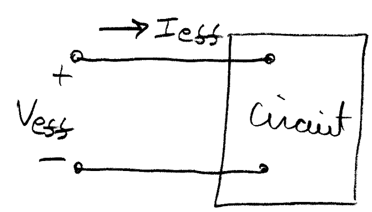
$\sin(-\theta) = -\sin\theta$

so $$I_{eff}\, e^{-j\theta_i} = I_{eff}\cos(-\theta_i) + j\, I_{eff}\sin(-\theta_i)$$
$$= I_{eff}\cos\theta_i - j\, I_{eff}\sin\theta_i$$
$$= I_{eff}^{*}$$ as stated before.

we can also write S in terms of V_m + I_m

$$S = \frac{1}{2} V I^{*}$$

All work has been derived by the passive sign convention.



For example

Let $V = 100\angle 15^\circ$ volts

$I = 4\angle -105^\circ$ Amps

so $I^* = 4\angle +105^\circ$ A

$$S = \frac{1}{2}(100\angle 15^\circ)(4\angle 105^\circ) = \frac{400}{2}\angle 15^\circ + 105^\circ = 200\angle 120^\circ \text{ VA}$$

$$= 200\cos 120^\circ + j\,200\sin 120^\circ$$

$$= -100 + j\,173.2 \text{ Volt}\cdot\text{Amps}$$

$$= P + jQ$$

so $P = -100$ watts

$Q = 173.2$ VAR

<u>Alternate ways to express complex Power</u>

$V_{eff} = Z\, I_{eff}$ Ohm's Law (again)

thus

$$S = V_{eff}\, I^*_{eff} = Z\, I_{eff}\, I^*_{eff} = Z\,|I_{eff}|^2$$

we can write the impedance in terms of the real & imag parts (reactance)

$Z = R + jX$

so $S = |I_{eff}|^2\,(R + jX)$

$$= \underbrace{|I_{eff}|^2 R}_{P} + j \underbrace{|I_{eff}|^2 X}_{Q}$$

so

$$P = |I_{eff}|^2 R = \frac{1}{2} I_m^2 R$$

$$Q = |I_{eff}|^2 X = \frac{1}{2} I_m^2 X$$

where X is the reactance of either an inductor (positive) or a capacitor (negative)

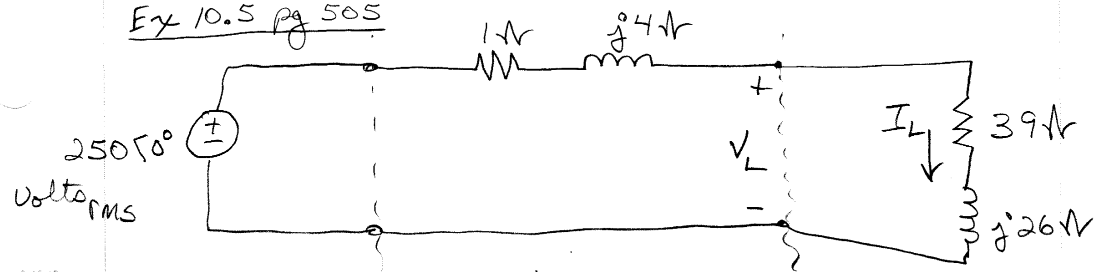
<u>Second Variation</u>

$$S = V_{eff} \left(\frac{V_{eff}}{Z} \right)^* \quad \text{which follows from} \quad I_{eff}^* = \left(\frac{V_{eff}}{Z} \right)^*$$

$$= \frac{|V_{eff}|^2}{Z^*}$$

$$= \underbrace{\frac{|V_{eff}|^2}{R}}_{P} + j \underbrace{\frac{|V_{eff}|^2}{X}}_{Q}$$

Ex 10.5 pg 505



a) find load current I_L & voltage V_L

$$Z_{eq} = (1+j4) + (39+j26) = 40 + j30 \ \Omega$$
$$= 50\angle 36.9° \ \Omega$$

$$I_L = \frac{250\angle 0° \ V_{rms}}{50\angle 36.9° \ \Omega} = 5\angle -36.9° \ A_{rms}$$
$$= 4 - j3 \ A$$

$$V_L = I_{L(rms)} Z_L = (4-j3)(39+j26) = 234 - j13 \ V_{rms}$$

$$= 234.36\angle -3.18° \ V_{rms}$$

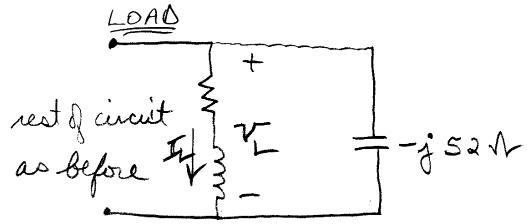
b) P_{avg} & Q for the load

$$S = V_{eff} I^*_{eff} = (234-j13)(4+j3) = \underbrace{975}_{P} + j\underbrace{650}_{Q} \ VA$$

absorbing both

D.E. 10.6 pg 511

Now add a capacitor shunt to the last exercise



a) find the new phasors V_L & I_L (rms)

$Z_L = (39 + j26) \,||\, (-j52) =$ (much math later) $= 52\angle -22.62\,\Omega$

$= 48 - j20$

so Z_{eq} for the circuit is $= (48 - j20)(1 + j4) = 49 - j16$

$= 51.55\angle -18.08°\ \Omega$

$I_{Load\,TOTAL}$ (Total for parallel branch) $= \dfrac{V_{rms}}{Z_{eq}} = \dfrac{250\angle 0°}{51.55\angle -18.08°} = 4.85\angle 18.08°\ A_{rms}$

So $V_{Load} = Z_L \cdot I_{LOAD\ TOTAL} = (4.85\angle 18.08°A)(52\angle -22.62\Omega)$

$= \underline{252.20\angle -4.54°\ V_{rms}}$

I_L (original branch) $= \dfrac{V_L}{39 + j26} = \dfrac{252.20\angle -4.54°}{46.87\angle 33.69}$

$= \underline{5.38\angle -38.23\ A_{rms}}$

b) find the new P & Q for the original load now that the shunt is in place.

$$S_{Load} = \underbrace{(252.2\angle -4.54)}_{V_L}\ \underbrace{(5.38\angle +38.23^\circ)}_{I_L^*}$$

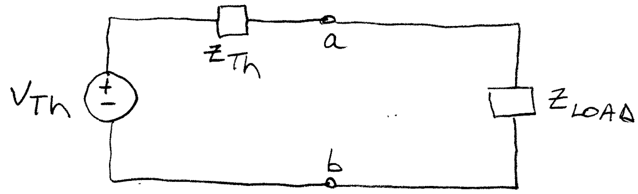
$$= 1357\angle 33.69^\circ \text{ VA}$$

$$= \underbrace{1129.09}_{P} - j\,\underbrace{752.73}_{Q} \text{ VA}$$

compare without the shunt $950 + j\,650$ VA

Next Time Sec 10.6 Max Power

Sec 10.6 Max Power Transfer



we want to find the load impedance Z_L that results in the delivery of the max average power at terminals ab.

we will show max avg transfer occurs when $Z_L = Z_{Th}^*$
Z_{Th}^* = complex conjugate of the Thevenin impedance.

$Z_{Th} = R_{Th} + j X_{Th}$

$Z_L = R_L + j X_L$

reactance
positive for inductor
negative for capacitor

Since we are making avg power calculation - we will express V_{Th} as its rms value.

thus

$$I_{rms} = \frac{V_{Th}(rms)}{(R_{Th}+jX_{Th}) + (R_L + jX_L)}$$

$$= \frac{V_{Th}(rms)}{(R_{Th}+R_L) + j(X_{Th}+X_L)}$$

recall Power at the load $P_L = |I|^2 R_L$ recall $|I|^2 = I \cdot I^*$

$$P_L = \frac{|V_{Th}|^2 R_L}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

the 2 independent variables are R_L & X_L (ie can change)
all others are "fixed" by the source Thévenin circuit.
thus max P is when $\frac{\partial P}{\partial R_L} = 0 = \frac{\partial P}{\partial X_L}$

$$\frac{\partial P}{\partial X_L} = \frac{|V_{Th}|^2 R_L \, (-2)(X_L+X_{Th})}{\left[(R_L+R_{Th})^2 + (X_L+X_{Th})^2\right]^2}$$

Be careful with the quotient rule here!

$= 0$ only when $X_L = -X_{Th}$

$$\frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2 \left[(R_L+R_{Th})^2 + (X_L+X_{Th})^2 - 2R_L (R_L+R_{Th})\right]}{\left[(R_L+R_{Th})^2 + (X_L+X_{Th})^2\right]^2}$$

Product & Quotient rule here

$= 0$ when $(R_L+R_{Th})^2 + (X_L+X_{Th})^2 - 2R_L(R_L+R_{Th}) = 0$

Shuffle the terms

$$\cancel{2}R_L^2 + \cancel{2R_L R_{Th}} - (\cancel{R_L^2} + \cancel{2R_L R_{Th}} + R_{Th}^2) = (X_L + X_{Th})^2$$

$$R_L^2 = R_{Th}^2 + (X_L + X_{Th})^2$$

$$R_L = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2} \quad \text{when } \frac{\partial P}{\partial R_L} = 0$$

So these two conditions are simultaneously met when

$Z_L = Z_{Th}^*$

To verify this let $Z_{Th} = R_{Th} + j X_{Th}$

$$Z_{Th}^* = R_{Th} - j X_{Th}$$

and $Z_L = R_L + j X_L$

thus it directly follows $X_L = -X_{Th}$ as found for $\frac{\partial P}{\partial X_L} = 0$

and $R_L = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$

$$= \sqrt{R_{Th}^2 \cancel{(-X_{Th} + X_{Th})^2}} = \sqrt{R_{Th}^2}$$

$$= R_{Th} \qquad \text{as found by } \frac{\partial P}{\partial R_L} = 0$$

<u>Max Power Absorbed</u>

when $Z_L = Z_{Th}^*$ then $I_{LOAD} = \frac{V_{Th}}{(R_{Th} + \cancel{jX_{Th}})(R_{Th} - \cancel{jX_{Th}})}$

ie $R_L = R_{Th}$.

$$= \frac{V_{Th}}{2R_{Th}} = \frac{V_{Th}}{2R_L}$$

so max Power is

$$P_{max} = \frac{|V_{Th}|^2 R_L}{4 R_L^2} \qquad P_{max} = |I|^2 R_L$$

$$= \frac{1}{4} \frac{|V_{Th}|^2}{R_L}$$

$$= \frac{1}{4} \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R_L} = \frac{1}{8} \frac{V_m^2}{R_L}$$

max Power when $Z_L \neq Z^*_{Th}$

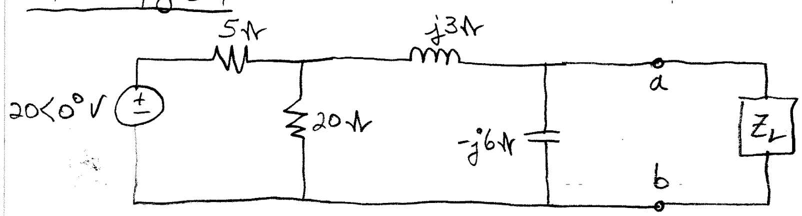
① we want to adjust $X_L \rightarrow -X_{Th}$ near as possible

& $R_L \rightarrow \sqrt{R^2_{Th} + (X_L + X_{Th})^2}$

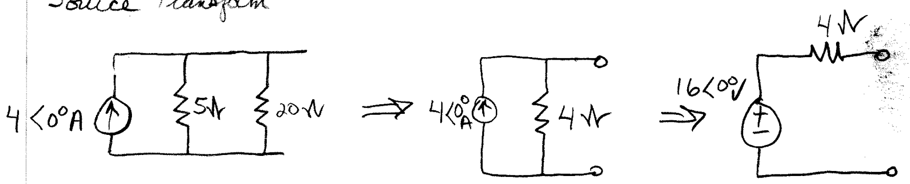
② what about when $|Z_L|$ can be varied but $\angle\theta$ is fixed?

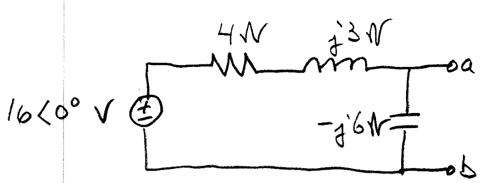
in this case set $|Z_L| \rightarrow |Z_{Th}|$ as close as possible.

Ex 10.8 pg 514



Source Transform





Z_{Th} seen at ab $\Rightarrow Z_{Th} = (4 + j3)\Omega \,||\, (-j6\Omega) = \dfrac{(-j6\Omega)(4+j3)}{-j6+4+j3}$

$Z_{Th} = \dfrac{18 - j24}{4 - j3}\Omega = \dfrac{144 - j42}{25}\Omega = (5.76 - j\,1.68)\,\Omega$

Thus $Z_L = Z^*_{Th} = 5.76 + j\,1.68\,\Omega$ for max avg power

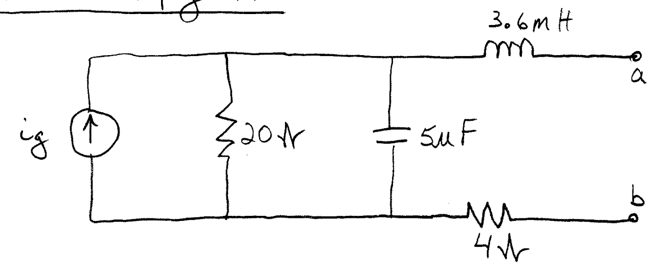
$$V_{Th} = \frac{(-j6)\Omega}{(4+j3-j6)\Omega}(16\angle 0^\circ \text{ Volts}) = \frac{(18-j24)}{25}(16\angle 0^\circ V)$$

$$= 11.52 - j\,15.36 \text{ Volts}$$

$$= 19.2 \angle -53.13^\circ \text{ Volts}$$

thus $P_{max} = \frac{1}{8}\frac{V_m^2}{R_L} = \frac{1}{8}\frac{(19.2\,V)^2}{5.76} = \underline{\underline{8 \text{ watts}}}$

<u>D.E. 10.9 pg 517</u>

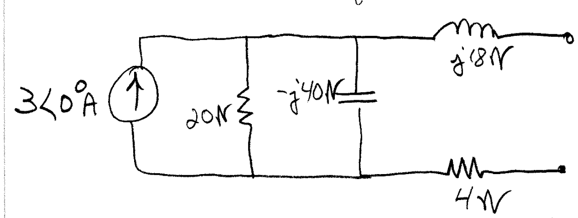


where $i_g = 3\cos 5000t$ Amps

$Z_{inductor} = j\omega L = j(5000\,\tfrac{rad}{s})(3.6mH) = j\,18\,\Omega$

$Z_{cap} = j\left(\frac{-1}{\omega C}\right) = j\left(\frac{-1}{5000 \cdot 5uF}\right) = -j\,40\,\Omega$

so Phasor Domian Equiv Circuit is



Now find The Thévenin Equivalent.

the impedance seen at terminals ab is

$$Z_{Th} = [20\,\Omega \,||\, (-j\,40)]\,\Omega + j\,18\,\Omega + 4\,\Omega$$

$$= (16 - j8)\,\Omega + j\,18\,\Omega + 4\,\Omega$$

$$= 20 + j\,10\,\Omega$$

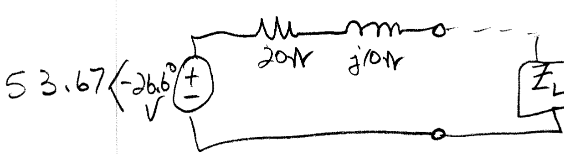
a) so max avg power transfer when $Z_L = Z_{Th}^*$

$$Z_L = 20 - j\,10\,\Omega$$

b) what is P_{max} in part a)? $(V = IR)$

$$V_{Th} = \underbrace{\frac{(20\,\Omega)(-j\,40\,\Omega)}{20 - j\,40\,\Omega}}_{\text{already found above}}(3\,\text{Amp}) = (16 - j8\,\Omega)(3A) = 48 - j24\,V$$

$$= 53.67\angle -26.57^\circ \text{ Volts}$$



$Z_{LOAD} = 20 - j10\,\Omega$ $\quad Z_{eq} = (20+j10) + (20-j$ $\quad = 40\,\Omega$

$$I = \frac{V_{Th}}{(20+20)\,\Omega} = 1.34\angle -26.57^\circ \text{ Amps} \qquad I_{rms} = \frac{I}{\sqrt{2}}$$

$$P_{max} = \left(\frac{|I|}{\sqrt{2}}\right)^2 (20\,\Omega) = \frac{1.34^2}{2}(20\,\Omega) = 17.956 \text{ watts}$$

at the load!