Philosophy 160 Practice In-class Test 1 Name: ___________________________

1. (5 pts.) Use a tree to determine whether the following formula is a tautology. Precisely explain your conclusion at the bottom of the page.

√ $\sim(\sim\exists x\exists y\ x=y \rightarrow \forall x\forall y\ f(x) = f(y))$

√$\sim\exists x\exists y\ x=y$

$\sim\forall x\forall y\ f(x) = f(y)$

$\forall x\sim\exists y\ x=y$

√$\sim\exists y\ f(a) =y$

$\forall y\ \sim f(a) =y$

$\sim f(a)=f(a)$

$\sim f(a)= f(b)$

x

Valid, negated formula closes. Note: It turns out there is no need to instantiate to f(a), f(b). Any two names or functions would work.

2. (5pts.) Derive the following in propositional logic. Use only basic rules (hypothetical and non hypothetical), <u>not</u> derived rules or equivalences. If you get stuck, you may use any rules except for $\sim\to$ for a maximum of 3 pts.

Exam contained a typo. Consequent was missing brackets which rendered it ~wff. Everyone received 5 pts.

$$\vdash \sim(S \to T) \to (S \,\&\, \sim T)$$

5 pt. solution

1.	$\sim(S \to T)$	H $\to$I
2.	T	H ~I
3.	S	H $\to$I
4.	$S \,\&\, T$	3,4 &I
5.	T	4,&E
6.	$S \to T$	3-5, $\to$I
7.	$(S \to T) \,\&\, \sim(S \to T)$	1,6 &I
8.	$\sim T$	2-7 ~I
9.	$\sim S$	H ~I
10.	S	H $\to$I
11.	$S \,\&\, \sim T$	8,10 &I
12.	$\sim T$	11, &E
13.	$S \to \sim T$	10-12 $\to$I
14.	$(S \to T) \,\&\, \sim(S \to T)$	1,13 &I
15.	$\sim\sim S$	9-14 ~I
16.	S	15 ~E
17.	$S \,\&\, \sim T$	8,16 &I
18.	$\sim(S \to T) \to (S \,\&\, \sim T)$	2-18, $\to$I

3 pt. solution.

1.	$\sim(S \to T)$	H $\to$I
2.	$\sim(\sim S \vee T)$	1, MI
3.	$\sim\sim S \,\&\, \sim T$	2, DM
4.	$S \,\&\, \sim T$	3, DN
5.	$\sim(S \to T) \to (S \,\&\, \sim T)$	1-4, $\to$I

3. (5pts.) Derive the following in predicate logic.

(Exam contained a typo. Relation F was miswritten as a two-place function rather than a relation. Everyone received 5 pts.)

$\exists xFxx \vdash (\sim\exists x\exists yFxy \rightarrow \forall zFzz)$

1.	$\exists xFxx$	A
2.	$\sim\exists x\exists y\ Fxy$	H $\rightarrow$I
3.	$\sim\forall zFzz$	H ~I
4.	$\forall x\forall y\sim Fxy$	3, QE(twice)
5.	Faa	H $\exists$E
6.	$\sim Faa$	4, $\forall$E(twice)
7.	$P\ \&\ \sim P$	5,6 CON
8.	$P\ \&\ \sim P$	1, 5-7 $\exists$E
9.	$\sim\sim\forall zFzz$	3-8 ~I
10.	$\forall zFzz$	9, DN
11.	$\sim\exists x\exists yFxy\rightarrow \forall zFzz$	2-10 $\rightarrow$I

4. (5 pts. ½ pt. each.) Evaluate each of the following predicate logic formulas as either true or false in the model. by writing either T or F beneath the formula. Study the definition of the model carefully. If you believe the formula is not a wff or an accepted variation of a wff, write "~wff," make the minimal correction that would render it a wff, and evaluate it on that basis.

p	q	9	z
1	3	r	e
a	j	8	2
f	b	5	m

Universe = All and <u>only</u> numbers and letters in the grid to the left.
Names = all the letters and numbers in the grid.

One place predicates
- N = number
- L = letter
- V = vowel
- C = consonant
- E = even
- O = odd
- P = prime

Two place predicates
- A = above
- U = under
- R = right of
- F= left of
- G = greater than
- S = subsequent to

Notes: Spatial predicates do not imply "directly". E.g., Apm is true even though p is not directly above m.

Of the numbers on the grid, only 2, 3 and 5 are prime.

G is a relation between numbers only. This entails, e.g., that $\mathcal{V}$(Ga1) = F, since a is not a number.

S is a relation between letters only. $\mathcal{V}$(Sea) = T because e comes after a in the alphabet, <u>not</u> the grid. This entails, e.g., that $\mathcal{V}$(S1a1) = F, since 1 is not a letter.

1. ∀xPx → ∀yEy

True

Antecedent s false because not everything is prime.

2. ∃x∃y(Cx ↔ (Vy ↔ y=b))

True

a=x and a=y satisfies thiis formula because Ca is false and (Va ↔ a=b) is false, making the entire biconditional true.

3. ∀x(Lx v (~Lx → ~Gx9))

True

Everything that is not a letter is a number that is not greater than 9.

4. Ser ↔ Re1

False

Ser is false and Re1 is true.

5. ∀x(Px → ∃y(Gyx & Nyx))

~wff (but any answer accepted)

N is a one place predicate.

6. ∀x∀y(Rxy v Ryx)

False

Nothing is to the right of itself.

7. ∀x(Axx → (P4 → ~∃zRzx))

True

Nothing is above itself, so the antecedent is false.

8. ∃x∃y(∀z(~Sxz →x=z) & ∀w(~Gyw → y=w))

True (any answer accepted due to ambiguity of z's reference)

This says that there is something subsequent to everything but itself and something greater than everything than iself. z and 9 respectvely satisfy this formula.

9. ∀x∀y((Cx & Vy) → (Axy → ∃z(~(z=y v z=x) & (Axz & Uyz))))

False (any answer accepted due to ambituity of z's reference)

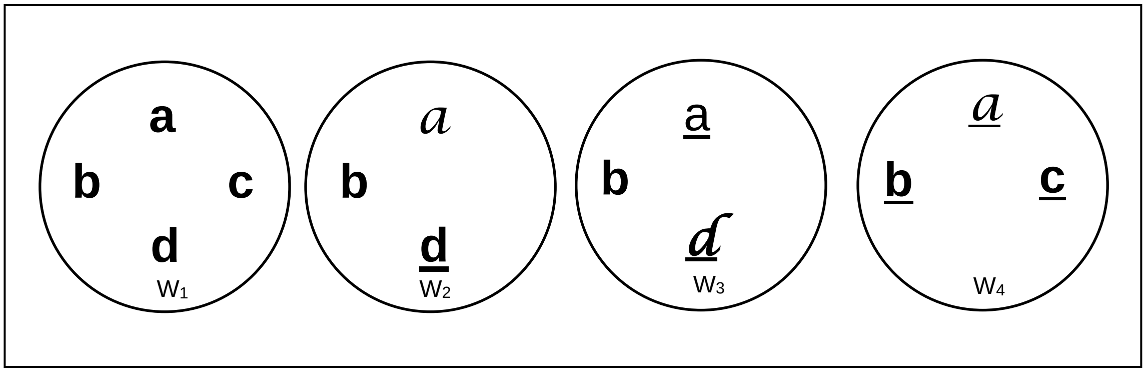
This says that for every consonant and every vowel, if the consonant is above the vowel, then there is something else that is between them. This is false, e.g., of x=z and y = e.

10. ∃x∃y((Ex & Lyx) & ∀z(Lz & (~Szy → z=y)))

~wff (any answer accepted due to ambiguity of z's reference.)

L is a one-place predicate.

5. (5pts.) The model $\mathcal{V}$ below depicts a universe of four possible worlds $\mathcal{W}_{\mathcal{V}} = \{w_1, w_2, w_3, w_4\}$. The styles of the name letters indicate the properties of <u>distinct</u> objects to which they refer, designated B, U and S for Bold, Underlined and Script. The table below supplies exactly the same information. Check it if you are uncertain about a visual interpretation. Notice that these are not mutually exclusive properties.



$\mathcal{V}(B,w_1) = \{a,b,c,d\}$	$\mathcal{V}(B,w_2) = \{b,d\}$	$\mathcal{V}(B,w_3) = \{b,d\}$	$\mathcal{V}(B,w_4) = \{b,c\}$
$\mathcal{V}(U,w_1) = \{\ \}$	$\mathcal{V}(U,w_2) = \{d\}$	$\mathcal{V}(U,w_3) = \{a,d\}$	$\mathcal{V}(U,w_4) = \{a,b,c\}$
$\mathcal{V}(S,w_1) = \{\ \}$	$\mathcal{V}(S,w_2) = \{a\}$	$\mathcal{V}(S,w_3) = \{d\}$	$\mathcal{V}(S,w_4) = \{a\}$

(1/2 pt. each) Evaluate the following as true, false or no truth value of the model above in Leibnizian modal logic. Write your answers as T or F or N directly below the corresponding formula. If you believe the formula is not a wff, write "~wff," make the minimal correction that would render it a wff, and evaluate it on that basis.

1. $\mathcal{V}(\Diamond Ba, w_1)$

True

a is bold at w1 so it is possible that a is bold at w1.

2. $\mathcal{V}(\Box(\sim Sa \rightarrow Ud), w_3)$

False

$(\sim Sa \rightarrow Ud)$ is false at w1, therefore not true in every possible world.

3. $\mathcal{V}((Ud \rightarrow \Box Ud), w_3)$

False

The antencendent is true because d is underlined at w3. But it is not true that Ud in every possible world, so the conditional is false.

4. $\mathcal{V}(\Box\exists x(x{=}a \ \&\ x{=}b), w_4)$

False

This says that in every possible world there is one thing that is both a and b. According to the model, every name refers to a distinct object

.

5. $\mathcal{V}(\sim\Diamond\sim\exists x\exists y(x{=}a \ \&\ y{=}b), w_1)$

True

This says that a and b exist in every possible world.

6. $\mathcal{V}((\sim\Box Bd \vee \Diamond Ua), w_2)$

True

Since a is underlined at w3 it is possible that a is underlined at w2, making the disjunct true.

7. $\mathcal{V}(\Box(\Box\sim\forall x Bx \rightarrow Sa), w_3)$

True

Since all objects are bold at w1 the antecedent is false in every possible world making the conditional true in every possible world.

8. $\mathcal{V}(\Box(\exists x\ x{=}b \rightarrow \Diamond\exists x Sx), w_1)$

True

The antecedent and consequent are true in every possilbe world, making the conditional true in every possibe world.

9. $\mathcal{V}(\Box(\Diamond Bd \leftrightarrow Sa), w_2)$

False

The biconditional is not true at every world because it has no determinate truth value at w4, due to the non existence of d. If you did not catch this, and assigned true in every world to ◊Bd, the formula is still false because Sa is false in w1 and w3.

10. $\mathcal{V}(\Box\exists x\exists y\exists z\ (\sim x{=}y \ \&\ \sim y{=}z), w2)$

True

There are three distinct objects at every possible world.

6. Extra credit: 2 pt.

Finish the following semantic rules from Leibnizian modal logic. Remember since you are stating the conditions under which a particular type of proposition is true, what you write must make <u>no reference to truth</u>.

- Given any Leibnizian valuation $\mathcal{V}$ for any world *w* in $\mathcal{W}_{\mathcal{V}}$

If *P* is a one-place predicate then:

$\mathcal{V}$(∀x*P*x,*w*) = T iff for every name *a* in *w*, $\mathcal{V}$(a) ∈ $\mathcal{V}$(*P*)

$\mathcal{V}$(□∀x*P*x,*w*) = T iff for every name *a* <u>in every *w in* $\mathcal{W}_{\mathcal{V}}$</u>, $\mathcal{V}$(a) ∈ $\mathcal{V}$(*P*)