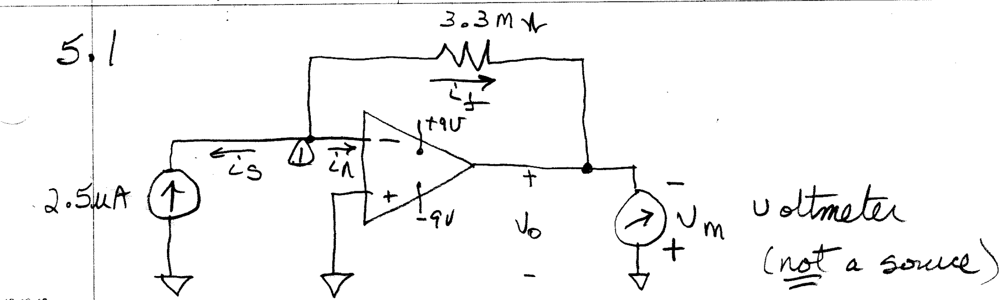
Engr 17 - CSUS Instructor – Russ Tatro

Chapter 5 The Operational Amplifier – Homework Solutions
James W. Nilsson and Susan A. Riedel, *Electric Circuits*, 6th Edition, 2001

Problems 1, 2, 3, 10 part a only, 16 and 21

1	Vm = 8.25 V
2	i_L = -200 μA
3	a) Vo = -12 V b) Vo = -18V c) Vo = 10 V d) Vo = -14 V e) Vo = 18 V f) 2.8 V ≤ Va ≤ 7.3 V
10	a) Vo = -2.4 V
16	a) i_L = 2 mA b) R_L = 3 kΩ max c) Hint – is the op-amp in saturation?
21	a) Vo = Vs [(R2 + R1)/R1] b) Vo = Vs c) No hint

5.1



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① Asume ideal op-amp

$i_p = i_n = 0$ and $v_p = v_n$

② note $v_o = -v_m$ (look a references on circuit)

find v_m

I have drawn all currents away from node ①

$$i_s + \cancel{i_n}^{0} + \frac{\cancel{v_n}^{0} - v_o}{R_f} = 0$$

i_s reference was away from node but indep current source is "into" thus $i_s = -2.5\,\mu A$

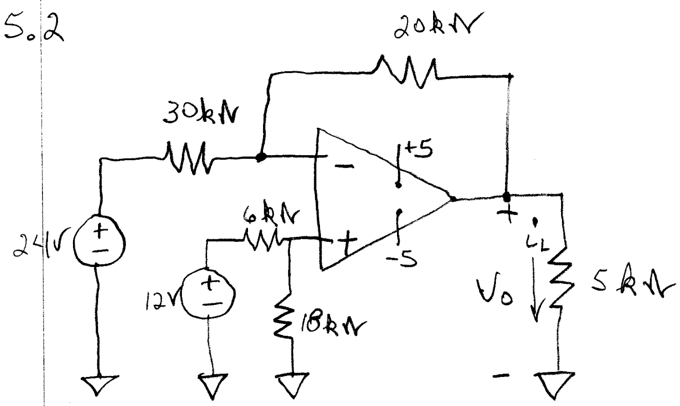
$$i_s - \frac{v_o}{R_f} = 0$$

$$v_o = i_s R_f = (-2.5\,\mu A)(3.3\,M\Omega) = -8.25V$$

less than negative power supply!

$$v_m = -v_o = \underline{\underline{8.25V}}$$

5.2



① v_o is not dependent on R_L, thus op-amp sets v_o and then R_L determines i_L

② $v_p = v_n$ but $v_p \neq 0$ it is set by voltage divider.

$$v_p = \frac{18k\Omega}{(6k + 18k)\Omega} 12V = 9.0V = v_n$$

at inverting node ($v_n \neq 0$!)

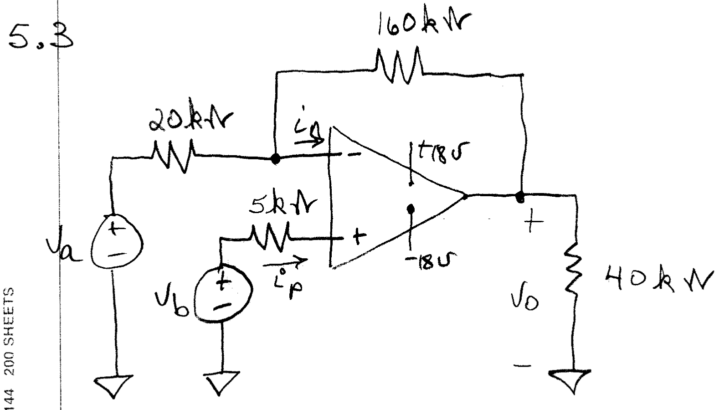
$$\frac{v_n - 24V}{30k\Omega} + \frac{v_n - v_o}{20k\Omega} = 0$$

$$\frac{9V - 24V}{30k\Omega} + \frac{9V - v_o}{20k\Omega} \Rightarrow v_o = 20k\Omega\left(\frac{-18V}{30k\Omega} + \frac{9V}{20k\Omega}\right)$$

$$v_o = -1V = i_L R_L = i_L (5k\Omega)$$

$$i_L = \frac{-1V}{5k\Omega} = -0.2 \times 10^{-3} A = \underline{\underline{-200\,\mu A}}$$

5.3



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Note $i_p = i_n = 0$ thus $\underline{v_p = v_b} = v_n$

a) find v_o for $v_a = 1.5V$ & $v_b = 0V$ (solution is general form of inverting op-amp)

$$v_o = -\frac{R_f}{R_s} v_s = -\frac{160k}{20k}(1.5V) = \underline{\underline{-12V}}$$

b) find v_o for $v_a = 3.0V$ & $v_b = 0$

$v_o = -8(3V) = -24V$ but outside linear region!

so op-amp in saturation

$$\underline{\underline{v_o = -18V}}$$

c) find v_o for $v_a = 1.0V$ & $v_b = 2.0V = v_n$

$$\frac{v_n - v_a}{20k\Omega} + i_n + \frac{v_n - v_o}{160k\Omega} = 0 \Rightarrow v_o = +160k\Omega\left(\frac{v_n - v_a}{20k\Omega} + \frac{v_n}{160k}\right)$$

$$v_o = +160k\Omega\left(\frac{2V - 1V}{20k\Omega} + \frac{2V}{160k}\right) = \underline{\underline{+10V}}$$

5.3 continued

d) find v_o for $v_a = 4V$ & $v_b = 2V = v_n$

use eqn from part c)

$$v_o = 160k\Omega\left(\frac{2V-4V}{20k\Omega} + \frac{2V}{160k\Omega}\right) = \underline{\underline{-14V}}$$

e) find v_o for $v_a = 6V$ & $v_b = 8V$

$$v_o = 160k\Omega\left(\frac{8V-6V}{20k\Omega} + \frac{8V}{160k}\right) = 24V$$ not possible! op-amp in saturation

so $\underline{\underline{v_o = 18V}}$

f) Given $v_b = 4.5V$ find v_a for linear operation of op-amp

again note $v_b = v_n$

$$\frac{v_b - v_a}{20k\Omega} + \frac{v_b - v_o}{160k\Omega} = 0 \Rightarrow v_o = 9v_b - 8v_a$$

limits of v_o are $\pm 18V$

so

$$-18V \leq 9v_b - 8v_a \leq 18V$$

$$-18V \leq 9(4.5V) - 8v_a \leq 18V$$

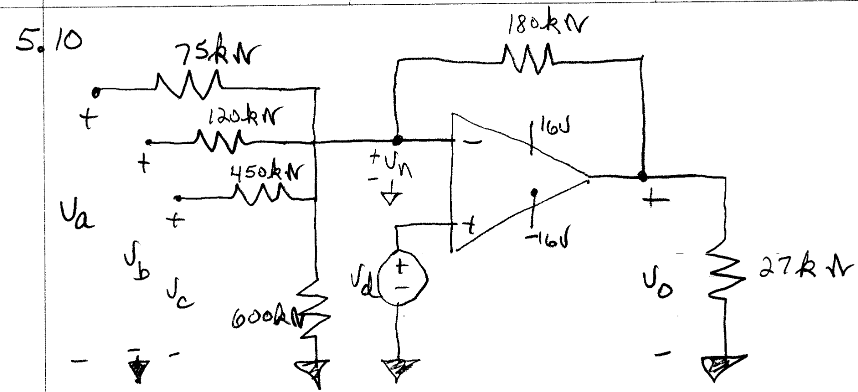
$$-18V \leq 40.5V - 8v_a \leq 18V$$

$$-58.5 \leq -8v_a \leq -22.5$$

$$7.3125 \geq v_a \geq 2.8125$$

thus $\underline{\underline{2.8125V \leq v_a \leq 7.3125V}}$

5.10



a) find v_o for $v_a = 18V$

$v_b = 6V$

$v_c = -15V$

$v_d = 8V \;=\; v_p = v_n$

Treat 600 kΩ branch as a 4th Summing at zero volts

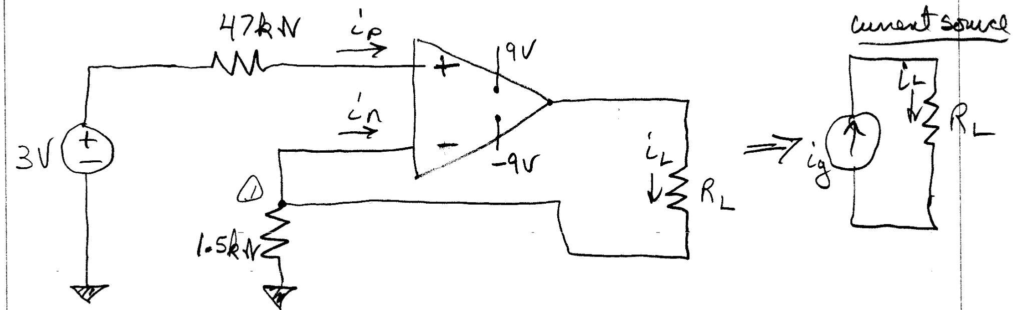
$$\frac{v_n - v_a}{72k\Omega} + \frac{v_n - v_b}{120k\Omega} + \frac{v_n - v_c}{450k\Omega} + \frac{v_n - 0}{600k\Omega} + \frac{v_n - v_o}{180k\Omega} = 0$$

in the above eqn the only unknown is v_o

$$v_o = 180k\Omega\left(\frac{8V - 18V}{72k\Omega} + \frac{8V - 6V}{120k\Omega} + \frac{8V-(-15V)}{450k\Omega} + \frac{8V}{600k\Omega} + \frac{8V}{180k\Omega}\right)$$

$v_o = -2.4V$

5.16



a) find i_L for $R_L = 2.5k\Omega$

$$v_p = 3V = v_n$$

at node ①

$$\frac{v_n - 0}{1.5k\Omega} + \cancel{i_n}^{0} + (-i_L) = 0$$

$$i_L = \frac{v_n}{1.5k\Omega} = \frac{3V}{1.5k\Omega} = \underline{2mA}$$

b) if $i_L = 2mA$ then find max R_L

$$v_o = i_L(R_L + 1.5k\Omega) \qquad \text{where } v_o \le 9V$$

$$R_L = \frac{v_o}{i_L} - 1.5k\Omega = \frac{9V}{2mA} - 1.5k\Omega = 4.5k\Omega - 1.5k\Omega$$

$$\underline{R_L = 3.0k\Omega \text{ max}}$$

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5.16 continued

c) given $R_L = 6.5\,k\Omega$. Explain the operation of the circuit.

since $R_L > 3\,k\Omega$ op-amp is in saturation $\rightarrow$ $\underline{\underline{v_O = 9V}}$

$$i_L = \frac{v_O}{(1.5+6.5)k\Omega} = \frac{9V}{8k\Omega} = \underline{\underline{1.125\,mA}}$$

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Note the following discussion is to show the "ideal op-amp" model is quite good.

The above results are valid only if $i_p = i_n = 0$.
or equivalently $i_p = i_n \ll i_L$

if we assume ① $i_p \to 0$ ie current thru $47k\Omega$ resistor is negligible.

and ② $R_{in} \geq 500\,k\Omega$ (typically $R_{in} \geq 1\,M\Omega$ for real op-amps)

$$v_n = i_L(1.5\,k\Omega) = (1.125\,mA)(1.5\,k\Omega) = 1.6875\,V$$

thus $$i_p = i_n = \left(\frac{v_p - v_n}{R_{in}}\right) = \frac{3V - 1.6875V}{500\,k\Omega}$$

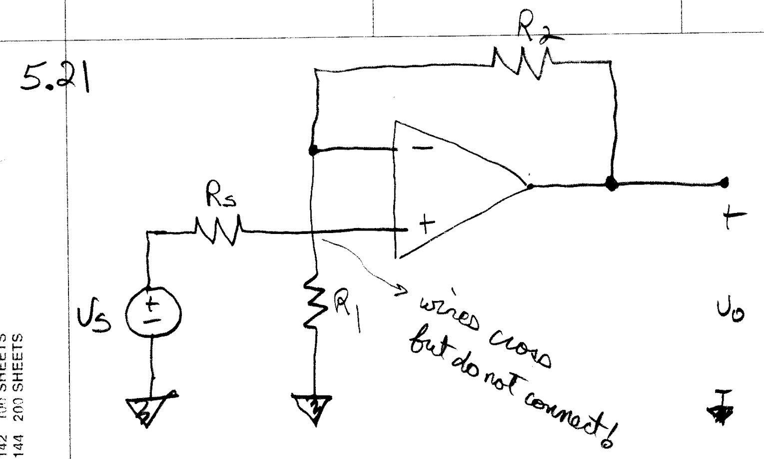
$$= 2.6\,\mu A = 2.6\times10^{-6}A$$

$v_p = 3V - (2.6\mu A)(47k\Omega)$
$= 2.9V$

thus our assumption $i_p = i_n = 0$ is quite good!!

or equally $\underline{\underline{i_p = i_n \ll i_L}}$

5.21



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a) show $v_o = \dfrac{R_1 + R_2}{R_1} v_s$

at node v_n

$$i_n + \frac{v_n - 0}{R_1} + \frac{v_n - v_o}{R_2} = 0 \qquad \text{but } v_n = v_p = v_s$$

$$\frac{v_s - 0}{R_1} + \frac{v_s - v_o}{R_2} = 0 \Rightarrow v_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v_o}{R_2}$$

$$v_o = v_s\left(\frac{R_2 + R_1}{R_1 \cancel{R_2}}\right)\cancel{R_2} = \underline{v_s\left(\frac{R_2 + R_1}{R_1}\right)}$$

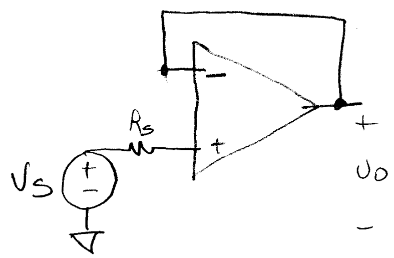
b) What happens as $R_1 \to \infty$ & $R_2 \to 0$

$$\frac{R_2 + R_1}{R_1} \to 1$$

thus $v_o = v_s$ ie <u>voltage follower</u>

5.21 continued

c) when $R_1 = \infty$ & $R_2 = 0$
we have this circuit



thus

$$\left.\begin{aligned} v_n &= v_p = v_s \\ \& \quad v_n &= v_o \end{aligned}\right\} \quad v_o = v_s$$

as found in part b)

So up to the limit of the positive power supply voltage V_{out} will "follow" v_s.

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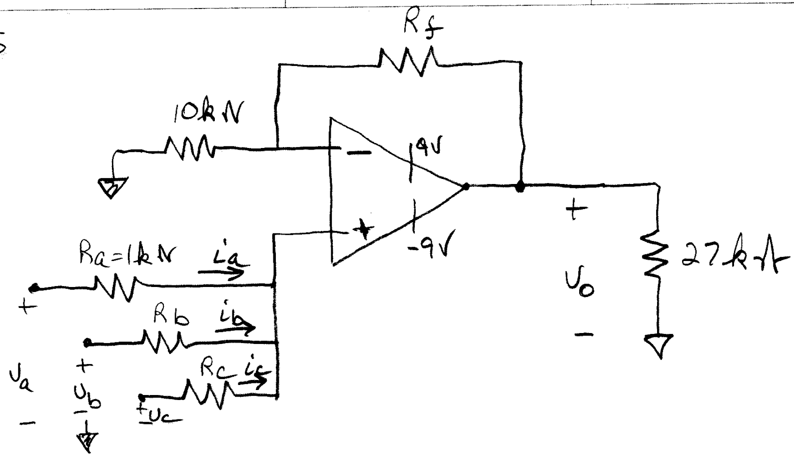
5.25



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Design the circuit so that $v_o = 5v_a + 4v_b + v_c$

a) find R_b, R_c & R_f

start by writing the node equations at v_p and then at v_n

$$\frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} = 0$$

$$v_p\left(\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c}\right) = \frac{v_a}{R_a} + \frac{v_b}{R_b} + \frac{v_c}{R_c}$$

$$v_p = \left(\frac{v_a}{R_a} + \frac{v_b}{R_b} + \frac{v_c}{R_c}\right)\left(\frac{R_a R_b R_c}{R_b R_c + R_a R_c + R_a R_b}\right)$$

now at v_n

$$\frac{v_n - 0}{10k\Omega} + \cancel{i_p}^{0} + \frac{v_n - v_o}{R_f} \Rightarrow v_n\left(\frac{1}{10k\Omega} + \frac{1}{R_f}\right) = \frac{v_o}{R_f}$$

$$v_o = v_n\left(\frac{R_f}{10k\Omega} + 1\right)$$

$$= v_p\left(\frac{R_f}{10k\Omega} + 1\right)$$ by ideal op-amp $v_n = v_p$

5.25 cont

so we found 2 eqns for v_o now combine those with the given constraint $v_o = 5v_a + 4v_b + v_c$

first let us gather up some common terms so we can see them clearly.

rewrite

$$v_p = \frac{R_b R_c}{D} v_a + \frac{R_a R_c}{D} v_b + \frac{R_a R_b}{D} v_c$$

where $D = R_b R_c + R_a R_c + R_a R_b$

and let $k = \frac{R_f}{10k\Omega} + 1$

thus

$$v_o = k\, v_p = \frac{k}{D}\left(R_b R_c v_a + R_a R_c v_b + R_a R_b v_c\right)$$

now equate common coefficients

$$\frac{k}{D} R_b R_c = 5 \quad \Rightarrow R_b = 5 \frac{D}{k R_c}$$

$$\frac{k}{D} R_a R_c = 4 \quad \Rightarrow R_a = 4 \frac{D}{k R_c}$$

$$\frac{k}{D} R_a R_b = 1 \quad \Rightarrow R_a = \frac{D}{k R_b} \Rightarrow \frac{1}{R_a} = \frac{k}{D} R_b$$

or also

$$\frac{1}{R_b} = \frac{k}{D} R_a$$

5.25 cont

Plug relationships & find a ratio to R_a ($= 1k\Omega$ remember)

$$\underbrace{\frac{k}{D}}_{1/R_a} R_b R_c = \frac{R_c}{R_a} = 5 \quad \text{thus } R_c = 5R_a = \underline{\underline{5\,k\Omega = R_c}}$$

$$\underbrace{\frac{k}{D}}_{1/R_b} R_a R_c = \frac{R_c}{R_b} = 4 \Rightarrow R_b = \frac{R_c}{4} = \underline{\underline{1.25\,k\Omega = R_b}}$$

Now all that is left is R_f.

$$\frac{k}{D} R_b R_c = 5 \Rightarrow k = \frac{5D}{R_b R_c}$$

where
$$D = [1.25(5) + 1(5) + 1.25(1)]\, M\Omega = 12.5\, M\Omega^2$$

so
$$k = \frac{5(12.5 M\Omega^2)}{(1.25k\Omega)(5k\Omega)} = 10 = \frac{R_f}{10k\Omega} + 1$$

$$\frac{R_f}{10k\Omega} = 9 \Rightarrow \underline{\underline{R_f = 90\,k\Omega}}$$

thus $R_a = 1k\Omega$ (given)

$R_b = 1.25k\Omega$

$R_c = 5k\Omega$

$R_f = 90k\Omega$

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5.25 cont.

b) find i_a, i_b & i_c in μA

for $v_a = 0.5V$

$v_b = 1.0V$

$v_c = 1.5V$

$$v_o = 5(0.5V) + 4(1.0V) + 1.5V = 8V$$

$$\frac{v_n - 0}{10k\Omega} + \cancel{i_n}^{0} + \frac{v_n - v_o}{90k\Omega} = 0 \Rightarrow v_n\left(\frac{1}{10k\Omega} + \frac{1}{90k\Omega}\right) = \frac{v_o}{90k\Omega}$$

$$v_n\left(\frac{90k\Omega}{10k\Omega} + \frac{90k\Omega}{90k\Omega}\right) = v_o$$

$$v_n(9+1) = v_o \Rightarrow v_n = \frac{v_o}{10} = \frac{8V}{10} = 0.8V = v_p$$

(you could have found v_p from the non-inverting amp solution $v_o = \left(1 + \frac{R_f}{R_s}\right)v_p \Rightarrow v_p = \frac{v_o}{1+\frac{90k}{10k}} = \frac{v_o}{1+9} = \frac{v_o}{10}$)

now we can find the currents

$$i_a = \frac{v_a - v_p}{1k\Omega} = \frac{(0.5-0.8)V}{1k\Omega} = \frac{-0.3V}{1k\Omega} = \underline{\underline{-300\mu A}}$$

$$i_b = \frac{v_b - v_p}{1.25k\Omega} = \frac{(1.0-0.8)V}{1.25k\Omega} = \underline{\underline{160\mu A}}$$

$$i_c = \frac{v_c - v_p}{5k\Omega} = \frac{(1.5-0.8)V}{5k\Omega} = \underline{\underline{140\mu A}}$$