EEE 117 Laboratory
Instructor: [redacted]

LAB 4: ACTIVE AND PASSIVE LOW PASS FILTER (TIME DOMAIN)

Lab Report by: [redacted]

Lab Session: Monday

Due Date of the lab: 11/03/2014

Date(s) of the lab: 10/20/214 and 10/27/2014

Lab partner(s): [redacted]

[redacted]

[redacted]

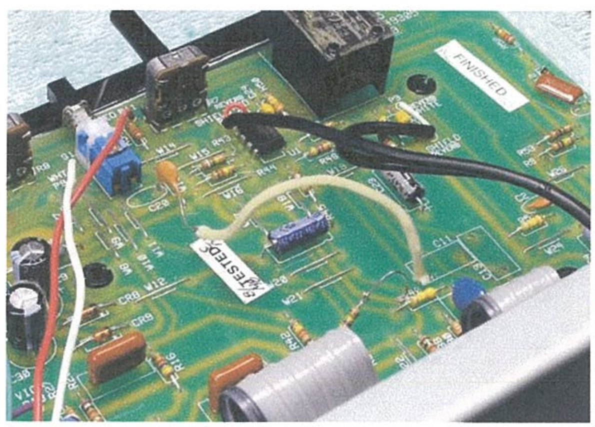
I. INTRODUCTION:

In this lab, we will learn (or re-learn) and become familiar with the concept of an Active and Passive Low Pass filter in a given circuit. From each of the given circuits that we are to build, or connect, we are to develop the transfer function and construct a Bode plot for each. After we obtain the data for each of the circuits that can either be loaded or unloaded, we will compare the similarities and differences of the outputs. In addition, we will compare the similarities and differences between a circuit with a capacitor based filter and a circuit with an operation amplifier based filter. Any excess information regarding Active and Passive Low-Pass Filters can be found on YouTube or Wikipedia.

II. PURPOSE:

The purpose of this lab is to consider the time domain response of both a passive and active first order low pass filter. In **LAB 10**, we will again look at first order filters with respect to the frequency domain. We will use the pre-built printed circuit board (which will be provided by our lab instructor) for this two week lab. We will find the exact part value of our pre-built circuit by checking the printed circuit board serial number versus the values posted on the course website.

Below, a circuit similar to the pre-built circuit board that my lab partners and I used for this experiment.



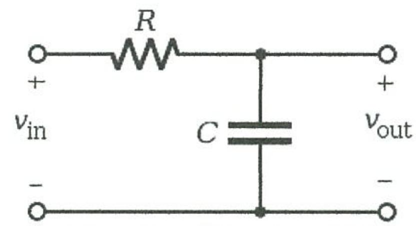
III. DISCUSSION AND RESULTS:

<u>Part I:</u>

For this part of the lab, we connected the proper circuit on the pre-built circuit board and we analyzed a simple low pass filter containing only a resistor and a capacitor. The circuit board my lab partners and I obtained contains the number "1." So all values for the "Printed Circuit Board Component Values" included in this lab report will be based off the values for Pre-built circuit board 1. This data can be provided the lab instructor. I did noticed that when my lab partners and I measured the resistance for each resistor on our pre-built circuit board, each of the resistors were off by a very minimal amount. On the following page, I have provided a table comparing the pre-determined values vs. the measured values we obtained through the DMM (Digital Multimeter). As I have said, the values between the pre-determined values and the measured values aren't too different.

TABLE 1: PRINTED CIRCUIT BOARD COMPONENT VALUES (PRE-DETERMINED VS. MEASURED)									
	Board #	R1	R2	R3	R4	C1	C2	C3
Pre-Determined Values	1	10,041 Ω	10,034 Ω	10.0 Ω	9,995 Ω	481 pF	946 nF	465 pF
Measured Values	1	10,033 Ω	10,027 Ω	9.96 Ω	9,988 Ω	481 pF	946 nF	465 pF

Below, the image of the standard low pass filter in a simple RC circuit can be observed.



Standard Low Pass Filter for a RC circuit

Below, is the circuit diagram for the RC circuit.

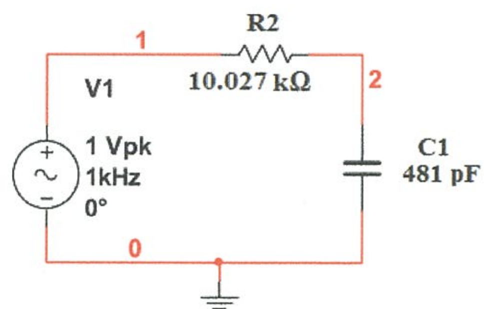


FIGURE 1: Lowpass Filter.

To create a Bode plot of the Low Pass Filter in a RC circuit, we first start with a voltage divider equation and applying KVL to the circuit seen above.

$$-v_s + iR_1 + iR_2 = 0$$

$$v_s = iR_1 + iR_2$$

$$v_s = i(R_1 + R_2)$$

$i = \frac{v_s}{R_1+R_2}$, then pluging in $i = \frac{v_1}{R_1}$

$\frac{v_1}{R_1} = \frac{vs}{R_1+R_2}$ which gives you $v_1 = \frac{R1vs}{R_1+R_2}$

$$v_o = \frac{R_{eq}}{R_1 + R_{eq}} v_s$$

Now that we have the basic voltage divider equation we can substitute in values for the low pass filter. First we will change V_s to V_l and R_{eq} to Z_{C1}.

$$H(s) = \frac{V_0}{V_l} = \frac{Z_{C1}}{R_2 + Z_{C1}}$$

In the frequency domain, s = jω.

$$\boxed{H(s) = \frac{Z_{C1}}{R_2+Z_{C1}} = \frac{\frac{1}{sC_1}}{R_2+\frac{1}{sC_1}} = \frac{\frac{1}{R_2C_1}}{s+\frac{1}{R_2C_1}} = \frac{\omega_c}{s+\omega_c}}$$

[Transfer Function]

Since the Cornering frequency (ω_c) equals $\frac{1}{R_2C_1}$ we can substitute into the equation above

$$\omega_c = \frac{1}{R_2C_1} = \frac{1}{(10027\Omega)(481pF)} = 207340.39 \text{ rads/sec}$$

[Corner Frequency]

$$H(s) = \frac{\omega_c}{s+\omega_c} = \frac{207340.39}{s+207340.39} = \frac{1}{1+\frac{j\omega}{207340.39}}$$

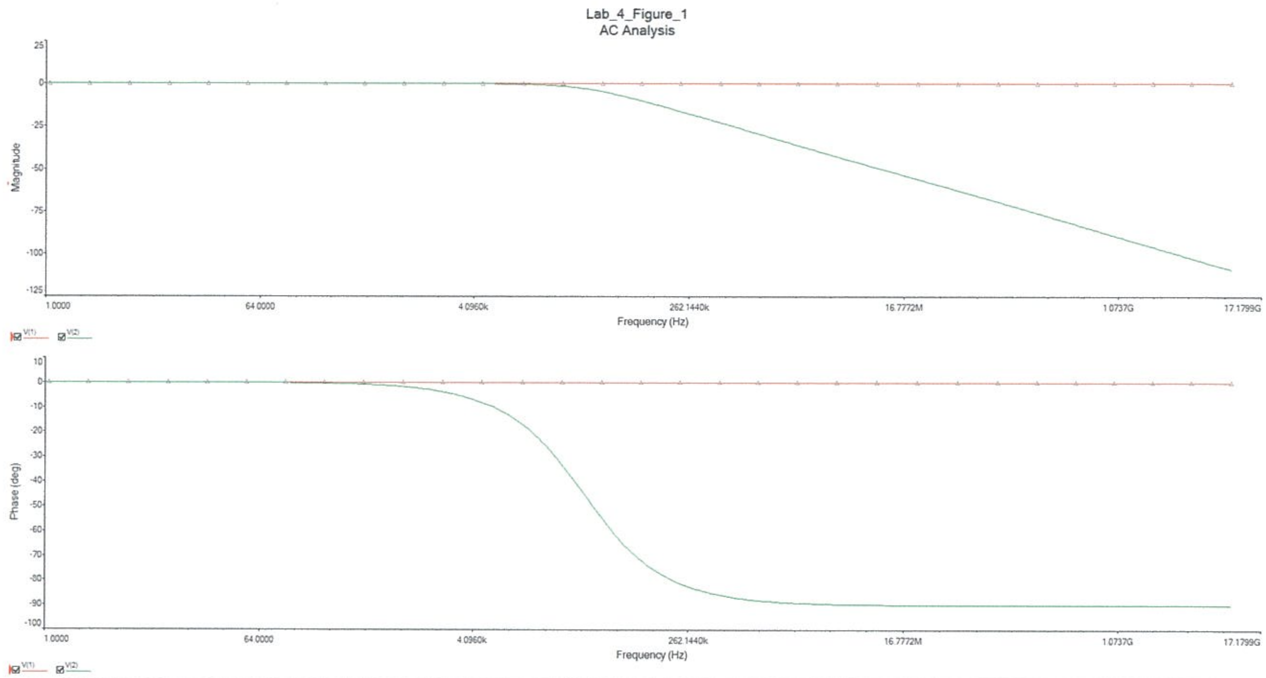
$$|\,H(s)\,| = 20\log(1) - 20\log\left|\,1 + \frac{j\omega}{207340.39}\right|$$

$$\Phi_{\text{phase}} = \arctan\left(\frac{0}{1}\right) + \arctan\left|\frac{\omega}{207340.39}\right|$$

Below, is a table consisting of amplitude and phase response values obtained through theoretical analysis.

THEORETICAL ANALYSIS		
Frequency (ω)	**H(dB)**	**Phase (°)**
100	0	-0.58
500	0	-0.94
1.00E+03	0	-1.6
5.00E+03	0	-10
1.00E+04	0	-16
5.00E+04	-5.67993	-56
1.00E+05	-10.4576	-69
5.00E+05	-23.098	-85
1.00E+06	-29.897	-86
5.00E+06	-43.098	-80
1.00E+07	-46.3752	-69

After our theoretical analysis, we went to simulate the circuit in FIGURE 1 and we obtained the following results.

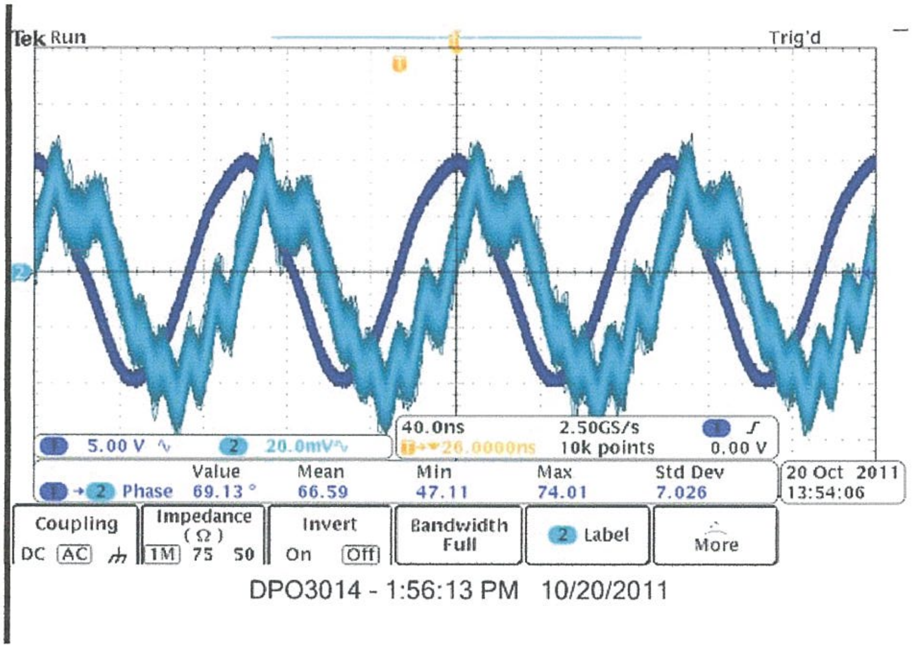


Above, is the Bode Plot (or AC analysis) we obtained with the circuit in FIGURE 1.

SIMULATION ANALYSIS		
Frequency (ω)	**H(dB)**	**Phase (°)**
100	0	-0.16
500	0	-0.84
1.00E+03	0	-1.67
5.00E+03	-0.1	-8.29
1.00E+04	-0.4	-16.2
5.00E+04	-5	-55.6
1.00E+05	-9.8	-71
5.00E+05	-23.3	-86
1.00E+06	-29.3	-88
5.00E+06	-43.27	-89.6
1.00E+07	-49.27	-89.8

Above are the values for the amplitude and phase response obtained through simulation on MultiSim.

After completing the simulation, we moved onto setting up the circuit by connecting it to the waveform generator and taking measurements using the oscilloscope. We obtained a waveform similar to the one below.



Below, are the values we obtained through measuring the constructed circuit board.

ACTUAL CIRCUIT MEASUREMENTS		
Frequency (ω)	H(db)	Phase (°)
100	0	-0.58
500	0	-0.94
1.00E+03	0	-1.6
5.00E+03	0	-10
1.00E+04	0	-16
5.00E+04	-5.67993	-56
1.00E+05	-10.4576	-69
5.00E+05	-23.098	-85
1.00E+06	-29.897	-86
5.00E+06	-43.098	-80
1.00E+07	-46.3752	-69

Below is the data regarding the Corner Frequency for the RC Filter.

Data points	Theoretical Analysis	Simulation	Measured	%Difference Theoretical vs Measured	% Difference Sim Vs Measured
ω_c	32,329 Hz	32,319 Hz	32,000 Hz	1.02%	1.00%
Magnitude	1 V	1 V	0.99 V	1%	1%

I used the following equation(s) to find the %Difference for theoretical vs. measured and the %Difference for simulation vs. measured.

$$\%differnce\ (Theoretical\ Analysis\ vs\ Measured) = \frac{TA - Measured}{Measured} x100$$

$$\%differnce\ (Simulated\ vs\ Measured) = \frac{Sim - Measured}{Measured} x100$$

We found that the Corner frequency for the theoretical analysis was close to a phase of -44°, but simulation gave us a -44.98° and the measured value was -43.78°.

The %Difference between measured and theoretical and simulated is very close to exactly 1 %. This result is due to the limitations of the oscilloscope and human capacity of determining the precision of the values. We found that it is somewhat challenging to obtain a phase shift of exactly 45°. Of course, my lab partners and I could have messed up somewhere during the experiment by recording the wrong values or connecting the circuit given in FIGURE 1 incorrectly. Since the Corner Frequency values for simulated and calculated are given in the units of radians per second, we need to manually convert the values to hertz. Since we tend to round the values we record, it can account for some errors in this part of the lab. The frequencies chosen as a sweep needed to capture both the phase shift and the change in magnitude, which is approximately one tenth and ten times the calculated Corner Frequency. The magnitude of the output signal equals in magnitude of the inputs signals. When we compare each of the tables involving theoretical, simulated, and measured amplitude and phase values, we noticed that they are close to consistency to each other.

Part II:

For this part, we are to experiment with the given circuit below (FIGURE 2).

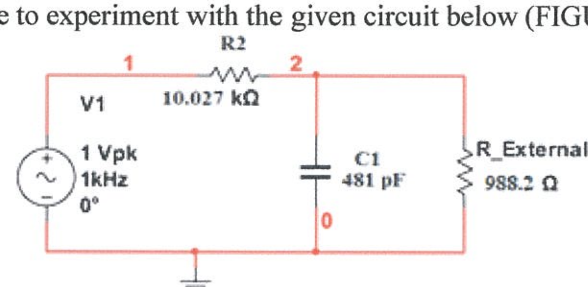


FIGURE 2: Loaded Low-Pass Filter.

To create a Bode plot of the Loaded RC Low Pass Filter, once again we start with a voltage divider equation and then apply KVL (clockwise)

$$-v_s + iR_1 + iR_{||} = 0 \qquad [\,\sum V = 0\,]$$

$$\text{where } R_{||} = \frac{R_3 R_2}{R_2 + R_3}$$

$$v_s = iR_1 + iR_{||}$$

$$v_s = i(R_1 + R_{||})$$

$$i = \frac{v_s}{R_1 + R_{||}}$$

then we plug in $i = \frac{v_1}{R_1}$

and we get $v_o = \frac{R_{eq}}{R_1 + R_{eq}} v_s$

Then, we will change v_s *to* v_l and R_{eq} *to* $R_{external} || Z_{C1}$.

$$\boxed{H_{(S)} = \frac{V_o}{V_l} = \frac{R_{external} \,||\, Z_{C1}}{R_2 + R_{external} \,||\, Z_{C1}}} \qquad \text{[Transfer Function]}$$

In the frequency domain, s = jω.

$$V_{out} = V_{in}\left(\frac{Z_{C1}//R_{external}}{R_2 + Z_{C1}//R_{external}}\right)$$

$$Z_{C1}//R_{external} = \left(\frac{Z_{C1} \times R_{external}}{Z_{C1} + R_{external}}\right) = \left(\frac{\left(\frac{1}{j\omega C_1}\right) \times R_{external}}{\left(\frac{1}{j\omega C_1}\right) + R_{external}}\right) = \left(\frac{R_{external}}{1 + j\omega C_1 R_{external}}\right)$$

$$V_{out} = V_{in}\left(\frac{\left(\frac{R_{external}}{1 + j\omega C_1 R_{external}}\right)}{R_2 + \left(\frac{R_{external}}{1 + j\omega C_1 R_{external}}\right)}\right)$$

$$\frac{V_{out}}{V_{in}} = \left(\frac{\left(\frac{R_{external}}{1 + j\omega C_1 R_{external}}\right)}{R_2 + \left(\frac{R_{external}}{1 + j\omega C_1 R_{external}}\right)}\right)\left(\frac{\frac{1 + j\omega C_1 R_{external}}{R_{external}}}{\frac{1 + j\omega C_1 R_{external}}{R_{external}}}\right)$$

$$H(j\omega) = \left(\frac{1}{R_2\left(\frac{1 + j\omega C_1 R_{external}}{R_{external}}\right) + 1}\right)$$

$$H(j\omega) = \left(\frac{1}{R_2 j\omega C_1 + \frac{R_2}{R_{external}} + 1}\right)$$

$$H(j\omega) = \left(\frac{1}{j\omega C_1 R_2 + \frac{R_2}{R_{external}} + \frac{R_{external}}{R_{external}}}\right)$$

$$H(j\omega) = \left(\frac{1}{j\omega C_1 R_2 + \left(\frac{R_2 + R_{external}}{R_{external}}\right)}\right)\left(\frac{R_{external}}{R_{external}}\right)$$

$$H(j\omega) = \left(\frac{R_{external}}{\left((j\omega C_1 R_2 R_{external}) + (R_2 + R_{external})\right)}\right)$$

$$H(j\omega) = \left(\frac{R_{external}}{(R_2 + R_{external})\left(1 + \left(\frac{j\omega C_1 R_2 R_{external}}{(R_2 + R_{external})}\right)\right)}\right)$$

$$H(j\omega) = \left(\frac{R_{external}}{(R_2 + R_{external})\left(1 + \left(\frac{j\omega}{\left(\frac{R_2 + R_{external}}{R_2 R_{external} C_1}\right)}\right)\right)}\right)$$

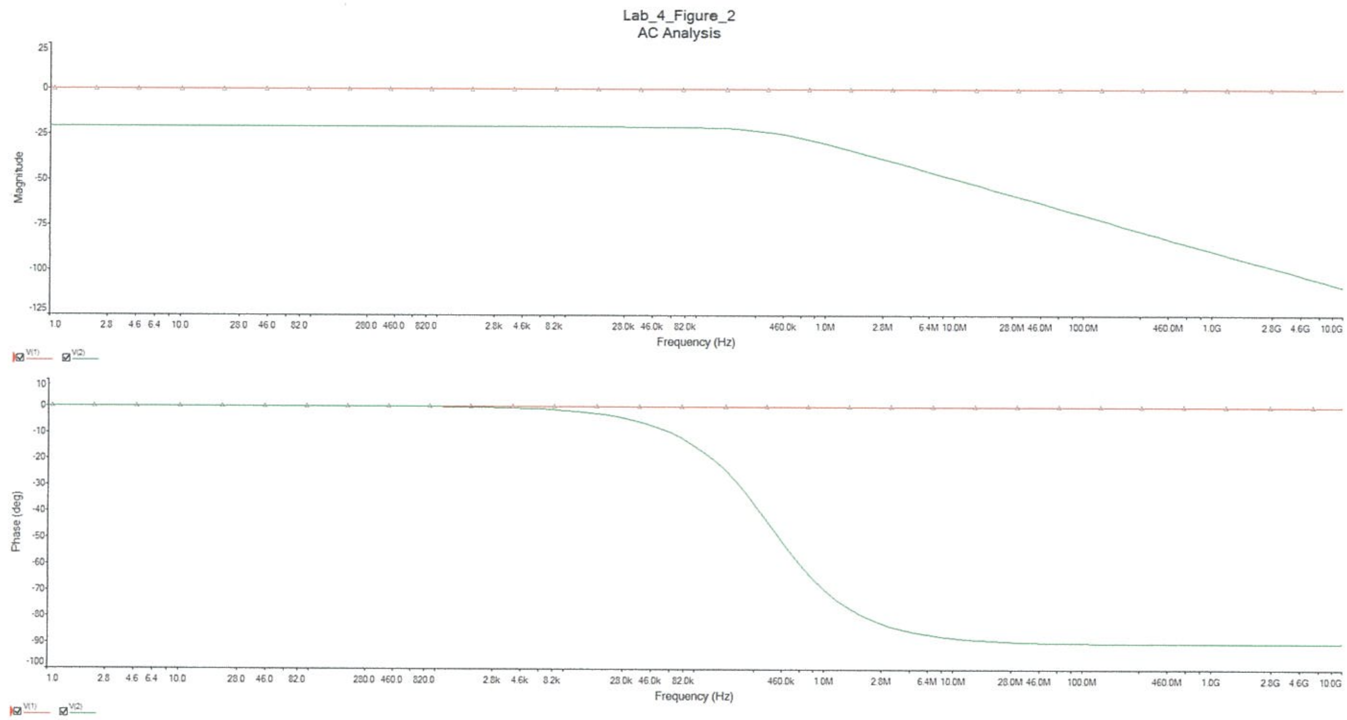
$$H(j\omega) = \left(\frac{988.2\Omega}{(11015.2\ \Omega)\left(1 + \left(\frac{j\omega}{(2311167.6)}\right)\right)}\right)$$

$\omega_c = \frac{1}{R_{eq} C_1} = \frac{1}{(10027\Omega || 988.2)(470pF)} =$ 2311167.6 rads/s [Corner Frequency]

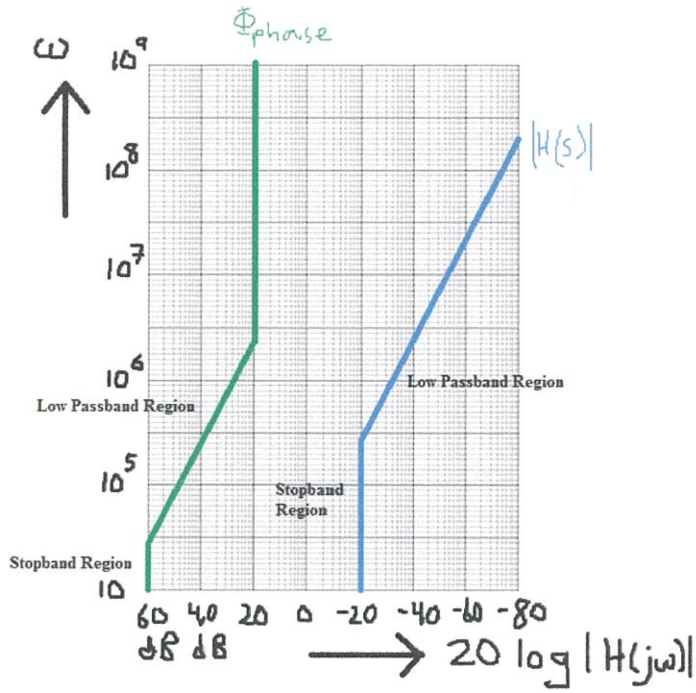
$$|H(s)| = 20\log(0.0897) - 20\log(1 + \frac{j\omega}{2311167.6})$$

$$\Phi_{phase} = \arctan\left(\frac{0}{0.0897}\right) + \arctan\left(\frac{\omega}{2311167.6}\right)$$

Below, is the Bode plot (AC Analysis) simulation for RC loaded low pass filter in the circuit from FIGURE 2.



On the next page, you will see a Bode Plot that my lab partners and I have calculated. The green line (left plot) represents the Φ_{phase} and the blue line (right plot) represents $|H(s)|$.



Below is a table with the data regarding the Cornering Frequency for Loaded RC Filter.

Data points	Theoretical Analysis	Simulation	Measured	%Difference Theoretical vs Measured	% Difference Sim Vs Measured
ω_c	367,834Hz	369,864 Hz	370,000 Hz	0.59%	0.04%
Magnitude	0.09066 V	0.09065 V	0.09200 V	1.5%	1.5%

I used the following equation(s) to find the %Difference for theoretical vs. measured and the %Difference for simulation vs. measured, which is similar to PART I.

$$\%differnce\ (Theoretical\ Analysis\ vs\ Measured) = \frac{TA - Measured}{Measured} x100$$

$$\%differnce\ (Simulated\ vs\ Measured) = \frac{Sim - Measured}{Measured} x100$$

We found that the Corner frequency for the theoretical analysis was close to a phase of -45°, but simulation gave us a -44.98° and the measured value was -43.80°.

I have calculated the Cornering Frequency for the given circuit in FIGURE 2 to be 2311167.6 rads/sec, but I converted that value into Hz, instead of rads/sec. Just as in PART I, the frequencies needed to capture the phase shift and the change in magnitude will be approximately one tenth and ten times the Cornering Frequency.

If you were to compare the data regarding the Corner Frequency for PART I and PART II, we see that the magnitude of the circuit in PART II is significantly smaller than the circuit in PART I. The reason for this is due to the external resistor that has been added for PART II, which is close to one tenth of the size of the resistor 2. If we were to apply a voltage divider for the voltage across the capacitor, we would find that it is one tenth the input voltage. The phase of the circuit with the external resistor also has a significant change from the circuit without the external resistor. The reason for this is due to the external resistance that is connected in parallel with the capacitor.

Part III:

For this part of the lab, we will construct and use a circuit similar to the one below. As you can see, we will be applying an operational amplifier to the circuit in addition to the capacitor.

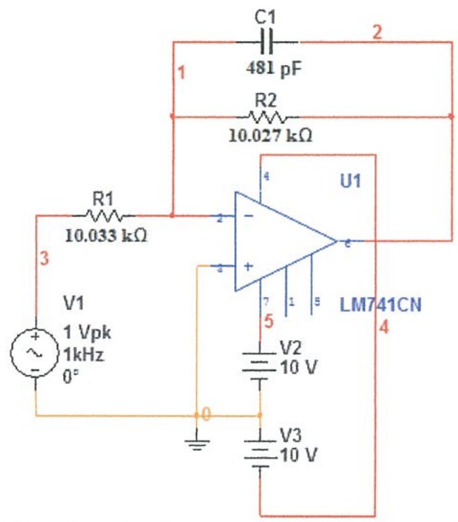


FIGURE 3: Op-amp based Low-Pass Filter.

To create a Bode plot for the Op-amp based Low-Pass Filter, we apply the node voltage equation. Before we proceed with the node voltage analysis, here are some conditions for an ideal op-amp:

$$V_n = V_p$$

$$I_n = I_p = 0$$

$$\text{Voltage Gain} = \infty$$

Below, we will apply node voltage method at the inverting node, which will have three currents:

$$\frac{V_n - V_1}{10\ k\Omega} + I_n + \frac{V_n - V_0}{10\ k\Omega} + \frac{V_n - V_0}{Z_{C1}} = 0$$

Since the non-inverting terminal of the Amplifier is connect to ground $V_p = 0$, the voltage at the inverting terminal is also zero $V_n = 0$. So,

$$I_n = I_p = 0$$

$$\frac{-V_1}{10\ k\Omega} + \frac{-V_0}{10k\Omega} + \frac{-V_0}{Z_{C1}} = 0$$

$$\frac{-V_1}{10\ k\Omega} - V_0\left(\frac{1}{10k\Omega} + \frac{1}{Z_{C1}}\right) = 0$$

$$\frac{-V_0}{V_1} = \frac{\frac{1}{10k\Omega}}{\left(\frac{1}{10k\Omega} + \frac{1}{Z_{C1}}\right)}$$

$$\frac{-V_0}{V_1} = \frac{\frac{1}{10k\Omega}}{\left(\frac{1}{10k\Omega} + \frac{1}{\frac{1}{j\omega C_1}}\right)}$$

$$\frac{-V_0}{V_1} = \frac{\frac{1}{10k\Omega}}{\left(\frac{1}{10k\Omega} + j\omega C_1\right)}$$

Below is the transfer function for the given circuit in FIGURE 3.

$$H(j\omega) = \frac{-V_0}{V_1} = \frac{\frac{1}{10k\Omega C_1}}{\left(\frac{1}{10k\Omega C_1} + j\omega\right)}$$ [Transfer Function]

Below is the Corner Frequency for the given circuit in FIGURE 3.

$$\omega_C = \frac{1}{RC_1} = \frac{1}{(10000\Omega)(481pF)} = 207900.2 \text{ rads/s}$$ [C o r n e r Frequency]

$$f_c = \frac{\omega_C}{2\pi} = \frac{207900.2 \text{ rads/s}}{2\pi} = 33088.3\ Hz$$

So, the transfer function becomes

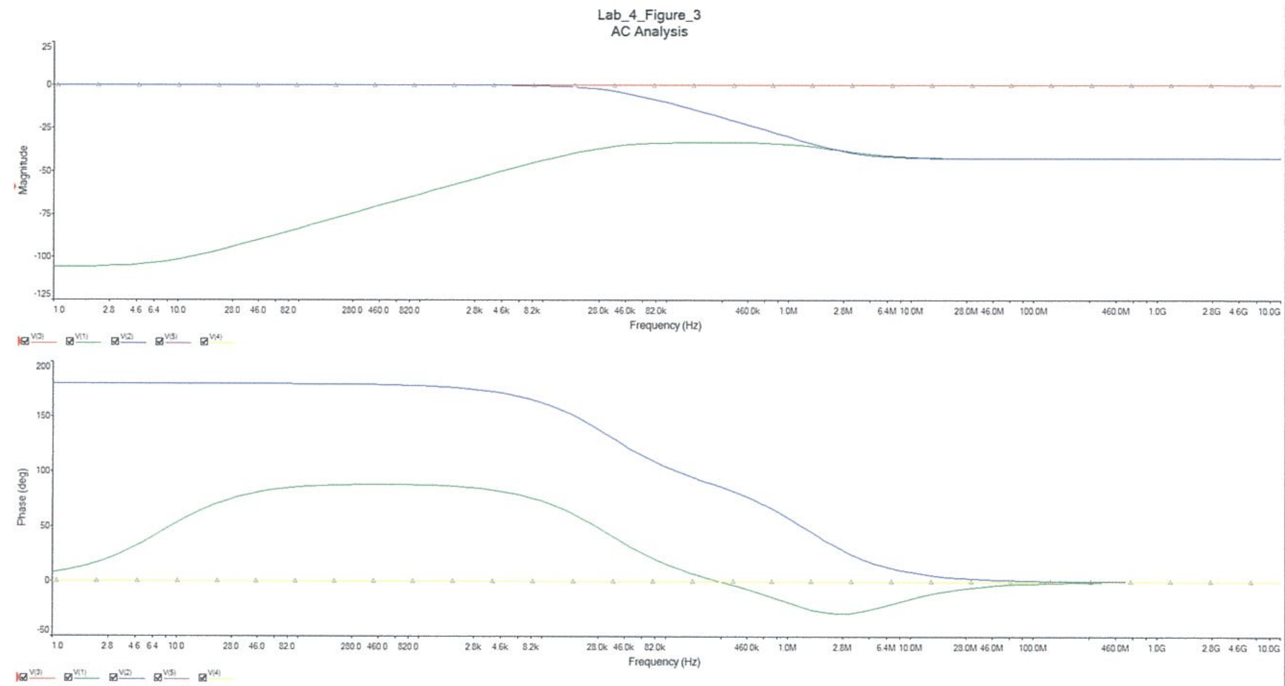
$$H(j\omega) = \frac{207900.2}{j\omega + 207900.2} = \frac{1}{1 + \frac{j\omega}{207900.2}}$$

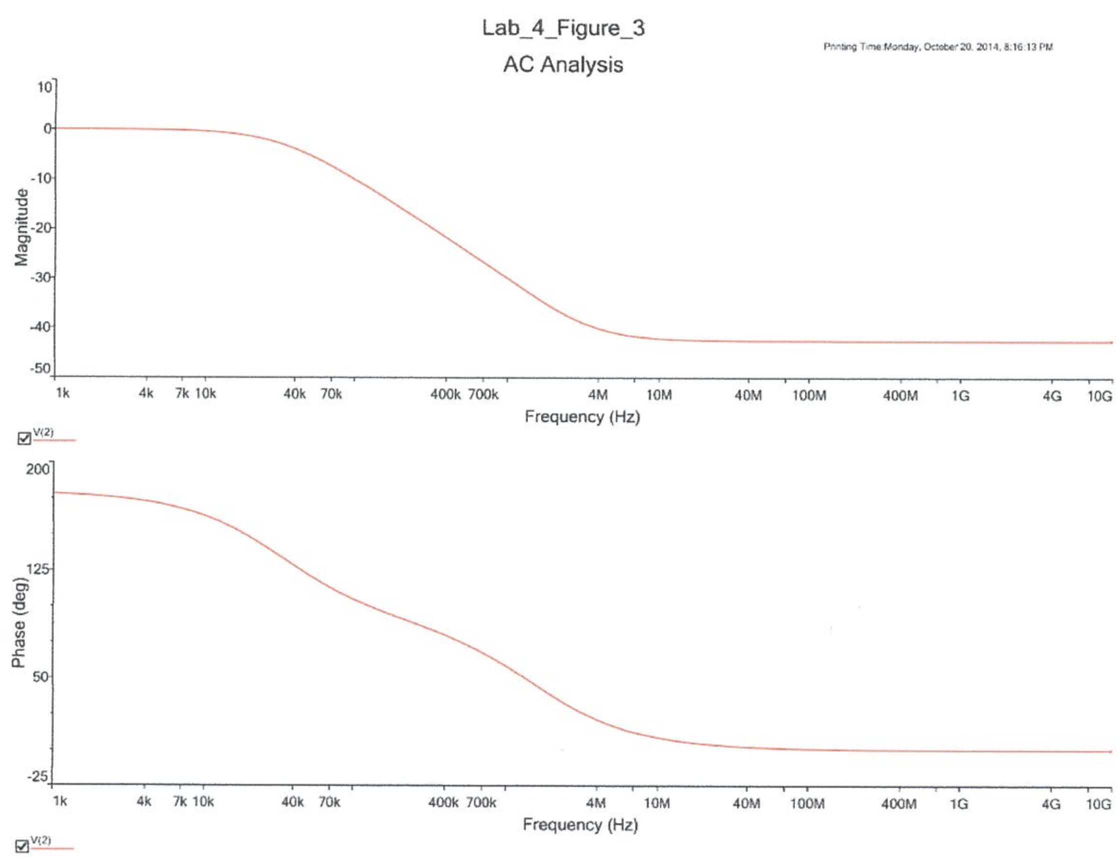
Thus,

$$|H(s)| = 20\log(1) - 20\log\left(1 + \frac{j\omega}{207900.2}\right)$$

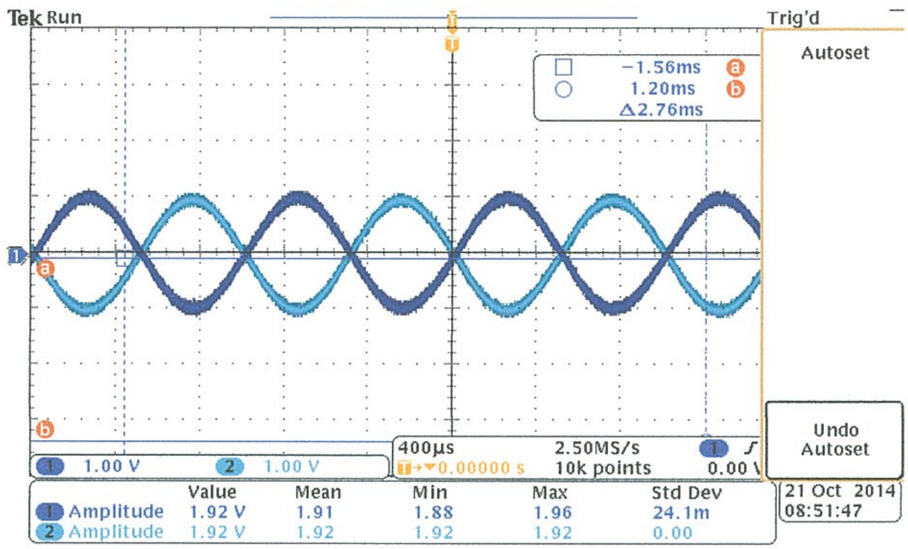
$$\Phi_{\text{phase}} = \arctan\left(\frac{0}{1}\right) + \arctan\left(\frac{\omega}{207900.2}\right)$$

Below, is the Bode plot (AC Analysis) simulation for Op-amp based Low-Pass Filter in the circuit from FIGURE 3.





Below, is the waveform we obtained for the input voltage and output voltage for the circuit in FIGURE 3.



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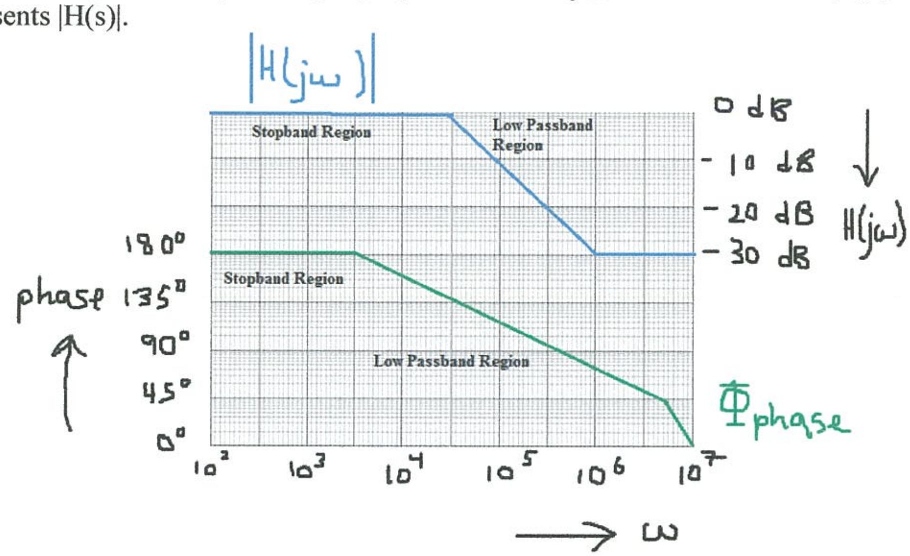
Below, you will see a Bode Plot that my lab partners and I have calculated. Just as in PART II, the green line (bottom plot) represents the Φ_{phase} and the blue line (top plot) represents $|H(s)|$.



Below is a table with the data regarding the Cornering Frequency for Op-amp based Low-Pass Filter.

Data points	Theoretical Analysis	Simulation	Measured	%Difference Theoretical vs Measured	% Difference Sim Vs Measured
ω_c	33,088.3 Hz	31,179.4 Hz	31,000 Hz	6.74%	0.58%
Magnitude	1V	1V	0.98 V	2.04%	2.04%

We found that at the inverting terminal the Corner frequency for the theoretical analys has a phase of approximately 135°, but simulation gave us a 134.98° and the measured value was very close to -134.8°.

In our analysis, my lab partners and I found that the reason for the huge error between theoretical and measured (%Difference) is due to our assumptions for the conditions o an ideal op-amp when we calculated the response of the circuit given in FIGURE 3. W were advised to obtained the supply power for the LM741 op-amp, which was ±18 vol This information can be obtained by looking up the data sheet for the LM741 op-amp online. After experimenting with the circuit, we found the Gain to be close to one. The voltage supply that we have chosen for the circuit in FIGURE 3 was ±10 volts. Based our result for Corner Frequency for this part, the frequencies required to obtain the pha shift and the change in magnitude will be very close to one tenth and ten times the Cor Frequency, similar to PART I and II. As you can see, the theoretical, simulated, and measured values of the magnitude and phase do match. The only significant errors occurred with ideal op-amp conditions.

Part IV:

For this part of the lab, we will construct and use a circuit similar to the one below. As you can see, we will be using the given circuit similar to the one in FIGURE 3, but nov with an external resistance added into the circuit.

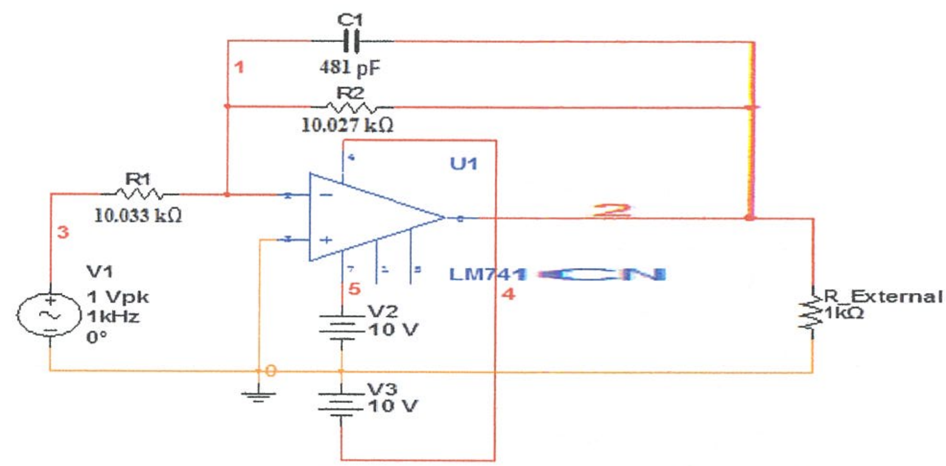


FIGURE 4: Loaded Op-amp based Low-Pass Filter.

To create a Bode plot of the Opamp based Low-Pass Filter we will use a similar setup from PART III (Refer to PART III for the theoretical analysis for this part).

On the following page, you will see a Bode plot (AC Analysis) of the given circuit fror FIGURE 4. My lab partners and I found that the Bode plot calculations (plotting by har were nearly identical to the Bode plot calculations from PART III (Please refer to it for analysis).

we find that it is not affected by time constants, so the output of the given circuit in this part is similar to the output of the circuit in PART III.

IV. CONCLUSION:

In this lab, we learned about the concept of a passive and active low pass filter function and the effects of a loaded and unloaded circuit. We learned that the LM741 op-amp can be used for limiting a frequency range on any given load. So when we conduct the experiments in this lab, it is necessary to understand how the LM741 op-amp operates, so that we can understand what is going on in the circuit whether we use an active or passive for the low-pass frequency filtered circuits. We found out that the passive low pass filter needs to be constructed for the load value if current across the load is higher than the op-amp's limitation of the amount of current it can handle, which is 25 mA, or if we need to include frequencies past the 1.5 MHz mark. Thus, we can conclude that the active and passive low-pass filters have their advantages and disadvantages.

I felt like there were some uncertainties, or error, when conducting our experiments in each of the four parts for this lab. I didn't know if the data that my lab partners and I obtained were correct. We, of course, went to compare our data and results with other lab groups and we found that their data and results were somewhat similar to our data and results. Since I was working fast, perhaps I have made a calculation error somewhere in my theoretical analysis. Or maybe we set up the incorrect connections on the pre-built circuit board, or maybe the lab equipment wasn't functioning to ideal conditions. No matter the reason, if there are any discrepancies in any of our data and or results, then the circumstances that I have described may potentially be the cause for any error in our data and or results.