SINUSOIDAL ANALYSIS

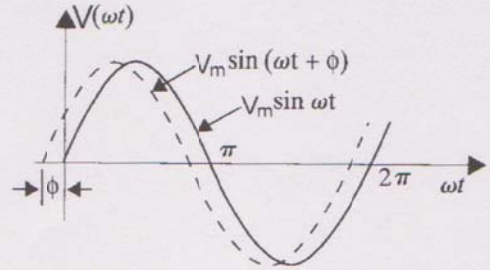
$V_m = wave\ amplitude$

$\omega = radial\ frequency\ in\ \dfrac{radians}{\sec}$

$\phi = phase\ shift$

$T = period\ of\ V(t)\ in\ \text{seconds} = \dfrac{2\pi}{\omega}$

$f = frequency\ \ in\ \text{Hertzs} = \dfrac{1}{\text{T}} = \dfrac{\omega}{2\pi}$



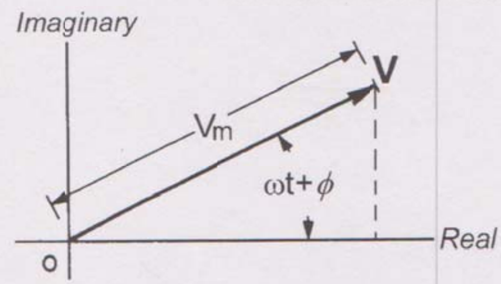
$$\boxed{\begin{array}{l} EULER'S\ \ FORMULA \\ e^{j\theta} = \cos\theta + j\sin\theta \end{array}}$$

$$V_m \cos(\omega t + \phi) + j\sin(\omega t + \phi) = V_m e^{j(\omega t + \phi)} = V_m e^{j\omega t} e^{j\phi}$$

Real part of the complex number = Re

$\text{V}_\text{m} \cos(\omega t + \phi) = \text{Re}\{V_m e^{j\omega t} e^{j\phi}\}$

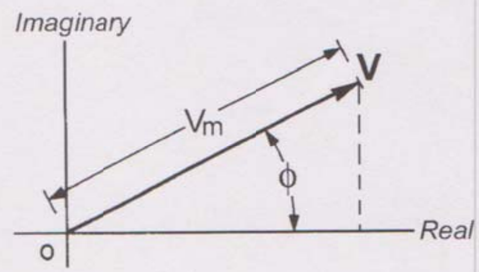
$$\text{V}_\text{m} \cos(\omega t + \phi) = \text{V}_\text{m}\,\text{Re}\{\cos(\omega t + \phi) + j\sin(\omega t + \phi)\}$$



Phasor (frequency) Domain R*epresentation*:

$$\underline{X} = X_m \cos\phi + \underbrace{X_m\ j\sin\phi}_{\textbf{Imaginary}} = X_m e^{j\phi}$$

$e^{j\omega t} \Rightarrow$ always there, but not written



$V_m e^{j\phi} \quad \Rightarrow V_m \angle\phi \quad$ *Phasor representation*

Sinusoidal Analysis

A sinusoid is a signal that has the form of the sine or cosine function.

Circuit Responses caused by sinusoidal input functions

• Sinusoidal waveforms are commonly used (e.g. wall outlets)

- Any other waveform can be expressed as a sum of sinusoidal waveforms.

• A sinusoidal forcing function is the standard waveform used in electrical power systems. An AC steady-state analysis determines the steady state response for a circuit when the inputs are sinusoidal forcing functions.

$$V(t) = V_m \cos(\omega t + \phi)$$

$\boxed{6.1}$

a) $V(t) = 12\cos(50t + 10^\circ)\,V$

$$V_m = 12 \quad \phi = 10^\circ \quad T = \frac{2\pi}{50} = 0.12$$

7.9 $\Rightarrow f = \frac{1}{T} = 8.3$

b) $i(t) = 5\cos(4\pi t - 60^\circ)\,A$

$v(t) = V_m \cos(\omega t + \phi)$

phasors

A phasor is a complex number that represents the amplitude and phase of a sinusoid

- A complex representation, known as a phasor, is used to transform a set of differential equations for some specific currents and voltages in a network, in the time domain, to a set of algebraic equations containing complex numbers in the frequency domain. Phasors can be manipulated like vectors. The solution of an equation is then transformed from the frequency domain back to the time domain

- Circuits exited with sinusoidal signals,
- Sources are on for a very long time.
- only one frequency (same frequency)

$\boxed{6.2}$

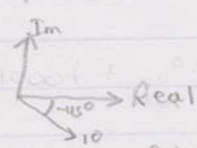
$\boxed{6.3}$ a) $v_1(t) = 10\cos(1000t - 45^\circ)\ V$

$V_m = 10 \quad \phi = -45^\circ \quad \therefore = 10e^{-j45} \quad -10\cos 45 - 10j\sin 45$

$-5\sqrt{2} - 5j\sqrt{2}$

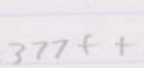
$\downarrow$

$10\angle -45^\circ$



b) $v_2(t) = 5\cos(1000t + 30^\circ)V$

$V_m = 5 \quad \phi = 30 \quad \therefore = 5e^{j30} \rightarrow 5\angle 30^\circ$

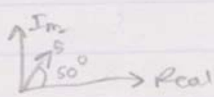


$5\cos 30 + 5j\sin 30$

$\frac{5\sqrt{3}}{2} + \frac{5}{2}j$

c) $i_2(t) = 5\cos(377t + 50^\circ)\ A$

$V_m = 5 \quad \phi = 50^\circ \quad \therefore = 5e^{j50} \rightarrow 5\angle 50^\circ$

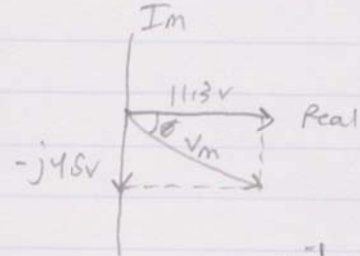


$5\cos 50^\circ + 5j\sin 50^\circ$

$\boxed{6.4}$ $V_1 = 7V - j\,7V$

$V_2 = 4.3V + j\,2.5V$

$11.3V - j4.5$



$-\tan^{-1}\left(\frac{4.7}{11.3}\right) = -21.8^\circ$

$v(t) = 12.3\cos(1000t - 21.8^\circ)\,V$

$V_m = \sqrt{(4.5)^2 + (11.3)^2} = 12.3$

Impedance and Admittance

Impedance in AC analysis is defined exactly like resistance in a DC analysis: Impedance is the ratio of the phasor voltage to the phasor current. The impedance of capacitors and inductors are frequency dependent, and a distinct phase relationship exists between the current in these elements and the voltage across them. Impedance (Z) serves the same role as resistance, but in the phasor domain. It is a constant like resistance, but turns out to be a complex value and depends on the frequency of the signal involved. (The inverse of impedance is admittance)

$$Z = \frac{V}{I}$$

Resistor: $Z = R$

Inductor: $Z = j\omega L$

Capacitor: $Z = \frac{1}{j\omega C} = \frac{-j}{\omega C}$