Chapter 11

Balanced Three-Phase Circuits

Text: *Electric Circuits* by J. Nilsson and S. Riedel

Prentice Hall

EEE 117 Network Analysis

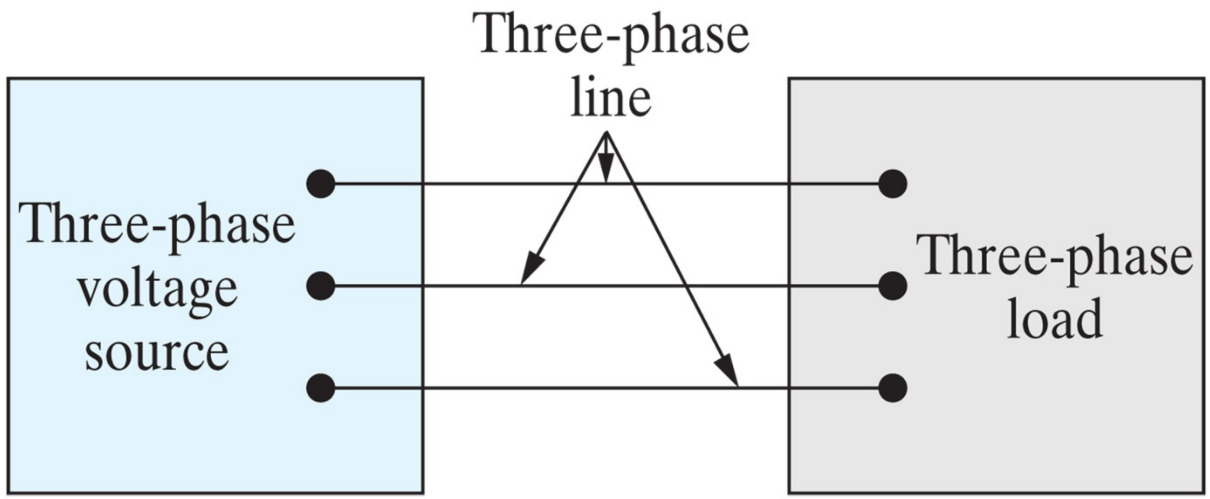
Instructor: Russ Tatro

Section 11.1

Balanced Three-Phase Voltages

Overview

Generating, transmitting, distributing, and using large blocks of electric power is accomplished with three-phase power.



This chapter is a general overview and only the steady-state sinusoidal behavior is presented.

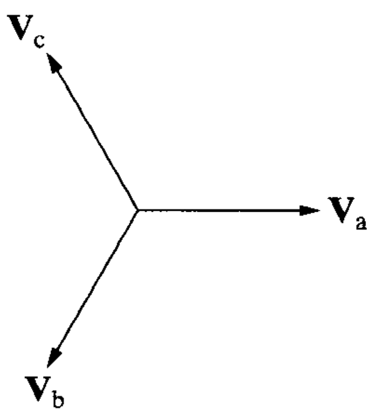
A set of balanced three-phase voltages consist of three sinusoidal voltages that have:

Identical amplitudes,

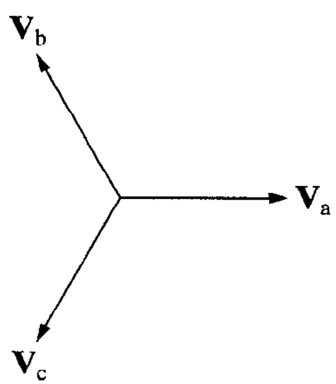
Identical frequencies,

and are out of phase with each other by 120°.

Only two possible phase orientations may exist.



abc sequence



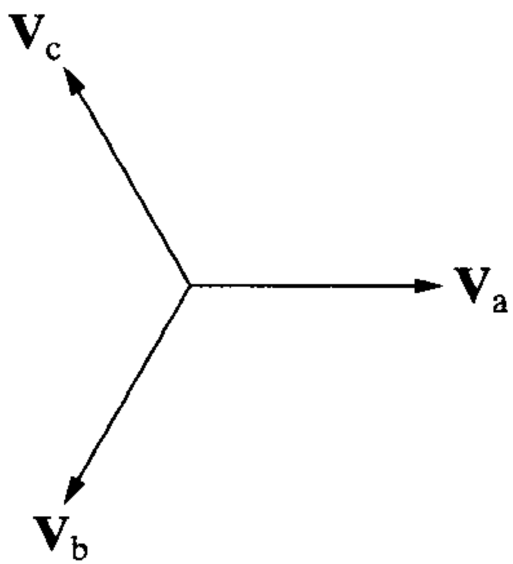
acb sequence

Section 11.1 Balanced Three-Phase Voltages

The abc sequence is known as the positive phase sequence.

Label the a, b and c phases going in clockwise order starting with V_a.

$$V_c = V_m \angle +120^\circ$$



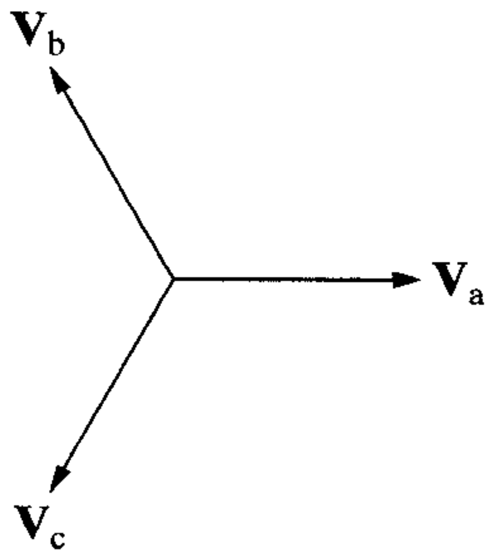
$$V_a = V_m \angle 0^\circ$$

$$V_b = V_m \angle -120^\circ$$

The acb sequence is known as the negative phase sequence.

Label the a, c and b phases going in counterclockwise order starting with V_a.

$$V_b = V_m \angle +120^\circ$$



$$V_a = V_m \angle 0^\circ$$

$$V_c = V_m \angle -120^\circ$$

Section 11.1 Balanced Three-Phase Voltages

A property of balanced three-phase voltages is that the sum of the voltages is zero.

$V_a + V_b + V_c = 0$ in the phasor form.

$v_a + v_b + v_c = 0$ in the instantaneous (time domain) form.

Confirm this fact on your calculators!

Let $V_m = 120\ V_{rms}$ and let t = zero.

$$V_{sum} = V_a \cos(\omega t + 0^\circ) + V_b \cos(\omega t - 120^\circ) + V_b \cos(\omega t + 120^\circ) = ?$$

$$V_{sum} = 120V_{rms}\left[\cos(0) + \cos(-2.0944) + \cos(2.0944)\right] = 120V_{rms}\left[1 - 0.5 - 0.5\right] = 0$$

We have a rigid relationship of the voltages and phase to each other in a balanced three-phase system.

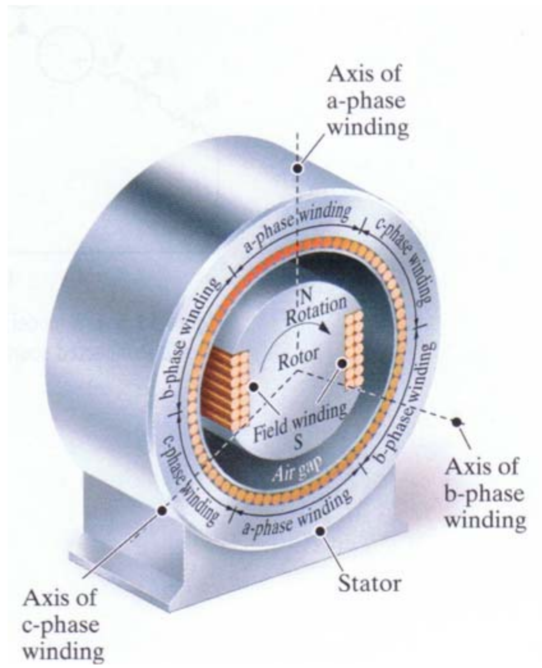
Thus if we know one voltage and it's phase then the others can be quickly found.

So we will focus on finding the voltage (or current) in one phase.

Section 11.2

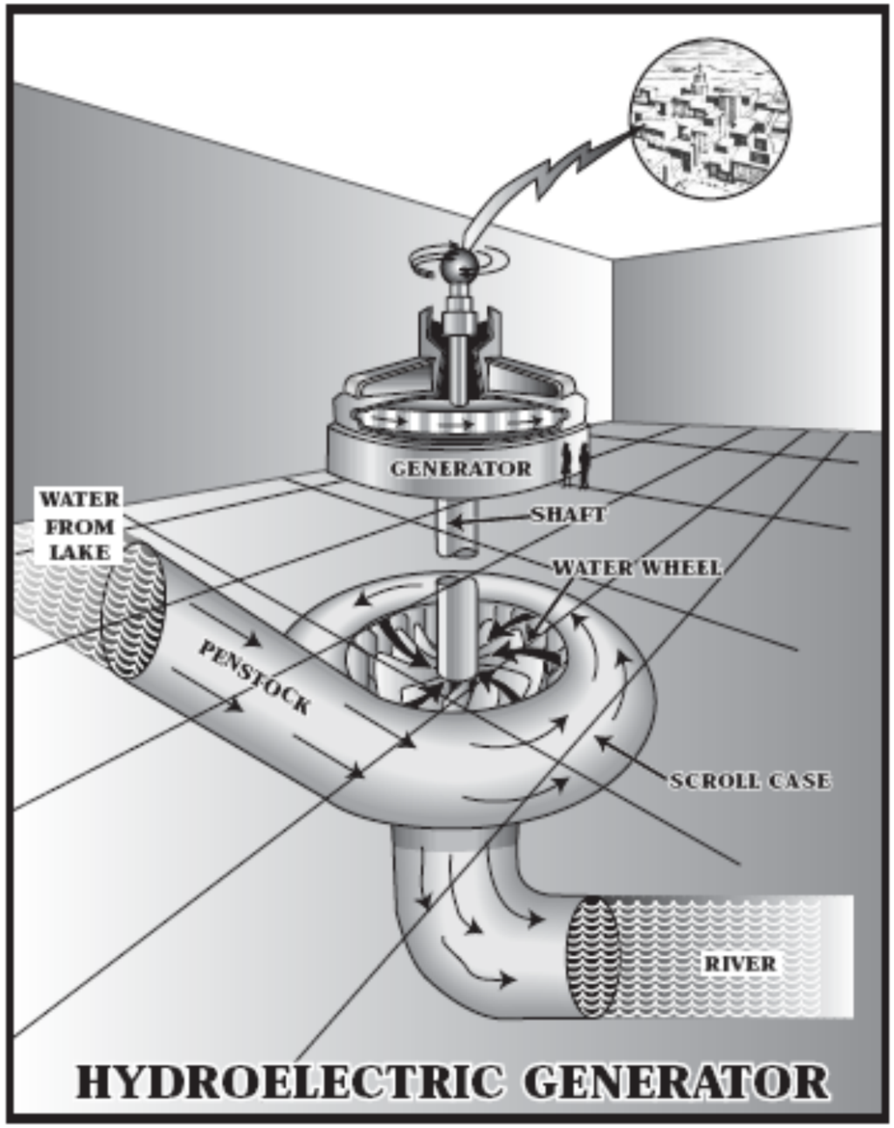
Three-Phase Voltage Sources

A three-phase voltage source is a generator with three separate windings distributed around the periphery of the stator.



The phase windings are designed so that the sinusoidal voltages are equal in magnitude and out of phase by 120°.

Schematic of primary generator at the Hoover Dam

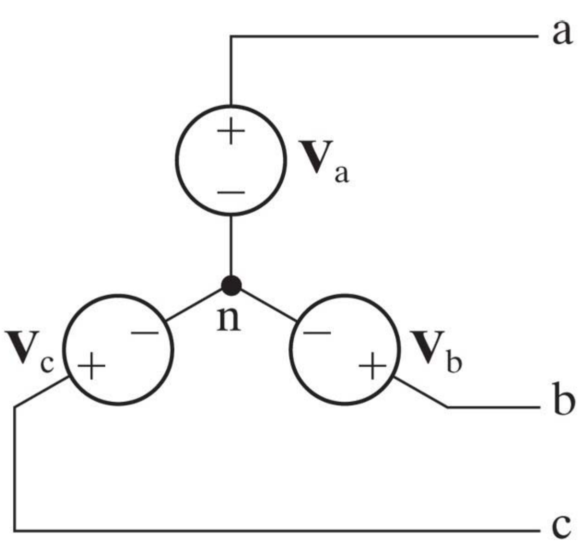


Credit: Hoover Dam Learning Packet

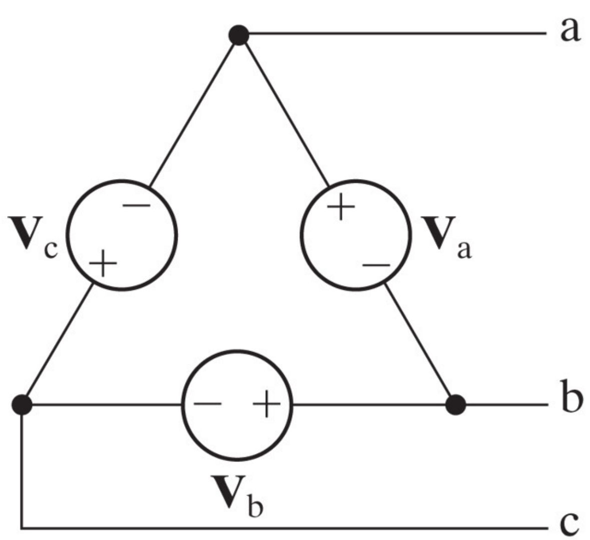
Hoover Dam Generators

The upper generator room at Hoover Dam, on the Nevada side

The separate phase windings may form a three-phase source by two types of interconnections: Wye (Y) or delta (Δ).

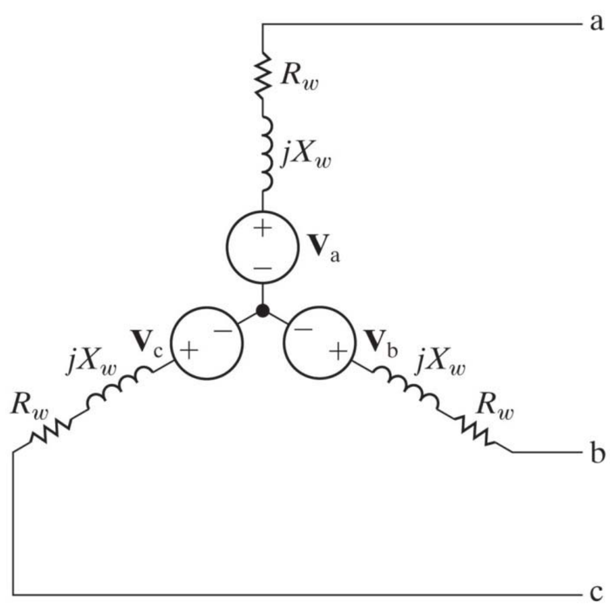


Wye

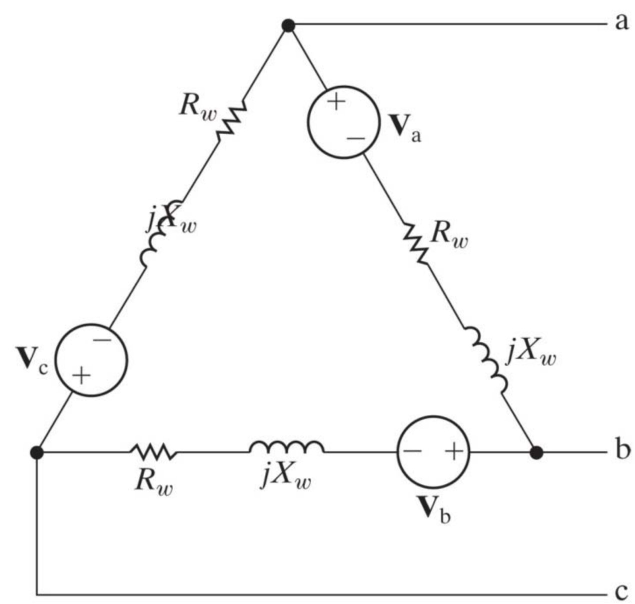


Delta

We can include the inductive reactance and resistance of the windings. Note that X_w and R_w are equal in each winding by definition.



Wye



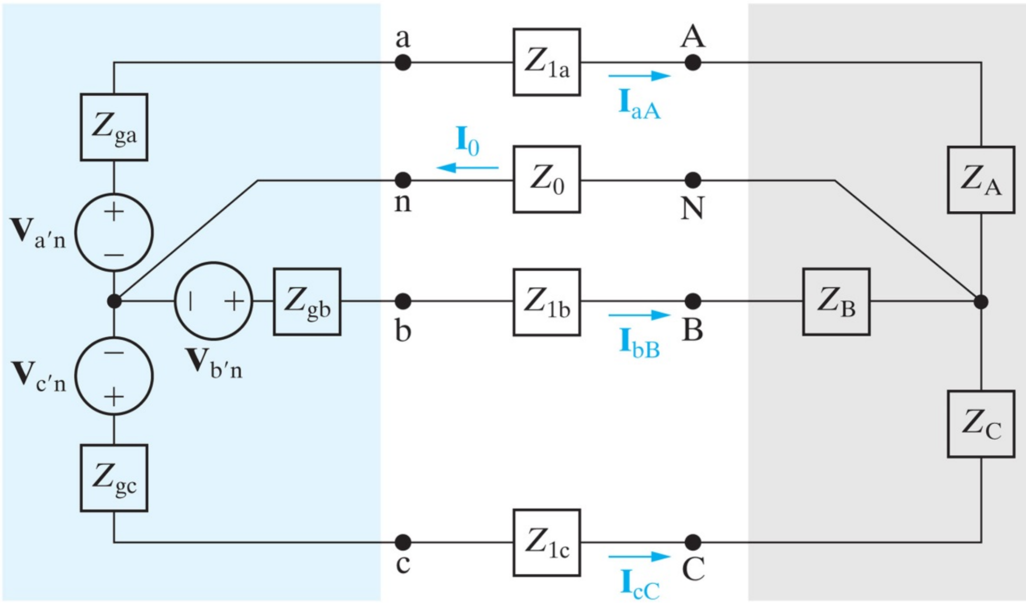
Delta

Because three-phase sources and loads can be either Y-connected or Δ-connected, there are four different circuit configurations.

Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Section 11.3

Analysis of the Wye-Wye Circuit



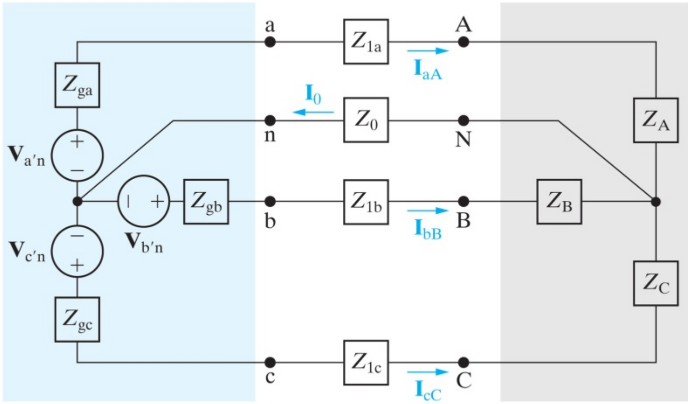
The above circuit is in the Y to Y configuration.

By KCL at node N, $I_0 - I_{aA} - I_{bB} - I_{cC} = 0.$

In terms of voltage the node equation is

$$\frac{V_N}{Z_0} + \frac{V_N - V_{a'n}}{Z_A + Z_{1a} + Z_{ga}} + \frac{V_N - V_{b'n}}{Z_B + Z_{1b} + Z_{gb}} + \frac{V_N - V_{c'n}}{Z_C + Z_{1c} + Z_{gc}} = 0$$

Section 11.3 Analysis of the Wye-Wye Circuit

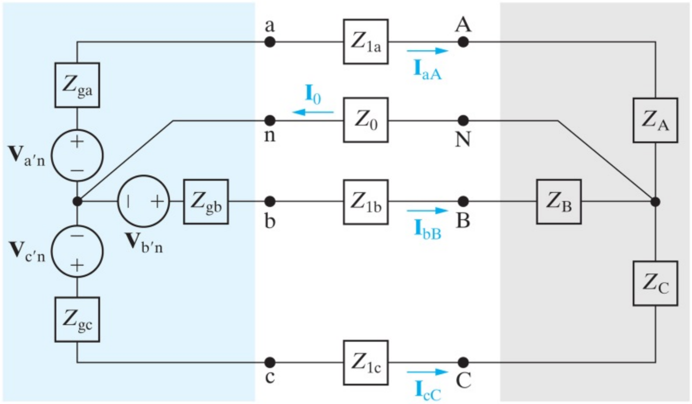


For a balanced 3ϕ system we have

1. $V_{a'n}$, $V_{b'n}$, and $V_{c'n}$ are a set of balanced 3ϕ voltages.
2. The impedance of each source phase is the same.
   $$Z_{ga} = Z_{gb} = Z_{gc}$$
3. The impedance of each line conductor is the same.
   $$Z_{1a} = Z_{1b} = Z_{1c}$$
4. The impedance of each phase of the load is the same.
   $$Z_A = Z_B = Z_C$$

Thus let Z_ϕ be the impedance of each phase.

$$Z_\phi = Z_A + Z_{1a} + Z_{ga} = Z_B + Z_{1b} + Z_{gb} = Z_C + Z_{1c} + Z_{gc}$$



Thus for this balanced 3ϕ the node equation at node N is

$$\frac{V_N}{Z_0}+\frac{V_N-V_{a'n}}{\underbrace{Z_A+Z_{1a}+Z_{ga}}_{Z_\phi}}+\frac{V_N-V_{b'n}}{\underbrace{Z_B+Z_{1b}+Z_{gb}}_{Z_\phi}}+\frac{V_N-V_{c'n}}{\underbrace{Z_C+Z_{1c}+Z_{gc}}_{Z_\phi}}=0$$

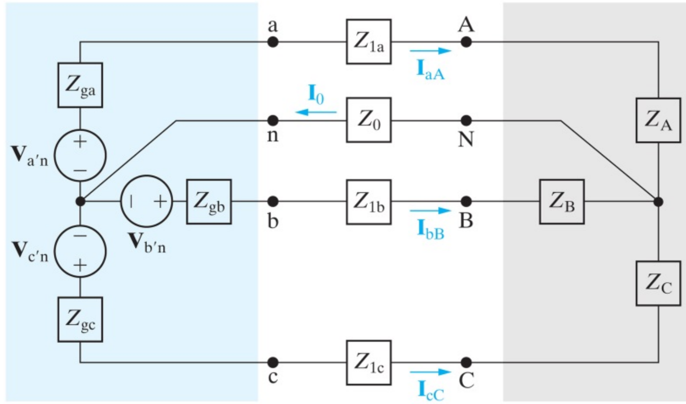
Combine the equal terms

$$V_N\left(\frac{1}{Z_0}+\frac{3}{Z_\phi}\right)-\frac{V_{a'n}+V_{b'n}+V_{c'n}}{Z_\phi}=0$$

Thus

$$V_N\left(\frac{1}{Z_0}+\frac{3}{Z_\phi}\right)=\frac{V_{a'n}+V_{b'n}+V_{c'n}}{Z_\phi}$$

Section 11.3 Analysis of the Wye-Wye Circuit



$$V_N\left(\frac{1}{Z_0}+\frac{3}{Z_\phi}\right)=\frac{V_{a'n}+V_{b'n}+V_{c'n}}{Z_\phi}$$

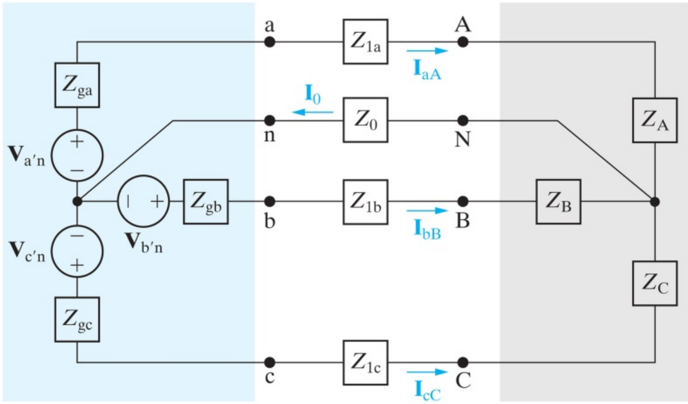
For a balanced system, source voltages sum to zero.

$$V_{a'n}+V_{b'n}+V_{c'n}=0$$

Thus V_N must also equal zero as required by the math.

Then the current in the neutral return branch I_0 must also be zero.

This condition allows us to examine each line (phase) independently!



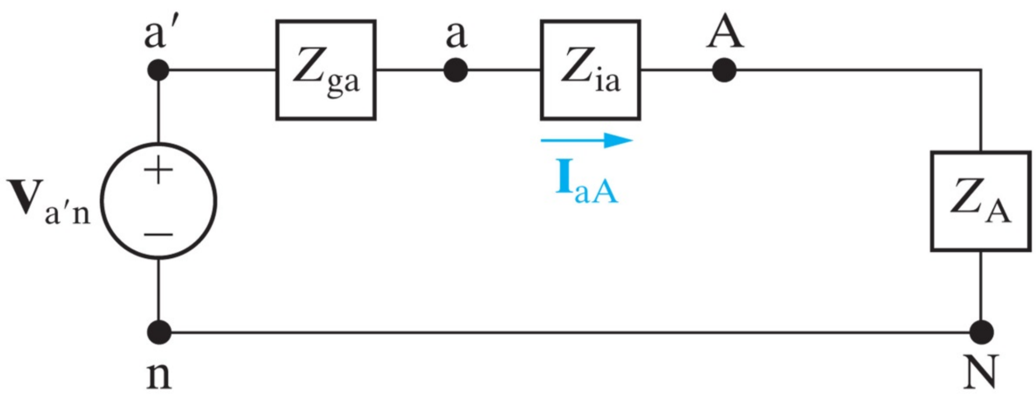
The three line currents can now be expressed as

$$I_{aA} = \frac{V_{a'n} - V_N}{Z_A + Z_{1a} + Z_{ga}} = \frac{V_{a'n} - 0}{Z_\phi} = \frac{V_{a'n}}{Z_\phi}$$

$$I_{bB} = \frac{V_{b'n} - V_N}{Z_B + Z_{1b} + Z_{gb}} = \frac{V_{b'n}}{Z_\phi}$$

$$I_{cC} = \frac{V_{c'n} - V_N}{Z_C + Z_{1c} + Z_{gc}} = \frac{V_{c'n}}{Z_\phi}$$

With the previous simplifications we can now write each phase as a single-phase equivalent circuit.



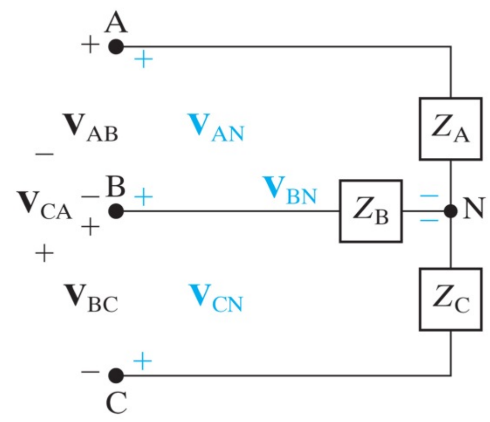
Caution: $I_0 \neq I_{aA}$ in this single-phase equivalent circuit!

Remember that it is the sum of the three phase currents that equals zero.

$$I_{aA} + I_{bB} + I_{cC} = 0$$

The next definitions we need to state is the relationship between the voltages which are the *line to neutral* and *line to line* voltages.

Line to neutral voltages: V_{AN}, V_{BN}, V_{CN}



Line to line voltages: V_{AB}, V_{BC}, V_{CA}

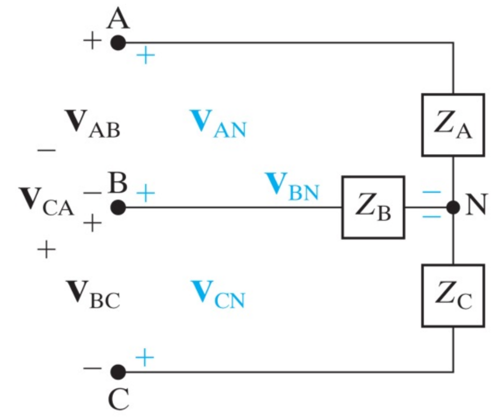
The relationship between line to line and line to neutral is given by KVL.

$$-V_{AB} + V_{AN} - V_{BN} = 0 \qquad \Rightarrow V_{AB} = V_{AN} - V_{BN}$$

$$-V_{BC} + V_{BN} - V_{CN} = 0 \qquad \Rightarrow V_{BC} = V_{BN} - V_{CN}$$

$$+V_{CA} + V_{AN} - V_{CN} = 0 \qquad \Rightarrow V_{CA} = V_{CN} - V_{AN}$$

Section 11.3 Analysis of the Wye-Wye Circuit



For the positive phase sequence *abc*:

$$V_{AN} = V_\phi \angle 0° \quad V_{BN} = V_\phi \angle -120° \quad V_{CN} = V_\phi \angle +120°$$

Thus the line to line voltage V_{AB} is

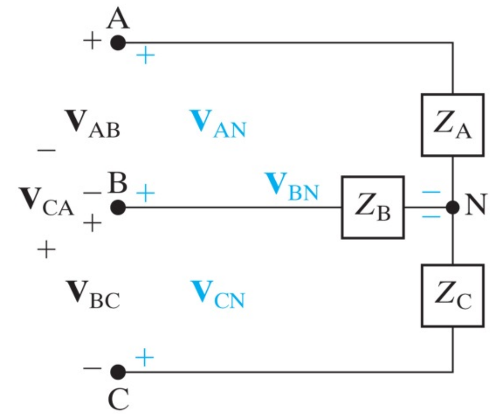
$$V_{AB} = V_{AN} - V_{BN} = V_\phi \angle 0° - V_\phi \angle -120°$$

We can use Euler's Identity to solve the last phasor equation.

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$V_\phi \angle 0° = V_\phi(\cos 0° + j\sin 0°) = V_\phi(1 + j0) = V_\phi$$

$$V_\phi \angle -120° = V_\phi\left[\cos(120°) + j\sin(-120°)\right] = V_\phi(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2})$$



Thus the line to line voltage V_{AB} is

$$V_{AB} = V_{AN} - V_{BN} = V_\phi\angle 0° - V_\phi\angle -120°$$

$$= V_\phi(1 + j0) - V_\phi(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2})$$

$$= V_\phi(1 + j0) + V_\phi(\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2})$$

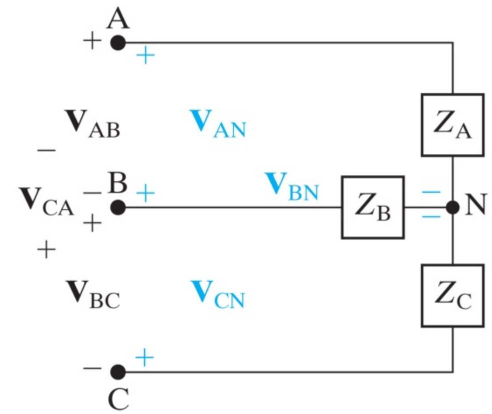
$$= V_\phi(\tfrac{3}{2} + j\tfrac{\sqrt{3}}{2})$$

Now convert back to polar (phasor) form. $x + jy = \sqrt{x^2 + y^2}\angle\tan^{-1}\left(\dfrac{y}{x}\right)$

$$V_{AB} = V_\phi\sqrt{\left(\tfrac{3}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}\right)^2}\angle\tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right)$$

$$= V_\phi\sqrt{\frac{12}{4}}\angle\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) \quad = V_\phi\sqrt{3}\angle 30°$$

For the positive phase sequence *abc!*



Now we can use the balanced 3ϕ condition to find the other two line to line voltages.

$$V_{AB} = \sqrt{3}\, V_{\phi} \angle 30^{\circ}$$

$$V_{BC} = \sqrt{3}\, V_{\phi} \angle (30^{\circ} - 120^{\circ}) \;\; = \sqrt{3}\, V_{\phi} \angle -90^{\circ}$$

$$V_{CA} = \sqrt{3}\, V_{\phi} \angle (30^{\circ} + 120^{\circ}) \;\; = \sqrt{3}\, V_{\phi} \angle +150^{\circ}$$

Line to line voltage = $\sqrt{3}$ Line to neutral voltage.

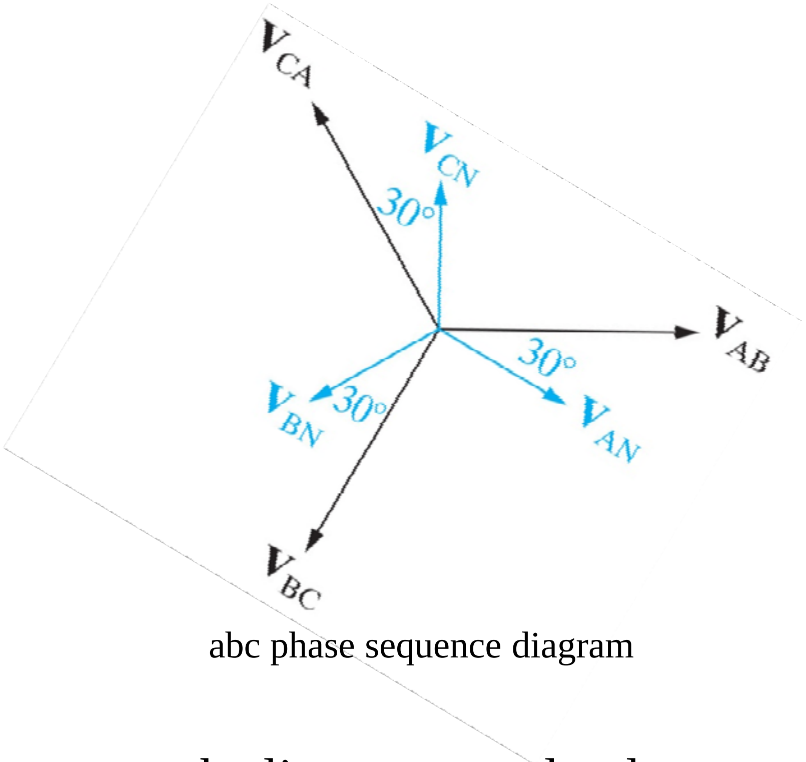
Line to line phase LEADS line to neutral phase for the positive (abc) sequence.

Line to line phase LAGS line to neutral phase for the negative (acb) sequence.

Example (P11.2_8ed)

The voltage from A to N in a balanced three-phase circuit is 240∠-30°. The phase sequence is positive (abc).

What is the value of V_{BC}?



abc phase sequence diagram

$$V_{AN} = 240\angle -30°$$

$$V_{BN} = 240\angle(-30° - 120°) \quad = 240\angle -150°$$

$$V_{CN} = 240\angle(-30° + 120°) \quad = 240\angle +90°$$

Now we can write the line to line voltages with respect to the line to neutral voltages.

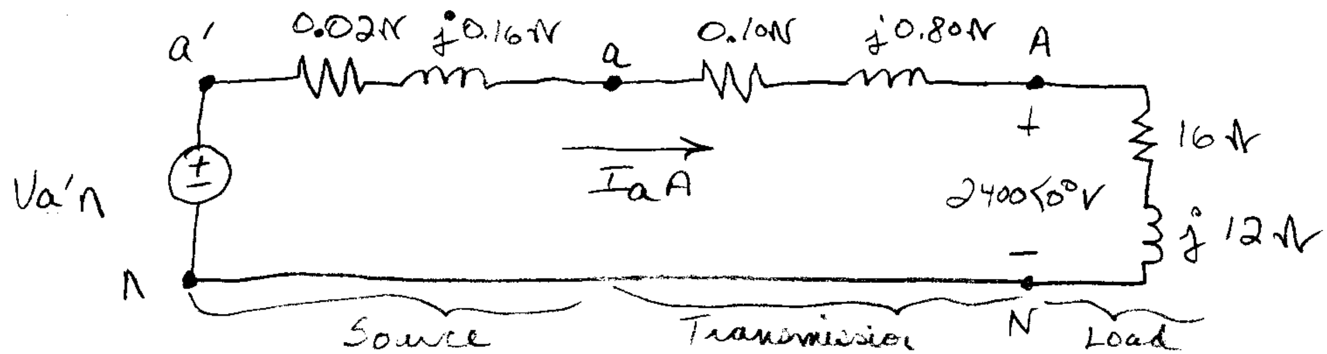
$$V_{AB} = \sqrt{3}\angle +30°(V_{AN}) \quad = (\sqrt{3}\angle 30°)(240\angle -30°) \quad = 240\sqrt{3}\angle 0° Volts$$

$$V_{BC} = \sqrt{3}\angle +30°(V_{BN}) = \sqrt{3}\angle 30°(240\angle -150°) \quad = 240\sqrt{3}\angle -120° Volts$$

$$V_{CA} = \sqrt{3}\angle +30°(V_{CN}) = \sqrt{3}\angle 30°(240\angle +90°) \quad = 240\sqrt{3}\angle +120° Volts$$

Example

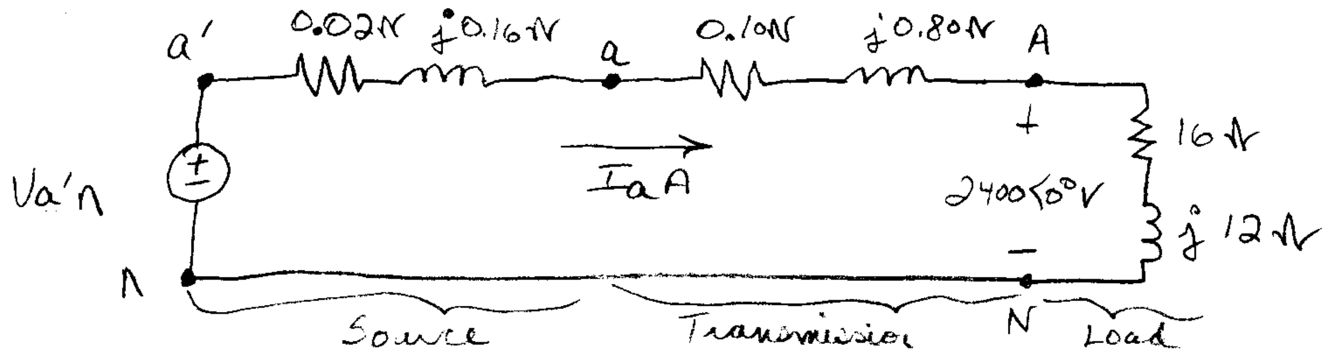
A balanced 3ϕ system is represented by this equivalent single phase circuit.



Given: phase sequence is negative (acb).

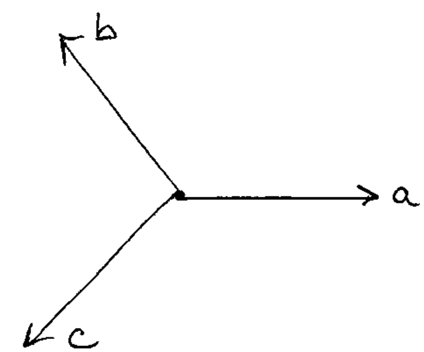
Calculate the three phase currents I_{aA}, I_{bB}, I_{cC}.

Example



We can immediately calculate I_{aA} since the voltage across the load was given.

$$I_{aA} = \frac{2400\angle 0° V}{16 + j12\Omega} = \frac{2400\angle 0° V}{20\angle 36.87°\Omega} = 120\angle -36.87° A$$

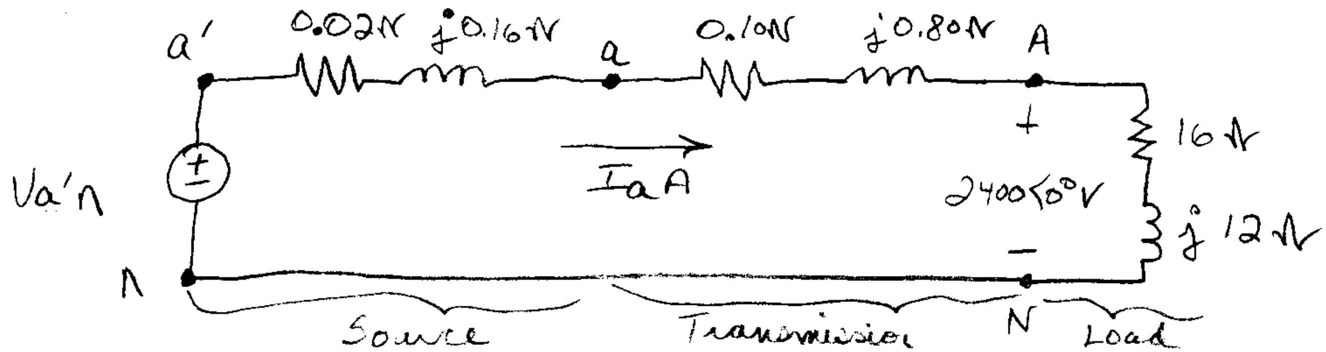


The other two currents can be written with respect to I_{aA}.

$$I_{bB} = |I_{aA}|\angle(\phi_A + 120°)A = 120\angle +83.13° A$$

$$I_{cC} = |I_{aA}|\angle(\phi_A - 120°)A = 120\angle -156.87° A$$

Example



Now calculate the phase voltages (at the source's terminal a,b,c) V_{an}, V_{bn}, V_{cn}.

$$V_{aN} = I_{aA} Z_{transmission} + V_{AN} \qquad Z_{Trans} = 0.1 + j0.80\,\Omega = 0.80623\angle 82.87°\Omega$$

$$= (120\angle -36.87° A)(0.80623\angle 82.87°\Omega) + 2400\angle 0° V$$

$$= 96.75\angle 46° V + 2400\angle 0° V \quad = 96.75(\cos 46° + j\sin 46°)V + 2400(1 + j0)V$$

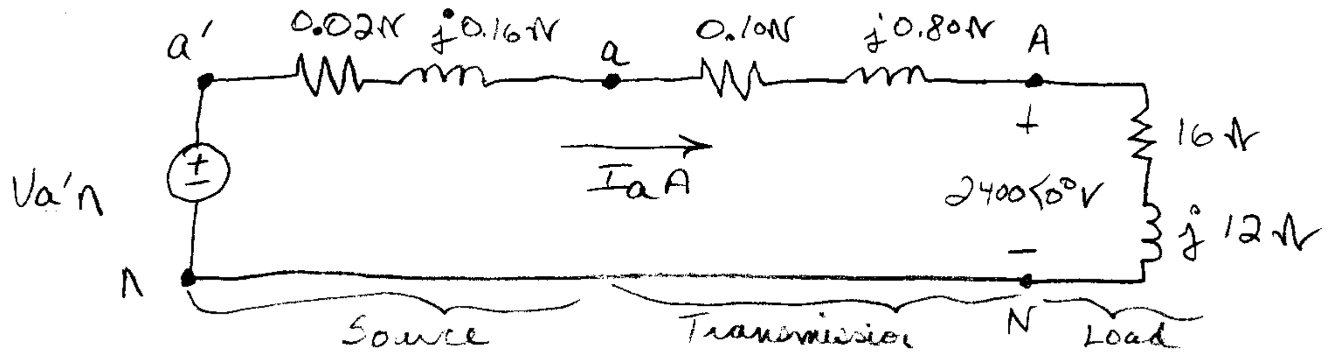
$$= 67.21 + j69.6V + 2400V \quad = 2467.21 + j69.6V \quad = 2468.2\angle 1.62°\ Volts$$

The other two phase voltages can be written with respect to V_{aN}.

$$V_{bN} = |V_{aN}|\angle(\phi_{aN} + 120°)V \quad = 2468.2\angle 121.62°\ Volts$$

$$V_{cN} = |V_{aN}|\angle(\phi_{aN} - 120°)V \quad = 2468.2\angle -118.38°\ Volts$$

Example



Now calculate the line to line voltages (at the source's terminal a,b,c) V_{ab}, V_{bc}, V_{ca}.

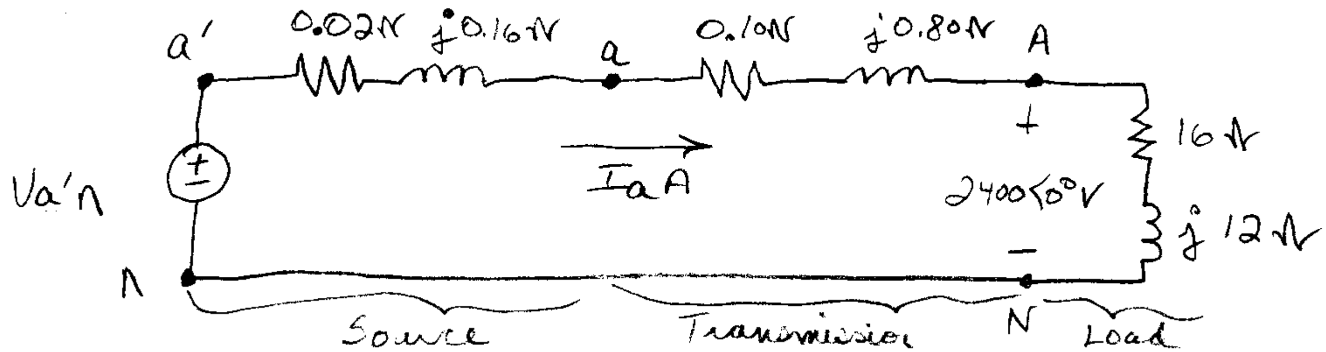
$$V_{ab} = \left(\sqrt{3}\angle -30^\circ\right)V_{aN} \quad = \left(\sqrt{3}\angle -30^\circ\right)2468.2\angle 1.62^\circ\ Volts \quad = 4275.05\angle -28.38^\circ\ Volts$$

The other two line to line voltages can be written with respect to V_{ab}.

$$V_{bc} = |V_{ab}|\angle\left(\phi_{ab} + 120^\circ\right)V \quad = 4275.05\angle 91.62^\circ\ Volts$$

$$V_{ca} = |V_{ab}|\angle\left(\phi_{ab} - 120^\circ\right)V \quad = 4275.05\angle -148.38^\circ\ Volts$$

Example



Now calculate the internal phase voltage of each source. $V_{a'n}$ $V_{b'n}$ $V_{c'n}$

$$V_{a'n} = I_{aA}\left(Z_{source} + Z_{transmission}\right) + V_{AN}$$

$$= \left(120\angle -36.87° A\right)\left(0.12 + j0.96\Omega\right) + 2400\angle 0° \, Volts$$

$$= 116.1\angle 46° V + 2400\angle 0° \, Volts \quad = 2480.65 + j83.52 \, Volts$$

$$= 2482.06\angle 1.93° \, Volts$$

The other two internal phase voltages can be written with respect to $V_{a'n}$.

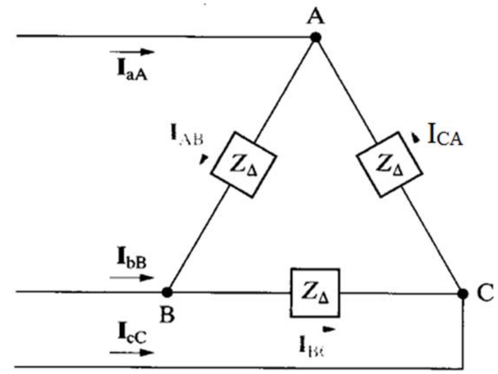
$$V_{b'n} = \left|V_{a'n}\right|\angle\left(\phi_{a'n} + 120°\right)V \quad = 2482.06\angle 121.93° \, Volts$$

$$V_{c'n} = \left|V_{a'n}\right|\angle\left(\phi_{a'n} - 120°\right)V \quad = 2482.06\angle -118.07° \, Volts$$

Section 11.4

Analysis of the Wye-Delta Circuit

Section 11.4 Analysis of the Wye-Delta Circuit

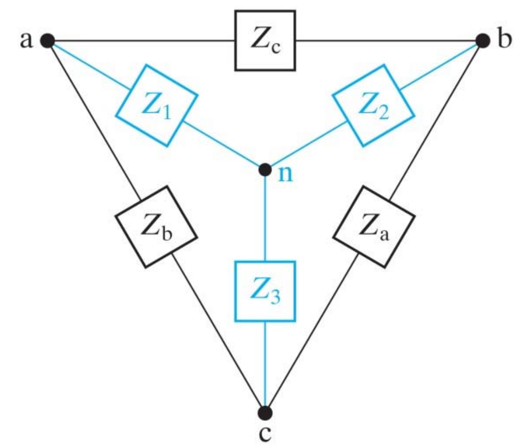


When the Y-source is connected to a Δ-load, the load can be transformed by the delta-to-wye transform of the text’s chapter 9, section 9.6.

When the load is balanced, the equal impedances in each leg of the wye transform is (see equation 9.51):

$$Z_y = Z_1 = Z_2 = Z_3$$

$$Z_y = Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} = \frac{Z_\Delta Z_\Delta}{Z_\Delta + Z_\Delta + Z_\Delta} = \frac{Z_\Delta^2}{3Z_\Delta} = \frac{Z_\Delta}{3}$$

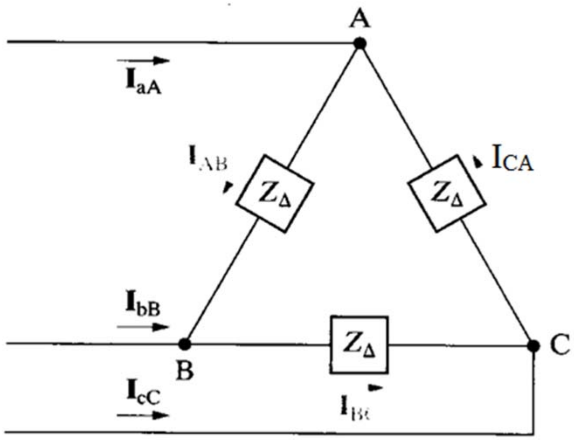


$$Z_\Delta = Z_a = Z_b = Z_c$$

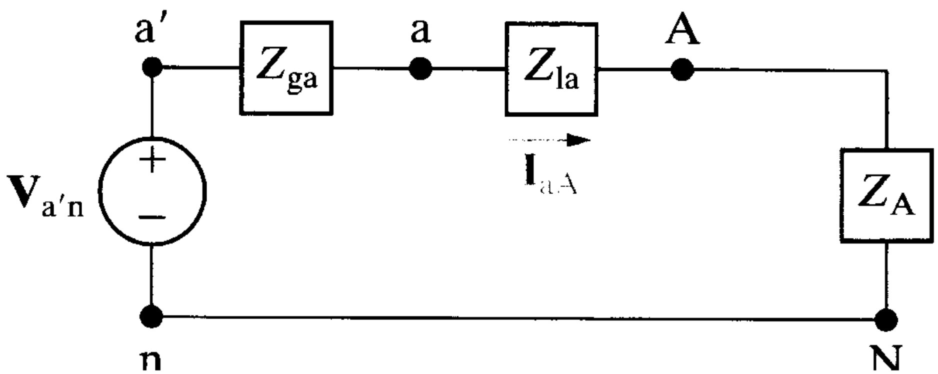
The rigid *balanced condition* allows us to write the wye impedance for all legs with one equation.

Section 11.4 Analysis of the Wye-Delta Circuit

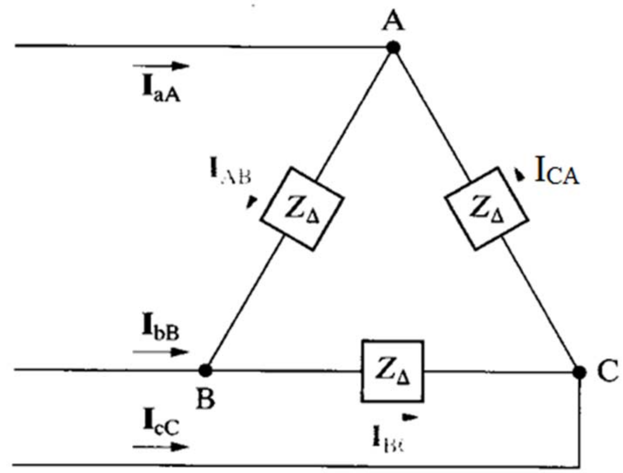
$$Z_Y = \frac{Z_\Delta}{3}$$



We can now use the single-phase equivalent circuit to find the line currents.



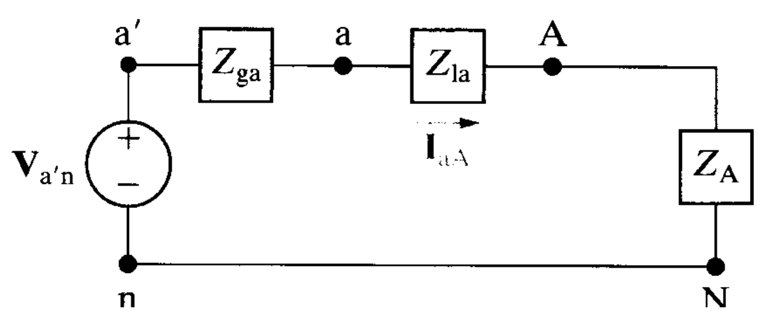
$$Z_A = Z_y = \frac{Z_\Delta}{3}$$

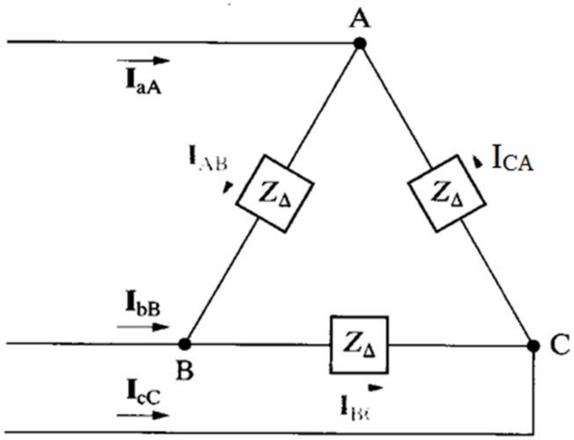


For the Δ configuration:
The phase voltage of a leg of the delta equals the line to line voltage at the source.

The current in each leg of the load is the phase current.
But note from the figure above that the phase current is not equal to the line current with respect to the line currents at the source.

In the single-phase equivalent circuit below, we find the line current w.r.t. to the source! So we need to know the relationship of line currents to phase currents in the delta load.





Given the circuit uses the positive (abc) phase sequence.

Let I_ϕ equal the phase current in each delta leg of the load.

$$I_{AB} = I_\phi \angle 0° \, Amps$$

I_{AB} is the assumed reference

$$I_{BC} = I_\phi \angle -120° \, Amps$$

$$I_{CA} = I_\phi \angle +120° \, Amps$$

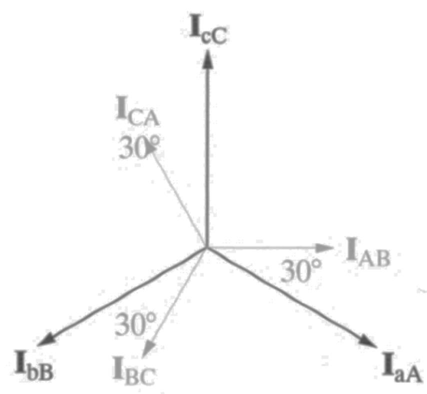
By KCL at node A, we have

$$-I_{aA} + I_{AB} - I_{CA} = 0$$

$$I_{aA} = I_{AB} - I_{CA} \quad = I_\phi \angle 0° - I_\phi \angle 120° \quad = I_\phi \sqrt{3} \angle -30°$$

$$I_{bB} = I_\phi \sqrt{3} \angle -150°$$

$$I_{cC} = I_\phi \sqrt{3} \angle +90°$$



abc phase sequence diagram

Example (AP11.4_8ed)

Given that the current I_{CA} in a balanced 3ϕ Δ-connected load is $8\angle -15°$ A

The phase sequence is positive (abc).

Calculate the value of wye current I_{cC}?

$$I_{cC} = I_{CA}\left(\sqrt{3}\angle -30°\right) = I_{\phi,\Delta}\left(\sqrt{3}\angle -30°\right)$$

$$= \left(8\angle -15°\right)(\sqrt{3}\angle -30°)$$

$$= 8\sqrt{3}\angle -45°$$

$$= 13.86\angle -45° \, Amps$$

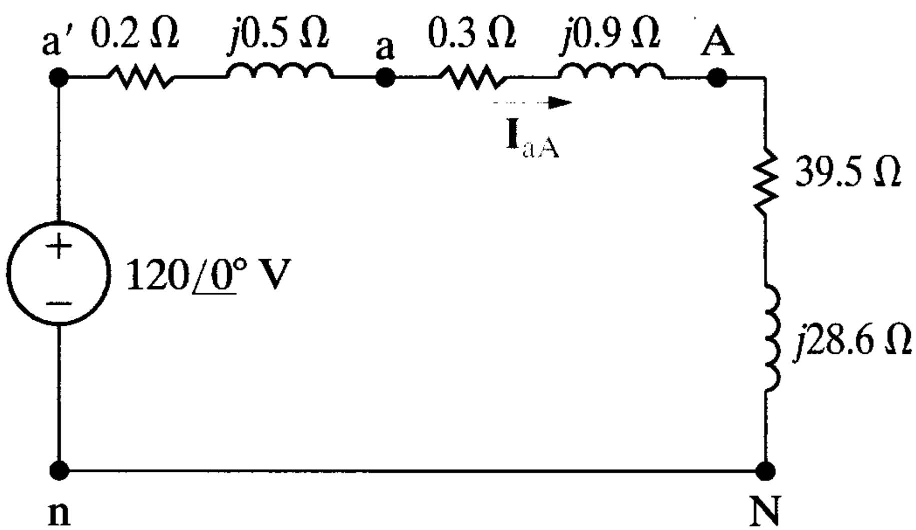
Example 11.2_8ed

Given a wye-connected source (use source and transmission line impedances from example 11.1 as shown below) feeds a Δ-connected load where Z_{Δ}= 118.5 + j 85.8 Ω.

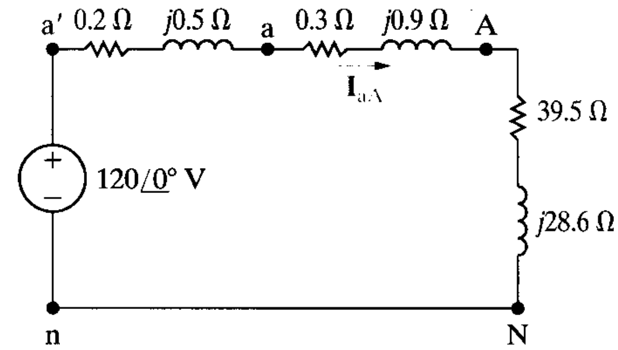
The phase sequence is positive (abc).

Construct a single-phase equivalent circuit.

$$Z_y = \frac{Z_{\Delta}}{3} = \frac{118.5 + j85.8}{3} = 39.5 + j28.6\,\Omega$$



Example



Find the line current I_{aA}.

By KVL:

$$-120\angle 0° + I_{aA} Z_{total} = 0$$

Solve for I_{aA}

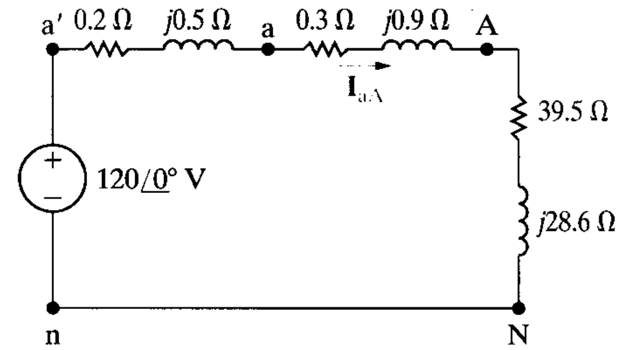
$$I_{aA} = \frac{120\angle 0°}{Z_{total}} = \frac{120\angle 0°}{(0.2+0.3+39.5) + j(0.5+0.9+28.6)} = \frac{120\angle 0°}{40 + j30}$$

$$= \frac{120\angle 0°}{50\angle 36.87°} = 2.4\angle -36.87°\,Amps$$

$$Thus\;\; I_{bB} = 2.4\angle -156.87°\,Amps$$

$$And\;\; I_{cC} = 2.4\angle 83.13°\,Amps$$

Example



Find the phase voltage V_{AB}.

First find V_{AN}

Recall line voltage (wrt source) = phase voltage (wrt load) when Δ-connected.

$$Z_{Load} = 39.5 + j28.6 = 48.77\angle 35.91^\circ\ \Omega$$

$$V_{AN} = I_{aA} Z_{Load} = \left(2.4\angle -36.87^\circ A\right)\left(48.77\angle 35.91^\circ\ \Omega\right) = 117.05\angle -0.96^\circ Volts$$

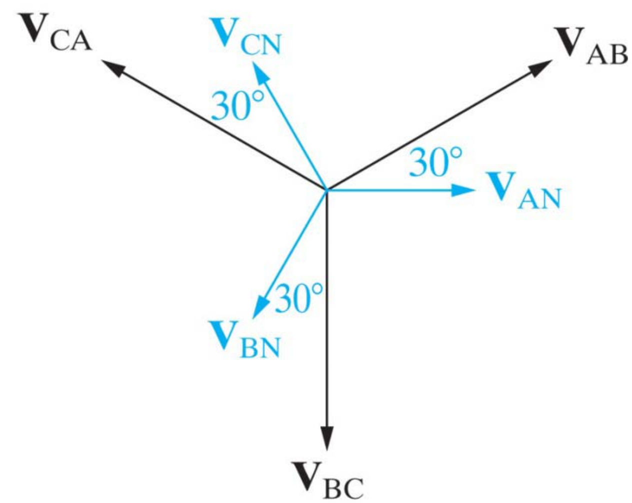
The line to line voltage V_{AB} can be found from the phase voltage V_{AN}.

$$V_{AB} = V_{AN}\left(\sqrt{3}\angle 30^\circ\right) = \left(117.05\angle -0.96^\circ V\right)\left(\sqrt{3}\angle 30^\circ\right)$$

$$= 202.74\angle 29.04^\circ\ Volts$$

$$Thus\ V_{BC} = 202.74\angle -90.96^\circ\ Volts$$

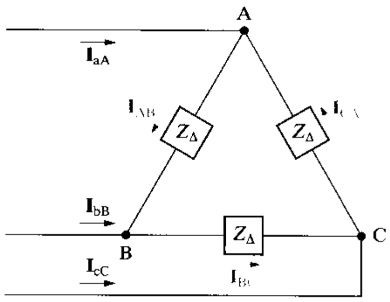
$$And\ V_{CA} = 202.74\angle 149.04^\circ\ Volts$$

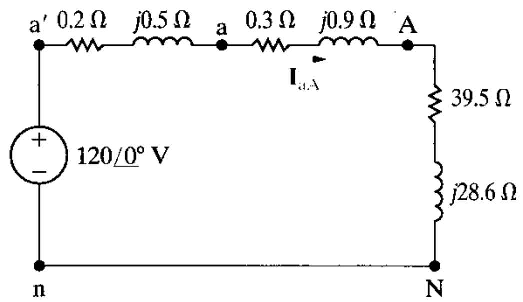


abc phase sequence diagram

Example

Find the Δ “phase” current I_{AB}.





We calculated the wye “phase” current I_{aA}.

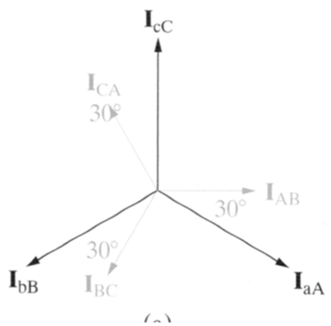
We also know the relationship between the singe-phase equivalent current and the load phase current.

$$I_{aA} = I_{\phi,\Delta}\sqrt{3}\angle -30^\circ$$

$$I_{\phi,\Delta} = I_{AB} = \frac{I_{aA}}{\sqrt{3}\angle -30^\circ} = \frac{2.4\angle -36.87^\circ}{\sqrt{3}\angle -30^\circ} = 1.386\angle -6.87^\circ \ Amps$$

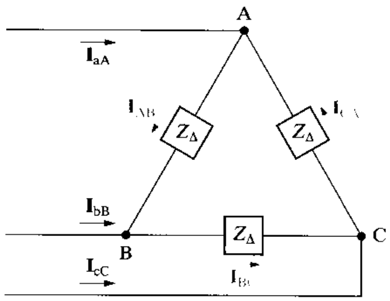
$$Thus \ I_{BC} = 1.386\angle -126.87^\circ \ Amps$$

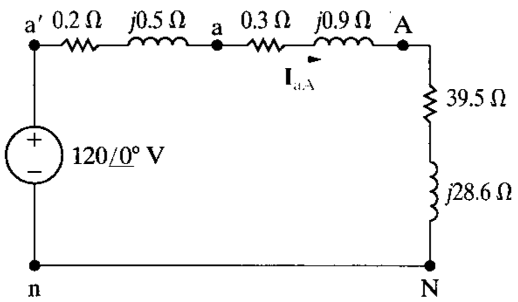
$$And \ I_{CA} = 1.386\angle 113.13^\circ \ Amps$$



Example

Find the line voltage V_{ab}.





This question is asking "what is the line to line voltage at the accessible nodes of the source. This is at terminal **an** for the wye-based a phase source.

By KVL: $-120\angle 0° + I_{aA}Z_{source} + V_{an} = 0 \qquad \Rightarrow V_{an} = 120\angle 0° - I_{aA}Z_{source}$

$$V_{an} = 120\angle 0° - \left(2.4\angle -36.87° A\right)\left(0.2 + j0.5\Omega\right)$$

$$= 120\angle 0° - \left(2.4\angle -36.87° A\right)\left(0.539\angle 68.2°\Omega\right)$$

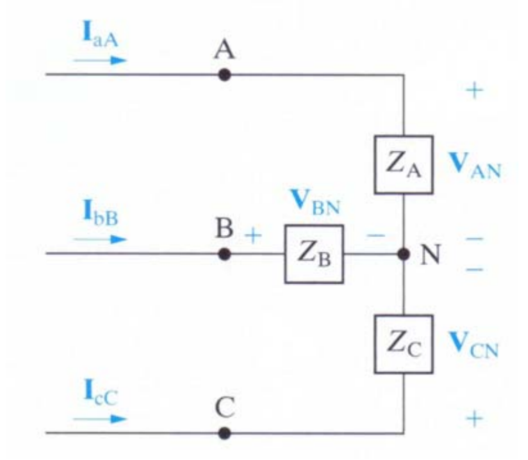
$$= 120\angle 0° - 1.294\angle 31.33° V \quad = 118.9\angle -0.32° \, Volts$$

$$V_{ab} = V_{an}\left(\sqrt{3}\angle 30°\right) \quad = 118.9\angle -0.32° \left(\sqrt{3}\angle 30°\right) \quad = 205.94\angle 29.68° \, Volts$$

$$Thus \; V_{bc} = 205.94\angle -90.32° \, Volts$$

$$And \; V_{ca} = 205.94\angle 149.68° \, Volts$$

Section 11.5
Power Calculations in
Balanced Three-Phase Circuits



The power for each phase in a balanced Wye-connected load is given by

$$P_A = |V_{AN}||I_{aA}|\cos(\theta_{vA} - \theta_{iA})$$

$$P_B = |V_{BN}||I_{bB}|\cos(\theta_{vB} - \theta_{iB})$$

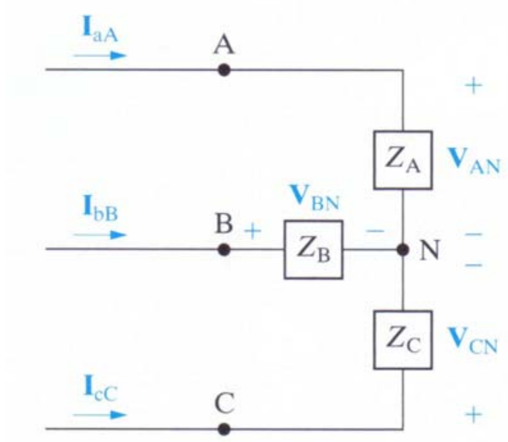
$$P_C = |V_{CN}||I_{cC}|\cos(\theta_{vC} - \theta_{iC})$$

The above equations are true when the voltage and current are written in **rms values**.

Note the magnitude of the voltages is the same for all three phases.

Also the magnitude of the currents is the same for all three phases.

Thus the power must be the same in all three phases.



Thus we can take the magnitudes voltages and phases and define a single power equation.

$$Let\ V_\phi = |V_{AN}| = |V_{BN}| = |V_{CN}|$$

$$Let\ I_\phi = |I_{AN}| = |I_{BN}| = |I_{CN}|$$

$$Let\ \theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}$$

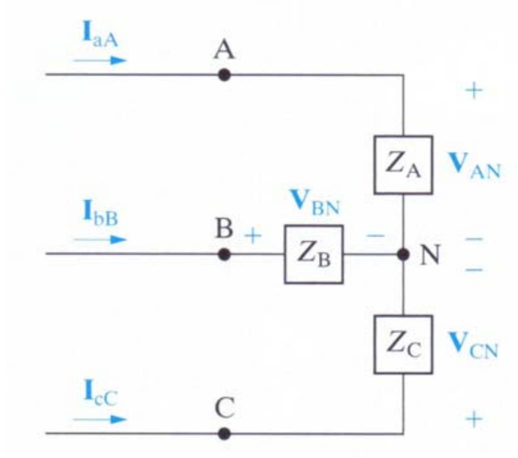
Then in terms of phase voltages: $P_\phi = P_A = P_B = P_C = V_\phi I_\phi \cos\theta_\phi$

The total average power delivered to the three-phase load is

$$P_{Total} = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi$$

In terms of line to line voltages with the line current equal the phase current, the total power is given by

$$P_{Total} = 3P_\phi = 3\frac{V_L}{\sqrt{3}} I_L \cos\theta_\phi \quad = \sqrt{3} V_L\, I_L \cos\theta_\phi$$



The reactive power for a Wye-connection is given by (review chap 10) .

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi$$

The total reactive power is

$$Q_{Total} = 3Q_\phi = 3V_\phi I_\phi \sin\theta_\phi \quad = \sqrt{3} V_L \, I_L \sin\theta_\phi$$

The complex power for a balanced load is

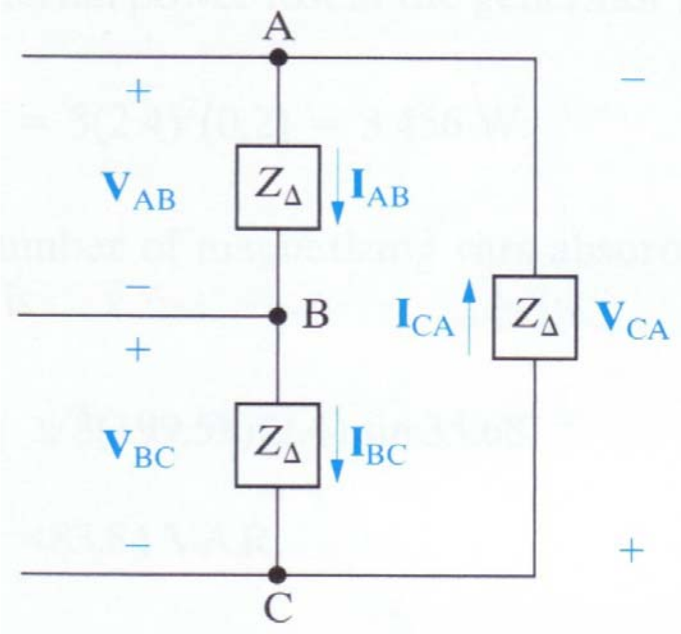
$$S_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^* = \mathbf{V}_{AN}\mathbf{I}_{aA}^* = \mathbf{V}_{BN}\mathbf{I}_{bB}^* = \mathbf{V}_{CN}\mathbf{I}_{cC}^*$$

$$= P_\phi + jQ_\phi$$

The total complex power is

$$S_{Total} = 3S_\phi = \sqrt{3} V_L I_L \angle \theta_\phi^*$$

The voltage and current definitions for the Δ-connected load.



$$P_A = |V_{AB}||I_{AB}|\cos(\theta_{vAB} - \theta_{iAB})$$

$$P_B = |V_{BC}||I_{BC}|\cos(\theta_{vBC} - \theta_{iBC})$$

$$P_C = |V_{CA}||I_{CA}|\cos(\theta_{vCA} - \theta_{iCA})$$

Re-define the magnitudes and phases similar to the wye-connected circuit.

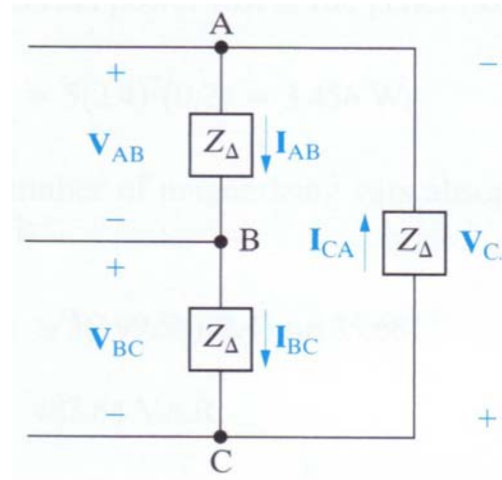
$$Let\ V_\phi = |V_{AB}| = |V_{BC}| = |V_{CA}|$$

$$Let\ I_\phi = |I_{AB}| = |I_{BC}| = |I_{CA}|$$

$$Let\ \theta_\phi = \theta_{vAB} - \theta_{iAB} = \theta_{vBC} - \theta_{iBC} = \theta_{vCA} - \theta_{iCA}$$

The power can now be stated as

$$P_\phi = P_A = P_B = P_C = V_\phi I_\phi \cos\theta_\phi$$



At this point, it is readily apparent that the reactive power for a Δ-connected load is the same as already given. $Q_\phi = V_\phi I_\phi \sin\theta_\phi$

The total reactive power is

$$Q_{Total} = 3Q_\phi = 3V_\phi I_\phi \sin\theta_\phi \quad = \sqrt{3} V_L \, I_L \sin\theta_\phi$$

The complex power for a phase is

$$S_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^* = \mathbf{V}_{AN}\mathbf{I}_{aA}^* = \mathbf{V}_{BN}\mathbf{I}_{bB}^* = \mathbf{V}_{CN}\mathbf{I}_{cC}^*$$

$$= P_\phi + jQ_\phi$$

The total complex power is

$$S_{Total} = 3S_\phi = \sqrt{3} V_L I_L \angle\theta_\phi^*$$

The trick was to follow the V & I definitions for the Δ-connected circuit!

Section 11.5 Instantaneous Power Calculations in 3ϕ Circuits

Let the instantaneous line to neutral reference voltage be v_{aN}.

Let the instantaneous line to neutral reference current be i_{aA}.

$$P_A = v_{aN} i_{aA} = V_{\max} I_{\max} \cos \omega t \cos\left(\omega t - \theta_\phi\right)$$

Then for positive phase sequence (abc).

$$P_B = v_{bN} i_{bB} = V_{\max} I_{\max} \cos\left(\omega t - 120°\right)\cos\left(\omega t - \theta_\phi - 120°\right)$$

$$P_C = v_{cN} i_{cC} = V_{\max} I_{\max} \cos\left(\omega t + 120°\right)\cos\left(\omega t - \theta_\phi + 120°\right)$$

The total instantaneous power is (after much trig manipulation!)

$$P_{Total} = P_A + P_B + P_C = 1.5 V_{\max} I_{\max} \cos\theta_\phi$$

Example (AP11.8_8ed)

A three-phase CPU on a mainframe computer has an average power consumption rating of 22,659 Watts.

Power supply has a line voltage of 208 V_{rms} and a line current of 73.8 A_{rms}.

The computer absorbs magnetizing VARs. This is an inductive load.

Find the reactive power absorbed.

$$|S_{Total}| = \sqrt{3} V_L I_L \quad = \sqrt{3}\left(208 V_{rms}\right)\left(73.8 A_{rms}\right) \quad = 26,587.7\ VA$$

$$= \sqrt{P_{avg}^2 + Q_{avg}^2}$$

Recall that reactive power is not dissipated.
It exchanges energy with the source at 2ω. So the average power consumption of the computer is the real power consumed.

$$|Q| = \sqrt{S_{Total}^2 + P_{avg}^2} \quad = \sqrt{26,587.7^2 + 22,659^2} \quad = 13,909.5\, VAR$$

Example (AP11.8_8ed)

Calculate the power factor for this computer system.

$$pf = \cos(\theta_v - \theta_i)$$

We do not know θ_v or θ_i in this case. But we do know the relationship between the total complex power and the real power dissipated.

$$P = |S|\cos(\theta_v - \theta_i) \qquad \Rightarrow \cos(\theta_v - \theta_i) = \frac{P}{|S|}$$

And recall the definition of the power factor for sinusoidal inputs.

$$pf = \cos(\theta_v - \theta_i)$$

Thus

$$\cos(\theta_v - \theta_i) = \frac{P}{|S_{Total}|} \quad = \frac{22,659}{26,587.7} \quad = 0.8522 \ \text{ Lagging}$$

The current lags the voltage in an inductive circuit.

Another Example (AP11.9_8ed)

The complex power for each phase of a balanced load is 384 + j 288 kVA.

The line voltage at the terminals of the load is 4,160 V_{rms}.

a. What is the magnitude of the line current feeding the load?

$$\text{phase voltage } V_\phi = \frac{V_L}{\sqrt{3}} \quad = \frac{4160\angle 0^\circ (\text{assumed phase!})}{\sqrt{3}} \quad = 2401.78\angle 0^\circ \, V_{rms}$$

Recall the formula for the complex power.

$$S_\phi = \mathbf{V}_{AN}\mathbf{I}^*_{aA} \quad \Rightarrow \mathbf{I}^*_{aA} = \frac{S_\phi}{\mathbf{V}_{AN}}$$

$$\mathbf{I}^*_{aA} = \frac{384,000 + j288,000}{2,401.78\angle 0^\circ} \quad = \frac{480,000\angle 36.87^\circ}{2,401.78\angle 0^\circ} \quad = 199.85\angle 36.87^\circ \, A_{rms}$$

$$I_{aA} = 199.85\angle -36.87^\circ \, A_{rms}$$

Example (AP11.9_8ed)

b. The load is delta connected and consists of a parallel reactance and resistance.

$$\frac{1}{Z_\Delta} = \frac{1}{jX} + \frac{1}{R} \quad \Rightarrow Z_\Delta = \left[\frac{1}{jX} + \frac{1}{R}\right]^{-1}$$

Calculate the reactance X and resistance R.

Use the power relations to solve for R and X.

$$P_\phi = \frac{V_L^2}{R} \;\Rightarrow R = \frac{V_L^2}{P_\phi} \quad = \frac{4160^2}{384,000} \quad = 45.07\,\Omega$$

$$Q_\phi = \frac{V_L^2}{X} \;\Rightarrow X = \frac{V_L^2}{Q_\phi} \quad = \frac{4160^2}{288,000} \quad = 60.09\,\Omega$$

Example (AP11.9_8ed)

c. The load is now wye connected and consists of a series reactance and resistance.

$$Z_\phi = R + jX$$

Calculate the reactance X and resistance R.

$$\mathbf{V}_{AN} = \mathbf{I}_{aA} Z_\phi \quad \Rightarrow Z_\phi = \frac{\mathbf{V}_{AN}}{\mathbf{I}_{aA}} = \frac{\frac{4160\angle 0°}{\sqrt{3}}}{199.85\angle 36.87°} \quad = 12.02\angle -36.87° \Omega$$

$$Z_\phi = 9.616 - j7.21\Omega$$

$$R = 9.616\Omega$$

$$X = 7.21\ \Omega$$

Chapter 11

Balanced Three-Phase Circuits

Text: *Electric Circuits* by J. Nilsson and S. Riedel

Prentice Hall

EEE 117 Network Analysis

Instructor: Russ Tatro