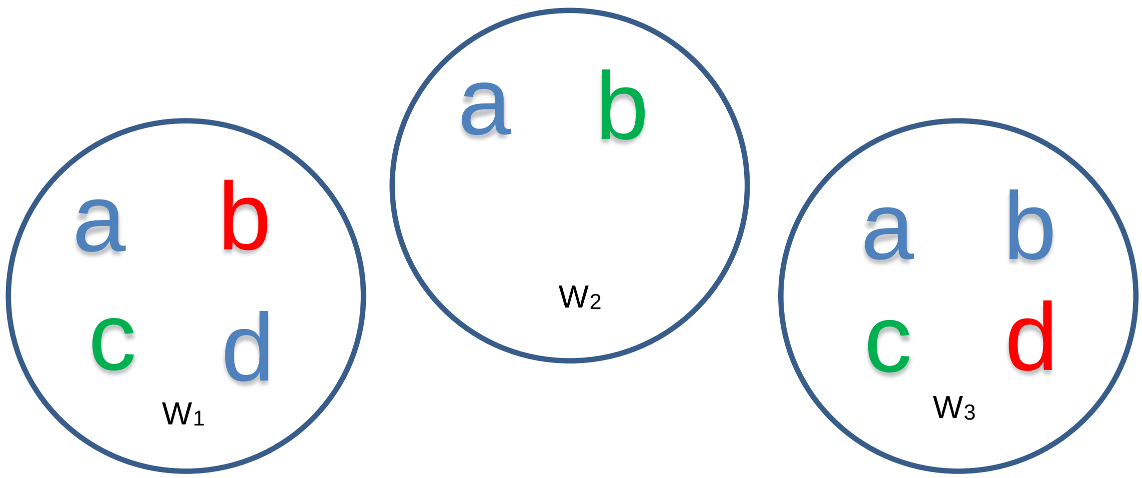
Philosophy 160
Evaluating modal logic formulas

The model $\mathcal{V}$ below depicts a universe of three possible worlds $\mathcal{W}_{\mathcal{V}} = \{w_1, w_2, w_3\}$. The colors of the name letters are indicating the properties of the objects to which they refer, designated R, G, B for red, green, blue. Follow the instructions below.



Notes: 1-4 are valuations of predicates, which will always be a set of objects. The rest are evaluations of propositions, which will always be a truth value.

Evaluate the following:

1. $\mathcal{V}(R, w_1) = \{b\}$
2. $\mathcal{V}(B, w_1) = \{a,b\}$
3. $\mathcal{V}(G, w_2) = \{b\}$
4. $\mathcal{V}(R, w_2) = \emptyset$
5. $\mathcal{V}(\exists x Bx, w_2) = T$
6. $\mathcal{V}(\forall x(Bx \rightarrow x=b), w_2) = F$
7. $\mathcal{V}(\forall x({\sim}Gx \rightarrow x=b), w_3) = F$
8. $\mathcal{V}(\forall x(Gx \vee Bx), w_2) = T$
9. $\mathcal{V}({\sim}\exists x Gx \rightarrow Bb), w_1) = T$
10. $\mathcal{V}(\exists x \forall y {\sim}(Rx \rightarrow By), w_3) = F$
11. $\mathcal{V}({\sim}Rd, w_2) =$ no truth value

Proof: The valuation rules require that for a formula containing a name *a* to have a truth value at w, $\mathcal{V}(a)$ must be a member of $\mathcal{D}_w$.

Evaluate the following:

1. $\mathcal{V}(Ba, w_3) = T$

   Proof: $\mathcal{V}(Ba, w_3)$ is true iff $\mathcal{V}(a) \in \mathcal{V}(B, w_3)$. In other words the object designated by a is a member of the extension of the predicate B in w_3. In other words iff Ba is true in w3. Ba is true in w3.

2. $\mathcal{V}(\Diamond Ba, w_3) = T$

   Proof: Ba is possible in w3 iff Ba is true some world of the model. Ba is true in every world of the model.

3. $\mathcal{V}(\Box Ba, w_3) = T$

   Proof: Ba is necessary in w3 iff Ba is true in every world of the model. Ba is true in every world of the model.

4. $\mathcal{V}(\Box\Diamond Gb, w_2) = T$

   Proof: $\Box\Diamond Gb$ is true in w2 iff $\Diamond Gb$ is true in every world. $\Diamond Gb$ is true in every world iff Gb is true in some world. Gb is true in w2.

5. $\mathcal{V}(\Box Rb \rightarrow Ra, w_1) = T$

   Proof: $\Box Rb \rightarrow Ra$ is false in w1 iff $\Box Rb$ is true and Ra is false in w1. If $\Box Rb$ is true in w1, then Rb is true in every world. Rb is not true in every world. Therefore $\Box Rb$ is false in w1 and $(\Box Rb \rightarrow Ra)$ is true in w1.

6. $\mathcal{V}(\Box Ba \rightarrow \Diamond Gd, w_1) = F$

   Proof: $(\Box Ba \rightarrow \Diamond Gd, w_1)$ is false in w1 iff $\Box Ba$ is true and $\Diamond Gd$ is false in w1. Ba is true in every world, so $\Box Ba$ is true in w1. $\Diamond Gd$ is true in w1 iff Gd is true in some world. Gd is false in every world. Therefore, $(\Box Ba \rightarrow \Diamond Gd)$ is false in w1.

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7. $\mathcal{V}(\Box(\Diamond Ra \rightarrow \Diamond Ga), w_2) = T$

Proof: □(◊Ra → ◊Ga) is true in w2 iff ◊Ra → ◊Ga is true in every world. ◊Ra → ◊Ga is true in a world unless there exists a world in which ◊Ra → ◊Ga is false. ◊Ra → ◊Ga is false in a world iff ◊Ra is true and ◊Ga is false. ◊Ra is true in a world iff Ra is true in some world. Ra is not true in any world. Therefore ◊Ra is false in every world. Therefore, (◊Ra → ◊Ga) is true in every world.

8. $\mathcal{V}(\Diamond\Box Gc, w_3) = F$

Proof: ◊□Gc is true in w3 iff □Gc is true in some world. □Gc is true in some world iff Gc is true in every world. Gc is not true in w3 since c does not exist in w3. (Note: Gc is not false in w3 for the same reason. Gc has no truth value in w3 at all. So, because Gc is neither true nor false in w3 both, □Gc and □~Gc are false in w3, or any other world for that matter.)

9. For any world *w* in $\mathcal{W}_{\mathcal{V}}$, $\mathcal{V}(\Diamond Bb, w) = T$

Proof: This is true iff ◊Bb is true in every world. ◊Bb is true in every world iff Bd is true in some world. Bb is true in w3.

10. For any world *w* in $\mathcal{W}_{\mathcal{V}}$, $\mathcal{V}(\sim\Box\exists xGx, w) = F$

Proof: □∃xGx is true iff ∃xGx is true in every world. ∃xGx is true in a world iff for some *a*, G*a* in that world. This is the case in every world, since Gc in w1, Gb in w2 and Gc in w3. Therefore □∃xGx is true in every world. Therefore ~□∃xGx is false.

11. $\mathcal{V}(\Box Gc, w_2) =$ no truth value

Proof: According to the valuation rules of Leibnizian modal logic, for □Gc to be true or false at w2, c must exist at w2. c does not exist at w2, so this formula has no truth value at w2.

12. $\mathcal{V}(\Box(Gc \rightarrow Ba), w_3) = F$

Proof: According to the valuation rules of Leibnizian modal logic, for □(Gc → Ba) to be true at w3, (Gc → Ba) must be true in every world. Gc has no truth value at w2, hence (Gc → Ba) is is not true at w2. Hence □(Gc → Ba) is false at w3.