Homework 6: Solving Equations in Python

For the following exercises, write your code in one `.py` *file for each problem or part.*

When printing an answer to the screen, make sure to also write some description of what the numbers correspond to, and use appropriate formatting.

Make sure that you scripts run without error in order to get credit. Do not hesitate to ask for help if needed!

Problem 1 (10 pts)

Solve the following equations and systems of equations using the `scipy` module.

In all cases, find all solutions, or at least 5 solutions (whichever is smaller). For parts (a) and (b), limit yourselves to positive (i.e., not including 0) values of x.

For each case, make a plot, adding dots at each solution you have solved for. Try annotating the solutions using the commands you learned in previous lecture notes!

Note: I recommend a separate script file for each part.

(a)
$$\ln(x) = \pi \sin(x)$$

(Beware: ln(0) blows up!)

(b)
$$x = \frac{1}{2}\tan(x)$$

(Note: when plotting, the vertical lines are asymptotes; they are not part of the tangent function so don't search for solutions at their intersection with the LHS.
Also, zero is not positive so it doesn't count as one of your five solutions!)

(c)
$$\begin{cases} x^5 - y^3 + y^2 = \sin(x) \\ y^5 - x^3 = -\frac{1}{4}\left(\cos^2(x) - 0.98\right) \end{cases}$$

Do not try to solve any part of this system of equations analytically. The goal here is to use purely numerical methods.

Problem 2 (5 pts)

Solve the following system of linear equations using the `numpy` module:

$$\begin{cases} 11a + 1 &= b + 3c \\ 9c &= 8 + 4a \\ 2a + 5b &= 7 + 15c \end{cases}$$

Problem 3 (10 pts)

Going back to the projectile problem (see previous notes), use the `scipy` module to solve numerically for the x-coordinate of the landing point for the two different ground functions defined below.

Note that you will have use the `args` parameter when employing the numerical solver as shown in the notes (remember the Gaussian example...)

Note: I recommend a separate script file for each part.

(a) The ground is given by

$$h_1(x) = \begin{cases} 0, & \text{when } x < d, \\ (x-d)\tan(\alpha), & \text{when } x \geq d \text{ and } (x-d)\tan(\alpha) < H, \\ H, & \text{otherwise} \end{cases}$$

where α, d, and H are constants.
You should be able to reuse the ground function you defined in Homework 5 for this.

For this part, use $d = 5\,\text{m}$, $H = 2\,\text{m}$, $\alpha = 30°$, $h_0 = 1.54\,\text{m}$, $\theta_0 = 25°$, and solve for $v_0 = 5\,\text{m/s}, 10\,\text{m/s}$, and $15\,\text{m/s}$.

(b) Ground is given by

$$h_2(x) = 1 - e^{-x^2/1000} + 0.28\left(1 - e^{-0.03x^2}\right)\left(1 - \cos(20x^{0.2})\right)$$

In this case, use $\theta_0 = 25°$ and solve for $v_0 = 6\,\text{m/s}, 16\,\text{m/s}$, and $26\,\text{m/s}$.

As in previous homeoworks, create plots that include the ground and the trajectories like you did in Homework 5. Use those to make educated guesses for the initial values in whatever solve functions you use.

As before, make sure to interrupt the trajectories at the landing points.