

Propositional logic basic inference rules

Negation Elimination ($\sim E$) $\sim\sim A \vdash A$ <u>Example</u> 1. $\sim\sim P$ 2. P 1 $\sim E$	Conditional Elimination ($\rightarrow E$) $A \rightarrow B, A \vdash B$ <u>Example</u> 1. $P \rightarrow Q$ 2. P 3. Q 1,2 $\rightarrow E$
Conjunction Introduction ($\&I$) $A, B \vdash A \& B$ $A, B \vdash B \& A$ <u>Example</u> 1. P 2. Q 3. $P \& Q$ 1,2 $\&I$	Conjunction Elimination ($\&E$) $A \& B \vdash A$ $A \& B \vdash B$ <u>Example</u> 1. $P \& Q$ 2. Q 1 $\&E$
Disjunction Introduction ($\vee I$) $A \vdash A \vee B$ $A \vdash B \vee A$ <u>Example</u> 1. P 2. $P \vee Q$ 1,2 $\vee I$	Disjunction Elimination ($\vee E$) $A \vee B, A \rightarrow C, B \rightarrow C \vdash C$ <u>Example</u> 1. $P \vee Q$ 2. $P \rightarrow R$ 3. $Q \rightarrow R$ 4. R 1,2,3 $\vee E$
Biconditional Elimination ($\leftrightarrow E$) $A \leftrightarrow B \vdash A \rightarrow B$ $A \leftrightarrow B \vdash B \rightarrow A$ <u>Example</u> 1. $P \leftrightarrow Q$ 2. $P \rightarrow Q$ 1 $\leftrightarrow E$	Biconditional Introduction ($\leftrightarrow I$) $A \rightarrow B, B \rightarrow A \vdash A \leftrightarrow B$ $A \rightarrow B, B \rightarrow A \vdash B \leftrightarrow A$ <u>Example</u> 1. $P \rightarrow Q$ 2. $Q \rightarrow P$ 3. $P \leftrightarrow Q$ 1,2 $\leftrightarrow I$

Important propositional logic equivalences

De Morgan's Rules (DM) $\sim(A \& B) \vdash \sim A \vee \sim B$ $\sim(A \vee B) \vdash \sim A \& \sim B$	Transposition (TRANS) $A \rightarrow B \vdash \sim B \rightarrow \sim A$
Material Implication (MI) $A \rightarrow B \vdash \sim A \vee B$	Negation Conditional ($\sim\rightarrow$) $\sim(A \rightarrow B) \vdash A \& \sim B$
Commutation (COM) $A \vee B \vdash B \vee A$ $A \& B \vdash B \& A$	Double Negation (DN) $A \vdash \sim\sim A$
Association (ASS) $A \& (B \& C) \vdash (A \& B) \& C$ $A \vee (B \vee C) \vdash (A \vee B) \vee C$	Distribution (DIST) $A \& (B \vee C) \vdash (A \& B) \vee (A \& C)$ $A \vee (B \& C) \vdash (A \vee B) \& (A \vee C)$

Important propositional logic derived rules

Hypothetical Syllogism (HS) $A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$ <u>Example</u> 1. $P \rightarrow Q$ 2. $Q \rightarrow R$ 3. $P \rightarrow R$ 1,2 HS	Modus Tolens (MT) $A \rightarrow B, \sim B \vdash \sim A$ <u>Example</u> 1. $P \rightarrow Q$ 2. $\sim Q$ 3. $\sim P$ 1,2 MT
Disjunctive Syllogism (DS) $A \vee B, \sim A \vdash B$ $A \vee B, \sim B \vdash A$ <u>Example</u> 1. $P \vee Q$ 2. $\sim P$ 3. Q 1,2 DS	Contradiction (CON) $A, \sim A \vdash B$ <u>Example</u> 1. P 2. $\sim P$ 2. Q 1,2 CON
Repeat (RE) $A \vdash A$ <u>Example</u> 1. P 2. P 1 RE	Constructive Dilemma (CD) $A \vee B, A \rightarrow C, B \rightarrow D \vdash C \vee D$ <u>Example</u> 1. $P \vee Q$ 2. $P \rightarrow R$ 3. $Q \rightarrow S$ 4. $R \vee S$ 1,2,3 CD

Theorem Introduction (TI) Any substitution instance of a theorem you have explicitly derived may be introduced at any time.

Summary of refutation tree rules.

$\sim\sim \quad \vee \quad \sim\sim A$ A	
$\& \quad \vee \quad A \& B$ A B	$\sim\& \quad \vee \quad \sim(A \& B)$ $\sim A$ $\sim B$
$\vee \quad \vee \quad A \vee B$ A B	$\sim\vee \quad \vee \quad \sim(A \vee B)$ $\sim A$ $\sim B$
$\rightarrow \quad \vee \quad A \rightarrow B$ $\sim A$ B	$\sim\rightarrow \quad \vee \quad \sim(A \rightarrow B)$ A $\sim B$
$\leftrightarrow \quad \vee \quad A \leftrightarrow B$ A B $\sim A$ $\sim B$	$\sim\leftrightarrow \quad \vee \quad \sim(A \leftrightarrow B)$ A $\sim A$ $\sim B$ B