Directions: Derive each of the following using any rules and equivalences.

Solutions

1. $(P \vee R) \rightarrow (S \vee T) \vdash (\sim P \ \& \sim R) \vee (\sim S \rightarrow T)$

1.	$(P \vee R) \rightarrow (S \vee T)$	A
2.	$\sim\sim(P \vee R) \rightarrow (S \vee T)$	1, DN
3.	$\sim(\sim P \ \& \sim R) \rightarrow (S \vee T)$	2. DM
4.	$\sim(\sim P \ \& \sim R) \rightarrow (\sim\sim S \vee T)$	3, DN
5.	$\sim(\sim P \ \& \sim R) \rightarrow (\sim S \rightarrow T)$	4, MI
6.	$\sim\sim(\sim P \ \& \sim R) \vee (\sim S \rightarrow T)$	5, MI
7.	$(\sim P \ \& \sim R) \vee (\sim S \rightarrow T)$	6, DN

2. ~(P → (S & R)) ⊢ (P & ~S) v (P & ~R)

1.	~(P → (S & R))	A
2.	P & ~(S &R)	1, ~→
3.	P & (~S v ~R)	2, DM
4.	(P & ~S) v (P & ~R)	3, DIST

1.	~(P → (S & R))	A (Alternative method)
2.	P & ~(S &R)	1, ~→
3.	P & (~S v ~R)	2, DM
4.	P	3, &E
5.	~S v ~R	3, &E
6.	| ~S	H →I
7.	| (P & ~S)	4,6, &I
8.	| (P & ~S) v (P & ~R)	7, vI
9.	~S → ((P & ~S) v (P & ~R))	6-8, →I
10.	| ~R	H →I
11.	| P & ~R	4,10 &I
12.	| (P & ~S) v (P & ~R)	11, vI
13.	~R → ((P & ~S) v (P & ~R))	10-12 →I
14.	(P & ~S) v (P & ~R)	5,9,13 vE

3. ~M & K, ~(K→ M) → (~P → Q) ├ ~Q → P

1.	~M & K	A
2.	~(K → M) → (~P → Q)	A
3.	(K & ~M) → (~P → Q)	2, ~→
4.	K & ~M	1, COM
5.	~P → Q	3,4 →E
6.	~Q → ~~P	5, TRANS
7.	~Q → P	6, DN

4. ⊢ (S → (Q → M)) → (~M → (~S v ~Q))

1.	S → (Q → M)	H →I
2.	~M	H→I
3.	S	H →I
4.	Q → M	1,3 →E
5.	~Q	2,4, MT
6.	S → ~Q	3-5, →I
7.	~S v ~Q	6, MI
8.	(~M → (~S v ~Q))	2-7, →I
9.	(S → (Q → M)) → (~M → (~S v ~Q))	1-8, →I

Pay attention to this method of re-conceiving f the disjunction you are trying to arrive at as a conditional and employing →I, then converting using MI. It is very useful!

5. ⊢ (~(P → ~P) & ~(R → ~R)) → ~(~P v ~R)

1.	(~(P → ~P) & ~(R → ~R))	H
2.	~(P → ~P)	1, &E
3.	~(R → ~R)	1, &E
4.	P & ~~P	2, ~→
5.	R & ~~R	3, ~→
6.	~~P	4, &E
7.	~~R	4, &E
8.	~~P & ~~R	6,7 &I
9.	~(~P v ~R)	8, DM
10.	(~(P → ~P) & ~(R → ~R)) → ~(~P v ~R)	1-9 →I

6. ⊢ (Q v ~T) → ((T& S) → (Q v L))

1.	Q v ~T	H
2.	T & S	H
3.	T	2, &E
4.	S	3, &E
5.	~~T	3, DN
6.	Q	1,5 DS
7.	Q v L	6, vI
8.	(T& S) → (Q v L)	2-7, →I
9.	(Q v ~T) → ((T& S) → (Q v L))	1-8, →I

7. ⊢ ((P v T) & (P↔T)) → (P & T)

1.	(P v T) & (P↔T)		H
2.	P v T		1, &E
3.	P ↔ T		1, &E
4.	P → T		3, ↔E
5.	T → P		3, ↔E
6.		P	H
7.		T	6, 4 →E
8.		P & T	6,7 &I
9.	P → (P & T)		6-8, →I
10.		T	H
11.		P	5,10 →E
12.		P & T	10,11 &I
13.	T → (P & T)		10-12, →I
14.	P & T		2,9,13 v E
15.	((P v T) & (P↔T)) → (P & T)		1-14, →I

8. $P \rightarrow (M \vee Q), \sim P \rightarrow (M \vee Q) \vdash \sim M \rightarrow Q$

1.	$P \rightarrow (M \vee Q)$	A
2.	$\sim P \rightarrow (M \vee Q)$	A
3.	$\sim(P \vee \sim P)$	H ~I
4.	$\sim P \ \& \sim\sim P$	3, DM
5.	$\sim\sim(P \vee \sim P)$	3-4 ~I
6.	$P \vee \sim P$	5, ~E
7.	$M \vee Q$	6,1,2 vE
8.	$\sim\sim M \vee Q$	7, DN
9.	$\sim M \rightarrow Q$	8, MI

This is not the most obvious way to do this proof, but it is an interesting way. Another way that works easily is to Hypothesize the negation of the conclusion. With ~I and DM, you will generate ~(M v Q) which will create a contradiction by two applications of MT.

9. ~(~P & Q), (P v R) → (~T & S), Q ↔ S ├ T → ~(Q v S)

1.	~(~P & Q)	A
2.	(P v R) → (~T & S)	A
3.	Q ↔ S	A
4.	Q → S	3, ↔E
5.	S → Q	3, ↔E
6.	~~P v ~Q	1, DM
7.	| T	H
8.	| | ~~P	H
9.	| | P	8, DN
10.	| | P v R	9, vI
11.	| | ~T & S	2,10 →E
12.	| | ~T	11, &E
13.	| | ~(Q v S)	7,12 CON
14.	| ~~P → ~(Q v S)	8-13, →I
15.	| | ~Q	H
16.	| | ~S	5,15 MT
17.	| | ~Q & ~S	15,16 &I
18.	| | ~(Q v S)	17, DM
19.	| ~Q → ~(Q v S)	15-18, →I
20.	~(Q v S)	6,14,19 vE
21.	T → ~(Q v S)	7-20, →I

10. $(P \rightarrow Q) \rightarrow (P \vee Q) \vdash (\sim P \vee Q) \rightarrow \sim(\sim P \,\&\, \sim Q)$

1.	$(P \rightarrow Q) \rightarrow (P \vee Q)$	A
2.	$\sim P \vee Q$	H →I
3.	$P \rightarrow Q$	2, MI
4.	$P \vee Q$	1,3 →E
5.	$\sim\sim(P \vee Q)$	4, DN
6.	$\sim(\sim P \,\&\, \sim Q)$	5, DM
7.	$(\sim P \vee Q) \rightarrow \sim(\sim P \,\&\, \sim Q)$	2-6, →I

11. ~((P → Q) v (S v ~R)) ├ (~S & R) & ~(~Q → ~P)

1.	~((P → Q) v (S v ~R))	A
2.	~((S v ~R) v (P → Q))	1, COM
3.	~(S v ~R) & ~(P →Q)	2, DM
4.	(~S & ~~R) & ~(P → Q)	3, DM
5.	(~S & R) & ~(P → Q)	4, DN
6.	(~S & R) & ~(~Q → ~P)	5, TRANS

12. ~(R & ~Q), ~R → S, Q → T ⊢ S v T

1.	~(R & ~Q)	A
2.	~R → S	A
3.	Q → T	A
4.	~S	H (for →I, to derive T)
5.	~~R	2,4 MT
6.	~R v ~~Q	1, DM
7.	~~Q	4,6 DS
8.	Q	7, DN
9.	T	3,8 →E
10.	~S → T	4-9, →I
11.	~~S v T	10, MI
12.	S v T	11, DN

13. (P v Q) v (S v M), (~Q & ~M) ├ P v S

1.	(P v Q) v (S v M)	A
2.	(~Q & ~M)	A
3.	~Q	2, &E
4.	~M	2, &E
5.	~P	H
6.	~P & ~Q	3,5 &I
7.	~(P v Q)	6, DM
8.	S v M	1,7 DS
9.	S	4,8 DS
10.	~P → S	5-9, →I
11.	~~P v S	10, MI
12.	P v S	11, DN

14. $S \rightarrow (R \,\&\, Q), P \rightarrow (\sim R \,\&\, \sim T) \vdash (P \,\&\, S) \rightarrow N$

1.	$S \rightarrow (R \,\&\, Q)$	A
2.	$P \rightarrow (\sim R \,\&\, \sim T)$	A
3.	$(P \,\&\, S)$	H
4.	P	3, &E
5.	S	3, &E
6.	$R \,\&\, Q$	1,5 →E
7.	$\sim R \,\&\, \sim T$	2,4 →E
8.	R	6, &E
9.	$\sim R$	7, &E
10.	N	8,9 CON
11.	$(P \,\&\, S) \rightarrow N$	3-10, →I

15. ~(~Q & P) & (Q ↔ ~P) ├ Q ↔ (P → R)

1.	~(~Q & P) & (Q ↔ ~P)	A
2.	~(~Q & P)	1, &E
3.	(Q ↔ ~P)	1, &E
4.	~~Q v ~P	2, DM
5.	Q	H
6.	P	H
7.	Q → ~P	3, ↔E
8.	~P	5,7 →E
9.	R	6,8 CON
10.	P → R	6-9, →I
11.	Q → (P → R)	5-10, →I
12.	P → R	H
13.	~Q	H for ~I
14.	~~~Q	13, DN
15.	~P	4,14 DS
16.	~P → Q	3, ↔E
17.	Q	15,16 →E
18.	Q & ~Q	13, 17 & I
19.	~~Q	13-18, ~I
20.	Q	19, DN
21.	(P → R) → Q	12-20 →I
22.	Q ↔ (P → R)	11,21 ↔I