Commutation

$$A + B = B + A$$

Distributive

$$A * (B + C) = AB + AC$$

$$p \overset{\text{OR}}{\vee} (q \overset{\text{AND}}{\wedge} r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Associative

$$A + (B + C) = (A + B) + C$$

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

De Morgan's

$$\neg (p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg (p \wedge q) \equiv \neg p \vee \neg q$$

$p \to q \not\equiv p \wedge q$

$x - 5 = 40$

Predicate

$x \to$ variable

$P(x)$: $x - 5 = 40$

$P \to$ predicate

$P(35) \to$ false

$P(45) \to$ true

$P(x)$: $5x = 15$

$P(1) \to$ false $\quad P(3) \to$ true

$P(x, y, z)$: $x + y > z$

$P(1, 1, 1) \to$ true $\quad P(1, 2, 10) \to$ false

Consider p(x): x+3 = 6
if x is an element from the set of real numbers
then p(x) is a proposition?

true

Given Predicates:

T F
$p \to q$

- p(x): x ≤ 3
- q(x): x+1 is odd
- r(x): x > 0

1. p(2) 2 <= 3 true
2. ¬q(4) false
3. p(−1)∧q(1) → false
4. p(0)→q(0) true
5. p(4)∨(q(1)∧r(2)) false

q(4) ⇒ 4+1 ⇒ 5 odd
↳ true

p(4) → 4 <= 3 → false

q(1) → 1+1 is odd → false

r(2) → 2 > 0 → true

false ∨ (false ∧ true)
⇒ false ∨ false
⇒ false

Quantifiers

1. All squares are rectangle. True

2. Each prime number is odd. False

3. There exists atleast one integer whose square is equal to itself. True

$1^2 = 1$

Universal ($\forall$)	Existential ($\exists$)
All, each, for every, for each	There exists atleast one
for all natural numbers $x>0$	

Domain

$\forall x \; P(x) > 0$ $\quad x \in N$ (natural number)

Negation of Quantified expression.

Find the negation of the following statements:

1. All citizens vote during election.
2. Someone in the class has a height greater than 5ft.

1. Atleast one citizen does not vote during elections.

2. Everyone in the class has a height less than or equal to 5ft.

Negation of Universal Quantifier

$$\neg\left(\forall x\, P(x)\right) \equiv \exists x\, \neg P(x)$$

Negation of Existential

$$\neg\left(\exists x\, P(x)\right) \equiv \forall x\, \neg P(x)$$

Rules of Inference using proposition →

Modus Ponens → method of affirming

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

If p is true and $p \to q$ is true then q must be true

$p \to q$: If it is sunny, then I will wear a hat.

p : It is sunny

$\therefore q$ → I will wear a hat

Modus Tollens → method of denying

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p \end{array}$$

$\neg q$ → true

q → false

$p \to q$ ⇒ true

false false

$\neg p$ → true

Hypothetical Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

⇒ True
⇒ True
⇒ True

$p \to q$ ⇒ If John goes to school, then Harry goes to school.

$q \to r$ ⇒ If Harry goes to school, then Seema goes to school

$p \to r$ ⇒ If John goes to school, then Seema goes to school

Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore\ q \end{array}$$

false true

$p \vee q \to$ true

$\neg p \to$ true

$p \to$ false

Addition

$$\begin{array}{l} p \\ \hline \therefore\ p \vee q \end{array}$$

Simplification

$$\begin{array}{l} p \wedge q \\ \hline \therefore\ p \end{array}$$

Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Resolution

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

$q \rightarrow$ false

true
$\Uparrow$
$p \vee q \rightarrow$ true

$\neg p \Rightarrow$ false

$\Rightarrow$ true
$\neg p \vee r \Rightarrow$ true

$r \rightarrow$ false

$\neg p \vee r \rightarrow$ true
$\Downarrow$
true

$p \rightarrow$ false

$p \vee q \rightarrow$ true
$\Downarrow$
true

Rules of Inference using predicate and Quantifiers.

Universal Instantiation

$$\frac{\forall x P(x)}{\therefore P(c) \text{ where c is a member of the domain}}$$

Examples

$P(x)$: All squares are rectangle

x : any square

$x \in$ set of squares

$\forall x \, P(x)$

$P(x)$: x is studying discrete maths

c : Srinidhi

$x \in$ Group1

$P(c) \Rightarrow$ true

Universal Generalisation

$$\frac{P(c) \text{ for an arbitrary c in domain}}{\therefore \forall x P(x)}$$

Existential Instantiation

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element c in the domain}}$$

$P(x)$: $x + 10 > 50$ $\qquad x \in N$

$P(60)$: true

$c = 60$

$P(c) \rightarrow$ true

Existential Generalisation

$$\frac{P(c) \text{ for some element c in the domain}}{\therefore \exists x P(x)}$$

Problems

Q1.

If it snows, the school will be closed
If the school is closed, the classes will be cancelled.
Therefore, if it snows, then the classes will be cancelled.

Hypothetical Syllogism

Given the hypotheses “I play outside or I study well,” “I don’t study well or I pass the exam,” and “I don’t play outside or I pass the exam,” is the conclusion, “Therefore, I passed the exam” valid?

p : I play outside
q : I study well
r : I pass the exam

Premise 1 : $p \vee q$
Premise 2 : $\neg q \vee r$
Premise 3 : $\neg p \vee r$

$\therefore r$

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array} \qquad \begin{array}{l} p \vee q \\ \neg q \vee r \\ \hline \therefore p \vee r \end{array}$$

$p \vee q$
$q \vee r$
$p \vee r$

If p = True

$$\begin{array}{l} q \vee r \\ \hline r \end{array}$$

If q = True

$$\begin{array}{l} p \vee r \\ \hline r \end{array}$$

Premise 1 : $p \vee q$

Premise 2 : $\neg q \vee r$

Premise 3 : $\neg p \vee r$

$\therefore r$ — T

r — F

r — F

$\neg p \vee r$ — T

r — F

$\neg p$ — T

p — F

$\neg q \vee r$ — T

r — F

$\neg q$ — T

q — F

$p \vee q \rightarrow T$

r

$r \rightarrow$ false

$\neg p \vee r \Rightarrow$ true

false

true

$p \Rightarrow$ false

$\neg q \vee r \Rightarrow$ true

false

true

$q \Rightarrow$ false

$p \vee q \Rightarrow$ true

Not possible