009x_63

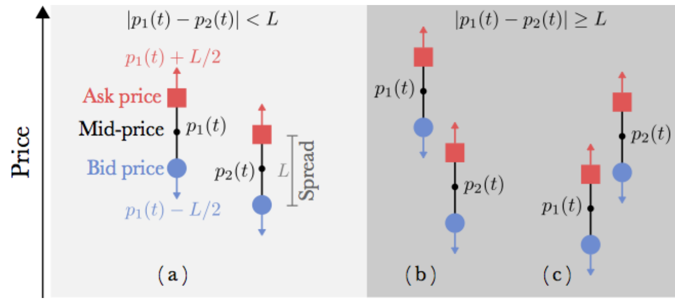
March 18, 2017

Stochastic Processes: Data Analysis and Computer Simulation

Stochastic processes in the real world
-A Stochastic Dealer Model II-

```
In [1]: % matplotlib inline
        import numpy as np  # import numpy library as np
        import math         # use mathematical functions defined by the C standard
        import matplotlib.pyplot as plt # import pyplot library as plt
        import pandas as pd # import pandas library as pd
        from datetime import datetime
        from pandas_datareader import data as pdr
        from pandas_datareader import wb   as pwb
        plt.style.use('ggplot') # use "ggplot" style for graphs
        pltparams = {'legend.fontsize':16,'axes.labelsize':20,'axes.titlesize':20,
                     'xtick.labelsize':12,'ytick.labelsize':12,'figure.figsize':(7.
        plt.rcParams.update(pltparams)

In [2]: # Logarithmic return of price time series
        def logreturn(St,tau=1):
            return np.log(St[tau:])-np.log(St[0:-tau]) # Eq.(J2) : G_tau(t) = log(S
        # normalize data to have unit variance (<(x - <x>)^2> = 1)
        def normalized(data):
            return ((data)/np.sqrt(np.var(data)))
        # compute normalized probability distribution function
        def pdf(data,bins=50):
            hist,edges=np.histogram(data[~np.isnan(data)],bins=bins,density=True) #
            edges   = (edges[:-1] + edges[1:])/2.0 # get bar center
            nonzero = hist > 0.0                   # non-zero points
            return edges[nonzero], hist[nonzero]
        # add logarithmic return data to pandas DataFrame data using the 'Adjusted
        def computeReturn(data, name, tau):
            data[name]=pd.Series(normalized(logreturn(data['Adj Close'].values, tau
        end_time   = datetime.now()
        start_time = datetime(end_time.year - 20, end_time.month, end_time.day)
        toyota     = pdr.DataReader('7203','yahoo',start_time,end_time)  # import t
        computeReturn(toyota, 'Return d1', 1)
```



0.1 The Dealer Model

- K. Yamada, H. Takayasu, T. Ito and M. Takayasu, Physical Revew E 79, 051120 (2009).

- Transaction criterion

$$|p_i(t) - p_j(t)| \geq L \tag{L1}$$

- Market price of transaction

$$P = \frac{1}{2}(p_1 + p_2) \tag{L2}$$

- Logarithmic price return

$$G_\tau(t) \equiv \log P(t+\tau) - \log P(t) \tag{L3}$$

1 The dealer model with memory (model 2)

- To improve the model, Yamada et al. (PRE 79, 051120, 2009) added the effect of "trend-following" predictions.

- Dynamics is captured by a random walk with a drift/memory term

$$p_i(t + \Delta t) = p_i(t) + d\langle \Delta P \rangle_M \Delta t + c f_i(t), \qquad i = 1, 2 \tag{L4}$$

$$f_i(t) = \begin{cases} +\Delta p & \text{prob.}1/2 \\ -\Delta p & \text{prob.}1/2 \end{cases} \tag{L5}$$

- The constant d determines whether the dealer is a "trend-follower" ($d > 0$) or a "contrarian" ($d < 0$)

- The added term represents a moving average over the previous price changes

$$\langle \Delta P \rangle_M = \frac{2}{M(M+1)} \sum_{k=0}^{M-1} (M-k)\Delta P(n-k) \tag{L6}$$

$$\Delta P(n) = P(n) - P(n-1) : \text{Market price change at the n-th tick}$$

- $\langle \Delta P \rangle_M$ is constant during the Random-Walk process, it is only updated at the transaction events

1.1 Dealer model as a 2D random walk

- The dealer model can again be understood as a standard 2D Random walk with absorbing boundaries.
- Introduce the price difference $D(t)$ and average $A(t)$

$$D(t) = p_1(t) - p_2(t) \tag{L7}$$

$$A(t) = \frac{1}{2}\left(p_1(t) + p_2(t)\right) \tag{L8}$$

- Dynamics of D and A describe a 2D random walk

$$D(t+\Delta t) = D(t) + \begin{cases} +2c\Delta p & \text{probability } 1/4 \\ 0 & \text{probability } 1/2 \\ -2c\Delta p & \text{probability } 1/4 \end{cases} \tag{L9}$$

$$A(t+\Delta t) = A(t) + d\langle\Delta P\rangle_M \Delta t + \begin{cases} +c\Delta p & \text{probability } 1/4 \\ 0 & \text{probability } 1/2 \\ -c\Delta p & \text{probability } 1/4 \end{cases} \tag{L10}$$

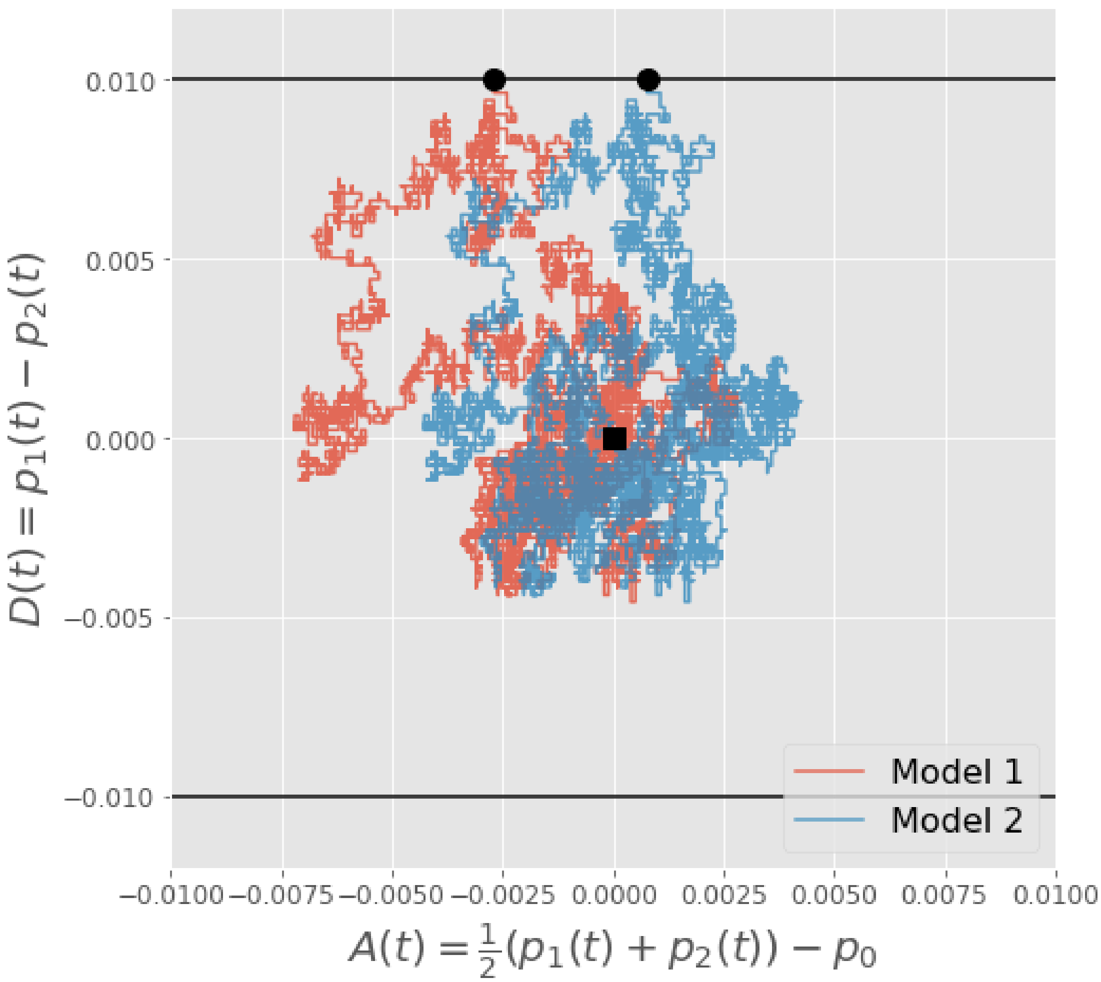
- When $D(t) = \pm L$ a transaction occurs and the random walk ends, the "particle" is absorbed by the boundary.

```
In [3]: params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2, 'd':1.25, 'M':1} # define
        def model2RW(params,p0,deltapm):          # simulate Random-Walk for 1 transa
            price = np.array([p0[0], p0[1]])      # initialize mid-prices for dealers
            cdp   = params['c']*params['dp']      # define random step size
            ddt   = params['d']*params['dt']      # define trend drift term
            Dt    = [price[0]-price[1]]           # initialize price difference as em
            At    = [np.average(price)]           # initialize avg price as empy list
            while np.abs(price[0]-price[1]) < params['L']:
                price=price+np.random.choice([-cdp,cdp],size=2) # random walk step
                price=price+ddt*deltapm           # Model 2 : add trend-following ter
                Dt.append(price[0]-price[1])
                At.append(np.average(price))
            return np.array(Dt),np.array(At)-At[0] # return difference array and av
```

```
In [4]: fig,ax=plt.subplots(figsize=(7.5,7.5),subplot_kw={'xlabel':r'$A(t) = \frac{
        p0 = [100.25, 100.25]
        params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2, 'd':1.25, 'M':1} # define
        for deltapm,lbl in zip([0, 0.003], ['Model 1', 'Model 2']):
            np.random.seed(123456)
            Dt,At = model2RW(params, p0, deltapm)
            ax.plot(At,Dt,alpha=0.8,label=lbl) #plot random walk trajectory
            ax.plot(At[-1],Dt[-1],marker='o',color='k', markersize=10) #last point
            print(lbl+' : number of steps = ',len(At),', price change = ', At[-1])
        ax.plot(0, 0, marker='s', color='k', markersize=10) # starting position
        ax.plot([-0.01,0.03],[params['L'],params['L']],color='k') #top absorbing bo
        ax.plot([-0.01,0.03],[-params['L'],-params['L']],color='k') #bottom absorbi
```

```
        ax.set_ylim([-0.012, 0.012])
        ax.set_xlim([-0.01, 0.01])
        ax.legend(loc=4,framealpha=0.8)
        plt.show()
```

```
Model 1 : number of steps =  9248 , price change =  -0.00270000000009
Model 2 : number of steps =  9248 , price change =  0.000767624991155
```



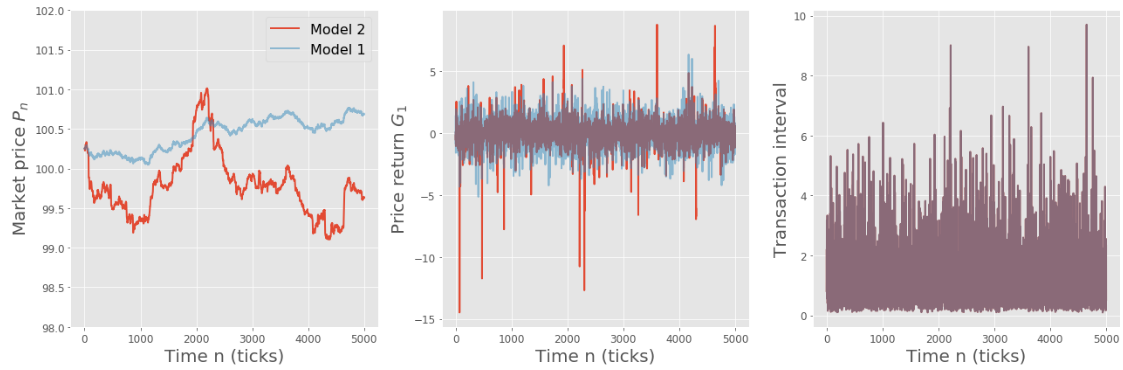

```
In [5]: params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2, 'd':1.25, 'M':1} # define
        def model2(params,p0,numt):
            def avgprice(dpn): # compute running average Eq.(L6)
                M = len(dpn)     #
                weights  = np.array(range(1,M+1))*2.0/(M*(M+1))
                return weights.dot(dpn)

            mktprice = np.zeros(numt)    # initialize market price P(n)
```

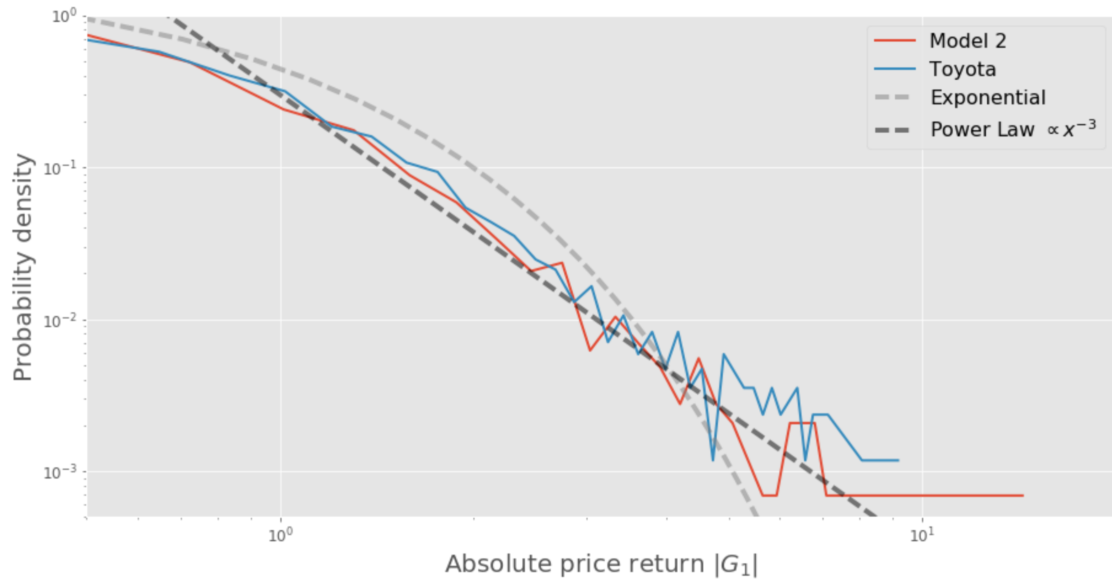
```
        dmktprice= np.zeros(numt)   # initialize change in price dP(n) needed f
        ticktime = np.zeros(numt,dtype=np.int) #initialize array for tick times
        price    = np.array([p0[0], p0[1]])    #initialize dealer's mid-price
        time,tick= 0,0 # real time(t) and time time (n)
        deltapm  = 0.0 # trend term d <dP>_m dt for current random walk
        cdp      = params['c']*params['dp'] # define random step size
        ddt      = params['d']*params['dt'] # define amplitude of trend term
        while tick < numt: # loop over ticks
            while np.abs(price[0]-price[1]) < params['L']: # transaction criter
                price = price + deltapm + np.random.choice([-cdp,cdp], size=2)
                time += 1 #update ral time
            price[:]        = np.average(price) #set mid-prices to new market pr
            mktprice[tick] = price[0]  # save market price
            dmktprice[tick]= mktprice[tick] - mktprice[np.max([0,tick-1])] # sa
            ticktime[tick] = time # save transaction time
            tick += 1 #update ticks
            tick0 = np.max([0, tick - params['M']]) #compute tick start for run
            deltapm = avgprice(dmktprice[tick0:tick])*ddt #compute updated tren
        return ticktime,mktprice
```

```
In [ ]: np.random.seed(0)
        ticktime2,mktprice2 = model2(params, [100.25, 100.25], 5000)
        np.savetxt('model2.txt',np.transpose([ticktime2, mktprice2]))
```

```
In [6]: ticktime,mktprice=np.loadtxt('model1.txt',unpack=True) # read saved data fr
        ticktime2,mktprice2=np.loadtxt('model2.txt', unpack=True)
        timeinterval=normalized((ticktime[1:]-ticktime[0:-1])*params['dt']) # compu
        timeinterval2=normalized((ticktime2[1:]-ticktime2[0:-1])*params['dt'])
        dprice=normalized(logreturn(mktprice,1)) # compute logarithmic return of th
        dprice2=normalized(logreturn(mktprice2,1))
        fig,[ax,bx,cx]=plt.subplots(figsize=(18,6),ncols=3,subplot_kw={'xlabel':r'T
        ax.plot(mktprice2, lw=2, label='Model 2')
        ax.plot(mktprice, alpha=0.5, lw=2, label='Model 1')
        ax.legend()
        ax.set_ylim(98,102)
        ax.set_ylabel(r'Market price $P_n$')
        bx.plot(dprice2, lw=2)
        bx.plot(dprice, alpha=0.5, lw=2)
        bx.set_ylabel(r'Price return $G_1$')
        cx.plot(timeinterval2,  lw=2)
        cx.plot(timeinterval, alpha=0.5, lw=2)
        cx.set_ylabel(r'Transaction interval')
        fig.tight_layout() # get nice spacing between plots
        plt.show()
```




```
In [7]: fig,ax,=plt.subplots(figsize=(15,7.5),subplot_kw={'xlabel':r'Absolute price
        for data,lbl in zip([dprice2, toyota['Return d1']], ['Model 2', 'Toyota']):
            edges, hist = pdf(np.abs(data), bins=50)
            ax.plot(edges, hist, label=lbl, lw=2)
        x = np.linspace(0.1, 10)
        ax.plot(x, 2*np.exp(-1.5*x),lw=4,color='gray',ls='--',alpha=0.5,label=r'Exp
        ax.plot(x, 0.3*x**(-3), lw=4, color='k', ls='--', alpha=0.5, label=r'Power
        ax.set_xlim(5e-1, 2e1)
        ax.set_ylim(5e-4, 1)
        ax.legend()
        ax.semilogx()
        ax.semilogy()
        plt.show()
```



In [8]:
```
params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2, 'd':1.00, 'M':10} # defin
def model2t(params,p0,numt):
    def avgprice(dpn): # compute running average Eq.(L6)
        M = len(dpn)     #
        weights  = np.array(range(1,M+1))*2.0/(M*(M+1))
        return weights.dot(dpn)
    def dtime(i, dpm): # return time varying d-coefficient
        if i <= 1000:  # contrarians
            return -params['d']
        elif i <= 2000:# random walkers: no memory
            return 0.0
        elif i <= 3000:# trend-followers
            return params['d']
        elif dpm >= 0.0: # trend-followers if running average increasing
            return params['d']
        else: # contrarians if running average decreasing
            return -params['d']
    mktprice = np.zeros(numt)    # initialize market price P(n)
    dmktprice= np.zeros(numt)    # initialize change in price dP(n) needed f
    ticktime = np.zeros(numt,dtype=np.int) #initialize array for tick times
    price    = np.array([p0[0], p0[1]])    #initialize dealer's mid-price
    time,tick= 0,0 # real time(t) and time time (n)
    deltapm  = 0.0 # trend term d <dP>_m dt for current random walk
    cdp      = params['c']*params['dp'] # define random step size
    while tick < numt: # loop over ticks
        ddt      = dtime(tick, deltapm)*params['dt'] # define amplitude of
        while np.abs(price[0]-price[1]) < params['L']: # transaction criter
            price = price + deltapm + np.random.choice([-cdp,cdp], size=2)
            time += 1 #update ral time
        price[:]       = np.average(price) #set mid-prices to new market pr
        mktprice[tick] = price[0]  # save market price
        dmktprice[tick]= mktprice[tick] - mktprice[np.max([0,tick-1])] # sa
        ticktime[tick] = time # save transaction time
        tick += 1 #update ticks
        tick0 = np.max([0, tick - params['M']]) #compute tick start for run
        deltapm = avgprice(dmktprice[tick0:tick])*ddt #compute updated tren
    return ticktime,mktprice
```

In [ ]:
```
np.random.seed(0)
ticktime2t,mktprice2t = model2t(params, [100.25, 100.25], 4001)
np.savetxt('model2t.txt',np.transpose([ticktime2t, mktprice2t]))
```

In [9]:
```
ticktime2t,mktprice2t=np.loadtxt('model2t.txt',unpack=True) # read saved da
fig,[ax,bx]=plt.subplots(figsize=(15,7.5),nrows=2, sharex=True)
for i,lbl in zip(range(4), [r'$d=-1$', r'$d=0$', r'$d=+1$', r'$d=d(\left\la
    n0,n1 = i*1000, (i+1)*1000
    dprice=normalized(logreturn(mktprice2t[n0:n1],1))
    ax.plot(range(n0,n1), mktprice2t[n0:n1])
```

```
        ax.plot([n0,n0],[100,102], color='gray')
        ax.text(n0+500, 100.05, lbl, fontsize=22)
        bx.plot([n0,n0],[-6,6],color='gray')
        bx.plot(range(n0+1,n1), dprice)
    ax.set_ylim(100,101)
    bx.set_ylim(-6,6)
    ax.set_ylabel(r'Market price $P_n$')
    bx.set_ylabel(r'Price Return $G_1$')
    bx.set_xlabel(r'Time $n$ (ticks)')
    fig.tight_layout()
    plt.show()
```