009x_22

March 18, 2017

Stochastic Processes: Data Analysis and Computer Simulation

Distribution function and random number
-Generating random numbers with Gaussian distribution-

1 Preparations

1.1 Python built-in functions for random numbers (numpy.random)

1. `seed(seed)`: Initialize the generator with an integer "seed".
2. `rand(d0,d1,...,dn)`: Return a multi-dimensional array of uniform random numbers of shape (d0, d1, ... , dn).
3. `randn(d0,d1,...,dn)`: The same as above but from the standard normal distribution.
4. `binomial(M,p,size)`: Draw samples from a binomial distribution with "M" and "p".
5. `poisson(a,size)`: Draw samples from a Poisson distribution with "a".
6. `choice([-1,1],size)`: Generates random samples from the two choices, -1 or 1 in this case.
7. `normal(ave,std,size)`: Draw random samples from a normal distribution.
8. `uniform([low,high,size])`: Draw samples from a uniform distribution.

- See the Scipy website for details https://docs.scipy.org/doc/numpy-dev/reference/routines.random.html

1.2 Import common libraries

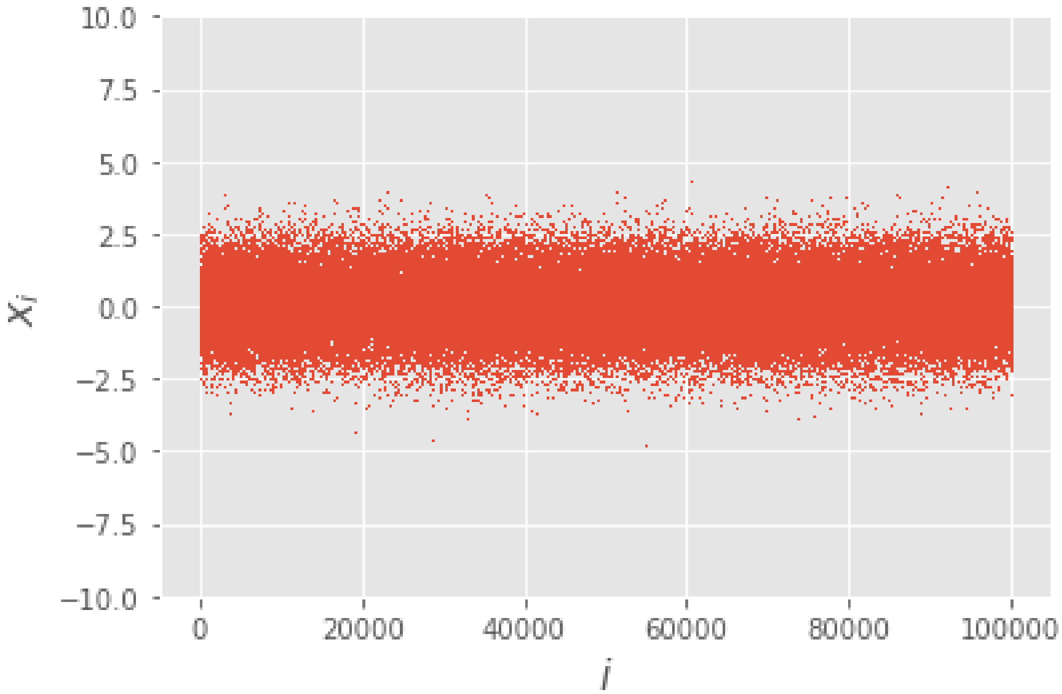
```
In [2]: % matplotlib inline
        import numpy as np # import numpy library as np
        import math # use mathematical functions defined by the C standard
        import matplotlib.pyplot as plt # import pyplot library as plt
        plt.style.use('ggplot') # use "ggplot" style for graphs
```

2 Normal / Gaussian distribution

2.1 Generate random numbers, $x_0, x_1, \cdots, x_N$

```
In [3]: ave = 0.0 # set average
        std = 1.0 # set standard deviation
        N = 100000 # number of generated random numbers
```

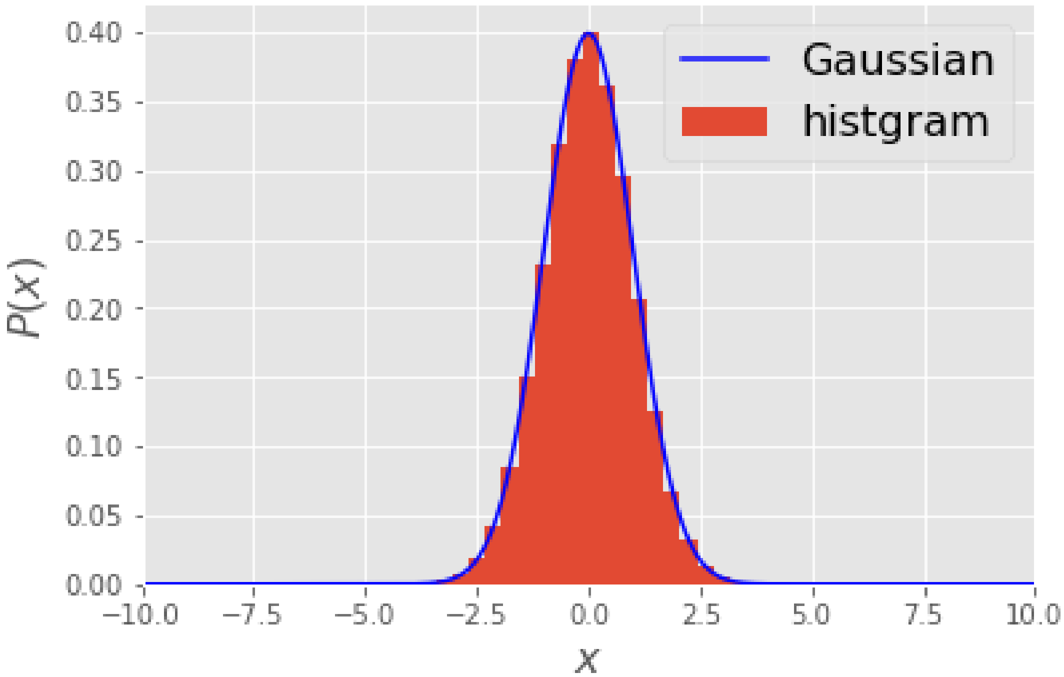
```
np.random.seed(0) # initialize the random number generator with seed=0
X = ave+std*np.random.randn(N) # generate random sequence and store it as X
plt.ylim(-10,10) # set y-range
plt.xlabel(r'$i$',fontsize=16) # set x-label
plt.ylabel(r'$x_i$',fontsize=16) # set y-label
plt.plot(X,',') # plot x_i vs. i (i=1,2,...,N) with dots
plt.show() # draw plots
```



2.2 Compare the distribution with the normal distribution function

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\langle X\rangle)^2}{2\sigma^2}\right] \tag{D1}$$

```
In [4]: plt.hist(X,bins=25,normed=True) # plot normalized histgram of R using 25 bi
        x = np.arange(-10,10,0.01) # create array of x from 0 to 1 with increment (
        y = np.exp(-(x-ave)**2/(2*std**2))/np.sqrt(2*np.pi*std**2) # create array o
        plt.xlim(-10,10) # set x-range
        plt.plot(x,y,color='b') # plot y vs. x with blue line
        plt.xlabel(r'$x$',fontsize=16) # set x-label
        plt.ylabel(r'$P(x)$',fontsize=16) # set y-label
        plt.legend([r'Gaussian',r'histgram'], fontsize=16) # set legends
        plt.show() # display plots
```



2.3 Calculate the auto-correlation function $\varphi(i)$

2.3.1 The definition

$$\varphi(i) = \frac{1}{N}\sum_{j=1}^{N}(x_j - \langle X\rangle)(x_{i+j} - \langle X\rangle) \tag{D2}$$

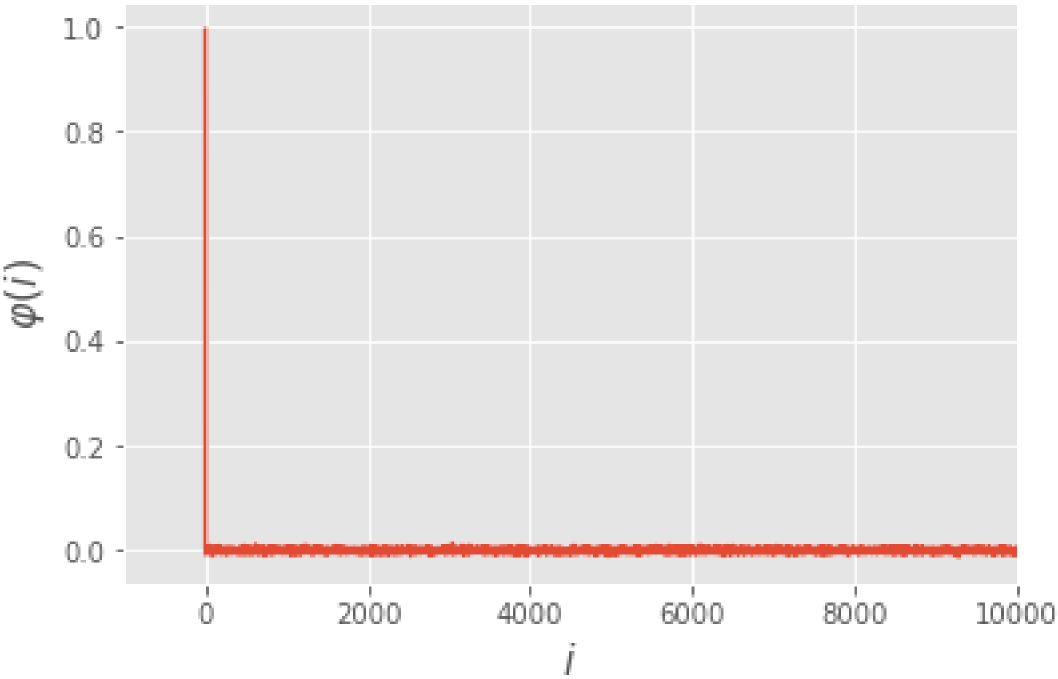
$$\varphi(i=0) = \frac{1}{N}\sum_{j=1}^{N}(x_j - \langle X\rangle)^2 = \langle x_j - \langle X\rangle\rangle^2 = \sigma^2 \tag{D3}$$

$$\varphi(i \neq 0) = \langle x_j - \langle X\rangle\rangle\langle x_{i\neq j} - \langle X\rangle\rangle = 0 \quad (\rightarrow \text{White noise}) \tag{D4}$$

2.3.2 A code example to calculate auto-correlation

```
In [5]: def auto_correlate(x):
            cor = np.correlate(x,x,mode="full")
            return cor[N-1:]
        c = np.zeros(N)
        c = auto_correlate(X-ave)/N
        plt.plot(c)
        plt.xlim(-1000,10000)
        plt.xlabel(r'$i$',fontsize=16)
        plt.ylabel(r'$\varphi(i)$',fontsize=16)
        print('\sigma^2  =',std**2)
        plt.show()
```

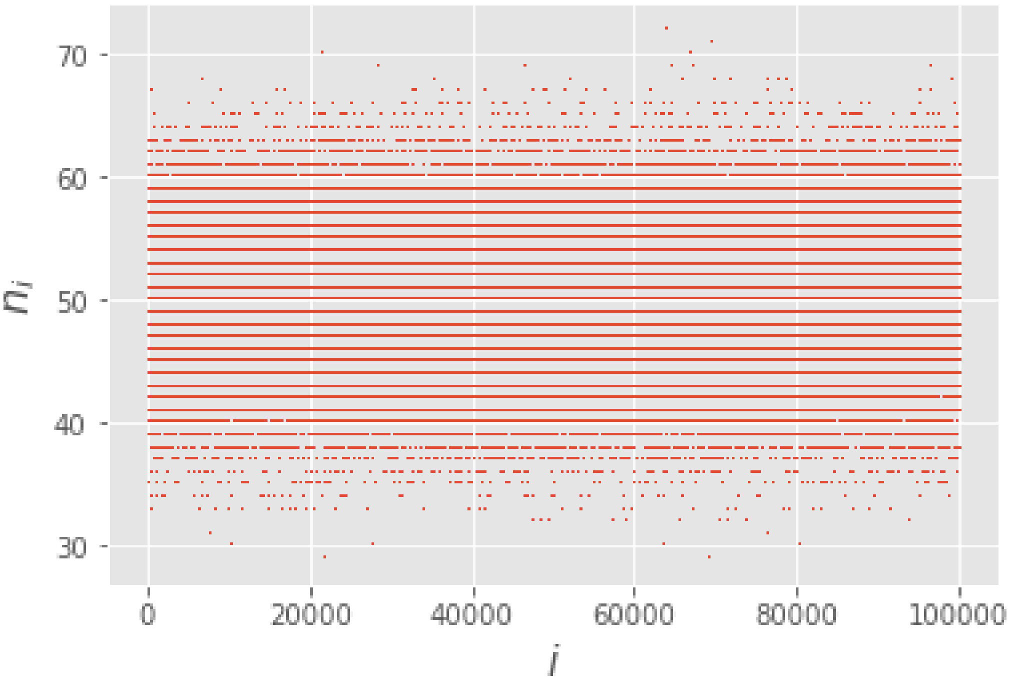
```
\sigma^2  = 1.0
```



3 Binomial distribution

3.1 Generate random numbers, $n_0, n_1, \cdots, n_N$

```
In [6]: p = 0.5 # set p, propability to obtain "head" from a coin toss
        M = 100 # set M, number of tosses in one experiment
        N = 100000 # number of experiments
        np.random.seed(0) # initialize the random number generator with seed=0
        X = np.random.binomial(M,p,N) # generate the number of heads after M tosses
        plt.xlabel(r'$i$',fontsize=16) # set x-label
        plt.ylabel(r'$n_i$',fontsize=16) # set y-label
        plt.plot(X,',') # plot n_i vs. i (i=1,2,...,N) with dots
        plt.show() # draw plots
```



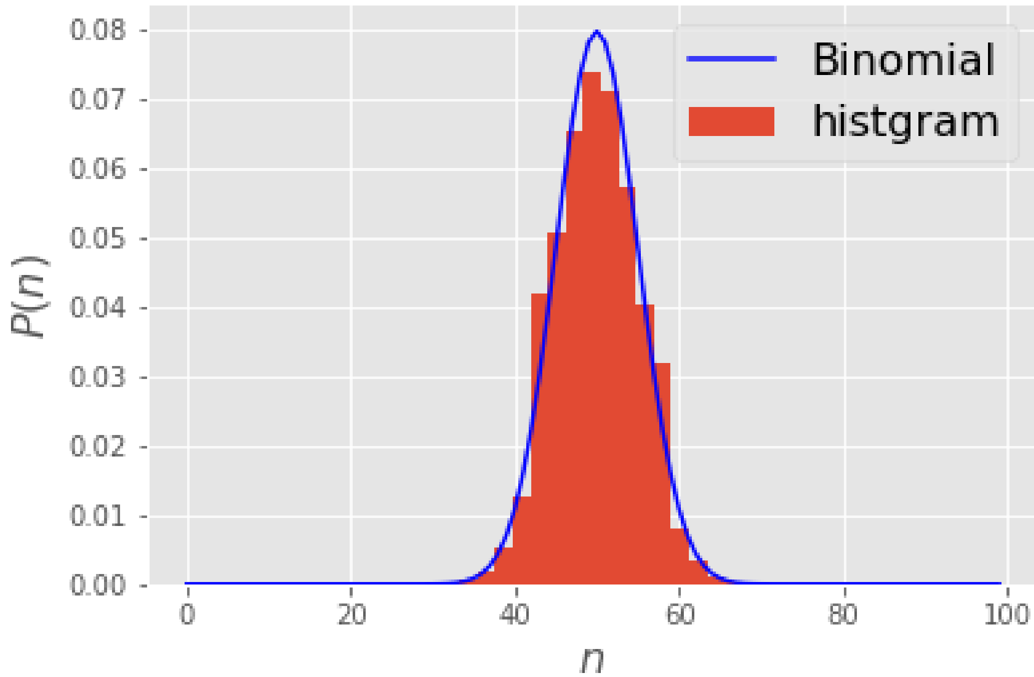
3.2 Compare the distribution with the Binomial distribution function

$$P(n) = \frac{M!}{n!(M-n)!} p^n (1-p)^{M-n} \tag{D5}$$

$$\langle n \rangle = Mp \tag{D6}$$

$$\sigma^2 = Mp(1-p) \tag{D7}$$

```
In [7]: def binomial(n,m,p):
            comb=math.factorial(m)/(math.factorial(n)*math.factorial(m-n))
            prob=comb*p**n*(1-p)**(m-n)
            return prob
        plt.hist(X,bins=20,normed=True) # plot normalized histgram of R using 22 bi
        x = np.arange(M) # generate array of x values from 0 to 100, in intervals o
        y = np.zeros(M) # generate array of y values, initialized to 0
        for i in range(M):
            y[i]=binomial(i,M,p) # compute binomial distribution P(n), Eq. (D5)
        plt.plot(x,y,color='b') # plot y vs. x with blue line
        plt.xlabel(r'$n$',fontsize=16) # set x-label
        plt.ylabel(r'$P(n)$',fontsize=16) # set y-label
        plt.legend([r'Binomial',r'histgram'], fontsize=16) # set legends
        plt.show() # display plots
```



3.3 Calculate the auto-correlation function $\varphi(i)$

3.3.1 The definition

$$\varphi(i) = \frac{1}{N}\sum_{j=1}^{N}(n_j - \langle n\rangle)(n_{i+j} - \langle n\rangle) \tag{D8}$$

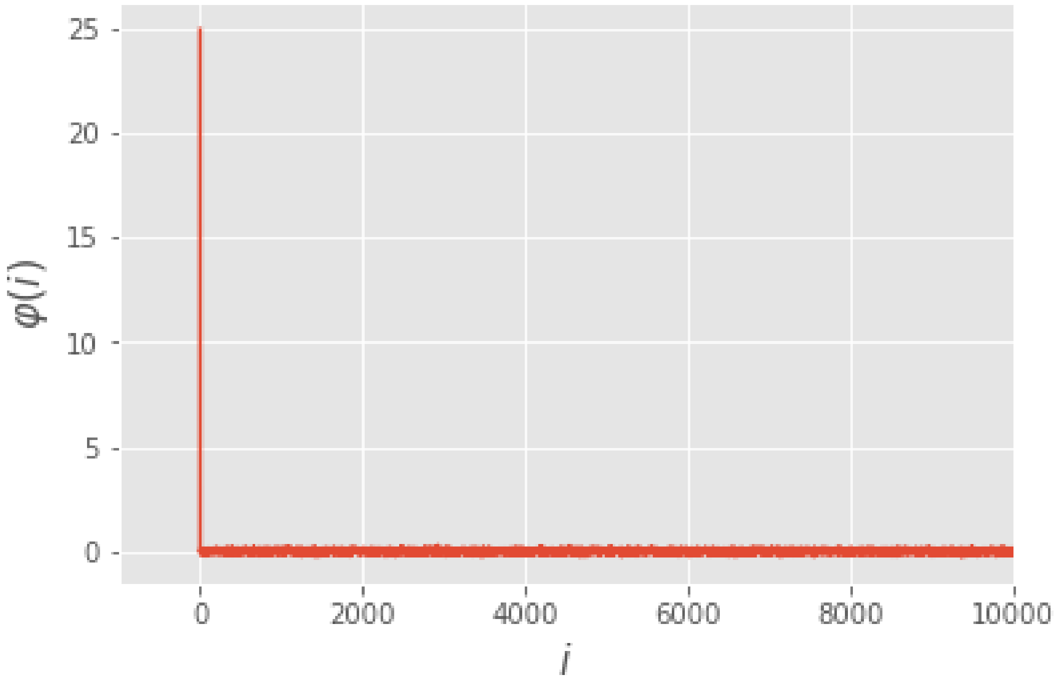
$$\varphi(i=0) = \frac{1}{N}\sum_{j=1}^{N}(n_j - \langle n\rangle)^2 = \langle n_j - \langle n\rangle\rangle^2 = \sigma^2 = Mp(1-p) \tag{D9}$$

$$\varphi(i \neq 0) = \langle n_j - \langle n\rangle\rangle\langle n_{i\neq j} - \langle n\rangle\rangle = 0 \qquad (\rightarrow \text{White noise}) \tag{D10}$$

3.3.2 A code example to calculate auto-correlation

```
In [7]: def auto_correlate(x):
            cor = np.correlate(x,x,mode="full")
            return cor[N-1:]
        c = np.zeros(N)
        c = auto_correlate(X-M*p)/N
        plt.plot(c)
        plt.xlim(-1000,10000)
        plt.xlabel(r'$i$',fontsize=16)
        plt.ylabel(r'$\varphi(i)$',fontsize=16)
        print('\sigma^2  =',M*p*(1-p))
        plt.show()
```

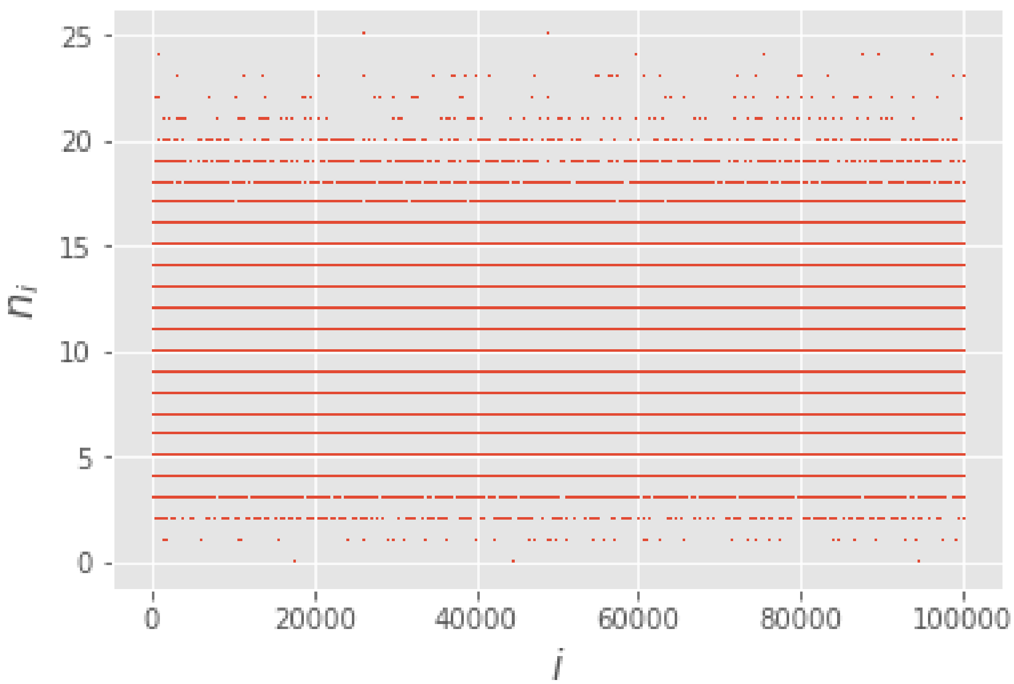
```
\sigma^2  = 25.0
```



4 Poisson distribution

4.1 Generate random numbers, $n_0, n_1, \cdots, n_N$

```
In [8]: a = 10.0 # set a, the expected value
        N = 100000 #  number of generated random numbers
        np.random.seed(0) # initialize the random number generator with seed=0
        X = np.random.poisson(a,N) # generate randon numbers from poisson distribut
        plt.xlabel(r'$i$',fontsize=16) # set x-label
        plt.ylabel(r'$n_i$',fontsize=16) # set y-label
        plt.plot(X,',') # plot n_i vs. i (i=1,2,...,N) with dots
        plt.show() # draw plots
```



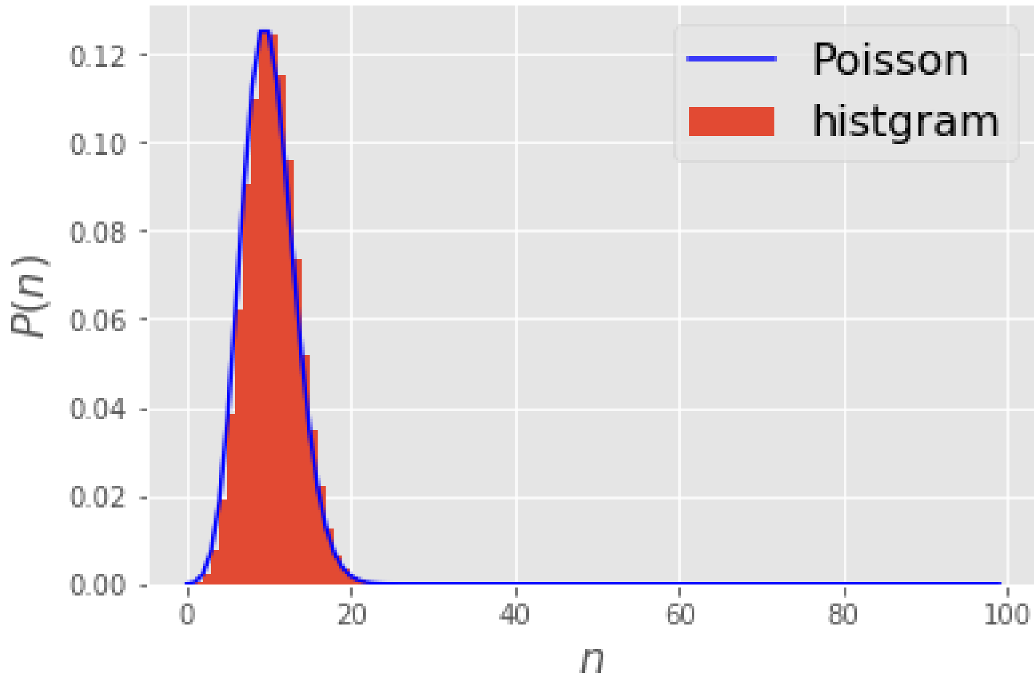
4.2 Compare the distribution with the binomial distribution function

$$P(n) = \frac{a^n e^{-a}}{n!} \tag{D11}$$

$$\langle n \rangle = a \tag{D12}$$

$$\sigma^2 = a \tag{D13}$$

```
In [9]: def poisson(n,a):
            prob=a**n*np.exp(-a)/math.factorial(n)
            return prob
        plt.hist(X,bins=25,normed=True) # plot normalized histgram of X using 25 bi
        x = np.arange(M) # generate array of x values from 0 to 100, in intervals o
        y = np.zeros(M) # generate array of y values, initialized to zero
        for i in range(M):
            y[i]=poisson(i,a) # Compute Poisson distribution for n, Eq. (D11)
        plt.plot(x,y,color='b') # plot y vs. x with blue line
        plt.xlabel(r'$n$',fontsize=16) # set x-label
        plt.ylabel(r'$P(n)$',fontsize=16) # set y-label
        plt.legend([r'Poisson',r'histgram'], fontsize=16) # set legends
        plt.show() # display plots
```



Poisson
histgram
$P(n)$
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
20
40
60
80
100
n