KyotoUx-009x (/github/ryo0921/KyotoUx-009x/tree/master)
/ 06 (/github/ryo0921/KyotoUx-009x/tree/master/06)

Stochastic Processes: Data Analysis and Computer Simulation

Stochastic processes in the real world

4. A Stochastic Dealer Model III

4.1. Preparation

In [1]:

```
% matplotlib inline
import numpy as np  # import numpy library as np
import math         # use mathematical functions defined by the C standa
import matplotlib.pyplot as plt # import pyplot library as plt
import pandas as pd # import pandas library as pd
from numpy import fft
from datetime import datetime
plt.style.use('ggplot') # use "ggplot" style for graphs
pltparams = {'legend.fontsize':16,'axes.labelsize':20,'axes.titlesize':2
             'xtick.labelsize':12,'ytick.labelsize':12,'figure.figsize':
plt.rcParams.update(pltparams)
```

In [2]:

```
# Logarithmic return of price time series
def logreturn(St,tau=1):
    return np.log(St[tau:])-np.log(St[0:-tau]) # Eq.(J2) : G_tau(t) = lc
# normalize data to have zero mean (<x> = 0) and unit variance (<(x - <x
def normalized(data):
    return ((data-np.average(data))/np.sqrt(np.var(data)))
# compute self-correlation of vector v
def auto_correlate(v):
    # np.correlate computes C_{v}[k] = sum_n v[n+k] * v[n]
    corr = np.correlate(v,v,mode="full") # correlate returns even array
    return corr[len(v)-1:]/len(v) # take positive values and normalize t
```

4.2. The dealer model with memory (model 2)

- Price dynamics given by a random-walk with extra memory or drift term to incorporate "trend-following" behavior.

$$p_i(t+\Delta t) = p_i(t) + d\langle\Delta P\rangle_M \Delta t + c f_i(t), \qquad i = 1,2 \tag{M1}$$

$$f_i(t) = \begin{cases} +\Delta p & \text{prob.}1/2 \\ -\Delta p & \text{prob.}1/2 \end{cases} \tag{M2}$$

- Trend-following term depends on the moving-average over the previous M ticks

$$\langle\Delta P\rangle_M = \frac{2}{M(M+1)}\sum_{k=0}^{M-1}(M-k)\Delta P(n-k) \tag{M3}$$

- Transaction takes place when the bid price of one dealer matches the ask price of the other.

$$|p_i(t) - p_j(t)| \geq L \tag{M4}$$

- Price return computed as

$$G_\tau(t) \equiv \log P(t+\tau) - \log P(t) \tag{M5}$$

In [3]:
```
def model2(params,p0,numt):
    def avgprice(dpn): # compute running average Eq.(L6)
        M = len(dpn)    #
        weights  = np.array(range(1,M+1))*2.0/(M*(M+1))
        return weights.dot(dpn)

    mktprice = np.zeros(numt)   # initialize market price P(n)
    dmktprice= np.zeros(numt)   # initialize change in price dP(n) neede
    ticktime = np.zeros(numt,dtype=np.int) #initialize array for tick ti
    price    = np.array([p0[0], p0[1]])    #initialize dealer's mid-pric
    time,tick= 0,0 # real time(t) and time time (n)
    deltapm  = 0.0 # trend term d <dP>_m dt for current random walk
    cdp      = params['c']*params['dp'] # define random step size
    ddt      = params['d']*params['dt'] # define amplitude of trend term
    while tick < numt: # loop over ticks
        while np.abs(price[0]-price[1]) < params['L']: # transaction cri
            price = price + deltapm + np.random.choice([-cdp,cdp], size=
            time += 1 #update real time
        price[:]       = np.average(price) #set mid-prices to new market
        mktprice[tick] = price[0]  # save market price
        dmktprice[tick]= mktprice[tick] - mktprice[np.max([0,tick-1])] #
        ticktime[tick] = time # save transaction time
        tick += 1 #update ticks
        tick0 = np.max([0, tick - params['M']]) #compute tick start for
        deltapm = avgprice(dmktprice[tick0:tick])*ddt #compute updated t
    return ticktime,mktprice
```

In [ ]:
```
params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2, 'd':1.00, 'M':10} # de
price = np.zeros((5,5000))
# run 5 simulations with different d paramaters
for i,d,lbl in zip(range(5),[-2.0, -1.25, 0.0, 1.25, 2.0], ['d-2.0', 'd-
    np.random.seed(0)
    params['d'] = d
    print(params['d'])
    time,price[i] = model2(params, [100.25, 100.25], 5000)
np.savetxt('model2_M10_5d.txt', np.transpose(price), header="d-2\t d-1\t
```

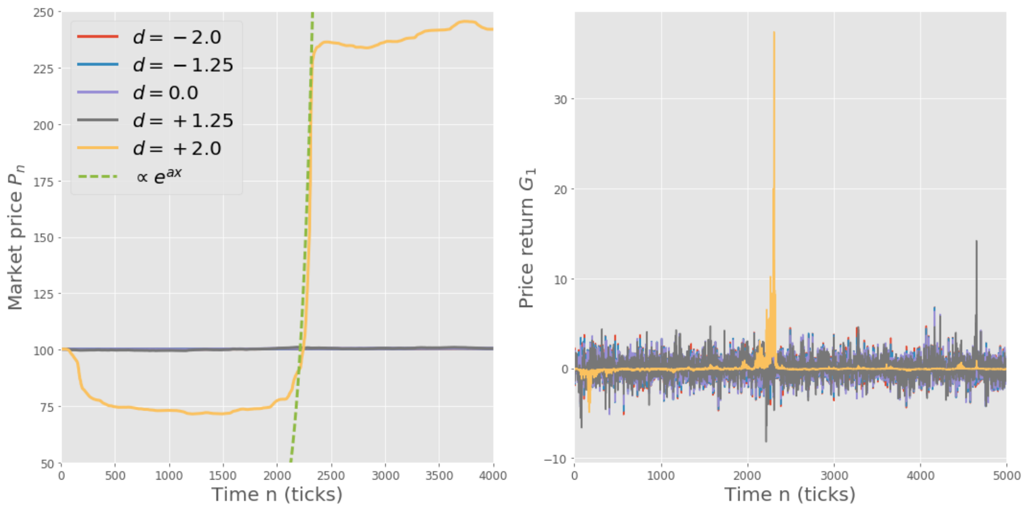
In [4]:
```
price  = pd.read_csv('model2_M10_5d.txt', header=0, delim_whitespace=Tru
dprice = pd.DataFrame()
for lbl in price.columns:
    dprice[lbl] = normalized(logreturn(price[lbl].values, 1)) #price ret
dprice.head()
```

Out[4]:

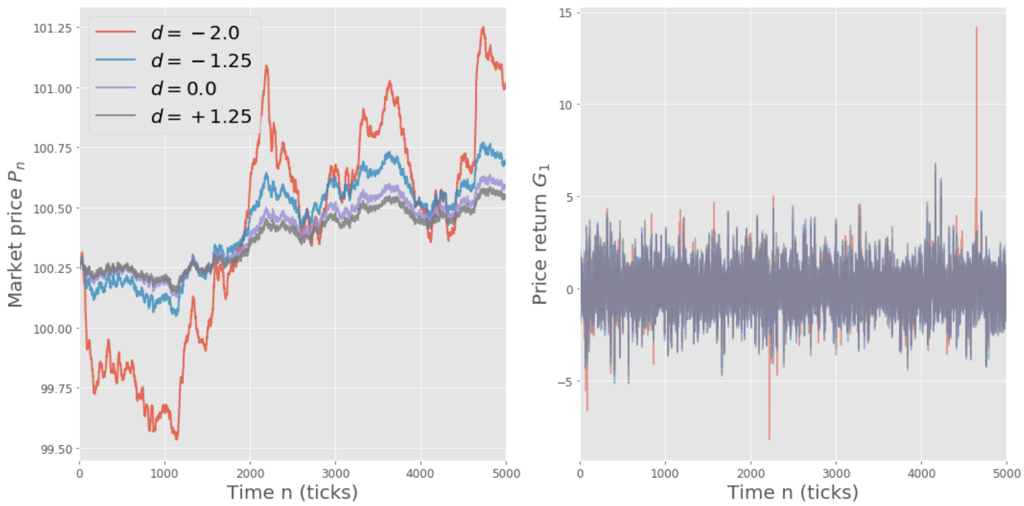
	d-2	d-1	d0	d+1	d+2
0	0.064483	0.064803	0.062363	0.042186	-0.093576
1	-0.249174	-0.245864	-0.235129	-0.193000	-0.100403
2	-0.759109	-0.824618	-0.909471	-0.824092	-0.120481
3	2.264443	1.994706	1.411017	0.612311	-0.087087
4	-0.453285	-0.333167	-0.215293	-0.201743	-0.102947

4.3. Price dynamics of the dealer model

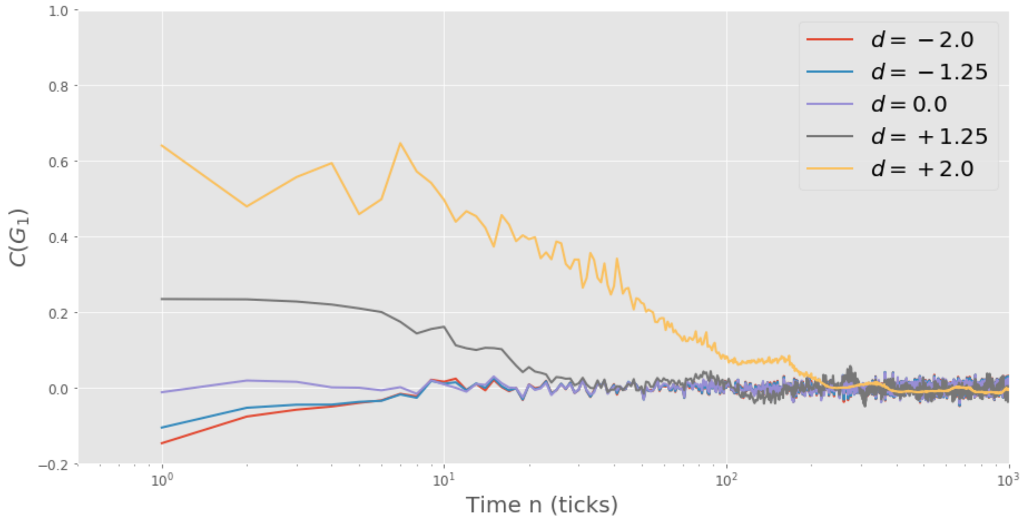
In [5]:
```
fig,[ax,bx]=plt.subplots(figsize=(15,7.5),ncols=2,subplot_kw={'xlabel':r
price.plot(ax=ax, lw=3)
dprice.plot(ax=bx, legend=False)
x = np.arange(1000,2550)
ax.plot(x,0.03*np.exp(8e-3*(x-1200)),lw=3,ls='--',label='exponential')
ax.set_ylim(50,250)
ax.set_xlim(0,4000)
ax.legend([r'$d=-2.0$', r'$d=-1.25$', r'$d=0.0$', r'$d=+1.25$', r'$d=+2.
ax.set_ylabel(r'Market price $P_n$')
bx.set_ylabel(r'Price return $G_{1}$')
fig.tight_layout() # get nice spacing between plots
plt.show()
```



In [6]:
```
fig,[ax,bx]=plt.subplots(figsize=(15,7.5),ncols=2,subplot_kw={'xlabel':r
cols = ['d+1','d0','d-1','d-2']
price[cols].plot(ax=ax, lw=2, alpha=0.8)
dprice[cols].plot(ax=bx, alpha=0.5, legend=False)
ax.legend([r'$d=-2.0$', r'$d=-1.25$', r'$d=0.0$', r'$d=+1.25$', r'+$d=2.
ax.set_ylabel(r'Market price $P_n$')
bx.set_ylabel(r'Price return $G_{1}$')
fig.tight_layout() # get nice spacing between plots
plt.show()
```



In [7]:
```
pricecor = pd.DataFrame()
for lbl in dprice.columns:
    ct = auto_correlate(dprice[lbl].values)
    pricecor[lbl] = ct/ct[0]
fig,ax=plt.subplots(figsize=(15,7.5),subplot_kw={'xlabel':r'Time n (tick
pricecor.plot(ax=ax, lw=2)
ax.semilogx()
ax.set_xlim(5e-1,1e3)
ax.set_ylim(-0.2, 1.0)
ax.legend([r'$d=-2.0$', r'$d=-1.25$', r'$d=0.0$', r'$d=+1.25$', r'$d=+2.
plt.show()
```



4.4. Dynamics of real data

In [8]:
```
def computeReturn(data, pname, dname, tau):
    data[dname]=pd.Series(normalized(logreturn(data[pname].values, tau))

# https://www.quantshare.com/sa-426-6-ways-to-download-free-intraday-and
dateparse = lambda x: pd.datetime.strptime(x, '%Y%m%d %H:%M:%S')
appletick = pd.read_csv('US2.AAPL_170301_170301_tick.csv', parse_dates={
applemin  = pd.read_csv('US2.AAPL_170301_170301_min.csv',  parse_dates={
computeReturn(appletick, 'Last', 'Return d1', 1)
computeReturn(applemin, 'Close', 'Return d1', 1)
```

In [9]:
```
appletick.head()
```

Out[9]:

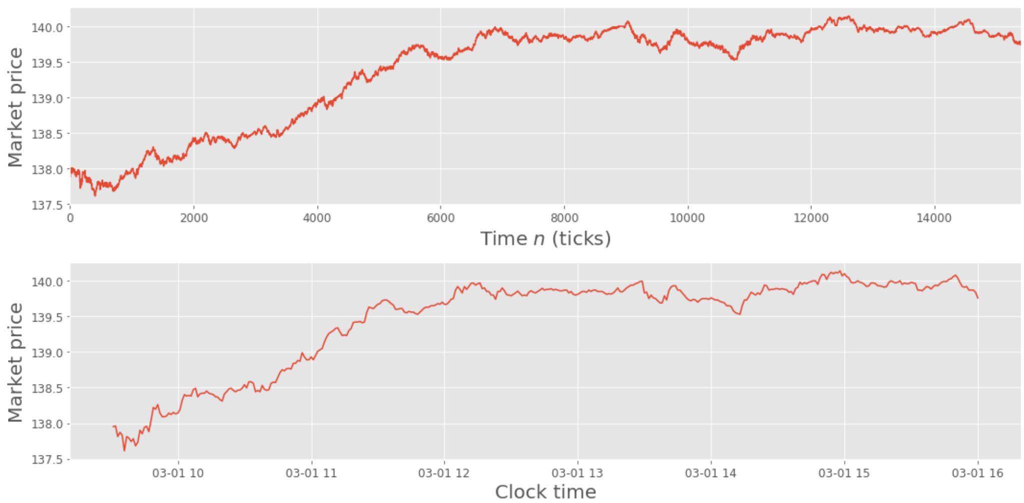
	datetime	Ticker	Per	Last	Vol	Return d1
0	2017-03-01 09:30:00	US2.AAPL	0	137.89	100	-1.503504
1	2017-03-01 09:30:00	US2.AAPL	0	137.88	100	1.467665
2	2017-03-01 09:30:00	US2.AAPL	0	137.89	100	7.408387
3	2017-03-01 09:30:00	US2.AAPL	0	137.94	100	-0.017920
4	2017-03-01 09:30:01	US2.AAPL	0	137.94	100	-0.017920

In [10]:
```
applemin.head()
```

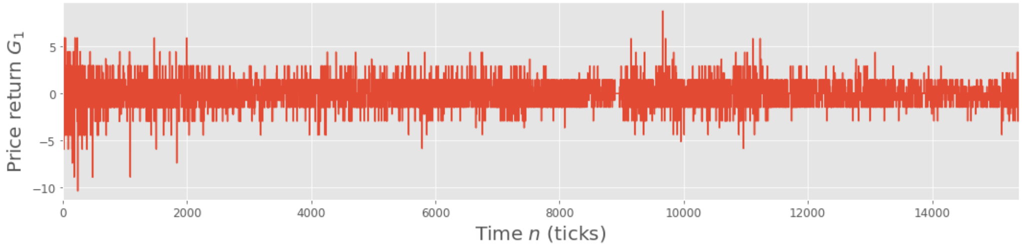
Out[10]:

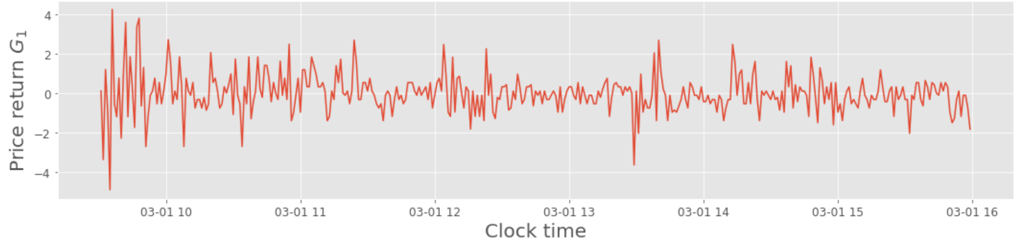
	datetime	Ticker	Per	Open	High	Low	Close	Vol	Return d1
0	2017-03-01 09:31:00	US2.AAPL	1	137.89	138.00	137.88	137.95	12370	0.116571
1	2017-03-01 09:32:00	US2.AAPL	1	137.95	137.98	137.88	137.96	8738	-3.353714
2	2017-03-01 09:33:00	US2.AAPL	1	137.96	137.96	137.72	137.81	5005	1.201604
3	2017-03-01 09:34:00	US2.AAPL	1	137.84	137.97	137.83	137.87	7138	-0.968038
4	2017-03-01 09:35:00	US2.AAPL	1	137.88	137.88	137.79	137.83	9587	-4.877729

In [11]:
```
fig,[ax,bx]=plt.subplots(figsize=(15,7.5), nrows=2, subplot_kw={'ylabel'
ax.plot(appletick.index, appletick['Last'])
bx.plot(applemin['datetime'], applemin['Close'])
ax.set_xlim(0,15398)
ax.set_xlabel('Time $n$ (ticks)')
bx.set_xlabel('Clock time')
plt.tight_layout()
plt.show()
```

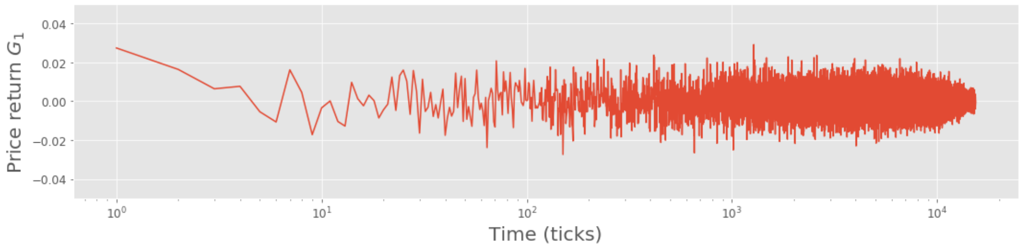


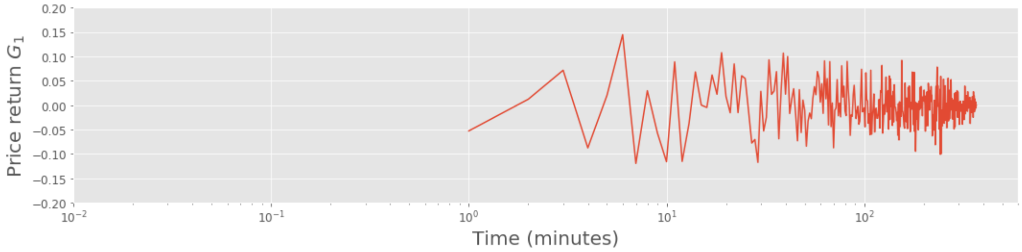
In [12]:
```
fig,[ax,bx]=plt.subplots(figsize=(15,7.5), nrows=2)
ax.plot(appletick.index, appletick['Return d1'])
bx.plot(applemin['datetime'], applemin['Return d1'])
ax.set_xlim(0,15398)
ax.set_ylabel(r'Price return $G_{1}$')
bx.set_ylabel(r'Price return $G_{1}$')
ax.set_xlabel('Time $n$ (ticks)')
bx.set_xlabel('Clock time')
plt.tight_layout()
plt.show()
```



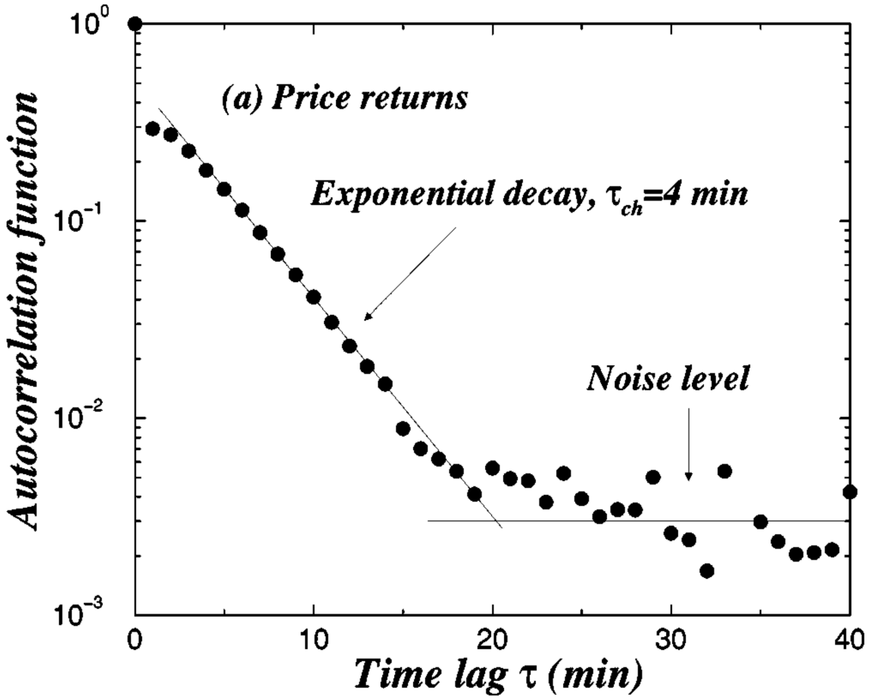


In [13]:
```
def computeCt(data, name):
    ct = auto_correlate(data[name].values[:-20]) # ignore NaN last point
    data['Ct'] = pd.Series(ct, index=data.index[:-20])
computeCt(appletick, 'Return d1')
computeCt(applemin, 'Return d1')
fig,[ax,bx]=plt.subplots(figsize=(15,7.5), nrows=2, subplot_kw={'ylabel'
ax.plot(appletick['Ct'])
bx.plot(applemin['Ct'])
ax.set_xlabel('Time (ticks)')
bx.set_xlabel('Time (minutes)')
ax.semilogx()
bx.semilogx()
ax.set_ylim(-0.05, 0.05)
bx.set_ylim(-0.2, 0.2)
bx.set_xlim(1e-2,6e2)
plt.tight_layout()
plt.show()
```





Time-correlation of the S&P 500 returns



- P. Gopikrishnan, V. Plerou, L. Amaral, M. Meyer and H. Stanley *Physical Revew E* **60**, 5305 (1999).
- Time auto-correlation of $G_{\Delta t}(\tau)$ with $\Delta t = 1\mathrm{min}$

4.5. Conclusions

- We have shown how a simple-model stochastic model, built borrowing concepts from statistical physics, can reproduce many behaviors seen in real-world stock markets.
- While we considered the simplest possible version, with only two dealers and constant and equal trend-following characteristics, you can easily remove these restrictions. You can try to simulate for hundreds of dealers, with non-constant d values. The main results still hold, it just becomes more complicated to analyze as the number of parameters increases.
- For the dealer model we presented, one can recover the non-trivial power law decay of the price returns, but only for a specific set of parameter values. If the parameters of the model are changed, then the nature of the distribution can also change.
- While this type of modeling can help you understand complex real-world systems, you should be very careful when trying to make precise quantitative predictions based on them.