Supplemental note for Week 3 Part 2

ver. 20180107-01

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1 Derivation of the mean square displacement

$$
\begin{aligned}
\langle |\Delta \mathbf{R}(t)|^2 \rangle &= \frac{6\tilde{D}}{\zeta m} \int_0^t dt_1 \left[\exp\left(\frac{\zeta}{m} t_1 \right) \int_{t_1}^t dt_2 \exp\left(-\frac{\zeta}{m} t_2 \right) \right] && (1) \\
&= \frac{6\tilde{D}}{\zeta m} \int_0^t dt_1 \left[\exp\left(\frac{\zeta}{m} t_1 \right) \left(-\frac{m}{\zeta} \left(\exp\left(-\frac{\zeta}{m} t \right) - \exp\left(-\frac{\zeta}{m} t_1 \right) \right) \right) \right] && (2) \\
&= \frac{6\tilde{D}}{\zeta^2} \int_0^t dt_1 \left[\exp\left(\frac{\zeta}{m} t_1 \right) \left(\exp\left(-\frac{\zeta}{m} t_1 \right) - \exp\left(-\frac{\zeta}{m} t \right) \right) \right] && (3) \\
&= \frac{6\tilde{D}}{\zeta^2} \int_0^t dt_1 \left[1 - \exp\left(\frac{\zeta}{m} t_1 \right) \exp\left(-\frac{\zeta}{m} t \right) \right] && (4) \\
&= \frac{6\tilde{D}}{\zeta^2} \left[t - \exp\left(-\frac{\zeta}{m} t \right) \int_0^t dt_1 \exp\left(\frac{\zeta}{m} t_1 \right) \right] && (5) \\
&= \frac{6\tilde{D}}{\zeta^2} \left[t - \exp\left(-\frac{\zeta}{m} t \right) \frac{m}{\zeta} \left(\exp\left(\frac{\zeta}{m} t \right) - 1 \right) \right] && (6) \\
&= \frac{6\tilde{D}}{\zeta^2} \left[t - \frac{m}{\zeta} + \frac{m}{\zeta} \exp\left(-\frac{\zeta}{m} t \right) \right] && (7) \\
&\simeq \frac{6\tilde{D}}{\zeta^2} t \qquad (t \to \infty) && (8)
\end{aligned}
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