009x_64

March 18, 2017

Stochastic Processes: Data Analysis and Computer Simulation

Stochastic processes in the real world
-A Stochastic Dealer Model III-

```
In [1]: % matplotlib inline
        import numpy as np  # import numpy library as np
        import math         # use mathematical functions defined by the C standard
        import matplotlib.pyplot as plt # import pyplot library as plt
        import pandas as pd # import pandas library as pd
        from numpy import fft
        from datetime import datetime
        plt.style.use('ggplot') # use "ggplot" style for graphs
        pltparams = {'legend.fontsize':16,'axes.labelsize':20,'axes.titlesize':20,
                     'xtick.labelsize':12,'ytick.labelsize':12,'figure.figsize':(7.
        plt.rcParams.update(pltparams)
```

```
In [2]: # Logarithmic return of price time series
        def logreturn(St,tau=1):
            return np.log(St[tau:])-np.log(St[0:-tau]) # Eq.(J2) : G_tau(t) = log(S
        # normalize data to have zero mean (<x> = 0) and unit variance (<(x - <x>)^
        def normalized(data):
            return ((data-np.average(data))/np.sqrt(np.var(data)))
        # compute self-correlation of vector v
        def auto_correlate(v):
            # np.correlate computes C_{v}[k] = sum_n v[n+k] * v[n]
            corr = np.correlate(v,v,mode="full") # correlate returns even array [0:
            return corr[len(v)-1:]/len(v) # take positive values and normalize by n
```

1 The dealer model with memory (model 2)

- Price dynamics given by a random-walk with extra memory or drift term to incorporate "trend-following" behavior.

$$p_i(t + \Delta t) = p_i(t) + d\langle\Delta P\rangle_M \Delta t + cf_i(t), \qquad i = 1, 2 \tag{M1}$$

$$f_i(t) = \begin{cases} +\Delta p & \text{prob.}1/2 \\ -\Delta p & \text{prob.}1/2 \end{cases} \tag{M2}$$

- Trend-following term depends on the moving-average over the previous M ticks

$$\langle \Delta P \rangle_M = \frac{2}{M(M+1)} \sum_{k=0}^{M-1} (M-k)\Delta P(n-k) \tag{M3}$$

- Transaction takes place when the bid price of one dealer matches the ask price of the other.

$$|p_i(t) - p_j(t)| \geq L \tag{M4}$$

- Price return computed as

$$G_\tau(t) \equiv \log P(t+\tau) - \log P(t) \tag{M5}$$

In [3]:
```
def model2(params,p0,numt):
    def avgprice(dpn): # compute running average Eq.(L6)
        M = len(dpn)     #
        weights  = np.array(range(1,M+1))*2.0/(M*(M+1))
        return weights.dot(dpn)

    mktprice = np.zeros(numt)   # initialize market price P(n)
    dmktprice= np.zeros(numt)   # initialize change in price dP(n) needed f
    ticktime = np.zeros(numt,dtype=np.int) #initialize array for tick times
    price    = np.array([p0[0], p0[1]])    #initialize dealer's mid-price
    time,tick= 0,0 # real time(t) and time time (n)
    deltapm  = 0.0 # trend term d <dP>_m dt for current random walk
    cdp      = params['c']*params['dp'] # define random step size
    ddt      = params['d']*params['dt'] # define amplitude of trend term
    while tick < numt: # loop over ticks
        while np.abs(price[0]-price[1]) < params['L']: # transaction criter
            price = price + deltapm + np.random.choice([-cdp,cdp], size=2)
            time += 1 #update real time
        price[:]        = np.average(price) #set mid-prices to new market pr
        mktprice[tick] = price[0]   # save market price
        dmktprice[tick]= mktprice[tick] - mktprice[np.max([0,tick-1])] # sa
        ticktime[tick] = time # save transaction time
        tick += 1 #update ticks
        tick0 = np.max([0, tick - params['M']]) #compute tick start for run
        deltapm = avgprice(dmktprice[tick0:tick])*ddt #compute updated tren
    return ticktime,mktprice
```

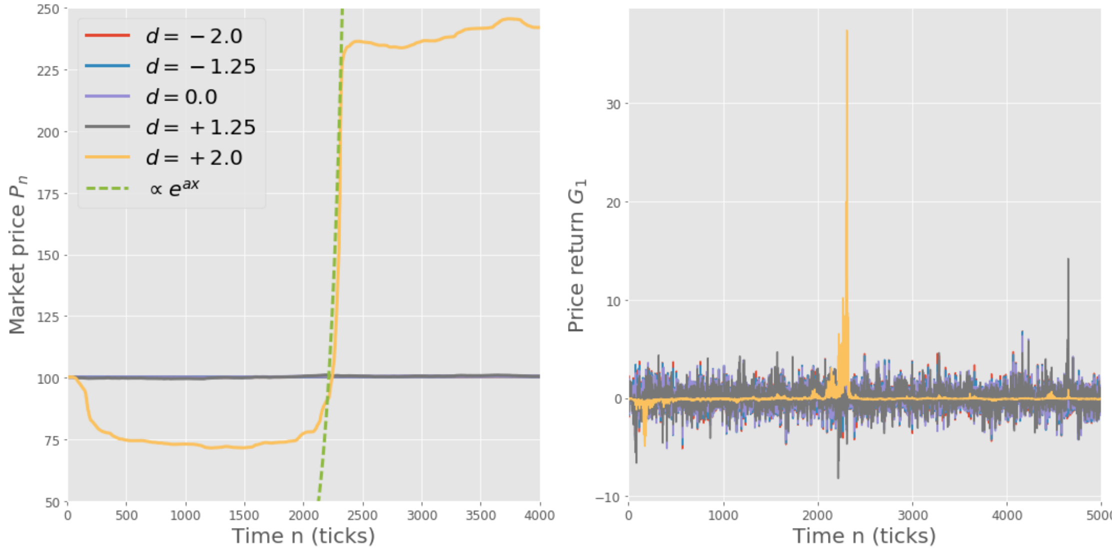
```
In [ ]: params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2, 'd':1.00, 'M':10} # defi
        price = np.zeros((5,5000))
        # run 5 simulations with different d paramaters
        for i,d,lbl in zip(range(5),[-2.0, -1.25, 0.0, 1.25, 2.0], ['d-2.0', 'd-1.2
            np.random.seed(0)
            params['d'] = d
            print(params['d'])
            time,price[i] = model2(params, [100.25, 100.25], 5000)
        np.savetxt('model2_M10_5d.txt', np.transpose(price), header="d-2\t d-1\t d0
```

```
In [4]: price  = pd.read_csv('model2_M10_5d.txt', header=0, delim_whitespace=True,
        dprice = pd.DataFrame()
        for lbl in price.columns:
            dprice[lbl] = normalized(logreturn(price[lbl].values, 1)) #price retur
        dprice.head()
```

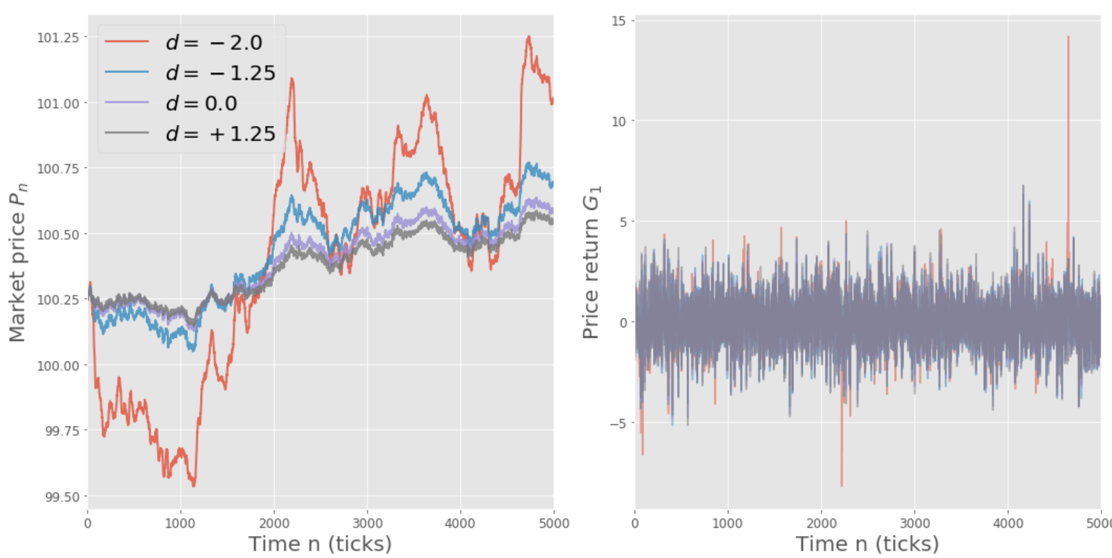
```
Out[4]:         d-2       d-1        d0       d+1       d+2
        0  0.064483  0.064803  0.062363  0.042186 -0.093576
        1 -0.249174 -0.245864 -0.235129 -0.193000 -0.100403
        2 -0.759109 -0.824618 -0.909471 -0.824092 -0.120481
        3  2.264443  1.994706  1.411017  0.612311 -0.087087
        4 -0.453285 -0.333167 -0.215293 -0.201743 -0.102947
```

2 Price dynamics of the dealer model

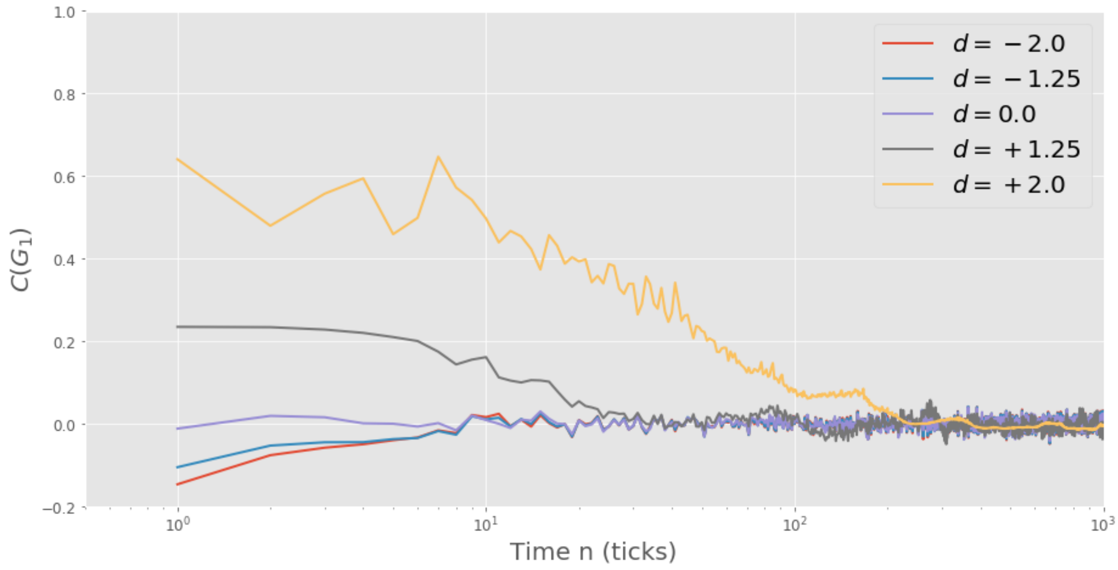
```
In [5]: fig,[ax,bx]=plt.subplots(figsize=(15,7.5),ncols=2,subplot_kw={'xlabel':r'Ti
        price.plot(ax=ax, lw=3)
        dprice.plot(ax=bx, legend=False)
        x = np.arange(1000,2550)
        ax.plot(x,0.03*np.exp(8e-3*(x-1200)),lw=3,ls='--',label='exponential')
        ax.set_ylim(50,250)
        ax.set_xlim(0,4000)
        ax.legend([r'$d=-2.0$', r'$d=-1.25$', r'$d=0.0$', r'$d=+1.25$', r'$d=+2.0$'
        ax.set_ylabel(r'Market price $P_n$')
        bx.set_ylabel(r'Price return $G_{1}$')
        fig.tight_layout() # get nice spacing between plots
        plt.show()
```




```
In [6]: fig,[ax,bx]=plt.subplots(figsize=(15,7.5),ncols=2,subplot_kw={'xlabel':r'Ti
        cols = ['d+1','d0','d-1','d-2']
        price[cols].plot(ax=ax, lw=2, alpha=0.8)
        dprice[cols].plot(ax=bx, alpha=0.5, legend=False)
        ax.legend([r'$d=-2.0$', r'$d=-1.25$', r'$d=0.0$', r'$d=+1.25$', r'+$d=2.0$'
        ax.set_ylabel(r'Market price $P_n$')
        bx.set_ylabel(r'Price return $G_{1}$')
        fig.tight_layout() # get nice spacing between plots
        plt.show()
```



```
In [7]: pricecor = pd.DataFrame()
        for lbl in dprice.columns:
            ct = auto_correlate(dprice[lbl].values)
            pricecor[lbl] = ct/ct[0]
        fig,ax=plt.subplots(figsize=(15,7.5),subplot_kw={'xlabel':r'Time n (ticks)'
        pricecor.plot(ax=ax, lw=2)
        ax.semilogx()
        ax.set_xlim(5e-1,1e3)
        ax.set_ylim(-0.2, 1.0)
        ax.legend([r'$d=-2.0$', r'$d=-1.25$', r'$d=0.0$', r'$d=+1.25$', r'$d=+2.0$'
        plt.show()
```



3 Dynamics of real data

```
In [8]: def computeReturn(data, pname, dname, tau):
            data[dname]=pd.Series(normalized(logreturn(data[pname].values, tau)),in

        # https://www.quantshare.com/sa-426-6-ways-to-download-free-intraday-and-ti
        dateparse = lambda x: pd.datetime.strptime(x, '%Y%m%d %H:%M:%S')
        appletick = pd.read_csv('US2.AAPL_170301_170301_tick.csv', parse_dates={'da
        applemin  = pd.read_csv('US2.AAPL_170301_170301_min.csv',  parse_dates={'da
        computeReturn(appletick, 'Last', 'Return d1', 1)
        computeReturn(applemin, 'Close', 'Return d1', 1)

In [9]: appletick.head()

Out[9]:              datetime    Ticker  Per    Last  Vol  Return d1
        0 2017-03-01 09:30:00  US2.AAPL    0  137.89  100  -1.503504
```

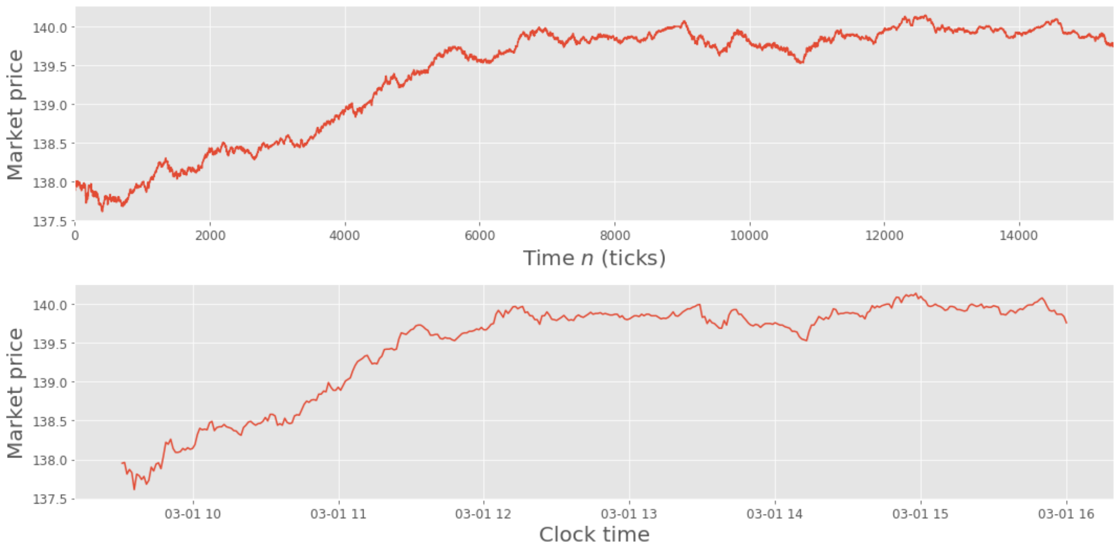
```
1 2017-03-01 09:30:00  US2.AAPL    0  137.88  100   1.467665
2 2017-03-01 09:30:00  US2.AAPL    0  137.89  100   7.408387
3 2017-03-01 09:30:00  US2.AAPL    0  137.94  100  -0.017920
4 2017-03-01 09:30:01  US2.AAPL    0  137.94  100  -0.017920
```

```
In [10]: applemin.head()
```

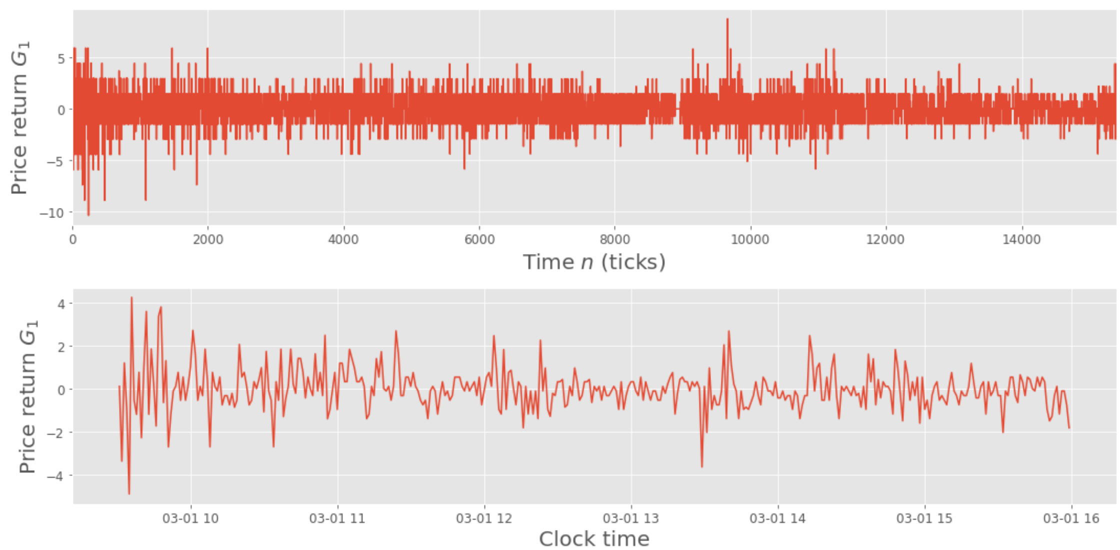
```
Out[10]:               datetime    Ticker  Per    Open    High     Low   Close    Vo
         0 2017-03-01 09:31:00  US2.AAPL    1  137.89  138.00  137.88  137.95  1237
         1 2017-03-01 09:32:00  US2.AAPL    1  137.95  137.98  137.88  137.96   873
         2 2017-03-01 09:33:00  US2.AAPL    1  137.96  137.96  137.72  137.81   500
         3 2017-03-01 09:34:00  US2.AAPL    1  137.84  137.97  137.83  137.87   713
         4 2017-03-01 09:35:00  US2.AAPL    1  137.88  137.88  137.79  137.83   958

            Return d1
         0   0.116571
         1  -3.353714
         2   1.201604
         3  -0.968038
         4  -4.877729
```

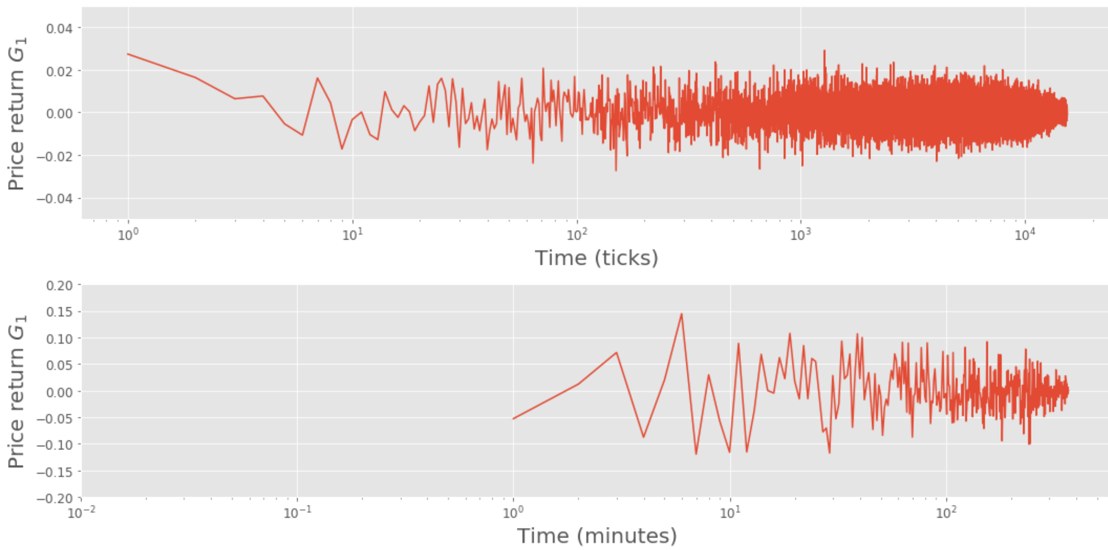
```
In [11]: fig,[ax,bx]=plt.subplots(figsize=(15,7.5), nrows=2, subplot_kw={'ylabel':r
         ax.plot(appletick.index, appletick['Last'])
         bx.plot(applemin['datetime'], applemin['Close'])
         ax.set_xlim(0,15398)
         ax.set_xlabel('Time $n$ (ticks)')
         bx.set_xlabel('Clock time')
         plt.tight_layout()
         plt.show()
```



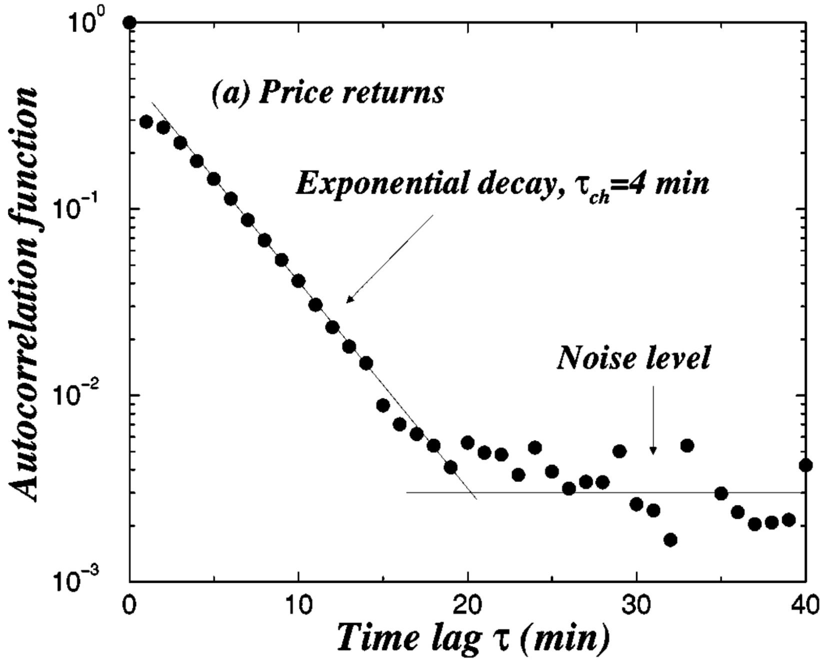
```
In [12]: fig,[ax,bx]=plt.subplots(figsize=(15,7.5), nrows=2)
         ax.plot(appletick.index, appletick['Return d1'])
         bx.plot(applemin['datetime'], applemin['Return d1'])
         ax.set_xlim(0,15398)
         ax.set_ylabel(r'Price return $G_{1}$')
         bx.set_ylabel(r'Price return $G_{1}$')
         ax.set_xlabel('Time $n$ (ticks)')
         bx.set_xlabel('Clock time')
         plt.tight_layout()
         plt.show()
```




```
In [13]: def computeCt(data, name):
             ct = auto_correlate(data[name].values[:-20]) # ignore NaN last point
             data['Ct'] = pd.Series(ct, index=data.index[:-20])
         computeCt(appletick, 'Return d1')
         computeCt(applemin, 'Return d1')
         fig,[ax,bx]=plt.subplots(figsize=(15,7.5), nrows=2, subplot_kw={'ylabel':r
         ax.plot(appletick['Ct'])
         bx.plot(applemin['Ct'])
         ax.set_xlabel('Time (ticks)')
         bx.set_xlabel('Time (minutes)')
         ax.semilogx()
         bx.semilogx()
         ax.set_ylim(-0.05, 0.05)
         bx.set_ylim(-0.2, 0.2)
         bx.set_xlim(1e-2,6e2)
         plt.tight_layout()
         plt.show()
```



3.0.1 Time-correlation of the S&P 500 returns



- P. Gopikrishnan, V. Plerou, L. Amaral, M. Meyer and H. Stanley Physical Revew E 60, 5305 (1999). - Time autocorrelation of $G_{\Delta t}(\tau)$ with $\Delta t = 1$min

4 Conclusions

- We have shown how a simple-model stochastic model, built borrowing concepts from statistical physics, can reproduce many behaviors seen in real-world stock markets.

- While we considered the simplest possible version, with only two dealers and constant and equal trend-following characteristics, you can easily remove these restrictions. You can try to simulate for hundreds of dealers, with non-constant d values. The main results still hold, it just becomes more complicated to analyze as the number of parameters increases.
- For the dealer model we presented, one can recover the non-trivial power law decay of the price returns, but only for a specific set of parameter values. If the parameters of the model are changed, then the nature of the distribution can also change.
- While this type of modeling can help you understand complex real-world systems, you should be very careful when trying to make precise quantitative predictions based on them.