KyotoUx-009x (/github/ryo0921/KyotoUx-009x/tree/master)
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Stochastic Processes: Data Analysis and Computer Simulation

Distribution function and random number

2. Generating random numbers with Gaussian/binomial/Poisson distributions

2.1. Preparations

Python built-in functions for random numbers (numpy.random)

1. `seed(seed)`: Initialize the generator with an integer "seed".
2. `rand(d0,d1,...,dn)`: Return a multi-dimensional array of uniform random numbers of shape (d0, d1, ... , dn).
3. `randn(d0,d1,...,dn)`: The same as above but from the standard normal distribution.
4. `binomial(M,p,size)`: Draw samples from a binomial distribution with "M" and "p".
5. `poisson(a,size)`: Draw samples from a Poisson distribution with "a".
6. `choice([-1,1],size)`: Generates random samples from the two choices, -1 or 1 in this case.
7. `normal(ave,std,size)`: Draw random samples from a normal distribution.
8. `uniform([low,high,size])`: Draw samples from a uniform distribution.

- See the Scipy website for details https://docs.scipy.org/doc/numpy-dev/reference/routines.random.html (https://docs.scipy.org/doc/numpy-dev/reference/routines.random.html)

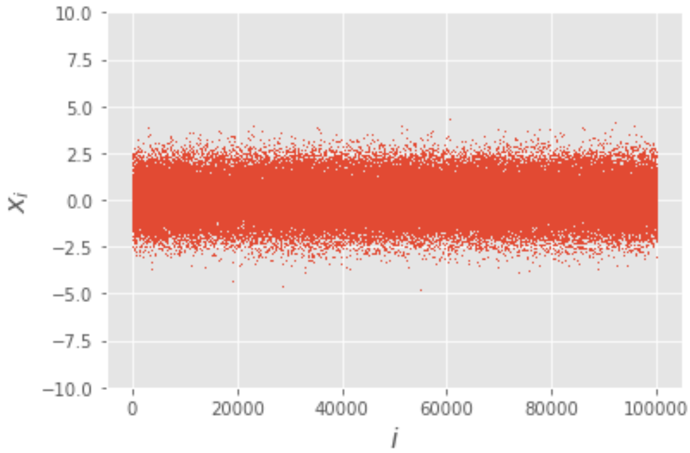
Import common libraries

In [2]:
```
% matplotlib inline
import numpy as np # import numpy library as np
import math # use mathematical functions defined by the C standard
import matplotlib.pyplot as plt # import pyplot library as plt
plt.style.use('ggplot') # use "ggplot" style for graphs
```

2.2. Normal / Gaussian distribution

Generate random numbers, $x_0, x_1, \cdots, x_N$

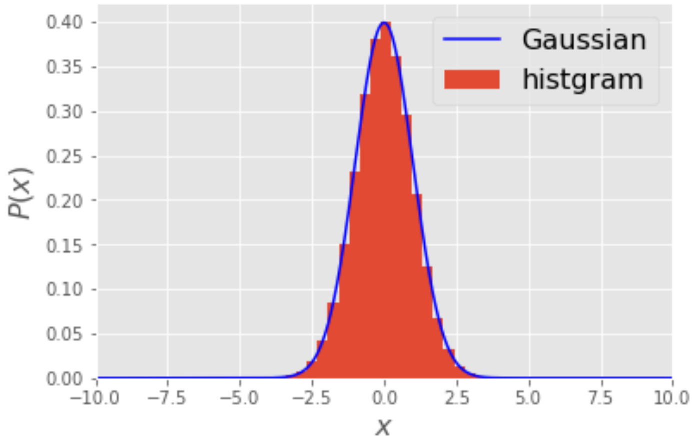
In [3]:
```
ave = 0.0 # set average
std = 1.0 # set standard deviation
N = 100000 # number of generated random numbers
np.random.seed(0) # initialize the random number generator with seed=0
X = ave+std*np.random.randn(N) # generate random sequence and store it a
plt.ylim(-10,10) # set y-range
plt.xlabel(r'$i$',fontsize=16) # set x-label
plt.ylabel(r'$x_i$',fontsize=16) # set y-label
plt.plot(X,',') # plot x_i vs. i (i=1,2,...,N) with dots
plt.show() # draw plots
```



Compare the distribution with the normal distribution function

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x - \langle X \rangle)^2}{2\sigma^2}\right] \tag{D1}$$

In [4]:
```
plt.hist(X,bins=25,normed=True) # plot normalized histgram of R using 25
x = np.arange(-10,10,0.01) # create array of x from 0 to 1 with incremen
y = np.exp(-(x-ave)**2/(2*std**2))/np.sqrt(2*np.pi*std**2) # create arra
plt.xlim(-10,10) # set x-range
plt.plot(x,y,color='b') # plot y vs. x with blue line
plt.xlabel(r'$x$',fontsize=16) # set x-label
plt.ylabel(r'$P(x)$',fontsize=16) # set y-label
plt.legend([r'Gaussian',r'histgram'], fontsize=16) # set legends
plt.show() # display plots
```



Calculate the auto-correlation function $\varphi(i)$

The definition

$$\varphi(i) = \frac{1}{N}\sum_{j=1}^{N}\left(x_j - \langle X\rangle\right)\left(x_{i+j} - \langle X\rangle\right) \tag{D2}$$

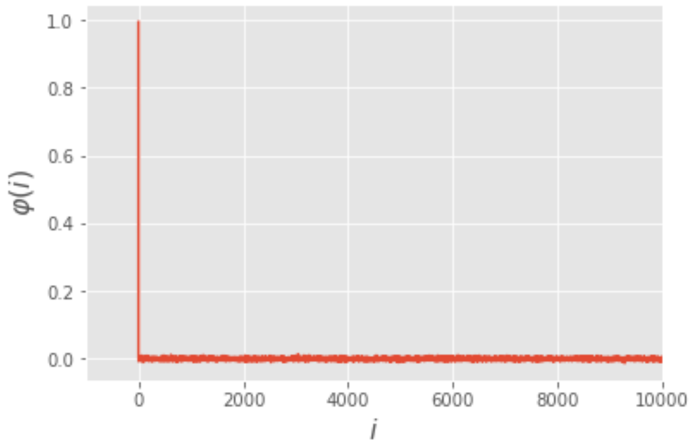
$$\varphi(i=0) = \frac{1}{N}\sum_{j=1}^{N}\left(x_j - \langle X\rangle\right)^2 = \langle x_j - \langle X\rangle\rangle^2 = \sigma^2 \tag{D3}$$

$$\varphi(i \neq 0) = \langle x_j - \langle X\rangle\rangle\langle x_{i\neq j} - \langle X\rangle\rangle = 0 \quad (\to \text{White noise}) \tag{D4}$$

A code example to calculate auto-correlation

In [5]:
```
def auto_correlate(x):
    cor = np.correlate(x,x,mode="full")
    return cor[N-1:]
c = np.zeros(N)
c = auto_correlate(X-ave)/N
plt.plot(c)
plt.xlim(-1000,10000)
plt.xlabel(r'$i$',fontsize=16)
plt.ylabel(r'$\varphi(i)$',fontsize=16)
print('\sigma^2  =',std**2)
plt.show()
```

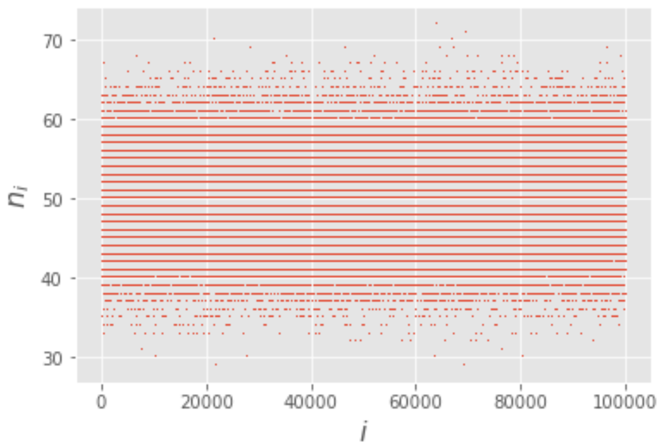
```
\sigma^2  = 1.0
```



2.3. Binomial distribution

Generate random numbers, $n_0, n_1, \cdots, n_N$

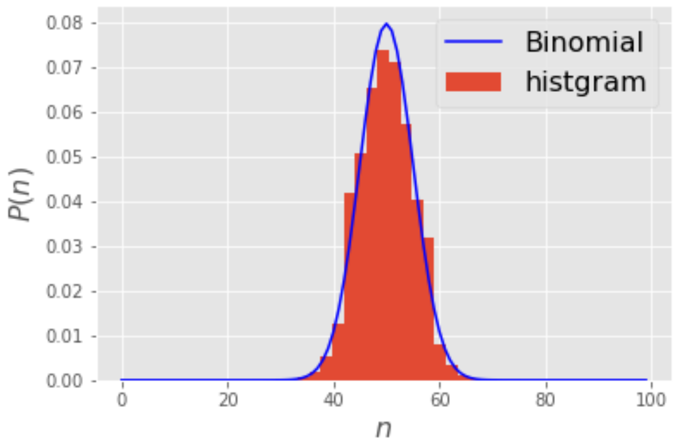
In [6]:
```
p = 0.5 # set p, propability to obtain "head" from a coin toss
M = 100 # set M, number of tosses in one experiment
N = 100000 # number of experiments
np.random.seed(0) # initialize the random number generator with seed=0
X = np.random.binomial(M,p,N) # generate the number of heads after M tos
plt.xlabel(r'$i$',fontsize=16) # set x-label
plt.ylabel(r'$n_i$',fontsize=16) # set y-label
plt.plot(X,',') # plot n_i vs. i (i=1,2,...,N) with dots
plt.show() # draw plots
```



Compare the distribution with the Binomial distribution function

$$P(n) = \frac{M!}{n!(M-n)!} p^n (1-p)^{M-n} \tag{D5}$$
$$\langle n \rangle = Mp \tag{D6}$$
$$\sigma^2 = Mp(1-p) \tag{D7}$$

In [7]:
```
def binomial(n,m,p):
    comb=math.factorial(m)/(math.factorial(n)*math.factorial(m-n))
    prob=comb*p**n*(1-p)**(m-n)
    return prob
plt.hist(X,bins=20,normed=True) # plot normalized histgram of R using 22
x = np.arange(M) # generate array of x values from 0 to 100, in interval
y = np.zeros(M) # generate array of y values, initialized to 0
for i in range(M):
    y[i]=binomial(i,M,p) # compute binomial distribution P(n), Eq. (D5)
plt.plot(x,y,color='b') # plot y vs. x with blue line
plt.xlabel(r'$n$',fontsize=16) # set x-label
plt.ylabel(r'$P(n)$',fontsize=16) # set y-label
plt.legend([r'Binomial',r'histgram'], fontsize=16) # set legends
plt.show() # display plots
```



Calculate the auto-correlation function $\varphi(i)$

The definition

$$\varphi(i) = \frac{1}{N}\sum_{j=1}^{N} \left(n_j - \langle n \rangle\right)\left(n_{i+j} - \langle n \rangle\right) \tag{D8}$$

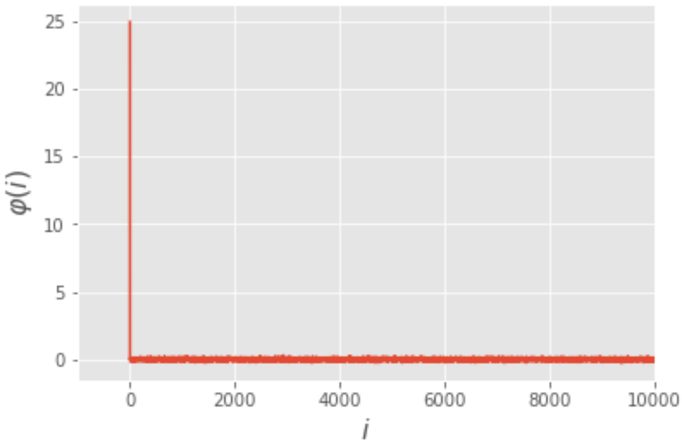
$$\varphi(i=0) = \frac{1}{N}\sum_{j=1}^{N} \left(n_j - \langle n \rangle\right)^2 = \langle n_j - \langle n \rangle \rangle^2 = \sigma^2 = Mp(1-p) \tag{D9}$$

$$\varphi(i \neq 0) = \langle n_j - \langle n \rangle \rangle \langle n_{i \neq j} - \langle n \rangle \rangle = 0 \qquad (\to \text{White noise}) \tag{D10}$$

A code example to calculate auto-correlation

In [7]:
```
def auto_correlate(x):
    cor = np.correlate(x,x,mode="full")
    return cor[N-1:]
c = np.zeros(N)
c = auto_correlate(X-M*p)/N
plt.plot(c)
plt.xlim(-1000,10000)
plt.xlabel(r'$i$',fontsize=16)
plt.ylabel(r'$\varphi(i)$',fontsize=16)
print('\sigma^2  =',M*p*(1-p))
plt.show()
```

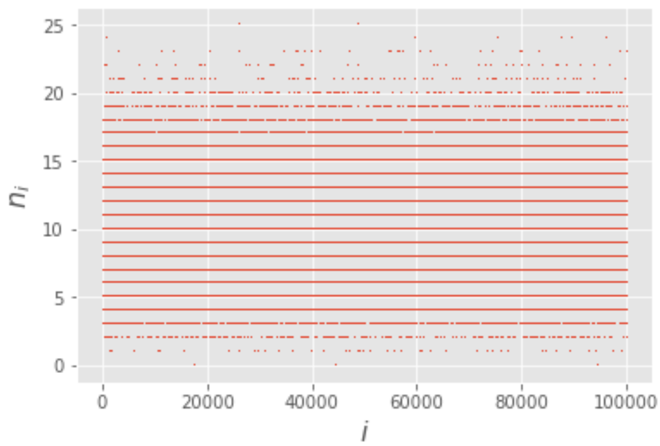
```
\sigma^2  = 25.0
```



2.4. Poisson distribution

Generate random numbers, $n_0, n_1, \cdots, n_N$

In [8]:
```
a = 10.0 # set a, the expected value
N = 100000 #  number of generated random numbers
np.random.seed(0) # initialize the random number generator with seed=0
X = np.random.poisson(a,N) # generate randon numbers from poisson distri
plt.xlabel(r'$i$',fontsize=16) # set x-label
plt.ylabel(r'$n_i$',fontsize=16) # set y-label
plt.plot(X,',') # plot n_i vs. i (i=1,2,...,N) with dots
plt.show() # draw plots
```



Compare the distribution with the binomial distribution function

$$P(n) = \frac{a^n e^{-a}}{n!} \tag{D11}$$
$$\langle n \rangle = a \tag{D12}$$
$$\sigma^2 = a \tag{D13}$$

In [9]:
```
def poisson(n,a):
    prob=a**n*np.exp(-a)/math.factorial(n)
    return prob
plt.hist(X,bins=25,normed=True) # plot normalized histgram of X using 25
x = np.arange(M) # generate array of x values from 0 to 100, in interval
y = np.zeros(M) # generate array of y values, initialized to zero
for i in range(M):
    y[i]=poisson(i,a) # Compute Poisson distribution for n, Eq. (D11)
plt.plot(x,y,color='b') # plot y vs. x with blue line
plt.xlabel(r'$n$',fontsize=16) # set x-label
plt.ylabel(r'$P(n)$',fontsize=16) # set y-label
plt.legend([r'Poisson',r'histgram'], fontsize=16) # set legends
plt.show() # display plots
```