009x_62

March 18, 2017

Stochastic Processes: Data Analysis and Computer Simulation

Stochastic processes in the real world
-A Stochastic Dealer Model I-

```
In [1]: % matplotlib inline
        import numpy as np  # import numpy library as np
        import math         # use mathematical functions defined by the C standard
        import matplotlib.pyplot as plt # import pyplot library as plt
        plt.style.use('ggplot') # use "ggplot" style for graphs
        pltparams = {'legend.fontsize':16,'axes.labelsize':20,'axes.titlesize':20,
                     'xtick.labelsize':12,'ytick.labelsize':12,'figure.figsize':(7.
        plt.rcParams.update(pltparams)
```

```
In [2]: # Logarithmic return of price time series
        def logreturn(St,tau=1):
            return np.log(St[tau:])-np.log(St[0:-tau]) # Eq.(J2) : G_tau(t) = log(S
        # normalize data to have unit variance (<(x - <x>)^2> = 1)
        def normalized(data):
            return ((data)/np.sqrt(np.var(data)))
        # compute normalized probability distribution function
        def pdf(data,bins=50):
            hist,edges=np.histogram(data[~np.isnan(data)],bins=bins,density=True) #
            edges    = (edges[:-1] + edges[1:])/2.0 # get bar center
            nonzero = hist > 0.0                    # non-zero points
            return edges[nonzero], hist[nonzero]
```

0.1 The Dealer Model

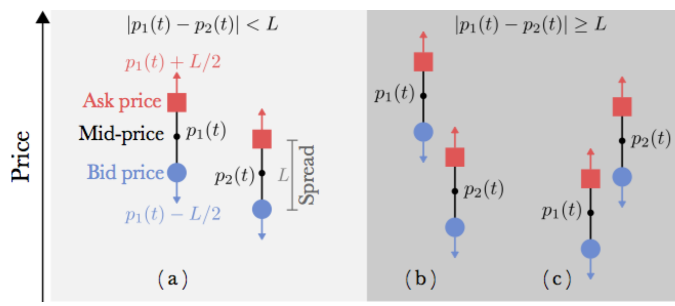
- K. Yamada, H. Takayasu, T. Ito and M. Takayasu, Physical Revew E 79, 051120 (2009).
- Simple Stochastic model with only two dealers offering buying and selling options.
- The mid-price $p_i(t)$ of dealer i at time t is the average of his bid and ask price.

$$|p_i(t) - p_j(t)| \geq L : \text{Transaction Criterion} \tag{K1}$$

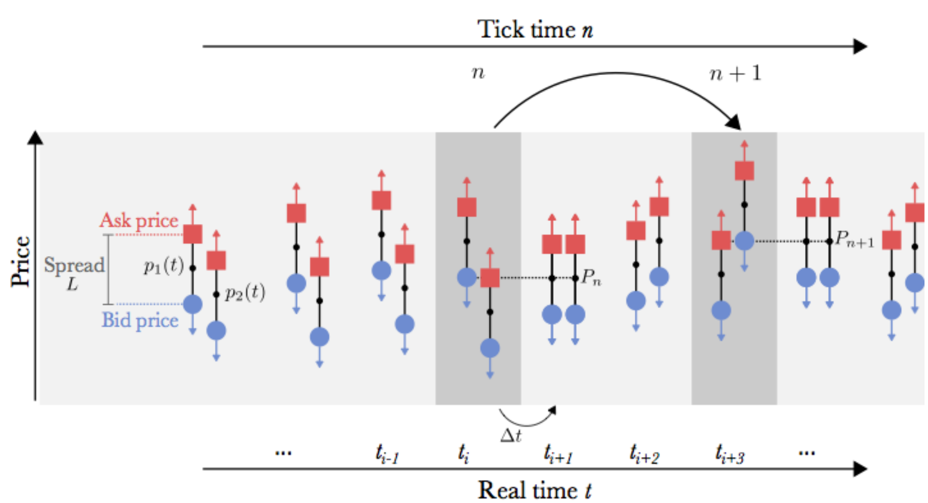
0.2 Dynamics of the dealer model

- Prices carry out a 1D random walk in 'price' space until a transaction takes place.

$$p_i(t + \Delta t) = p_i(t) + cf_i(t), \qquad i = 1, 2 \tag{K2}$$



Price
$|p_1(t) - p_2(t)| < L$
$p_1(t) + L/2$
Ask price
Mid-price
$p_1(t)$
Bid price
$p_1(t) - L/2$
$p_2(t)$
L
Spread
(a)
$|p_1(t) - p_2(t)| \geq L$
$p_1(t)$
$p_2(t)$
(b)
$p_1(t)$
$p_2(t)$
(c)



Tick time n
n
$n+1$
Price
Ask price
Spread
L
Bid price
$p_1(t)$
$p_2(t)$
P_n
P_{n+1}
Δt
...
t_{i-1}
t_i
t_{i+1}
t_{i+2}
t_{i+3}
...
Real time t

$$f_i(t) = \begin{cases} +\Delta p & \text{probability} 1/2 \\ -\Delta p & \text{probability} 1/2 \end{cases}$$

- The "Market" price P of a transaction is given by the average of the mid-prices

$$P = (p_1 + p_2)/2 \tag{K3}$$

0.3 Dealer model as a 2D random walk

- The dealer model can be understood as a standard 2D Random walk with absorbing boundaries.
- Perform a change in variables, from $p_1(t)$ and $p_2(t)$, to the price difference $D(t)$ and average $A(t)$

$$D(t) = p_1(t) - p_2(t) \tag{K4}$$

$$A(t) = \frac{1}{2}\big(p_1(t) + p_2(t)\big) \tag{K5}$$

- Dynamics of D and A describe a 2D random walk

$$D(t+\Delta t) = D(t) + \begin{cases} +2c\Delta p & \text{probability } 1/4 \\ 0 & \text{probability } 1/2 \\ -2c\Delta p & \text{probability } 1/4 \end{cases} \tag{K6}$$

$$A(t+\Delta t) = A(t) + \begin{cases} +c\Delta p & \text{probability } 1/4 \\ 0 & \text{probability } 1/2 \\ -c\Delta p & \text{probability } 1/4 \end{cases} \tag{K7}$$

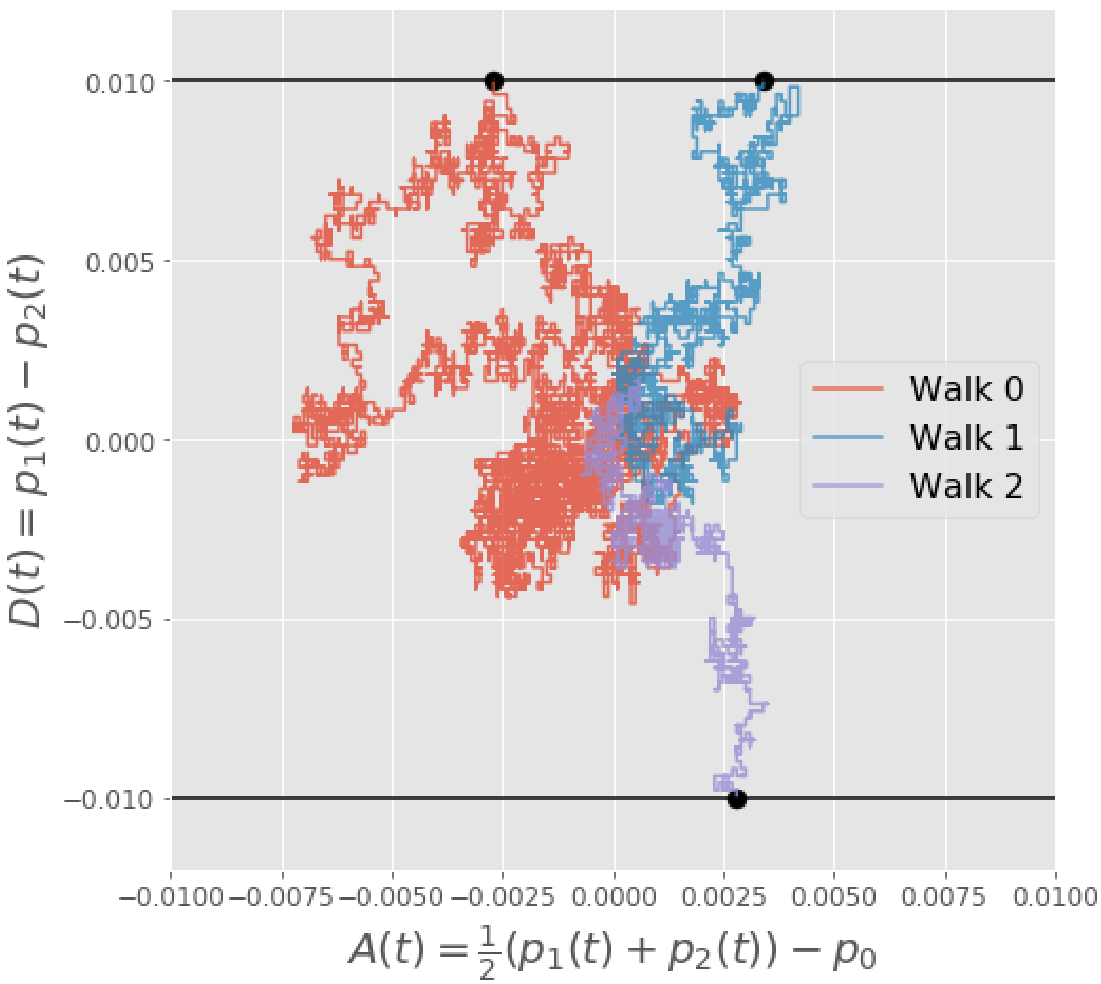
- When $D(t) = \pm L$ a transaction occurs and the random walk ends, the "particle" is absorbed by the boundary.

```
In [3]: params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2} # define model parameters
        def model1RW(params,p0):                       # simulate Random-Walk for 1 transa
            price = np.array([p0[0], p0[1]])     # initialize mid-prices for dealers
            cdp   = params['c']*params['dp']     # define random step size
            Dt    = [price[0]-price[1]]          # initialize price difference as em
            At    = [np.average(price)]          # initialize avg price as empy list
            while np.abs(price[0]-price[1]) < params['L']:
                price=price+np.random.choice([-cdp,cdp],size=2) # random walk step
                Dt.append(price[0]-price[1])
                At.append(np.average(price))
            return np.array(Dt),np.array(At)-At[0] # return difference array and av

In [4]: np.random.seed(123456)
        fig,ax=plt.subplots(figsize=(7.5,7.5),subplot_kw={'xlabel':r'$A(t) = \frac{
        p0 = [100.25, 100.25]
        for i in range(3):
            Dt,At = model1RW(params, p0)
            ax.plot(At,Dt,alpha=0.8,label='Walk '+str(i)) #plot random walk traject
            ax.scatter(At[-1],Dt[-1],marker='o',s=80,color='k') #last point
```

```
        print('Walk ', i,' : number of steps = ',len(At),', price change = ', A
    ax.plot([-0.01,0.03],[params['L'],params['L']],color='k') #top absorbing bo
    ax.plot([-0.01,0.03],[-params['L'],-params['L']],color='k') #bottom absorbi
    ax.set_ylim([-0.012, 0.012])
    ax.set_xlim([-0.01, 0.01])
    ax.legend(loc=5,framealpha=0.8)
    plt.show()
```

```
Walk  0  : number of steps =  9248 , price change =  -0.00270000000009
Walk  1  : number of steps =  2201 , price change =  0.00340000000011
Walk  2  : number of steps =  1629 , price change =  0.00280000000009
```

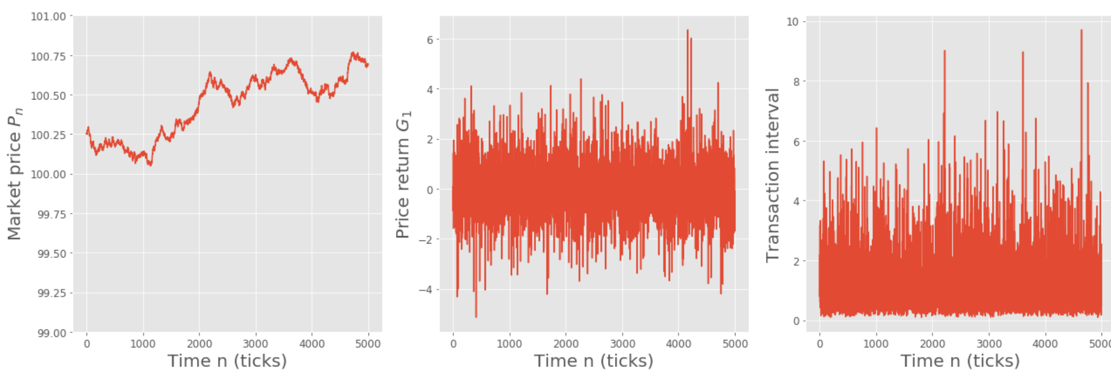



```
In [5]: params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2} # define model parameters
        def model1(params,p0,numt):                       # simulate dealer model for numt
            mktprice = np.zeros(numt)                     # initialize array for market pr
            ticktime = np.zeros(numt,dtype=np.int) # initialize array for tick time
```

```
        price     = np.array([p0[0], p0[1]])     # initailize dealer's mid-price
        time,tick = 0,0                          # real time (t) and tick time (n
        cdp       = params['c']*params['dp']     # define random step size
        while tick < numt:                       # loop over ticks
            while np.abs(price[0]-price[1])< params['L']: # perform one RW for
                price=price+np.random.choice([-cdp,cdp],size=2) # random walk s
                time += 1 # update time t
            price[:] = np.average(price)         # set mid-prices to new market p
            mktprice[tick] = price[0]            # save market price
            ticktime[tick] = time                # save tick time
            tick += 1                            # updat ticks
        return ticktime,mktprice
```

In [ ]:
```
np.random.seed(0)
ticktime,mktprice=model1(params,[100.25,100.25],5000)
np.savetxt('model1.txt',np.transpose([ticktime,mktprice]))
```

In [6]:
```
ticktime,mktprice=np.loadtxt('model1.txt',unpack=True) # read saved data fr
timeinterval=normalized((ticktime[1:]-ticktime[0:-1])*params['dt']) # compu
dprice=normalized(logreturn(mktprice,1)) # compute logarithmic return of th
fig,[ax,bx,cx]=plt.subplots(figsize=(18,6),ncols=3,subplot_kw={'xlabel':r'T
ax.plot(mktprice)
ax.set_ylim(99,101)
ax.set_ylabel(r'Market price $P_n$')
bx.plot(dprice)
bx.set_ylabel(r'Price return $G_1$')
cx.plot(timeinterval)
cx.set_ylabel(r'Transaction interval')
fig.tight_layout() # get nice spacing between plots
plt.show()
```



In [7]:
```
fig,[ax,bx]=plt.subplots(figsize=(15,7.5),ncols=2,subplot_kw={'ylabel':r'Pr
edges,hist=pdf(np.abs(dprice),bins=25) # probability density of price chang
ax.plot(edges, hist, lw=3, label='Dealer Model')
```

```
x = np.linspace(0, 5)
ax.plot(x,2*np.exp(-x**2/2)/np.sqrt(2*np.pi),lw=6,ls='--',color='gray',alph
ax.plot(x, 2*np.exp(-1.5*x),lw=6,color='k',ls='--',alpha=0.8,label=r'Expone
ax.set_xlabel(r'Absolute price return $G_1$')
ax.set_ylabel(r'Probability density')
ax.set_ylim([5e-4,1])
ax.semilogy()
ax.legend()
edges,hist=pdf(timeinterval,bins=100) # probability density of transaction
bx.plot(edges,hist, lw=2)
bx.set_xlabel(r'Transaction interval')
bx.set_ylabel(r'Probability distribution')
bx.semilogy()
plt.show()
```