

STAT 221 - 11S1 (C)

Semester One 2011

15 points

Due: May. 05, 2011 at 10:00AM in Forestry 252

**Monte Carlo Methods
Assessment 2a Cover Sheet**

Student ID # :

Surname or Family Name :

First Name or Names :

Course Coordinator: Raazesh Sainudiin

STATEMENT REGARDING DISHONEST PRACTICE

(relating to work submitted for assessment)

The University has a clear interpretation of what constitutes dishonest practice as described in you Calendar. Dishonest practice includes the following:

1. **Plagiarism**, being the presentation of any material (text, data or figures, on any medium including computer files) from any other source without clear and proper acknowledgement of the source of that material.
2. **Collusion**, being work performed in whole or in part in conjunction with another person or persons, but submitted as if it had been completed by the named author alone (or joint authors if a group item of work).
3. **Copying**, being the use of material (in any medium, including computer files) produced by another person or persons, with or without their knowledge and approval.
4. **Ghost writing**, being the use of another party (with or without any form of payment) to prepare all or part of an item of work submitted for assessment.

Under the University regulations, evidence of any of these or other forms of dishonest practice by any student(s) represents grounds for disciplinary action and may result in penalties ranging from denial of credit for the item of work in question to exclusion from the University.

- This interpretation of the dishonest practice of collusion is not intended to discourage students from having discussions with each other about how to approach a particular assigned task, and incorporating general ideas coming out of such discussions into their own individual submissions.

DECLARATION:

In signing below, I confirm that I have read and fully understand the statement regarding dishonest practice, as detailed in the University Calendar and briefly outlined above, and hereby certify that this work submitted for assessment is entirely my own work.

Signed :

Date :

ENQUIRIES

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See Course Syllabus for other details.

STAT 221 Assessment 2a

Due: Hand in your solutions on May. 05, 2011 at 10:00AM in Forestry 252 (or TBA)

There are two parts to Assessment 2. This is part a. Each problem is worth 3% of your course grade. There are five problems in Assessment 2a. The required computations and drawings have to be done by hand with pencil/pen and paper. You do not need a computer to do these problems. Please use white paper and pencil/pen for the solutions. Please write legibly and show all workings for full credit.

1. Let $A = \{a, b, c, d, e\}$ be the set of offspring or children of the set of parents $B = \{\alpha, \beta, \gamma, \delta\}$. The offspring-parent relation can be summarised by the following set of ordered pairs:

$$\{(a, \alpha), (b, \alpha), (c, \beta), (e, \gamma), (d, \delta)\} .$$

- (a) Draw a picture of the function or map $f : A \rightarrow B$ depicting the above offspring-parent relations (you need to make a picture with two sets A and B and 5 arrows between them).
- (b) What is $(\{f(a)\} \cap \{f(b)\}) \cup \{f(e)\}$?
- (c) What is $f^{[-1]}(\alpha) \cup f^{[-1]}(\gamma)$?
- (d) Will the following set of ordered pairs:

$$\{(a, \alpha), (b, \alpha), (c, \beta), (e, \gamma), (a, \delta), (d, \delta)\}$$

constitute a function from A to B ? Explain your answer.

- (e) Will the following set of ordered pairs:

$$\{(a, \alpha), (b, \beta), (c, \gamma), (d, \delta)\}$$

constitute a function from A to B ? Explain your answer.

2. Suppose we model the NZ Lotto as follows:

$$X_1, X_2, \dots, X_n \stackrel{IID}{\sim} \text{de Moivre}(1/40, 1/40, \dots, 1/40)$$

Show that the probability of any subset A of the sample space $\{1, 2, \dots, 40\}$ is $|A|/40$. This implies that $P(\{2, 4, 6, \dots, 38, 40\}) = 20/40 = 1/2$, for instance. You have to explain each step in your general derivation that $P(A) = |A|/40$ using the basic laws of probability. Recall that $|A|$ is the number of elements in A .

3. Suppose we roll a fair dice twice in an independent and identical manner. Let A be the event that the sum is 5 and B be the event that the first die is 2. What is $P(B|A)$, the conditional probability that the first die is 2 given that the sum is 5? Show each step in your computation.
4. Suppose you observe the following five data points from some product experiment:

$$0, 2, 1, 0, 3.$$

- (a) compute the sample mean.
- (b) compute the sample variance.

- (c) compute the order statistics.
 - (d) draw a graph of the empirical mass function.
 - (e) draw a graph of the empirical distribution function.
5. Consider the following random variable X that is a discrete mixture of two $\text{Uniform}(\theta_1, \theta_2)$ RVs. The PDF of X can be expressed as follows:

$$f(x) = \frac{1}{2}3 \mathbf{1}_{[0,1/3]}(x) + \frac{1}{2}3 \mathbf{1}_{[2/3,1]}(x)$$

Show each step in your computation of the following quantities.

- (a) Draw the graph of the PDF $f(x)$.
- (b) Find the CDF $F(x) := \int_{-\infty}^x f(y)dy$ of X .
- (c) Draw the graph of the CDF $F(x)$.
- (d) Find the Expectation $E(X) := \int_{-\infty}^{\infty} xf(x)dx$.
- (e) What is $P(X \in [2/3, 3/4])$?