STAT 221 - 11S1 (C)	Semester One 2011
15 points	Due: June 10, 2011 at 10:00AM in Tutorial

Monte Carlo Methods
Assessment 3a Cover Sheet

Student ID # :
Surname or Family Name :
First Name or Names :

Course Coordinator: Raazesh Sainudiin

STATEMENT REGARDING DISHONEST PRACTICE

(relating to work submitted for assessment)

The University has a clear interpretation of what constitutes dishonest practice as described in you Calendar. Dishonest practice includes the following:

1. **Plagiarism**, being the presentation of any material (text, data or figures, on any medium including computer files) from any other source without clear and proper acknowledgement of the source of that material.
2. **Collusion**, being work performed in whole or in part in conjunction with another person or persons, but submitted as if it had been completed by the named author alone (or joint authors if a group item of work).
3. **Copying**, being the use of material (in any medium, including computer files) produced by another person or persons, with or without their knowledge and approval.
4. **Ghost writing**, being the use of another party (with or without any form of payment) to prepare all or part of an item of work submitted for assessment.

Under the University regulations, evidence of any of these or other forms of dishonest practice by any student(s) represents grounds for disciplinary action and may result in penalties ranging from denial of credit for the item of work in question to exclusion from the University.

- This interpretation of the dishonest practice of collusion is not intended to discourage students from having discussions with each other about how to approach a particular assigned task, and incorporating general ideas coming out of such discussions into their own individual submissions.

DECLARATION:

In signing below, I confirm that I have read and fully understand the statement regarding dishonest practice, as detailed in the University Calendar and briefly outlined above, and hereby certify that this work submitted for assessment is entirely my own work.

Signed : .....

Date : .....

ENQUIRIES

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See Course Syllabus for other details.

STAT 221 Assessment 3a

Due: Hand in your solutions on June 10, 2011 at 10:00AM in Tutorial

Each problem is worth 5% of your course grade. There are three problems in Assessment 3a. The required computations and drawings may be done by hand with pencil/pen and paper. You do not need a computer to do these problems. However, having a computer can be helpful. Please use white paper and pencil/pen for the solutions. Please write legibly and show all workings for full credit.

1. The distribution of ocean wave heights, H, may be modeled with the Rayleigh(α) RV with parameter α and probability density function,

$$f(h;\alpha) = \frac{h}{\alpha^2}\exp\left(-\frac{1}{2}(h/\alpha)^2\right), \qquad h \in [0,\infty) \ .$$

   The parameter space for α is $(0,\infty)$. Suppose that the following measurements $h_1, h_2, \ldots, h_{10}$ of wave heights in meters were observed to be

$$1.50,\ 2.80,\ 2.50,\ 3.20,\ 1.90,\ 4.10,\ 3.60,\ 2.60,\ 2.90,\ 2.30 \ ,$$

   respectively. Under the assumption that the 10 samples are IID realisations from a Rayleigh(α^*) RV with a fixed and unknown parameter α^*, find the ML estimate $\widehat{\alpha}_{10}$ of α^*. You need to set up the log-likelihood function, find its derivative, and solve it to find the MLE. Then, you need to plug in the data to obtain the maximum likelihood estimate.

2. The Distribution function for the Rayleigh(α) RV with parameter α is:

$$F(h;\alpha) = 1 - \exp\left(-\frac{1}{2}(h/\alpha)^2\right), \qquad h \in [0,\infty) \ .$$

   Derive in detail an algorithm that can transform samples from a PRNG to those from a Rayleigh(α) RV with a given parameter α.

3. Suppose a frog jumps between three lily pads labelled by the integers in $\{0,1,2\}$. If the frog is in lily pad i then it jumps to the lily pad $(i+1) \mod 3$ with probability $1/3$ and to the lily pad $(i-1) \mod 3$ with probability $1/3$.

   (a) Draw the flow diagram for a Markov chain model of the frog jumps over the state space of the three labelled lily pads.

   (b) Produce the 3×3 transition probability matrix $\mathbf{P}$ for the model.

   (c) Compute the probability of being in state 2 in time-step 2 if you started from lily pad 1 at the initial time-step 0.

   (d) What is the steady-state probability vector $\mathbf{s}$ for this Markov chain?

   (e) Describe in words and in pseudo-code an algorithm that can produce sample paths (sequence of jumps) for this Markov chain with transition probability matrix $\mathbf{P}$ if you started at time 0 according to the initial probability vector $\mathbf{p}^{(0)} = (1/3, 1/3, 1/3)$ over the state space $\{0,1,2\}$.